3/04/2019

जय माँ सरस्वती

~~Organic Sho~~ Unit and Dimension

Unit and Dimension

* Physical quantity ⇒

The quantity by which we can describe the law of physics are called physical quantities.

Base quantity	Derived quantity	Supplement quantity
quantities which don't depends on eachother are called base quantity	quantities which are derives from the base quantities are called derived quantities	→ plane angle → solid angle

* Unit ⇒ standard of measurement

* ~~Dimension~~ Dimension of a physical quantity are the power to which base quantity must be raise to represent that quantity

Q) which can be assumed as personal base quantities?

Solⁿ

~~length~~ ~~area = l^2~~ ✗	length
	current
mass	velocity = $\frac{l}{t}$] but it can't be defined by using length and current. There is need to Time

नोट ⇒ जिस अलग-अलग quantity में आपस में कोई Relation नहीं हो और वह Personaly Independent हो उसे Personal base quantity कहा जाता है।

☆ Base quantity	S.I unit
① Length	metre (m)
② mass	kilogram (kg)
③ Time	second (s)
④ Electric current	Ampere (Amp)
⑤ luminous Intensity of light	candla (cd)
⑥ Temperature	kelvin (K)
⑦ Amount of substance	mol

☆ Dimension of some Important Physical Quantity

$[Force] = [ML^1T^{-2}]$

$[work] = [Force][displacement] = ML^2T^{-2}$

$[P] = [m][v] = [MLT^{-1}]$

$acc^n = \frac{v}{T} = [LT^{-2}]$

[illegible] / Torque = $R \times F = [F][length]$

$= [ML^2T^{-2}]$

Angular momentum $= [m][v][r]$

$= [ML^2T^{-2}]$

Angular Impulse = Torque × time

Angular velocity (ω) $= \frac{[angular\ disp]}{[time]} = T^{-1}$

Angular displacement $M^0L^0T^0$

Note → A dimension less quantity may have a

unit [eg. Angle].

- while a unitless quantity can never have dimensions

eg :- refractive index, dielectric constant.

(i) Emf / voltage / potential difference (V)

$$[V] = \frac{[W]}{[Q]} = \frac{M^1L^2T^{-2}}{[AT]}$$

$$V = [M^1L^2T^{-3}A^{-1}]$$

(ii) Electric field intensity $(E) = \frac{[F]}{[Q]} = \frac{[M^1L^1T^{-2}]}{[AT]}$

$$= [MLT^{-3}A^{-1}]$$

(iii) Permitivity $(\varepsilon_0) =$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$ → physical relation

$$[\varepsilon_0] = \frac{q_1q_2}{Fr^2}$$ → Dimensional relation

$$[\varepsilon_0] = \frac{[A^2T^2]}{[MLT^{-2}L^2]} \rightarrow [A^2T^4M^{-1}L^{-3}]$$

(VII)

(iv) Relative permittivity (ε_r) / Di-electric constant

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

$$\varepsilon_r = [M^0 L^0 T^0]$$

(v) Resistance (R):

$$[R] = \frac{[V]}{[I]} = \frac{[ML^2 T^{-3} A^{-1}]}{[A]}$$

$$\Rightarrow [ML^2 T^{-3} A^{-2}]$$

(vi) Capacitance (C) =

$$\cancel{q = \frac{C}{V}}$$

$$\cancel{C = q.V}$$

$$\cancel{\Rightarrow [A.T].[ML^2 T^{-3} A^{-1}]}$$

$$\cancel{\Rightarrow [T^2 M L^2]}$$

(vii) Inductance (L) =

[illegible]

$$\cancel{\phi = LI}$$

[illegible]

$$[L] = \frac{[Emf]}{(di/dt)} = \frac{ML^2 T^{-3} A^{-1}}{(AT^{-1})} \Rightarrow ML^2 T^{-2} A^{-2}$$

(vii) Permiability (μ_0) =

$$C = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

speed of light

$$[\mu_0] = \frac{1}{\cancel{C}\,[\varepsilon_0]\,[L^2T^{-2}]}$$

$$= \frac{1}{\cancel{M^{-1}}\cancel{T}\,m^{A}L^{-3}}$$

or

$$B = \frac{\mu_0}{4\pi}\frac{i\,dl\sin\theta}{r^2}$$

$$[\mu_0] = \frac{[B][r^2]}{[i][dl]}$$

(viii) magnetic field/ magnetic induction/ magnetic field intensity (B)

$$[F] = [q\overset{\text{velocity}}{v}B]$$

$$[B] = \frac{[F]}{[qv]} = \frac{[M^1L^1T^{-2}]}{[AT][LT^{-1}]}$$

(ix) Universal gravitational constant [G]

$$F = G\frac{m_1m_2}{r^2}$$

$$[G] = \frac{Fr^2}{m_1m_2} \Rightarrow \frac{MLT^{-2}L^3}{M^2}$$

$$\Rightarrow M^{-1}L^3T^{-2}$$

(x) Gas constant (R) ⇒

$$PV = nRT$$

$$R = \frac{PV}{nT} \Rightarrow \frac{Pa\cdot\cancel{J}}{K} = \frac{ML^2T^{-2}}{mol\,K} \Rightarrow$$

$$= ML^2T^{-2}\cdot mol^{-1}K^{-1}$$

(XI) Boltzman constant (K) :-

$$PV = K T$$

$$K = \frac{PV}{T} = mL^2T^{-2}K^{-1}$$

PV → dimension of energy के बराबर होगा।

(XII) Pressure/ Stress/ elastic constant/ energy density.

→ Young's modulus (Y)

→ ~~modulus~~ Bulk modulus (B)

~~[P] =~~

$$\text{Stress} = [mL^{-1}T^{-2}]$$

Modulus

(XIII) $[\text{Strain}] = [m^0L^0T^0]$

Note :-

$$\boxed{\text{Young's modulus } (Y) = \frac{\text{Stress}}{\text{Strain}}}$$

Q. Dimensions of the following are:

(a) $\frac{L}{R} = \frac{\cancel{mL^2T^{-2}A^{-2}}}{\cancel{mL^2T^{-3}A^{-2}}} = T$

(b) RC ✓ ⒷⓀ $\sqrt{LC}$

(c) $\frac{L}{RCV}$

(d) $\frac{1}{\sqrt{\mu_0\epsilon_0}}$

(e) $\frac{\omega_0 L}{R}$

(f) $\frac{1}{\omega_0 RC}$

(g) $\frac{1}{2}\epsilon_0 E^2$

(h) $\frac{B^2}{2\mu_0}$

(i) $\frac{q^2}{2C}$ (J) $\frac{1}{2}Li^2$

Sol^n

(A) $RC = [m L^2 T^{-3} A^{-2}]\ T^{-2} m L^2$

$= m^2 L^4 T^{-6} A^{-2}$

Ⓑ $[RC] = [R][C]$

(A) $= [T]$

Capacitance

(C) $\left[\frac{L}{RCV}\right] = \frac{1}{RC} \times \frac{1}{V/L}$

Note $\Rightarrow$ $[\mathcal{E}] = \left[L \frac{di}{dt}\right]$

$\left[\frac{L}{RCV}\right] = \frac{1}{T} \times \frac{1}{AT^{-1}} = A^{-1}$

Potential difference

(d) $C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} =$

$\left[\frac{1}{\mu_0 \epsilon_0}\right] = [L T^{-1}]$

(e) $\omega_0 \frac{L}{R}$

(f) $= \frac{1}{\omega RC} \Rightarrow \frac{1}{T^{-1} T} = 0 = [M^0 L^0 T^0]$

(g) $\frac{1}{2}\varepsilon_0 E^2 =$

$\cancel{[\varepsilon_0 E^2] = [A^2 T^4 M^{-1} L^{-3}]}$

$\frac{1}{2}\varepsilon_0 E^2 =$ (energy density or energy per unit volume.

↳ Very Important

$[\frac{1}{2}\varepsilon_0 E^2] = [M^1 L^{-1} T^{-2}]$

(h) $\frac{B^2}{2\mu_0} = [M^1 L^{-1} T^{-2}]$

↳ (energy density of magnetic field.

(i) $\frac{q^2}{2C}$

↳ Potential energy

$\left[\frac{q^2}{2C}\right] = [M^1 L^2 T^{-2}]$

(j) $\frac{1}{2} L i^2$

↳ Potential energy of Inductor

$[\frac{1}{2} L i^2] = [M^1 L^2 T^{-2}]$

(k) $[\sqrt{LC}] = [T]$

classmate
Date
Page

Note →

(i) sin(----)
(----) → Dimensionless
sin(----) → Dimension less

(ii) cos(---)
(---) → Dimens. less
cos(---) → D.L

(iii) tan(---)
(---) → D.L
tan(---) → D.L

(iv) $a^{(---)}$
(---) → D.L
$a^{(---)}$ → numerical value.

(v) $e^{(---)}$
(---) → D.L
$e^{(---)}$ → D.L

(vi) log(---)
(---) → D.L
log → D.L

Q.)

→ alpha

$$P = \frac{\alpha}{\beta} \frac{-\alpha Z}{k\theta}$$

P → Pressure
Z → distance
K → Boltzmann constant
θ → temp.

Dimension ~~[illegible]~~ of β is

$$\beta = \frac{\alpha}{P}\, e^{\frac{-dZ}{d\theta}}$$

→ D.L

→ D.L

$$\cancel{\frac{dZ}{d\theta} = \frac{K}{K} \to K K^{-1}}$$

$$\cancel{\beta = \frac{\alpha}{[M L^{-1} T^{-2}]}}$$

$$\frac{\alpha}{\beta} = P \Rightarrow \left[\frac{\alpha}{\beta}\right] = [M L^{-1} T^{-2}]$$

(i)

$$\frac{dZ}{d\theta} = M^0 L^0 T^0$$

→ PV = KT = Kθ → given that in term

energy

$$[\alpha] = [M^1 L^1 T^{-2}]$$

$$[\beta] = \frac{[\alpha]}{[M^1 L^{-1} T^{-2}]}$$

quantity similar & its means
their dimensions
are equal..

Q) $\left(P + \frac{a}{V^2}\right)(V-b) = RT$

p→ Pressure
v→ volume

Dimensions of a and b are

$\frac{PV}{n} = RT \to m^1L^{-1}T^{-2}$

Concept:-

A physical quantity can be added or subtracted by with similar quantity.

$\left[\frac{a}{V^2}\right] = [P]$

$a = [m^1L^{-1}T^{-2}]\,[L^6]$

$[V] = [b] = L^3$

Uses of Dimensions:-

- (I) homogonity of Dimensions
- (II) Conversion of unit
- To decide a relation b/w Physical quantities.

(a) Homogenity of Dimensions

Principle of homogenity → A/q to the principle each term on L.H.S and R.H.S of an equation has same dimension.

eg.) $v = at + \frac{bt}{l^2} + c$

find dimensions of a and b

t → time
l → length
v → speed

Sol^n

$LT^{-2} = a$ ✓

$\frac{bt}{l^2} = LT^{-1}$

$b = L^3T^{-2}$ ✓

$c \rightarrow LT^{-1}$

Concept

$$\therefore [v] = [at] = \left[\frac{bt}{l^2}\right] = [c]$$

∴ A/q homogenity each term has same dimension eg.) $y = A\sin(kx - ct)$

$y \rightarrow$ displacem.
$x \rightarrow$ position
$t \rightarrow$ time

for find Dimension of A, k & c

Sol^n ~~Asin to~~

~~$kx-ct/z$~~

$[A] = [L]$ ✓

$$\therefore y = A\sin\underbrace{(\overset{D.L}{kx} - \overset{D.L}{ct})}_{U.D.L}$$

$$[kx] = [ct] = [m^0L^0T^0]$$

$k = L^{-1}$ | $c = T^{-1}$

* Important point :

(i) A dimensionaly correct equation may or may not be actually correct

(ii) A dimensionaly incorrect equation must be actually Incorrect

⑥ Conversion of units :-

$$\underbrace{5}_{\text{numerical value}}\ \underbrace{m/s}_{\text{units}} \rightarrow \text{magnitude}$$

* magnitude of a physical quantity does not depend upon system of unit

Note

$$\boxed{\underbrace{n}_{\text{numerical value}}\ \underbrace{u}_{\text{units}} \Rightarrow \text{Constant}}$$

i.e. $\boxed{n \propto \frac{1}{u}}$

Larger the unit smaller will be the numerical value and vice-versa.

i.e. $\boxed{n_1 u_1 = n_2 u_2}$

eg.) $\underbrace{1 \text{ newton}}_{\text{S.I unit of force}} = ?\ \underbrace{\text{dyne}}_{\text{C.G.S unit of force}}$

Sol^n $[F] = [m^1 L^1 T^{-2}]$

$1 \text{ newton} = \frac{kg \cdot m}{s^2}$

ans. $1 \text{ dyne} = gm \cdot cm/s^2$ Note → यहाँ पर 'n' मान कर question को solve करें

$1 \, kg \, m/s^2 \Rightarrow n \, gm \, cm/s^2$

$\cancel{\frac{1000 \times 100}{s^2}}$ $\frac{1000 \, (gm) \times 100 \, (cm)}{s^2} = n \frac{gm \, cm}{s^2}$

$\boxed{n = 10^5}$

eg.) $1 \text{ Joule} = ? \text{ erg}$

sol^n own

$\frac{kg \, m^2}{s^2} = n \frac{g \, cm^2}{s^2}$

$1000 \times (100)^2 = n$

$\boxed{n = 10^7}$

Teache

$[W] = [m^1 L^2 T^{-2}]$

$1 \text{ Joule} = \frac{kg \, m^2}{s^2}$

$1 \text{ erg} = \frac{g \, cm^2}{s^2}$

~~Tea~~

$\cancel{\frac{1 \, kg \cdot m^2}{s^2}}$

eg.) If unit of mass(m) and velocity(v) is doubled then unit of linear momentum is

sol^n $p = \cancel{m^2 v^2}$ $\cancel{= (mv)^2}$ $\cancel{\Rightarrow kg^2 m^2 s^{-2}}$

$P = \cancel{4mv}$

$P_{final} \Rightarrow 4mv$

unit of P_{final} 4 times of initial

Q) In the above question number value of momentum will be

Ans $\because nu = \text{Constant}$

$n \propto \frac{1}{4}$

So,

Numerical value becomes $\frac{1}{4}$

ⓒ To deduce a relation b/w physical quantity

Time period of simple pendulum depends on its length and acc^n due to gravity find the relation time period, length and ~~acc^n~~ g.

Sol

$T \propto l^a g^b$

$T = k l^a g^b$

$[T] = [l]^a [g]^b$

$l^{1/2}$ $g^{-1/2}$

$\sqrt{L}$ $\frac{1}{\sqrt{g}}$

$[m^0 L^0 T^1] = [L]^a [LT^{-2}]^b$

$[m^0 L^0 T^1] = [m^0 L^{a+b} T^{-2b}]$

$a+b=0$

$a=-b$

$a = \frac{1}{2}$

$-2b = 1$

$-b = \frac{-1}{2}$

$L^0 \to 1$

$T = T = T$

Now

$T = l^{1/2} g^{-1/2}$

$\Rightarrow \sqrt{l} \times \frac{1}{\sqrt{g}}$

$$\boxed{T = K\sqrt{\frac{l}{g}}}$$

$\to$ experimentally determined

Q. Given that mass depends on force, length and time find physical relation b/w these quantities

Sol/n

$m \propto [F]^a [L]^b [T]^c$

$\to [mLT^{-2}]^a [L]^b [T]^c$

$\Rightarrow m^a L^{a+b} T^{-2a+c}$

$[m] \Rightarrow [m^a L^{a+b} T^{-2a+c}]$

$[M^0 L^0 T^0]$ ~~x~~ $= [M^a L^{a+b} T^{-2a+c}]$

$a+b=0$ $\quad | -2a+c=0$

$a=1$

$m \propto F L^{-1} T^2$

$\propto \dfrac{FT^2}{L}$

$m = \dfrac{kFT^2}{L}$

Units of Same Important Physical Quantities

Physical Quantities	S.I units
① velocity	m/s
② Accn	m/s^2
③ force	newton
④ work / Energy	Joule
⑤ Torque	N-m
⑥ Impulse	N-s
⑦ linear momentum	kg m/s

⑧	Power	watt (W) or J/s
⑨	ang. displacement	rad
⑩	ang. velocity	rad/sec
⑪	Angular accⁿ	radian/sec²
⑫	Angular Impulse	N-m-s or J-s
⑬	M.O.I	kg·m²
⑭	Surface tension	
	→ force / length	→ N/m
	→ Energy / Area	→ J/m²

Note →

$$1\frac{N}{m} = 1\frac{J}{m^2}$$

⑮	Elastic coefficient → young modulus → bulk modulus → shear modulus	$\frac{N}{m^2}$
⑯	Pressure	Pascal (Pa)
⑰	Stephens constant	watt/m²K⁴
⑱	Resistance	ohm (Ω)
⑲	Capacitance	farad (F)
⑳	Inductance	henry (H)
㉑	specific resistance (ρ) or Resistivity	ohm·m or Ωm

(22)	Electric field	V/m or N/C
(23)	Electric potential	volt (V)
(24)	magnetic field	tesla (T)
(25)	magnetic flux	weber (wb)
(26)	specific ~~[illegible]~~ heat	J/kg K or J/kg °C
(27)	Latent heat	J/kg
(28)	Redberg constant (R)	m^{-1}

④ Some practical units

1 femto = 10^{-15} m

1 a° = 10^{-10} m

1 light year = $3\times10^8 \times 365 \times 86400$ s

= $3\times10^8 \times 365 \times 24 \times 60 \times 60$

=

1 parsec = 3.26 light year.

1 ev = 1.6×10^{-19} Joule

(Note → 1 coulomb volt → 1 Joule)

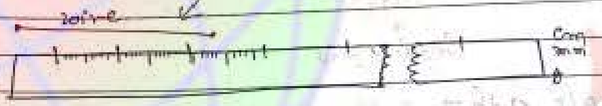
1 kilo watt hour → (10^3 watt)(3600 s)

$= 3.6 \times 10^6$ J

1 Joule = 1 watt · s

Significant digit

Note:- किसी भी wire का लंबाई हम least से least जितने हो सकता हम छोटी-छोटी unit को लेकर करते हैं।



m.S.D

l = 3.4 7 cm

L.S.D

3 cm

$\frac{1}{10}$ th of cm

$\frac{1}{100}$ th of cm

m.S.D → most significant digit

L.S.D → least " "

No of significant digit = 3

✱ Rule's for finding significant digit

Rule 1st ⇒ All non-zero digits are significant

eg 1 2 3 7 8 , S.D ⇒ 5

Rule 2nd :- Zero's b/w non-zero digit are significant

eg 1 2 0 4 5 , S.D ⇒ 5

→ Rule 3rd :

In a number with decimal, zero left to the non-zero digit are insignificant

eg

0.0027 , digit =

└ Insignificant digits or leading zero

→ Rule 4th :

In a number with decimal, zero's right to the non-zero digit (trailing zero's) are significant

eg

12.270 S.D = 5

└ their zero is significant or trailing zero is counted

eg

0.001330 , S.D = 3

12400.0 , S.D = 6

→ Rule 5th

In a number without decimal, trailing zeros are not significant

eg, 12400 S.D = 3 (without decimal)

12400.0 S.D → 6 (with decimal)

→ Rule 6th →

If we change the unit of measurement number of significant digit's do not changed

Significant digits in calculatory

① In division or multiplication :-

eg → $a = 7.22$
$b = 2.2$

Rule → final Answer में question का smallest significant digit जितना होगा उतना ही significant digit answer में consider करना है।

322×22
644
644×
708 4

$ab = ?$

$ab = 7.0\underline{8}\,\underline{4}$ ← L.S.D, m.s.D → Insignificant

~~final answer~~

Rounding off of 7.084 is 7.1
so
$ab = 7.1$

AIEEE 8

eg ② $m = 3.513$ kg
$v = 5.00$ m/s
$p = ?$

(A) 17.565 kg m/s
(B) 17.56 "
(C) 17.57 "
(D) 17.6

82/u

$p = 17.6$

Ans = D

3513×5
~~17565~~
$17.5\textcircled{65}$
m.s.D, L.S.D, Insignificant

Note→ In division or multiplication number of significant digits in the answer is equal to the (given) quantity which has least number of significant digits

Note

~~imp~~ Counting का significant digit Infinite होता है

eg→ 7 सेब, 5 व्यक्ति etc

Note→ अगर 5 की case में significant digit लेना हो तो उस case में 7

Rule 1st→ odd number के case में +1 करना है

eg→ find three S.D in following number?

7.37592 → 7.38

+1 in case of odd

Rule 2nd→ even number के case में "0" add करना होता है अर्थात् कोई change नहीं।

q→ find 3 S.D in following number

738572 → 7.38

even के case में no change

③ In addition or substraction.

eg:- a = 2.2
b = 2.323
c = 3.2

a+b+c = ?

soln 2.2
2.323
3.

* Step 1st

arrange the number one below the other w.r.t decimal

2.2
2.323
3.2

* Step 2nd

Identify the column with doubtful digit

1st column (Rounding off) with doubtful digit → जो सबसे कम significant digit वाला है उसी के Respect से Rounding off निकाले |

2.2
2.323 ⟶ 2.3
3.2

2.2
2.3
3.2

$\boxed{7.7}$ ↳ find answer

* Step's followed in this Question
(i) arrange the number one-below the other
(ii) Identify the 1st column with doubtful digit

गुणा → odd → ~~add~~ add 1
even → 0 (not add)

③

(iii). ~~Rcc~~ Rounding off is done to [illegible]
column with doubtful digit

(iv) Addition is take place

Q) Round off the following to three significant digits
Soln:
(a) 12730 → 12700 → बिना Decimal की काउंटी number को Zero कर देते हैं
(b) 127.30 → 127 → Decimal के कारण number को हटा देते हैं
(c) 4301 → 4300 = 4.30×10³
(d) 43.273 → 43.3
(e) 12780 → 12800

10 की power को insignificant ही count किया जाता है।

Note Round off upto two significant digits
(f) 107.8431 → 110

Note → Error में हम always maximum error निकालते हैं। कि अधिक-से अधिक कितना गलती होने की संभावना है। इसलिए इस बात का ध्यान रखें कि -ve को भी modules से लेकर +ve कर देना है अन्यथा maximum value नहीं निकाला जा सकता है।

Absolute error, relative error and percentage error.

Let observation of taken of a physical quantity are,

$a_1, a_2, a_3 ----- a_n$

$$a_{mean} = \frac{a_1 + a_2 + a_3 + ----- + a_n}{n}$$

True value or mean value.

① Absolute error→

$\Delta a_1 = a_{mean} - a_1$

$\Delta a_2 = a_{mean} - a_2$

$\Delta a_n = a_{mean} - a_n$

Note
mean absolute error (Δa) → maximum possible error

$$\Delta a_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + --- + |\Delta a_n|}{n}$$

so,

$$\boxed{a = a_{mean} \pm \Delta a_{mean}}$$

eg :- $R = (5.5 \pm 0.5)\Omega$

$R_{mean} = 5.5\Omega$

$\Delta R_{mean} = 0.5\Omega$

$(5.5 - 0.5)\Omega \leq R \leq (5.5 + 0.5)\Omega$

② Relative error (fractional error)

$$\text{Relative error} = \pm \frac{\Delta a_{mean}}{a_{mean}}$$

③ Percentage error $= \pm \frac{\Delta a_{mean}}{a_{mean}} \times 100$

Error determination in mathematical operations: I

(a) In multiplication

$$A = xy$$

$$\log A = \log x + \log y$$

$$\boxed{\frac{\Delta A}{A} = \frac{\Delta x}{x} + \frac{\Delta y}{y}}$$

Relative error of A,

and also
Percentage error in A

$$\frac{\Delta A}{A} \times 100 = \frac{\Delta x}{x} \times 100 + \frac{\Delta y}{y} \times 100$$

(b) In division

$$A = \frac{x}{y}$$

$$\log A = \log x - \log y$$

$$\frac{\Delta A}{A} = \frac{\Delta x}{x} - \frac{\Delta y}{y}$$

max. possible relative error in A is

$$\boxed{\frac{\Delta A}{A} = \frac{\Delta x}{x} + \frac{\Delta y}{y}}$$

Note :-

$$A = \frac{K x^a y^b}{z^c}$$

$$\log A = \log K + a \log x + b \log y - c \log z$$

$$\frac{\Delta a}{a} = 0 + a \cdot \frac{\Delta x}{x} + b \cdot \frac{\Delta y}{y} - c \cdot \frac{\Delta y}{y}$$

Sol

maximum possible relative error is

$$\frac{\Delta a}{a} = a \cdot \frac{\Delta x}{x} + b \frac{\Delta y}{y} + |c| \frac{\Delta y}{y}$$

eg :- % error in the length of simple pendulum is 2% ; and error in the measurement of g is 3% ; find % error in the measurement of Time period.

Sol/n

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$T = 2\pi\, l^{1/2} g^{-1/2}$$

$$\frac{\Delta T}{T} \times 100 = \frac{1}{2} \cdot \frac{\Delta l}{l} \times 100 + \left|\frac{-1}{2}\right| \times \frac{\Delta g}{g} \times 100$$

Ex 1 → 24 to 42, Ex 2 → 3, 6, 7, 8, 9 to 17, 21, 24, 25
↳ 012
↳ correction
Ans → $\frac{1}{4}$ times होगा

Ex 4 → complete
(A)

$$\frac{\Delta T}{T} \times 100 = \frac{1}{2} \times (\pm 2)\% + \frac{1}{2} \times (\pm 3)\%$$

$$= \pm 1\% \pm \frac{3}{2}\%$$

$$\Rightarrow \pm 2.5\%$$

① Concept ASG

V = 10.0 ± 0.1

I = 5.0 ± 0.1 A

% of R = ?

Sol^n ~~R → 15.0 ± 0.2 A~~

~~Explanation~~

~~$R = \frac{V}{I}$~~

~~$\log R = \log V - \log I$~~

~~$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 - \frac{\Delta I}{I} \times 100$~~

~~$= (10.0 \pm 0.1) + (5.0 \pm 0.1)$~~

$$\Rightarrow (15.0 \pm 0.2) A$$

$$R = \frac{V}{I}$$

$$= \frac{10}{5} = 2$$

Now

% error =

$R = \frac{V}{I}$

$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$

$= \frac{0.1}{\cancel{10}} \times \overset{10}{\cancel{100}} + \frac{0.1}{\cancel{5}} \times \overset{20}{\cancel{100}}$

$= 1 + 2$

$= \pm 3\%$

या

$\boxed{R = 2.0 \pm 3\%}$

② Concept 2nd

In addition and subtraction

Note → ① $Z = xy$, $Z = \frac{x}{y}$ } % error of "Z" both case में same आएगा

लेकिन % में $\boxed{\frac{\Delta Z}{Z} \times 100 = (\text{const})}$

लेकिन error को "Z" दोनों में same नहीं होगा

② $A = x + y$

$\Delta x = \Delta x + \Delta y$

$\frac{\Delta x}{A} = \frac{\Delta x + \Delta y}{A}$

$$\frac{\Delta A}{A} = \frac{\Delta x+\Delta y}{x+y}$$

$$\boxed{\frac{\Delta A}{A} = \frac{\Delta x}{x+y} + \frac{\Delta y}{x+y}}$$

and

$$\boxed{\frac{\Delta A}{A}\times 100 = \frac{\Delta x}{x+y}\times 100 + \frac{\Delta y}{x+y}\times 100}$$

(ii) Substraction

$$A = x-y$$

$$\Delta x = \Delta x - \Delta y$$

maximum possible error

$$\Delta x = \Delta x + \Delta y$$

and

$$\boxed{\frac{\Delta A}{A} = \frac{\Delta x+\Delta y}{x+y}}$$

$$\boxed{\frac{\Delta A}{A}\times 100 = \frac{\Delta x}{x+y}\times 100 + \frac{\Delta y}{x+y}\times 100}$$

eg 1 $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

relative error in R is

Sol^n $\cancel{\frac{1}{R} = \frac{R_1+R_2}{R_1 R_2}}$ $\cancel{\frac{\Delta R}{R}\times 100 = \frac{\Delta R_1 R_2}{R_1+R_2}\times 100}$

$\cancel{R = \frac{R_1 R_2}{R_1+R_2}}$

$$R = \frac{R_1 R_2}{(R_1 + R_2)}$$

$$R = R_1 R_2 (R_1 + R_2)^{-1}$$

$$\frac{\Delta R}{R} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + |-1| \frac{\Delta (R_1 + R_2)}{R_1 + R_2}$$

$$\boxed{\frac{\Delta R}{R} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}}$$

eg 2).

$R_1 = (10.0 \pm 0.1)\Omega$

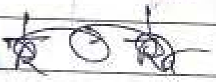
$R_2 = (5.0 \pm 0.2)\Omega$

find % error in the measurement of net resistance if

(i) R_1 and R_2 are in parallel combined

(ii) R_1 and R_2 are in series combine

30/7



$$\frac{\Delta R}{R} \times 100 = \frac{0.1}{\cancel{10}} \times \cancel{100}^{10} + \frac{0.2}{\cancel{5}} \times \cancel{100}^{2} + \frac{0.3}{\cancel{15}_{3}} \times \cancel{100}$$

$$\Rightarrow 1 + 4 + \frac{30}{150} \Rightarrow 1 + 4 + 2 \Rightarrow 7\,\%$$

$\frac{1}{R} = \frac{1}{10} + \frac{1}{5}$

$\Rightarrow \frac{1+2}{10}$

$R = \frac{10}{3}$

$\Rightarrow 1 + 10 + 2 \Rightarrow 13$

$R_{net} = \left(\frac{10}{3} \pm 7.0\right)\Omega$

H.W → complete question discuss

classmate
Date ______
Page ______

(R) ~~$R = R_1 + R_2$~~

~~$\rightarrow \frac{0.1}{10} \times 100 + \frac{0.2}{5} \times 100$~~ , $R = 10 + 5$

~~$\rightarrow 1 + 4$~~

~~$= 5\%$~~

~~$R_{net} = (15.0 \pm 5\%)$~~

$\log R = \log(R_1 + R_2)$

must be solve again

$\frac{\Delta R}{R} \times 100 = \frac{\Delta(R_1 + R_2)}{R_1 + R_2}$

$\rightarrow \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2} \times 100 = \frac{0.3}{15} \times \frac{100}{10}$

$= \pm 2\%$

so $(R = 15 \pm 2\%)$

Gravitation

Newton law of gravitation :→

According to this law gravitational force of attraction b/w two particle is directly proportional to the product of their masses and inversely proportional to the square of the separation b/w them

$$F \propto m_1 m_2$$

$$F \propto \frac{1}{r^2}$$

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

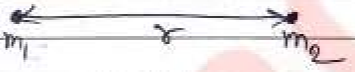
where

G is universal gravitational constant

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

Note

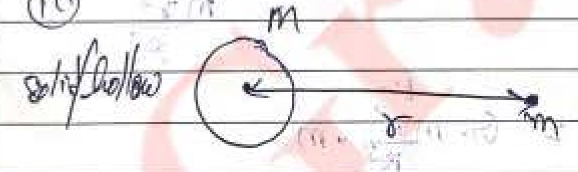
①



(point mass)

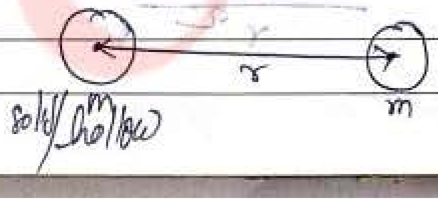
$$F = G\frac{m_1 m_2}{r^2}$$

(ii)



$$F = G\frac{Mm}{r^2}$$

(iii)

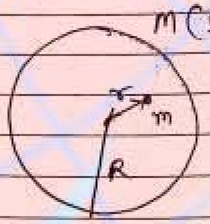


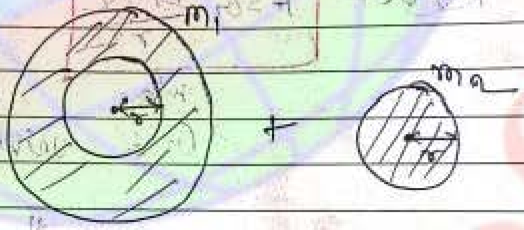
$$F = \frac{G Mm}{r^2}$$

(iv) spherical shell

$$\boxed{F=0}$$

(v) M (solid)





F=0
(concept of hollow)

$$F = \frac{G\, M \cdot m_2}{r^2}$$

Note

$$m_2 = \frac{4}{3}\pi r^3 \cdot \rho$$

$$= \frac{4}{3}\pi r^3 \left(\frac{M}{\frac{4}{3}\pi R^3}\right)$$

$$= \frac{M r^3}{R^3}$$

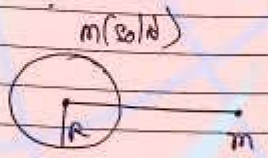
$$F = \frac{G \cdot \frac{M r^3}{R^3} \cdot m}{r^2}$$

very Important

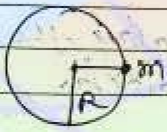
$$\boxed{F = \frac{G M m r}{R^3}}$$

M → mass of big body
m → mass of point body.

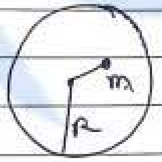
Special casez



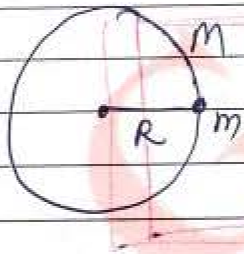
$F = \frac{G M m}{r^2}$, r > R



$F = \frac{G M m}{R^2}$, r = R

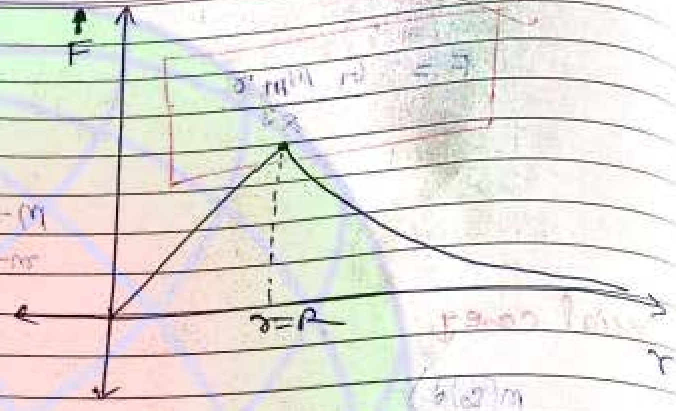


$F = \frac{G M m r}{R^3}$, r < R

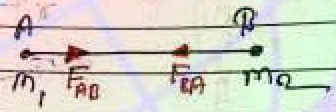


when ~~[illegible]~~ "m" is on the surface of M then F is max.

#



Vector Representation :



Note → Gravitation force is along the line joining the two Particles

F_{AB} → force on "A" due to "B"

F_{BA} → force on "B" due to "A"

$$F_{AB} = F_{BA} = \frac{G m_1 m_2}{r^2}$$

but in vector form

$$\vec{F}_{AB} = (-)\vec{F}_{BA}$$

Q) Force experience by a particle is "F" when it is kept on the surface of sphere of radius 2m

find
(i) force experienced by it in 1m below the surface
(ii) Force experienced by it in 1m above the surface

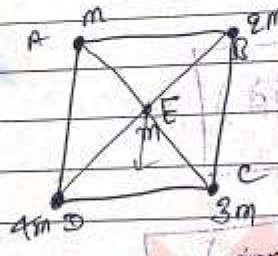
30/9

$$F = \frac{G m_1 m_2}{4} \quad so,\ G m_1 m_2 = 4F$$

(a)
$$F_1 = \frac{G m_1 m_2}{8} \Rightarrow \frac{4F}{8} = \frac{F}{2}$$

(b)
$$F_2 = \frac{G m_1 m_2}{9} \Rightarrow \frac{4F}{9}$$

Q)



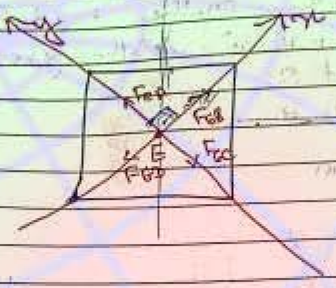
find force experience by E due to A, B, C and D

2m

$$F = \frac{G\, 4m^2}{} \vec{r}$$

10/10

$\sqrt{x^2+y^2}$



$$\vec{F}_E = \vec{F}_{EA} + \vec{F}_{EB} + \vec{F}_{EC} + \vec{F}_{ED}$$

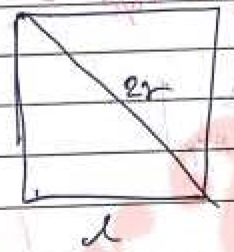
$$\vec{F}_E = \frac{Gm^2}{r^2}(-\hat{i}) + \frac{Gm^2}{r^2}(\hat{j})$$

$$+ \frac{Gm^2}{r^2}(-\hat{i}) + \frac{Gm^2}{r^2}(-\hat{j})$$

$$= \frac{-2Gm^2(\hat{i})}{r^2} + \frac{-2Gm^2}{r^2}\hat{j}$$

$$F_E = \sqrt{\left(\frac{2Gm^2}{r^2}\right)^2 + \left(\frac{2Gm}{r^2}\right)^2}$$

$$= \frac{2\sqrt{2}Gm^2}{r^2}$$



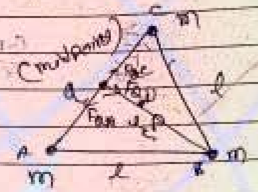
where

$(2r)^2 = l^2 + l^2$

$r = \frac{l}{\sqrt{2}}$

$$F_E = \frac{2\sqrt{2}Gm^2}{l^2/2} = \frac{4\sqrt{2}Gm^2}{l^2}$$

Q) Find gravitational force on the particle on another particle of mass "m" by the given system if (i) it is kept at Q
(ii) it is kept at P



$\frac{\sqrt{3}}{4} \times a^2$

or

$\frac{1}{2} \times a \times h =$

$= \frac{al}{2} =$

Sol^n ~~(i) (m) $F = \frac{Gm^2}{}$~~

① $\vec{F}_{net} = \vec{F}_{Q/A} + \vec{F}_{Q/B} + \vec{F}_{Q/C}$

$F_{Q/C} = F_{Q/A}$

and

$F_{Q/C} = (-)\ F_{Q/A}$

$\vec{F}_{Q/A} + \vec{F}_{Q/C} = 0$

$|\vec{F}_{net}| = |\vec{F}_{Q/B}|$

$F_Q = F_{Q/B}$

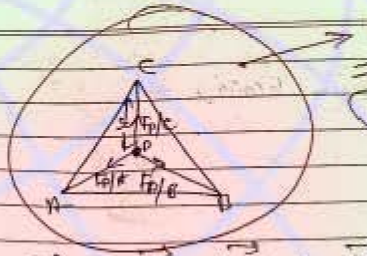
$= \frac{Gm^2}{r^2}$

where

$r = \sqrt{l^2 - \left(\frac{l}{2}\right)^2}$

$$\boxed{r = \frac{\sqrt{3}l}{2}}$$

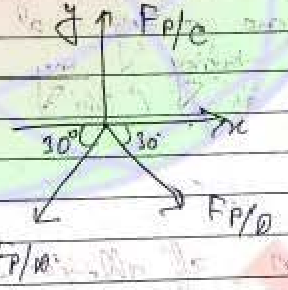
⑪



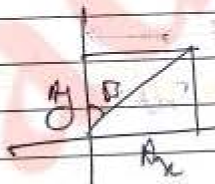
Note
Plane के case में x-y Coordinate system मान कर solve करें

$\vec{F}_{net} = \vec{F}_{P/A} + \vec{F}_{P/B} + \vec{F}_{P/C}$

$F_{P/A} = F_{P/B} = F_{P/C} = \frac{Gm^2}{r^2}$



Note



$\cos\alpha = \frac{A_x}{A}$

$\sin\alpha = \frac{A_y}{A}$

$\therefore A_x = A\cos\alpha$

$A_y = A\sin\alpha$

$A_x = A\sin\theta$

$A_y = A\cos\theta$

"n" $\vec{F_p} = (- F\cos 30^\circ \hat{i}$

Theorem 1st

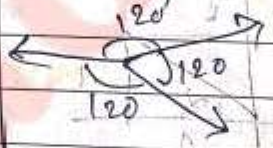
If "n" co-planar forces of equal magnitude are acting on a particle in such a way that any force makes an angle 360° by "n" $\left(\frac{360^\circ}{n}\right)$ when it neighbour force then,

vector sum of all the forces is zero

eg- if (n=2)

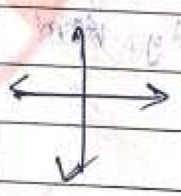
Fnet=0

n=3



Fnet=0

n=4



Fnet=0

[illegible] - ए Image [illegible]

Q) Two Particles each of mass "m" are moving in a circle of radius "r" under mutual force of attraction with same speed.
speed of each particle is

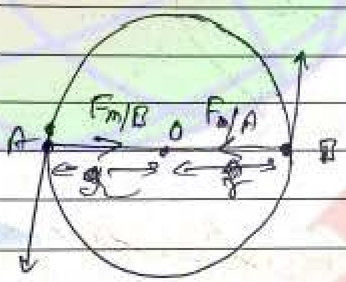
So/n

$F_1 = \frac{G m_1 m_2}{4r^2}$

$F_2 = \frac{G m_1 m_2}{4r^2}$

Now, $v = r\omega$

$F = \frac{mv^2}{r}$



Note

$F_{centripetal} = m\, a_{centripetal}$

and

$a_{centri} = \frac{v^2}{r} = \omega^2 r$

For A

$$F_{m/B} = \frac{mv^2}{r}$$

$$\frac{Gm^2}{(2r)^2} = \frac{mv^2}{r}$$

$$\cancel{m}v^2 = \frac{G\cancel{m^2}\cancel{r}}{4\cancel{r^2} \times \cancel{m}}$$

$$V = \sqrt{\frac{Gm}{4r}}$$

~~9/9/2013~~

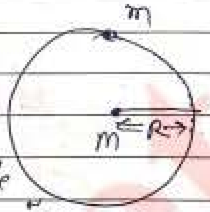
Acceleration due to ~~[illegible]~~ gravity (g) and factor affecting ~~[illegible]~~ it →

$$\boxed{\text{Eq } g = \frac{F}{m}}$$

where
~~[illegible]~~ force
F is gravitation force on the particle of "m" by the earth.

* On the surface of Earth

$$g_0 = \frac{\frac{G M \cancel{m}}{R^2}}{\cancel{m}}$$

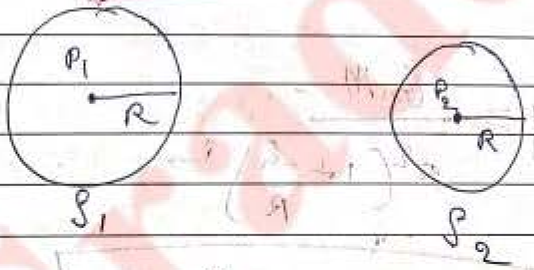


$$\boxed{g_0 = \frac{G m}{R^2} = 9.8 \, m/s^2}$$

mass of Earth or planet etc.

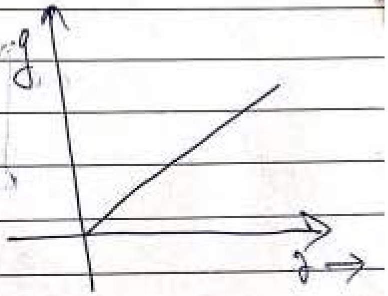
→ This is only on the surface of earth.

Note To find density (and concentrate ~~[illegible]~~ radius)



$$g_m = \frac{G \frac{4}{3}\pi R^3 \rho}{}$$

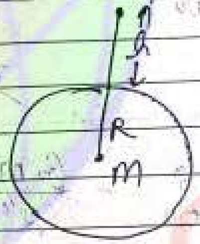
$$\boxed{g_m \propto R\rho}$$



$$\boxed{\frac{(g_m)_{P_1}}{(g_m)_{P_2}} = \frac{R_1 \rho_1}{R_2 \rho_2}}$$

Accⁿ due to gravity at height "h"

$$g = \frac{F}{m}$$



$$g = \frac{\frac{GMm}{r^2}}{m}$$

$$\boxed{g = \frac{GM}{r^2} = \frac{GM}{(R+h)^2}}$$

$$g = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2}$$

or $$\boxed{g = g_0\left(1+\frac{h}{R}\right)^{-2}}$$

Special case

if $\frac{h}{R} <<<<< 1$

then

$$\boxed{g = g_0\left(1-\frac{2h}{R}\right)}$$

Note

$$\boxed{(1+x)^n = 1+nx \quad \text{if } x <<<<< \text{"}1\text{"}}$$

"x" एक से बहुत छोटा होना चाहिए।

$$\boxed{\% \text{ decrease in acc}^n \text{ due to gravity} = \frac{g_0-g}{g_0}\times 100}$$

if $\frac{h}{R} <<<< 1$

then

$$\% \text{ decrease} = \frac{g_0 - g_0\left(1-\frac{2h}{R}\right)\times 100}{g_0}$$

$$\boxed{\% \text{ decrease} = \frac{2h}{R}\times 100}$$

Q) A particle is taken to a height $h = \frac{R}{2}$ from the surface of earth. find % decrease in "g"

Sol^n

~~% decrease =~~ $$\cancel{\frac{\frac{GM}{R^2} - \frac{4GM}{(R/2)^2}}{\frac{GM}{R^2}} \times 100}$$

$$\cancel{\Rightarrow \frac{\frac{GM}{R^2} - \frac{4GM}{R^2}}{\frac{GM}{R^2}} \times 100}$$

$$\cancel{= \frac{-3GM}{GM} \times 100}$$

$$\cancel{\Rightarrow 300}$$

$$\cancel{\Rightarrow 3\%}$$

$h = \frac{R}{2}$

$$g = g_0\left(1 - \frac{h}{R}\right)^{-2}$$

$$\boxed{= g_0\left(1 + \frac{R/2}{R}\right)^{-2} \Rightarrow \frac{4}{9} g_0}$$

% decrease = $\frac{g_0 - \frac{4}{9}g_0}{g_0} \times 100$

$\Rightarrow 55\%$

Q) A particle is taken to some height from the surface of earth. h = 1% of radius of earth. find % decrease g

Sol:- R =

$g_h = g_0\left(1 + \frac{\frac{1}{100}R}{R}\right)$

$\Rightarrow g_0\left(1 + \frac{1}{50}\right)$

$\Rightarrow g_0\left(\frac{50}{51}\right)$

$\Rightarrow \frac{50}{51}g_0$

$R + \frac{R}{50}$

$\frac{51R}{50R}$

% decrease = $\frac{2 \times \frac{R}{100}}{R} \times 100$

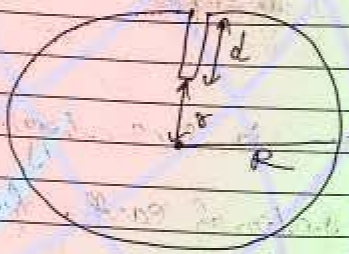
$\Rightarrow \frac{1}{50} \times 100$

$= 2\%$

use this formula upto **5%**

Acc^n due to gravity at a depth "d"

$$g = \frac{F}{m}$$



$$g = \frac{GM\cancel{m}r}{R^3\cancel{m}}$$

$$\boxed{g = \frac{GMr}{R^3}}$$

r → distance from the centre of earth
d → depth from the surface of earth.

$$g = \left(\frac{GM}{R^2}\right)\left(\frac{r}{R}\right)$$

or

$$\boxed{g = g_0\left(1 - \frac{d}{R}\right)}$$

$$\% \text{ decrease in } g = \frac{g_0 - g}{g_0} \times 100$$

So

$$\boxed{\% \text{ decrease in } g = \frac{d}{R} \times 100}$$

Q) Acc^n due to gravity at a height from the earth surface is same as at a distance "d" below the earth surface. where $\frac{h}{R} <<<<< 1$

which of the following is correct

(i) $2h = d$ ✓

(ii) $h = 2d$

(iii) $h = d$

(iv) N.O.T

Soln

$$g_h = g_d$$

$$\cancel{g_0}\left(1 - \frac{2h}{R}\right) = \cancel{g_0}\left(1 - \frac{d}{R}\right)$$

$$\boxed{2h = d}$$

Q) At what depth below the earth surface acc^n due to gravity is same as at a height $h = \frac{R}{2}$ from the earth surface.

Soln

$$\cancel{g_0}\left(1-\frac{d}{R}\right) = \cancel{g_0}\left(1-\frac{h}{R}\right)^{-2}$$

$$\left(1-\frac{d}{R}\right) = \left(1+\frac{R}{2R}\right)^{-2}$$

$$\left(1-\frac{d}{R}\right) = \left(\frac{3}{2}\right)^{-2}$$

$$\frac{d}{R} = \frac{5}{9}$$

$$d = \frac{5}{9}R$$

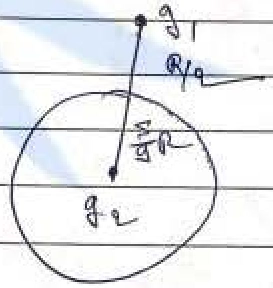
$$1-\frac{d}{R} = \frac{g_1}{g}$$

$$\frac{d}{R} = \frac{g_1}{g} - 1$$

$$= 1-\frac{g_1}{g}$$

$$\frac{d}{R} = \frac{5}{9}$$

$$d = \frac{5R}{9}$$



ⓒ Rotation of earth about its axis:

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{24 \times 3600}$ rad/s

Imp

W → Actual weight = mg

N → app. weight

↳ free falling body का app. weight zero होता है क्योंकि उसमें कोई Normal force नहीं लगता है

Acc due to gravity = $\frac{\overset{\text{weight}}{\uparrow}}{}$ $\frac{W}{m}$

App. weight = $\frac{N}{m}$

$$N + mr\omega_0^2 \cos\lambda = mg_0$$

Note

$$N = mg_0 - mr\omega_0^2 \cos\lambda$$

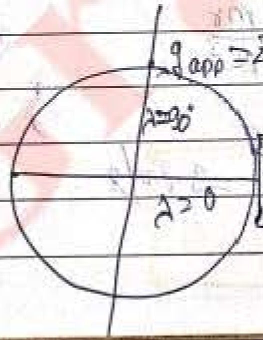
$$= mg_0 - mR\omega_0^2 \cos^2\lambda$$

$$= \text{app. weight}$$

But

$$g_{app} = \frac{N}{m} = g_0 - R\omega_0^2 \cos^2\lambda$$

$$\boxed{g_{app} = g_0 - R\omega_0^2 \cos^2\lambda}$$



g_{app} = Zero (0) ✗ $\boxed{g_{app} = g_0}$ (maximum)

↑ g_{app} is Increase

$\boxed{g_{app} = g_0 - \omega_0^2 R}$ (~~maximum~~) (minimum)

Q) what must be the time period of earth so that app. weight of the object at equator become zero

S/n $N = 0$ (at $\lambda = 0°$)

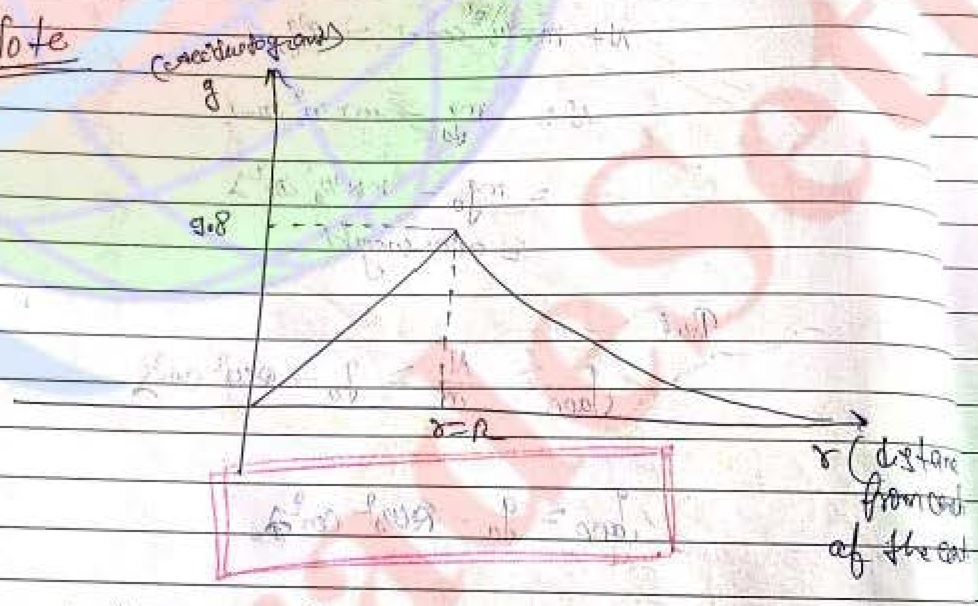
$$mg_0 - mR\omega^2 = 0$$

$$\omega = \sqrt{\frac{g_0}{R}}$$

and

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g_0}} \cong 1.41 \text{ hour}$$

Note



(i) if $r < R$, $g = \frac{GMr}{R^3}$

$g \propto r$

(ii) if $r = R$, $g = \frac{GM}{R^2} = 9.8 \text{ m/s}^2$

(iii) if ~~g~~ $r > R$

$g = \frac{GM}{r^2}$

$g \propto \frac{1}{r^2}$

Note

- direction of "Acc" due to gravity is always towards the centre of earth.

\# strength or

\# Intensity of gravitational field (E)

\#

It is defined as

$$\boxed{E = \frac{F}{m}} \xrightarrow{\text{force exerted by any body}} \boxed{g = \frac{F}{m}}$$

force exerted by earth,

where,

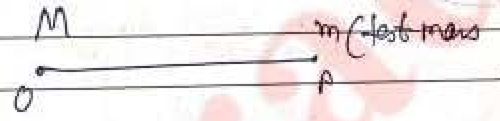
F = force experienced by the particle of mass "m" in gravitational field. (test mass)

~~S.I unit → N~~

- S.I unit of E is $\frac{N}{Kg}$
- S.I unit of g is $\frac{m}{s^2}$

Note :- "g" is a special case of "E" (that is Intensity of earth's gravitational field.)

\# Intensity due to point mass (M)



$$E = \frac{F}{m} \quad \frac{G M \cancel{m}/r^2}{\cancel{m}} = \frac{G M}{r^2}$$

so,

$$\boxed{E = \frac{G M}{r^2}}$$

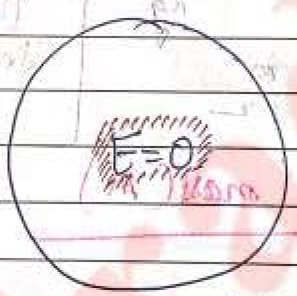
$$\boxed{\vec{E} = \frac{GM}{r^2} \text{ (from p to 0)}}$$

★ Intensity due to the spherical shell

$$E = \frac{F}{m} = \frac{GMm/r^2}{m}$$

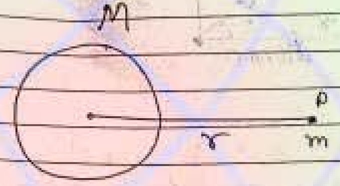
$$E = \frac{GM}{r^2}$$

Note



(c) Intensity due to solid sphere

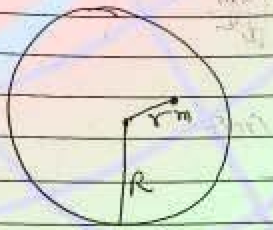
(i)



$$E = \frac{GM\cancel{m}/r^2}{\cancel{m}}$$

$$\boxed{E = \frac{GM}{r^2}}$$

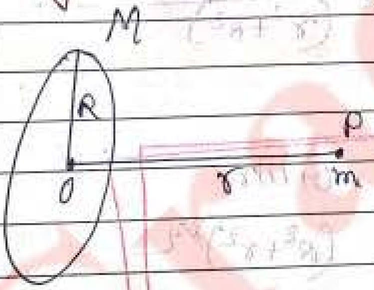
(ii)



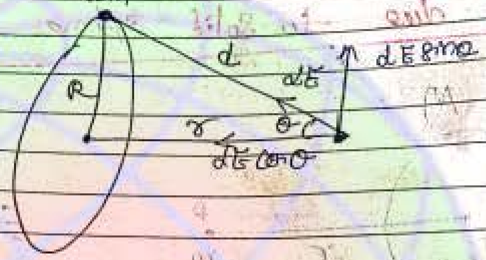
$$E = \frac{F}{m} = \frac{GM\cancel{m}r/R^3}{\cancel{m}}$$

$$\boxed{E = \frac{GMr}{R^3}}$$

(d) Intensity due to Ring



M



$$E = \int dE \cos\theta$$

where

$$dE = \frac{G\,dm}{d^2}$$

$$E = \int \frac{G\,dm}{d^2} \cos\theta$$

$$E = \frac{G\cos\theta}{d^2} \int dm$$

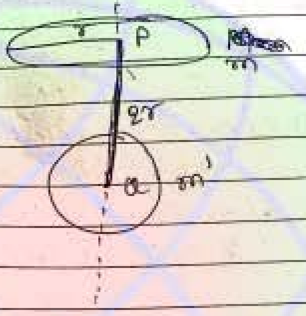
$$E = \frac{G\left(r/\sqrt{r^2+R^2}\right)\cdot M}{(r^2+R^2)} = \frac{GMr}{(r^2+R^2)^{3/2}}$$

$$\boxed{E = \frac{GMr}{(R^2+r^2)^{3/2}}}$$

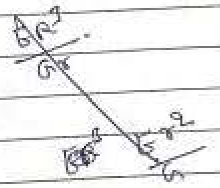
L-1 → 100...
L-2 → 3, 5, 8, 9, 12, 14, 16, 17, 20, 21, 24, 25, 26, 27, 30,

$F = \frac{GMr}{R^3}$

Q.



F = G



Intensity due to Ring at the point Q. is

$$E = \frac{Gmr}{(R^2+r^2)^{3/2}}$$

$$= \frac{Gm(2r}{((2r)^2+r^2)^{3/2}}$$

and

$F = E\, m'$ Ans

Gravitational Potential energy

Energy of a particle by virtue of its position in conservative force field is called potential energy.

Conservative force field

If workdone by the field in a round trip of a particle is always zero than field is called conservative.

eg :- Gravitational field, Electric field etc.

Note

In Conservative field workdone by the field on the particle does not depend upon path. it depends on initial and final ~~pos~~ position

~~According~~

Change in P.E = $(-) W_{\text{conservative field}}$

Change in P.E = (−) workdone by the conservative field

$$\Delta U = (-) W_{\text{conservative field}}$$

if $W_{\text{cons. force}} > 0$ then $\Delta U < 0$

$W_{\text{cons. force}} < 0$ then $\Delta U > 0$

$W_{\text{cons}} = 0$ then $\Delta U = 0$

$$\boxed{W = \Delta K}$$ → work energy theorem.

\# Change in gravitational potential energy (G.P.E) = (−)ve work done by the gravitational force

So $$\boxed{\Delta U_g = (-)\, W_g}$$

Note

Imp

$$W = \int \vec{F} \cdot \vec{dr}$$

if $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$\vec{dr} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$W = \int (F_x dx + F_y dy + F_z dz)$$

if

$F = f(x)$

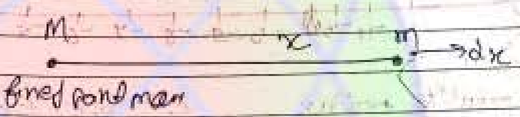
$$W = \int F_x dx$$

work

has

#

$$\boxed{U_f - U_i = (-)\int_i^f \vec{F_g}\cdot\vec{dr}}$$



$$U_\infty - U_x = (-)\int_x^\infty F\,dx$$

$$U_\infty - U_x = (-)\int_x^\infty \frac{-GMm}{x^2}\,dx$$

$$U_\infty - U_x = GMm\left(\frac{-1}{x}\right)_x^\infty$$

$$U_\infty - U_x = \frac{GMm}{x}$$

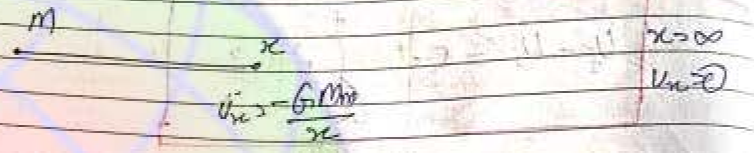
or

$$U_x - U_\infty = \frac{-GMm}{x}$$

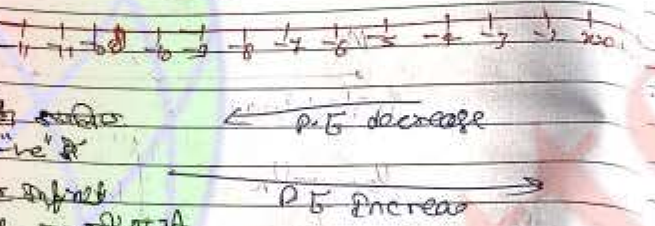
if $U_\infty = 0$ then

$$\boxed{U_x = \frac{-GMm}{x}}$$

★ Concept of Energy

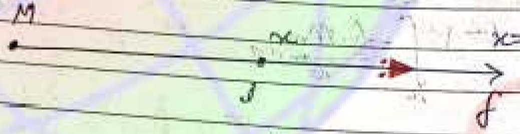


$$V_x = -\frac{GMm}{x}$$



इस Graph से पता चलता है कि Infinity (∞) पर Energy को zero (0) माना जाता है इसलिए Binding energy का value "-ve" हो जाता | लेकिन यह Infinite Generally माना ही पर ही मान सकते हैं

①

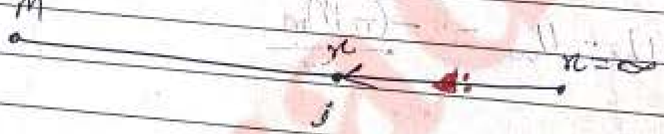


$$\Delta U = +\frac{GMm}{x}$$

Note → इसको Conc याद रखें जाना

$$U_g = \frac{(-)GMm}{x}$$

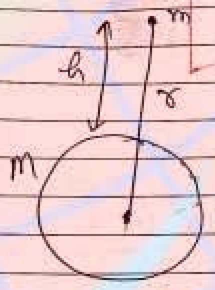
②



$$\Delta U = -\frac{GMm}{x}$$

$$\boxed{U_g = \frac{GMm}{x}}$$

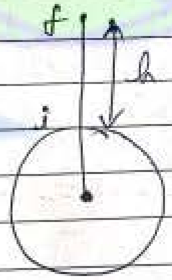
Potential energy due to earth



$$U = \frac{-GMm}{r}$$

$$U = -\frac{GMm}{R+h}$$

Special case:1
If particle is taken from earth surface to a height "h" → "h" is from the earth surface



$$\Delta U = \left(\frac{-GMm}{R+h}\right) - \left(\frac{-GMm}{R}\right)$$

$$\Delta U = \frac{GMm}{R} - \frac{GMm}{R+h}$$

$$\Delta U = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$\Delta U = GMm\left[\frac{h}{(R)(R+h)}\right]$$

$$\Delta U = \frac{GMmh}{R^2\left(1+\frac{h}{R}\right)}$$

$$\boxed{\Delta U = \frac{mgh}{\left(1+\frac{h}{R}\right)}}$$ → जब P.E नीचे से ऊपर को जाते है

Note

If

$\frac{h}{R} <<< 1$

$$\boxed{\Delta U = mgh}$$

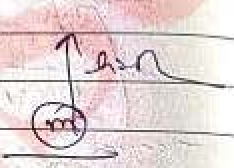
Note

Q) A particle of mass 'm' is taken from earth surface to a height h = R, where R is radius of earth. ΔU = ?

Sol

$$\Delta U = \left(\frac{mgR}{1+\frac{R}{R}}\right)$$

$$\frac{mgR}{2}$$



Q) In the above question w. done by gravity is

$$\frac{mgR}{2}$$

$$\therefore (\Delta U = -W_{gra})$$

Q) In the above question w. done by ext agent

$$\frac{mgR}{2}$$

$$\boxed{W_{ext.agent} = W_{\text{~~gravity~~ agent}}}$$

Note

w done by tracking object from earth to height 'R' / w done by ext agent

Note

① $\boxed{W_{all} = \Delta K}$

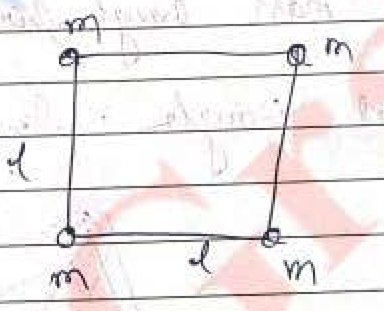
w done by all force

② $\boxed{W_{ext} = \Delta U + \Delta K}$

$W_{ext} = \Delta U + \Delta K$

$= \frac{mgR}{2}$

Q-

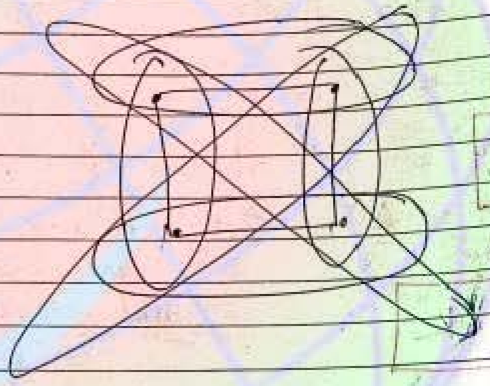


$\Delta U = ?$ if $U_\infty = 0$

~~mgh 0 + mgh~~

Note

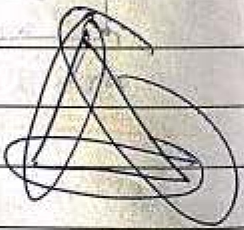
$U = \frac{-GMm}{r}$ if $U_{\infty} = 0$



$$U_{system} = 4\left(\frac{-Gm^2}{l}\right) + 2\left(\frac{-Gm^2}{\sqrt{2}l}\right)$$

$$= \frac{-Gm^2}{l}(4+\sqrt{2})$$

Q) three particles of same mass brought from ∞ to vertices of an equilateral triangle. length of a side is 'l'.



$W_{ext} = 3\left(\frac{-Gm^2}{l}\right)$

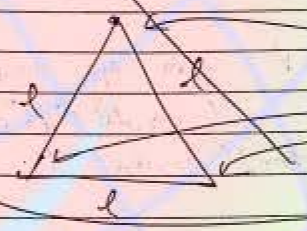
$$\boxed{\text{Note} \rightarrow \; W_g = \frac{3Gm^2}{l}}$$

So

$$W_{ext} = \cancel{\frac{3Gm^2}{l}}$$

$$\Rightarrow 0 + \left(\frac{-Gm^2}{l}\right) - \frac{2Gm^2}{l} = \cancel{\frac{-3Gm^2}{l}}$$

$$\cancel{\frac{3Gm^2}{l}}$$



Q) A particle of mass "m" is released from a height h = R, R is Rad. of earth.
[illegible]
And its speed when it reaches the earth surface

$$\cancel{\frac{1}{2}mv^2} = \cancel{\frac{mgR}{2}}$$

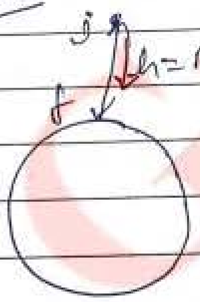
$$\left\{ \begin{array}{l} mgh \\ \rightarrow \dfrac{mgR}{1+1} \end{array} \right.$$

$$= \frac{\cancel{m}gR}{2}$$

$$\boxed{v = \sqrt{gR}}$$

Teach

V ∝ m



$$W_{ext} = \Delta U + \Delta K$$

$$0 = \frac{-mgh}{\left(1+\frac{h}{R}\right)} + \left(\frac{1}{2}mv^2 - 0\right)$$

Here,

$h = r$

$R \approx \frac{h}{6}0$

$v = \sqrt{gR}$

$= 8\ km/s$

Q) A particle is released from very large distance compare to Radius of earth (at mean ∞) find its speed when it reaches the earth surface

Ans

$\frac{GM\cancel{m}}{R} = \frac{1}{2}\cancel{m}v^2$

$v = \sqrt{\frac{2GM}{R}}$

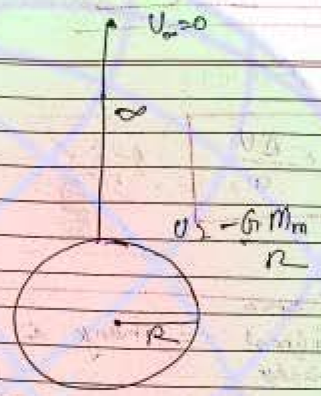
$\Delta W_{ext} = mg$

$\frac{mgx}{1 + \frac{x}{R}} = \frac{1}{2}mv^2$

$\frac{R\cancel{m}gx}{R+x} = \frac{1}{2}\cancel{m}v^2$

$V = \sqrt{\frac{Rx}{R+x}}$

(V) - Escaping Velocity



$$\Delta W_{ext} = \Delta U + \Delta K$$

$$0 = \left(-\frac{GMm}{R} - 0\right) + \left(\frac{1}{2}mv^2 - 0\right)$$

$$v = \sqrt{\frac{2GM}{R}} = \sqrt{2g_0R}$$

$$= 11.2\,km/s$$

Note

Imp

$$W_{ext} = \Delta U + \Delta K$$

$$0 = \frac{-mg_0}{\frac{1}{\infty} + \frac{1}{R}} + \left(\frac{1}{2}mv^2 - 0\right)$$

∞

[illegible]

$$v = \sqrt{2Rg_0}$$

Gravitational Potential (V)

$$\boxed{\Delta V = \frac{\Delta U}{m}}$$

~~change in potential~~ change in gravitational potential = change in potential energy per unit mass

Note

SI unit J/kg

$$\Delta U = \frac{(-)\int \vec{F}\cdot d\vec{r}}{m}$$

$$\therefore \Delta U = -\int \vec{F}\cdot d\vec{r}$$

so

$$\boxed{\Delta V = (-)\int \vec{E}\cdot d\vec{r}}$$

potential diffe

gravitational field intensity

if $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

and

$d\vec{r} = dx\hat{i} \; dy\hat{j} \; dz\hat{k}$

$$\boxed{\Delta V = (-)\int E_x dx + E_y dy + E_z dz}$$

If $E = f(x)$ (In 1D)

~~$\Delta V = (-)\int E_x dx$~~

$$\boxed{\Delta V = (-)\int E_x\, dx}$$

and

$$\frac{dv}{dx} = -E_x$$

or

$$\boxed{E_x = \frac{-dv}{dx}}$$

If $E = f(x, y, z)$ (in 3D)

$$\boxed{\Delta V = (-)\int(E_x\, dx + E_y\, dy + E_z\, dz)}$$

and

$$\boxed{E_x = \frac{-\partial v}{\partial x}} \; ; \; \boxed{E_y = \frac{-\partial v}{\partial y}} \; ; \; \boxed{E_z = \frac{-\partial v}{\partial z}}$$

→ partial diff

for very Imp

$$\boxed{E = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}}$$

eg.) $V = \underbrace{K}_{\text{constant}}(x+y)$

find intensity at any point (x,y)

$E_x = -\frac{\partial V}{\partial x} \Rightarrow -K$

$E_y = -\frac{\partial V}{\partial y} = -K$

so

$E = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j}$

$E = -K\hat{i} - K\hat{j}$

$F = \frac{a}{m}$

$E = \frac{F}{q}$ Recall

eg) In the above question find force experienced by a particle of mass "m" kept at the point $P(x,y)$.

From here $E = \frac{F}{m}$ Intensity of gravitational field

$F = Em$

$= \sqrt{2}\,m|k|$

$= \sqrt{2}\,m|k|$

$= \sqrt{2}\,|k|m$

Q) if $\vec{E} = (3\hat{i} + 4\hat{j})\ \frac{N}{kg}$

find gravitational potential at a point P (3, 4) m if V at origin = 0

Sol:

$\Delta V = -\int E\,dx$

$- [3x + 4y]$

$= -[9 + 16]$

$= -25$

Recall

$\Delta V = -\left(\int 3\,dx + \int 4\,dy\right)$

$V_f - V_i = (-)\left(\int_0^3 3\,dx + \int_0^4 4\,dy\right)$

$V_p = -25\ \frac{J}{kg}$

E (Intensity)

$\downarrow$ Integration

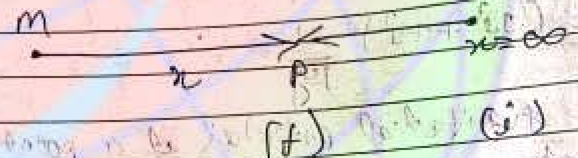
V (pot. diff)

$E = -\frac{dV}{dr}$

$\boxed{E = \frac{-dV}{dr}}$

~~ΔV [illegible] g~~

☆ Gravitational potential due to a point mass



$$\Delta V = \frac{\Delta U}{m}$$

$$V_x - V_\infty = \frac{U_x - U_\infty}{m}$$

$$= \frac{-G\,M\cancel{m}/x}{\cancel{m}}$$

$$\boxed{V_x - V_\infty = \frac{-GM}{x}}$$

if $V_\infty = 0$ (assumption)
↑ Reference point

then

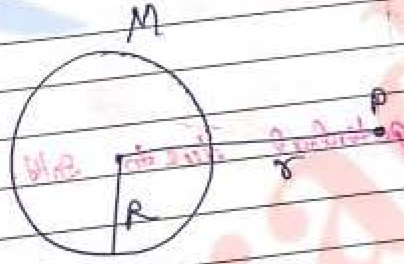
Imp formula

$$\boxed{V_x = \frac{-GM}{x}}$$

Note

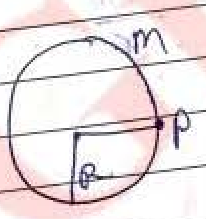
Reference point : point at which potential is taken to be zero.

2 Gravitational potential due to thin spherical shell :-



$$V_{out} = \frac{-GM}{r}$$

taken $V_\infty = 0$

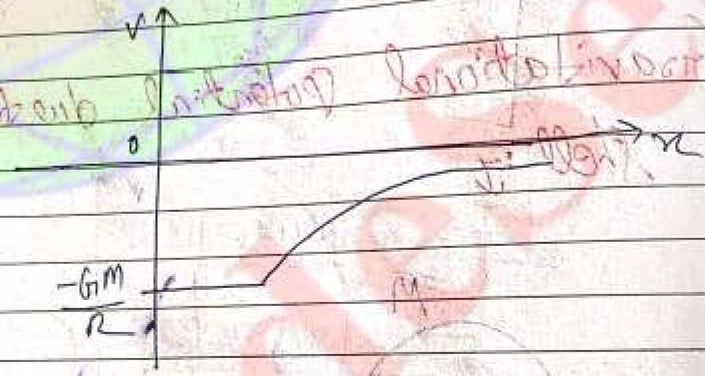


$$V_{surface} = \frac{-GM}{R}$$

$V_{inside} = \frac{-GM}{R}$

Note
$$\boxed{V_{inside} = V_{surface}}$$

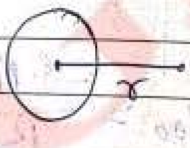
Graph :-



★ Gravitational potential due to solid sphere

① if $r > R$

$$V = \frac{-GM}{r} \quad \text{if } V_\infty = 0$$



(ii) If $r = R$

$$V_{surf} = \frac{-GM}{R}$$

(iii) if $r < R$

$$\Delta V = (-)\int E_x\, dx$$

$$V_x - V_R = (-)\int_R^r (-)\left(\frac{GMx}{R^3}\right) dx$$

$$V_r - V_R = \frac{GM}{R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

$$V_r = \frac{GM}{R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right] + V_R$$

$$= \boxed{\frac{GM}{R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right]\left(\frac{-GM}{R}\right)}$$

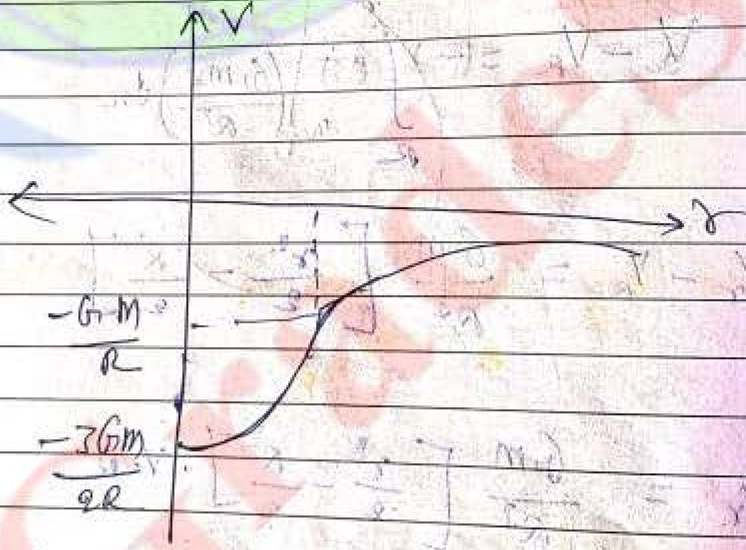
Hw
L-1 → 25 to 33
L-2 → 4,7, 22, 28, 31,
4A → 1, 4, 9, 11, 12, 13, 15, 6, 17, 18,

$$= \frac{GM}{R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right] + \left[\frac{-GM}{R}\right]$$

$$= \frac{GM}{R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right] + \left[\frac{-GMR^2}{R^3}\right]$$

$$V_r = \frac{GM}{R^3}\left[\frac{r^2}{2} - \frac{3R^2}{2}\right]$$

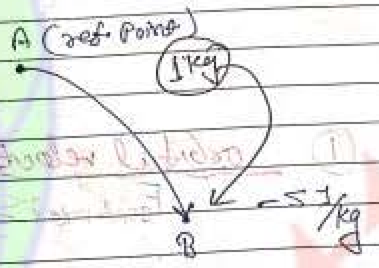
$$\boxed{V_r = \frac{-GM}{2R^3}\left[3R^2 - r^2\right]}$$



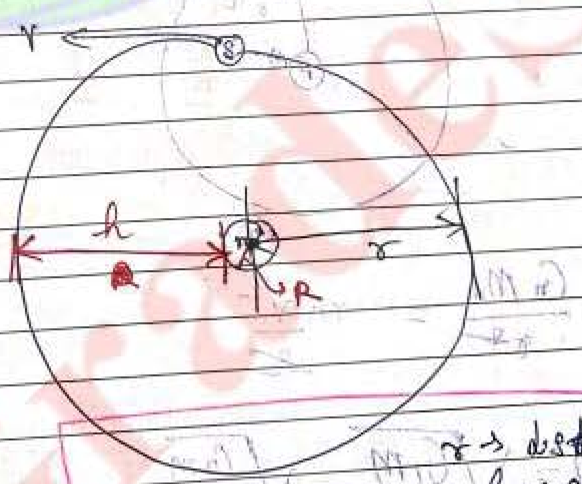
$$\boxed{V_{centre} = \frac{-3GM}{2R}}$$

Note Gravitational potential at a point is equal to the change in gravitation potential energy per unit mass as the mass is brought from ref. point to the given point.

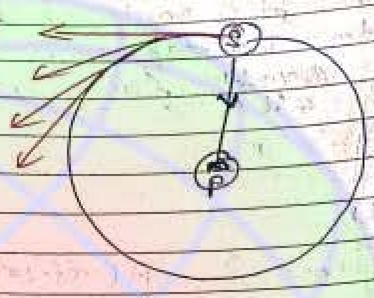
$V_{ref.\ point} = 0$



Satellite Motion



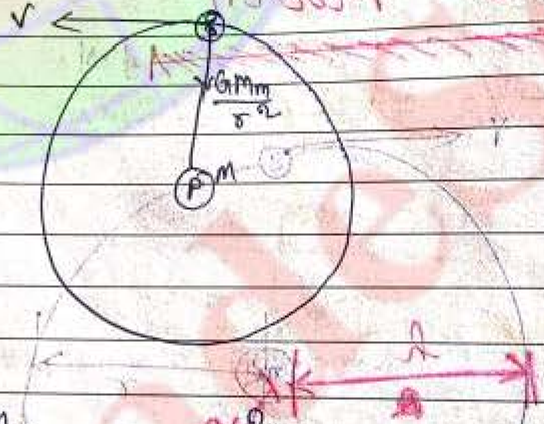
$r \rightarrow$ distance of satellite from centre of earth



① orbital velocity →

$F_{centripetal} = m\, a_{centripetal}$ → $\frac{v^2}{r}$

Satellite Motion



$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}\,v^2}{\cancel{r}}$$

$$\boxed{\underbrace{v_o}_{\text{orbital speed}} = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+r}}}$$

Note orbital speed of satellite ~~per~~ does not depends on it's own mass.

If Force is always ⊥r to ~~the body~~ velocity of the body then workdone by ~~per body~~ force is Zero

② Time period.

$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi r}{\sqrt{GM/r}} = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

$$\boxed{T^2 = \frac{4\pi^2 r^3}{GM}}$$

or

$$\boxed{T^2 \propto r^3}$$ if $M = \text{const}$

③ kinetic energy

$$K = \frac{1}{2} m v^2$$

$$K = \frac{1}{2} m \left(\frac{GM}{r}\right)$$

$$\boxed{K.E = \frac{GMm}{2r}}$$

④ Potential energy

$$\boxed{U = \frac{-GMm}{r}}$$

⑤ Total mechanical energy

$$T = K.E + P.E$$

$$\boxed{T = -\frac{GMm}{2r}}$$

⑥ Binding energy

$$\boxed{B.E = \frac{GMm}{2r}}$$

Note

Standard Relation

$$\boxed{T = -K.E = \frac{1}{2} P.E}$$

Special case :

① If $h \simeq 0$, that a satellite revolves near the earth surface

then,

$$v = \sqrt{\frac{GM}{R}} = \sqrt{Rg_0}$$

$= 8$ km/sec (near the earth surface)

② Time period (T) $= \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi R}{\sqrt{Rg_0}} = 2\pi\sqrt{\frac{R}{g_0}}$

$\simeq 1.41$ hou

③ kinetic energy (K.E) $= -\frac{GMm}{2R} = +\frac{mg_0R}{2}$

④ Potential energy (P.E) $= -\frac{GMm}{R} = -mg_0R$

⑤ $\boxed{\text{Total energy (T.E)} = \frac{-mg_0R}{2}}$

⑥ Binding energy = $-$(total energy)

$= \frac{mg_0R}{2}$

$$\boxed{\sqrt{\frac{M}{m}}\,{}^{3/2}\left(\frac{g}{g}\right) = \frac{T}{T}}$$

motion of Two planets]

Some Important case]

(A)

$h \simeq 0$

① $v = \sqrt{\frac{Gm}{R}}$

$$\boxed{\frac{v}{v'} = \sqrt{\frac{m}{m'}} \times \sqrt{\frac{R'}{R}}}$$

② Time period (T) →

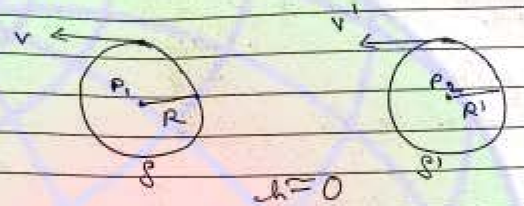
$$\frac{2\pi R}{\sqrt{\frac{Gm}{R}}} = \frac{2\pi R^{3/2}}{\sqrt{Gm}}$$

$$T \propto \frac{R^{3/2}}{\sqrt{m}}$$

$$\boxed{\frac{T}{T'} = \left(\frac{R}{R'}\right)^{3/2} \sqrt{\frac{M_1}{m}}}$$

Ⓑ

⑥



$h = 0$

① $$V = \sqrt{\frac{GM}{R}} = \sqrt{\frac{G\cdot\frac{4}{3}\pi R^3 \rho}{R}}$$

$$\boxed{V \propto R\sqrt{\rho}}$$

$$\boxed{\frac{V}{V'} = \frac{R}{R'}\sqrt{\frac{\rho}{\rho'}}}$$

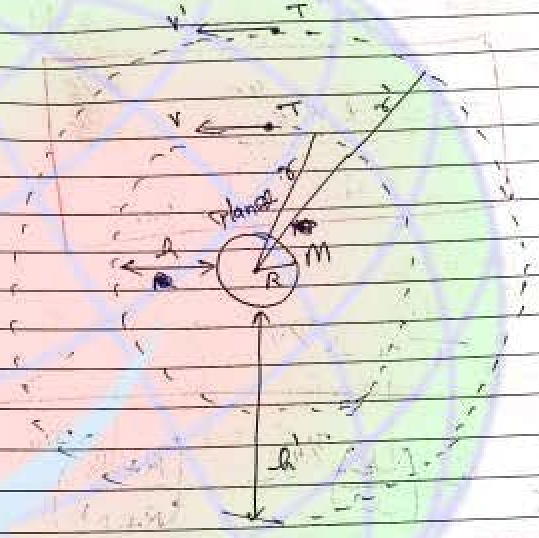
② $$T = \frac{2\pi R^{3/2}}{\sqrt{GM}}$$

$$T = \frac{2\pi R^{3/2}}{\sqrt{G\cdot \cancel{R}\frac{4}{3}\pi R^3 \rho}}$$

$$\boxed{T \propto \frac{1}{\sqrt{\rho}}}$$

$$\boxed{\frac{T}{T'} = \sqrt{\frac{\rho'}{\rho}}}$$

☆ Motion of two satellite about same planet: I



① $$\boxed{V = \sqrt{\frac{GM}{r}}} \rightarrow \text{"M" is same in both case of satellite}$$

$$\boxed{v \propto \frac{1}{\sqrt{r}}}$$

$$\boxed{\frac{v}{v'} = \sqrt{\frac{r'}{r}} = \sqrt{\frac{R+h'}{R+h}}}$$

②

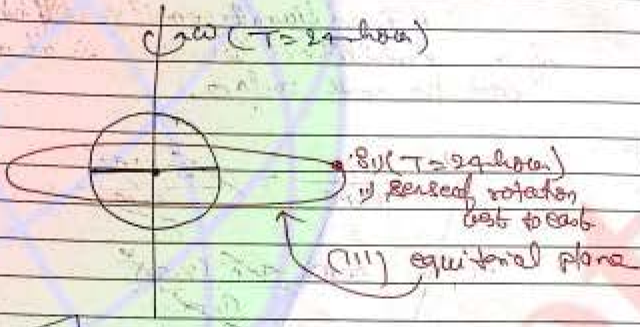
$$\boxed{T = \frac{2\pi r^{3/2}}{\sqrt{G \textcircled{M}}}} \rightarrow \text{same}$$

$$T \propto r^{3/2}$$

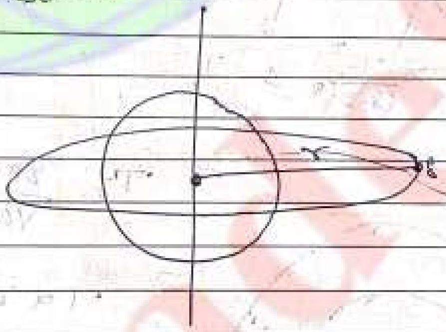
$$\boxed{\frac{T}{T'} = \left(\frac{r}{r'}\right)^{3/2} = \left(\frac{R+h}{R+h'}\right)^{3/2}}$$

Geo stationary sattelite

A sattelite which remain at rest with respect to earth is called Geo-stationary sattelite.



Radius of earth 6,000 km
[illegible]



$$T^2 = \frac{4\pi r^3}{GM}$$

$$\boxed{r = \left(\frac{T^2 GM}{4\pi^2}\right)^{1/3}}$$

$r \simeq 42,000$ km from centre of earth

36,000 km from surface of earth

Q.) A geo-stationary sattelite is at a height 6R from the earth surface where R is radius of earth find time period of a satellite which is at a height 2.5R from the earth surface.

Sol/n

$$T^2 = \frac{4\pi r^2}{GM} = \frac{4\pi (7R)^3}{GM}$$

$$T_1^2 = \frac{4\pi (3.5R)^3}{GM}$$

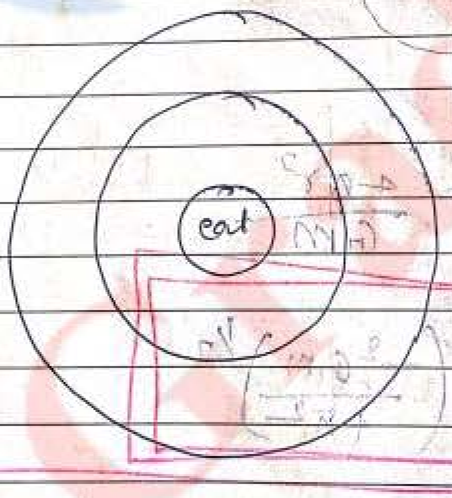
$$\left(\frac{T}{T_1}\right)^2 = 2 \times 2.5 \times 3.5$$

$$(T_1) = \frac{T^2}{(3.5)^2} \quad T$$

~~Procedure.~~

$$T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

$$T \propto r^{3/2}$$



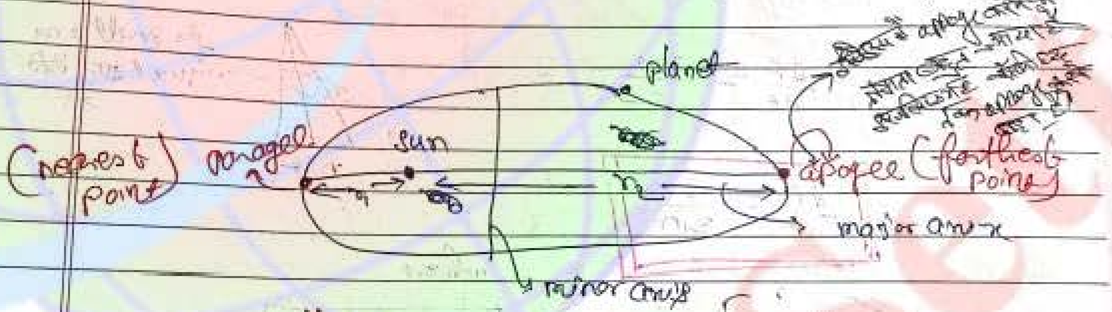
$$\frac{T'}{T} = \left(\frac{r'}{r}\right)^{3/2}$$

$$\Rightarrow T' = (24)\left(\frac{2.5R}{7R}\right)^{3/2}$$

$$\frac{1}{2\sqrt{2}} 24 \quad 6\sqrt{2}\ \text{hr}$$

Keeplers law
- → law of orbit
- → law of area
- → law of time period.

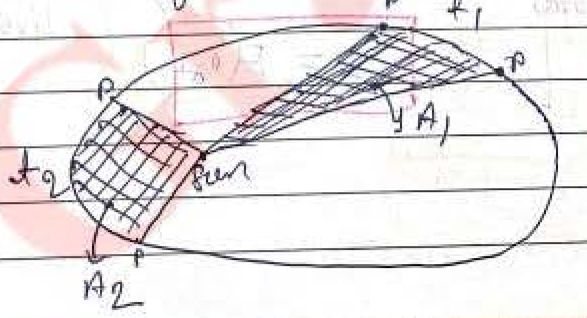
① law of orbit :→ each planet moves about the elliptical path about the sun on the focus.



$$\boxed{\text{Semi major } (a) = \frac{r_1 + r_2}{2}}$$

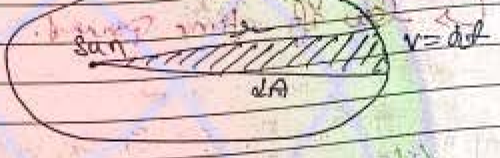
② law of Area :-

A/q to this law a line joining planet to the sun swipps out equal area in equal Interval of time

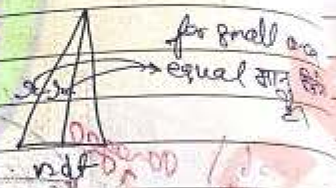


$A_1 = A_2$ if $t_1 = t_2$
and
$t_1 = t_2$ if $A_1 = A_2$

That is same area in same time.



$$dA = \frac{1}{2}(vdt)r$$



$$\frac{dA}{dt} = \frac{vr}{2}$$

$$\boxed{\frac{dA}{dt} = \frac{J}{2m}}$$

where

$$\boxed{J = mvr}$$

↳ Angular momentum

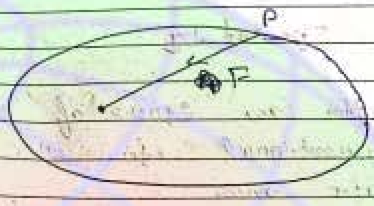
Note

① $\boxed{\tau = \frac{dJ}{dt}}$

and

$$\boxed{\tau = F r_{\perp}}$$

this is know as Liver arm



Here

$\tau = 0$

$\therefore \tau = 0$

or $\frac{dJ}{dt} = 0$

$J = \text{constant}$

Note - This law (law of area) is also known as Angular momentum Conservation Principle

① Law of Area is based on principle of Conservation of Angular momentum. during the motion angular momentum of the planet remain's unchanged.

②

$$mV_{max} \cdot r_{min} = mV_{min} r_{max}$$

$$\boxed{V_{max} \cdot r_{min} = V_{min} r_{max}}$$

Ⓒ Law of time period :-

According to this law square of time period is directly proportional to the third of semimajor axis.

$$\boxed{T^2 \propto a^3 \text{, where } a = \left(\frac{r_1+r_2}{2}\right)}$$

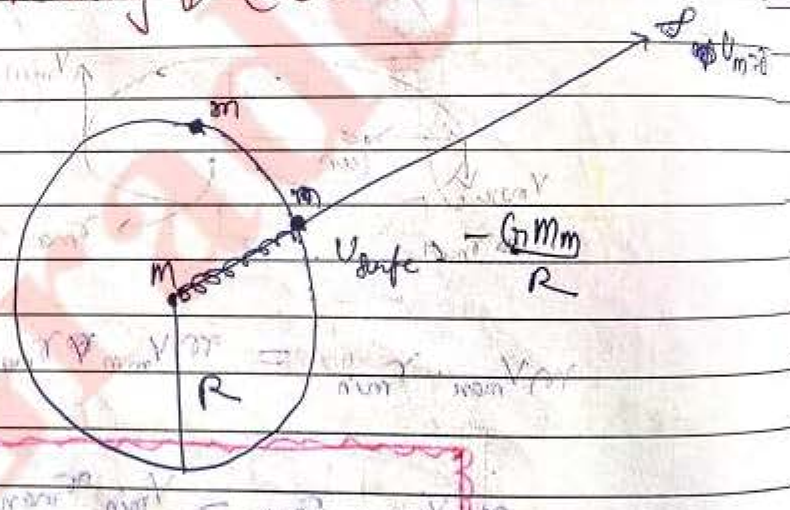
Note

In case of circular motion

$$\boxed{T^2 \propto r^3}$$

r → radius of orbit

☆ Escape velocity :- (v_e)



Now

$$T.E = \cancel{K.E} \; K.E + P.E$$

$$= \frac{1}{2}mv^2 + \left(\frac{-GMm}{R}\right) \quad (1)$$

a) if ~~T.E~~ T.E < 0 , object can not escape

b) if T.E ≥ 0 , object escape from the earth gravitation field.

$$\frac{1}{2}mv^2 - \frac{GMm}{R} \geq 0$$

$$\boxed{v \geq \sqrt{\frac{2GM}{R}}}$$

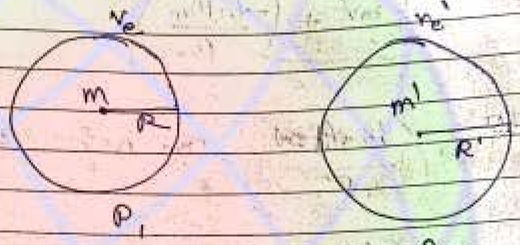
$$\boxed{V_{escape} = \sqrt{\frac{2GM}{R}} = \sqrt{2g_0R} = 11.2 \text{ km/s} \text{ (On the earth surface)}}$$

$$V_e = \sqrt{\frac{2G\,\frac{4}{3}\pi R^3 \rho}{R}}$$

$$\boxed{V_e = \sqrt{\frac{8G\pi R^2 \rho}{3}}}$$

Imp. Point :-

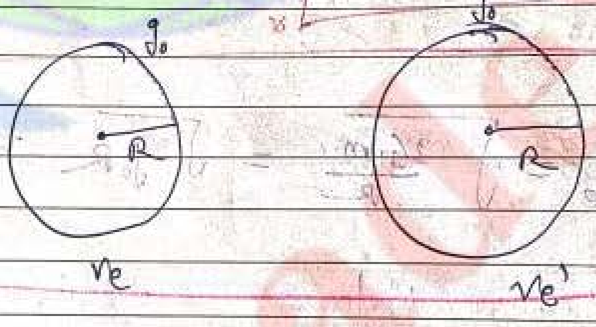
①



$$v_e \propto \sqrt{M/R}$$

$$\boxed{\frac{v_e}{v_e'} = \sqrt{\frac{M}{M'} \times \frac{R'}{R}}}$$

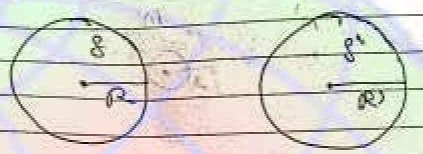
②



$$v_e \propto \sqrt{g_0 R}$$

$$\boxed{\frac{v_e}{v_e'} = \sqrt{\frac{g_0 R}{g_0' R'}}}$$

③



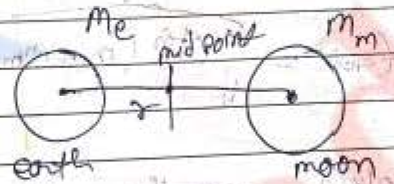
$$v_e \propto R\sqrt{\rho}$$

$$\boxed{\frac{v_e}{v_e'} = \frac{R}{R'}\sqrt{\frac{\rho}{\rho'}}}$$

① Escape velocity does not depend on the mass of body.

② Escape velocity does not depend upon angle of projection.

Q.)

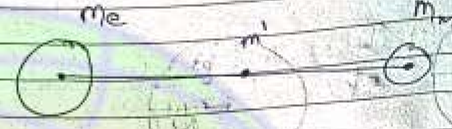


what minimum velocity must be given to a particle so that it may escape from the mid-point into the space.

Sol/ $\cancel{v = \sqrt{\frac{2GM}{R}}}$

$\sqrt{\frac{g}{2}} -$

Sol^n



U G.P.E

$$v_1 = \left(\frac{-G m_e m'}{r/2}\right) + \left(\frac{-G m_m m'}{r/2}\right)$$

$$\rightarrow \frac{-2Gm'}{r}(m_e + m_m)$$

$$T.E = \frac{1}{2}mv_1^2 + v_1'$$

$$T.E \geq 0$$

$$\frac{1}{2}mv_1^2 + \left(-\frac{2Gm}{r}\right)(m_e + m_m) \geq 0$$

$$v \geq \sqrt{\frac{4G}{r}(m_e + m_m)}$$

$$V_{conte} = \sqrt{\frac{4G}{r}(m_e + m_m)}$$

A satellite revolves near the earth surface find % increase in its velocity so that it may escape into the space

$$v = \sqrt{\frac{Gm}{R}} \qquad \frac{v}{v'} = \frac{1}{\sqrt{2}}$$

$$v' = \sqrt{\frac{2Gm}{R}} \qquad \frac{v'}{} = \sqrt{2}\,v \qquad = \sqrt{2}\,x$$

Sol^n

UGPE

$$V_1 = \left(\frac{-Gm_e m'}{r/2}\right) + \left(\frac{-Gm_m m'}{r/2}\right)$$

$$\to \frac{-2Gm'}{r}(m_e + m_m)$$

$$T.E = \frac{1}{2}mv^2 + V'$$

$$T.E \geq 0$$

$$\frac{1}{2}mv^2 + \left(\frac{-2Gm}{r}\right)(m_e + m_m) \geq 0$$

$$v \geq \sqrt{\frac{4G}{r}(m_e + m_n)}$$

$$V_{crit} = \sqrt{\frac{4G}{r}(m_e + m_m)}$$

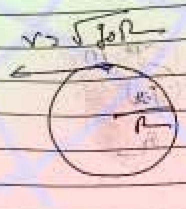
Q) A satellite revolves near the earth surface find % increase in its velocity so that it may escape into the space

Sol^n

$$v_c = \sqrt{\frac{Gm}{R}} \qquad \frac{v}{v'} = \frac{1}{\sqrt{2}}$$

$$v' = \sqrt{\frac{2Gm}{R}} \qquad \frac{v}{v'} = \sqrt{2}\,v \quad 2\sqrt{2}\,x$$

Soln



% Increase in velocity = $\frac{v_f - v_i}{v_i} \times 100$

$$= \frac{\sqrt{2g_0R} - \sqrt{g_0R}}{\sqrt{g_0R}} \times 100$$

$$\simeq 41\%$$

Q) In the above question find % increase in kinetic energy

Soln % Incr. = $\frac{K_f - K_i}{K_i} \times 100$

$$K_f = \frac{1}{2} m (v_0)^2$$

$$= \frac{1}{2} \times (\sqrt{2g_0R})^2$$

$$= m g_0R$$

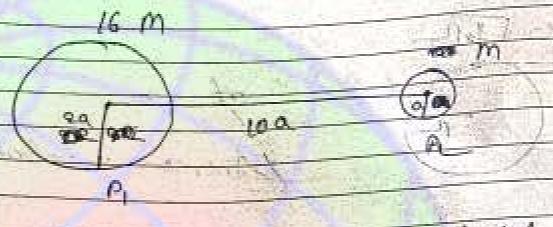
$$\% \uparrow = \frac{mg_0R - mg_0R/2}{mg_0R/2} \times 100$$

$$= 100\ \%$$

Note

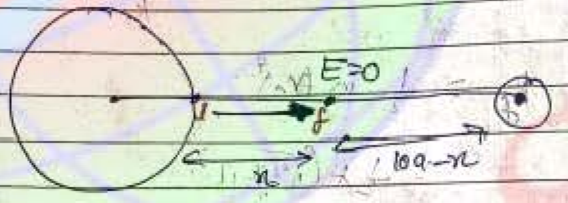
$$\boxed{\%\ \text{decrease} = \frac{A_i - A_f}{A_i} \times 100}$$

q)



A body is projected straight from the surface of bigger plane towards the smaller planet. what's min vel. must be given to it so that it reaches the surface of smaller plane

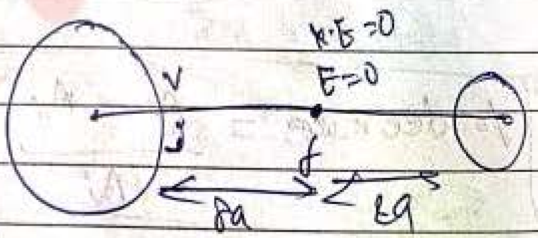
80 /n



$$\frac{G(16m)}{x^2} = \frac{Gm}{(10a-x)^2}$$

$$\frac{4}{x} = \frac{1}{10a-x}$$

$$\boxed{x=8a}$$



Vp = 0-0

NOW

~~HW [illegible]-2 6-7-3/2015~~

$$U_i = \frac{-G(16m)m'}{2a} + \frac{-G(m)(m')}{8a}$$

$$U_i = \frac{-65}{8}\frac{Gmm'}{a}$$

$$U_f = \frac{-G(16m)m'}{8a} + \frac{-G(m)(m')}{2a}$$

$$= \frac{-20}{8}\frac{Gmm'}{a}$$

$$\Delta U = U_f - U_i$$

$$= \frac{45}{8}\frac{Gmm'}{a}$$

but

$$\Delta U + \Delta K = W_{ext}$$

$$\frac{45}{8}\frac{Gm\cancel{m'}}{a} + \left(0 - \frac{1}{2}\cancel{m'}v^2\right) = 0$$

$$V 2 \sqrt{\frac{45}{4}\frac{Gm}{a}} = \frac{3}{2}\sqrt{\frac{5Gm}{a}}$$

98/6/2013 → Ra

Vector

\# vector quantities- The quantities which have magnitude and direction and can be added or subtracted According to triangle Rule are called vector Quantities

eg:→ displacement, velocity, Acc^n, force, etc.

Note symbol of vector: → $\vec{V}$, **V** ↳ old symbol (old में लिखा जाता था।)

$$\vec{A} = |\vec{A}|\hat{A}$$
$$= A\hat{A}$$

where

$|\vec{A}| = A$ = magnitude of $\vec{A}$

and

$\hat{A}$ = unit vector in the direction of $\vec{A}$

Note $|\hat{A}| = 1$

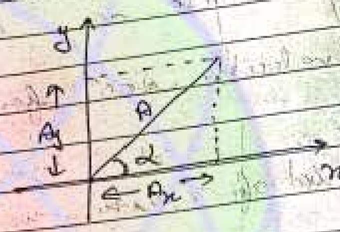
↳ modulus of unit vector

* unit vector !?

in +x direction = $\hat{i}$ | and in −x direction = $-\hat{i}$
" +y " = $\hat{j}$ | " −y " " = $-\hat{j}$
" +z " = $\hat{k}$ | " −z " " = $-\hat{k}$

Resolution of a vector :-

Concept



$A_x \rightarrow$ X-component of vector

$A_y \rightarrow$ Y-component of vector

$\frac{A_x}{A} = \cos\alpha$ $\boxed{A_x = A\cos\alpha}$

$\frac{A_y}{A} = \sin\alpha$ $\boxed{A_y = A\sin\alpha}$

$A = A_x\hat{i} + A_y\hat{j}$

so

$$\boxed{\vec{A} = A\cos\alpha\,\hat{i} + A\sin\alpha\,\hat{j}}$$

$A^2 = A_x^2 + A_y^2$

$$\boxed{A = \sqrt{A_x^2 + A_y^2}}$$

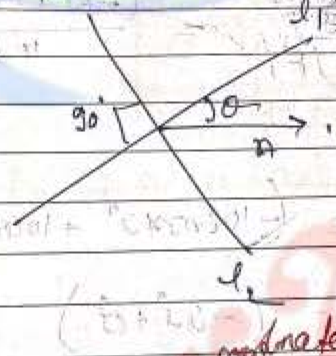
<u>Case 2nd</u>

$$\frac{A_x}{A} = \sin\theta \quad , \quad A_x = A\sin\theta$$

$$\frac{A_y}{A} = \cos\theta \quad , \quad A_y = A\cos\theta$$

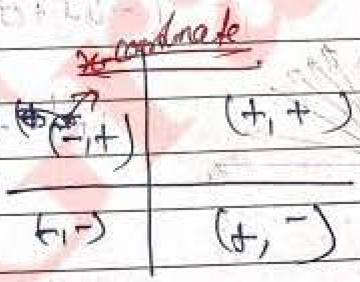
$$\vec{A} = A\sin\theta\,\hat{i} + A\cos\theta\,\hat{j}$$

<u>Case 3rd</u>

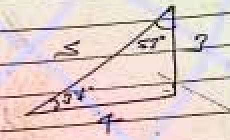


Projection of $\vec{A}$ along l_1 i.e. = A cos θ
" " " l_2 " = A sin θ

<u>Case 4th</u>



Ques 5th



$\sin 37° = \frac{3}{5}$

$\cos 37° = \frac{4}{5}$

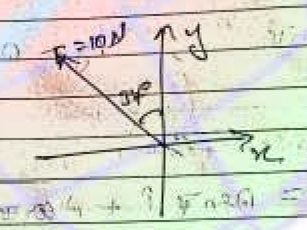
$\tan 37° = \frac{3}{4}$

$\sin 53° = 4/5$ ~~cos 53°~~

$\cos 53° = 3/5$

$\tan 53° = 4/3$

Q)



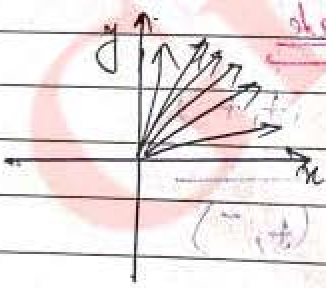
Unit vector in direction of $\vec{F}$ is

Sol: $\hat{F} = \frac{\vec{F}}{|\vec{F}|} = \frac{(-6\hat{i}+8\hat{j})N}{10N} = -\frac{3}{5}\hat{i}+\frac{4}{5}\hat{j}$

where $\vec{F} = (-10\sin 37\hat{i} + 10\cos 37\hat{j})\,N$

$= (-6\hat{i}+8\hat{j})$

Note



$|\hat{F}| = 1$ (Always)

Q) A block of mass
external force
a stationary
mag of magn

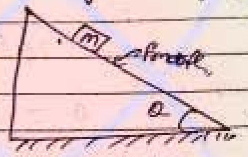
Sol

mg sin

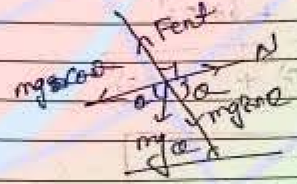
Q

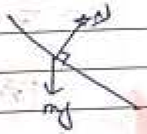
Que.

Q) A block of mass "m" is kept on smooth inclined plane. external force is ~~inclined~~ applied along inclined to make a stationary
~~mag of~~ magnitude of the force.

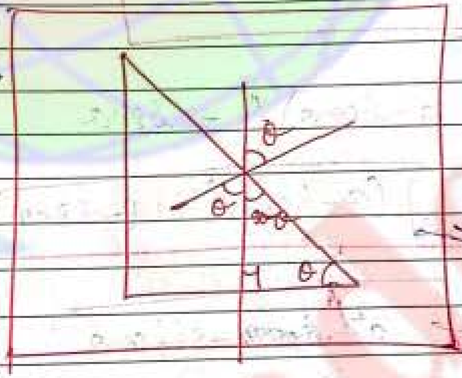


Sol





Geo



$F_{ext} = mg\sin\theta$

$N = mg\sin\theta$

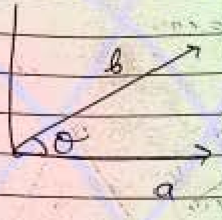
Addition and substraction of vectors →

(i) $|\vec{a}+\vec{b}| = ?$

(ii) $|\vec{a}-\vec{b}| = ?$

Note

①



$a = a\hat{i}$

$b = b\cos\theta\hat{i} + b\sin$

②

(i) $\vec{a}+\vec{b} = (a+b\cos\theta)\,\hat{i} + b\sin\theta\hat{j}$

$$|\vec{a}+\vec{b}| = \sqrt{(a+b\cos\theta)^2 + (b\sin\theta)^2}$$

$$\boxed{|\vec{a}+\vec{b}| = \sqrt{a^2+b^2+2ab\cos\theta}}$$

(ii) $\vec{a}-\vec{b} = (a-b\cos\theta)\,\hat{i} - b\sin\theta$

$$|\vec{a}-\vec{b}| = \sqrt{(a-b\cos\theta)^2 + (-b\sin\theta)^2}$$

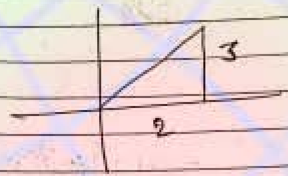
☆

$$\boxed{|\vec{a}-\vec{b}| = \sqrt{a^2+b^2-2ab\cos\theta}}$$

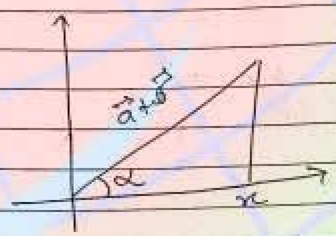
Pas

Note

① $A = 2\hat{i} + 3\hat{j}$



②



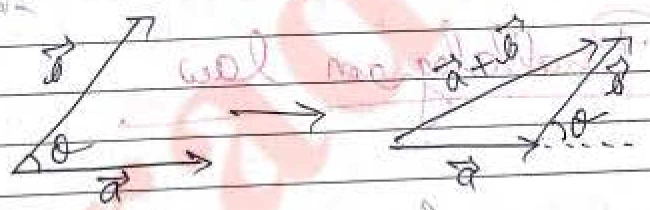
Proj

$$\tan\alpha = \frac{b\sin\theta}{a+b\cos\theta}$$

$$\alpha = \tan^{-1}\left(\frac{b\sin\theta}{a+b\cos\theta}\right)$$

= Angle b/w $\vec{a}+\vec{b}$ and $\vec{a}$

Triangle Rule



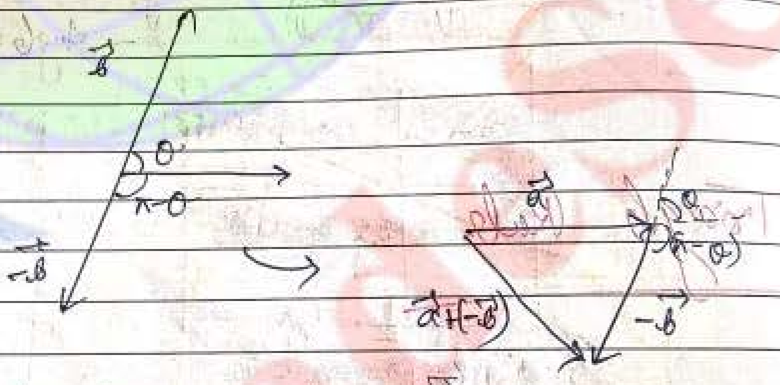
(ii)

$\vec{p} + \vec{q} + \vec{r} = 0$

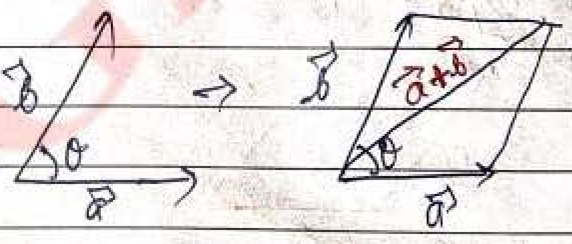
$-\vec{p} + (-\vec{q}) + -\vec{r} = 0$

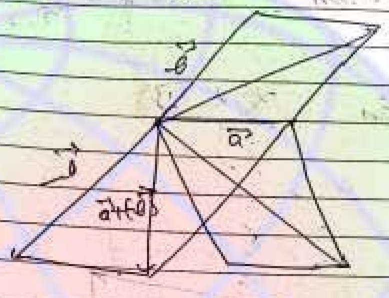
(iii) $\vec{a} - \vec{b}$

$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

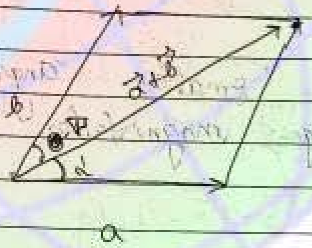


★ Parallelogram law





Note



$$\boxed{\tan\alpha = \frac{b\sin\theta}{a+b\cos\theta}}$$

if $a = b = k$

$$\tan\alpha = \frac{\sin\theta}{1+\cos\theta} = \frac{\cancel{2}\sin\left(\frac{\theta}{2}\right)\cancel{\cos\left(\frac{\theta}{2}\right)}}{\cancel{2}\cos^2\frac{\theta}{2}}$$

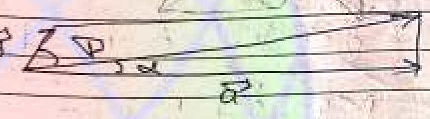
$$\tan\alpha = \tan\frac{\theta}{2}$$

Ans

$$\boxed{\alpha = \frac{\theta}{2}}$$

① if $a = b = k$, then $\alpha = \beta = \frac{\theta}{2}$

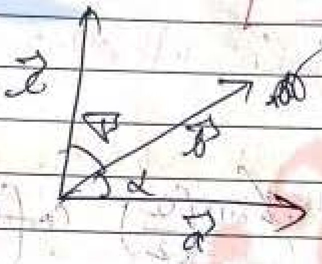
② if $a > b$, then $\alpha < \beta$



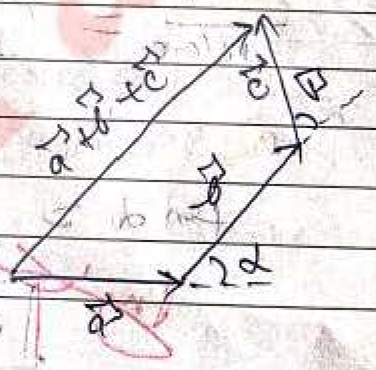
③ if $a < b$ then $\alpha > \beta$

④ Resultant makes smaller angle with vector of bigger magnitude.

✩ Polygon law

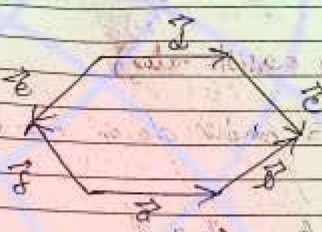


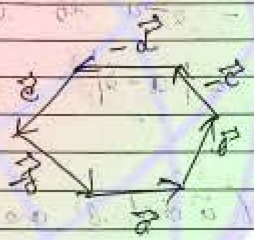
$\vec{a} + \vec{b} + \vec{c} = ?$



Note

①





$$\vec a+\vec b-\vec c-\vec d+\vec e-\vec f=0$$

$$\vec a+\vec b+\vec e = \vec c+\vec d+\vec f$$

Some Special case:-

(*) $|\vec a+\vec b| = \sqrt{\vec a^2+\vec b^2+2ab\cos\theta}$ (maximum possible value)

where θ is angle b/w $\vec a$ and $\vec b$

Case 1st, if $\theta=0^\circ$ ($\vec a$ and $\vec b$ are on the same line)

$$|\vec{a}+\vec{b}| = a+b = |\vec{a}|+|\vec{b}|$$

(ii) Case 2nd → (minimum possible value)

if $\theta = 180$ ($\vec{a}$ and $\vec{b}$ are in opp. directn)

$$|\vec{a}+\vec{b}| = a-b \quad \text{if } a>b$$

$$= b-a \quad \text{if } b>a$$

$$= |a-b| \quad \text{in general}$$ ⇐ q)

(iii) if $\theta = \frac{\pi}{2}$ ($\vec{a}$ and $\vec{b}$ are mutually ⊥r)

$$|\vec{a}+\vec{b}| = \sqrt{a^2+b^2}$$

q) $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

angle b/w $\vec{a}$ and $\vec{b}$ is

$$\cancel{a^2+b^2} + 2ab\cos\theta = \cancel{a^2+b^2} - 2ab\cos\theta$$

$$\cancel{\cos\theta} = \cancel{-\cos\theta}$$

$$4ab\cos\theta = 0$$

$$\theta = \frac{\pi}{2}$$

$\left[\because \theta = \cos^{-1}(0) = \frac{\pi}{2}\right.$

Note

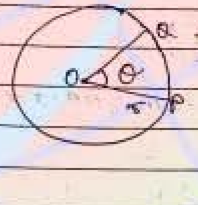
① $\vec{P} \pm \vec{Q}$ ⑩ $P = Q$

Implies

जरूरी नहीं

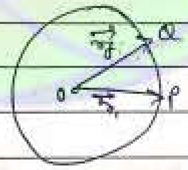
Here Ist Implies 2nd but 2nd does not imply Ist

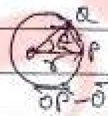
Q.



A particle moves in a circle of radius "r" find mag. of change in its position vect w.r.t. O as it moves from the point P to point Q

So





$\vec{OP} - \vec{OQ} = \vec{PQ}$

change of position vector $(\vec{\Delta r}) = \vec{r_f} - \vec{r_i}$

$$|\vec{\Delta r}| = |\vec{r_f} - \vec{r_i}|$$

$$|\vec{\Delta r}| = \sqrt{r_f^2 + r_i^2 - 2r_f r_i \cos\theta}$$

where,

$$r_f = r_i = r$$

$$\Delta r = \sqrt{r^2 + r^2 - 2r^2 \cos\theta}$$

$$= \sqrt{2}r\sqrt{(1-\cos\theta)}$$

$$= \sqrt{2}r\sqrt{2\sin^2(\theta/2)}$$

$$|\Delta\vec{r}| = 2r\sin(\theta/2)$$

$\because 1-\cos\theta$
$2\sin^2($

Special Point :-

If θ is very small $(\sin\theta \simeq \theta)$

so,

$$|\Delta\vec{r}| = 2r\sin\left(\frac{\theta}{2}\right)$$

$$= r\theta \longrightarrow \text{in radian}$$

Note

$$\Delta r = r_f - r_i$$

$$= r - r = 0$$

Note

$|\Delta\vec{r}| \rightarrow$ change in magnitude

$\Delta r \rightarrow$ change in magnitude.

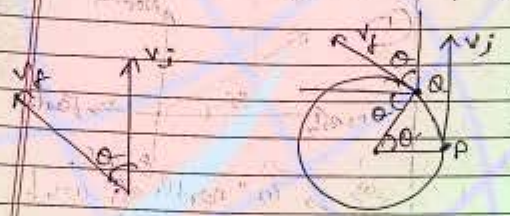
Q.44

Q) In the above question particle is moving with uniform speed "v" then magnitude of change in its velocity is

Sol

change in velocity $(\Delta\vec{v})$ = $\vec{v_f} - \vec{v_i}$

$$|\Delta\vec{v}| = |\vec{v_f} - \vec{v_i}|$$



$$v_f = v_i = v$$

$$|\Delta\vec{v}| = \sqrt{v_f^2 + v_i^2 - 2v_f v_i \cos\theta}$$

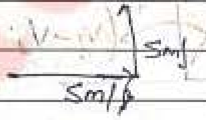
$$v_f = v_i = v$$

$$= 2v\sin\left(\frac{\theta}{2}\right)$$

Q.44

Q) A truck is moving towards east with speed of 5m/s after some time its velocity changes to 5m/s towards north find magnitude and direction of change in velocity.

Sol



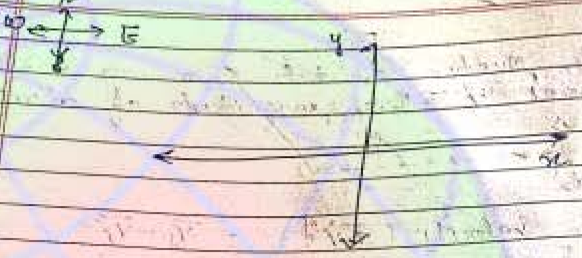
$5\sqrt{50}$

$\sqrt{\frac{10}{2}}$

$5\sqrt{2}$ m/s

$$\vec{\Delta v} = \vec{v_f} - \vec{v_i}$$

Ras , Qas

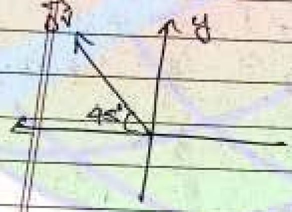


$\vec{\Delta v} = 5\,m/s(\hat{i}) - 5\,m/s\,\hat{j}$

$\vec{\Delta v} = (-5\hat{i} + 5\hat{j})\,m/s$

$|\vec{\Delta v}| = \sqrt{(-5)^2 + (5)^2}\,m/s$

$= 5\sqrt{2}\,m/s$



Rs

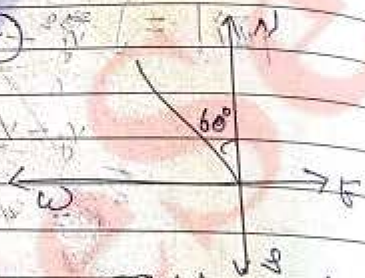
$\vec{\Delta v} = (5\sqrt{2}\,m/s)$ due N-W

Rule of direction in vector

① ,

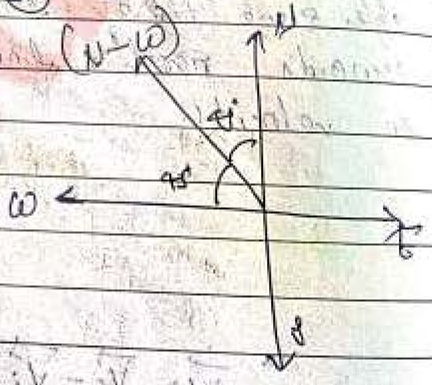
Ras → 30° north of west

②



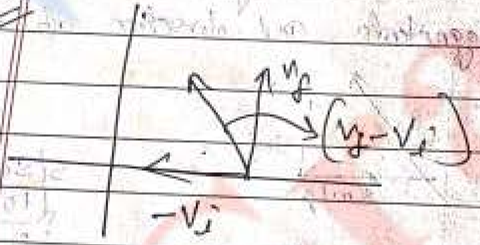
60° west of North

③



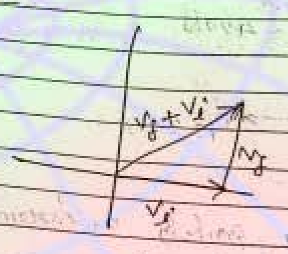
Note

Ras →

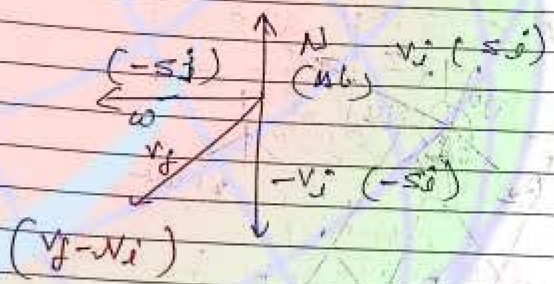


while

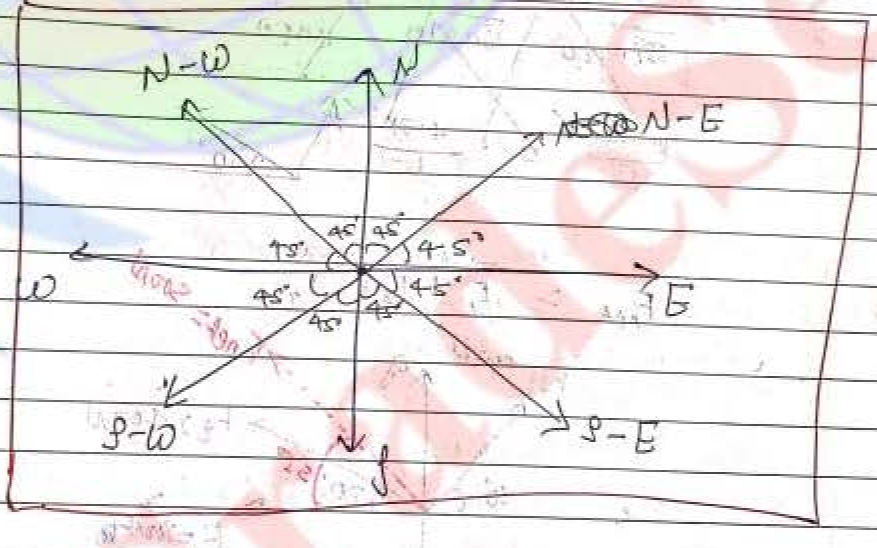
(1)



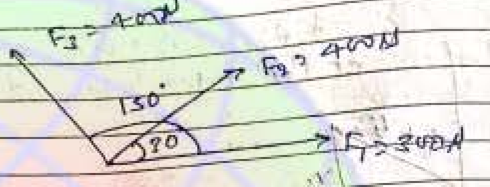
(2)



Note

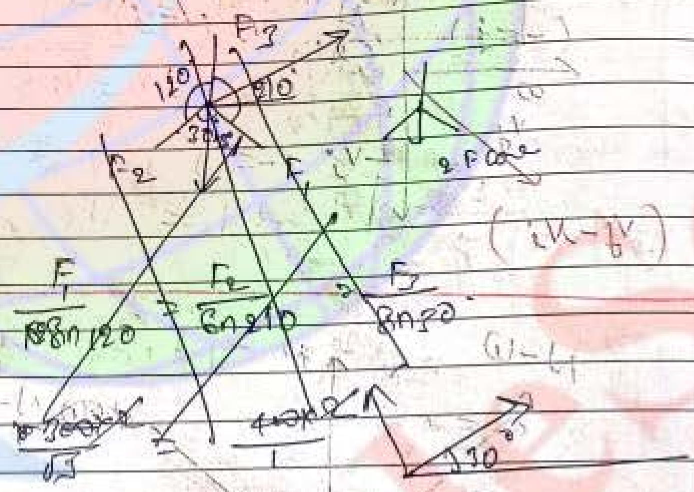


Q.)



3 forces are acting on a particle as shown in figure mag. of resultant for is

Sol



$$\frac{F_1}{\sin 120} = \frac{F_2}{\sin 210} = \frac{F_3}{\sin 90}$$

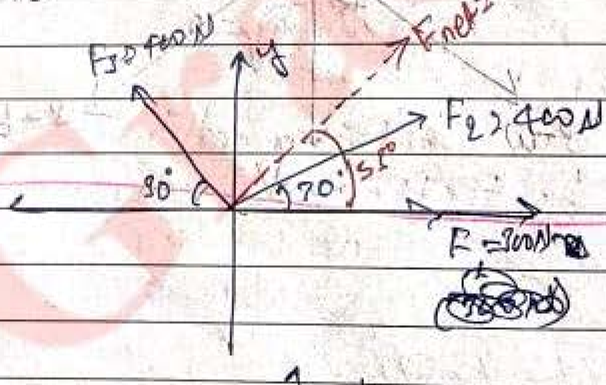
~~300~~ / √3 ~~400~~ / 1

180/150

6R5√3/4

Note

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$



$F_1 = 300\,\hat{i}\,N$

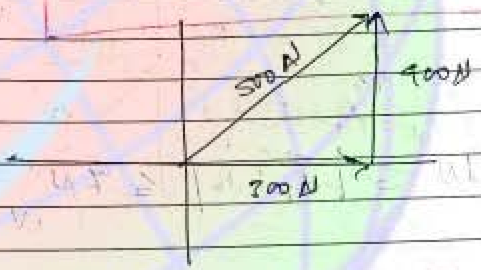
$F_2 = 400 \cos 30 \hat{i} + 400 \sin 30^\circ \hat{j}$

$\vec{F_1} = (-400 \cos 30 \hat{i} + 400 \sin 30^\circ \hat{j})$ N

$\vec{F_{net}} = (300 \hat{i} + 400 \hat{j})$ N

$|\vec{F_{net}}| = \sqrt{(300)^2 + (400)^2}$ N

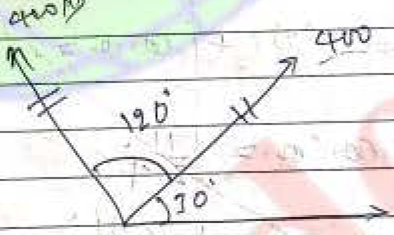
$= 500$ N



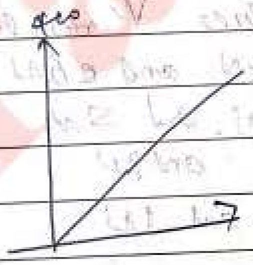
$\tan\theta = \frac{4}{3}$

$\theta = 53^\circ$

Note



$$\sqrt{(400)^2 + (400)^2 + 2(400)(400)\cos 120^\circ}$$



eg.) If $F_1 = 3N$ and $F_2 = 4N$ which of the following can not be the magnitude of resultant force
(i) 6N (ii) 5N
(iii) 8N (iv) 1N

S/n (iii)

Note

$$\boxed{|A-B| \le |\vec{A}+\vec{B}| \le A+B}$$

$$1N \le |\vec{F_1}+\vec{F_2}| \le 7N$$

Note

If $|\vec{F_1}+\vec{F_2}| = 6N$ (Squaring both)

$$3^2+4^2+2(3)(4)\cos\theta = 36$$

$$\cos\theta = \frac{11}{24}$$

$$\theta = \cos^{-1}\left(\frac{11}{24}\right)$$

Q.) which of the following set of coplanar forces can not produce resultant of zero magnitude
(i) 3N, 4N and 6N
(ii) 3N, 4N, and 5N
(iii) 3N, 4N, and 8N
(iv) 3N, 4N, and 1N

Sol^n

$1N \le 1N + 2N \le 3N$

So Ans = (1N)

Notes If F_1, F_2 and F_3 are co-planar then they can produce resultant of zero magnitude

$$F_1 - F_2 \le F_3 \le F_1 + F_2$$

ok

[illegible]

Q) If $\vec{A} + \vec{B} = \vec{C}$ where, A = 3unit

B = 4unit

C = 5unit

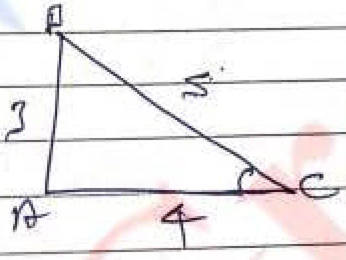
angle b/w $\vec{C}$ and $\vec{A}$

~~$9 + 16 + 2 \times 3 \times 4 \times \cos\theta = 25$~~

~~$\cos\theta = 0$~~

~~$\theta = \cos^{-1} 0$~~

~~$\theta = \pi/2$~~



$\vec{B} = |\vec{C} - \vec{A}|$

$B^2 = C^2 + A^2 - 2AC\cos\theta$

where θ = angle b/w $\vec{C}$ and $\vec{A}$

Gravitat

L-3 → Rama

L-4 → Ramesh

L-1 → 1 to 8, 11 to 21

L-2 → 1 to 4, 6 to 14

$$\cos\theta = \frac{c^2 + A^2 - B^2}{2AB}$$

$$\cos\theta = \frac{18}{50} = \frac{3}{5}$$

$$\theta = 53^\circ$$

Note

$$\boxed{1\,kgf = 10N}$$

Products of vectors

Date
Page

(i) Scalar product or dot product
(ii) vector product or cross product.

① Scalar product :-

$$\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$$
$$= AB\cos\theta$$

where
θ is angle b/w $\vec{A}$ and $\vec{B}$

* if $\theta = 0$, $\vec{A}\cdot\vec{B} = AB$
* If $\theta = 180°$, $\vec{A}\cdot\vec{B} = -A\cdot B$ VVI
if $\theta = \frac{\pi}{2}$, $\vec{A}\cdot\vec{B} = 0$ Imp

Scalar product of two vector quantities is = Scalar quantity

eg(1)
$\omega = \vec{F}\cdot\vec{s}$
$\omega = \vec{F}\cdot\vec{V}$

* $\vec{A}\cdot\vec{A} = |\vec{A}|\,|\vec{A}| = A^2$
* $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
* $\hat{j}\cdot\hat{j} = \hat{i}\hat{k} = \hat{k}\hat{i} = 0$
* $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

* if $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
and
$B = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

~~$\vec{A}\cdot\vec{B}$ = [illegible]~~

$$\boxed{\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y}$$

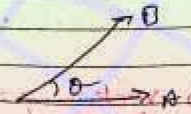
* $\vec{A}\cdot\vec{A} = |\vec{A}|\,|\vec{A}|$

~~$\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{A}$~~

Angle b/w arbitrary

~~$\cos\theta$~~

$$\boxed{\cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\;\sqrt{B_x^2 + B_y^2 + B_z^2}}}$$



[illegible]

① Comp of $\vec{A}$ along $\vec{B}$ = $A\cos\theta$

$= \frac{\vec{A}\cdot\vec{B}}{B}$

in vector form $= \left(\frac{\vec{A}\cdot\vec{B}}{B}\right)\cdot\hat{B}$

⑪ comp. of $\vec{B}$ along $\vec{A}$ = $B\cos\theta$

$= \frac{\vec{A}\cdot\vec{B}}{A}$

in vector form = $\frac{\vec{A}\cdot\vec{B}}{A}(\hat{A})$

Eg) if $\vec{A} = \hat{i}+\hat{j}+\hat{k}$

$\vec{B} = \hat{i}-\hat{j}+\hat{k}$

comp of $\vec{A}$ along $\vec{B}$ = ?

Sol comp of $\vec{A}$ along $\vec{B}$ = $\frac{\vec{A}\cdot\vec{B}}{B}$

$= \frac{\vec{A}\vec{B}}{B}$

$= \frac{1-1+1}{\sqrt{1^2+(-1)^2+1^2}}$

$= \frac{1}{\sqrt{3}}$

In vector form:

$\frac{1}{\sqrt{3}}\hat{B}$

$= \frac{1}{\sqrt{3}}\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

$= \frac{1}{3}(\hat{i}-\hat{j}+\hat{k})$

Note

① if $\vec{A} \perp \vec{B}$

$\vec{A} \cdot \vec{B} = 0$ → Note Two vectors are ⊥ dot product $\vec{A} \cdot \vec{B}$ ∵ $\cos 90 = 0$

$A_x B_x + A_y B_y + A_z B_z = 0$

Q.) if $|\vec{A}| = |\vec{B}|$

then

Angle b/w $(\vec{A} + \vec{B})$ and $|\vec{A} - \vec{B}|$

~~$2\cos^2 90$ ... $\cos\theta$~~

~~$2A^2 + 2A^2 \cos\theta = 0$~~

~~$(1 - \cos\theta) = 0$~~

~~$2\sin^2\frac{\theta}{2} = 0$~~

Sol^n

$$\cos\theta = \frac{(\vec{A} + \vec{B})(\vec{A} - \vec{B})}{|\vec{A} + \vec{B}|\,|\vec{A} - \vec{B}|}$$

$$\cos\theta = \frac{A^2 - \cancel{\vec{A}\vec{B}} + \cancel{\vec{B}\vec{A}} - B^2}{|\vec{A} + \vec{B}|\,|\vec{A} - \vec{B}|}$$

$$\cos\theta = \frac{A^2 - B^2}{|\vec{A} + \vec{B}|\,|\vec{A} - \vec{B}|}$$

if

Q.) if $\vec{A}$ $\vec{B}$

Q.) if

80|^n

if $|\vec{A}| = |\vec{B}|$

$\cos\theta = 0$

$\boxed{\theta = \frac{\pi}{2}}$ Ans

eg)

if $\vec{A} = \hat{i} + \hat{j} + \hat{k}$

$\vec{B} = -\hat{i} + \hat{j} + \hat{k}$

angle b/w $\vec{A}$ and $\vec{B}$ is

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{-1+1+1}{\sqrt{3}\sqrt{3}} = \frac{1}{3}$$

$\boxed{\theta = \cos^{-1}\left(\frac{1}{3}\right)}$ Ans

eg.) if $\vec{P} = (2\sin\omega t\,\hat{i} + 2\cos\omega t\,\hat{j})$ unit

ω = constant

t = time

P = linear momentum

angle b/w $\vec{F}$ and $\vec{P}$ is

80/n

$$\vec{F} = \frac{dp}{dt} = 2\cos\omega t\cdot\frac{d\theta}{dt} + \omega\cot$$

$$\vec{F} = 2\omega\cos\omega t\,\hat{i} - 2\omega\sin\omega t\,\hat{j}$$

$$\cos\theta = \frac{\vec{F}\cdot\vec{P}}{|\vec{F}||\vec{P}|} = \frac{F_xP_x + F_yP_y}{|\vec{F}||\vec{P}|} = \frac{0}{|\vec{F}||\vec{P}|}$$

$\theta = \frac{\pi}{2}$ Ans

(B) Vector product or cross product:-

vector product of two vector quantity is a vector quantity.

e.g. (i) $\vec{F} = q(\vec{V}\times\vec{B})$

(ii) $\vec{\tau} = \vec{r}\times\vec{F}$

(iii) $\vec{V} = \vec{\omega}\times\vec{r}$

* $\vec{A}\times\vec{B} = |\vec{A}|\,|\vec{B}|\,\cancel{co}\sin\theta\,\hat{n}$

and

$|\vec{A}\times\vec{B}| = |\vec{A}|\,|\vec{B}|\sin\theta$

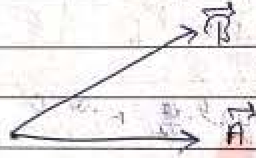
and

$\hat{n}$ ⇒ unit vector in the direction of $\vec{A}$

Note

(i) direction of $\hat{n}$ is determined by right thumb rule or screw rule

(ii)



$\vec{A}\times\vec{B}$ is directed ⊥r to the plane outward

and

$\vec{B}\times\vec{A} = (-)(\vec{A}\times\vec{B})$

and

$|\vec{B}\times\vec{A}| = |\vec{A}\times\vec{B}|$

(iii) $\vec{A} \times \vec{B}$ is $\perp r$ to the plane containing $\vec{A}$ and $\vec{B}$ both

$$\underbrace{(\vec{A}\times\vec{B})}\cdot\underbrace{\vec{A}} = 0$$

their two are $\perp r$

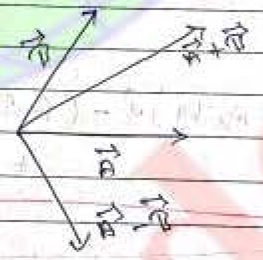
$$(\vec{A}\times\vec{B})\vec{B} = 0$$

$$(\vec{A}\times\vec{B})(\vec{A}+\vec{B}) = \vec{A}\times\vec{B}\cdot\vec{A} + \vec{A}\times\vec{B}\cdot\vec{B}$$

$$= 0 + 0$$

$$= 0$$

$$(\vec{A}\times\vec{B})\cdot(\vec{A}-\vec{B}) = 0 - 0 = 0$$

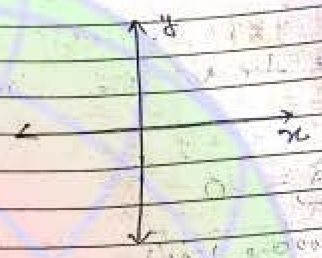


* $|\vec{A}\times\vec{A}| = 0$

* $|\hat{i}\times\hat{i}| = |\hat{j}\times\hat{j}| = |\hat{k}\times\hat{k}| = 0$

* $\hat{i}\times\hat{j} = \hat{k}$
$\hat{j}\times\hat{k} = \hat{i}$
$\hat{k}\times\hat{i} = \hat{j}$

RQ_b RQ



★ if $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{i} - (A_xB_z - A_zB_x)\hat{j} + (A_xB_y - A_yB_y$$

RQ

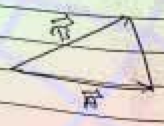
$$\boxed{|\vec{A} \times \vec{B}| = \sqrt{(A_yb_z - A_zb_y)^2 + (A_xA_z - A_zb_x)^2 + (A_xb_y - A}}$$

and

RQ_b →

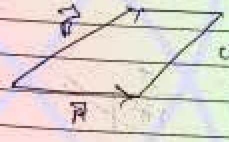
$$\boxed{\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}}$$

(i)



Area of triangle $= \dfrac{|\vec{A} \times \vec{B}|}{2}$

(ii)



Area of ||gm $= |\vec{A} \times \vec{B}|$

Components of a 3D-vector

① A = mag. of $\vec{A}$

α = angle b/w +x-axis and $\vec{A}$

β = " " +y " "

γ = " " +z " "

$A_x = A\cos\alpha$, $A_y = A\cos\beta$, $A_z = A\cos\gamma$

$\vec{A} = A(\cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k})$

and,

$\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

$$\boxed{\hat{A} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}}$$

Imp

Note

① $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called direction cosines

② if $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

then,

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

and

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

$$\hat{A} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

$$\cos\alpha = \frac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

$$\cos\beta = \frac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

$$\cos\alpha = \frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Note

$$\cos^2\alpha + \cos^2\beta + \cos^2\alpha = 1$$

eg) if $\vec{A} = \hat{i} - \hat{j} + \hat{k}$

then

Angle b/w $\vec{A}$ and +x-axis α

$$\cos\alpha = \frac{A_x}{|\vec{A}|} = \frac{1}{\sqrt{3}}$$

$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

eg) if $\vec{A} = \hat{i} + \hat{j} + \hat{k}$

and $\vec{B} = \hat{i} - \hat{j} + \hat{k}$

find a vector ⊥r to $\vec{A}$ and $\vec{B}$ both

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = 2\hat{i} - 2\hat{k}$$

$$|\vec{A} \times \vec{B}| = 2\sqrt{2}$$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$\hat{n} = \frac{\hat{i} - \hat{k}}{\sqrt{2}}$$

In general a vector ⊥r to $\vec{A}$ and $\vec{B}$ both $= \boxed{k}\,\hat{n}$

+ve or -ve

Imp Q.

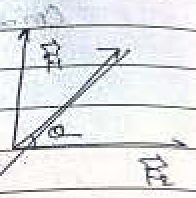
Sum of magnitude of two forces is 18 N and mag. of their resultant is 12N. Smaller force is ⊥r to the resultant force. find mag. of each force

Sol^n

~~$\vec{F_1} + \vec{F_2} = 18N$~~

~~$F_{res} = 12N$~~

~~$$\vec{F} \times \vec{F_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$~~



~~$F_1\cos\alpha$~~

~~$F_2\cos\alpha$~~

~~$F_1\cos\alpha + F_2\cos\theta$~~

marks प्रदायक की तीन eq. get करने की प्रक्रिया सर्वप्रथम होती है फिर करने के बाद eq^n सबको solve हो जाता है।

$$\vec{F_1} + \vec{F_2} = 18N \quad \text{—(i)}$$

and

$$|\vec{F_1} + \vec{F_2}| = 12N$$

$$F_1^2 + F_2^2 + 2F_1F_2\cos\theta = 144 \quad \text{—(ii)}$$

or

$$\vec{F_1} \perp (\vec{F_1} + \vec{F_2})$$

so

$$\vec{F_1} \cdot (\vec{F_1} + \vec{F_2}) = 0$$

$$F_1^2 + F_1F_2\cos\theta = 0 \quad \text{—(iii)}$$

θ = angle b/w $\vec{F_1}$ and $\vec{F_2}$

From eq(2) and (3)

~~$F_1^2 + F_2^2 +$~~ [illegible]

$$F_1^2 + F_2^2 + 2(-F_1^2) = 144$$

$$\left[F_1 F_2 \cos\theta = -F_1^2 \right]$$

$$F_2^2 - F_1^2 = 144$$

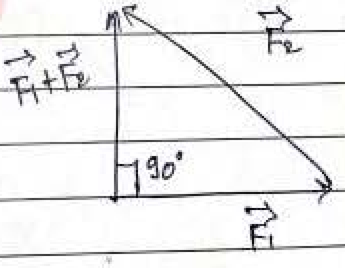
$$(F_2 - F_1)(F_2 + F_1) = 144$$

$$(F_2 - F_1)(\cancel{18}) = \overset{8}{\cancel{144}}$$

$$F_2 - F_1 = 8 \quad \longleftarrow (v)$$

solving eq(i) and (v)

$F_1 = 5\,N$
$F_2 = 13\,N$



Motion in One Dimensional

Distance and Displacement →

1D → motion in straight line
2D → motion in plane
3D → motion in space

Distance:-

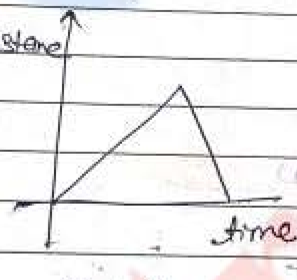
- → Scalar quantity
- → S.I unit metre
- → Dimension ⇒ $[M^0 L^1 T^0]$

(i) Actual length of path travells by particle is called Distance travelled by it.

(ii) It can never be -ve.

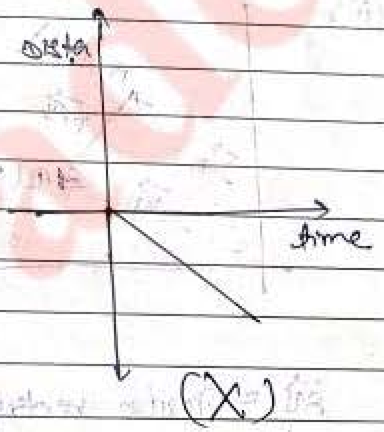
(iii) with increase in time distance can never decrease

(iv) Distance - time graph



(X)

(Not possible)



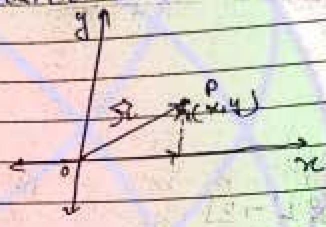
(X)

(Not possible.

(ii) ~~Displacements~~

(iii) Position vector and displacement vectors

(i) Position vector

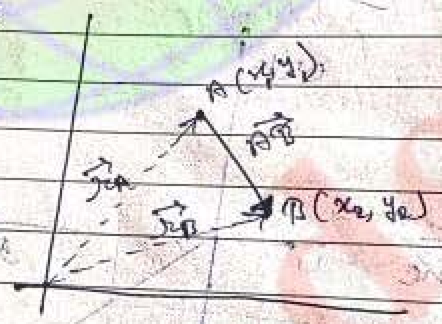


Position vector of P w.r.t. O $(\vec{OP}) = \vec{r} = x\hat{i} + y\hat{j}$

And

$|\vec{r}| = \sqrt{x^2+y^2}$

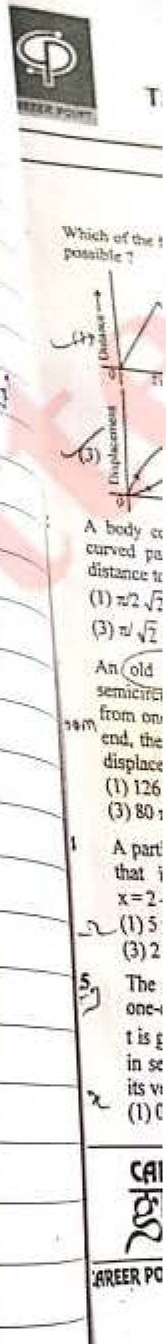
(ii)



$\vec{AB}$ (= Position vector of $\vec{B}$ w.r.t. A

but

$\vec{r}_A + \vec{AB} = \vec{r}_B$



$$\boxed{\vec{AB} = \vec{r}_B - \vec{r}_A}$$

$$\vec{AB} = (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$$

$$\boxed{\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}}$$

and,

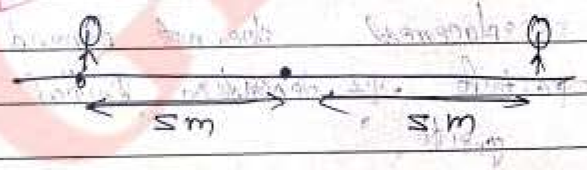
$$\boxed{|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

(ii) Displacement :-

- Vector quantity.
- SI unit : m
- Dimensions :- $[M^0L^1T^0]$

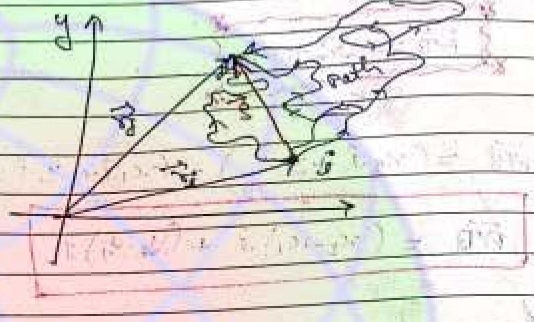
* change in position is defined as displacement

$$\boxed{\vec{S} = \vec{r}_f - \vec{r}_i}$$



L-1 → Com-AER

L-2 →



* shortest distance b/w final and Initial position is equal to magnitude of displacement and it is directed from initial position to final position

$$\vec{S} = \vec{r}_f - \vec{r}_i$$

$$\vec{S} = (x_f\hat{i} + y_f\hat{j}) - (x_i\hat{i} + y_i\hat{j})$$

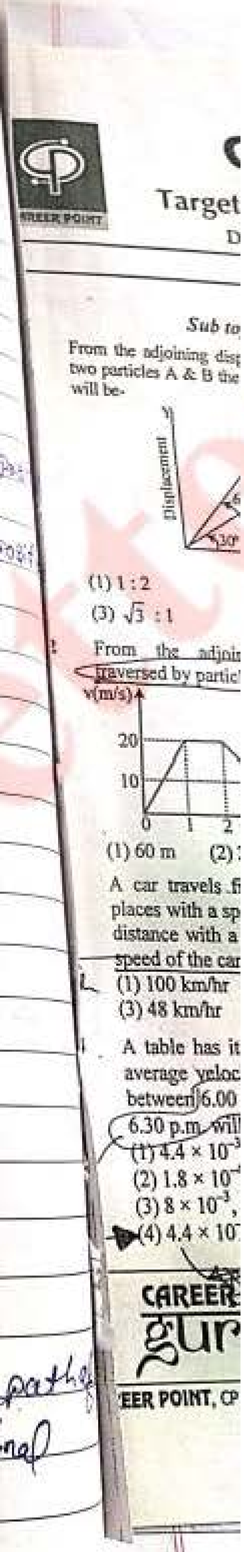
$$\vec{S} = (x_f - x_i)\hat{i} + (y_f - y_i)\hat{j}$$

or

$$\boxed{\vec{S} = \Delta x\hat{i} + \Delta y\hat{j}}$$

$$\boxed{|\vec{S}| = \sqrt{(\Delta x)^2 + (\Delta y)^2}}$$

* Displacement does not depend on path of the particle. It depends on Initial and final position.


Target

Sub to

From the adjoining disp two particles A & B the will be-

(1) 1 : 2

(3) $\sqrt{3}$: 1

From the adjoin traversed by partic

(1) 60 m (2)

A car travels fi places with a sp distance with a speed of the car

(1) 100 km/hr

(3) 48 km/hr

A table has it average veloc between 6.00 6.30 p.m. will

(1) 4.4 × 10

(2) 1.8 × 10

(3) 8 × 10

(4) 4.4 × 10

- If there is no change in direction of motion during the given time interval then

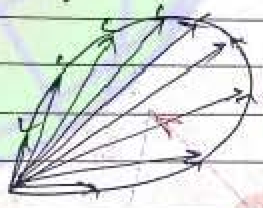
$$|disp| = \text{distance travelled}$$

- If direction of motion changes

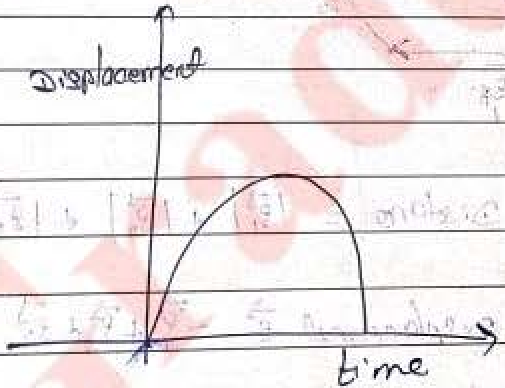
$$|displacement| < \text{distance travelled}$$

- displacement can never be greater than distance travelled.

- |displacement| may decrease with increase in time.



Note

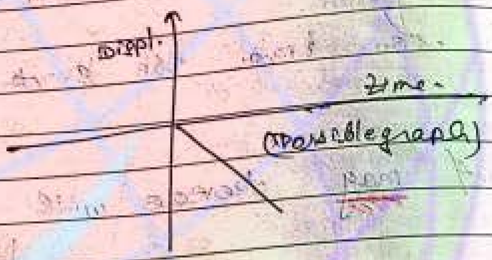


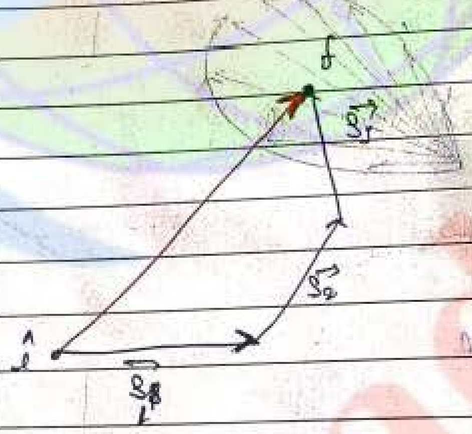
[illegible]

Possible

$\vec{S_A} = +5m$

$\vec{S_B} = -5m$



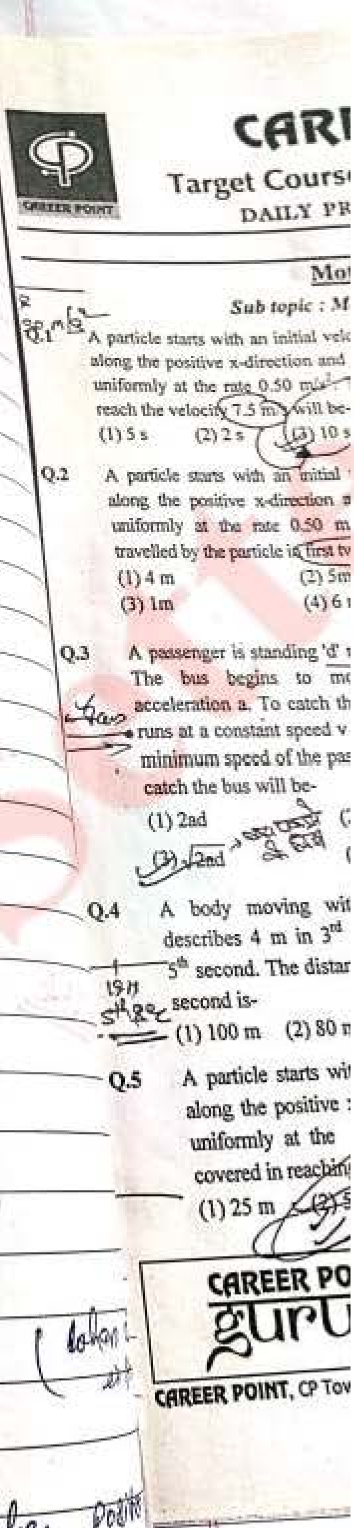


(i) Distance = $|\vec{S_1}| + |\vec{S_2}| + |\vec{S_3}|$

(ii) displacement $\vec{S}$ = $\vec{S_1} + \vec{S_2} + \vec{S_3}$ (when

Note

$\vec{S} = \vec{r_f} - \vec{r_i}$ (when Positi known



CARI

Target Cours

DAILY PR

Mot

Sub topic : M

Q.1 A particle starts with an initial velo along the positive x-direction and uniformly at the rate 0.50 m/s². reach the velocity 7.5 m/s will be-
(1) 5 s (2) 2 s (3) 10 s

Q.2 A particle starts with an initial along the positive x-direction uniformly at the rate 0.50 m travelled by the particle in first tw
(1) 4 m (2) 5m
(3) 1m (4) 6

Q.3 A passenger is standing 'd' The bus begins to mo acceleration a. To catch th runs at a constant speed v minimum speed of the pas catch the bus will be-
(1) 2ad
(3) $\sqrt{2ad}$

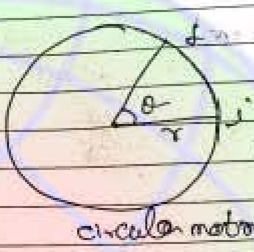
Q.4 A body moving wi describes 4 m in 3rd 5th second. The distan second is-
(1) 100 m (2) 80 m

Q.5 A particle starts wi along the positive uniformly at the covered in reaching
(1) 25 m (2) 5

CAREER PO
guru

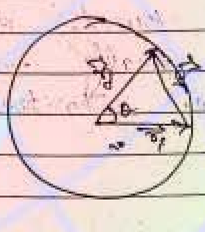
CAREER POINT, CP Tov

a)



circular motion.

$|disp| = ?$



$|\vec{s}| = |\vec{r_2} - \vec{r_1}|$

$|\vec{s}| = \sqrt{r^2 + r_2^2 - 2r^2\cos\theta}$

$= 2r\sin(\theta/2)$

and

Distance travelled $= r\theta$
θ is in radian

Special Case :-

(i) $\frac{1}{4}$ circle $(\theta = \pi/2)$

distance $= \frac{\pi r}{2}$ and $|\text{Displacement}| = \sqrt{2}r$

(ii) $\frac{1}{2}$ circle $(\theta = \pi)$

distance $= 2r$

$|\text{displace}| = 2r$

(iii) Complete circle $(\theta = 2\pi)$

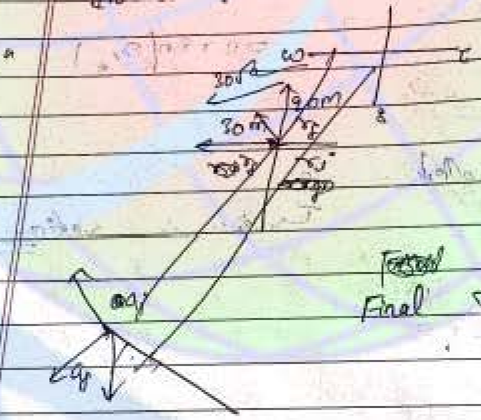
~~displacement = 2πr~~

distance $= 2\pi r$

displacement $= 0$

Q) A person moves 30m towards east then 20m towards north and finally $30\sqrt{2}$ m towards south west. find its final position w.r.t. initial position

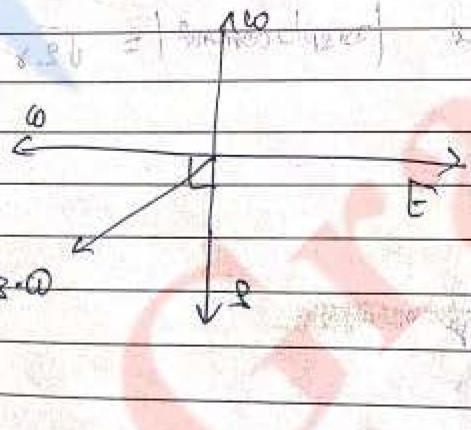
sol



Final position w.r.t. initial position

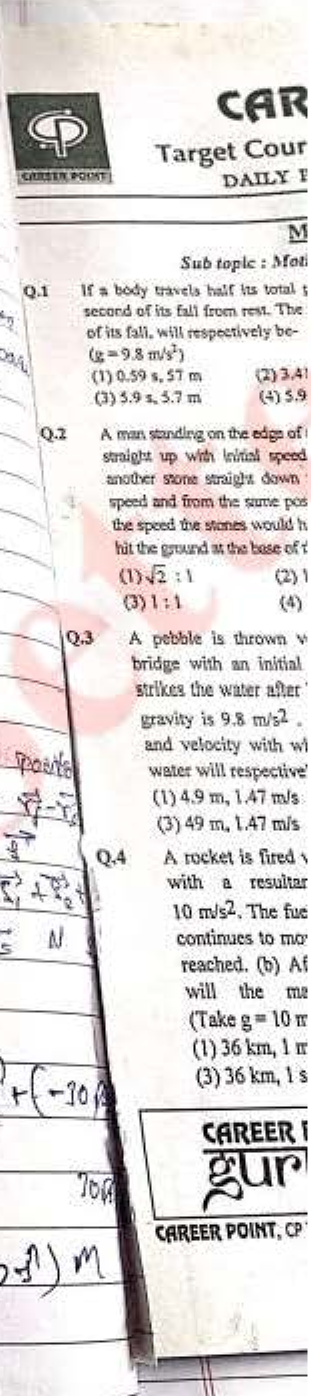
$= \vec{r}_f - \vec{r}_i$

$= \vec{r}_1 + \vec{r}_2 + \vec{r}_3$



$\vec{r} = 30\hat{i} + 20\hat{j} + (-30\hat{i}$

$= (-10\hat{j})$ m



CAR

Target Cour

DAILY

M

Sub topic : Mot

Q.1 If a body travels half its total
second of its fall from rest. The
of its fall, will respectively be-
$(g = 9.8 \text{ m/s}^2)$
(1) 0.59 s, 57 m (2) 3.4
(3) 5.9 s, 5.7 m (4) 5.9

Q.2 A man standing on the edge of
straight up with initial speed
another stone straight down
speed and from the same pos
the speed the stones would h
hit the ground at the base of
(1) $\sqrt{2}$: 1 (2)
(3) 1 : 1 (4)

Q.3 A pebble is thrown
bridge with an initial
strikes the water after
gravity is 9.8 m/s^2 .
and velocity with w
water will respective
(1) 4.9 m, 1.47 m/s
(3) 49 m, 1.47 m/s

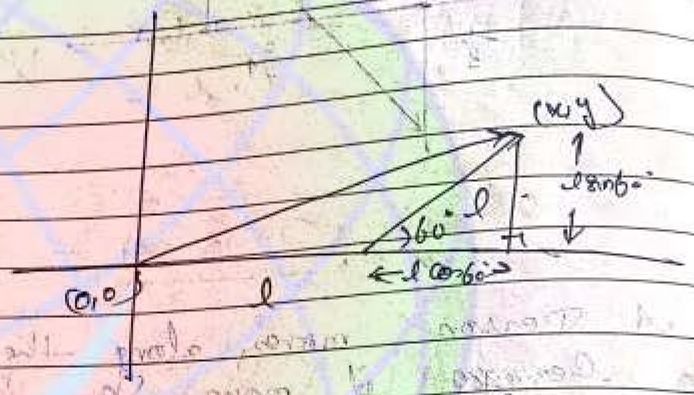
Q.4 A rocket is fired
with a resultan
10 m/s^2. The fue
continues to mo
reached. (b) Af
will the ma
(Take g = 10 m
(1) 36 km, 1 m
(3) 36 km, 1 s

CAREER
gur

CAREER POINT, CP

Sol

$\vec{S} = \vec{r_f} - \vec{r_i}$



$\vec{S} = \sqrt{x^2+y^2}$

where

$x = l + \frac{l}{2}$

$y = \frac{\sqrt{3}l}{2}$

$S = \sqrt{3}l$

Speed and velocity

① Speed :-

↳ scalar quantity

S.I unit = m/s

[Speed] = $[M^0L^1T^{-1}]$

(i) Average speed (V_{avg})

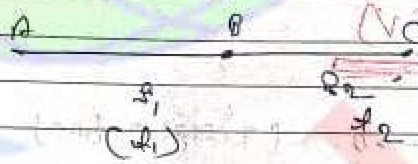
$$\boxed{V_{avg} = \frac{\text{total distance travelled}}{\text{total time taken}}}$$

* Cases :-

(i)

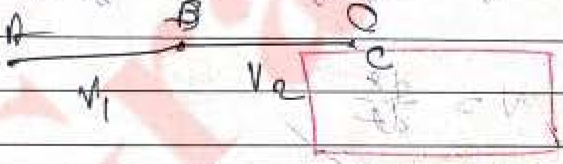
$$V_{avg} = \frac{s}{t}$$

(ii)



$$V_{avg} = \frac{s_1+s_2}{t_1+t_2}$$ (from A to C

(iii)



$\cancel{v_1+v_2 = \frac{s_1}{t_1} + \frac{s_2}{t_2}}$ $\cancel{= \frac{s_1t_2 + s_2t_1}{t_1t_2}}$

(Not able to find V_{avg} because Incomplete Informed

RA

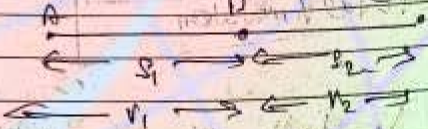
distance = $v_1 t_1$

distance = $v_2 t_2$

RA (iv)

$$V_{avg} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}$$ (from above)

(v)



$$V_{avg} = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}} \rightarrow \frac{(s_1+s_2)\, v_1 v_2}{s_1 v_2 + s_2 v_1}$$

Instantaneous speed (V)

$\Delta t \rightarrow 0$ (change in time)

at t = ~~[illegible]~~ (time at particular instant)

V → Rate of change of distance w.r.t time

$$\boxed{V = \frac{ds}{dt}}$$

3) Velocity :-

→ vector

→ SI un

→

(i) average

V_{avg}

Note

①

②

Rq

(B) Velocity :-

- → vector quantity
- → SI unit => m/s
- → [velocity] = $[M^0 L T^{-1}]$

(i) average velocity ($\vec{V}_{avg}$) :-

$$\boxed{\text{Avg velocity } (\vec{V}_{avg}) = \frac{\text{Total displacement}}{\text{total time taken}}}$$

Note

① If direction of motion does not change then

$$\boxed{|\text{Avg velocity}| = \text{Avg speed}}$$

$$\boxed{|\vec{V}_{avg}| = V_{avg}}$$

② If direction of motion changes

$$|\vec{V}_{avg}| < V_{avg}$$

average speed

Rq

(ii) $|\vec{V}_{avg}|$ can not be greater than V_{avg}

(iv) $|\vec{V}_{avg}|=0$

→ when particle does not move than

$V_{avg}=0$

→ when initial and final position of particle is same than

$$\boxed{V_{avg} \neq 0}$$

so,

Case 1st ⇒ we say that if $|\vec{V}_{avg}|=0$ then

$$\boxed{V_{avg} \text{ may or may not be zero}}$$

Case 2nd ⇒ if $V_{avg}=0$ then

$$\boxed{|\vec{V}_{avg}|=0}$$

✡ Instanta

✡ Spe

(i)

Rg

Instantaneous velocity

$\vec{V}$ = Rate of change of position w.r.t. time

$$= \left(\frac{d\vec{r}}{dt}\right)$$

$\boxed{\text{OR}}$

= Rate of change of displacement w.r.t. time,

$$= \left(\frac{d\vec{s}}{dt}\right)$$

Special cases

(1) If motion is along ~~[illegible]~~ x-axis :-

(a) $\vec{V} = \frac{dx}{dt}$

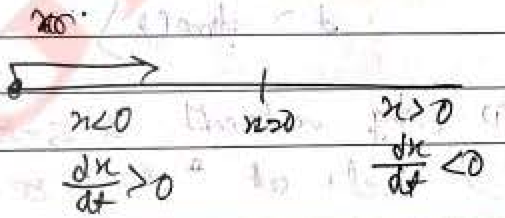
(b) $\vec{V}_{avg} = \frac{\Delta x}{\Delta t}$

Note

$x \rightarrow$ Represent location

$\frac{dx}{dt}$ $\xrightarrow{\text{Represent}}$ direction of motion

Note:



Rg

(2) If motion takes place in x-y plane

or $\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$

where $\frac{dx}{dt} = v_x$ = velocity along x-axis

and $\frac{dy}{dt} = v_y$ = velocity along y-axis

Note

(a) Instantaneous speed = $|\vec{v}|$

$$v = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

(b) $\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{dt}\hat{j}$

A particle moves along x-axis i.e.

$x = (t^2)$ m

x = Position (m)

t = time (s)

find

(i) Avg velocity at 5-sec

(ii) inst. vel. at t=5 sec

a) $\vec{V}_{avg} = \frac{\Delta x}{\Delta t}$

$= \frac{x_{t=5} - x_{t=0}}{\Delta t} = \frac{25-0}{5} = 5 m/s$

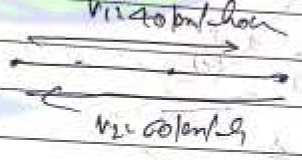
b) $\vec{V} = \frac{dx}{dt}$

$\vec{V} = (2t)$ m/s

$\vec{V}_{t=5} = 10 m/s$

⑪ A person travells from kota to ... with avg speed of 40 km/hour and return's back from the same path with avg. speed of 60 km/h. The avg. speed in comp journey

sol.



Avg = 40

1 h - 40 km

&

D = $\frac{v}{t}$

$V_{avg} = \frac{\Delta s}{\Delta t}$

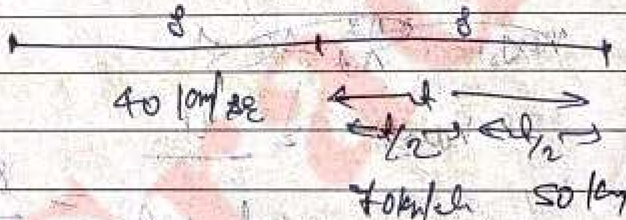
$V_{avg} = \frac{2d}{\frac{d}{40} + \frac{d}{60}} = \frac{2(40)(60)}{860+400} = 48 km/h$

$\vec{V}_{avg} = 0$

Q) A person travels ~~1st~~ half of total dist... with Avg speed of 40 km/h and next ... ~~start~~ at time t.

~~In first half of time "t" is Avg speed 40 km/h and in next half is Avg speed is 50 km/hr~~
~~Find Avg. speed for complete journey~~

Teacher



Vavg

$$V_{avg} = \frac{70(t/2) + 50(\frac{t}{2}}{t}$$

Q) A person travels south

fen

$V'_{avg} = \frac{2s}{\frac{s}{40}+\frac{s}{60}} \Rightarrow \frac{2\cancel{s}\times 2400}{100\cancel{s}}$

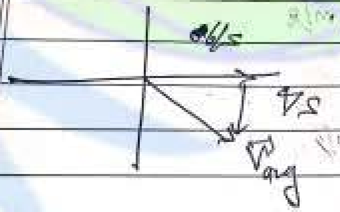
$= 480$

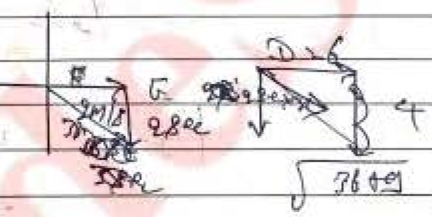
$= 48$ km/h

Q) A person moves with unifor speed of 2m/s but it travels towards east for 3 sec then towards south for two second $\vec V_{avg}$?

$|v_1+v_2|$

$s = 2m/s$





$\tan\theta = \frac{4/5}{6/5} = \frac{2}{3}$

$\theta = \tan^{-1}\left(\frac{2}{3}\right)$ Ans

$\Rightarrow \frac{6+4}{5}$

$= \frac{6\hat i - 4\hat j}{5}$

$= \sqrt{\left(\frac{6}{5}\right)^2+\left(\frac{4}{5}\right)^2}$

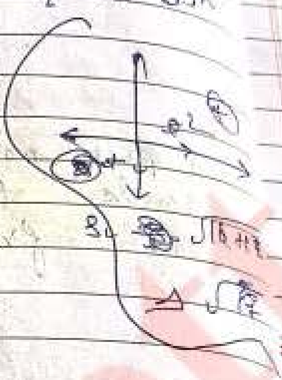
$= \frac{1}{5}\sqrt{52}$ m/s.

Q) A particle move in x-y plane $x = t^2$ and
$x, y \rightarrow$ position (m)
$t \rightarrow$ time (s)

Speed of the particle at $t = 2$ sec is $y = t$ m

Sol" Teach

$v_x = \frac{dx}{dt} = 2t =$
and
$v_y = 1$ m/s



Note

$\therefore$ Net $\vec{v} = (2t\hat{i} + \hat{j})$ m/s

$\vec{V}_{t=2} = (4\hat{i} + \hat{j})$ m/s

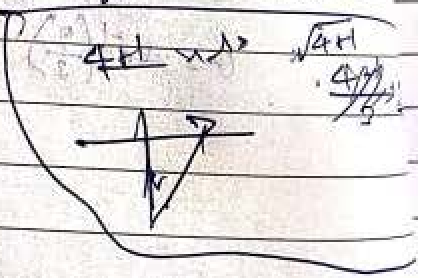
$|\vec{V}_{t=2}| = \sqrt{4^2 + 1^2}$ m/s

$= \sqrt{17}$ m/s

Q) In the above question average velocity in 2 sec is

Teach

$\frac{\Delta x}{\Delta t} = \frac{x_{t=2} - x_{t=0}}{2}$



$= \frac{4-0}{2} = 2\ m/s$

$\frac{\Delta y}{\Delta t} = \frac{2-0}{2}$

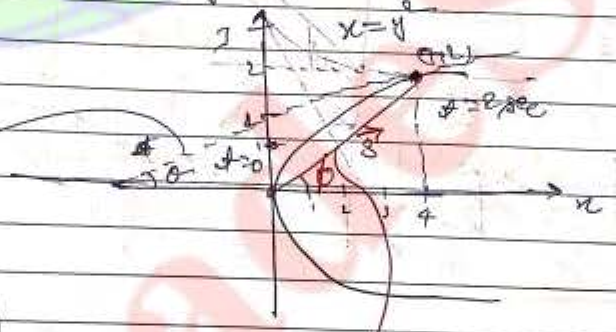
$V_{avg} = (2\hat{i} + \hat{j})\ m/s$

$|V_{avg}| = \sqrt{2^2+1^2}\ m/s = \sqrt{5}\ m/s$

Note

$x = t^2$ and $y = t$

$\therefore x = y^2$



$\tan\theta = \frac{1}{4}$

$\theta = \tan^{-1}\left(\frac{1}{4}\right)$

slope of target

$\frac{dy}{dx}$

$\tan\phi = \frac{1}{2}$

$\phi = \tan^{-1}\left(\frac{1}{2}\right)$

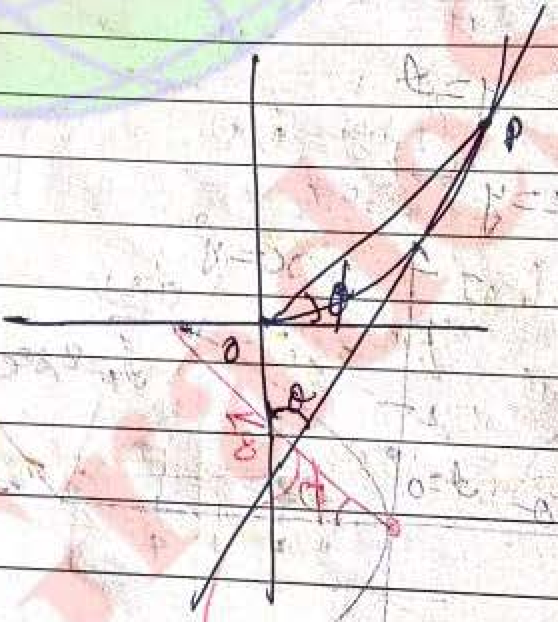
slope of chord $= \frac{\Delta y}{\Delta x}$

(i) $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\left(\frac{dy}{dx}\right)_{x=4} = \frac{1}{4}$$

(ii) $\frac{\Delta y}{\Delta x} = \frac{2-0}{4-0} = \frac{1}{2}$



Acceleration

→ vector quantity
→ SI unit → m/s²
$[acc] = [M^0 L^1 T^{-2}]$

- Average Acc^n ($\vec{a}_{avg}$)

$$\boxed{\vec{a}_{avg} = \frac{\text{change in vel } (\Delta \vec{v})}{\text{time interval } (\Delta t)}} = \boxed{\frac{\vec{v}_f - \vec{v}_i}{\Delta t}}$$

- Instantaneous Acc^n

$\vec{a}$ = rate of change of vel. w.r.t. time

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt}}$$

$$\boxed{= \frac{d^2\vec{r}}{dt^2} \text{ or } \frac{d^2 s}{dt^2}}$$

$$\boxed{v = \frac{dr}{dt} \text{ or } \frac{ds}{dt}}$$

SP case : 7

① If particle moves along x-axis

$$\vec{a}_{avg} = \frac{\Delta V_x}{\Delta t} \quad \text{where } V_x = \frac{dx}{dt}$$

and

$$\boxed{\vec{a} = \frac{dV_x}{dt} = \frac{d^2x}{dt^2}}$$

ii) If particle moves in xy plane

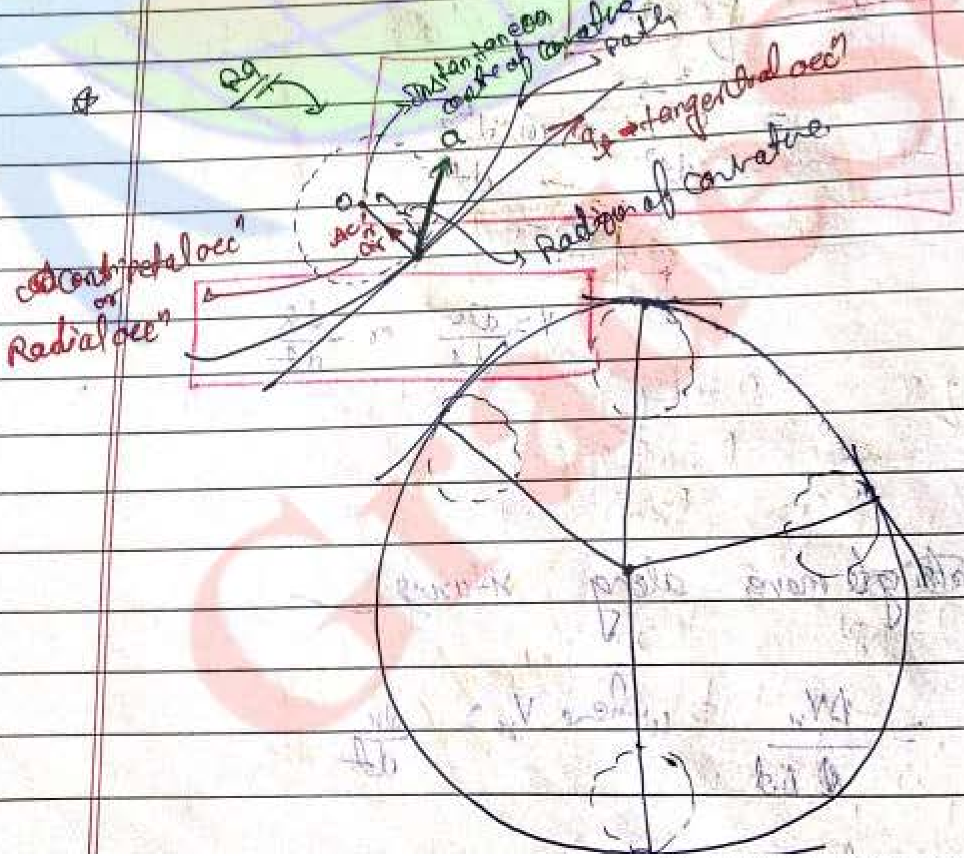
$$\boxed{\vec{a}_{avg} = \frac{\Delta V_x \hat{i}}{\Delta t} \left(+ \frac{\Delta V_y \hat{j}}{\Delta t} \right)}$$

$$\vec{a} = \frac{dV_x}{dt}\hat{i} + \frac{dV_y}{dt}\hat{j}$$

$$= \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

$$= a_x\hat{i} + a_y\hat{j}$$

Q.



a_t → tangential acc^n

a_c → ~~centr~~ centripetal acc^n

Notes

Centripetal acc^n is directed towards the Instantaneous centre of curvature.

(I) $a_t \Rightarrow$ rate of change of speed w.r.t. time

$$\boxed{a_t = \frac{d|\vec{v}|}{dt}}$$

(II) $$\boxed{a_c = \frac{v^2}{r}}$$

v = inst. speed

r = inst. radius of curvature.

Notes

~~Note~~ ~~$\vec{v}$~~ ~~velocity~~

$\vec{v}$ = velocity

$|\vec{v}|$ = mag. of velocity or speed

(III)

(iii) $a = \frac{d\vec{v}}{dt}$

$\vec{a} = \vec{a}_c + \vec{a}_t$

and

$a = \sqrt{a_c^2 + a_t^2}$

$$\boxed{\left|\frac{d\vec{v}}{dt}\right| = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{d|\vec{v}|}{dt}\right)^2}}$$

~~[illegible]~~ Special cases:

① If ~~a~~ particle move along straight line:

$a_c = 0$

i.e. $\boxed{a = a_t}$

Eg For a particle moving in a xy plane

$x = 2 \sin 2t$

$y = 2 \cos 2t$

when

x, y are position (m)

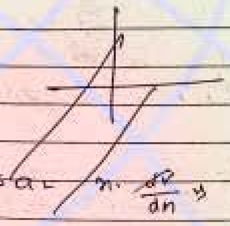
t is time (sec)

find i) ~~[illegible]~~ acc^n

ii) ~~[illegible]~~ tangential acc^n

(iii) ~~a_c~~ Centripetal acc^n

Q.4



~~a_c~~ $a_c = x \cdot \frac{dr}{dn} y$

$y = a \cdot \frac{dr}{dt}$

$\frac{dy}{dx}$

$\frac{x}{y} = 2\tan 2\theta$

$\frac{d}{dt}\tan 2\theta$

$\tan 2\theta = \frac{x}{2y}$

(i) $\frac{dx}{dt} = 4\cos 2\theta$ and

$\frac{dy}{dt} = -4\sin 2\theta$

Now

$\frac{d^2x}{dt^2} = -8\sin 2\theta$ and

$\frac{d^2y}{dt^2} = -4\times 2\cos 2\theta = -8\cos 2\theta$

$\vec{a} = (-8\sin 2\theta\,\hat{i} - 8\cos 2\theta\,\hat{j})$

or

$|\vec{a}| = 8\ m/s^2$

(ii) $a_t = \frac{d}{dt}|\vec{v}|$

$\vec{v} = 4\cos 2\theta\,\hat{i} - 4\sin 2\theta\,\hat{j}$

$|\vec{v}| = 4\ m/s$

$\frac{d|\vec{v}|}{dt} = 0$ so $a_t = 0$

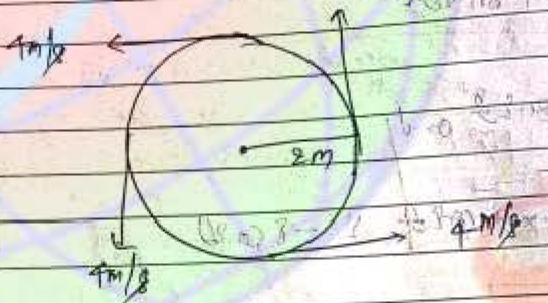
(iii) $a_c = \sqrt{a^2 - a_t^2}$

$\because a_t = 0$

$a_c = a = 8\,m/s^2$

(iv) $x^2 + y^2 = 2^2$

$\therefore (|\vec{v}| = 4\,m/s)$



$a_c = \frac{v^2}{r} = \frac{4^2}{2} = 8\,m/s^2$

Note

① Uniform circular motion

circular motion with ~~uniform~~ uniform speed

② Non-uniform circular motion

circular motion with non-uniform speed

Ra

Ra

(iii) uniform motion or uniform velocity

No change in speed and no change in direction of motion.

$\boxed{\text{Acc}^n \text{ in this case} = 0}$

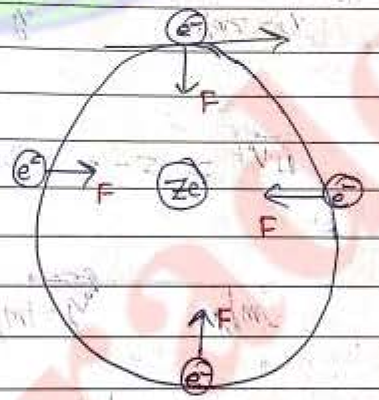
(iv) uniformly Accelerated motion

No change in mag. of acc^n and No change in direction of acc^n!

Note

$$\boxed{\vec{a} = \frac{\vec{F}}{m}}$$

Note eq^n of motion is valid only in case of uniformly accelerated motion



$$\boxed{F = \frac{K(Ze)(e)}{r^2}}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

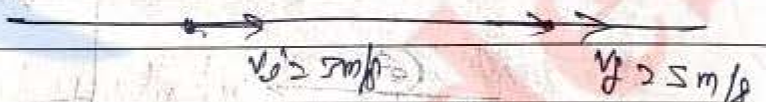
Note

$d\vec{v}$ → small change in velocity

$\vec{v}$ → velocity

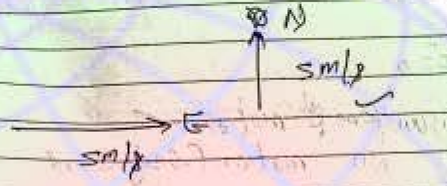
$\Delta\vec{v}$ → change in velocity in considerable time interval

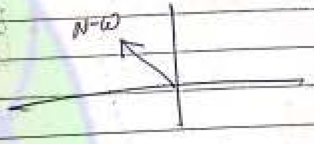
$d\vec{v}$ → instant



$\Delta\vec{v} = 5 - 3 = 2 m/s$

$\Delta V = -1m$





$\Delta V = 5\hat{i} - 5\hat{j}$

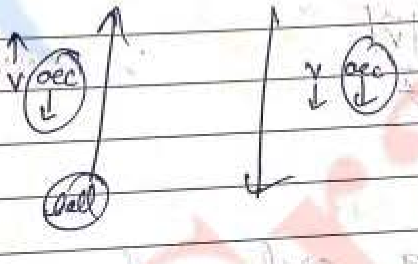
$|\Delta \vec{V}| = 5\sqrt{2}$ m/s

Note

$\vec{a} = \frac{d\vec{v}}{dt}$ → दोनों का direction same होना चाहिए

But

dire of $d\vec{v}$ direct of $\vec{a}$
{may or may not same}



Acc^n is not change

V is change

(Acc^n always in downward directn)

① Zero acc^n : I

if acc^n = 0

i) uniform ~~@~~ motion

ii) No motion (body is at rest)

we can say that

if body is either in rest or in uniform motion then acc^n is zero

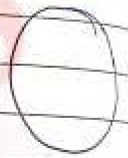
Q) A particle moves in a circle with uniform speed which of the foll. parameter is zero

(a) acc^n

(b) centripetal acc^n

(c) tangental acc^n

(d) None of these.



sol^n (c) tangental acc^n

$$a_t = \frac{d|\vec{v}|}{dt}$$

$\because |\vec{v}| = 0$

$= 0$

Q) A particle moves in a circle with non-uniform speed, which of the foll. parameter is zero

(a) acc^n ~~[illegible]~~ (c) ③

(b) centripetal acc^n

(c) tangental acc^n

(d) none of these

Ch (D) none of the

Q) For a particle moving along x-axis $V = \alpha\sqrt{x}$

α is constant $(\alpha > 0)$

$x \rightarrow$ Position $(x \geq 0)$

~~vel~~ $V \rightarrow$ velocity

Initially (t=0) particle is at $x=0$

Concept:

Concept that in all cases not take at initial zero all time any item by without understanding

Note

$\boxed{t=0}$ means when we start observation or those time when

Kota — Bundi — Jaipur

$x=0$ $x=45$ km $x=245$ km

Bus → $V=0$ $\rightarrow V=40$ km/h

A	B
In this case	in this case
at	at
$t=0$	$t=0$
$V=0$	$V=40$ km/h
	$x=45$

find
(i) vel. at any time 't'
(ii) distance travelled in time t
(iii) accn

8/a

$v = \alpha\sqrt{x}$

$a = \frac{dv}{dt}$

$= \alpha \times \frac{1}{2} \frac{}{\sqrt{x}}$

$= \frac{\alpha}{2\sqrt{x}}$

(i) $\frac{dx}{dt} = \alpha\sqrt{x}$

$$\int_{x=0}^{x} \frac{dx}{\sqrt{x}} = \int_{t=0}^{t} \alpha\, dt$$

$2\sqrt{x} = \alpha t$

$x = \frac{\alpha^2 t^2}{4}$

dist in time $= \frac{\alpha^2 t^2}{4}$

(iii) $\frac{dx}{dt} = \frac{\alpha^2 t}{2}$

$v = \frac{\alpha^2 t}{2}$

vector quantity = { Disp, vel, acc^n, time }

acc $a = \frac{dv}{dt}$

$\alpha = \frac{\alpha^2}{2}$

Note

$$\boxed{a_x = \frac{dv_x}{dt}}$$

$$a_x = \frac{dx}{dt} \cdot \left(\frac{dv_x}{dx}\right)$$

$$\boxed{a_x = v_x \cdot \frac{dv_x}{dx}}$$

$$\boxed{a = \frac{dv}{dt} = v \cdot \frac{dv}{dx}}$$

In the given question

$$v = \alpha\sqrt{x}$$

$$\frac{dv}{dx} = \frac{\alpha}{2\sqrt{x}}$$

$$a = v\frac{dv}{dx} = \alpha\sqrt{x} \quad 2\alpha\cancel{\sqrt{x}} \cdot \frac{\alpha}{2\cancel{\sqrt{x}}}$$

$$= \frac{\alpha^2}{2}$$

ex. For a particle moving along x-axis

$t = \alpha x^2 + \beta x$

$\alpha, \beta \longrightarrow$ constants $\begin{cases} \alpha > 0 \\ \beta > 0 \end{cases}$

$x \rightarrow$ Position

$t \rightarrow$ time

Retardation is ____

S/n

Note

~~$a = \frac{dv}{dt} = v\frac{dv}{dx}$~~

~~$v = \frac{dx}{dt}$~~ ~~$\frac{dt}{dx} = \frac{1}{v}$~~

$$\frac{dt}{dx} = 2\alpha x + \beta$$

$$\frac{dx}{dt} = \frac{1}{(2\alpha x + \beta)}$$

$$v = \frac{1}{(2\alpha x + \beta)}$$

$$\frac{dv}{dx} = \frac{-1}{(2\alpha x + \beta)^2} \cdot 2\alpha$$

$$a = v\frac{dv}{dx}$$

$$a = \frac{-2\alpha}{(2\alpha x + \beta)^3}$$

$$a = -2\alpha(2\alpha x + \beta)^{-3}$$

$$a = -2\alpha v^3$$

$$\text{Retardation} = 2\alpha v^3$$

Note: If acc^n is opp. to the vel. than its magnitude is called retardation.

Alt

$$\frac{dx}{dt} = \frac{1}{(2\alpha x + \beta)}$$

$$\frac{d^2x}{dt^2} = \frac{-1}{(2\alpha x + \beta)^2} \cdot 2\alpha \cdot \frac{dx}{dt}$$

$$= \frac{-2\alpha}{(2\alpha x + \beta)^2}\left(\frac{1}{2\alpha + \beta}\right)$$

Q) For a particle moving along x-axis

$f = kt$

where

$f \to$ acc^n, k → constant, t → time

Initially at rest and $x = x_0$

find :-

① vel. after time 't'

(ii) Position of the particle after time t

(iii) Displacement of the particle in time "t".

Sol^n (i) $\frac{dv}{dt} = kt$

$$\int_{v=0}^{v} dv = \int_{0}^{t} kt\,dt$$

$$v = \frac{kt^2}{2}$$

$\frac{t^2}{2}$ ×

$f > kt$

(ii) $\frac{dx}{dt} = \frac{kt^2}{2}$

$$\int_{x_0}^{x} dx = \int_{0}^{t} \frac{kt^2}{2} dt$$

$$x - x_0 = \frac{kt^3}{6}$$

$$x = x_0 + \frac{kt^3}{6}$$

(iii) Disp in time t $= x - x_0 = \frac{kt^3}{6}$

8/4/2017

★ Graphs:

Ra Ra

$$\begin{tikzcd} x = f(t) \arrow[d, "diff"] \\ v = f(t) \arrow[d, "diff"] \arrow[u, bend left, "Integral"'] \\ a = f(t) \arrow[u, bend left, "Int"'] \end{tikzcd}$$

① straight line:

$y = mx + c$

where, $m = \text{slope} = \tan\theta$

$c = \text{Intercept on } y\text{-axis}$

Ra

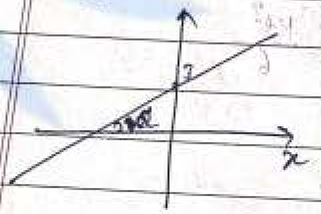
$m > 0 \Rightarrow 0 < \theta < 90$

$m < 0 \Rightarrow 90 < \theta < 180°$

$m = 0 \Rightarrow \theta = 0°$

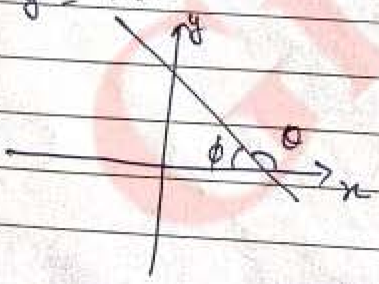
$m = \infty \Rightarrow \theta = 90°$

(i) $y = 2x + 3$



$\tan\theta = 2$

(ii) $y = -2x + 3$



$\left.\begin{aligned} \tan\theta &= -2 \\ \tan\theta &= 2 \end{aligned}\right\}$ Ra

↳ line का θ' → है y = m

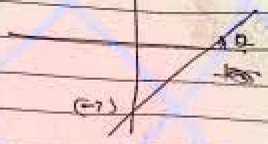
उपयोग

slope 'm'

$y = mx + c$

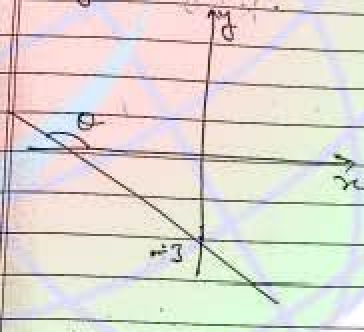
$ax + by + c = 0$ form में

(iii) $y = 2x - 1$



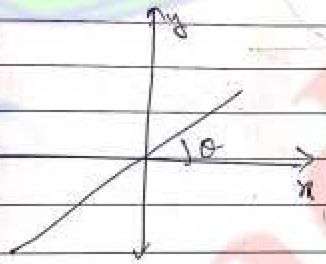
$\tan\theta = -2$

iv) $y = -2x - 3$



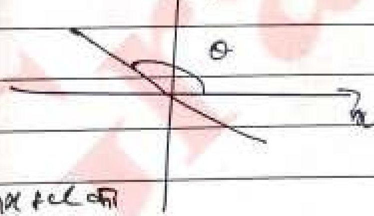
$\tan\theta = -1$

(v) $y = 2x$



$\tan\theta = 2$

vi)



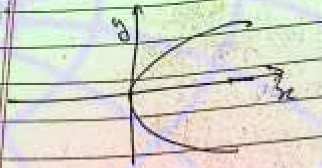
$\tan\theta = -2$

का eqⁿ → है $y = mx + c$ का प्रयोग करके slope "m" find करें

Parabolas

(i) $y^2 = Kx$

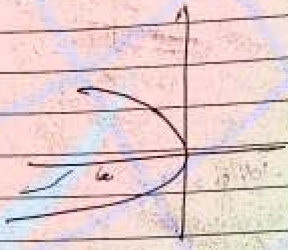
$K > 0$



(ii) $y^2 = Kx$

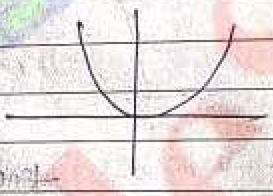
$y^2 = Kx$

$K < 0$



(iii) $x^2 = ky$

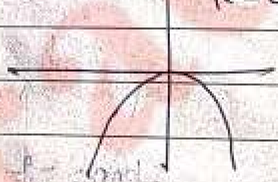
$k > 0$



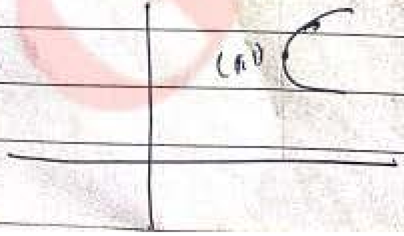
(iv) $x^2 = ky$

$k < 0$

$k < 0$

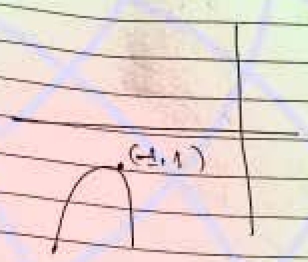


(v) $(y-1)^2 = 2(x-1)$



Ra

⑦ $(x+1)^2 = -2(y+1)$

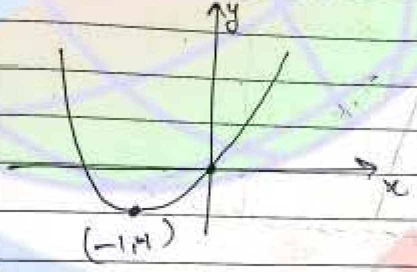


⑧ $y = 2x + x^2$

$y+1 = x^2+2x+1$

$(y+1) = (x+1)^2$

Ra

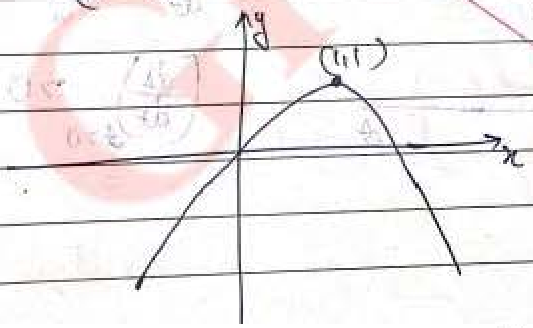


⑨ $y = 2x - x^2$

$-y = x^2 - 2x$

$-y + 1 = x^2 - 2x + 1$

$-(y-1) = (x-1)^2$

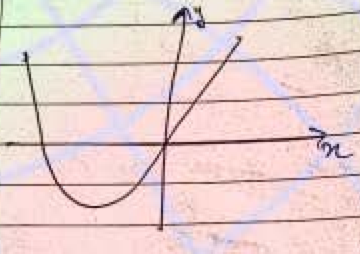


Note

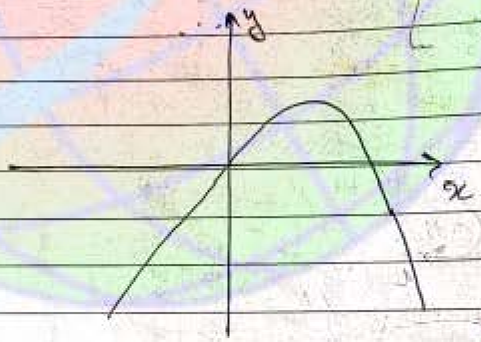
Imp

(i) $y = ax + bx^2$

$a > 0,\ b > 0$



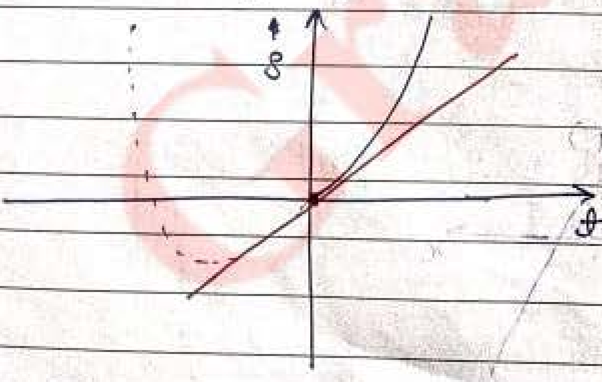
(ii) $y = ax - bx^2$, $a > 0$
$b > 0$



Important
(ii)

(i) $s = ut + \frac{1}{2}at^2$

$s \rightarrow$ displacement
$t \rightarrow$ time



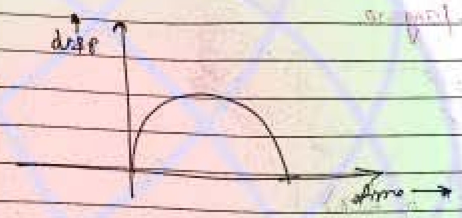
Note
→ General eqⁿ of this
i.e. $y = ax + bx^2$

$\frac{ds}{dt} > 0$ (at
or

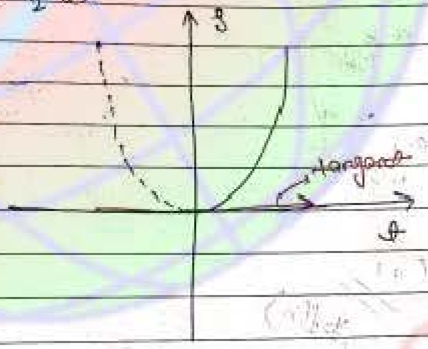
$\left(\frac{ds}{dt}\right)_{t=0} > 0$

↳ General eqn of the

(i) $s = ut - \frac{1}{2}at^2$

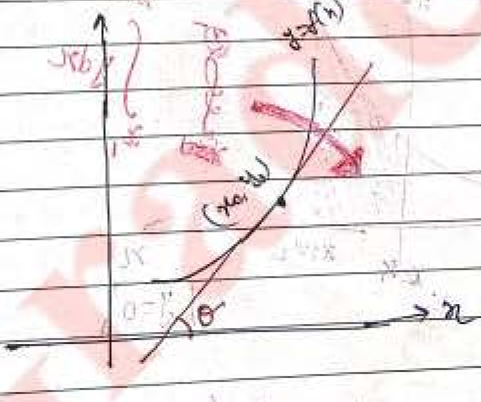


(ii) $s = \frac{1}{2}at^2$



$$\left(\frac{ds}{dt}\right)_{t=0} = 0$$

①

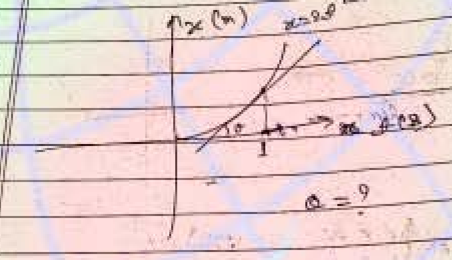


$$\left(\frac{dy}{dx}\right)_{x=x_0} = \tan\theta$$

<u>Note</u>

$\frac{dy}{dx}$ = slope of tangent

eg)



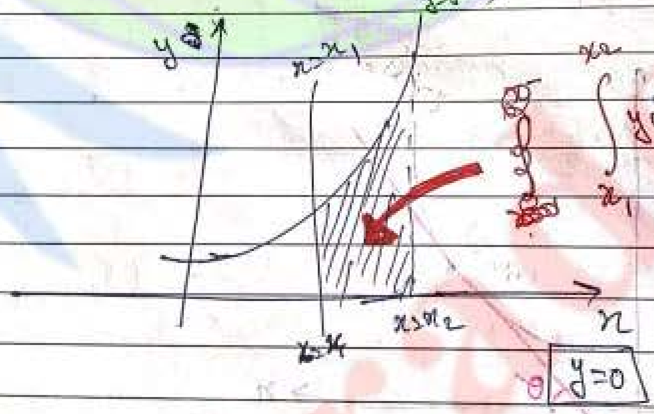
θ = ?

$\frac{dx}{dt} = 4t$

$\left(\frac{dx}{dt}\right)_{t=1} = 4$

tan θ = 4

θ = tan⁻¹(4)



$\int_{x_1}^{x_2} y\,dx$

$\boxed{y=0}$

$\int_{x_1}^{x_2} y\,dx$ = Area enclosed b/w $x=x_1$, $x=x_2$, y=f(x) and y=0

or

Area under the curve

eg)

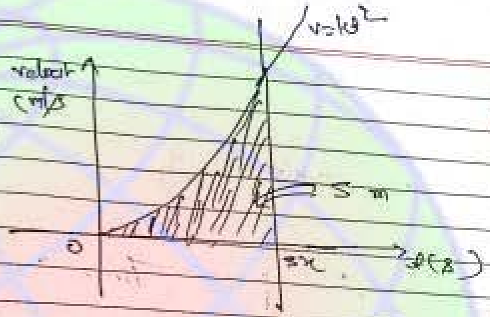
velocity (m/s)

$\frac{dx}{dt} = \tan$

Note

x =

eg.)



$\boxed{v = f'(t)}$

Sol

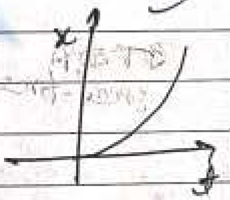
$$\int_0^5 v\,dt = 5$$

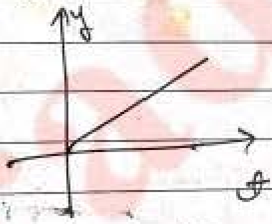
$$\int_0^5 kt^2\,dt = 5$$

$$K = \frac{3}{25}$$

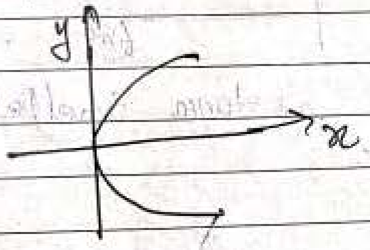
Note

$x = t^2$; $y = t$



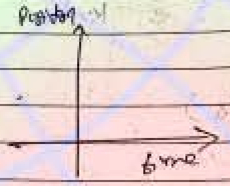


Now $x = y^2$



Notes Graphs

①



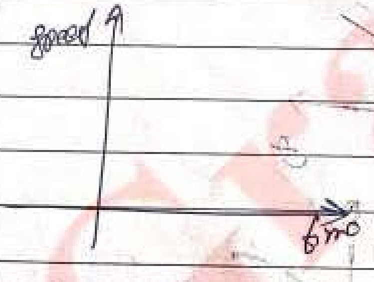
velocity = slope of tangent

②

(a) acceleration ⇒ slope of tangent

(b) displacement = area under the curve

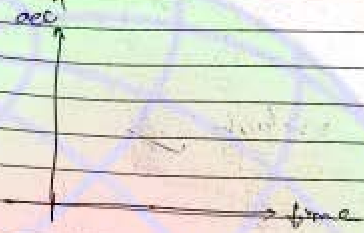
③



speed-time

distance travelled ⇒ area under the curve

④



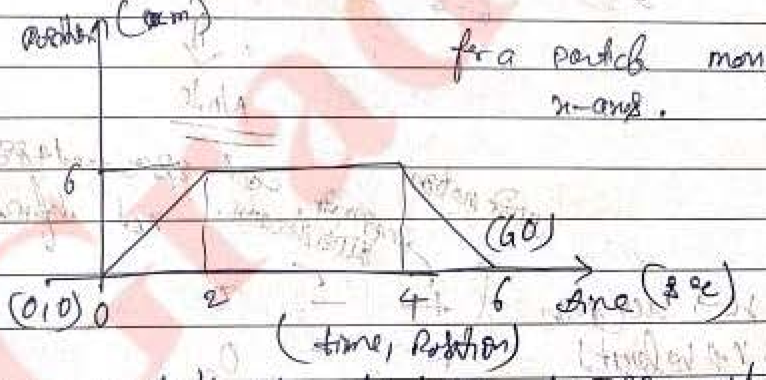
change in velocity = Area under the curve.

Note :- $\int_{\vec{v}_i}^{\vec{v}_f} d\vec{v} = \int_i^f \vec{a}\, dt$

$\vec{v}_f - \vec{v}_i = \int_i^f \vec{a}\, dt$

$$\boxed{\Delta \vec{v} = \int_i^f \vec{a}\, dt}$$

⑤ eg)



for a particle moving along x-axis.

(1) velocity at t = 1sec, t = 3sec and t = 5sec.

* at t = 1 sec

(1) $\underset{x_1\ y_1}{(0,0)}$, $\underset{x_2\ y_2}{(2,6)}$

slope = $\frac{6}{2}$ = 3 = velocity

* at t = 3 sec

slope = 0 =

* at t = 5 sec

$\underset{x_1,\ y_1}{(4,6)}$, $\underset{(x_2,\ y_2)}{(6,0)}$

$\frac{-6}{2} = -3$ = vel

Total

(time, position)

$v_{t=1} = \frac{6-0}{2-0} = 3$ m/s — Note (0 से 2 sec के अंतराल)

$v_{t=3} = \frac{6-6}{4-2} = 0$ — Note (2 से 4 sec के Interval में)

$v_{t=5} = \frac{0-6}{6-4} = -3$ m/s — Note 4 से 6 sec के Interval

⇓

Note

at 2 sec, and 4 sec vel. को not defined

Note

x → location + — 0

v → velocity + — 0

a → Acc^n → के sign को fix करके Relate करें

+ (बढ़ना) — (F नहीं) 0 (F = 0)

आगे motion

origin से पीछे location

displ

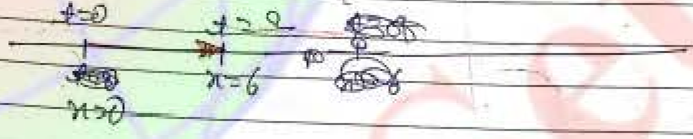
+ → location change left to right

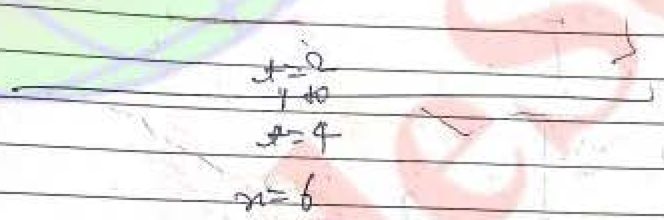
− → location change from right to left

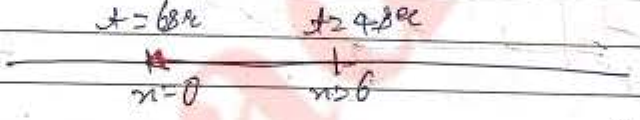
0

Note

i) Rests



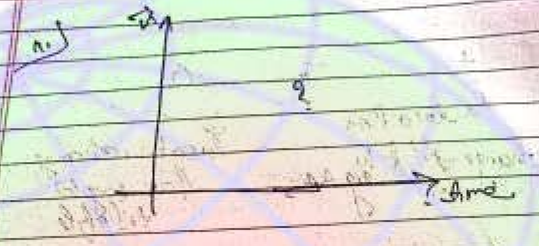




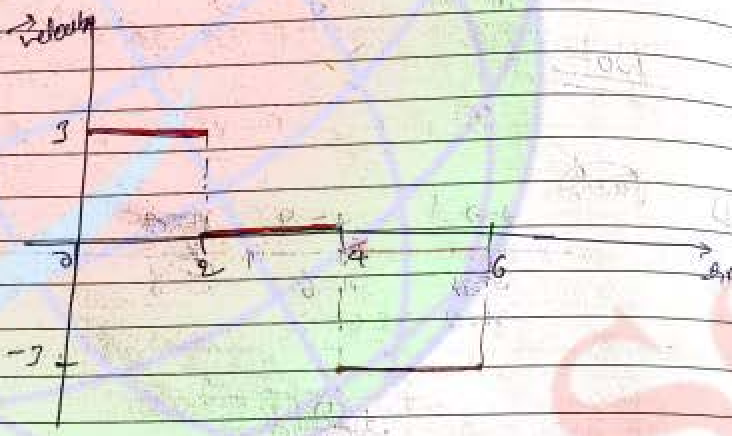
Note iii) in 6 s

$V_{avg} = \frac{12\,m}{6\,s} = 2\,m/s$ → total distance

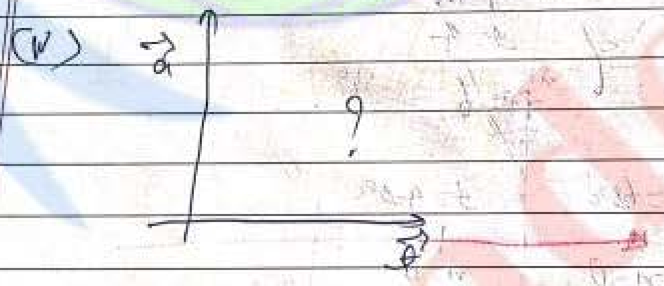
and $|\vec{V}_{avg}| = \frac{(0)}{6} = 0$ → total displacement

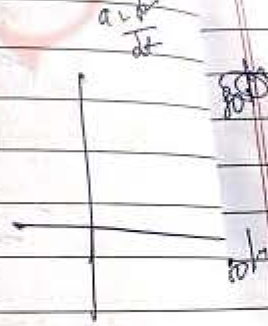


sol^n



(V)

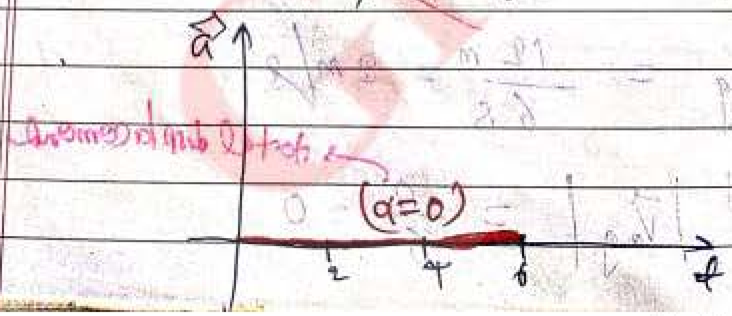




sol^n

sol^n

vel vs time graph નો slope લેવો



Here

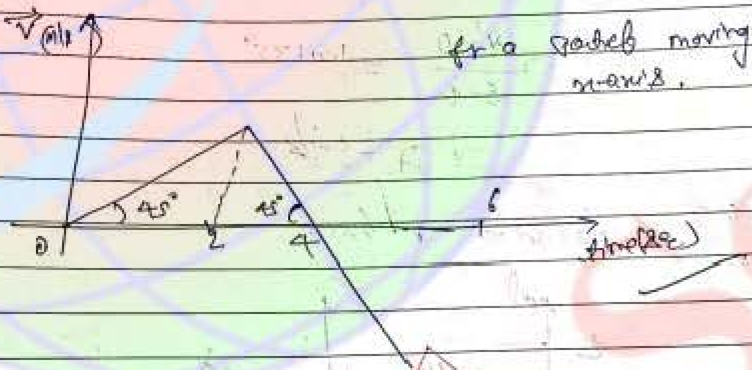
a=0

either at rest

or

is uniform motion.

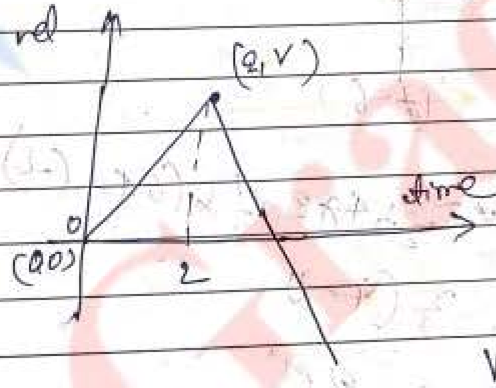
Q.2



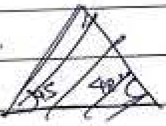
for a particle moving along x-axis.

Sol (i) vel. after 2 sec.

Sol



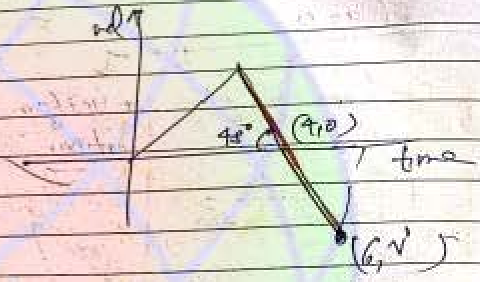
(time, velocity)



$$\frac{v-0}{2-0} = \tan 45^\circ$$

$$\Rightarrow v = +2\,m/s$$

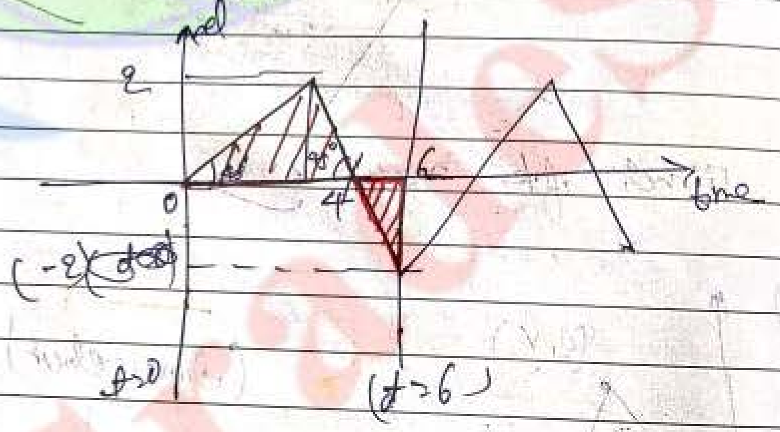
iv) vel at t = 6se is



$$\frac{v'-0}{6-4} = \tan 135°$$

$$v' = -2 \text{ m/s}$$

(Ans) Displacement in 6sec.



$$\text{Displacement} = \frac{1}{2} \times 4 \times 2 + \frac{1}{2} \times (6-4)(-2)$$

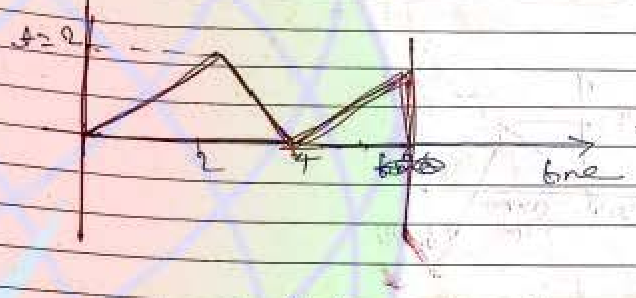
$$= (4-2)$$

$$= 2m$$

v) Distance

iv) Distance travelled in 6 sec में

speed = $|\vec{v}|$ ↪ speed ही है velocity का modulus है इसलिए vel. के speed वाली के लिए vel. का modulus लिये है



Distance travelled = 4+2 = 6m

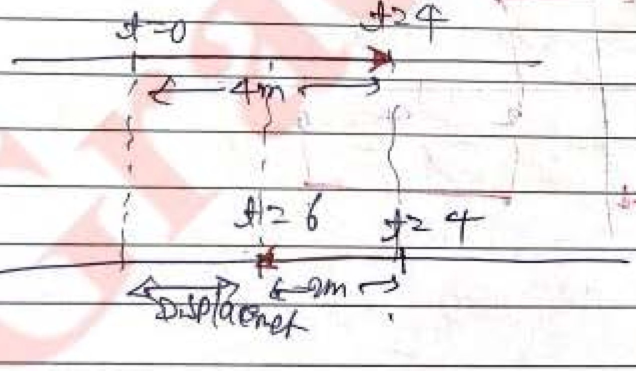
(v) $|\vec{V}_{avg}|$ and V_{avg} in 6 sec में —

$$|\vec{V}_{avg}| = \frac{2}{6} = \frac{1}{3}\ m/s$$

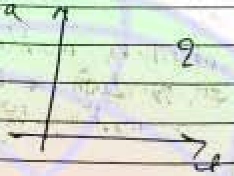
and

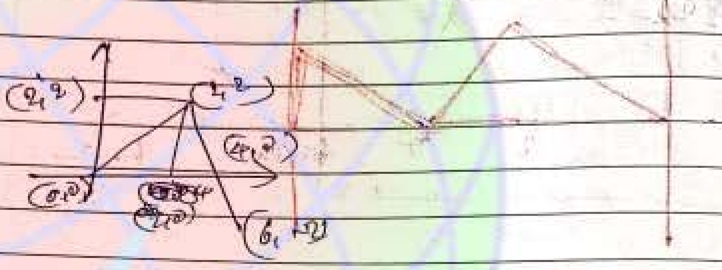
$$V_{avg} = \frac{6}{6} = 1\ m/s$$

vi)



(iv)

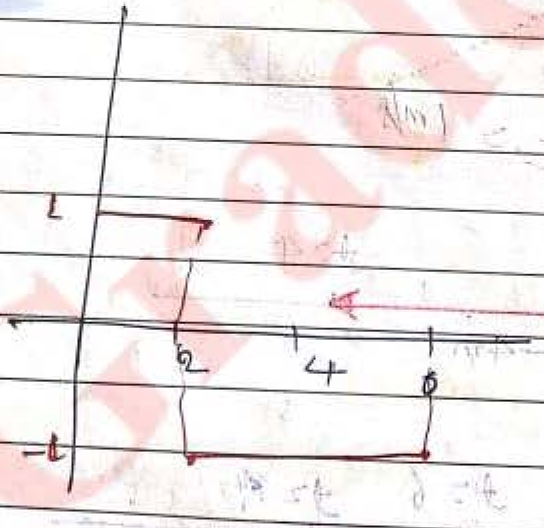




m = |1|

$\frac{-2}{2} = 1$

$= -1$



Note

time interval	velocity	accⁿ
$0 < t < 2$	> 0 (greater than zero)	> 0 (greater than zero)
$2 < t < 4$	> 0 (greater than zero)	< 0 (less than zero)
$4 < t < 6$	< 0 (less than zero)	< 0 (less than zero)

→ In this case velocity and accⁿ are in opp. sign so, deacceleration or retarding motion is take place.

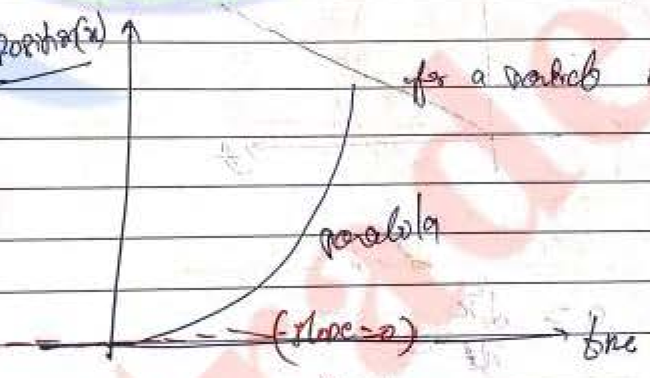
Note

$a \downarrow$

$$\left.\begin{matrix} a > 0 \\ a < 0 \end{matrix}\right\} \Delta a < 0$$

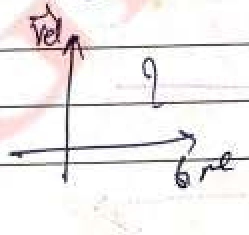
we say that retardation का sufficient condition vel. का greater than zero होता है

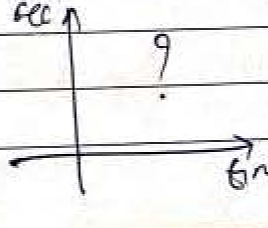
eg



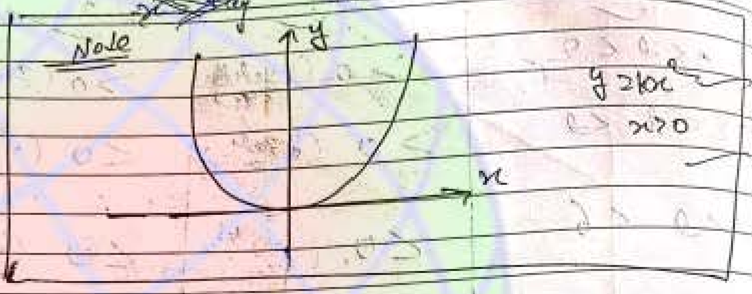
for a particle moving along x-axis

Draw its :





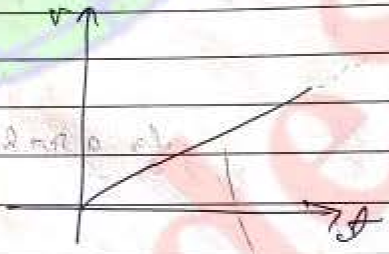
Sol



$x = kt$

$\frac{dx}{dt} = 2kt$

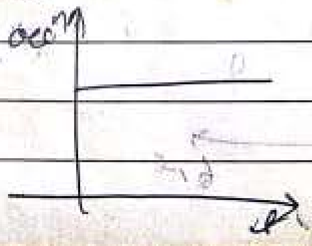
$v = 2kt$ → linea relatio



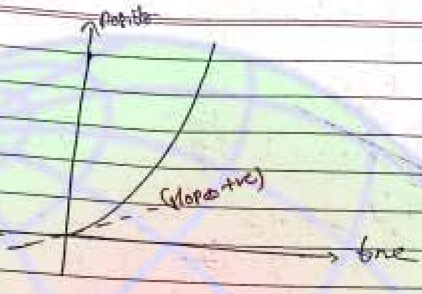
(ii)

$a = \frac{d^2x}{dt^2}$

$a = \boxed{2k}$ → constant

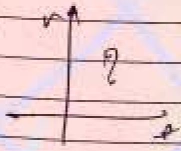


9a)

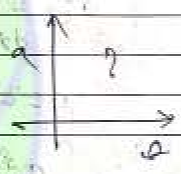


first

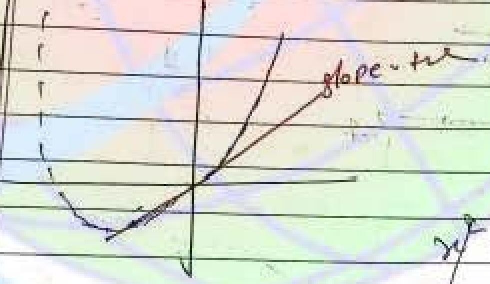
(i)



(ii)



9b)



$y = k_1x + k_2x^2$

~~$x^2 = kt$~~

~~$\frac{dx}{dt} = 2kt$~~

~~$v^2 = 2kt$~~

As

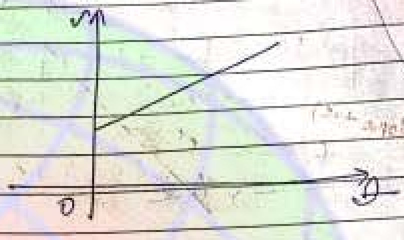
$x = k_1t + k_2t^2$

$k_1 > 0$ and $k_2 > 0$

(i) $v = \frac{dx}{dt}$

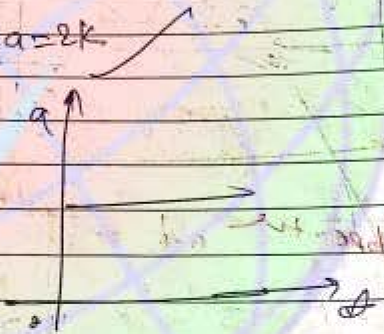
$v = 2k_2t + k_1$ ⟷ linearly dependent

Remark ↳ intercept (+ve)

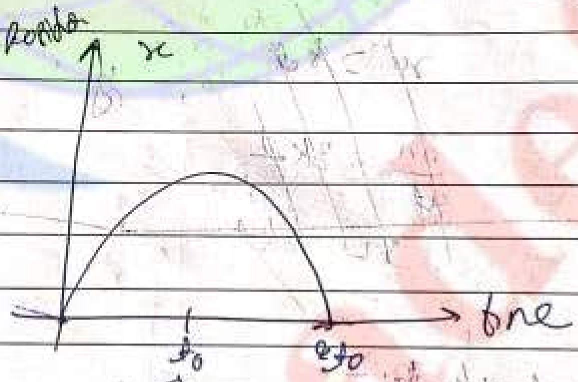


(ii) $a = \frac{d^2x}{dt^2}$

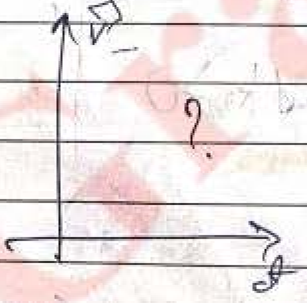
$a = 2k$



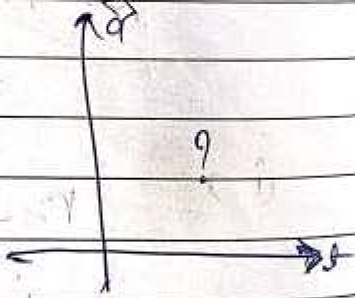
eg.



(i)



(ii)



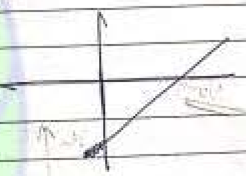
Ra

b)

$x = ut - \frac{1}{2}at^2$

$\frac{dx}{dt} = u - \frac{1}{\cancel{2}} \times \cancel{2} at$

$\frac{dx}{dt} = \cancel{at}\, u - at$



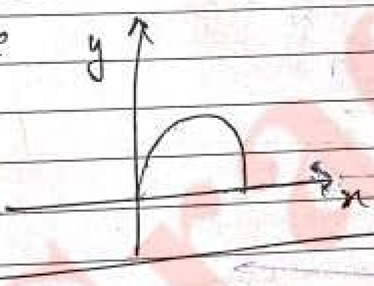
(c)

$\frac{d^2x}{dt^2} = -a$

Teach

Ra

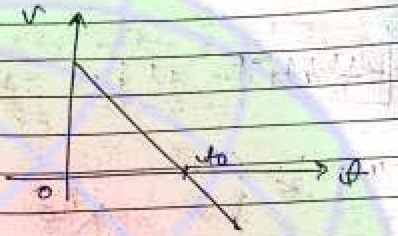
Note



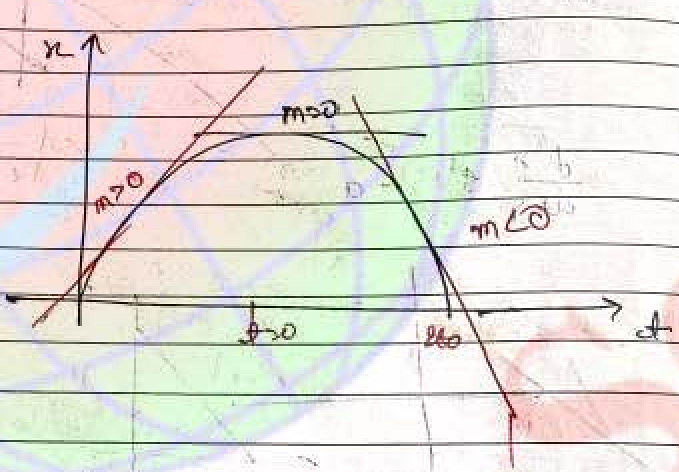
$y = k_1x - k_2x^2$

$v = \frac{dx}{dt}$

$v = -2k_2x + k_1$



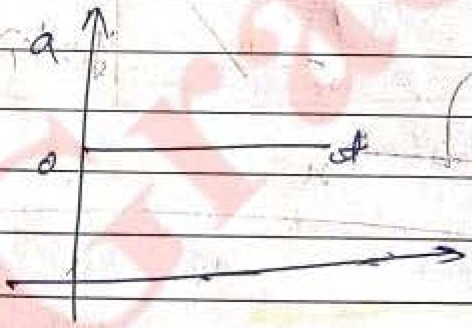
Note



(ii)

$$a = \frac{d^2x}{dt^2}$$

$$a = -2kx$$

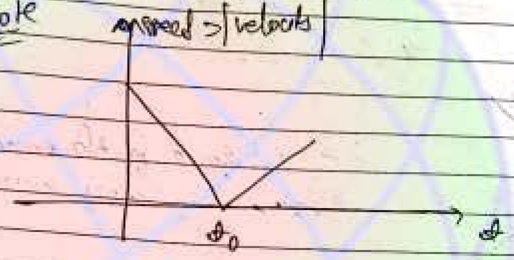


Note

Note

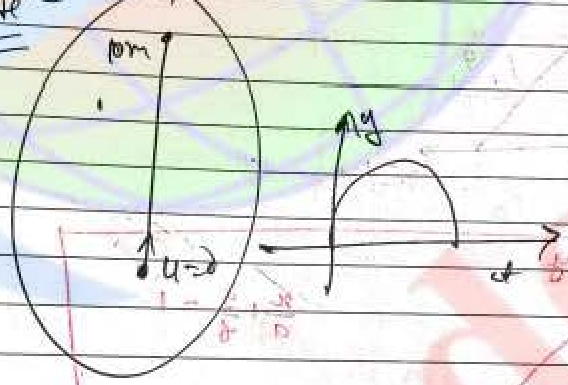
① speed decrease
② vel. or acc' sign अलग।

Note

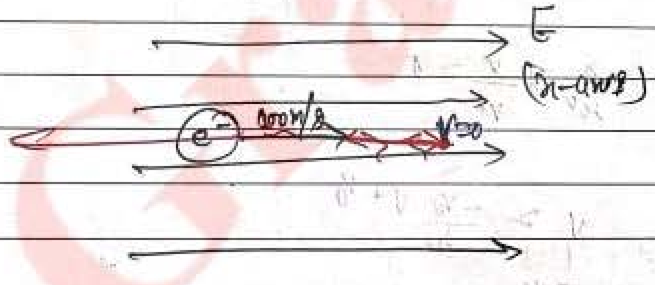


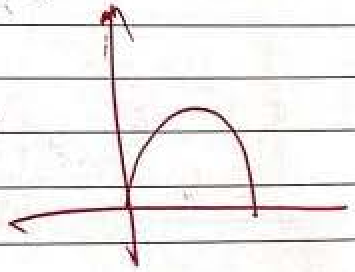
→ we say that ऐसे केस में Retarding motion है।

Note ②



(c)



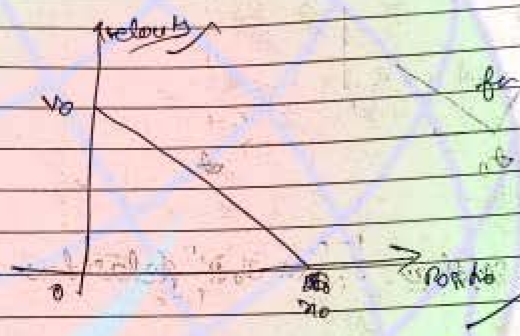


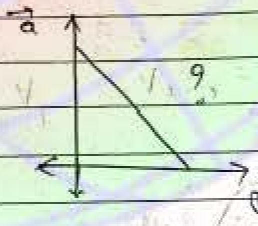
↳ L-1 → 1 to 7 3
↳ L-2 → 3 to 10, 15, 25 to 25
↳ L-3 → 2, 8, 15,
↳ L-4 → 9, 10, 11, 15

$$\boxed{x = a + bt + ct^2 + dt^3}$$

eg

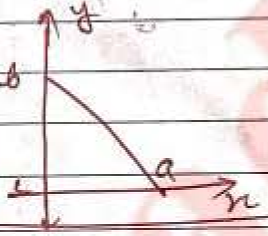
for a partb moving along x-axis





Sol^n

Note

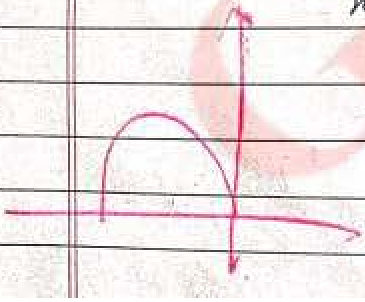


$\frac{x}{a} + \frac{y}{b} = 1$

Here,

$\frac{x}{x_0} + \frac{v}{v_0} = 1$

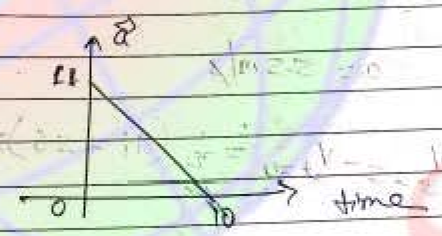
$v = -\frac{v_0}{x_0}v + v_0$



$\frac{dv}{dx} = \frac{F \cdot V_e}{x^0}$

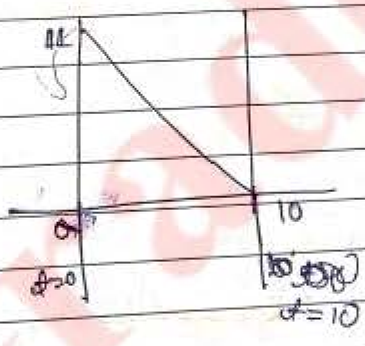
r per :

Q)



Q) Vel at $t = 5$ sec is

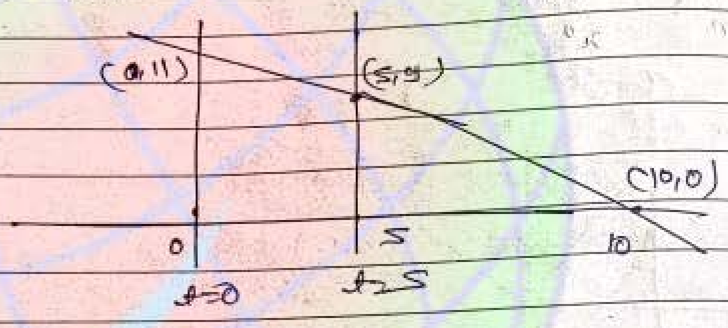
(1) find velocity of the particleal $t = 10$ sec its initial velocity is zero



$~~V_{t=10} - V_{t=0}~~$ $V_{t=10} - V_{t=0} = \frac{1}{2}(10)(11)$

$V_{t=10} = 55$ m/s

ii) Inthe above question velocity at $t=5$ sec is →



$$\frac{a-11}{5-0} = \frac{0-9}{10-5}$$

$a = 5.5$ m/s

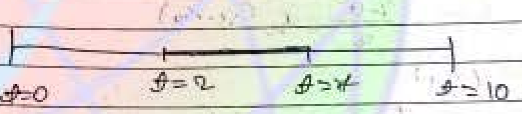
$$V_{t=5} - V_{t=0} = \frac{1}{2}(11+56)\times 5$$

$V_{t=5} = 41.25$ m/s

☆ Uniformly accelerated motion along a straight line ☆

$\vec{a} = \text{Constant}$

u = initial velocity
v = final velocity
a = acc^n
s = displacement
t = time interval



① First eq^n of motion :-

$$\frac{d\vec{v}}{dt} = \vec{a}$$

$$\int_{\vec{u}}^{\vec{v}} d\vec{v} = \int_{0}^{t} \vec{a}\, dt$$

$$\vec{v} - \vec{u} = \vec{a}\, t$$

$$\vec{a} = \frac{\vec{v} - \vec{u}}{t}$$

so,

$$\boxed{\vec{v} = \vec{u} + \vec{a}\, t}$$

② 2nd eq^n of motion :-

$$\frac{d\vec{s}}{dt} = \vec{v}$$

$$\int d\vec{s} = \int \vec{v}\, dt$$

$$\int_{0}^{\vec{s}} d\vec{s} = \int_{0}^{t} (\vec{u} + \vec{a}t)\, dt$$

$$\vec{s} = \vec{u}t + \frac{\vec{a}\, t^2}{2}$$

so

$$\boxed{\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2}$$

Here
s = change in position
(if motion takes
place along x-axis)

Rq →

$$\boxed{\begin{array}{l} \vec{s} = \Delta x \\ \vec{s} = x - x_0 \end{array}}$$

(iii) Third eqⁿ of motion

$$V_x \frac{dV_x}{dx} = a_x$$

$$\int_u^v V_x dV_x = \int_{x_0}^{x} a_x dx$$

$$\frac{v^2-u^2}{2} = a_x (x-x_0)$$

$$v^2-u^2 = 2a_x \Delta x$$

$$\boxed{v^2 = u^2 + 2a_x s}$$

(iv) Average velocity ($\vec{V}_{avg}$)

$$\vec{V}_{avg} = \frac{\vec{s}}{t}$$

or

$$\vec{V}_{avg} = \frac{\vec{u}t + \frac{1}{2}\vec{a}t^2}{t}$$

$$\vec{V}_{avg} = \vec{u} + \frac{\vec{a}t}{2}$$

$$= \vec{u} + \frac{\vec{v}-\vec{u}}{2}$$

for

$$\boxed{\vec{V}_{avg} = \frac{\vec{v}+\vec{u}}{2}}$$

vvi

This is ~~[illegible]~~ applica only when $\vec{a} = c$

(v)

Spee

(iii)

(iv)

(v) Displacement in $\boxed{nth}$ second ($\vec{S}_n$)

Disp. in 7 sec.

Disp in 7th sec.

$\vec{S}_n$ = Displacement in n sec − Disp. in (n-1) sec.

$$\vec{S_n} = \left[\vec{u}(n) + \frac{\vec{a}}{2}(n)^2\right] - \left[\vec{u}(n-1) + \frac{\vec{a}}{2}(n-1)^2\right]$$

$$\boxed{\vec{S}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)}$$

Special case: I

(A) If $u=0$ (Initially object is at rest)

(i) Velocity after time t ⇒

$$v = u + at$$

(ii) Distance travelled in time 't'

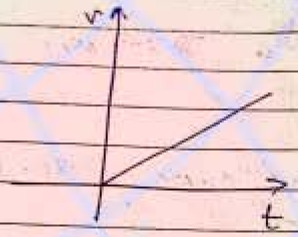
$$s = \frac{1}{2}at^2$$

(iii) velocity in travelling a distance 's'

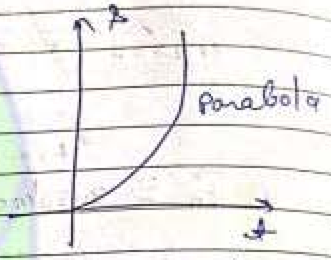
$$v^2 = 2as \text{ or } v = \sqrt{2as}$$

(iv) Disp. in nth sec.

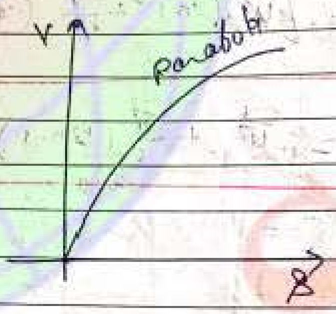
$$s_n = \frac{a}{2}(2n-1)$$



$v = at$



$s = \frac{1}{2}at^2$



$v^2 = 2as$

(B) If v=0 (final velocity is zero)

Let retarding $= \alpha$

(i) Velocity after time "t"

$$v = u - \alpha t$$

(ii) Displacement in time "t"

$$s = ut - \frac{1}{2}at^2$$

(iii) Velocity after displacement 's':

$$v^2 = u^2 - 2\alpha s$$

(iv) Displacement in n^{th} sec

$$s_n = u - \frac{\alpha}{2}(2n-1)$$

(v) stopping time

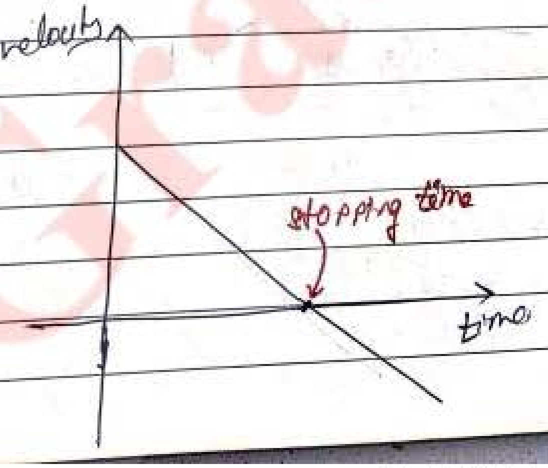
$$0 = u - \alpha t$$

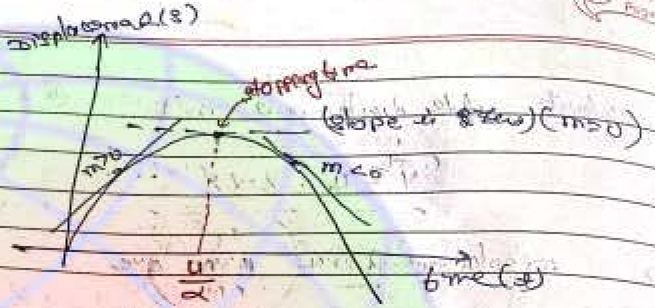
$$\boxed{t = \frac{u}{\alpha} = \frac{\text{Initial velocity}}{\text{Retardation}}}$$

(vi) stopping distance

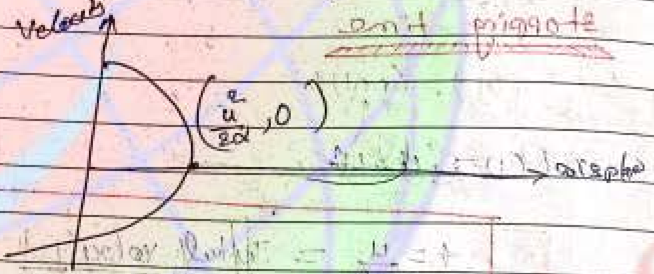
$$0 = u^2 - 2\alpha s$$

$$\boxed{s = \frac{u^2}{2\alpha} = \frac{(\text{Initial velocity})^2}{2(\text{retardation})}}$$





$$\vec{s} = ut - \frac{1}{2}\alpha t^2$$

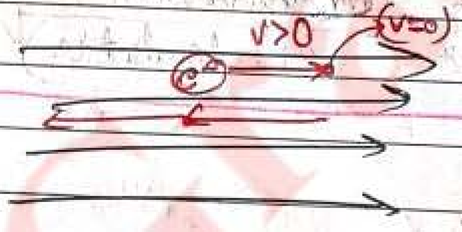


$$v^2 = u^2 - 2\alpha s$$

$$v^2 = -2\alpha\left(s - \frac{u^2}{2\alpha}\right)$$

$$(v-0)^2 = -2\alpha\left(s - \frac{u^2}{2\alpha}\right)$$

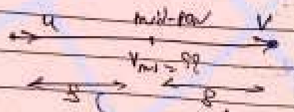
understand



vel. at mid-point

Q1 For a particle moving along a straight line under uniform acc^n initial and final velocities are "u" and "v" resp. Find vel. of the particle at the mid-point of its path

Sol:



v, t → I

s, t → II

v, s → III

$$v_{mid}^2 = u^2 + 2as$$

and

$$v^2 = v_{mid}^2 + 2as$$

$$v_{mid}^2 - v^2 = u^2 - v_{mid}^2$$

$$\boxed{v_{mid} = \sqrt{\frac{u^2+v^2}{2}}}$$

Q2 A particle initially at rest moves under uniform acc^n find ratio of distance travelled by it in 1st 10 sec to next 10 sec

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2} \times a \times (10)^2$$

$$s = 10 \times 5$$

$$= 50$$

$$s = 10 \times 10 + \frac{1}{2} \times 10 \times (10)^2$$

u = 0

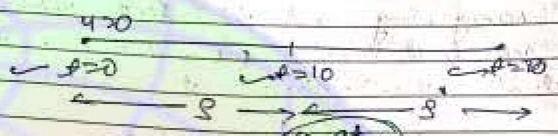
acc^n const

t = 10

s = 4

Ra

Tanhe



$v = at$
$= a\times10$
$= 10a$ → Rq

$\frac{S}{S'} = ??$

$S = \frac{1}{2}a(10)^2$

$S = 50a$

and

$S' = (10a)\cdot(10) + \frac{1}{2}a(10)^2$

vel. at t = 10 sec.

$\frac{S}{S'} = \frac{1}{3}$

Note

$S+S' = \frac{1}{2}a(20)^2$

Q. 03 Initial vel. of a particle is "u" it moves along straight line under uniform retardation. On loosing 50% of its initial vel. it travels distance "S". find distance travelled by it further to come to rest

Sol:

$u = u$ ✓

$a = -a$

$v = \frac{1}{2}u$

$S = S$ ✓

$v = 0$ ✓

$S = ?$ ✓

$\left(\frac{u}{2}\right)^2 = u^2 - 2\alpha S$

α → retardation

~~u, v, a, t~~

u, v, a, S

u, a, t, S

- ~~S~~

$$2aS = \frac{3}{4}u^2 \quad \text{—(1)}$$

and

$$0 = \left(\frac{u}{2}\right)^2 - 2aS'$$

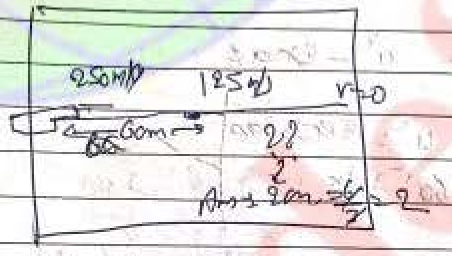
$$2aS' = \frac{u^2}{4}$$

$$\frac{S}{S'} = \frac{3}{1}$$

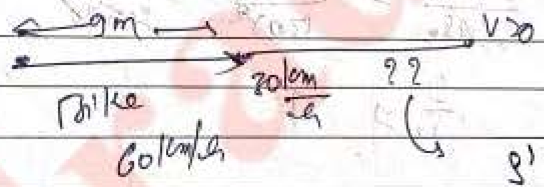
$$\boxed{S' = S/3}$$

Applications:-

(i)

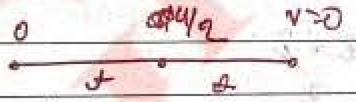


(ii)



$S' = \frac{9}{3} = 3m$

(iii)



Ans

$$\boxed{\frac{t}{t'} = 1}$$

Now

$$\frac{u}{2} = u - \alpha t$$

$$\alpha t = \frac{u}{2} \quad \text{—(i)}$$

and

$$0 = \frac{u}{2} - \alpha t'$$

$$\alpha t' = \frac{u}{2} \quad \text{—(ii)}$$

Q) Two identical cars A and B are moving along straight lines with vel. 60 km/hr and 30 km/hour resp. They are stopped by applying the same brakes find ratio of stopping distance A to B.

Sol^n

$$0 = u^2 - 2 \times a s$$

$$u^2 = 2as$$

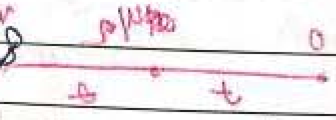
$$(60)^2 = 2as$$

$$\frac{2as_1}{2as_2} = \frac{(60)^2}{(30)^2} = \frac{60 \times 60}{30 \times 30}$$

$$\frac{s_1}{s_2} = 4$$

Teah

$$0 = u^2 - 2\alpha s$$



$$s = \frac{u^2}{2\alpha}$$

$$\boxed{\frac{1}{same} = \frac{t}{t}}$$

$$\alpha = \frac{F}{m}$$

$S = u^2$

$$\frac{S_A}{S_B} = \left(\frac{u_A}{u_B}\right)^2 = \left(\frac{60}{20}\right)^2 = 9$$

Q) A truck and a car are moving along straight line they are stopped by applying the same breaks which will come to rest at w.r.t. distance

$m_{truck} > m_{car}$

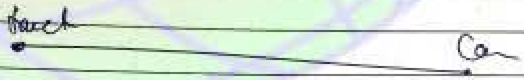
if

(i) Both have same initial vel.

(ii) Both have same initial K.E.

(iii) Both have same initial momentum

Soln



$0 = u^2 - 2\alpha x$

$$\boxed{\begin{array}{l} F = ma \\ \alpha = \frac{F}{m} \end{array}}$$

$S = \frac{u^2}{2\alpha}$

$S = \frac{mu^2}{2F}$

(i) $S = \frac{m\textcircled{u^2}}{2\textcircled{F}}$ → same → same

$S = m$

$\boxed{S_T > S_{car}}$ (car will stop in short distance

(i) $s = \frac{mu^2}{2F}$

$s = \frac{K_i}{f}$ → same → same

$$\boxed{S_T = S_{car}}$$

(ii) $s = \frac{K_i}{F}$

$s = \frac{P_i^2}{2mF}$ → same → same

$K.E = \frac{P^2}{2m}$

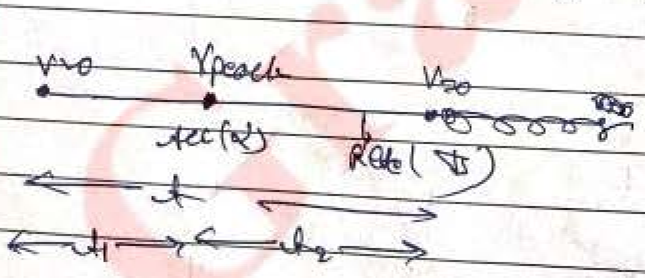
$s \propto \frac{1}{m}$

$$\boxed{S_T < S_{car}}$$

Ex: A body initially at rest acc^n at constant rat "α" then retards at constant rate "β" to come to rest motion is along straight line and total time is "t".

Find pick value of the velocity.

Sol^n



u=0

a = α

← t

$t = t_1 + t_2$

$V_{peak} = 0 + \alpha t_1$

and

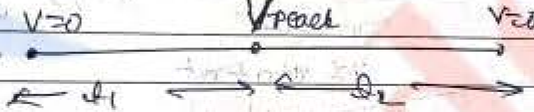
$0 = V_{peak} - \beta t_2$

$\frac{V_{peak}}{\alpha} + \frac{V_{peak}}{\beta} = t$

$$\boxed{V_{peak} = \frac{\alpha\beta t}{(\alpha+\beta)}}$$

(ii) Distance travelled by the object is ——

~~$V_1 = \frac{\alpha\beta t}{\alpha\beta}$ t_1~~ [illegible]



Total distance $= \left(\frac{0 + V_{peak}}{2}\right) t_1 + \left(\frac{V_{peak} + 0}{2}\right) t_2$

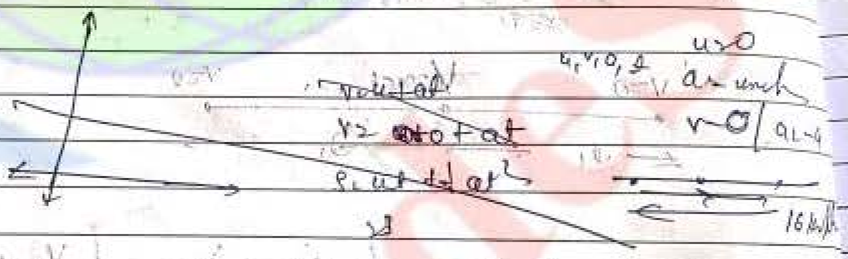
$= \frac{V_{peak}(t_1+t_2)}{2} = \frac{V_{peak}\, t}{2}$

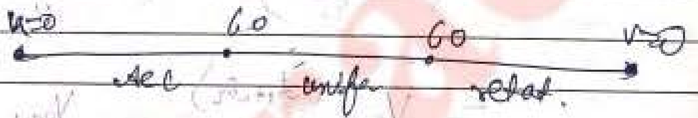
$$\boxed{S = \frac{\alpha\beta t^2}{2(\alpha+\beta)}}$$

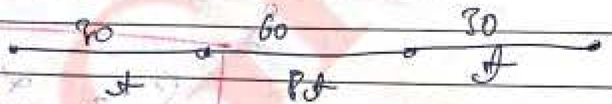
$$\boxed{s = \frac{1}{2} V_{peak} \cdot t}$$

Q) A body initially at rest moves under unifm acc^n ~~[illegible]~~
then with unifm velocity and finally comes to rest under uniform retardation
motion is along straight line
ratio of time taken are 1:8:1
peak velocity is 16km/hour
Average speed of the complete journey is

A





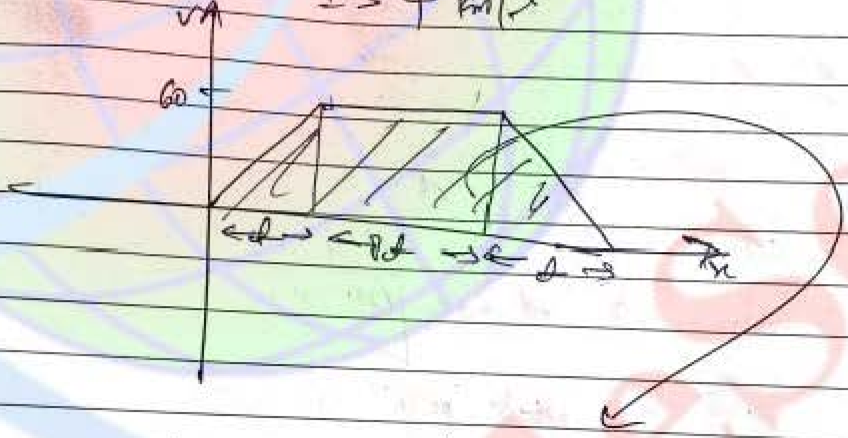


Distan 3/2
[illegible] 60t [illegible] 10t

[illegible]

$$\{V_{avg}\} = \frac{30(d) + 60(8d) + 30(d)}{10d}$$

$$= 54 \text{ km/h}$$



$$V_{avg} = \frac{\frac{1}{2}d(0) + (8d)(60) + \frac{1}{2}(d)(60)}{10d}$$

$$\Rightarrow 54 \text{ km/h}$$

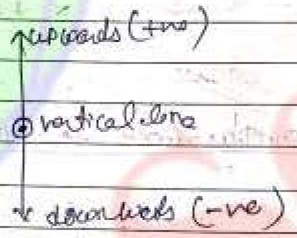
Motion under Gravity

Note

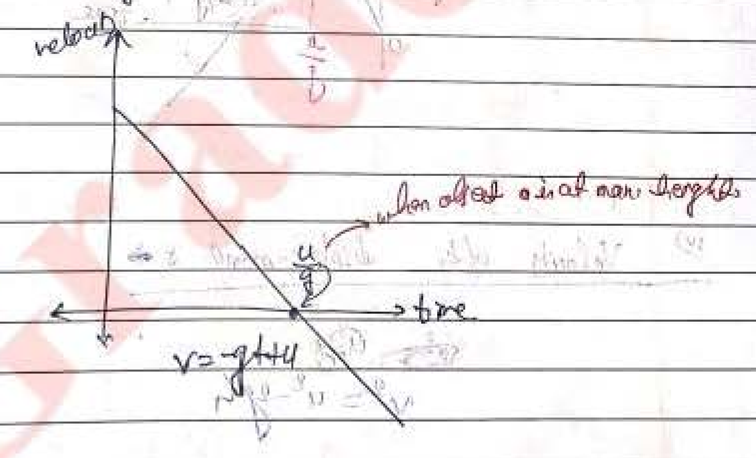
(i) object is near the earth's surface
$g = 9.8\ m/s^2 \simeq 10\ m/s^2$

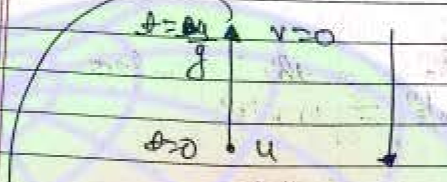
(ii) air friction force is neglected.

(A) If object is thrown vertically upwards:-



(i) velocity after time t:-
$v = u - gt.$





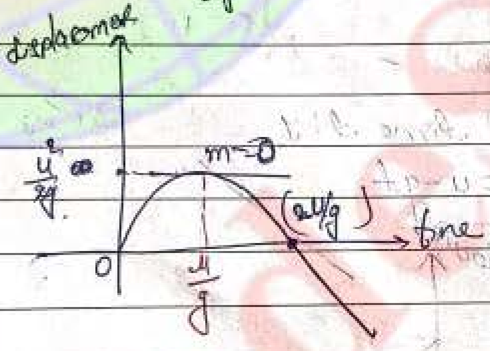
Notes stopping time

$$0 = u - gt$$

$$t = \frac{u}{g}$$

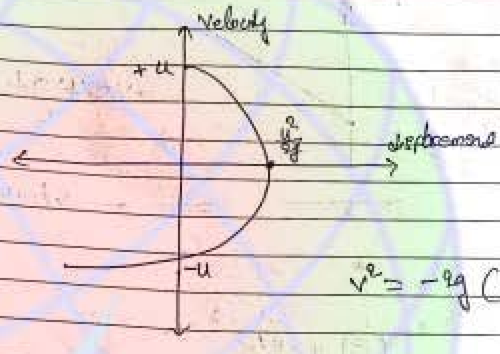
iii) Displacement at time "t":-

$$s = ut - \frac{1}{2}gt^2$$



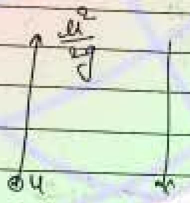
iv) Velocity after displacement s →

$$v^2 = u^2 - 2gh$$

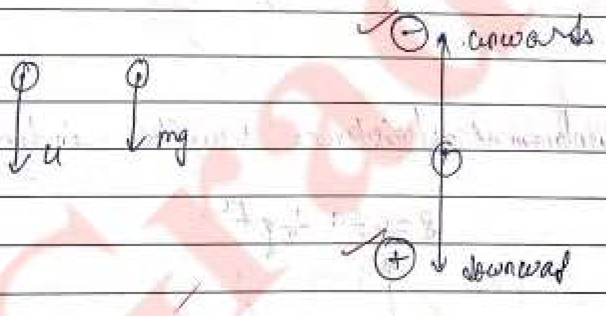


$$v^2 = -2g\left(s - \frac{u^2}{2g}\right)$$

$$(v-0)^2 = -2g\left(s - \frac{u^2}{2g}\right)$$

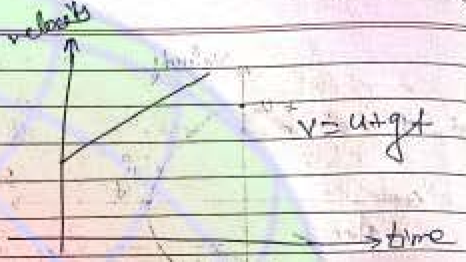


② If object is thrown vertically downwards from some height (H);



(i) Velocity after time "t".

$v = u + gt$

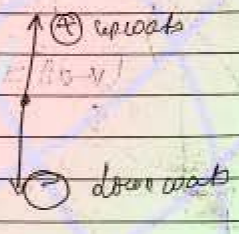


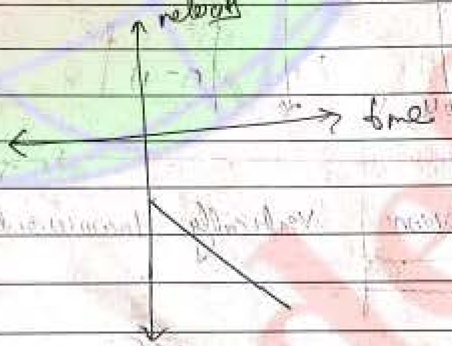
Note

(+ve standard sign)

+ve -ve sign

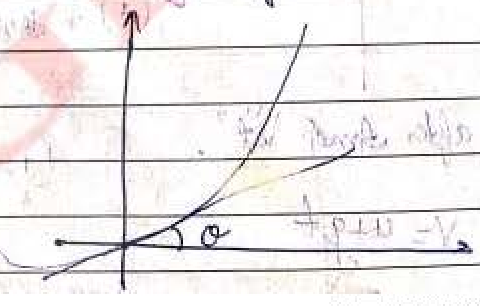
if not [illegible] is mentioned





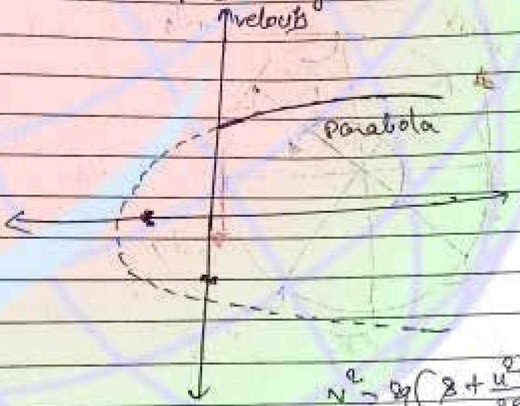
(ii) Displacement / Distance travelled in time 't'?

$$s = ut + \frac{1}{2}gt^2$$



velocity after displacement (s) :⇒

$$v^2 = u^2 + 2gs$$

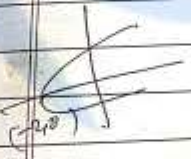


$x \geq 0$
$y > 0$
| $v \geq u$

$$v^2 = 2g\left(s + \frac{u^2}{2g}\right)$$

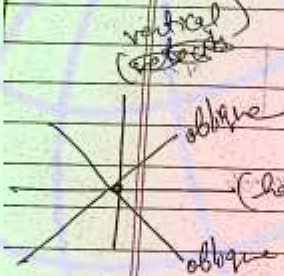
$$(v-0)^2 = 2g\left(s + \frac{u^2}{2g}\right)$$

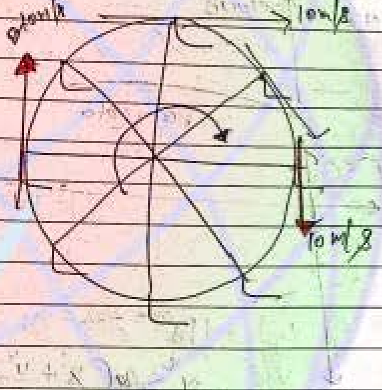
$(y-0)^2 = 2(x+a)$



Special cases

① If object is released or dropped from stationary platform

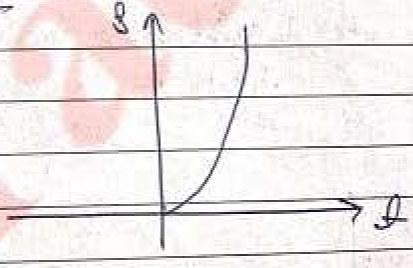




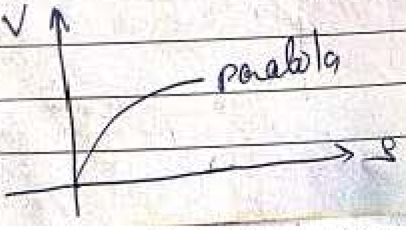
$\boxed{u=0}$

(i) $v = gt$

(ii) $s = \frac{1}{2}gt^2$

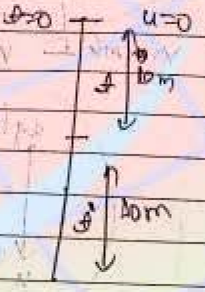


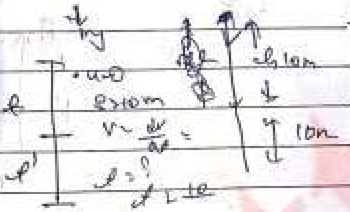
(iii) $v^2 = 2gs$ =



Q) A body is released from ~~same~~ some height. Find ratio of time ~~taken~~ taken in travelling 1st 10 meter and next 10 meter.

$$\frac{t}{t'} = \frac{1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{1}$$





$$10 = \frac{1}{2}gt^2$$

and

$$20 = \frac{1}{2}g(t+t')^2$$

$$\frac{1}{2} = \frac{t^2}{(t+t')^2}$$

$$\frac{1}{\sqrt{2}} = \frac{t}{(t+t')}$$

$$t' = (\sqrt{2}-1)t$$

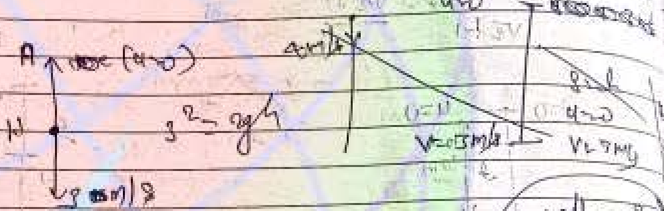
Now

$$\frac{t}{t'} = \frac{1}{\sqrt{2}-1}$$

$$= \frac{\sqrt{2}+1}{1} = \frac{2.4}{1}$$

Q) A body released from height 'h' reaches the ground with speed of 3 m/s and the body ~~the~~ is thrown vertically downwards with speed of 4 m/s the same height find its speed when it reaches the ground.

Sol



$3^2 = 2gh$

$2gH = 3^2$

$v^2 = 4^2 + 2gH$

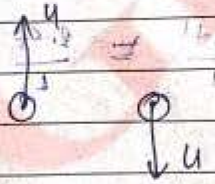
$v^2 = 4^2 + 2gH$

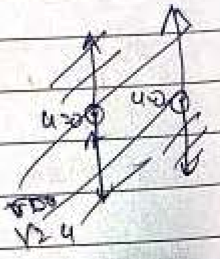
$v^2 = 4^2 + 3^2$

$v = 5$ m/s

Q.) Two objects "A" and "B" are thrown with same speed along vertical line from the same height but in opposite direction find ratio of their velocities when they hit the ground.

Sol





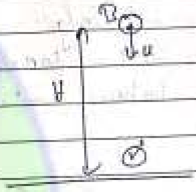
Q) A bod
from a
fin

Sol

$v^2 = u^2 - 2gs$ → displacement

$v^2 = u^2 - 2g(-H)$

$\boxed{v^2 = \sqrt{u^2 + 2gH}}$



$v'^2 = u^2 + 2gs$

$v'^2 = u^2 + 2gH$

$\boxed{v' = \sqrt{u^2 + 2gH}}$

$\frac{v}{v'} = \frac{\sqrt{u^2+2gH}}{\sqrt{u^2+2gH}} = 1$

Q) A body is thrown ~~to~~ vertically downwards ~~from~~ from a tower of height 20m with speed of 20m/s find time taken to reach the ground.

Solⁿ

~~$v^2 = u^2 + 2as$~~ $v = u + t$

~~$0 = 20\times20 + 2\times10\times20$~~

u, v, a, s
u, v, a, s
a, o, t, s
v

u = 20 m/s
s = 20m
t = ?
a = 10

20m

$s = ut + \frac{1}{2}gt^2$

$20 = 20\times t + \frac{1}{2}\times10\times t^2$

$t^2 + 4t - 4 = 0$

$t = \frac{-4 \pm \sqrt{4^2 + 4(4)}}{2}$

$t = \frac{-4 + 4\sqrt{2}}{2} = -2 + 2\sqrt{2} \simeq 0.82$ sec

Q) A body is vertically upward with speed of 20m/s from top of the tower of height 20m find time taken to reach the ground

Sol:-

$s = ut + \frac{1}{2}gt^2$

~~$20 = 20t - \frac{1}{2}\times10\times t^2$~~

$40 = 20t - \frac{1}{2}10\times t^2$

$-t^2 + 4t - 4 = 0$

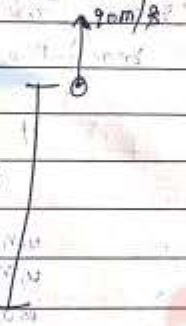
$t^2 - 4t + 4 = 0$

$t^2 - 2t - 2t + 4 = 0$

$t(t-2) - 2(t-2) = 0$

$(t-2)(t-2)$

Teacher



displacement

$s = ut - \frac{1}{2}gt^2$

$-20 = 20t - 5t^2$

~~$t^2 - 4t$~~

$t^2 - 4t - 4 = 0$

$t = \frac{4 \pm \sqrt{4^2 + 4(4)}}{2}$

$t = \frac{4 \pm 4\sqrt{2}}{2}$

$= 2 + 2\sqrt{2}$ or 4.8sec

Q) A body is
20m find

Sol

Note

Note

Q) A bo
total
Mode

Q) A body is released from top of a tower of height 20m find time taken by it to reach the ground

Sol

$s = \frac{1}{2}gt^2$ (u=0)

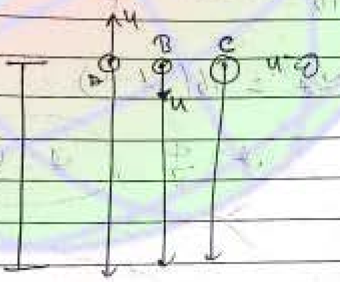
$t = \sqrt{\frac{2\times 20}{10}} = 2$ sec

[illegible]

Note

$(-2+2\sqrt{2})(2+2\sqrt{2}) = 8-4 = 4$

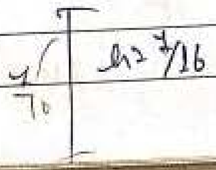
Note



$$\boxed{t_c^2 = t_A t_B}$$

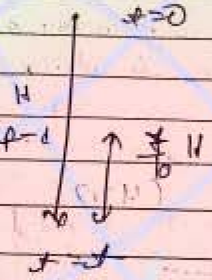
where $t =$ time taken to reach the ground

Q) A body released from some height travells $\frac{7}{16}$ part of the total height ~~so~~ in ~~its~~ last second of ~~its~~ its descend. [illegible]



L-1 → [illegible]
L-2 → 1, 2, 1(10) 4, 16
L-3 → 1, 3, 6, 8, 11, 12, 19, 21
L-4A → 2, 3, 4, 5, 7, 8, 9, 10, 11, 1, 17, 18

$t=0$, $a=0$



$H = \frac{1}{2}gt^2$ —(i)

and

$\frac{7}{16}H = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2$

$\frac{7}{16}H = \frac{g}{2}(2t-1)$ —(ii)

$\frac{7}{16}\left(\frac{1}{2}gt^2\right) = \frac{g}{2}(2t-1)$

$7t^2 = 16(2t-1)$

$t = \frac{4}{7}$ $\boxed{t = 4s}$

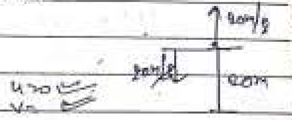
Q.) A body
from
time
with
what time

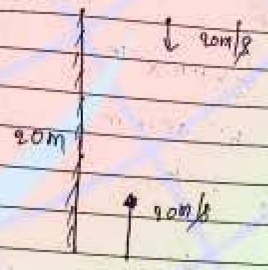
eg.>

1000

Q) A body is thrown vertically upwards with speed of 20m/s from foot of a tower of height 20m. at the same time another body is thrown vertically downwards with speed of 20 m/s from top of the tower. at what time will they cross each other

sol)



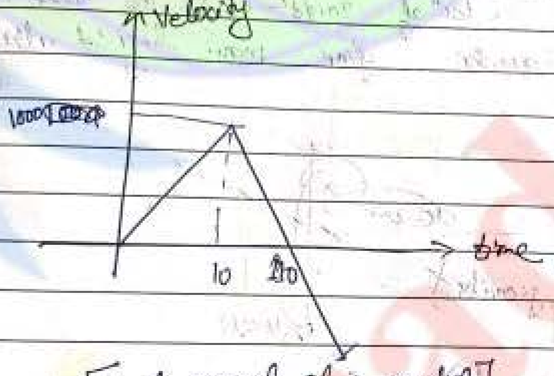


$S_A + S_B = 20$

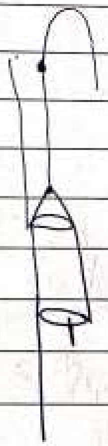
$\left(20t - \frac{1}{2}gt^2\right) + \left(20t + \frac{1}{2}gt^2\right) = 20$

$\boxed{t = \frac{1}{2}\ sec}$

eg.) A rocket is launched from ground level maximum height attained by the rocket is

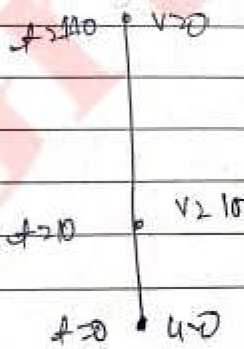


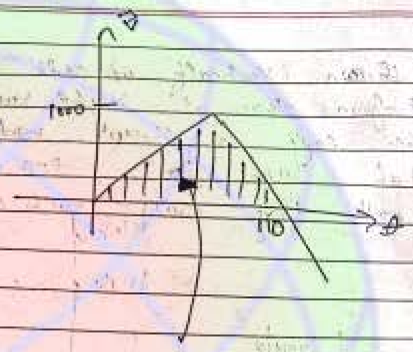
[V-t graph of a rocket]



80/4

t=10sec
v=1000
u=0

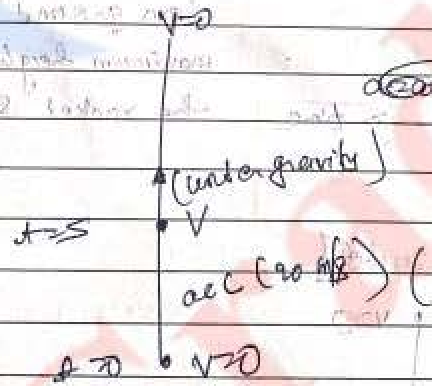




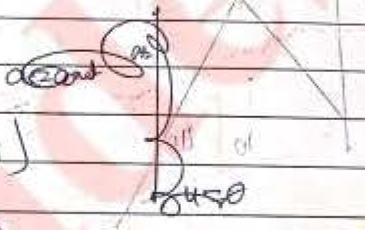
$$\text{max height} = \frac{1}{2} \times 110 \times 1000$$
$$= 55,000\ m$$
$$= 55\ km$$

RQ

A rocket is launched along vertical line from ground level it starts its motion from rest with the acc^n of 20 m/s^2 ~~yildh~~ @ 5 sec its fuel is exhausted find max. height attained by the rocket



(General eq^n of motion



Now

(i) velocity at $t = 5$ sec :-

$V = at$

$= (20)(5) = 100$ m/s

(ii) Distance travelled by the rocket in 5 sec

$s = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2}(20)(5)^2 = 250$

(iii) $0 = (100)^2 - 2hg$

$h = \frac{(100)^2}{20} = 500$ m

maximum height = 250m

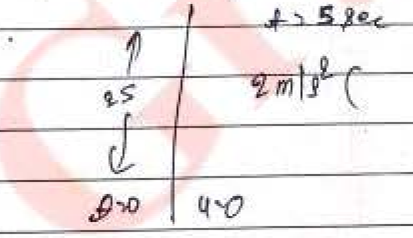
Q) A baloon starts raising vertically upwards from ground level with zero

After 5 sec and 60

Find time taken by the body in reaching the ground after him relea

Sol

(i)

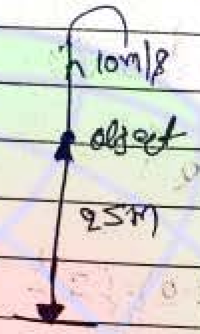


$V = at = 10$ m/s

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times 25$

$= 25$ m

(ii)

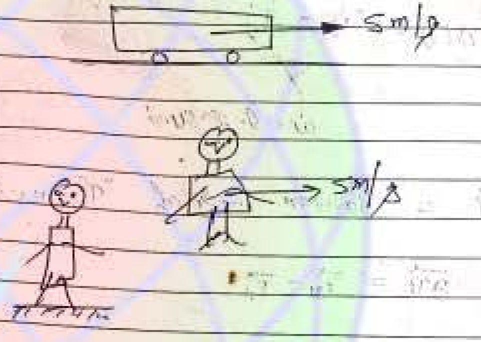


$$s = ut - \frac{1}{2}gt^2$$

$$-25 = 10t - 5t^2$$

$$t^2 - 2t - 5 = 0$$

Relative motion (change in ref of frame)



① motion ⇒ combined property of object and observer

② $\vec{V}_{A/B}$ ⇒ velocity of A w.r.t. B

$$\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B$$

where
$\vec{V}_A$ ⇒ vel. of "A" w.r.t. ground
$\vec{V}_B$ = vel. of B w.r.t. ground

③ $\vec{S}_{A/B}$ = disp. of A w.r.t. B

$$\vec{S}_{A/B} = \underbrace{\vec{S}_A - \vec{S}_B}_{\text{w.r.t. ground.}}$$

L-1 → 45 to 56
L-2 → 17 to 20
L-3 → 5, 7, 9, 10, 16
OAR → 6, 24

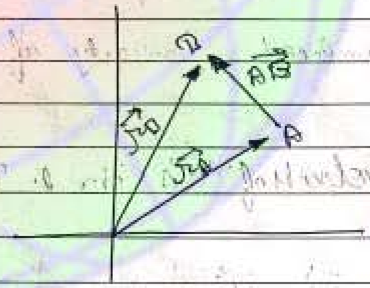
(v) $\vec{a}_{A/B}$ = acc of A w.r.t. B

[illegible] $\vec{a}_{A/B} = \vec{a}_A - \vec{a}_B$

w.r.t. ground

(iv) $\vec{AB}$ = Position vector of "B" w.r.t "A"

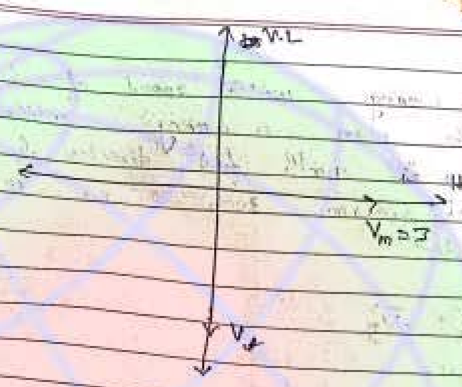
$\vec{AB} = \vec{r}_B - \vec{r}_A$



Q) Raindrops fall vertically downwards with speed of 4m/s. A man is walking on horizontal straight road with speed of 3m/s. In what direction man should hold his umbrella to ~~[illegible]~~ keep the rain drops away

sol^n $\vec{V}_{r/m} = \vec{V}_r - \vec{V}_m$

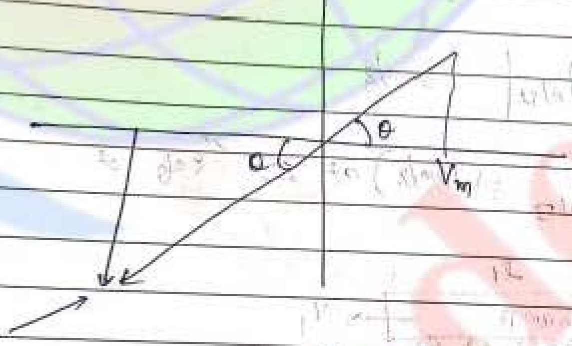
Now



$\vec{V}_{r/m} = (-4\hat{j}) - (3\hat{i})$

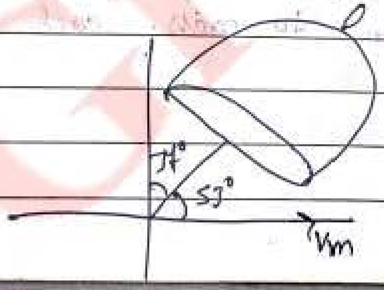
$\vec{V}_{r/m} = (-3\hat{i} - 4\hat{j})$ m/s

$|\vec{V}_{r/m}| = 5$ m/s



for θ = $\frac{4}{3}$

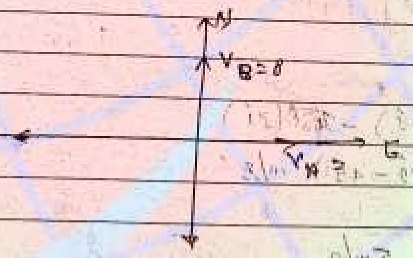
θ = 53° (along the horizontal line).



Q) A man is moving with speed of 6m/s towards east another man is moving with speed of 8m/s towards north find direction of ~~motion of~~ 1st person seen by the second person

Sol^n

$$\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B$$

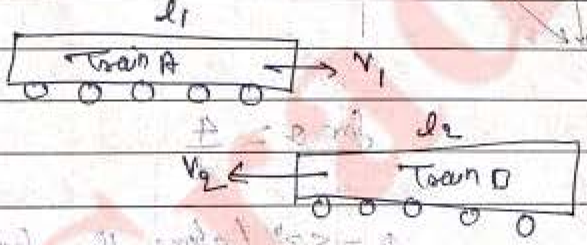


$$\vec{V}_{A/B} = (6\hat{i} - 8\hat{j}) m/s$$

$$|\vec{V}_{A/B}| = 10 m/s$$

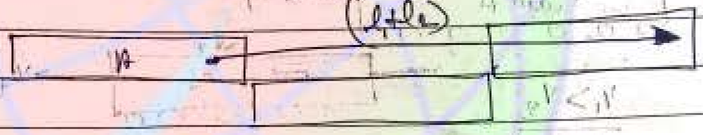
$$\vec{V}_{A/B} = (10m/s) \text{ at } 53° \text{ S of E}$$

Q)



Time taken to cross each other is —

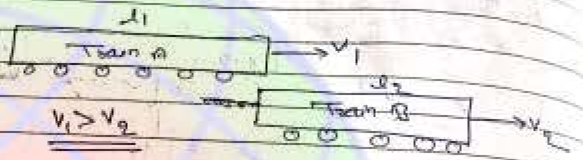
w.r.t. B



$\vec{V}_{A/B} = v_1 - (-v_2) = V_1 + V_2$

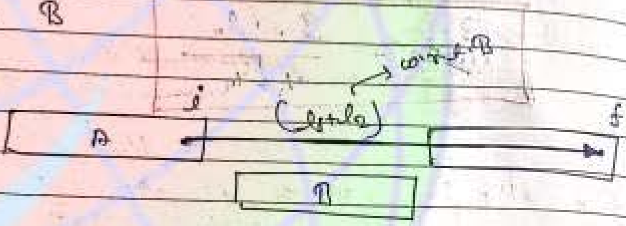
$$\boxed{\text{Time} = \frac{l_1+l_2}{v_1+v_2}}$$

eg.)



Time taken by "A" to overtake "B" is =

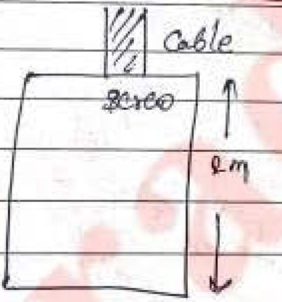
Sol^n w.r.t B



$$\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B = v_1 - v_2$$

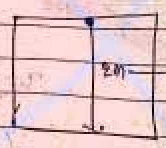
$$\boxed{\therefore \text{ time taken} = \frac{l_1+l_2}{(v_1-v_2)}}$$

eg.)



A ~~Stife~~ lift initially at rest start moving ~~up~~ upwards with an acc^n of $6 m/s^2$ at the same time a screw from this ring starts falling ~~do~~ down find time taken by the screw ~~to~~ when reaching the floor ~~of~~ of the lift

sol^n



→ w.r.t lift

$s = ut + \frac{1}{2}at^2$ → w.r.t same observer or same frame

$\vec{S}_{s/l} = \vec{U}_{s/l}\, t + \frac{1}{2}\vec{a}_{s/l}\, t^2$ → $(\vec{a}_s - \vec{a}_l)$ screw, lift

$2m = 0 + \frac{1}{2}(10-(-6))t^2$

$t^2 = \frac{1}{4}$

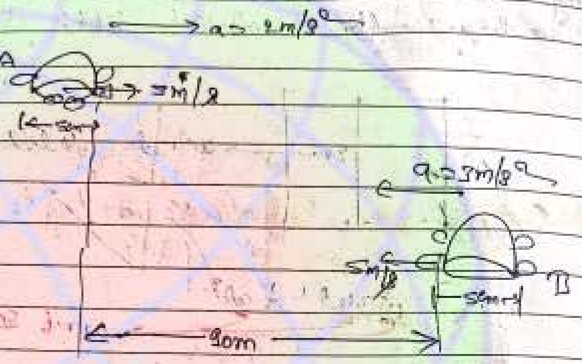
$\therefore t = \frac{1}{2}s$

Note

$-2 = 0 + \frac{1}{2}(-0-6)t^2$

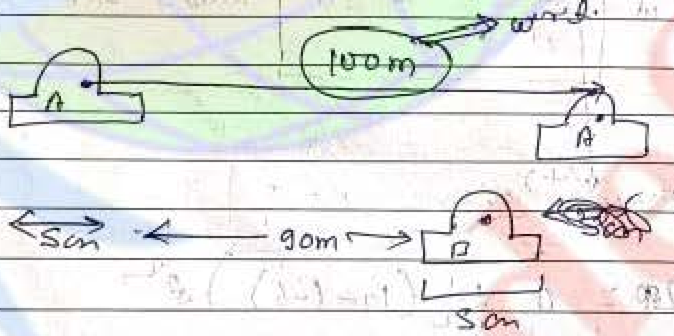
$t = \frac{1}{2}$ sec.

Q.)



Time taken to cross each other is —

Sol



$$\vec{S}_{A/B} = \vec{u}_{A/B}t + \frac{1}{2}\vec{a}_{A/B}t^2$$

$$\xleftarrow{\ominus} \quad \xrightarrow{\oplus}$$

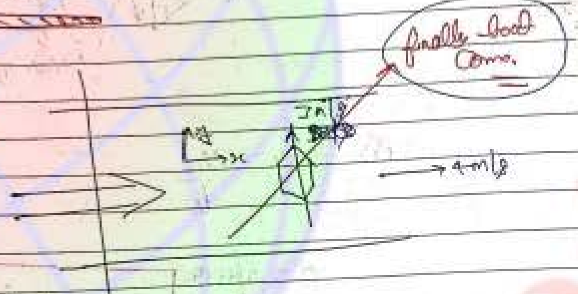
$$100 = (5-(-5))t + \frac{1}{2}(2-(-3))t^2$$

$$100 = 10t + \frac{1}{2}5t^2$$

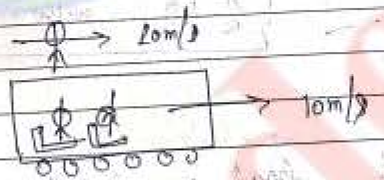
$x^2+4x-40=0$

$x=\checkmark$

Crossing of River



$|4\hat{i}+3\hat{j}| = 5m/s$



$V_b \Rightarrow$ vel. of boat or swimmer in still water
or
vel. of boat or swimmer w.r.t running water.

$V_w \rightarrow$ vel. of water

$w \rightarrow$ width of the river.

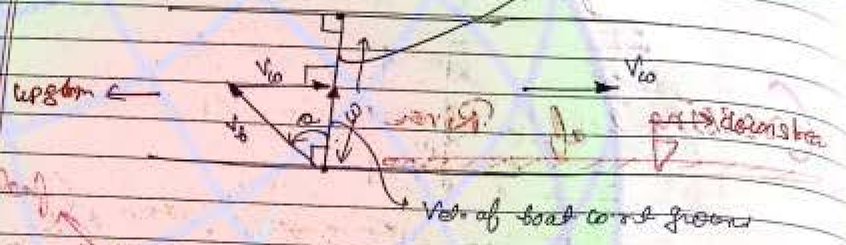
Q.1

min. Path

Q.

Note
- Time कम distance
कम length along
जा रहा है

(A) Crossing through shortest path →



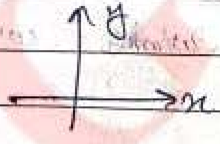
(i) $\sin\theta = \dfrac{V_\omega}{V_b}$

$$\boxed{\theta = \sin^{-1}\left(\frac{V_\omega}{V_b}\right)} \text{ with normal}$$

or $\boxed{\frac{\pi}{2} + \theta \text{ with down stream}}$

(ii) vel. of boat w.r.t ground →

vel. of boat w.r.t ground $= \vec{V_b} + \vec{V_\omega}$



$= (-V_b\sin\theta\,\hat{i} + V_b\cos\theta\,\hat{j}) + (V_$

$$= [(V_\omega - V_b \sin\theta)\hat{i} + V_b \cos\theta \hat{j}]$$

In magnitude $= \sqrt{(V_\omega - V_b \sin\theta)^2 + V_b^2 \cos^2\theta}$

$$= \sqrt{V_\omega^2 + V_b^2 - 2 V_b V_\omega \sin\theta \cdot \frac{V_\omega}{V_b}}$$

$$= \sqrt{V_b^2 - V_\omega^2}$$

So

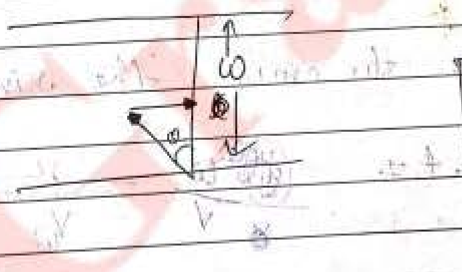
$$\boxed{|V_b + V_\omega| = \sqrt{V_b^2 - V_\omega^2}}$$

(iii) Time taken to cross the river ⇒

$$\boxed{\text{time taken to cross the river} = \frac{\omega}{\sqrt{V_b^2 - V_\omega^2}}}$$

$(V_b > V_\omega)$

Note



$$\boxed{t = \frac{\omega}{V\cos\theta}}$$

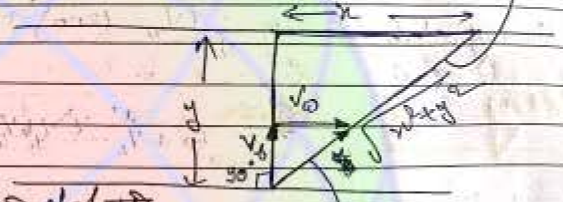
Ra.

(min time)

Ra.

Note
Time तथा
उस line के
ध्यान रखना
है।

(B) Crossing in shortest time.



vel. of boat w.r.t. ground

Note:- Time को short करें
यदि boatman की है वह
boat को River के ⊥r flow करवाएँ।
इस concept पर
Problem को solve करें।

Note

① $\boxed{\theta = 0}$ i.e. boatman has to row the boat along the normal. i.e. ⊥r to river flow

① vel. of boat w.r.t. ground = $\vec{V}_b + \vec{V}_\omega$

$\vec{V}$ in magnitude = $\sqrt{V_b^2 + V_\omega^2}$

② Time taken to cross the river

Note:- जिस line पर distance लें उसी line पर vel. को consider करना है

i.e. $t = \frac{\omega}{v} = \frac{x}{V_\omega} = \frac{\sqrt{x^2+y^2}}{\sqrt{V_b^2+V_\omega^2}}$

Note

$$\boxed{x = V_w t}$$

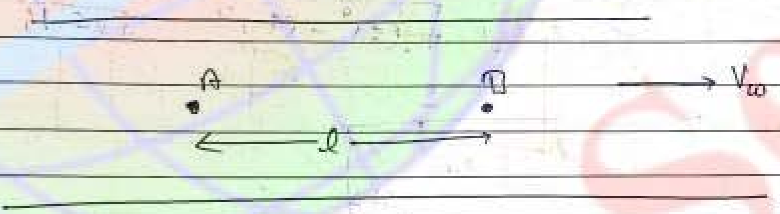
"Drifting"

जिसको minimise करना है

① by shortest path

② by shortest time

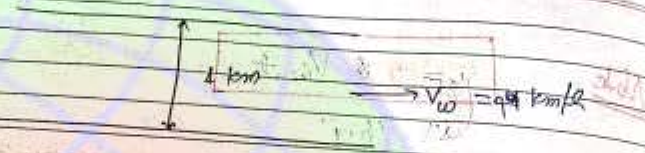
© motion along the river flow



(i) Time taken from "A" to "B" = $\frac{l}{V_b + V_w}$

(ii) Time taken from B to A ⇒ $\frac{l}{V_b - V_w}$

eg.

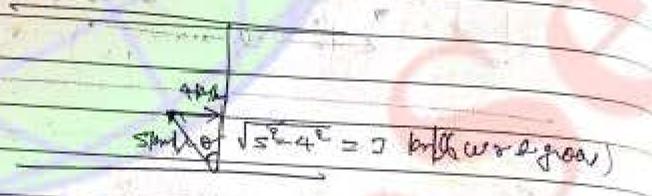


Boat man can row the boat at the rate 5 km/h in still water.

① find time taken to cross the river through shortest path

Sol(n)

$$t = \frac{1000}{\sqrt{(5)^2-(4)^2}} = \frac{1000}{\sqrt{25-16}} = \frac{1000}{3}$$



$\sqrt{5^2-4^2} = 3$ km/h (we r going)

$$t = \frac{1 km}{3 km/h} = \frac{1}{3} \text{ hours}$$

$$= \frac{1}{3} \times 60$$

$= 20$ min

Note –

$\theta = \sin^{-1}\left(\frac{4}{5}\right) = 37°$ with normal up the stream

(ii) what can be the minimum time taken to cross the river

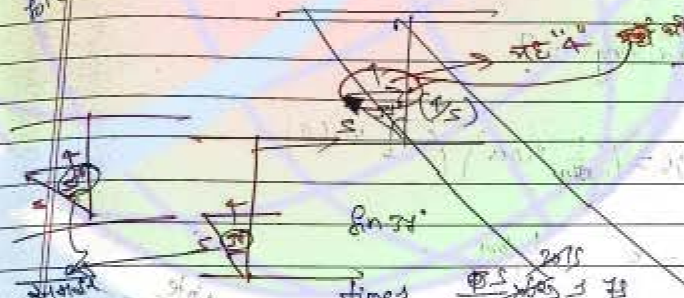
Sol(n)

Note जब भी time निकालेंगे तो [illegible] time पर तो distance का speed पता होना चाहिए।

$t = \frac{1\ km}{5\ km/h} = \frac{1}{5} h = \frac{1}{5} \times 60 = 12$ min

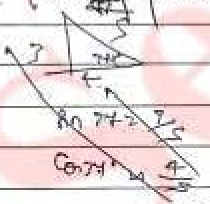
Q. 1 find time taken to cross the river if boatman row his ~~boat~~ boat at 37° with normal up the stream.

Sol.



यह "4" [illegible] की ओर है।

Note → इस तरह के question में सबसे पहले main angle (जैसे 37°) निकाल के देखने बाद question को proceed करें।



sin 37° = 3/5

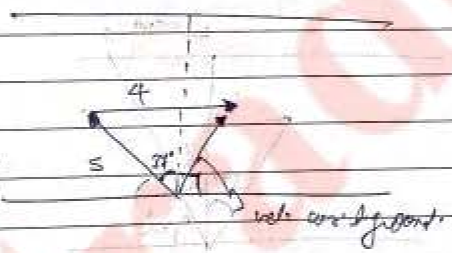
cos 37° = 4/5

times = [illegible]

Teacher's

Note → velocity वाले question को तो vector method से भी solve करना सीखो।

आसान पड़ेगा।



vel. of boat w.r.t ground $= \vec{v}_b + \vec{v}_w$

$= (-5 \sin 37° \hat{i} + 5 \cos 37° \hat{j}) + (4 \hat{i})$

$= 1\hat{i} + 4\hat{j}$

Note इसका उपयोग आगे drift velocity निकालने में किये हैं।

इसका उपयोग आगे time निकालने में किये हैं।

or

$|3\hat{i}+4\hat{j}| = \sqrt{1+16} = \sqrt{17}$

Time taken $= \frac{y}{v_y}$ $= \frac{1}{4}\frac{km}{km} = \frac{1}{4}$ hr

$= \frac{1}{4} \times 60 = 15$ min

Note

Drifting $= x$

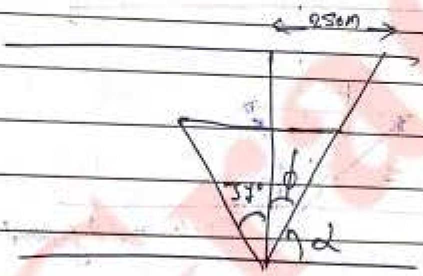
$x = \left(\frac{1}{4} \text{ hour}\right)\left(1 \text{ km/h}\right)$

$= \frac{1}{4}$ km

$= \frac{1}{4} \times 1000 = 250$ m

Note

Ans. के along
में Consider करना
है



$\phi = \tan^{-1}\left(\frac{1}{4}\right)$

$\alpha = \tan^{-1}(4)$

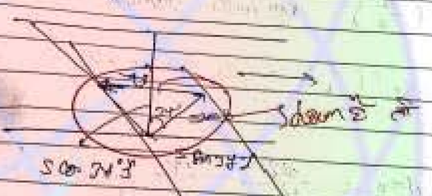
$\tan\alpha = \tan^{-1}\left(\frac{y}{x}\right)$

① find tim…

sol

① find time taken to cross the river of boat man 300 he boat at 37° with normal down the stream

sol^n



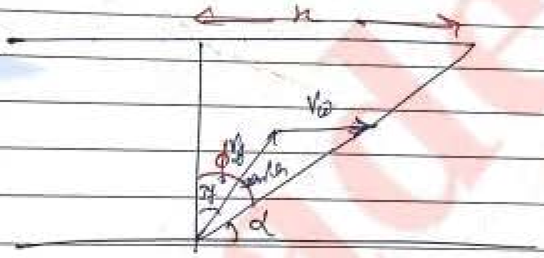
vel of w.r.t ground $\Rightarrow \vec{V}_b - \vec{V}_\omega$

$\Rightarrow (5\cos 37°\hat{j} - 5\sin 37°\hat{i}) - 4\hat{i}$

$\Rightarrow 5\times\frac{4}{5}\hat{j} - 5\times\frac{3}{5}\hat{i} - 4\hat{i}$

$\Rightarrow 4\hat{j} - 3\hat{i} - 4\hat{i}$

$\Rightarrow 4\hat{j} - 9\hat{i}$



vel of boat w.r.t ground $= V_b + V_\omega$

$\Rightarrow (5\sin 37°\hat{i} + 5\cos 37°\hat{j}) + (4\hat{i})$

$\Rightarrow (7\hat{i} + 4\hat{j})$ km/h

L-1 → 57 to 66
L-2 → 21 to 24, 37 to 42
L-7 → 2, 4, 13, 17 to 24
L-44 →

$|7\hat{i} + 4\hat{j}| = \sqrt{7^2+4^2} = \sqrt{65}$ km/h

$\alpha = \tan^{-1}\left(\frac{4}{7}\right)$ with downstream

$\theta = \tan^{-1}\left(\frac{7}{4}\right)$ with normal

½ mark

time taken $= \frac{d_y}{V_y} = \frac{1}{4} = \frac{1}{4}$ hr

and Drifting

$x = \left(\frac{1}{4}\right)\left(7\ \frac{km}{hour}\right)$

$= \frac{7}{4}$ km

$= 1750$ m

Overall approach to solve River-boat Problem

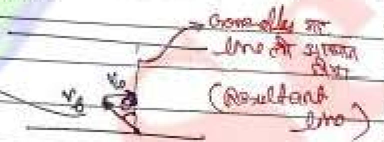
① सबसे पहले ~~सही~~ Diagram बनाएँ

② इसके बाद जो भी given हो उसको x-axis and y-axis के along Represent करें।

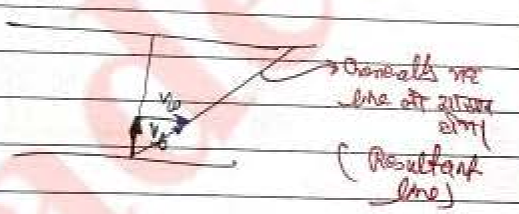
③ तब इसके बाद इस concept का प्रयोग करें।

(a) अगर shortest path है → ~~तो~~ नाव को ⊥ line से बाएँ कुछ angle से चलें

Note
इस case में यह Angle पता करें



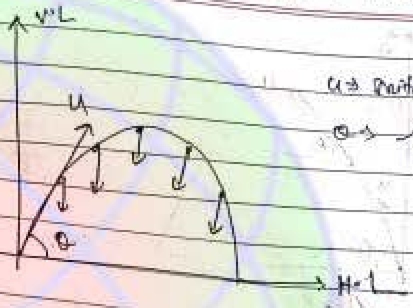
(b) अगर shortest time है → नाव को ⊥ line के along चलें।



④ इसके बाद किसी एक line के along अगर v, a, s में कोई भी दो पता हो तो तीसरे को निकाल लो।

Note → अगर axis पर directly पता ना हो तो vector की सहायता से उस line पर ला करके।

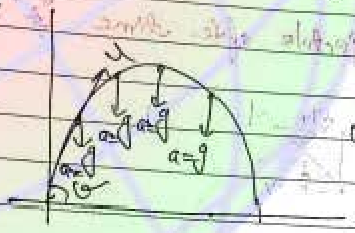
Projectile motion (motion in vertical plane under gravity)



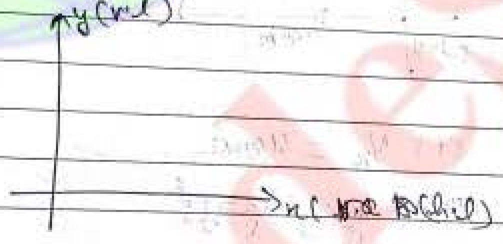
u ⇒ initial velocity.

θ ⇒ Angle of projection with horizontal line

$a = \frac{F}{m} = \frac{mg}{m} = g$



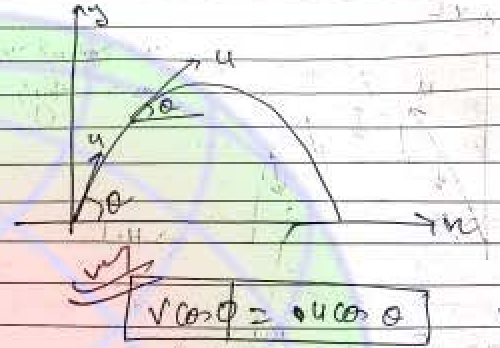
uniformly ~~acc~~ decelerated motion under gravity,



i) $u_x = u\cos\theta$ and $u_y = u\sin\theta$

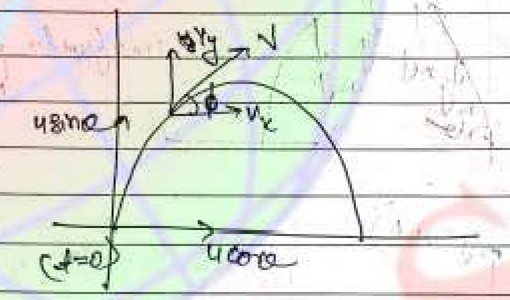
(ii) $a_x = acc^n_x = 0$ and $acc_y = a_y = (-g) = g_{downward}$

Note motion along ~~vert~~ horizontal line is uniform. horizontal velocity remains unchanged.



$\boxed{v\cos\phi = u\cos\theta}$ Imp

ⓐ Velocity of projectile after time 't' :-



(i) $v_x = u\cos\theta$

~~$v_y = u_y - gt$~~

(ii) $v_y = u_y - gt$

Qa, (ii) $\boxed{v_y = u\sin\theta - gt}$

(iii) $\vec{v} = v_x\hat{i} + v_y\hat{j}$

$= u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$

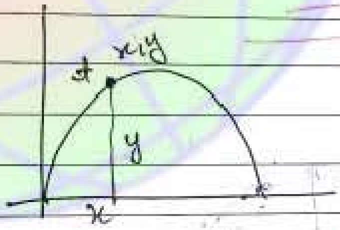
(iv) $|\vec{v}| = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$

= speed.

(v) $\tan\phi = \frac{v_y}{v_x}$

$$\boxed{\phi = \tan^{-1}\left(\frac{u\sin\theta - gt}{u\cos\theta}\right)}$$

⑧ Position after some time "t" →



(i) $x = u_x t$

$x = u\cos\theta\, t$

(ii) $s_y = u_y t - \frac{1}{2}gt^2$

$y = u\sin\theta t - \frac{1}{2}gt^2$

(iii) Position vector

$\vec{r} = x\hat{i} + y\hat{j}$

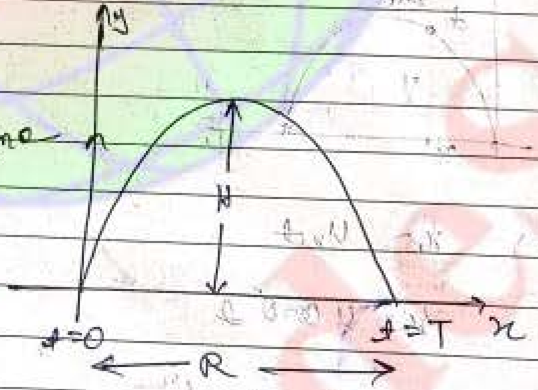
(iv) Equation of trajectory :-

$$y = u\sin\theta \cdot \frac{x}{u\cos\theta} - \frac{g}{2} \cdot \frac{x^2}{u^2\cos^2\theta}$$

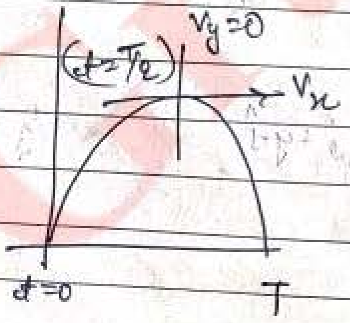
$$\boxed{y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}}$$

↳ [Parabolic motion]

(C) T, H, and R



(i) Time of flight (T) :→



$V_y = u\sin\theta - g t$

$0 = u\sin\theta - g\frac{T}{2}$

$$\boxed{T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}}$$

(ii) maximum height (H) :→

$V_y^2 = u_y^2 - 2g S_y$

$0 = u_y^2 - 2gH$

$$\boxed{H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}}\ \boxed{= \frac{(u\sin\theta)^2}{2g}}$$

(iii) Horizontal range (R):→

$x = u_x t +$

or $R = u_x \cdot T$

$$\boxed{R = u_x\cdot\left(\frac{2u_y}{g}\right) = \frac{2(u\cos\theta)(u\sin\theta)}{g}}$$

$$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$

Note

(i) $$\boxed{y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}}$$ → y in terms of u

$$y = x\tan\theta\left(1 - \frac{gx^2}{2u^2\cos^2\theta \cdot \cancel{x}\tan\theta}\right)$$

$$y = x\tan\theta\left(1 - \frac{x}{2u^2\sin\theta\cos\theta/g}\right)$$

$$\boxed{y = x\tan\theta\left(1 - \frac{x}{R}\right)}$$

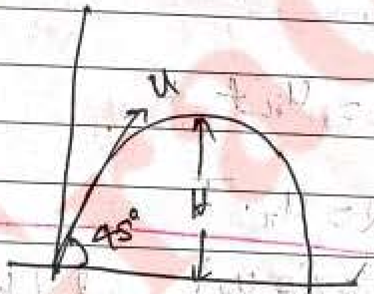
→ y in terms of R

(ii) if $\theta = 45°$

$$R = \frac{u^2}{g}$$ (maximum possible value)

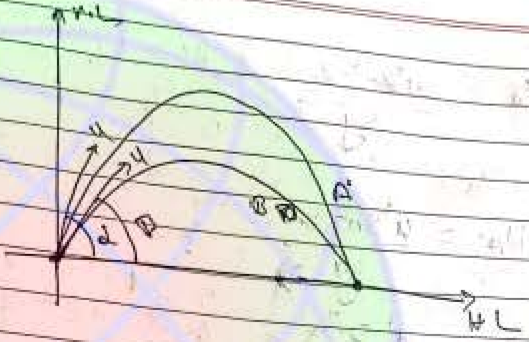
and

$$H = \frac{u^2}{4g}$$



$$\boxed{R = 4H}$$

②



$$\boxed{\alpha + \beta = \frac{\pi}{2}}$$

Note

$$\boxed{\begin{array}{c} \sin\alpha = \cos\beta \\ \hline \cos\alpha = \sin\beta \end{array}}$$

(i) $R_A = \frac{u^2 \sin 2\alpha}{g}$

and

$$R_B = \frac{u^2 \sin 2\beta}{g}$$

$$= \frac{u^2 \sin(\pi - 2\alpha)}{g}$$

$$= \frac{u^2 \sin 2\alpha}{g}$$

$\therefore R_A = R_B$

ii) $H_A = \frac{u^2\sin^2\alpha}{2g}$

any

$H_A = \frac{u^2\sin^2\beta}{2g}$

$H_B = \frac{u^2\cos^2\alpha}{2g}$

$$\boxed{\frac{H_A}{H_B} = \frac{\tan^2\alpha}{1} = \frac{1}{\tan^2\beta}}$$

and

$H_A H_B = \frac{u^2\sin^2\alpha}{2g} \times \frac{u^2\cos^2\alpha}{2g}$

$H_A H_B = \frac{(u^2\sin\alpha\cos\alpha)^2}{4g^2}$

but

$R_A = R_B = R$

$= \frac{2u^2\sin\alpha\cos\alpha}{g}$

$H_A H_B = \frac{\left(\frac{Rg}{2}\right)^2}{4g^2}$

$$\boxed{H_A H_B = \frac{R^2}{16}}$$

(iii) $T_A = \frac{2u\sin\alpha}{g}$

and

$T_B = \frac{2u\sin\beta}{g}$

$= \frac{2u\cos\alpha}{g}$

$$\therefore \boxed{\frac{T_A}{T_B} = \frac{\tan\alpha}{1} = \frac{1}{\tan\beta}}$$

$$\boxed{T_A \cdot T_B = \frac{4u^2\sin\alpha\cos\alpha}{g^2}}$$

$$= \frac{4\left(\frac{Rg}{2}\right)}{g^2} = \frac{2R}{g}$$

$$\boxed{T_A \cdot T_B = \frac{2R}{g}}$$

~~$T_A \cdot T_B \propto R$~~

$$\boxed{T_A \cdot T_B \propto R}$$

(E) change in momentum in time "t".

$$\Delta\vec{P} = \vec{P}_t - \vec{P}_0$$

↳ after time "t" ↳ initial

$$= m(\vec{v}_t - \vec{v}_0)$$

where,

$$\vec{v}_t = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$$

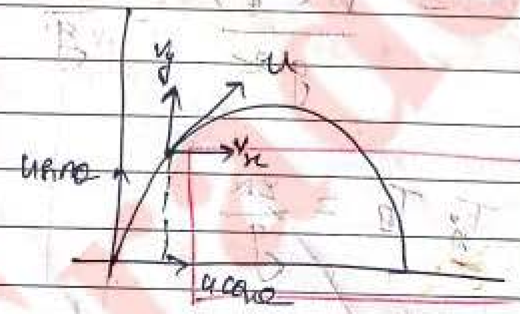
$$\vec{v}_0 = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$$

$$\Delta\vec{P} = m(-gt\,\hat{j}) = -mgt\,\hat{j}$$

$$\boxed{|\Delta\vec{P}| = mgt}$$ (vertically downwards

$\Delta P = mgt$

(f) kinetic energy at a height "h".



(i) $v_x = u\cos\theta$

(ii) $v_y^2 = u_y^2 - 2g\,s_y$

$$v_y^2 = u^2\sin^2\theta - 2gh$$

(iii) $\vec{v} = v_x\hat{i} + v_y\hat{j}$

$$v^2 = v_x^2 + v_y^2$$

or $v^2 = u^2\cos^2\theta + (u^2\sin^2\theta - 2gh)$

$$\boxed{v^2 = u^2 - 2gh}$$

(v) K.E $- k = \frac{1}{2}mv^2$

$$k = \frac{1}{2}m(v^2 - 2gh)$$

$$k = \frac{1}{2}mu^2 - mgh$$

$$\boxed{k = k_0 - mgh}$$

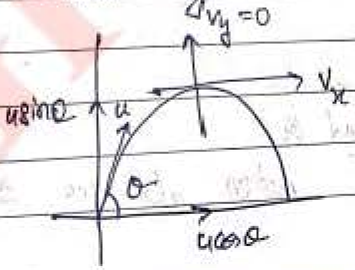
Note

(i) Increase in PE = mgh

(ii) No change in mechanical energy.

★ Special Points :-

① K.E at maximum height :-



$k = \frac{1}{2} m v_x^2 = \frac{1}{2} m (u\cos\theta)^2$

$$\boxed{k = \left(\frac{1}{2} m u^2\right)\cos^2\theta}$$

$$\boxed{k = k_0 \cos^2\theta}$$

minimum value of K.E

V.V.I important point

Note

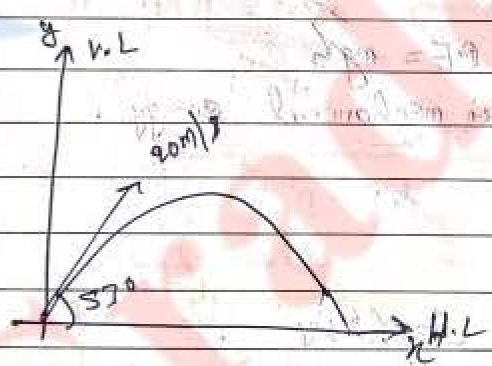
(1) Increase in P.E = mgH

$= mg\frac{u^2 \sin^2\theta}{2g}$

$$\boxed{= \left(\frac{1}{2} m u^2\right) \sin^2\theta}$$

$= K_0 \sin^2\theta$

eg.1



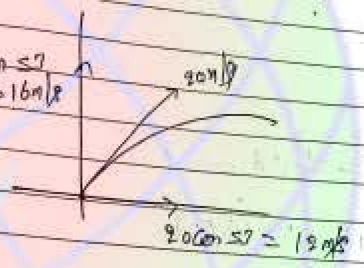
find

(i) Find T, H and R

(ii) Direction of motion after one sec. of projection.

(iii) Speed aft one sec of

~~T = 2u sinθ~~



(i) $T = \frac{2u\sin\theta}{g} = \frac{2\times 16}{g} = 3.2$ sec.

and $H = \frac{20(\sin 53)^2}{2g} = \frac{16^2}{20} = \frac{256}{20} = 12.8$ m

$$\text{Range} = \frac{2(20\cos 57^\circ)(20\sin 57)}{g}$$

$$= \frac{2(16)(12)}{10} = 38.4\ \text{m}$$

(ii) $\tan\phi = \frac{u\sin\theta - gt}{u\cos\theta}$

~~$\tan\phi = u\sin\theta$~~

~~$\cos\phi$~~ $=$ ~~$4\sin 57$~~

$\sin\phi =$

(iii) $\vec{v} = (20\cos 30^\circ)\hat{\imath}$

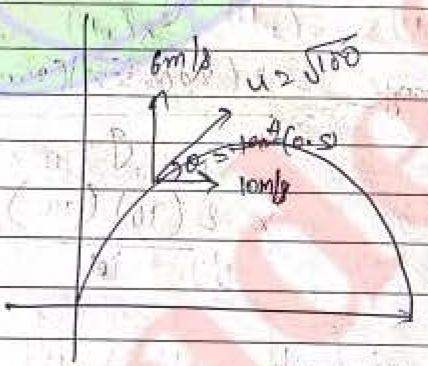
(iv) $y = u_y t - \frac{1}{2}gt^2$

$= 10(1) - \frac{1}{2}(10)(1)^2$

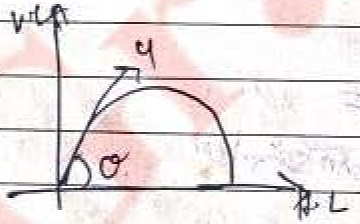
$y = 5\ m$

Direction of motion after one sec. of projection

Speed after one sec. of projection

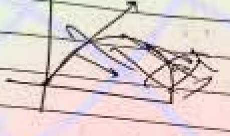


Q5



At what ~~stone~~ time after projection velocity is ⊥r to the Initial velocity.

Sol>



$$\vec{V}_t \perp \vec{u}$$

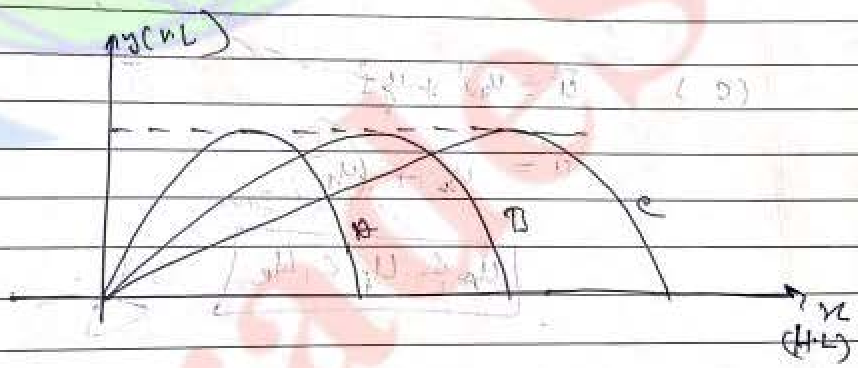
$$\Rightarrow (u\cos\theta\,\hat{i} + u\sin\theta - gt\hat{j})\cdot(u\cos\theta\,\hat{i} + u\sin\theta\hat{j}) = 0$$

$$u^2\cos^2\theta + (u\sin\theta - gt)(u\sin\theta) = 0$$

$$u^2\cos^2\theta + u^2\sin^2\theta - gt + u\sin\theta = 0$$

$$\boxed{t = \frac{u}{g\sin\theta} = \frac{u\,\mathrm{cosec}\,\theta}{g}}$$ Ans

Eg)



which of the following is correct option

(A) $T_A = T_B = T_C$ (B) $(u_x)_A < (u_x)_B < (u_x)_C$

(C) $u_A < u_B < u_C$ (D) All of these

sol^n

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{u_y^2}{2g}$$

$$\underbrace{u_y}_{\text{Same}} = \sqrt{2Hg}$$

(A) $$T = \frac{2u\sin\theta}{g} = \frac{2\overset{\text{Same}}{u_y}}{g}$$

$$\therefore T_A = T_B = T_C$$

(B) $$R = \frac{2(u\cos\theta)(u\sin\theta)}{g} = \frac{2u_x u_y}{g}$$

$$\boxed{\begin{gathered} R \propto u_x \\ \therefore (u_x)_A < (u_x)_B < (u_x)_C \end{gathered}}$$

(C) $$\vec{u} = u_x \hat{i} + u_y \hat{j}$$

$$u^2 = u_x^2 + \underbrace{u_y^2}_{\text{Same}}$$

$$\boxed{u_A < u_B < u_C}$$ Ans

eg) For a projectile thrown in vertical plane

$$y = \sqrt{3}x - \frac{gx^2}{2}$$

find (i) angle of projection
(ii) Initial speed

Solⁿ

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$\tan\theta = \sqrt{3}$ $\therefore \theta = 60°$

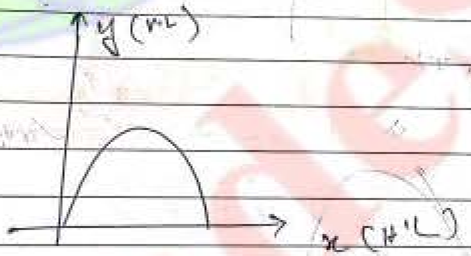
and

$u^2\cos^2\theta = 1$

$u = \frac{1}{\cos 60°} = 2\,m/s$

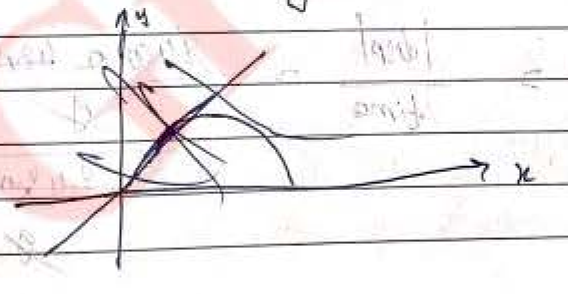
eg:) For a projectile thrown in vertical plane

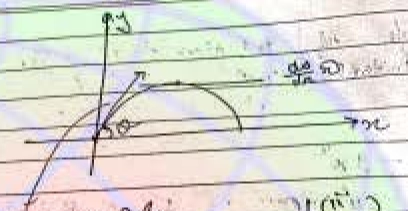
$y = ax - bx^2$



(i) find angle of projection
(ii) maximum height.

Solⁿ





(i) $\frac{dy}{dx} = a - 2bx$

$\left(\frac{dy}{dx}\right)_{x=0} = a$

$\tan\theta = a$

$\theta = \tan^{-1}(a)$

(ii) If $\frac{dy}{dx} = 0$

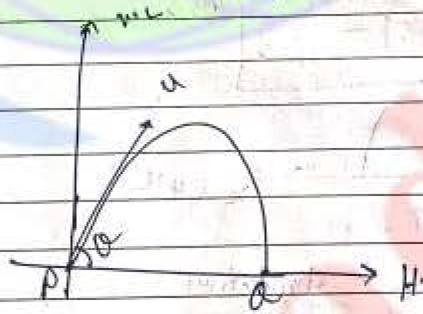
$a - 2bx = 0$

$x = \frac{a}{2b}$

$\therefore y = a\cdot\frac{a}{2b} - \cancel{b}\cdot\frac{a^2}{4b^{\cancel{2}}}$

$\boxed{y = \frac{a^2}{4b}}$ (maximum height)

eg.)



Avg. vel. b/w "P" and "Q" is ⇒

Sol)

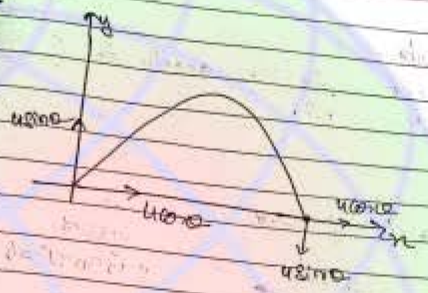
$$|\vec{v}_{avg}| = \frac{|disp|}{time} = \frac{\frac{\cancel{2}u\cos\theta\, u\sin\theta}{\cancel{g}}}{\frac{\cancel{2}u\sin\theta}{\cancel{g}}} = u\cos$$

Note

(ii)

eg.)

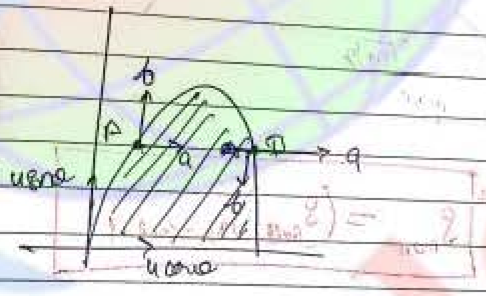
Note



$$|\vec{V}_{avg}| = \frac{(u\cos\theta\hat{i} + \cancel{u\sin\theta\hat{j}}) + (u\cos\theta\hat{i} - \cancel{u\sin\theta\hat{j}})}{2}$$

$$= u\cos\theta$$

(ii)

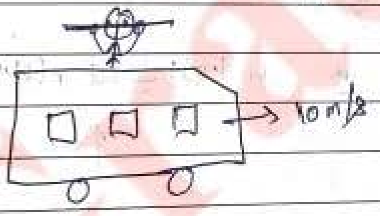


$$V_{avg} = \frac{(a\hat{i}+b\hat{j})+(a\hat{i}-b\hat{j})}{2}$$

$$= a\hat{i}$$

$$= u\cos\theta\hat{i}.$$

Eg)

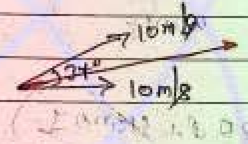


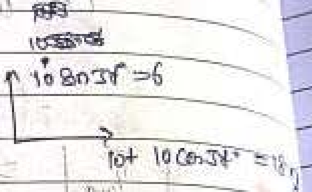
man throws a ball in forward direction at an angle of 37° above the horizontal line with

speed of 10 m/s.
what should be the acceleration of bus so that
he can catch the ball.

Sol^n

vel. of ball w.r.t. ground

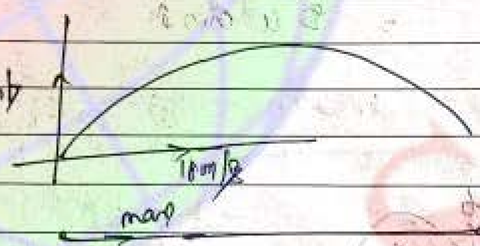




$t = \frac{2u\sin\theta}{g}$

$t = \frac{2(6)}{10}$

$= 1.2$ sec.



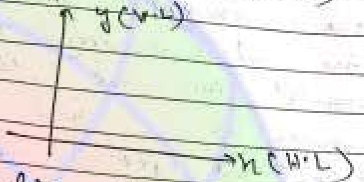
$$\boxed{S_{man} = (S_{ball})_{horizontal}}$$

$10(1.2) + \frac{1}{2} \times a \times (1.2)^2 = \frac{2(6)(18)}{g}$

a2L

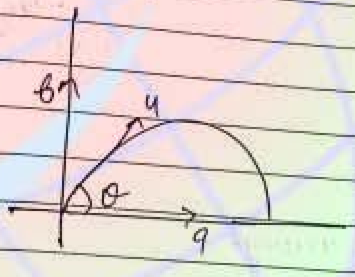
Δ-1 → 1 to 35
L-2 → 2,3, 4,5 6,8, 11, 12, 13 to 22
Page 9, 23, 24

Q) for a particle thrown in ~~vertical~~ vertical plane initial velocity is (ai + bj)



find relation b/w "a" and "b" if horizontal rang is twice of maximum height.

Sol:



$u\cos\theta = a$

$u\sin\theta = b$

$R = 2H$

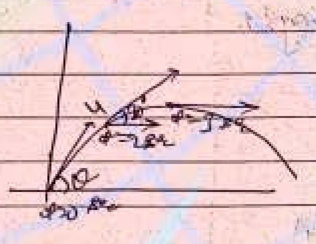
$$\frac{2(u\cos\theta)(u\sin\theta)}{g} = \frac{2\cdot(u\sin\theta)^2}{2g}$$

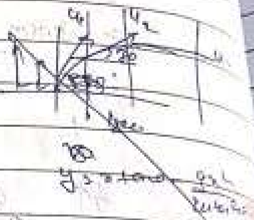
$$\frac{2(a)\cancel{(b)}}{\cancel{g}} = \frac{\cancel{2}(b^{\cancel{2}})}{\cancel{2g}}$$

$2g = \cancel{b}\ \boxed{2a = b}$ Ans

Q) A body is projected with some angle above the horizontal line, after two sec of projection it is moving at an angle of with horizontal line and after one more sec it is moving in horizontal direction find angle of projection and initial speed.

Sol





Note:

$$\tan\phi = \frac{V_y}{V_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

Now

at, $t = 2$

$$\tan 30^\circ = \frac{u\sin\theta - (10)(2)}{u\cos\theta}$$

$$\frac{u\cos\theta}{\sqrt{3}} = u\sin\theta - 20 \quad \text{—(ii)}$$

& at $t = 3$

$$\tan 0 = \frac{u\sin\theta - (10)(3)}{u\cos\theta}$$

$$\therefore u\sin\theta = 30 \quad \text{—(i)}$$

Now from eq (i)

$$\frac{u\cos\theta}{\sqrt{3}} = 30 - 20$$

$u \cos\theta = 10\sqrt{3}$ — eqⁿ 1

Solving eqⁿ no ① and ②

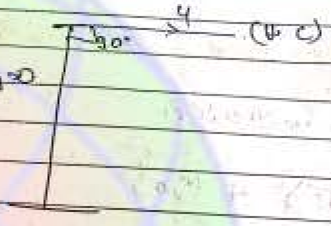
$u^2 = (10\sqrt{3})^2 + (10)^2$

$u = 20\sqrt{3}$ m/s

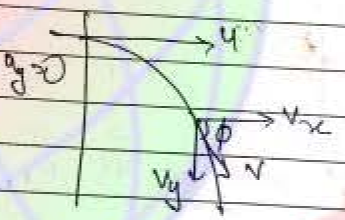
and $\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \pi/6$

Horizontal projection



(a) vel after time t ⇒



u, v, a, t

$v = \underset{u0}{u} + gt$

$\boxed{v = gt}$

(i) $v_x = u$

(ii) $v_y = gt$ (downwards)

(iii) $\vec{v} = v_x \hat{i} + v_y \hat{j}$

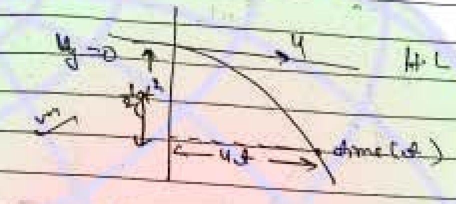
or

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\boxed{v = \sqrt{u^2 + g^2 t^2}}$$

(iv) $\tan \phi = \frac{gt}{u}$ ∴ $\phi = \tan^{-1}\left(\frac{gt}{u}\right)$

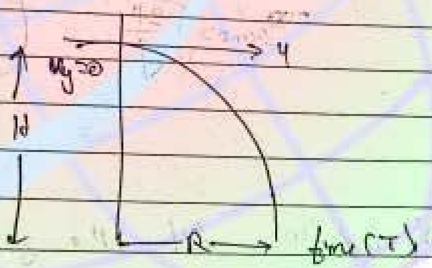
(b) Horizonal and vertical distance travelled in time "t".



vertical distance = $\frac{1}{2}gt^2$

Horizontal distance = ut

(c) "T" and "R"



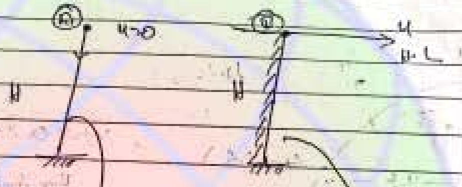
(i) $H = \frac{1}{2}gt^2$

$$\boxed{\therefore \quad T = \sqrt{\frac{2H}{g}}}$$

(ii) $R = uT$

$$\boxed{R = u\sqrt{\frac{2H}{g}}}$$

Concept A



दोनों body एक साथ नीचे गिरेंगे क्योंकि vertical acc^n को ~~[illegible]~~ change नहीं कर रहे हैं

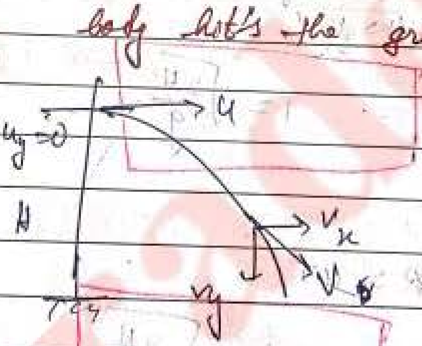
$H = \frac{1}{2}gt^2$

$t = \sqrt{\frac{2H}{g}}$

$t_{B} = \sqrt{\frac{2H}{g}}$ (No component of u)

i.e. "A" and "B" hit the ground simultaneously

(d) KE when body hit's the ground



(i) $v_x = u$

(ii) $v_y^2 = 2gH$

(iii) $v^2 = v_x^2 + v_y^2$

m,a,8

$v^2 = u^2 + 2gH$

$\boxed{v^2 = 2gH}$

$$\boxed{v^2 = u^2 + 2gH}$$

(iv) $K.E = \frac{1}{2}mv^2$

$$K = \frac{1}{2}mv^2 + mgH$$

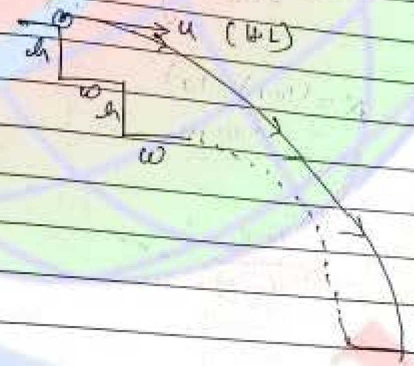
$$\boxed{K = K_0 + mgH}$$

Note

(i) P.E decreases by mgH

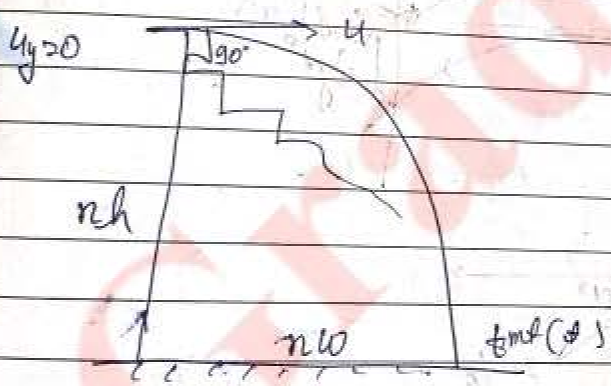
(ii) No change in mechanical energy.

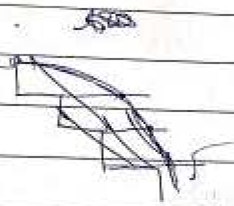
eg.)



Ball just hit the edge of nth step. ~~find the ball~~ Find 'n' in terms of 'h', 'w', 'u' and 'g'.

8/4





$nh = ut$ — (i)

and

$nh = \frac{1}{2}gt^2$

or $nh = \frac{1}{2}g\frac{n^2w^2}{u^2}$

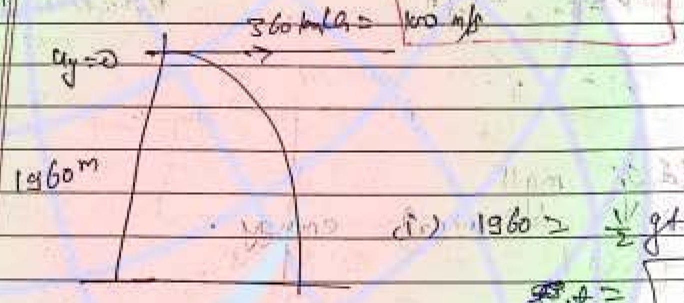
Ans

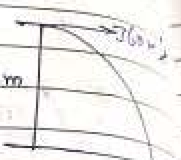
$$\boxed{n = \frac{2hu^2}{gw^2}}$$

$$\boxed{\frac{1\ km}{h} = \frac{5}{18}\frac{m}{s}}$$

eg1 A fight plane flying at a height of 1960 from ground level in horizontal direction with speed of 360 m/s its pilot release ~~drop~~ a packet find horizontal distance travelled by the packet in reaching the ground.

Sol^n

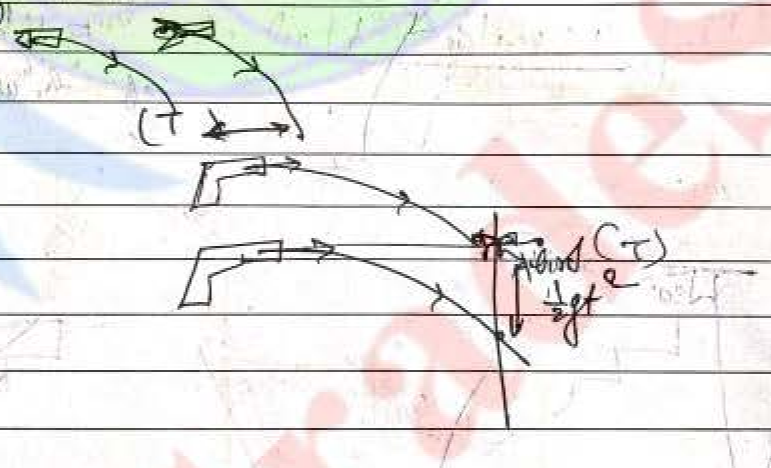


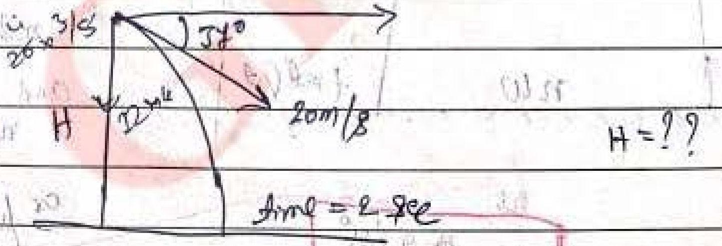


(i) $1960 = \frac{1}{2}gt^2$

$t = \sqrt{\frac{2 \times 1960}{9.8}} = 20\ sec$

(ii) $R = (100)(20)$
$= 2000\ m$





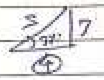
$H = ??$

time = 2 sec

$v = u + at$

$20 \times \cos 37°$

$20 \times \frac{4}{5} = \boxed{16}$



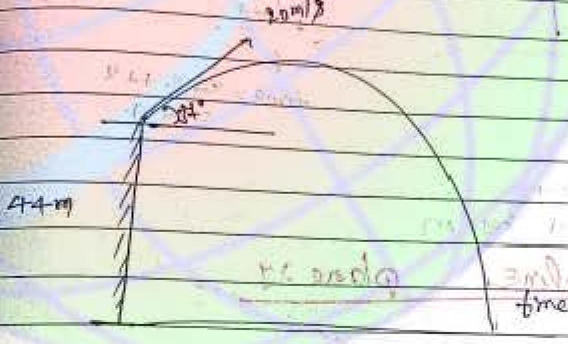
Teale

$H = ut + \frac{1}{2} g t^2$

$H = 12 \times 2 + \frac{1}{2} \times 10 \times 2 \times 2$

$= 24 + 20$

$= 44 m$



times?

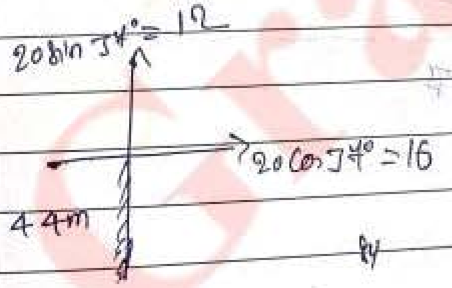
$v = u + at$

$s = ut + \frac{1}{2} a t^2$

$44 = 20t + \frac{1}{2} \times 10 t^2$

$5t^2 + 20t - 44 = 0$

Teal

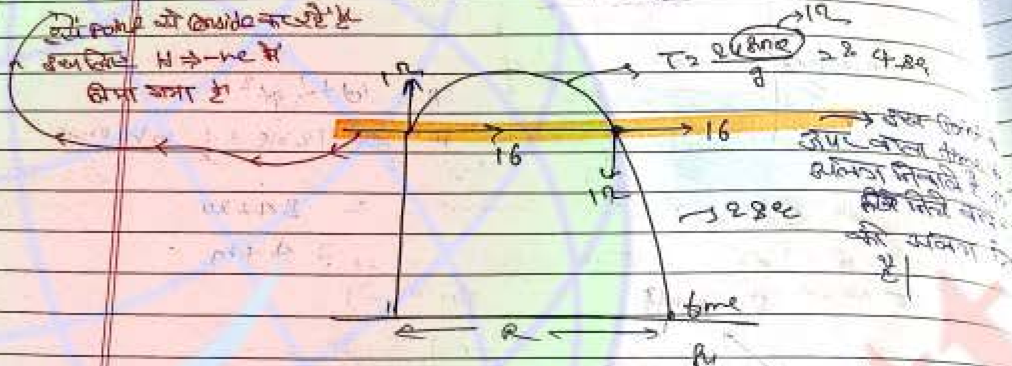


$S_y = 12t - \frac{1}{2}gt^2$

$\Rightarrow -44 = 12t - 5t^2$

$\Rightarrow 5t^2 - 12t - 44 = 0$

$t = 4.4$ sec



इस point को consider कर रहे है इसलिए H → -ve है लिया जाता है

$T = \frac{2(u\sin\theta)}{g} = 2.4$ sec, $u\sin\theta \rightarrow 12$

→ 2 sec

इसके लिए ऊपर वाला time अलग निकाले फिर नीचे वाले की अलग से है।

R,

time = 2.4+2

= 4.4

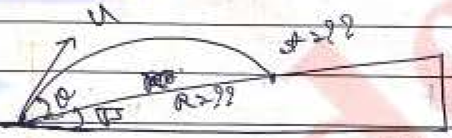
Note

R = 16 × 4.4

= 70.4 m

motion in Inclined plane :-

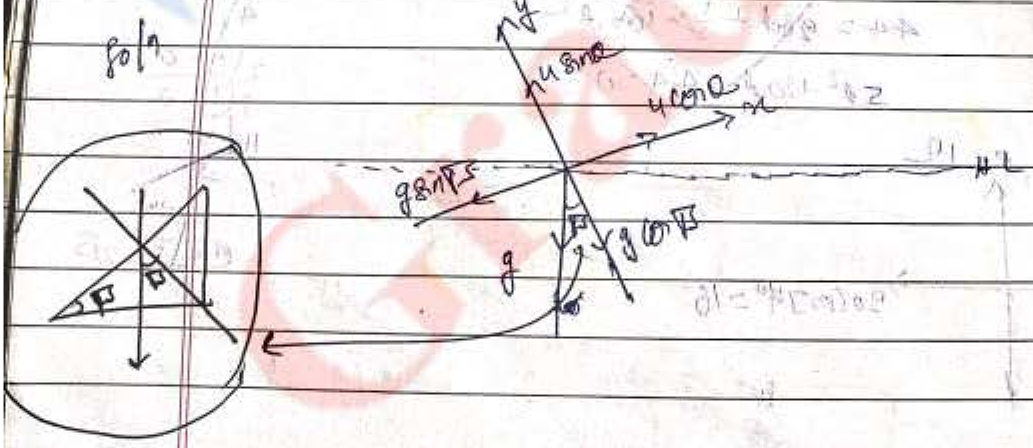
Q1)



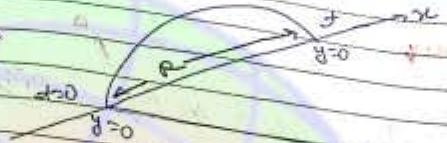
β → Angle of Inclination

θ → Angle of projection with incline.

Sol)



(i)



$S_y = u\sin\theta - \frac{1}{2} g(\cos\phi) t^2$

यहाँ → negative लिये क्योंकि u sinθ और g cos ϕ दोनों opp. direction में हैं।

$0 = u\sin\theta - \frac{1}{2} g\cos\phi\, t^2$

$t = \frac{2u\sin\theta}{g\cos\phi}$

$$\boxed{t = \frac{2u\sin\theta}{g\cos\phi}}$$

(ii) $S_x = u\cos\theta\, t - \frac{1}{2} g\sin\phi \cdot t^2$

Note: यहाँ -ve लिया क्योंकि u cosθ और g sin ϕ दोनों ही opp. direction में हैं।

$R = u\cos\theta \frac{2u\sin\theta}{g\cos\phi} - \frac{1}{2} g\sin\phi \cdot \frac{4u^2\sin\theta}{g^2\cos^2\phi}$

$\boxed{\cos(a+b) = \cos a\cdot\cos b - \sin a\,\sin b}$

$R = \frac{2u^2\sin\theta}{g\cdot\cos^2\phi}\left[\cos\theta\cdot\cos\phi - \sin\theta\cdot\sin\phi\right]$

$$\boxed{R = \frac{2u^2\sin\theta\cos(\theta+\phi)}{g\cos^2\phi}}$$

Special case:-

$$R = \frac{u^2}{g\cos^2\beta}\cdot\left(2\cos\overset{A}{(\theta+\beta)}\,\sin\overset{B}{\theta}\right)$$

$$R = \frac{u^2}{g\cos^2\beta}\left[\sin(2\theta+\beta) - \sin(\beta)\right]$$

if $2\theta+\beta = \frac{\pi}{2}$

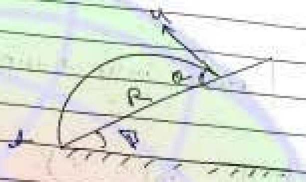
or

$$\boxed{\theta = \frac{\pi}{4} - \frac{\beta}{2}}$$ imp

so

$$\boxed{R_{max} = \frac{u^2(1-\sin\beta)}{g\cos^2\beta} = \frac{u^2}{g(1+\sin\beta)}}$$ (maxim

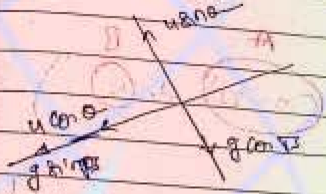
Q2.

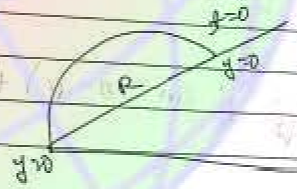


t = ??

R = ??

Sol:





$s_y = u\sin\theta t - \frac{1}{2} g(\cos\beta)t^2$

$0 = u\sin\theta \cancel{t} - \frac{1}{2} g\cos\beta . t^2$

$$\boxed{\therefore t = \frac{2u\sin\theta}{g\cos\beta}}$$

(ii) $S_x = u\cos\theta t + \frac{1}{2} g\sin\beta . t^2$

$R = u\cos\theta . \frac{2u\sin\theta}{g\cos\beta} + \frac{1}{2} \cancel{g}\sin\beta . \frac{4u^2\sin\theta}{g^{\cancel{2}}\cos^2\beta}$

(L-1 to L-2) ⇒ Complete

$$R = \frac{2u^2 \sin\theta}{g\cos^2\beta}\left[\cos\theta\cdot\cos\beta + \sin\theta\cdot\sin\beta\right]$$

$$R = \frac{2u^2 \sin\theta}{g\cos^2\beta}\cos(\theta-\beta)$$

Special case

$$R = \frac{u^2}{g\cos^2\beta}\left(2\cos\overset{A}{(\theta-\beta)}\sin\overset{B}{\theta}\right)$$

$$R = \frac{u^2}{g\cos^2\beta}\left[\sin(2\theta-\beta) - \sin(-\beta)\right]$$

$$= \frac{u^2}{g\cos^2\beta}\left(\sin(2\theta-\beta) + \sin\beta\right)$$

if

$$2\theta - \beta = \frac{\pi}{2}$$

or

$$\boxed{\theta = \frac{\pi}{4} + \frac{\beta}{2}}$$ v.v. Important

$$\boxed{R_{max} = \frac{u^2(1+\sin\beta)}{g\cos^2\beta} = \frac{u^2}{g(1-\sin\beta)}}$$

Newtons law of motion

↳ Dynamics (Cause of motion)

Force :→

Force is something by virtue of which we can change the linear motion of a particle.

Force or it is interaction b/w two bodies.

i) It is a vector quantity so added or subtracted according to triangle rule of vector addition

ii) S.I unit is Newton (N) or kg m/s²

iii) >

a_t → always due to force. a_c

F_t F_c

along tangent — ⊥r to tangent

(Responsible for speed) — Responsible for direction

Types of fundamental forces

i) → Electromagnetic force → force b/w charge.

ii) → Gravitational force → force b/w mass.

iii) → Nuclear force → force b/w nucleons. eg. P-P, N-P, N-N

iv) → Weak force.

→ β^+ — β^-

$e^+ + \nu$ — $e^- + \bar{\nu}$ — Nuclear force

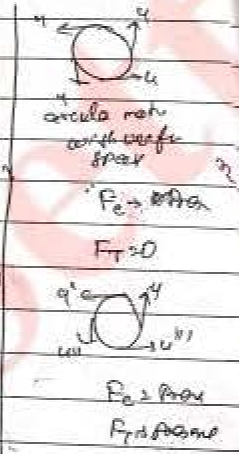
Nucleus → e^-

$n \rightarrow p^+ + e^- + \bar{\nu}$ ← Antineutrino

कहाँ आ गए

$p \rightarrow n + e^+ + \nu$

weak force is applicable for β-decay.



✡ Some electromagnetic forces used in mechanics.

① Tension in string → Pulling Nature

② Tension in Spring → pulling and ~~pushing~~ pushing

③ Normal contact force. } Contact force.

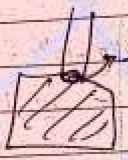
Note (I) ④ friction force

(II) ⑤ force by the gravity.

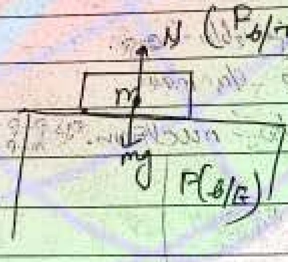
(Pushing force)

Compulsory

(Pulling

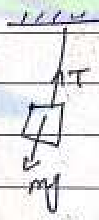


eg)



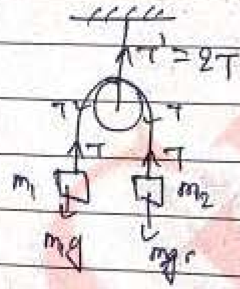
($N = mg$)

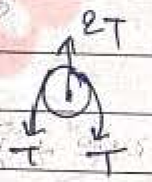
eg)



($T = mg$)

eg.)





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boat will remain stati

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$\frac{2h}{g} \cdot \frac{1}{\sin\theta}$

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* Some electromagnetic forces used in mechanics

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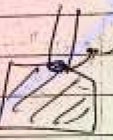
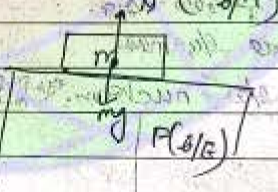


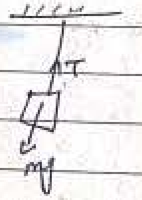
Attraction develop due to charge distribution.

Q1)



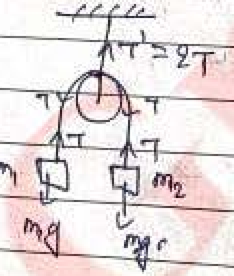
$(N = mg)$

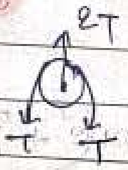
Q)



$(T = mg)$

Q2)





CAREER
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DAILY PRA

…n of linear Momentum

…ectric fan is placed on a boa…
…ary on the surface of water o…
…blown with it on the sails…
…of the following statement is…
…boat will move with uniform…
…boat will move with uniform…
…boat will remain stationary…
…boat will move irregularly…

…ck is released from the t…
…of height h and angle o…
…me taken by the block t…
…plane is given by -
$\sqrt{\frac{2h}{g}}$ (2)
$\sqrt{\frac{2h}{g}} \cdot \frac{1}{\sin\theta}$ (4)

…oins are placed on…
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…is the magnitude…
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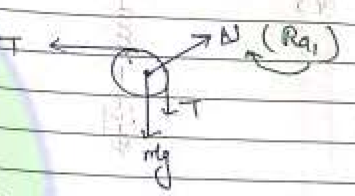
find Normal force



$\boxed{T = mg}$

Note:- सभी भी body के rest में रहने के लिए उस body पर लगने वाले सारा force zero होना चाहिए।

ये जो निकालना है उसकी इसी दिशा में direction में नहीं होता है इसलिए direction और component की सहायता से पता करेंगे

$\vec{N} + (-T\hat{i}) + (-(T + m'g)\hat{j})$

$\vec{N} = T\hat{i} + (T + m'g)\hat{j}$

$\vec{N} = mg\hat{i} + (mg + m'g)\hat{j}$

$N = \sqrt{m^2g^2 + (m+m')^2g^2}$

$= \sqrt{m^2 + (m+m')^2}\,g$

$\phi = \tan^{-1}\left(\frac{mg + m'g}{mg}\right)$ (क्योंकि $\tan\theta = \frac{y}{x}$ होता)

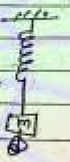
$= \tan^{-1}\left(\frac{m+m'}{m}\right)$

Step 1st :- सभी force को दर्शाइए।

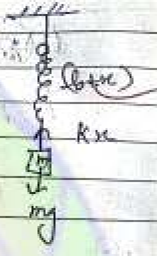
Step 2nd :- To test → sum of all force is Zero

Rq1 , Rq2 , Rq3

(v)



Soln



→ यह natural ... है रो, ... ने बढ़ ...

mg = kx

↳ this ...

Rq1 →

Note

Rq2 →

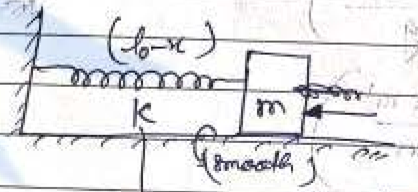
l_0 → ~~as~~ Natural length
K → ~~of~~ force constant
or
spring constant.

Sor

Here $T = kx$ → extension

y खींची हुई हो तो खींचने का काम ...

(vi)



→ Represents hardness of the spring.

Rq3

$N = mg$
$F = kx$

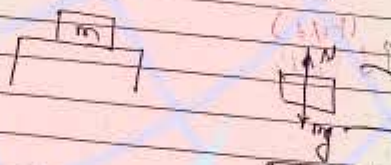
Third ... only ...

→ force of action अब सही तरीके से ...

ote

① F_{AB} → force exerting body.
↳ force experiencing body

②



$\boxed{N = mg}$

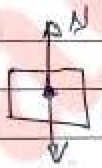
ये दोनो Action-reaction की pair में नहीं है

Condition →

① If two force action-reaction pair → (i) magnitude equal
[illegible] → (ii) direction opp.

② प्रतीत नहीं but two force of equal mag. and opp direction is not sufficient condition of Action-reaction pair

$F_{AB} \longleftrightarrow F_{B/A}$
Action reaction pair force.

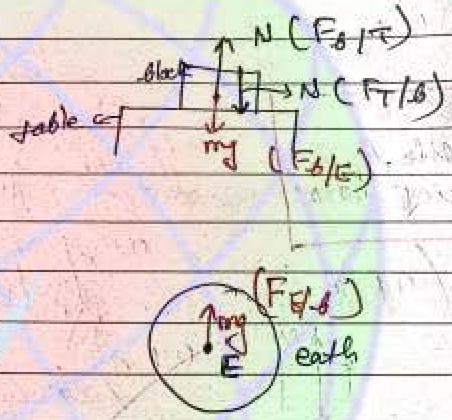


$(F_{b/T})$

$F_{b/E}$ (पृथ्वी के कारण लग रहा है table के कारण नहीं)

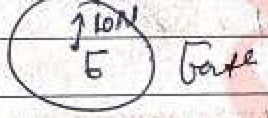
So, see the Action rex^n pair in true way.

Concept: Action and rex^n both not lie on the same particle!


N (F B/T)
block
N (F T/B)
table
mg
(F B/E)
(F E/B)
mg
E
earth

e.g.


Particle.
10N


10N
G
Force

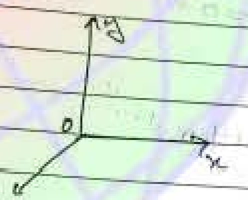
☆ Newtons 1st law of motion:
According to this law if acc^n of a particle is measured from Inertial frame of refrence is zero, then resultant force on the particle should be Zero

if

$a=0$ then $\vec{F}=0$ ↳ Resultant force

when object is in inertial frame at uniform motion = at $a=0$

Refrence frame is basically a co-ordinate



Observer is attached with the co-ordinate system.

Refrance frame (observer)

Inertial

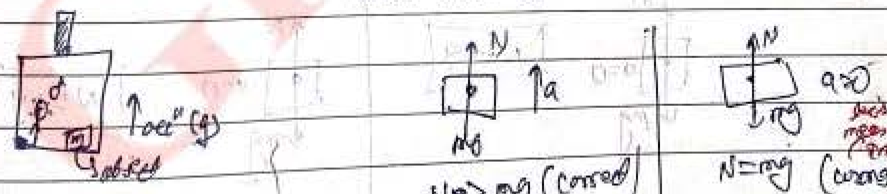
Non-Inertial

(If frame is either at rest or in uniform motion w.r.t ground)

(If frame is accelerated w.r.t. ground)

w.r.t (Inertial)

w.r.t O' (non-Inertial)



Jabzel

N > mg (correct)

N = mg (wrong)

law's not measured from Inertial frame of ref.

Note

① when observer is in Inertial frame of reference than

if $\boxed{a=0 \quad \text{and} \quad F=0}$

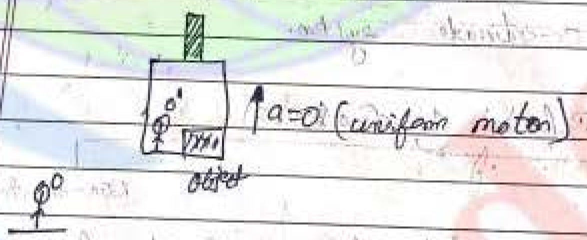
if $\boxed{a\neq0 \quad \text{and} \quad F\neq0}$

② when observer is in non-Inertial frame of reference then

if $a=0$ than यह जरूरी नहीं है कि $F=0$ है या नहीं है

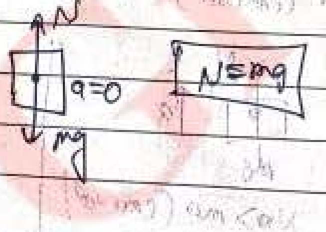
and also $a\neq0$ than यह जरूरी नहीं है कि $F\neq0$ होगा या नहीं होगा

(ii)



w.r.t O

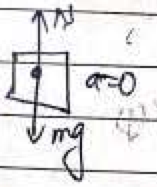
(Inertial)



$\boxed{N=mg}$

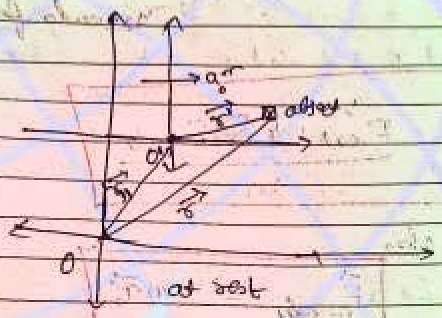
w.r.t O'

(Inertial
↳(क्योंकि acc^n uniform)



$\boxed{N=mg}$

★ Peudo force: →



$$\vec{r''} + \vec{r'} = \vec{r}$$

$$\frac{d^2\vec{r''}}{dt^2} + \frac{d^2\vec{r'}}{dt^2} = \frac{d^2\vec{r}}{dt^2}$$

Acc^n of object w.r.t. "O'"

Acc^n of object w.r.t. O $= \frac{d^2r}{dt^2}$

Acc^n of object w.r.t. O' $= \frac{d^2r'}{dt^2}$

Acc^n of O' w.r.t. O $= \frac{d^2\vec{r''}}{dt^2} = \vec{a_0}$

$\vec{a}_{\text{obj w.r.t } O'} = \vec{a}_{\text{obj w.r.t } O} - \vec{a_0}$

~~object/O'~~ = ~~Acc^n of~~

Ray

$$\vec{a}_{obj/O'} = m\,\vec{a}_{obj/O} - (m\vec{a}_0) \rightarrow \text{pseudo force}$$

$$\boxed{\vec{a}_{obj/O} = \frac{\vec{F}_{real} + \vec{F}_{pseudo}}{m}}$$

where

$$\boxed{\vec{F}_{pseudo} = -m\vec{a}_0}$$

i.e.

$$\boxed{F_{pseudo} = (\text{mass of the object})(\text{Acc. of non-inertial observer.})}$$

or

$$\boxed{F_{pseudo} = (\text{mass of the object})(\text{Acc}^n \text{ of reference frame})}$$

Note I:

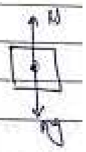
Ray → pseudo force is directed opposite to the obser
or ref. frame.

or

Direction of pseudo force is opp to the direction of motion of reference frame
of referen

When observation is a
reference that is
he needs to apply
addition to all
of motion the fa
force.

O



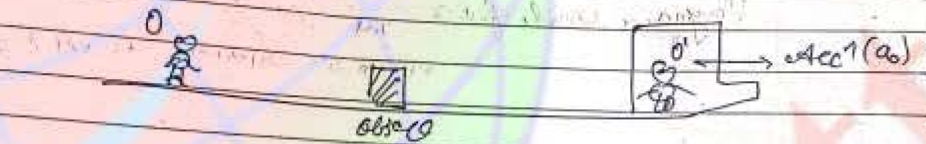
Newton's

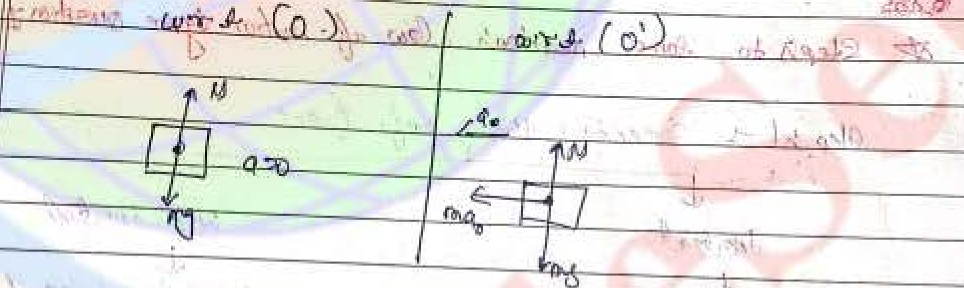
it oppose
ft

10

When observation's are taken from the non-inertial frame of reference that is by the non-inertial observer then he needs to apply an imaginary force on the particle in addition to all the real forces to use newton's laws of motion. this ~~force~~ is imaginary force is called pseudo force.

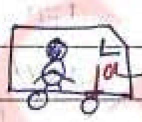
eg.





Newton's ~~1st~~ 1st law is also known as law of inertia.

that property of a massive body by virtue of which it opposes the change in its state of rest or ~~uniform~~ state of motion. is called inertia.



→ से Reason देखा Pseudo force का |

1° → से ~~क्या~~ Reason देखा Inertia का |

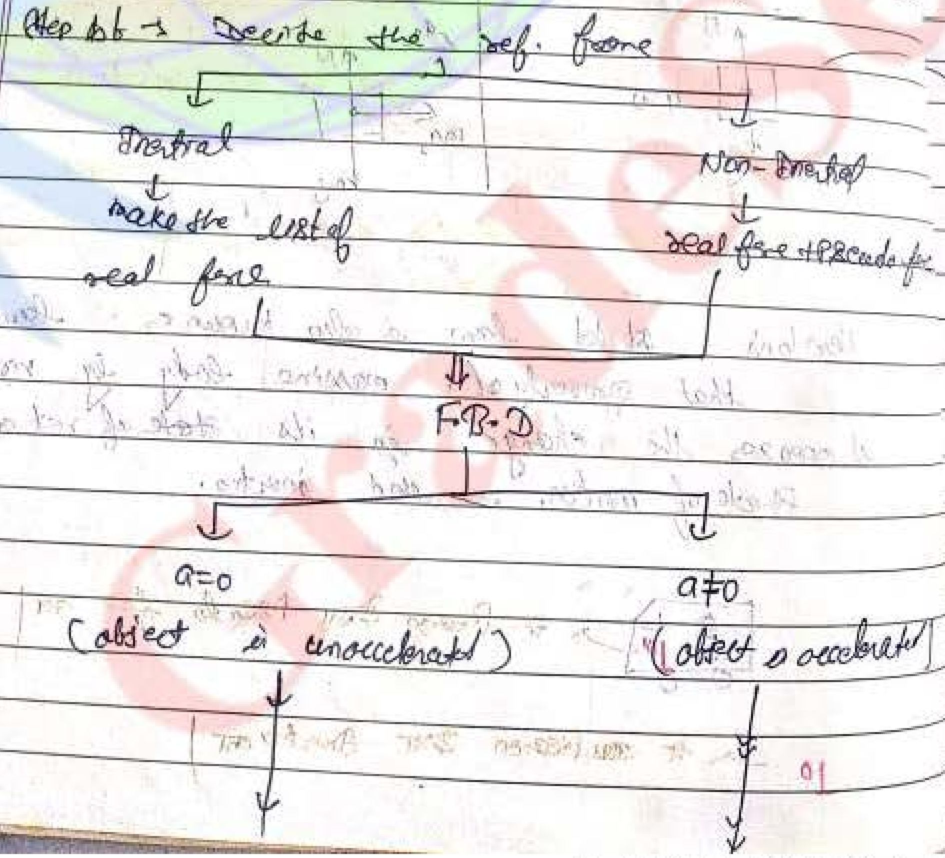
Difference b/w Real and Pseudo or Imaginary forces

Real force: ~~exerting and~~ action - Reaction law लगता
लगता है

force exerting body and force releasing ...
दोनो अलग अलग Real होते है।

Imagina or Pseudo force: जो force लगता रहता है उसका कोई
पता नहीं है
लेकिन उसको experience कर रहा है [illegible]

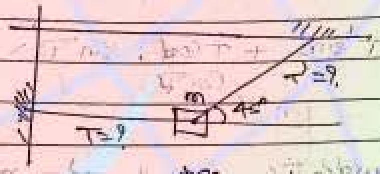
Steps to solve Newton's law of motion type questions



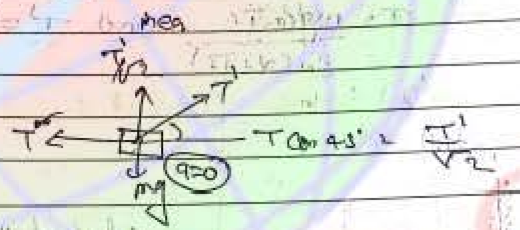
① use x-y co-ordinate system
(ii) apply 1st law along x only.
both.

① use x-y co-ordinate system
(ii) Assume x-axis ~~along~~ along line of acc^n
(iii) Apply 2nd law along x-axis.
apply 1st law along y-axis.

q.)



Sol. ocn



$T\cos45° = \frac{T'}{\sqrt{2}}$

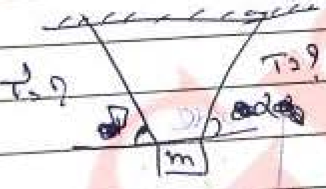
$T = \frac{T'}{\sqrt{2}}$ —(i)

$\frac{T'}{\sqrt{2}} = mg$ —(ii)

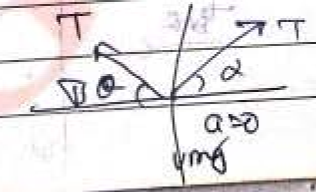
$T' = \sqrt{2}\,mg$

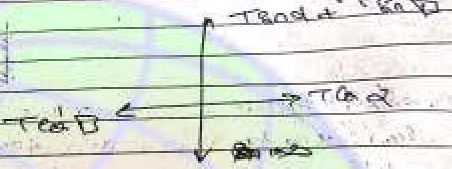
$T = mg$

q.)



Sol.





$T\cos\alpha = T'\cos\beta$

and

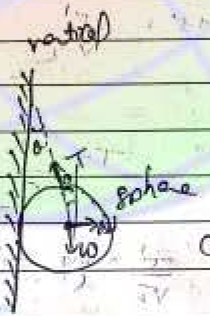
$T\sin\alpha + T'\sin\beta = mg$

or $T'\sin\alpha + \frac{T\cos\alpha}{\cos\beta}\cdot\sin\beta = mg$

∴

$T = \frac{mg\cos\beta}{\sin(\alpha+\beta)}$ and $T' = \frac{mg\cos\alpha}{\sin(\alpha+\beta)}$

eg)



(In eqm)

Which of the following is on

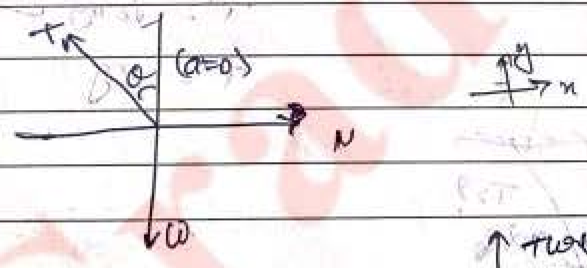
(i) $\vec{T}+\vec{N}+\vec{W}=0$

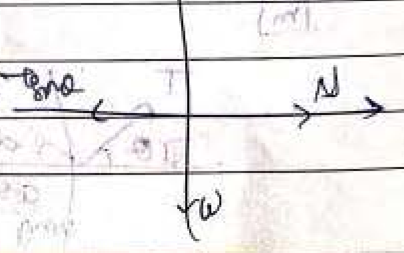
(ii) $\tan\theta = \frac{N}{W}$

(iii) $T^2 = N^2+W^2$

(iv) None of these

sol





$T\sin\theta = N$

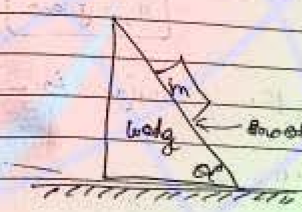
$T\cos\theta = W$

$T^2 = N^2 + W^2$

and $\tan\theta = \frac{N}{W}$

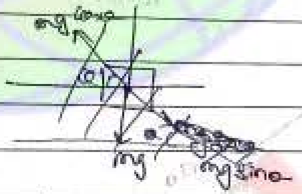
$\vec{N} + \vec{W} + \vec{T} = 0$

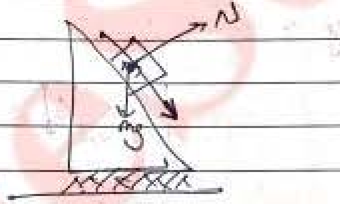
Q.



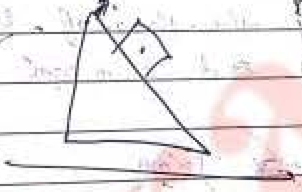
What should be the horizontal acc^n of wedge so that block remain stationary w.r.t. wedge.

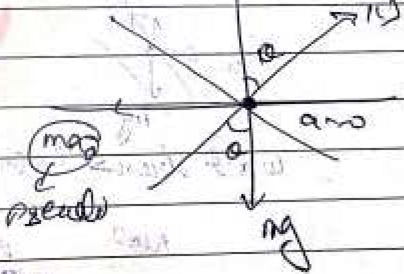
Sol^n



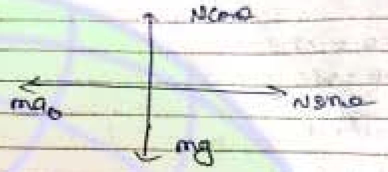


w.r.t. wedge (Non inertial)





where a_0 = Acc^n of the wedge.



$N\sin\theta = ma_0$

and

$N\cos\theta = mg$] $\tan\theta = \frac{a_0}{g}$

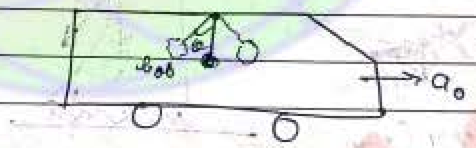
$\boxed{a_0 = g\tan\theta}$ Ans

(ii) For on the block by the wedge is —

$N\cos\theta = mg$

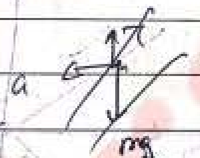
$N = \cancel{mg}\ \frac{mg}{\cos\theta}$

eg →



Find angle made by the thread of simple pendulum with vertical line when it is in eqm w.r.t. bus.

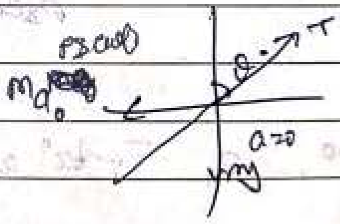
Sol

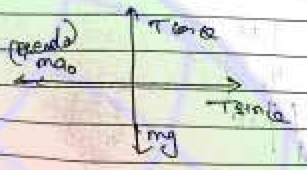


~~Tsinθ~~ ~~T=mg~~

w.r.t. bus (Non-Inertial)

Now





$T\sin\theta = ma_0$

$T\cos\theta = mg$

$\tan\theta = \frac{a_0}{g}$

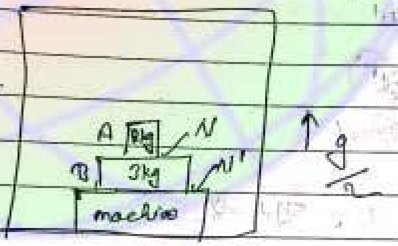
$\theta = \tan^{-1}\left(\frac{a_0}{g}\right)$

(ii) Tension in the thread during

$T^2 = m^2(a_0^2+g^2)$

$T = m\sqrt{a_0^2 + g^2}$

Q)

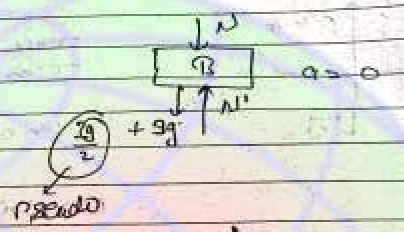


(i) force b/w A and B
(ii) Reading of the machine

Sol) w.r.t. the lift (Non inertial

$N = 2g + \frac{2g}{2}$

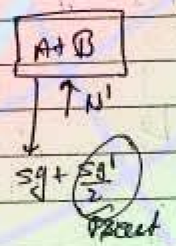
$= 30$ newton



$N' = 3g + 3g/_2 + N$

$= 45$ New

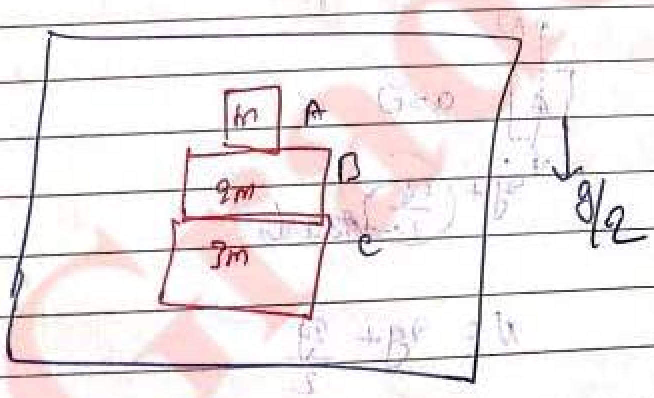
Note



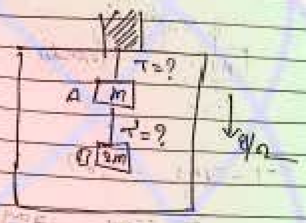
$N = 5g + 5g/_2$

$= 75$ Newton

Short trick

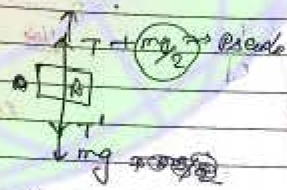


Q.)



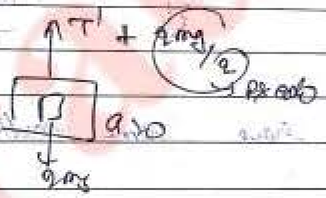
Sol

w.r.t lift (Non-inertial)



$T + \frac{mg}{2} = T' + mg$

$T = T' + mg/2$



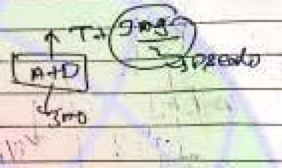
$T' + \frac{2mg}{2} = 2mg$

$T' = mg$

$\therefore T = \frac{3mg}{2}$

L-1 ⇒ 1,6,11,12 14,15,16,17,18, 10 to 24, 43 to 45
L-2

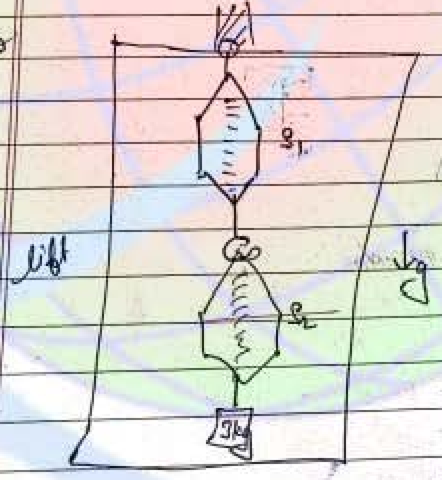
Note



$T + \frac{3mg}{2} = 3mg$

$T = 3mg - \frac{3mg}{2}$

eg

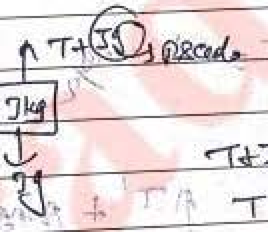


Reading of S₁ and S₂ a...

To lm

Note (Series combination of spring में tension same रहता है)

w.r.t lift



$T + 1g = 1g$

$T = 0$

Note

Both have same Reading

Note

lift

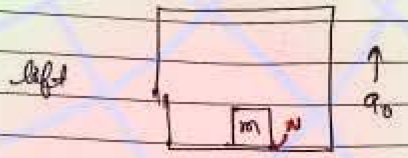
% che

(i) if lift

(ii) if l

(iii)

Note



N → apparent weight
ω → actual weight

$$\boxed{\% \text{ change in weight} = \frac{N-\omega}{\omega} \times 100}$$

(i) if lift is acc^n upward

$$\boxed{N = mg + ma_0}$$ $(N > \omega)$

(ii) if lift is acc^n downward

$$\boxed{N = mg - ma_0}$$ $(N < \omega)$

(iii) if $a_0 = g$ (free fall)

$$\boxed{N = 0}$$ (weightless.)

Newton
2nd law :-

According to this law acc^n of a particle as measured from the inertial frame of reference is equal to the vector sum of all the real forces acting on the particle divided by its mass.

~~$\vec{a}_{w.r.t\ inertial} = \vec{F}_{real+}$~~

$$\boxed{\vec{a}_{w.r.t\ inertial} = \frac{\vec{F}_{real}}{m}}$$

Note

$$\boxed{\vec{a}_{w.r.t\ non\text{-}inertial} = \frac{\vec{F}_{real} + \vec{F}_{pseudo}}{m}}$$

if $F=0$ then $a=0$

i.e. 1st law can be derived from 2nd law

\# According to 2nd law :-

$$\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$$

= Rate of change of linear momentum of a particle w.r.t. time

- Rate of change of linear momentum of a particle w.r.t time is equal to the net force acting on it

i.e. if $F=0$

$$\frac{d\vec{P}}{dt} = 0$$

$\vec{P}$ = momentum

$\Delta P = 0$

If force acting on a particle is zero then its momentum remains unchanged.

This is called linear momentum conservation because

$$\therefore \int dP = \int \vec{F} \cdot dt$$

$$\boxed{\Delta \vec{P} = \int \vec{F} \cdot dt}$$

= change in momentum

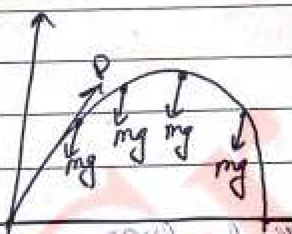
= Impulse

- If F = constant

$$\Delta \vec{P} = \vec{F}(\Delta t)$$

$$\boxed{|\Delta \vec{P}| = F(\Delta t)}$$

eg



change in momentum in time t: T

$$|\Delta \vec{P}| = mg\, t$$

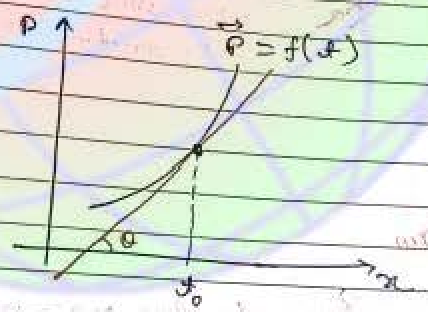
Note - Same
- $a_x =$
- $F_x =$
- ΔP
-

Note :- same linear force [illegible] consider करता है।

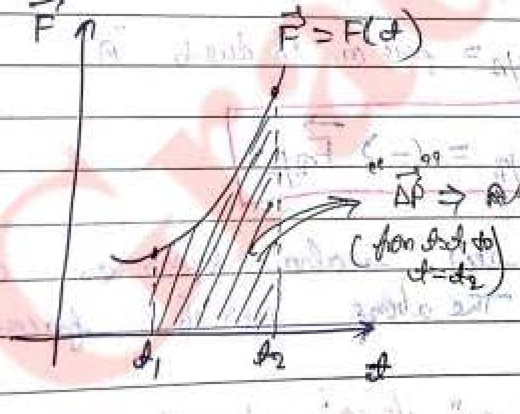
- $\boxed{a_x = \frac{\vec{F}_x}{m}}$ and $\boxed{a_y = \frac{F_y}{m}}$

- $\boxed{F_x = \frac{dP_x}{dt}}$ and $\boxed{F_y = \frac{dP_y}{dt}}$

- $\boxed{\Delta P_x = \int F_x \, dt}$ and $\boxed{\Delta P_y = \int F_y \, dt}$

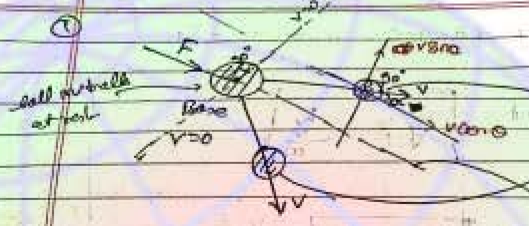
- 

$$\boxed{\left(\vec{F}\right)_{t=t_0} = \tan\theta}$$ (slope निकलनी होगा)

- 

$\vec{\Delta P} \Rightarrow$ Area represent change in momentum.

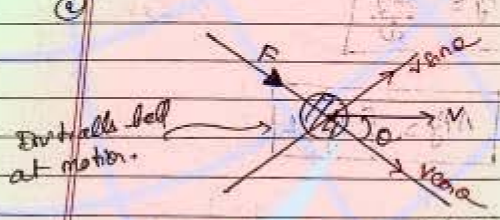
①



→ ये दोनों position में move नहीं

→ इस case में body किसी force direction में moving

②



→ इस case में body में जिस direc में force लगा रहा है उसके along जाएगा क्योंकि पहले से उस velocity उस direction में है।

Newton 3rd law

According to this law if force on a body "A" by the "B" is $\vec{F}$ then force on "B" by the "A" will be as

$$\vec{F}_{A/B} = \text{force on "A" due to "B"}$$

$$\vec{F}_{B/A} = \text{force on "B" due to "A"}$$

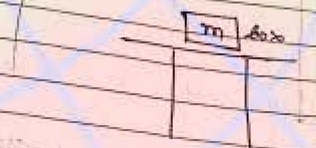
$$\boxed{\vec{F}_{A/B} = (-)\vec{F}_{B/A}}$$

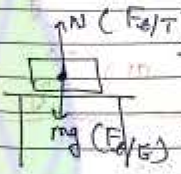
Note :- This law is also known as action reaction law. The above pair of forces is called action reaction pair.

* Action and reaction forces do not act on the same particle.

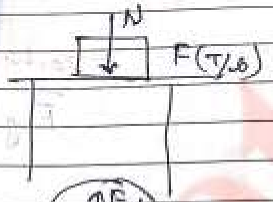
* These are equal in magnitude but opp. in direction.

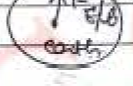
eg. →



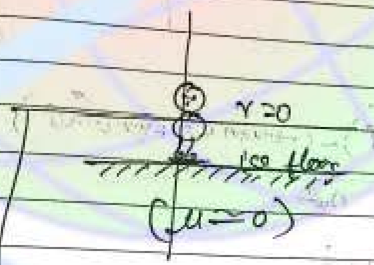


→ force equal in mag but opp in direction है पर action-reaction नहीं है





Q.)



Smooth surface फिसलने का नहीं है

Note: कमर-पैर हिलाना ✗
चलना ✗
Roll (लुढ़कना) ✗

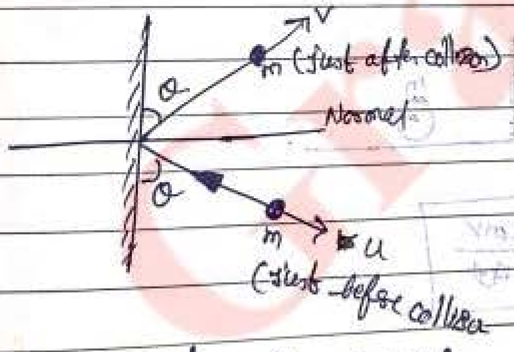
→ आगे या पीछे खिसक सकता है

~~किसी भी काम नहीं~~

① किसी चीज़को आगे या पीछे धकेलें
② झुके (आगे या पीछे)

means horizontal direction में force लगायें चाहे कितना भी हो।

eg.)



Find average force on the ball by the wall.

Contact time = Δt

8/7

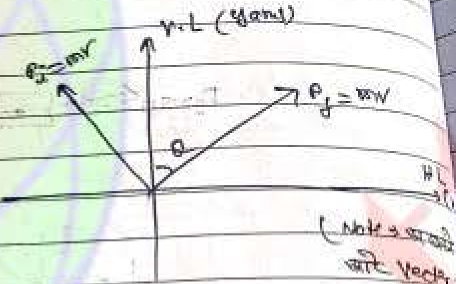
Concepts

① $$\vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t}$$

(ii) $$\vec{F}_{avg} = m\vec{a}_{avg}$$

Ques

Technique



(Note →
vect
angle पर shif

$$\vec{F}_{avg} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$$

$$\vec{F}_{avg} = \frac{(mv\sin\theta\hat{i} + \cancel{mv\cos\theta\hat{j}}) - (-mv\sin\theta\hat{i} + \cancel{mv\cos\theta\hat{j}})}{\Delta t}$$

$$\vec{F}_{avg} = \frac{\cancel{2mv\sin\theta} \; 2mv\sin\theta\,\hat{i}}{\Delta t}$$

$$\boxed{|\vec{F}_{avg}| = \frac{2mv\sin\theta}{\Delta t}}$$

Note

(1)

$$\boxed{F_{avg} = \frac{2mv}{\Delta t}}$$

(c) →

$$\boxed{F_{avg} = \frac{mv}{\Delta t}}$$

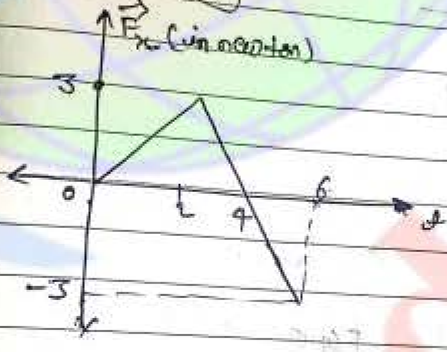
eg) A machine gun "n" bullet's per second mass of each bullet is "m" and velocity is "v". find avg. force on the gun by the bullet's.

sol Let, N = No of bullet fired in time "t".

$$F_{avg} = \frac{Nmv}{t} = n\,mv$$

$F_{avg} = ma$ \
$= m \times v$ \
$= \frac{mv}{t}$

eg)



A ball of mass 1kg is moving along y-axis with speed of 4m/s. force acting on the ball is as shown in the graph. find velocity of the ball at t = 6 sec.

Sol at t=0 , $v_x = 0$, $v_y = 4m/s$ \
at t = 6 s , $v_x = ?$ $v_y = 4m/s$

at t = 6

$$m\left(V_{t=6}\right)_x - 0 = \frac{1}{2}\times 4 \times 3 + \frac{1}{2}\times 2 \times (-3)$$

1kg

$$\left(V_{t=6}\right)_x = 3\ m/s$$

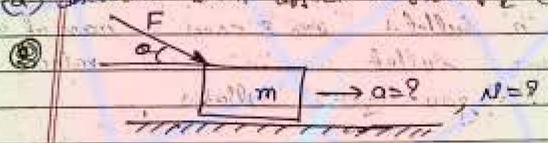
$\frac{1}{2}\times 2 \times 3$ \
$= 6 - 3 = 3$ \
$F = ma$ \
$F = m\frac{dv}{dt}$ \
$F_x = \frac{dp}{dt}$

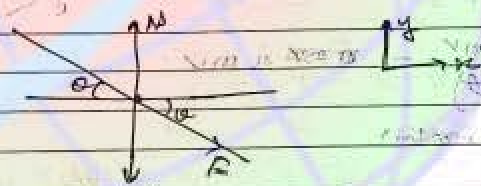
$$\vec{V}_{t=0} = (3\hat{i} + 4\hat{j})\ m/s$$

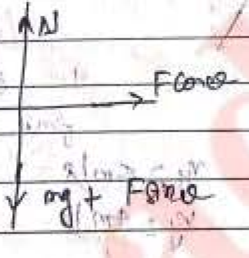
$$|\vec{V}_{t=0}| = 5m/sec.$$

Application of newton's law of motion →

(a) Case when object is pushing (ढेलना)







2nd law ⇒

$$\boxed{a = \frac{F\cos\theta}{m}}$$

1st law

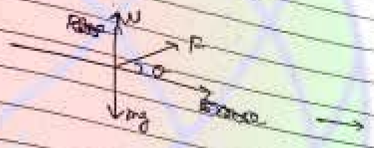
$$\boxed{N = mg + F\sin\theta}$$

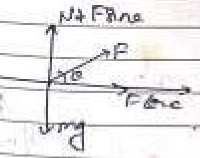
Note Pushing (ढेलना भारी है) → b/c cone of push Fsinθ support mg. so frictional force increase. So it is tough.

H.W. L-1 → 1 to 38, 42, 44, 45, 46

(b) when object is pulling ([illegible]) :-

N = ?



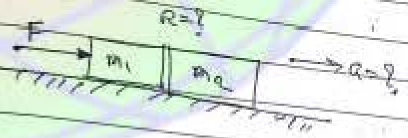


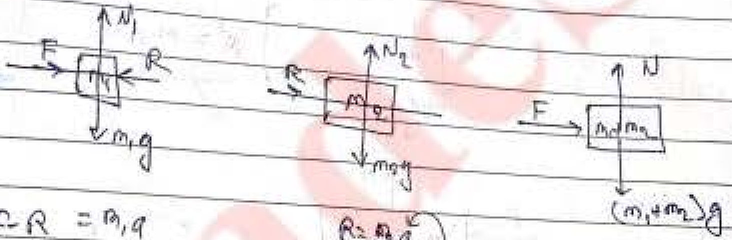
ans $\boxed{a = \frac{F\cos\theta}{m}}$

$N + F\sin\theta = mg$

$\boxed{N = mg - F\sin\theta}$

(c)





$F - R = m_1 a$

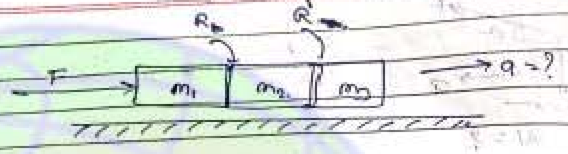
$R = m_2 a$

$F = (m_1+m_2)a$

$\therefore a = \frac{F}{m_1+m_2}$

$$\boxed{R = m_2 \cdot \frac{F}{m_1+m_2}}$$

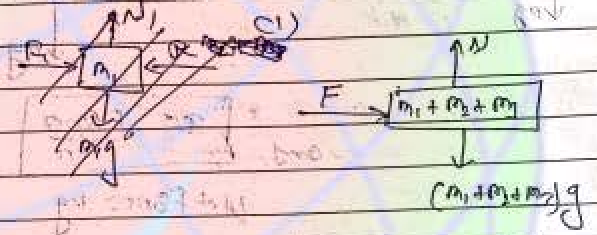
ⓓ



R = ? , R' = ? , a = ?

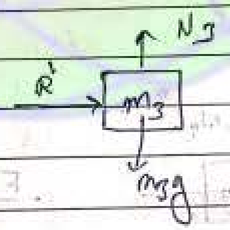
Sol^n

(i)



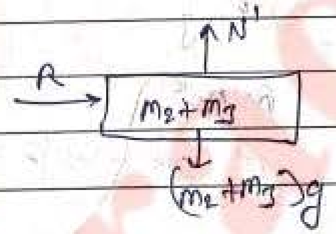
$$\boxed{a = \frac{F}{(m_1+m_2+m_3)}}$$

(ii)



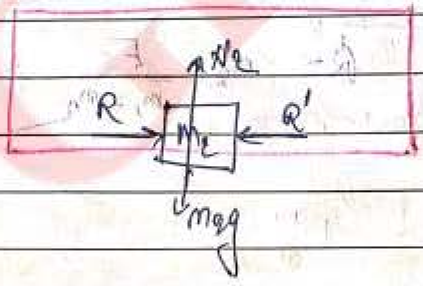
$$\boxed{R' = m_3a}$$

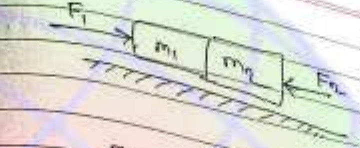
(iii)



$$\boxed{R = (m_2+m_3)a}$$

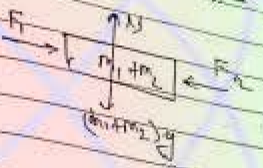
Note





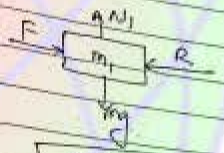
$(F_1 > F_2)$

(i)



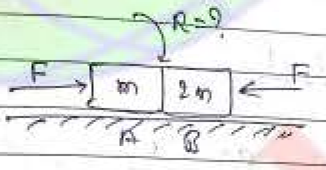
$$\boxed{a = \frac{F_1 - F_2}{(m_1 + m_2)}}$$

(ii) R = ?

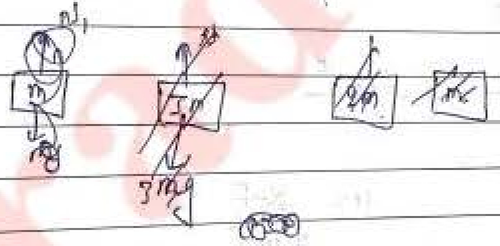


$$\boxed{F - R = m_1 a}$$

$$\boxed{R = \frac{m_1 F_2 + m_2 F_1}{(m_1 + m_2)}}$$

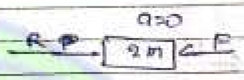


find contact force b/w A and B



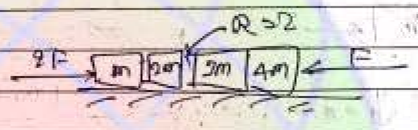
(i) $a = \frac{F - F}{3m} = 0$

NOW

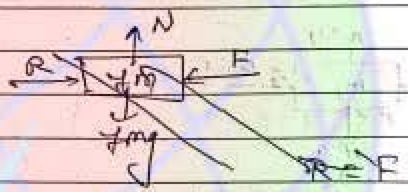


$$\boxed{R = F}$$

Que)

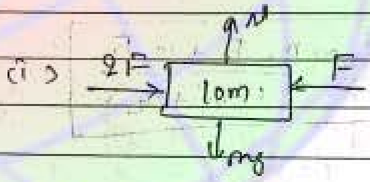


Sol



~~R = F~~

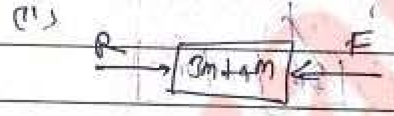
N = 7mg



$$a = \frac{2F - F}{10m}$$

$$a = \frac{F}{10m}$$

(ii)



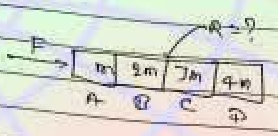
$$a = \frac{R - F}{7m}$$

$$R = F + 7m\left(\frac{F}{10m}\right)$$

$$= \frac{17}{10} F$$

Ans

eg→



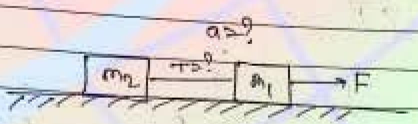
(i) $a = \frac{F}{10m}$

(ii)

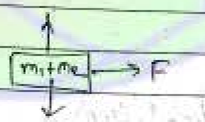
$R = 7ma$ ← Ans

$R = \frac{7F}{10}$

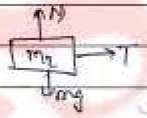
(f)



(i)



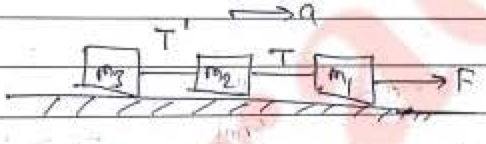
(ii)



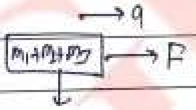
$$\boxed{a = \frac{F}{m_1+m_2}}$$

$$\boxed{T = m_2 a}$$

(g)



(i)



(ii)

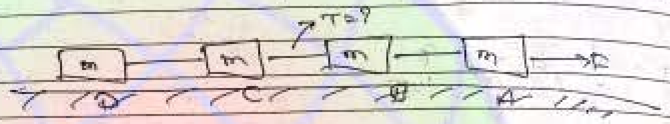
$T = (m_2 + m_3)a$

$$\boxed{a = \frac{F}{\Sigma m}}$$

(iii)

$\boxed{T' = mg}$

eg



soln

(i) $F = 4m \times a$

$a = \frac{F}{4m}$ —(i)

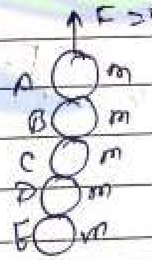
(ii) $\boxed{C+D} \longrightarrow T$

$T = 2m \times a$

$= 2m \times \frac{F}{4m}$ (from eq(i))

$= \frac{F}{2}$

eg) ↑ F = 10mg (vertical line)



Find tension at the joint of B and C

sol acm

$F = 5m \times a$

$a = \frac{F}{5m}$

$T = 3m\,a$

$= 3m \times \frac{F}{5m}$

$= \frac{3F}{5}$

$= \frac{3 \times 10}{5} = 6N$

eg

(i)

F=10

5m

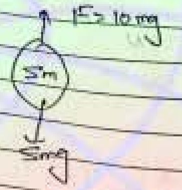
5mg

(ii)

eg

Tension

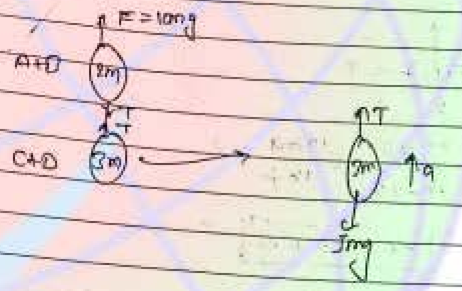
(i)



$a = \frac{F - 5mg}{5m}$

$\Rightarrow \boxed{a = g}$

(ii)



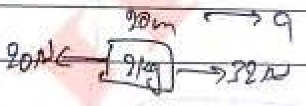
$T - 3mg = 3 \times m \times a$

$T = 3 mg$

eg

Tension in the rod at a distance of 20cm from the end "A" is —

Soln



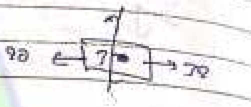
$F = m \times a$

$12N = 2 \times a$

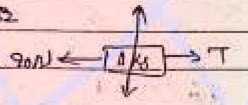
$a = \frac{12}{2} = 6$

Tension

(i) $a = \frac{32-20}{3} = 4\ m/s^2$



(ii)

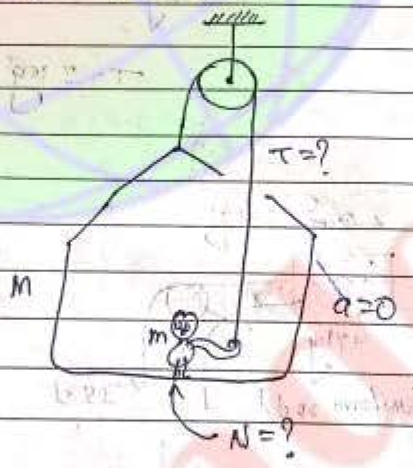


$T - 20 = m \times a$

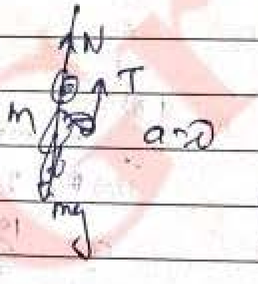
$T - 20 = 1 \times 4$

$\boxed{T = 24\ N}$

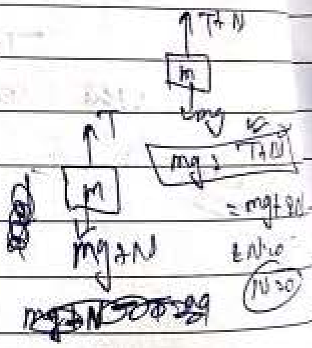
eg.)



Sol (i)



$\boxed{N+T = mg}$ (i)



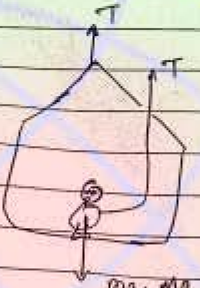
mg = T+N

= mg+20

N=0

N=0

Fi;



$mg + Mg = g(m+M)$

$2T = (M+m)g$

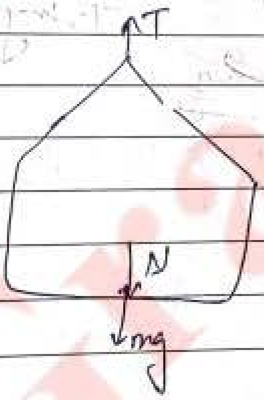
$T = \frac{(M+m)g}{2}$ —(ii)

$\therefore N = mg - \frac{(M+m)g}{2}$

or

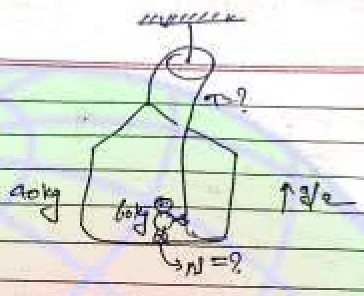
$N = \left(\frac{mg - mg}{2}\right)$ from eq ① and ②

<u>Note</u>

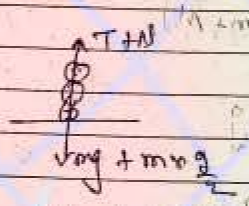


$T = N + mg$

Q)

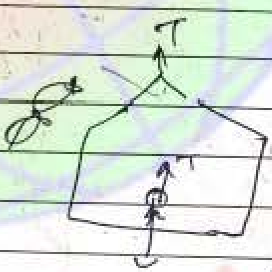


Sol(ution)



$\Rightarrow 60\left(g+\frac{g}{2}\right) = \cancel{60}^{30} \times \frac{3g}{\cancel{2}} = 90g$

$T+N = 90g$



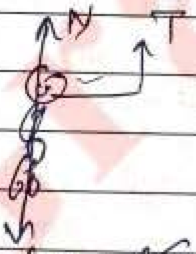
$T = 100g$

$100g + \frac{\cancel{100}^{50} \times g}{\cancel{2}}$

$150g$

Teacher

(i)



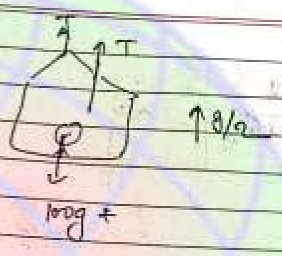
$a \neq 0$

$N + T = 900$ —(i)

$60g + \frac{\cancel{60} \times g}{\cancel{2}} = 90g$

T

(ii) →



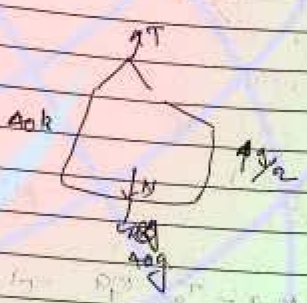
T

$2T - 100g = (100)\frac{g}{2}$

$T = 450$ N

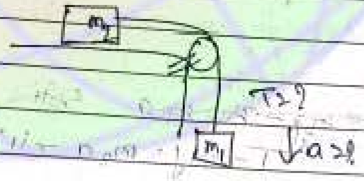
$\therefore N = 150$ Newton

Note

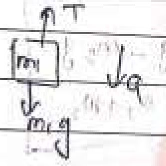


$T - (40 - N) = 40 \cdot \frac{g}{2}$

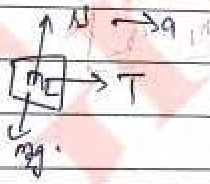
#



Sol^n



$m_1 g - T = ma$ —(i)



$\cancel{\boxed{N = m_2 g}}$ —(ii)

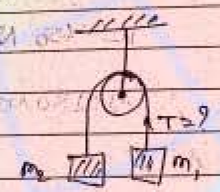
$\boxed{T = m_2 a}$ —(iii)

from eq ① and (iii)

H.W → L-1, 1 to 32 , 33, 34, 35 41, 43, 44 45, 46
L-2 → 1 to 6, 8, 9, 10, 11, 12, 13, 14, 15 13/15, 17,

$$\boxed{a = \frac{m_1 g}{m_1 + m_2}}$$

#

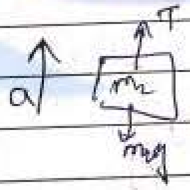


$\downarrow a \ge 1$

$(m_1 > m_2)$

80|-

$\downarrow a$ $m_1 g - T_0 = ma$ -(i)



$T - m_2 g = m_2 a$ -(ii)

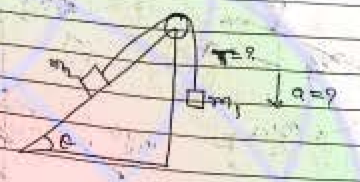
from ⓘ and (ii)

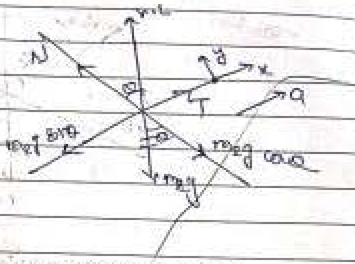
$$\boxed{a = \frac{m_1 g - m_2 g}{m_1 + m_2}}$$

$$\boxed{T = \frac{2 m_1 m_2 g}{m_1 + m_2}}$$

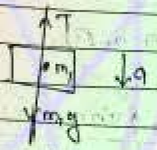
T_1

#





$T - m_2 g \sin\theta = m_2 a$



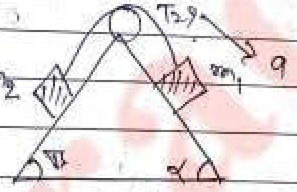
$m_1 g - T = m_1 a$ —(ii)

Adding eq ① and ②

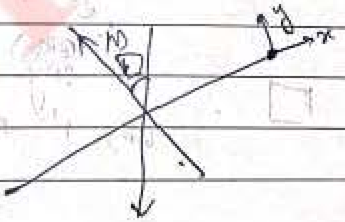
$\therefore a = \frac{m_1 g - m_2 g \sin\theta}{m_1 + m_2}$

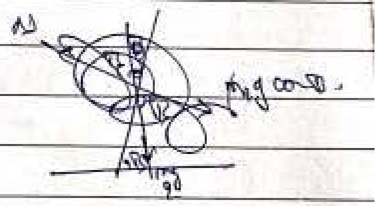
$T_1 \rightarrow$ and $T = \frac{m_1 m_2 g (1 + \sin\theta)}{m_1 + m_2}$

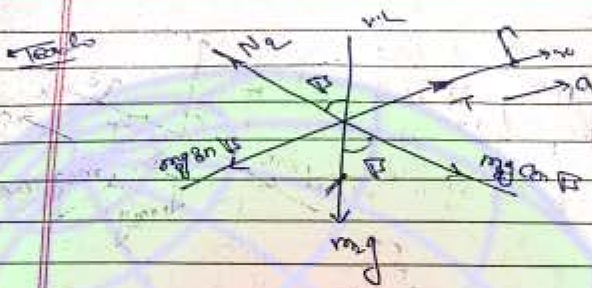
#

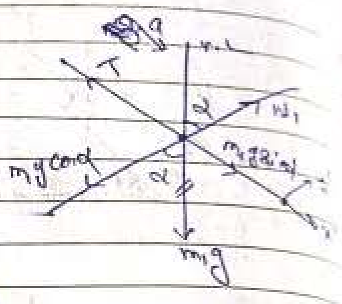


Sol^n/ Ans









$$T - m_2 g \sin\beta = m_2 a \quad -(1)$$

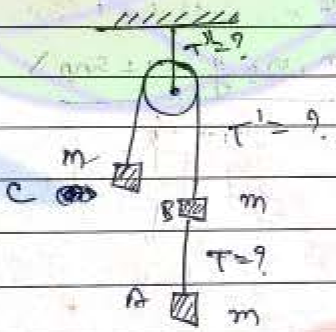
$$m_1 g \sin\alpha - T = m_1 a$$

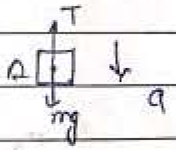
Add eq(1) and (ii)

$$a = \frac{m_1 g \sin\alpha - m_2 g \sin\beta}{m_1 + m_2}$$

and $$T = \frac{m_1 m_2 g (\sin\alpha + \sin\beta)}{(m_1 + m_2)}$$

eg.)



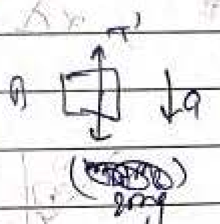


$mg - T = ma$

$T = mg - ma$
$= m(g - a)$

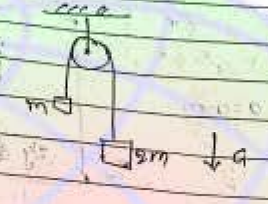
20/9

Ques



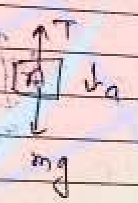
$2mg - T' = ma$ $\quad T' = 2mg - ma$
$= m(2g - a)$

Teacher (i) → for acc^n purpose



$$a = \frac{2mg - mg}{2m + m} = \frac{g}{3} \quad —(1)$$

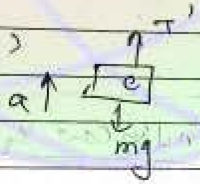
(ii)



$ma = mg - T$

$\boxed{T = \cancel{m}\ \frac{2}{3} mg}$ → from eq①

(iii)



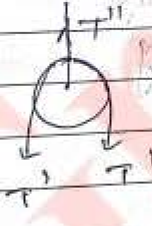
$T' - mg = ma$

$T' - mg = m\frac{g}{3}$

$T' = \frac{mg}{3} + mg$

$\boxed{T' = \frac{4}{3} mg}$

(iv)

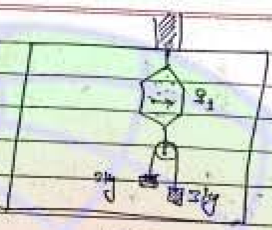


$T'' = 2T$

$\therefore \boxed{T'' = \frac{8}{3} mg}$ Ans

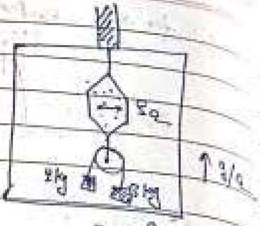
Q.)

(i)



$S_1 = ?$

(ii)



$S_2 = ?$

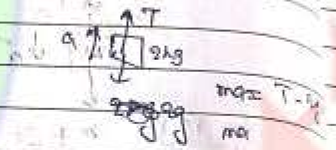
Sol/ⁿ (i) Case a=0

$\boxed{S_1 = 2T}$

$T = \frac{2(3)(2)}{3+2} \cdot g$

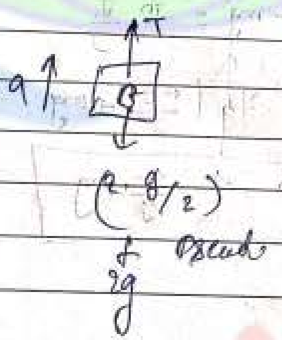
T = 24 newton ✓

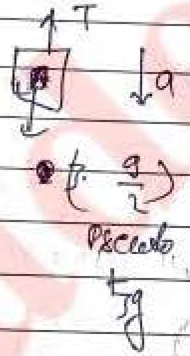
S_1 = 48 Newton



(ii) w.r.t lift (non-inertial)

$\boxed{\text{In this case } S_2 = 2T}$





acc' w.r.t lift.

T - 30 = 2a -(i)

and

45 - T = 3a -(ii)

solving

$a = 3\ m/s^2$

$T = 36$ newton

$\therefore\ 2T = 72$ newton.

Note

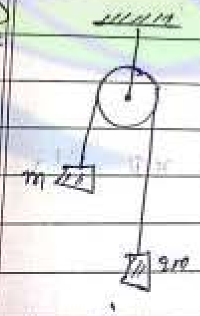
Acc^n of "A" w.r.t. ground = 3 (downwards) + $\frac{g}{2}$ (upward)

= 2 (upward)

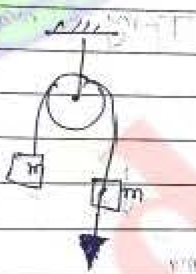
Acc^n of B w.r.t. ground = 3 (up) + $\frac{g}{2}$ up)

= 8 (up)

eg.)



(i)

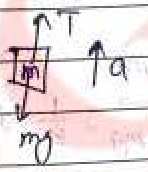


F = mg

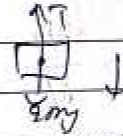
(ii)

(a) $\frac{a_1}{a_2} = ?$ (b) $\frac{T_1}{T_2} = ?$

So/n

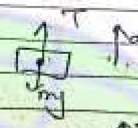


$T - mg = ma$



$2mg - T = 2ma$

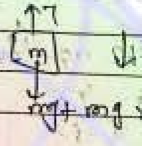
$a = \frac{mg - 2mg}{3m} = \frac{mg}{3mg} = \frac{g}{3}$



$T - mg = ma$ —(i)

~~[illegible]~~ —(i)

$T = mg = ma$



mg + mg = 2mg

~~[illegible]~~

$2mg - T = ma$ —(ii)

$2mg - mg = ma + ma$

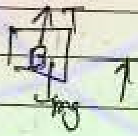
$\cancel{m}g = 2\cancel{m}a$

$\boxed{a = \frac{g}{2}}$

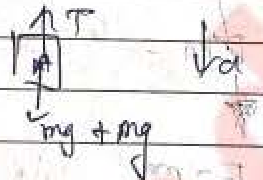
Each

(i) $a_1 = \frac{2mg - mg}{3m} = g/3$, $T_1 = \frac{2(2m)(m)}{3} = \frac{4}{3}mg$

(ii)



$T - mg = ma$ —(i)



$2mg - T = ma$ —(ii)

mg + mg

add eq (i) + eq (ii)

$a = \frac{g}{2}$

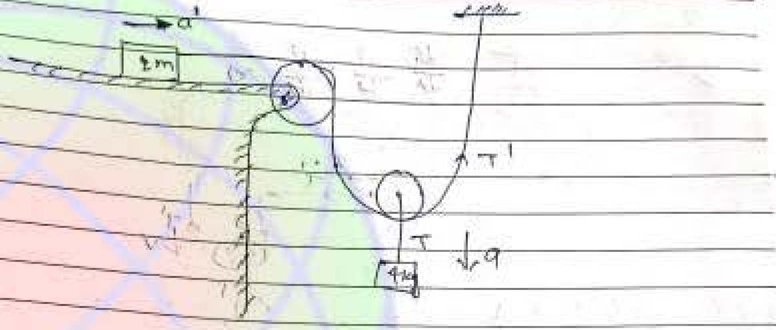
$T = \frac{3}{2}mg$

So

$\frac{a_1}{a_2} = \frac{g/3}{g/2} = \frac{2}{3}$; $\frac{T_1}{T_2} = \frac{\frac{4}{3}mg}{\frac{3}{2}mg} = \frac{8}{9}$ Ans

Special case (type problem of newton's law) →

Q10



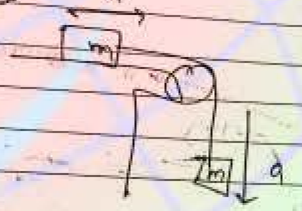
(ii)

a = ?

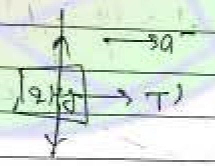
T' = ?

a' = ?

Sol. – Note one ab.

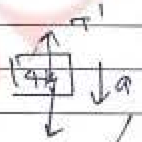


(i) →



(ii) →

T = 2T' —(a)



T' = ma' —(i)

4g = mg

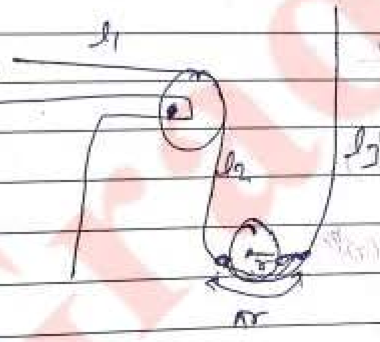
4g − T = 4a

from eq(a)

40 − 2T' = 4a

20 − T' = 2a (ii)

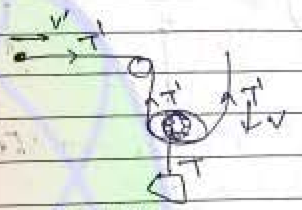
Note → जो distance बदलता [illegible] उसको निकालकर the "l" consider [illegible] [illegible]



$l_1 + l_2 + l_3 + \pi r = l$

T_1

$$\frac{dl_1}{dt} + \frac{dl_2}{dt} + \frac{dl_3}{dt} = 0$$



$-v' + v + v = 0$

$2v = v'$

$2a = a'$

$\boxed{2a = a'}$ — (III)

यह Relation याद ही रखो

$T_1 \rightarrow$

Note
Tension के Reverse
आनुपात में acc' का Relation
पता कर सकते है
$\left\{ \boxed{2T' = T} \atop 2a = a' \right.$

Now Yorma ② and ①

$20 - T = a'$ | $\therefore a' = \frac{20}{7}$ m/s² — (IV)

but

$T' = 2a'$

$\therefore 20 - 2a' = a'$

$a = \frac{10}{7}$ m/s²

and $T' = 2\left(\frac{20}{7}\right)$

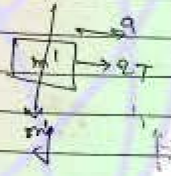
$= \frac{40}{7}$ newton

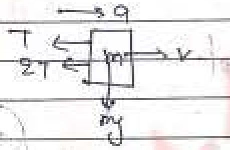
Q1→ T तथा V' का Ratio ज्ञात कीजिए

Sol.

$\frac{v'}{v} = ?$

Sol^n Gen^n





$7T > ma$

Trick

Note

① $|\vec{T_1} \cdot \vec{V_1}| = |\vec{T_2} \cdot \vec{V_2}|$

(ii) $|\vec{T_1} \cdot \vec{a_1}| = |\vec{T_2} \cdot \vec{a_2}|$

$T_P = 2T'$

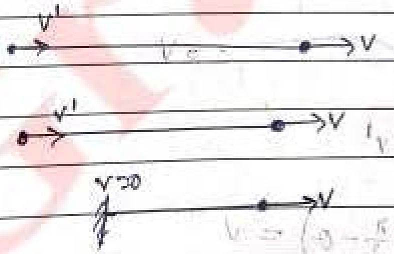
$T_Q = T+T'$

method 1st

$(2T) \cdot v' = (3T) v$

$\therefore \frac{v'}{v} = \frac{3}{2}$

method 2nd



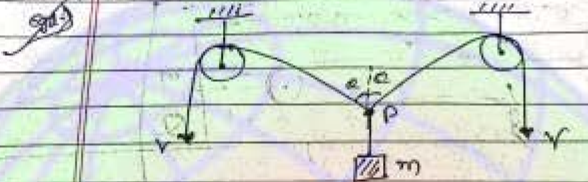
$-2v' + 3v = 0$

$\frac{v'}{v} = 3/2$

Questions based on velocity (v') in along the string →

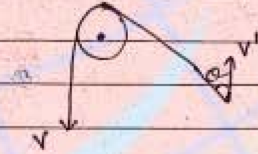
Concept:
String का velocity
अलग-अलग point
पर same होगा।
क्योंकि string fixed
होता है इसे बाला
का समान रहेगी
और इसी concept के
अलग-अलग point पर
velocity की equal
होती है।

Ques)



Velocity of the point "P" =

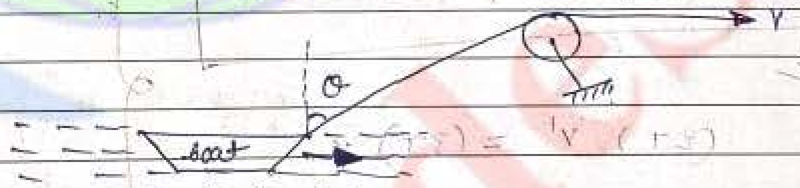
Sol/n



$v = v'\cos\theta$ → [v' को string के along लिये हैं]

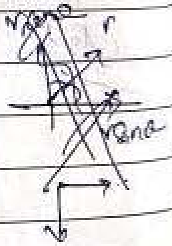
$v' = \frac{v}{\cos\theta}$

Q2)



Velocity of boat is ?

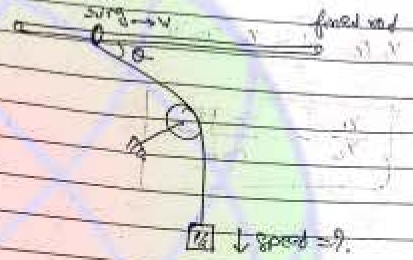
Sol



v' को string के along लिये है ← $v'\cos\left(\frac{\pi}{2}-\theta\right) = v$

$v' = \frac{v}{\sin\theta}$ A

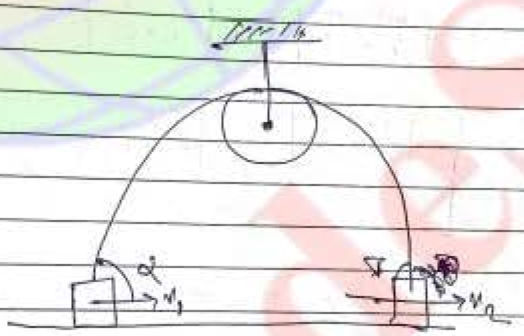
Q



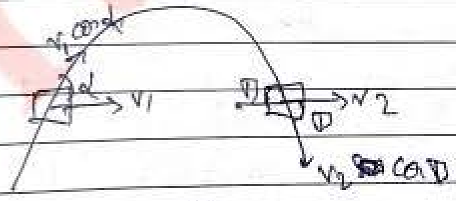
Soln

$v \cos\theta = v'$

Q



$v' \downarrow v_2 = ?$



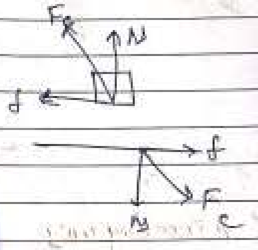
v_2

$$v_1 \cos \alpha = v_2 \cos \beta$$

$$\boxed{\frac{v_1}{v_2} = \frac{\cos \beta}{\cos \alpha}}$$

Start

Friction



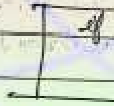
N → Normal contact force

f → friction force

f_c → Contact force

$$\vec{F_c} = \vec{N} + \vec{f}$$

$$\vec{F_c} = N^2 + f^2$$



$f > 0$, $\boxed{N = F_c}$

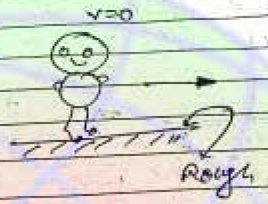
Note

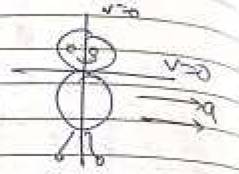
① Component of contact force ⊥r to the contact surface is called normal contact force.

② Component of contact force along the contact surface is called friction force.

(iii) "N" and "f" are mutually "⊥r".

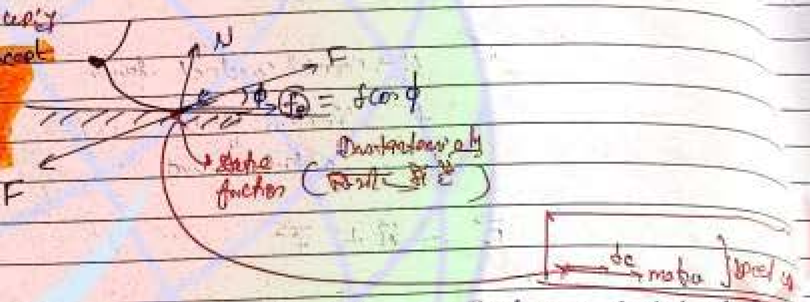
(iv)





speeding up'y

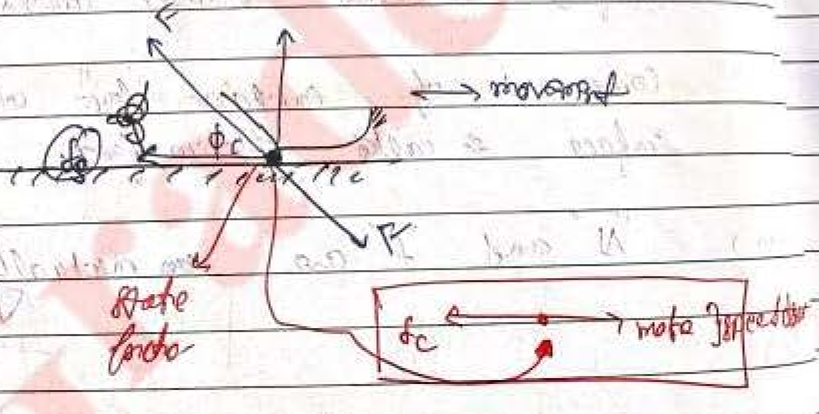
Note → यहां General concept जहां force है force के opp direction में friction force लगेगा।



static → Not slipping → speeding up (static friction का direction situation पर depend करेगा।)

kinetic → slipping → motion के opp friction force होगा।

speeding ~~opp~~ direction

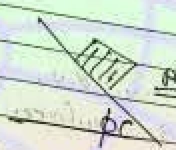


Rey ⇒

L-3 → 1 to 11

L-4A → 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 14, 15, 16, 20,

L-4D → 2, 4, 5, 6,



Rest (static frict)

(kinet

friction force

kinetic friction force (f_k)

static friction force (F_s)

1) kinetic friction force :-

when surfaces in contact slip over each other then force of friction b/w them is called kinetic friction

2) Static friction force :-

when surfaces in contact does not slip over each other then force of friction b/w them is called static friction//

Properties of kinetic friction (F_k)

① kinetic friction (f_k) is directly proportional to the normal contact force b/w the surfaces in contact

$$f_k \propto N$$

$$\boxed{f_k = \mu_k N}$$ where μ_k → Coeff. of kinetic friction

Rq1, Rq2

2) * "μ_k" depends on two factors.

(i) smoothness of the surface.

(ii) material of the contact surface.

3) * Kinetic friction force opposes the relative motion.

Rq1

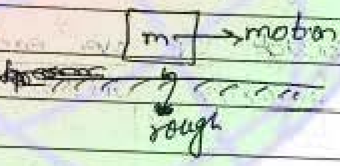
suppose object A and B ~~are~~ are in contact with each other ~~fricton~~ kinetic friction force on A due to "B" will be opposite to the $\vec{V}_{A/B}$ value.

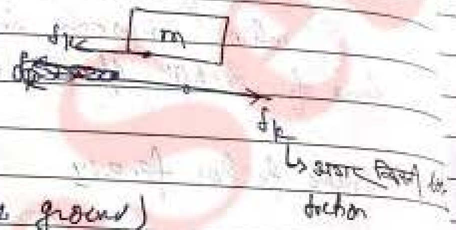
(kinetic friction force की दिशा Relative motion के उल्टा होता है)

$\vec{V}_{A/B} \Rightarrow$ vel of A w.r.t. B

eg: 1

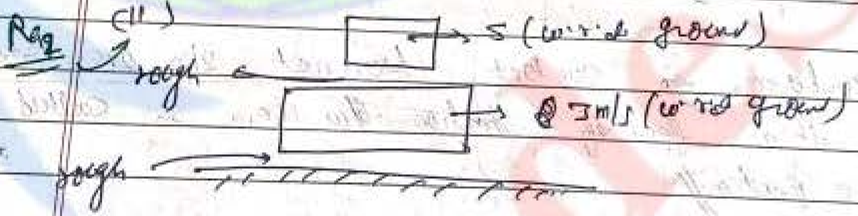
(i)



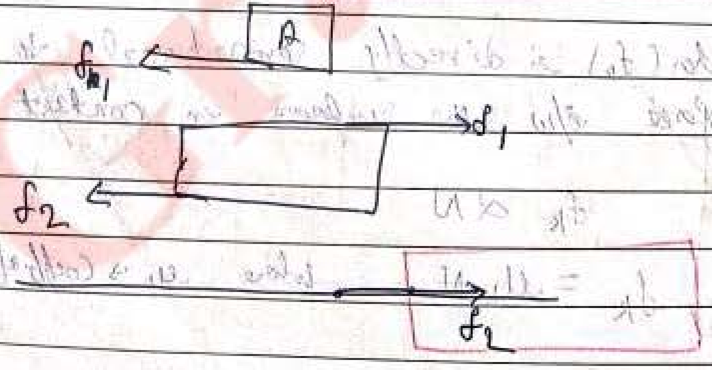


Rq2

(ii)

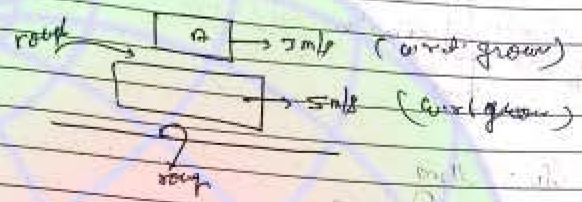


⇓



Ra1

(iii)



$\vec{V}_{A/B} = 3-5 = -2\,m/s$

↳ contact surface पर एक दूसरे के Respect में Relative motion देखना है

Ra1

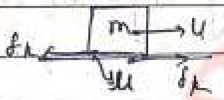
Q) A block of mass "m" is projected with velocity "u" along horizonal rough surface Coeff of friction is "μ" find →

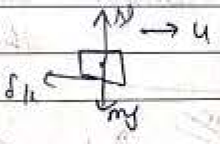
(i) retardation

(ii) stopping time

(iii) stopping distance

80/4 sol





(✓)

(i) $f_k = \mu_k \times N$

$= \mu \times mg$

$\mu = f_k$

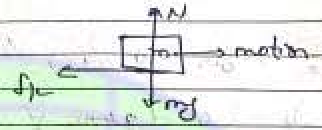
$\mu = \cancel{\mu} mg$

$mg = 1$

$\boxed{g = \frac{1}{m}}$

(i) g.

Teacher



$$f_k = \mu mg$$

(i) $a = \frac{f_k}{m} = \frac{\mu mg}{m} = \mu g$
↳ retardation

motion [illegible]
retarding is [illegible]
[illegible] motion

$v = 0$ (stopping time)

$v = u - at$

$0 = u - at$

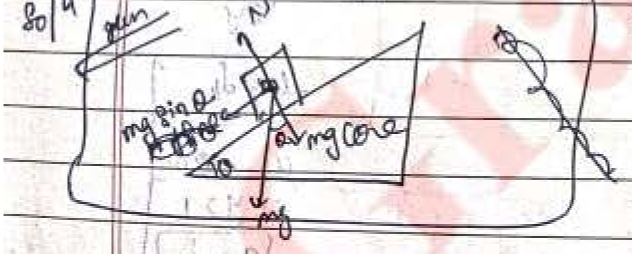
$$\boxed{t = \frac{u}{a}}$$

(ii) $0 = u^2 - 2as$

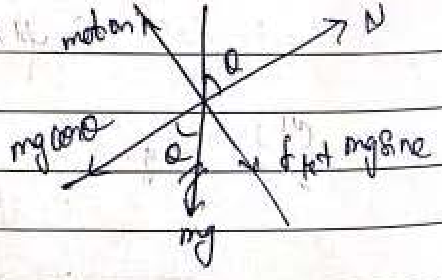
$$\boxed{s = \frac{u^2}{2a}}$$

eg.) In the above question if projection is along the rough inclined plane having angle of inclination "θ" then retardation is

Sol^n



Teacher



Ra)

$$\alpha = \frac{f_k + mg\sin\theta}{m}$$

$$\alpha = \frac{\mu \cancel{m}g\cos\theta + \cancel{m}g\sin\theta}{\cancel{m}}$$

$$\alpha = \mu g\cos\theta + g\sin\theta$$

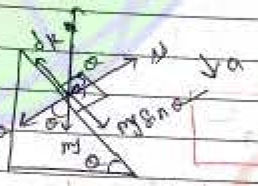
Note:

(i) $t = \frac{u}{\alpha}$ (stopping time)

(ii) $S = \frac{u^2}{2\alpha}$ (stopping distance)

Q) A block of mass "m" is kept at the top of rough inclined coeff of frict is "μ". Block slides downwards its acc^n is

Sol^n

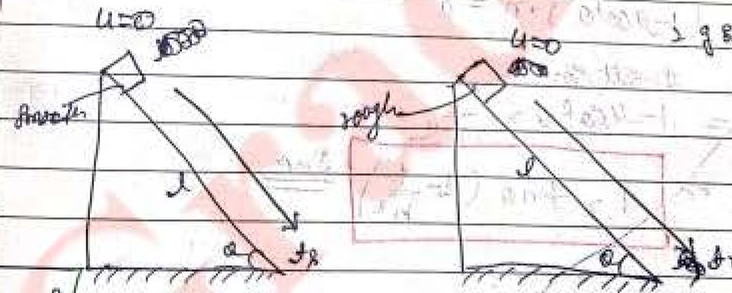


$$a = \frac{mg\sin\theta - f_k}{m}$$

$$= \frac{\cancel{m}g\sin\theta - \mu\times\cancel{m}g\cos\theta}{\cancel{m}}$$

$$= g\sin\theta - \mu g\cos\theta$$

Ra, eg.)



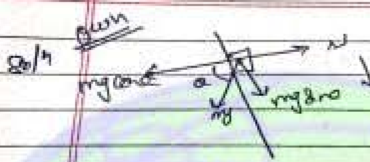
$\because S = \frac{1}{2}at^2$

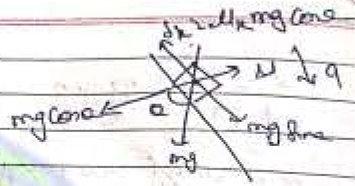
$$\boxed{a_r t_r^2 = a_s t_s^2}$$

given that, $t_r = n t_s$ (n>1)

coeff of friction is —

20/4 Ques





$N = mg\cos\theta$ —(i)

~~mgsinθ~~ $\frac{mg\sin\theta}{\cancel{m}} = a$

$a = g\sin\theta$ —(ii)

~~from eq (ii)~~

$\mu_k =$ ~~tanθ~~ $- \tan\theta$

$N = mg\cos\theta$ —(i)

~~a =~~ $\frac{\cancel{m}g\sin\theta - \mu_k \cancel{m}g\cos\theta}{\cancel{m}}$

$a = g\sin\theta - \mu_k g\cos\theta$

$\frac{a - g\sin\theta}{g\cos\theta} = -\mu_k$

$\mu_k = \frac{\cancel{g}\sin\theta}{\cancel{g}\cos\theta} - \frac{a}{g\cos\theta}$

$= g\tan\theta - \cancel{g}\,a\,g$ [illegible]

Trick

$S = \frac{1}{2}at^2$

$$\boxed{a_r t_r^2 = a_s t_s^2}$$

$$(\cancel{g}\sin\theta - \mu\cancel{g}\cos\theta)\left(\frac{t_r}{t_s}\right)^2 = \cancel{g}\sin\theta$$

$= (1-\mu\cot\theta)\cdot n^2 = 1$

~~1 − μ cos~~

$= 1 - \mu\cot\theta = \frac{1}{n^2}$

or $\boxed{\mu = \tan\theta\left(1-\frac{1}{n^2}\right)}$ Imp

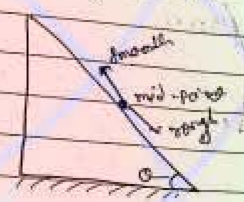
e.g. if $n = 2$

$\mu = \frac{3}{4}\tan\theta$

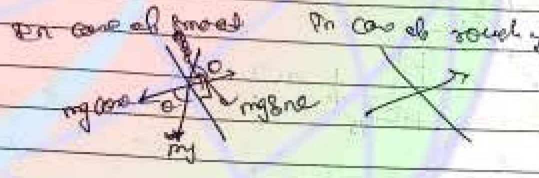
Qa

Q) A block is kept at the top of inclined plane and released it reaches the bottom with zero velocity find coeff. of friction

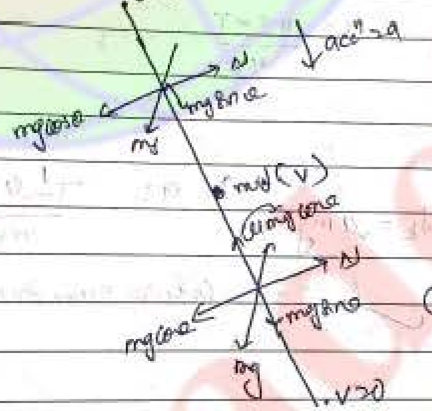
Qa



S/n Ans



Trad



↓ $acc^n = a$

$a = \frac{\cancel{m}g\sin\theta}{\cancel{m}} = g\sin\theta$

$(ret. = \alpha)$ ↑ $(acc^n$ & opp direction$)$

~~$\alpha = \mu mg\cos\theta - mg\sin\theta$~~

$\alpha = \frac{\mu \cancel{m}g\cos\theta - \cancel{m}g\sin\theta}{\cancel{m}}$

$= \mu g\cos\theta - g\sin\theta$

Now

on ~~the~~ smooth inclined

$v^2 = 2as$

Rq

~~on the rough~~

on the rough Inclined

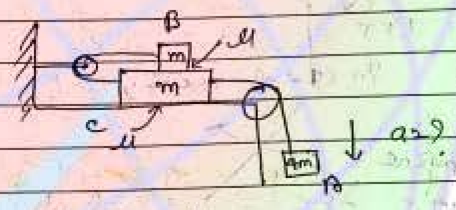
$0 = v^2 - 2aS$

$\therefore a = a'$

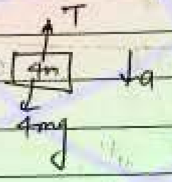
$\cancel{g}\sin\theta - \mu\cancel{g}\cos\theta - \cancel{g}\sin\theta$

$$\boxed{\mu = 2\tan\theta}$$

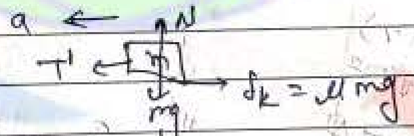
eg.)



Sol^n own (in some step)



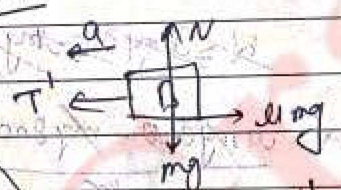
$a = \frac{mg - T}{m}$ —(i)



$a = \frac{T' - \mu mg}{m}$ —(ii)

acc^n दोनों blocks का same होगा।

Trick

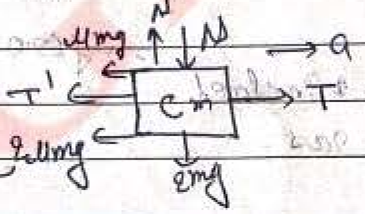


$T' - \mu mg = ma$ —(i)

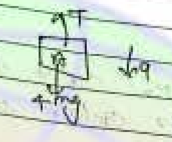
Remark

$N + mg = N'$

$N' = 2mg$



$T - T' - 3\mu mg = ma$ —(ii)



$$4mg - T = 4ma \quad —(III)$$

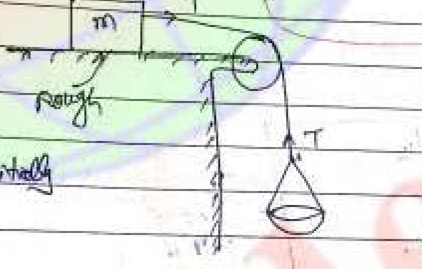
adding I, II, III a

$$4\cancel{m}g - 4\mu \cancel{m}g = 6\cancel{m}a$$

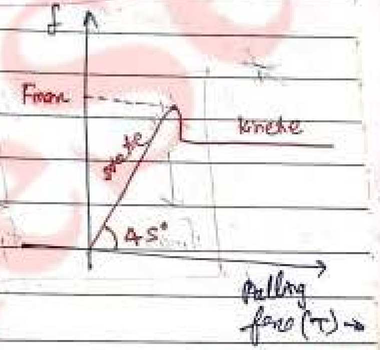
$$a = \frac{4g}{6}(1-\mu)$$

$$a = \frac{20}{3}(1-\mu)$$

✫ Properties of static friction force ⇒ (f_s)



Block is initially at rest



✱ Fmax ∝ N

$$\boxed{f_{max} = \mu_s N}$$

where
μ_s = coefficient of static friction.

Fmax = maximum possible value of static friction.
or
limiting friction force.

- μ_s depends on :-
  - ↳ smoothness of the contact surface
  - ↳ material of the contact surface

- $F_{max} > f_k$

  $\mu_s > \mu_k$

- Let static friction force ⇒ f_s

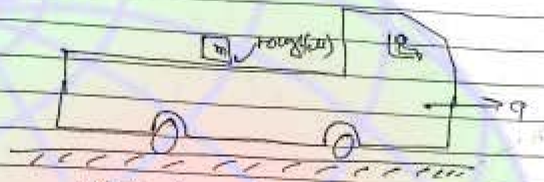
  $\boxed{f_s \le f_{max}}$ v.v.g Imp

★ **Static friction are self adjusting समान है ///**

★

$$\begin{tikzcd} \mu_s \arrow[d] & \mu_k \arrow[d] \\ f_{max} & f_k \end{tikzcd}$$

q)



Block does not slip ~~so~~ friction force is

~~(A) ma~~

(B) μmg

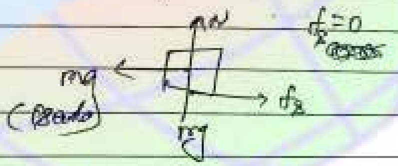
(C) zero

(D) Nothing can be predicted.

μmg

soln (A) ma (Pseudo force)

w.r.t. floor



$\boxed{f_s = ma}$

q2) In the above question what can be the max. acc^n of the truck so that block does not slip.

soln

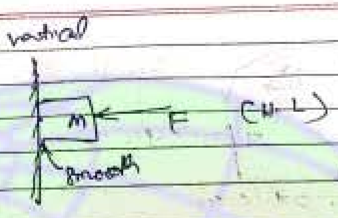
~~~~so~~ f_s~~ = ~~~~

$\boxed{f_s \le f_{max}}$

$\Rightarrow ma \le f_{max}$

$\Rightarrow \cancel{m}a \le \mu \cancel{m}g$ so $\boxed{a \le \mu g}$
~~~~

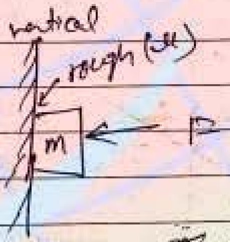
Q)



Sol

(smooth के case में कितना भी force लगालो नीचे को block slip होगा ही होगा)

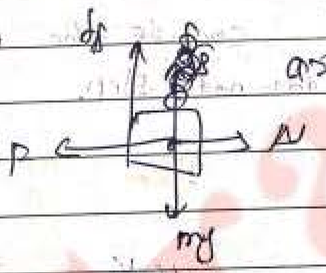
Q) (1)



Block does not slip friction force

- ~~(A)~~ mg
- (B) μP
- (C) zero
- (D) Nothing can ~~[illegible]~~ be predict.

Sol mg

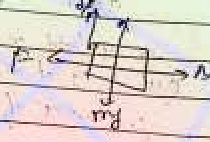


$f_s = mg$

(11) What should be minimum value of F so that block does not slip

Sol

Rq,



Rq, → $\boxed{f_s \le f_{max}}$

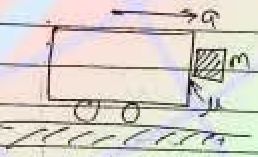
$mg \le \mu F$

~~$f_s = mg$~~

$F \geq \frac{mg}{\mu}$

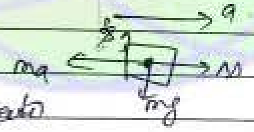
$\boxed{F_{min} = \frac{mg}{\mu}}$

Q)



what should be the minimum acc' of bus so that block does not fall down.

Sol



N = ma , mg = f_s — (1)

N = ma

and f_s = mg

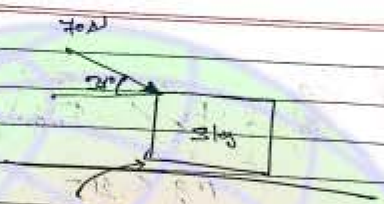
∴ $f_s \le f_{max}$

$mg \le \mu ma$

$\cancel{m}g \le \mu \cancel{m}a$

$a \geq \frac{g}{\mu}$

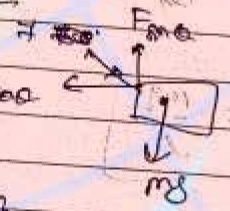
eg)

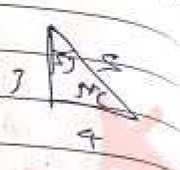


$\mu_s = 0.5$, $\mu_k = 0.4$

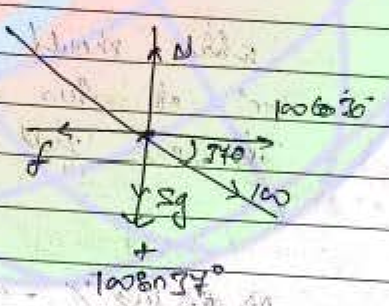
~~Block~~ A force of 100N is applied on the block (Initially at rest) as shown in fig. Acc^n of the block is

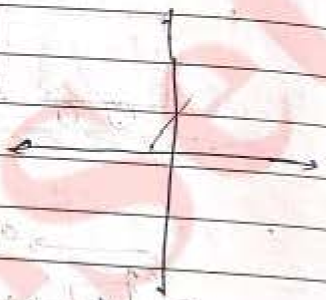
& M prev

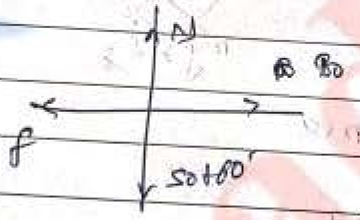




Teach







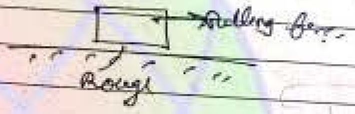
N = 110 Newto

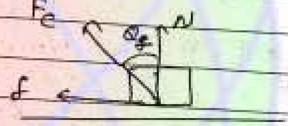
$f_{max} = \mu_s N$

$= 0.5 \times 110$

Angle of friction

Angle b/w Normal force and contact force is called friction angle.





$$\tan \phi_c = \frac{f}{N}$$

$$\boxed{\phi_f = \tan^{-1}\left(\frac{f}{N}\right)}$$

Special Case : if $f = f_{max}$

$$\phi_f = \tan^{-1}\left(\frac{f_{max}}{N}\right)$$

but

$$f_{max} = \mu_s N$$

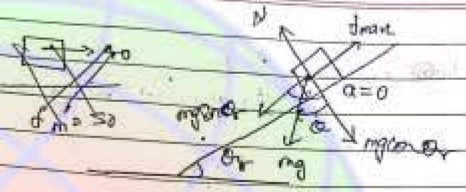
$$\therefore \quad \phi_f = \tan^{-1}(\mu_s)$$

= maximum angle of friction.

Note

$$\boxed{\text{Friction angle} \leq \tan^{-1}(\mu_s)}$$

Repose Angle (θ_r)



(Block is on the verge of sliding.

$mg \sin\theta_r = f_{max}$

$\cancel{mg} \sin\theta_r = \mu_s \cdot \cancel{mg} \cos\theta$

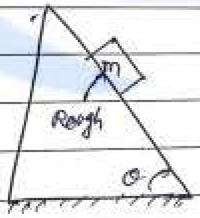
~~[illegible]~~

$\boxed{\tan\theta_r = \mu_s}$

$\boxed{\tan\theta_r = \mu_s}$

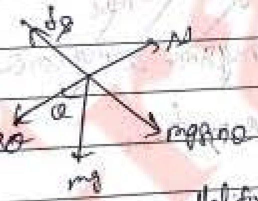
$\boxed{\theta_r = \tan^{-1}(\mu_s)}$

★



Block of mass m rests on Inclined plane.

~~a) friction force~~



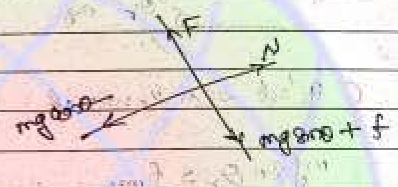
a) friction force is : ?

$\boxed{f_s = mg\sin\theta}$ A

b) minimum force parallel to the Inclined required to move the block up the Inclined : ?

(I)

sol ~~$f_{min} \geq f_s + mg\sin\theta + \mu mg\cos\theta$~~



To move the block up the inclined

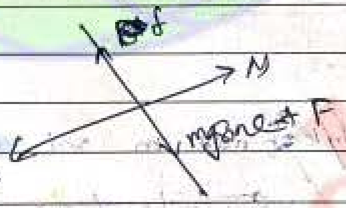
$F \geq mg\sin\theta + f_{max} \rightarrow (I_1)$

$$\boxed{F_{min} = (mg\sin\theta + \mu mg\cos\theta)}$$

(iii) minimum force (parallel to the inclined surface) required to move the block down the inclined

i.e.

sol



To move the block

~~$F = mg\sin\theta - f_{max}$~~

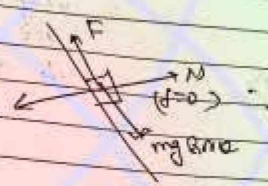
~~$= mg\sin\theta - \mu mg\cos\theta$~~

$F + mg\sin\theta \geq f_{max}$

$F \geq \mu mg\cos\theta - mg\sin\theta$

$F_{min} = \mu mg\cos\theta - mg\sin\theta$

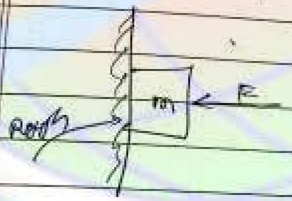
(iv) Force required (∥ to the incline) to make the friction force zero i.e.



F = mg sinθ

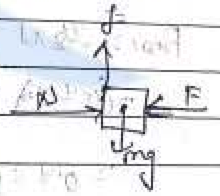
Note

g)



(1) block does not slip angle of friction is

Sol (1)



$f = mg$ —(i)

$N = F$ —(ii)

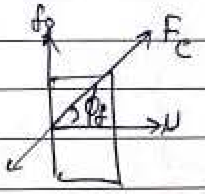
$$\phi_s = \tan^{-1}\left(\frac{f}{N}\right)$$

$$= \tan^{-1}\left(\frac{mg}{F}\right)$$

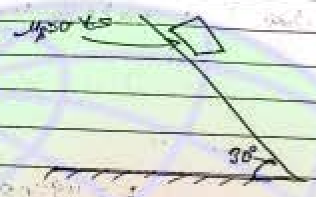
(ii) Contact force is

$$F_c = \sqrt{N^2+f^2}$$

$$= \sqrt{F^2+m^2g^2}$$

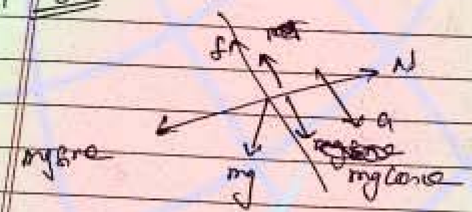


eg.)



A block is kept on the rough inclined plane and released find its acc?

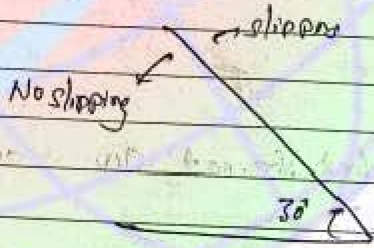
Sol^n ocn

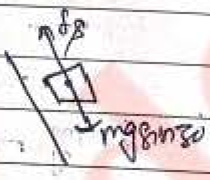


$a = mg\cos\theta - f$

$= mg\sin\theta - 0.75\, mg\sin\theta$

Teach





$F_{max} = \mu_s N$

$= (0.75)\, mg\sin 30°$

$= 0.75\, mg\sin 30°$

Here

$mg\sin 30° < F_{max}$

So No slipping

$a = 0$

and $f = mg\sin 30°$

Stat

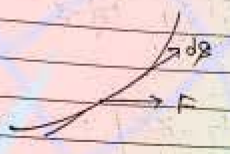
Work, Energy, and Power

work:

i) It is a scala quantity

ii) S.I unit = Joule (J)

$$1 J = 1 kg.\frac{m^2}{s^2}$$



$$dw = \vec{F}\cdot d\vec{r}$$

or

$$dw = \vec{F}\cdot d\vec{r}$$

and

$$\boxed{w = \int \vec{F}\cdot d\vec{r}}$$ → when force is variable//

where

$\vec{F}$ = force acting on the particle.

if $\vec{F}$ = constant

$$W = \vec{F}\cdot \int d\vec{r}$$

or

$$\boxed{W = \vec{F}\cdot \Delta\vec{r}}$$ → when force is constant//

☆ If $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

and

$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

then

$$W = \int \vec{F}\cdot d\vec{r}$$

$$\boxed{W = \int F_x dx + F_y dy + F_z dz}$$

if $\vec{F}$ = constant

$$W = F_x \int dx + F_y \int dy + F_z \int dz$$

$$\boxed{W = F_x \Delta x + F_y \Delta y + F_z \Delta Z}$$

if force is acting along x-axis

then $\boxed{W = \int F_x dx}$ → If F is variable

and

if F_x = Constant

$\boxed{W = F_x \Delta x}$ → If F is constant

CE

Target C

DAILY

ion based on : Work & Ener

A horizontal force of 5 N

velocity of 2 m/s for a blo

over a rough surface. The

in one minute is

(1) 600 J (2) 60 J

A cord is used to lower

M by a distance d

acceleration $\frac{g}{4}$. Work

block is

(1) $Mg\frac{d}{4}$ (2) 3Mg

A bullet of mass m m

suspended wooden b

rises to a height h, t

will be

(1) $\sqrt{2gh}$

(3) $\frac{m}{M+m}2gh$

The energy requir

to 20 m/s is how

accelerate the car

(1) Equal (2)

A body of mass

which is quadra

the surfaces are

rest, its speed a

(1) 4.43 m/sec

(3) 0.5 m/sec

CAREER

gur

ER POINT, CP

Kinetic energy and work energy theorem →

Energy by virtue of motion of the particle is defined as its kinetic energy.

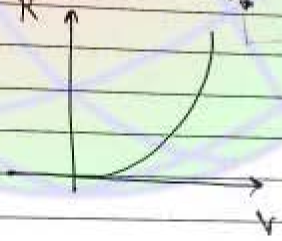
It is given by

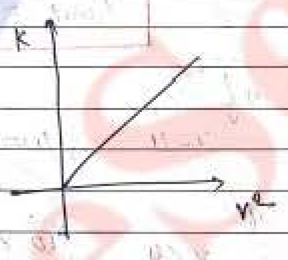
$$\boxed{K = \frac{1}{2}mv^2} \quad (v <<<< c)$$

So in form of momentum

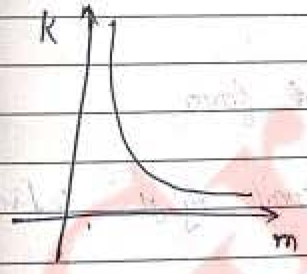
$$\boxed{k = \frac{P^2}{2m}} \quad \because P = mv$$

Case 1st → mass Constant if





Case 2nd → P = Constant if



Note: if $P_1 = P_2$

$$\boxed{\frac{k_1}{k_2} = \frac{m_2}{m_1}}$$

$$K \propto \frac{1}{m}$$

(B) Work energy theorem :-

According to the theorem total workdone on a partic is equal to the change in its kinetic en

$$\boxed{W_{total} = \Delta K}$$

$$W_{total} = K_f - K_i$$

$$\boxed{W_{total} = \frac{1}{2} m(v^2 - u^2)}$$

If $v = u$, $W = 0$

If $v < u$, $W < 0$

If $v > u$, $W > 0$

☆ W_{total} = workdone by resultant force

= Algebric sum of work by the individu force.

☆ $$\boxed{W_{total} = \int \vec{F_r} \cdot \vec{dr}}$$

$$= \int (\vec{F_1} + \vec{F_2} + \vec{F_3} + \dots + \vec{F_4}) \vec{dr}$$

CARE

Target Course

DAILY PRA

W

estion based on : Work & Energy

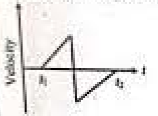
A particle of mass m is moving in a circle of radius r under a centripetal fo $-K/r^2$, where K is a constant. The to the particle is

(1) $\frac{K}{2r}$ (2) $-\frac{K}{2r}$ (3) $-\frac{K}{r}$

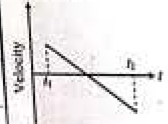
2 The displacement x of a particle dimension under the action of a co related to the time t by the equa where x is in meters and t is in sec done by the force in the first 6 sec

(1) 9 J (2) 6 J (3) 0 J

3 A batsman hits a sixer and th ground outside the cricket gro following graph describes th cricket ball's vertical velocity the time t_1 as it hits the bat touches the ground



(2)



(4)

CAREER POINT gurukul

AREER POINT, CP Tower, IPIA, R

$$W_{total} = \int \vec{F_1}\cdot d\vec{r} + \int \vec{F_2}\, d\vec{r} + \cdots + \int \vec{F}\, d\vec{r}$$

$$\boxed{W_{total} = W_1 + W_2 + W_3 + \cdots + W_n}$$

Note

① $\because$ $W_{total} = \Delta K$

or $\boxed{W_1 + W_2 + \cdots + W_n = \Delta K}$

② $W = \int \vec{F}\cdot d\vec{r}$

if $\vec{F} = \text{constant}$

$W = \vec{F}\cdot d\vec{r}$

$= \vec{F}\cdot\vec{S}$

$$\boxed{W = |\vec{F}|\cdot|\vec{S}|\cos\theta}$$

θ = angle b/w $\vec{F}$ and $\vec{S}$

if $0° < \theta < 90°$ $W > 0$

if $90° < \theta < 180$ $W < 0$

if $\theta = 0$ $W = FS$

if $\theta = 180$ $W = -FS$

Work done by gravity →

Note: object is near the earth surface ⇒

$g = 9.8 m/s^2$

$\approx 10 m/s^2$

- Workdone by gravity does not depends on path it depends on vertical height b/w final and initial position

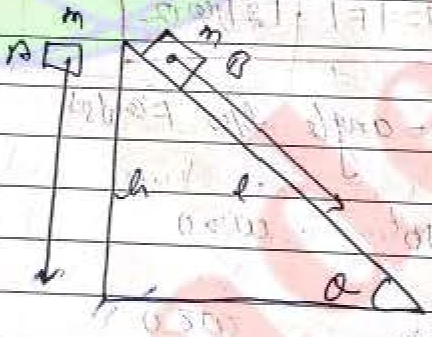
$W_g = +mgh$ (in descending motion)

and

$W_g = -mgh$ (in ascending motion)

and

$W_g = 0$ (in horizontal motion.



① Workdone by gravity on "A" $= F_g h \cos 0°$

$= mgh$

(ii) workdone by gravity on B $= F_g l \cos(90-\theta)$

$= mg l \sin\theta$

$= mgh$ $\left[\because \sin\theta = \frac{h}{l}\right.$

tion based on : Work

A particle free to mo

potential energy given b

for $-\infty \le x \le +\infty$, whe

appropriate dimension

(1) At point away fr

in unstable equili

(2) For any finite n

force directed a

(3) If its total mech

minimum kineti

(4) For small dis

motion is simp

The kinetic energ

traveling a certai

under the action

proportional to

(1) $\sqrt{m}$

(3) $1/\sqrt{m}$

An open knife

height 'h' on a w

upto the depth

resistance offer

(1) mg

(3) $mg\left(1+\frac{h}{d}\right.$

L-2 → 34-40-49

L-2 →

, L-1 → 45066//

q)

Block is released from the position shown in fig.

(I) workdone by gravity from $\overset{i}{P}$ to $\overset{f}{Q}$ is

Ans = mgr

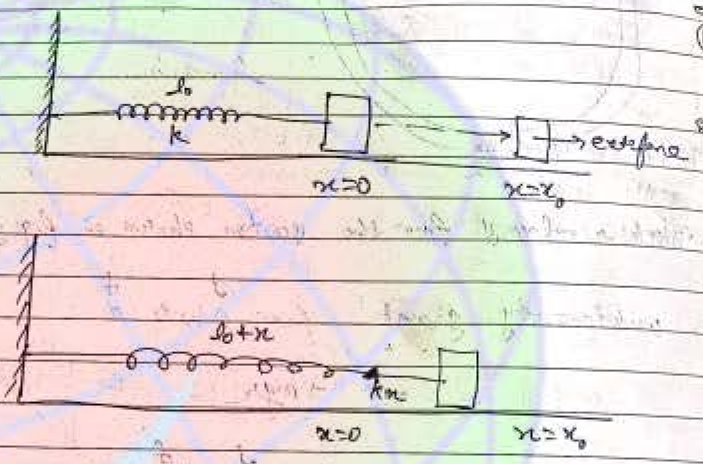
(II) workdone by gravity from $\overset{i}{Q}$ to $\overset{f}{R}$ is

Ans = -mgh

(III) workdone by gravity from $\overset{i}{P}$ to $\overset{f}{R}$ is

Ans = 0 //

workdone in streaching a spring →



$$dW_{sp.} = -kx\,dx$$

$$W_{sp.} = \int_0^{x_0} -kx\,dx$$

$$\boxed{W_{sp} = \frac{-kx_0^2}{2}}$$ from $x=0$ to $x=x_0$

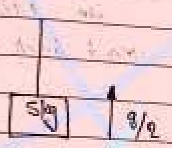
∴ work done against the spring force $= (-)W_{sp}$ (by ext agent) $= \dfrac{kx_0^2}{2}$

ote From $x=x_1$ to $x=x_2$

$$\boxed{W_{sp.} = \frac{kx_1^2}{2} - \frac{kx_2^2}{2} = \frac{k}{2}(x_1^2 - x_2^2)}$$ by spring force

or $$\boxed{W_{ext.force} = \frac{k}{2}(x_2^2 - x_1^2)}$$ Against spring force.

Eg.)

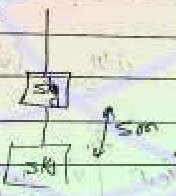


(i) Find workdone by the tension on the block in displacing it by 5m

(ii) Find workdone by the gravity on the block in the displacement of 5m

(iii) Total workdone on the block in the displacement of 5m

Sol^n own

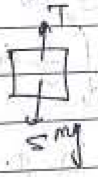


(i) $w = \frac{-k}{2}(x_1^2 - x_2^2)$

$= \frac{-k}{2}(5)^2 = \frac{-25k}{2} = 12.5k$

(ii)

Teache



$T - 50 = \frac{5g}{2}$

$T = 75N$

(i) $w_T = TS\cos 0°$

$= (75)(5) = 375$ joule

(ii) $W_g = -mgh$

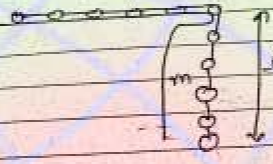
$= -(50)(5) = -250$ joule

(iii) $W_{total} = 375 - 250 = 125$ joule//.

\#

(uniform chain)

Eg:

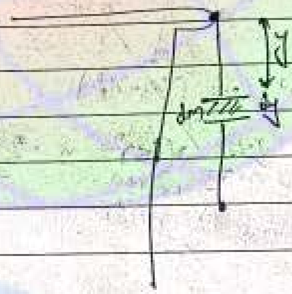


m = mass of hanging
l = length of the hanging chain

Find workdone against gravity in pulling the part back on the table

Soln own

Trick



$$dW_{ext} = (dm)g \cdot y$$

$$W_{ext} = \int_0^l dm \cdot g \cdot y$$

$$\therefore dm = \frac{m}{l} dy$$

$$W_{ext} = \int_0^l \frac{m}{l} dy \cdot gy$$

$$= \frac{m}{l} \cdot g \cdot \frac{l^2}{2} = \frac{mgl}{2}$$

Note

① If a particle of mass "m" is taken to a height work against gravity = mgh.

② If a body of mass "m" is taken to a height "h" work against gravity = mgh_{com} centre of mass

---

CA

Target Cou

DAILY

...tion based on : Collision &

of Momentum

Particle falls from a height

horizontal plane and reboun

coefficient of restitution, th

travelled before rebounding has

(1) $h\left(\frac{1+e^2}{1-e^2}\right)$ (2) $h\left(\frac{1}{1}\right.$

(3) $\frac{h}{2}\left(\frac{1-e^2}{1+e^2}\right)$ (4) $\frac{h}{2}\left(\frac{1}{1}\right.$

A body of mass M_1 collides another mass M_2 at rest. Th transfer of energy when

(1) $M_1 > M_2$

(2) $M_1 < M_2$

(3) $M_1 = M_2$

(4) Same for all values of M_1 a

Two particles having position ve metres and $\vec{r_2} = (-5\hat{i} - 3\hat{j})$ metres velocities $\vec{v_1} = (4\hat{i} + 3\hat{j})$ m/s and If they collide after 2 seconds, the v

(1) 2 (2) 4 (3) 6

A bag (mass M) hangs by a bullet (mass m) comes horizo v and gets caught in the combined (bag + bullet) system

(1) Momentum is $\frac{mvM}{M+m}$

(2) Kinetic energy is $\frac{mv^2}{2}$

(3) Momentum is $\frac{mv(M}{M}$

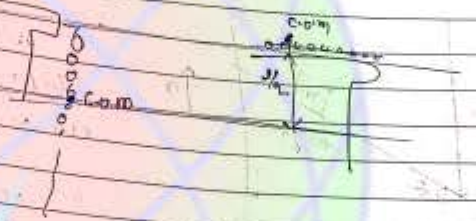
(4) Kinetic energy is $\frac{}{2}$

CAREER POINT

gurukul

CAREER POINT, CP Tower, IPIA

Shortcut (c.o.m)



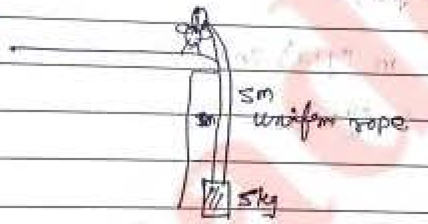
$W_{ext} = \frac{mgl}{2}$

q)

∴ c.o.m = $\frac{l}{2}$

$W_{ext} = \frac{mgl}{2}$

q)



mass per unit length of the rope = 1 kg/m

Find workdone by the mass in pulling hanging system on the platform.

80/4

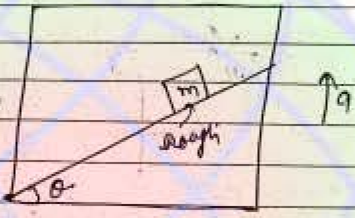
$W_{ext} =$ w done on rope + w done on block

$= 5 \cdot g \cdot \left(\frac{5}{2}\right) + 5g(5)$

~~$w = mgl\cos$~~ $= mg\left(\frac{l}{2}\right)$

$U_{tot} = 5 \times 5$ joule //

eg:)



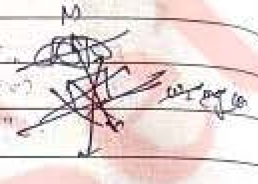
Block does not slip

find :-

(i) workdone by the friction in time t

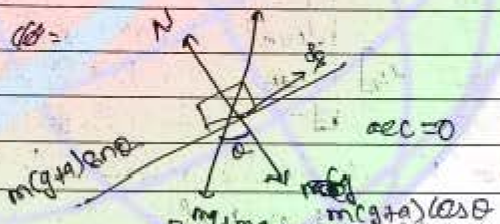
- w.r.t. lift
- w.r.t. ground

(ii) workdone by the Normal w.r.t. ground in time t



Sol^n//

w.r.t. lift



acc = 0

$= mg + ma$ (pseudo)

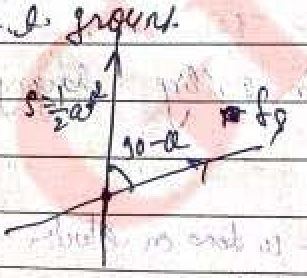
$= m(g+a)$

$f_s = m(g+a)\sin\theta$

$N = m(g+a)\cos\theta$

(i)(a) $(W_f)_{\text{w.r.t. lift}} = 0$

(b) w.r.t. ground

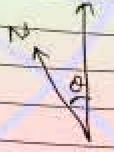


$$\omega = f_s \cdot s \cdot \cos(90-\theta)$$

$$= m(g+a)\sin\theta \cdot \left(\frac{1}{2}at^2\right)\sin\theta$$

$$= m(g+a)\cdot\frac{1}{2}at^2\sin^2\theta \, //$$

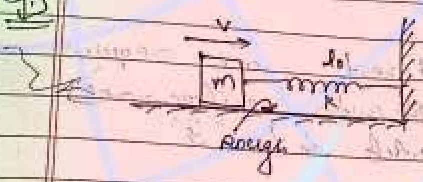
(iii) W. done by the ~~gravity~~ ~~of~~ Normal force w.r.t ground



$s = \frac{1}{2}at^2$

$W = N.S.\cos\theta$
$= m(g+a)\cos^2\theta \cdot \left(\frac{1}{2}at^2\right)$

Q.)



Maximum compression produced by the block in the spring is "x_0" which of the following relation is correct —

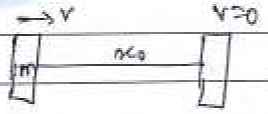
(A) $\frac{kx_0^2}{2} + \mu mgx = \frac{1}{2}mv^2$

(B) $\frac{kx_0^2}{2} - \mu mgx_0 = \frac{mv^2}{2}$

(C) $\mu mgx_0 - \frac{kx_0^2}{2} = \frac{mv^2}{2}$

(D) N.O.T

Sol/n



$W_N + W_f + W_g + W_{sp} = 0 - \frac{1}{2}mv^2$

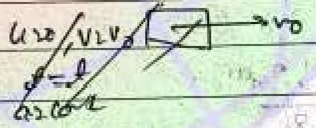
$0 + (-\mu mgx_0) + 0 + -\frac{kx_0^2}{2} = -\frac{1}{2}mv^2$

$\underbrace{\mu mgx_0}_{\text{w.done against friction}} + \underbrace{\frac{kx_0^2}{2}}_{\text{w.done against spring}} = \frac{1}{2}mv^2$

Note kinetic energy of a body represents capacity of doing work before coming to rest.

Q) A particle is initially at rest and acquires velocity "v_0" in time "t_0" under uniform acc^n find workdone on the particle in time "t".

Sol^n



(i) Let velocity after time "t" = v

$$W = \frac{1}{2}mv^2 - 0$$

where

W = work done in time t

(ii) $a = \frac{v_0 - 0}{t_0} = \frac{v_0}{t_0}$

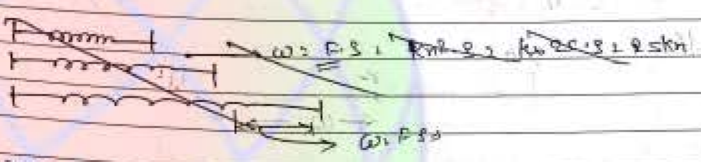
and

$$v = at = \left(\frac{v_0}{t_0} \times t\right)$$

$$\therefore \quad W = \frac{1}{2} m \cdot \frac{v_0^2}{t_0^2} \cdot t^2$$

Workdone in stretching a spring by 5cm from its natural length is W. find workdone to strech it further by another 5cm.

Soln



$w = \frac{k}{2}(5cm)^2$

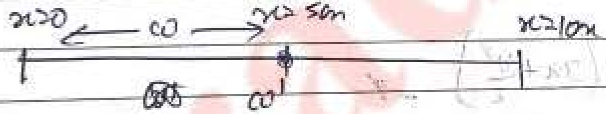
and $w' = \frac{k}{2}((10cm)^2 - (5cm)^2)$

$\frac{w}{w'} = \frac{1}{3}$

$\boxed{w' = 3w}$

Q) workdone in stretching a spring by 5cm from its natural length is W. find workdone to strech it 10cm from natural length.

Sol



$w = \frac{k(5cm)^2}{2}$

$w' = \frac{k(10cm)^2}{2}$

$w' = 4w$

$\frac{w}{w'} = \frac{1}{4}$

Q) A spring of force constant 'k' is initially ~~stre~~ stretched by x. Find workdone to stretch it further by another length y.

Sol^n

$$W = \frac{k(x+y)^2}{2} - \frac{kx^2}{2}$$

Q) A particle of mass 2kg is initially at rest. A force $\vec{F} = (3\hat{i} + y\hat{j})$ newton acts on it. Find workdone by this force in displacing the particle from origin to a point (x, y). Initially particle is at the origin.

Sol^n

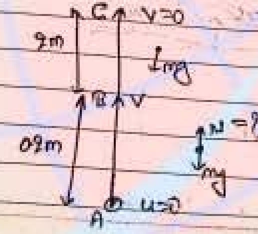
$$W = \int F_x dx + \int F_y dy$$

$$W = \int_0^x 3dx + \int_0^y y\,dy$$

$$= \left(3x + \frac{y^2}{2}\right)$$ Ans.

eg.) A ball of mass 200g ~~[illegible]~~ is thrown vertically upwards by applying a force by the hand. hand moves with the ball upto 0.2m for the distance travelled by the ~~ball~~ ball is 2m.
find force on the ball by the hand

Solⁿ



From A to C

$W_{total} = 0 - 0$

$W_N + W_g = 0$

$N(0.2) + (-mg)(2.2) = 0$

$N = \frac{mg(2.2)}{0.2}$

$N = \frac{0.2}{0.2}(-10)(2.2)$

$= 2.2$ Newton //

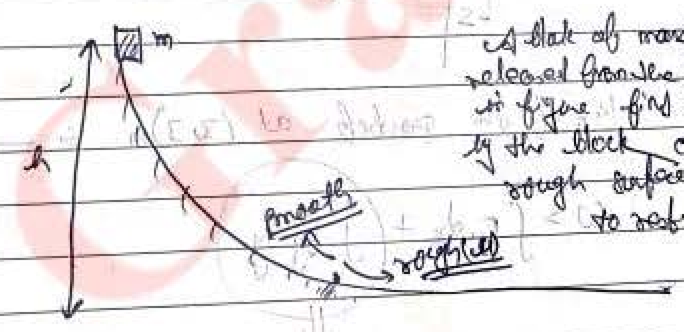
From A to B

$N(0.2) - mg(0.2) = \frac{1}{2}mv^2$

and

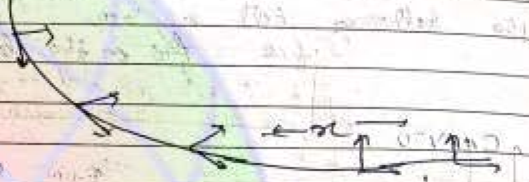
$0 = v^2 - 2g(2)$

eg.)



A block of mass "m" should be released from the position shown in figure. find distance travelled by the block on horizontal rough surface before coming to rest //.

sol^n ~~$\omega_N + \omega_f + \omega_g = 0$~~



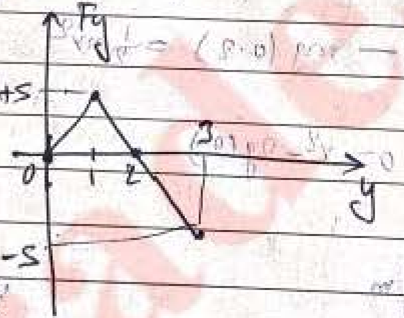
$\omega_N + \omega_g + \omega_f = 0$

$0 + [-\mu m g x] + mgh = 0$

Note:-

If a force is always ⊥r to the motion then workdone by that force will be zero//

Q) A particle is initially at rest, t=0, x=y=0 a force $\vec{F} = 3\hat{i} + F_y\hat{j}$ acts on the particle.



K.E of the particle at (3,3)m is ___.

sol^n //

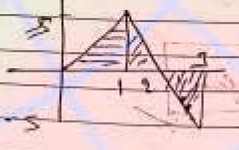
$$W = \int F_x dx + \boxed{\int_i^f F_y dy}$$

⇓

W = 3×3 + Area under the curve.

$$w = 3(3) + \frac{1}{2}(1)(5) \Rightarrow \frac{23}{2}$$

$$= 11.5 \text{ joule}//$$



$$K - 0 = w$$

$$K = 11.5 \text{ joule}//$$

Potential energy and Principle of Conservation of mechanical energy.//

① Potential energy

i) ~~This~~ This energy is defined only for the conservative force field.

ii) If workdone by a force in a round ~~trip~~ trip of a system is always zero then it is called conservative force. otherwise force will be non-conservative.//

iii) Conservative force → i) Gravitational force
ii) Coulomb force
(iii) spring force.

(iv) Non-conservative force → viscous force, friction force//

v) change in P.E of a system as it passes from initial configuration to final configuration is equal to the -ve times of workdone by the Internal conservative force.

$$\boxed{\Delta U = - W_{\text{conservative force}}}$$

(vi) P.E is by virtue of configuration
and
K.E is by virtue of motion.

If $\boxed{W_{\text{conservative force}} > 0}$ $\Delta U < 0$ P.E ↓

If $W_{\text{conservat force}} < 0$ $\Delta U > 0$ P.E ↑

if $W_{\text{conservat force}} = 0$ $\Delta U = 0$ P.E remains unchanged

Attract I

⟶ ⟵ P.E ↓

⟵ ⟶ P.E ↑

P.E ↑ ans false

Both P.E ↓

L-1 → 1 to 14

L-2 → 1, 2, 3, 4, 5, 11, 13,

L-3 → 2, 3, 5, 8, 10, 15

L-4A → 1 to 4, 9, 10, 22

4A L-4

Repulsive

⊕ → ← ⊖   P.E ↓

← ⊕   ⊖ →   P.E ↑

System P.E कम करना चाहता है

☆ Attractive force field | Repulsive force

sep ↓   P.E ↓ | sep ↓   P.E ↑

P.E ↓   ↑ | sep ↑   P.E ↓

S.P ↑   ↑

☆ $\boxed{\Delta U = -W_{cons}}$

Note workdone by the conservative force does not depends on mass it depends on initial and final state of the system//

~~work done depends on state~~

(V) $\boxed{\Delta U = (-)\int_{conservative} \vec{F} \cdot d\vec{r}}$

or

$$\boxed{\Delta U = (-)\int \left(F_x dx + F_y dy + F_z dz\right)}$$

Note

(iii) • $\frac{\partial U}{\partial x} = -F_x$ or $F_x = -\frac{\partial U}{\partial x}$

• $\frac{\partial U}{\partial y} = -F_y$ or $F_y = -\frac{\partial U}{\partial y}$

• $\frac{\partial U}{\partial z} = -F_z$ or $F_z = -\frac{\partial U}{\partial z}$

(iv) $\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$

$$\boxed{\vec{F} = -\frac{\partial u}{\partial x}\hat{i} + \frac{\partial u}{\partial y}\hat{j} - \frac{\partial u}{\partial z}\hat{k}}$$

(v) If force acting along x-axis

$$\boxed{\Delta U = (-)\int F_x\, dx}$$

and

$$\boxed{F_x = -\frac{\partial U}{\partial x}}$$

Gravitational Potential energy

Note

① Object is near the earth surface i.e.

(ii) $h << R_e$, $R_e \rightarrow$ Radius of earth

(3) Change in Gravitational potential energy = (−)(work done by the gravity)

By formula

$$\boxed{\Delta U = (-) W_{cons}}$$

(iv) * In ascending motion i.e.

$$\boxed{\Delta U = mgh}$$

* In descending motion

$$\boxed{\Delta U = -mgh}$$ [that is P.E is decreases by −mgh]

$\Delta U = -5$ Read as
↳ change in x is equal to "−5"
or
↳ potential energy decreases by "5"

* In horizontal motion $\boxed{\Delta U = 0}$

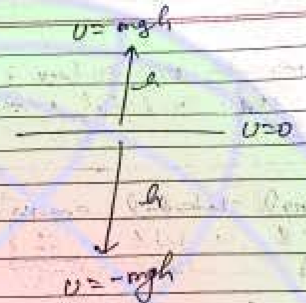
where, m = mass of the particle

Note

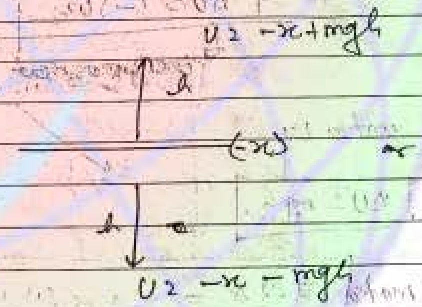
If, (m = mass of a continuous body)

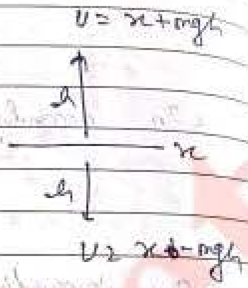
then $[h = h_{c.o.m}]$

(v)



(vi)

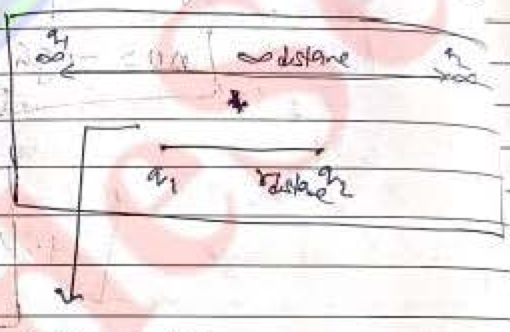




(vii)

$$\boxed{U_r - U_\infty = \frac{kq_1q_2}{r}}$$

$\because U_\infty = 0$



Then work done

$$\boxed{U_r = \frac{kq_1q_2}{r}}$$

(viii)

K.E $\xrightarrow{depends}$ $\boxed{\text{total ~~work~~done force}}$ के work done पर K.E depends

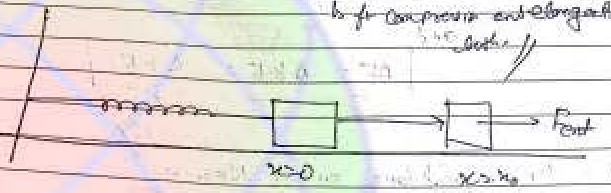
P.E $\xrightarrow{depends}$ $\boxed{\text{Conservative force}}$ के work done पर depend होता है

Elastic Potential energy (P.E of a spring)

① change in elastic P.E = (−) workdone by the elastic force (elastic force)

↳ for compression and elongation both //

(2)



$$\Delta U = (-)\left(\frac{-kx_0^2}{2}\right)$$

$$\boxed{U_{x=x_0} - U_{x=0} = \frac{kx_0^2}{2}}$$

⇒ for comp. and elongation both

Note ③

P.E of a spring is supposed to be zero in its natural length.

$$(U_{x=0})$$

(4)

$$\boxed{U_{x=x_1} = \frac{kx_0^2}{2}}$$

⇒ for compression and elongation both

* Principle of conservation of mechanical energy

① Let

E = mechanical energy

$\boxed{E = K.E + P.E}$

Note

conservative force mechanical energy को बदल नहीं सकता

and

$\boxed{\Delta E = \Delta K.E + \Delta P.E}$

(2) For workdone energy theorem

$\boxed{W_{all} = \Delta K.E}$

Conservative   other than conservative

$\boxed{W_{conservative} + W_{other\ force} = \Delta K.E}$

or

$-\Delta U + W_{other\ force} = \Delta K.E$

or

$\boxed{W_{other\ force} = \Delta K.E + \Delta P.E}$

or

$\boxed{W_{other\ force} = \Delta E}$

If $\boxed{W_{other} = 0}$

then $\Delta.E = 0$

or $\boxed{K_f = K_i}$

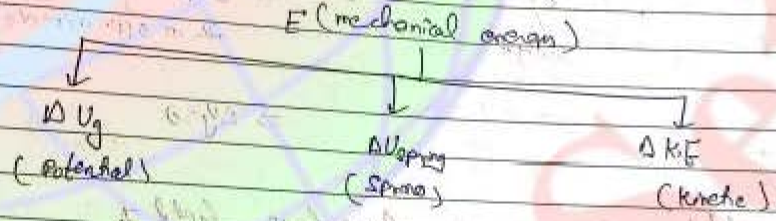
$\boxed{\Delta U = \Delta PE}$
↳ [illegible] zero K

i.e. If work done by ~~con~~ other than conservative force is zero than mechanical energy of the system remains unchanged. This is called principle of conservation of mechanical energy.

Note (3)

- $\boxed{W_{all} = \Delta K.E}$
- $\boxed{W_{conservative} = -\Delta U}$
- $\boxed{W_{other} = \Delta E}$

(4)

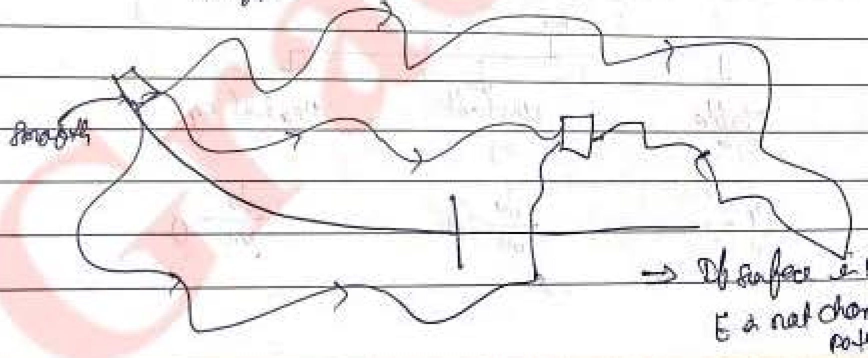


$\boxed{\Delta U_g + \Delta U_{sp} + \Delta K.E = \Delta E}$

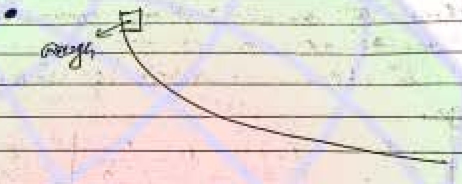
* If $\boxed{\Delta E = 0}$ or (if $W_{other} = 0$)
then

$\boxed{\Delta U_g + \Delta U_{sp} + \Delta K.E = 0}$
↳ gravitational potential energy.

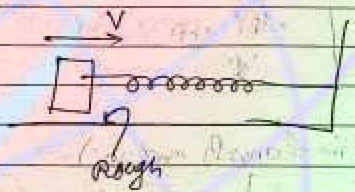
(5)



→ If surface is smooth E is not change for any path.

- 

In the Case "E↓" because Contral force friction force act//

- 

⇒ E↓ because fricti is in opp. directe

⇒ $\Delta U \geq 0$

★ Stability in Conservative force field :-

A particle is said to be in eqⁿ if net force acting on it is zero

if F=0 then $\frac{du}{dx}=0$

eqⁿ
- stable eqⁿ
- unstable eqⁿ
- Neutral eqⁿ

$\frac{dy}{dx}=0$ $\frac{du}{dx}=0$ $\frac{dy}{dx}=0$

① Stable

A part
energy
points.

(2) unstable
A p
energy

(3) Neutr

A
ener

⇒ Gra

① Stable eq^n :-

A particle is said to be stable eq^m if its potential energy is minimum as compare to the neighbouring points.

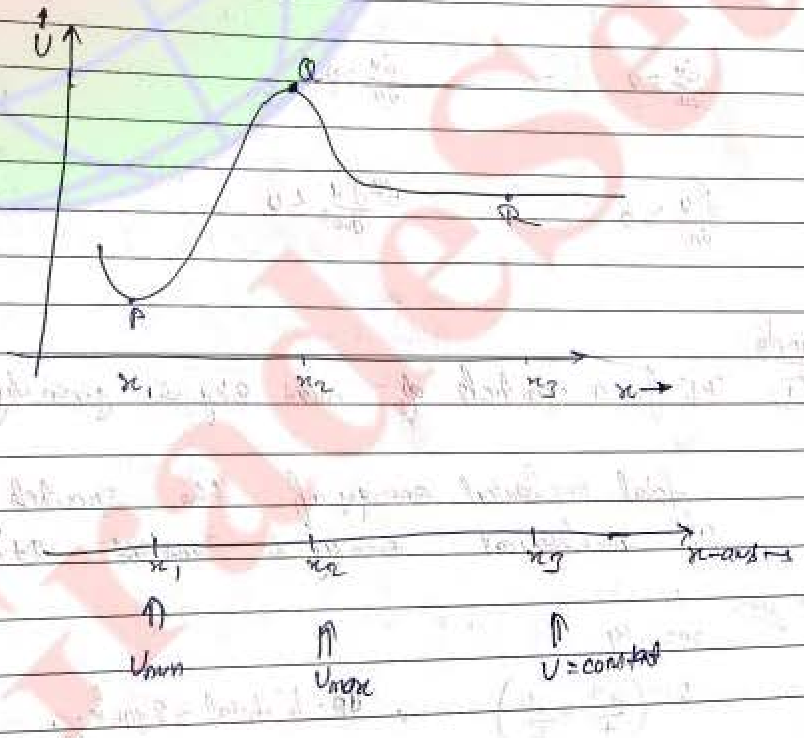
(2) Unstable eq^n :-

A particle is said to be unstable eq^m if its potential energy is maximum as compare to neighbouring points.

(3) Neutral eq^m.

A particle is said to be in neutral eq^m if its potential energy is same as that of the neighbouring point's.

⇒ Graph :-



at

$x = x_1$ (stable eq^m)

$x = x_2$ (unstable eq^m)

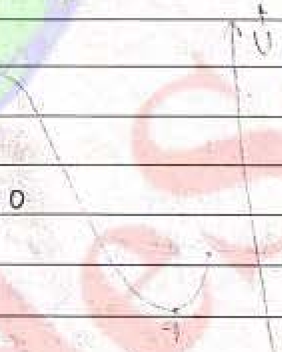
$x = x_3$ (Neutral eq^m)

so

$$\boxed{\frac{du}{dx} = 0 \quad \text{at} \quad \begin{array}{l} x = x_1 \\ x = x_2 \\ x = x_3 \end{array}}$$

Note

stable eq^n	Unstable eq^n	Neutral eq^n
U_{min}	U_{max}	$U = \text{const}$
$\frac{du}{dx} = 0$	$\frac{du}{dx} = 0$	$\frac{du}{dx} = 0$
$\frac{d^2u}{dx^2} > 0$	$\frac{d^2u}{dx^2} < 0$	$\frac{d^2u}{dx^2} = 0$



Example

① P.E of a particle of mass 2kg is given by $U = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$

so total mechanical energy of the particle is 2 joule. If mechanical energy is constant It's maximum speed.

Sol^n

$m = 2kg$

$U = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$, $E_{total} = 2$ joule

$\frac{x^4}{4} - \frac{x^2}{2} + 2\times0 +$

Teacha

$E = K.E + U$

$K.E =$ ~~[illegible]~~ $E - U$

$$U = \frac{x^4}{4} - \frac{x^2}{2}$$

$$\frac{du}{dx} = x^3 - x$$

If $\frac{du}{dx} = 0$, $x = 0$, $x = \pm 1$ m//

$$\frac{d^2U}{dx^2} = 3x^2 - 1$$

at $x = 0$ $\frac{d^2u}{dx^2} < 0$ (U_{max})

at $x = \pm 1$

$\frac{d^2u}{dx^2} > 0$ (U_{min})

$U_{min} = \frac{1}{4} - \frac{1}{2} = \frac{-1}{4}$ joule.//

$K_{max} = E - U_{min}$

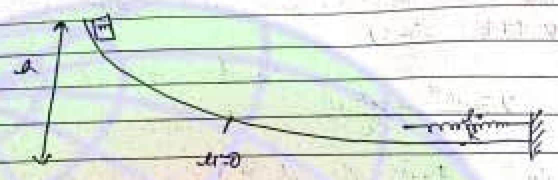
$= 2 - \left(-\frac{1}{4}\right) = \frac{9}{4}$ joule

$$\frac{1}{2} m v_{max}^2 = \frac{9}{4}$$

$$\frac{1}{2} \times 2 \times v_{max}^2 = \frac{9}{4}$$

$v_{max} = \frac{3}{2}$ m/s Ans

②



Block is released from the position shown in fig. ~~~~ maximum compression in the spring will be -

Sol/n

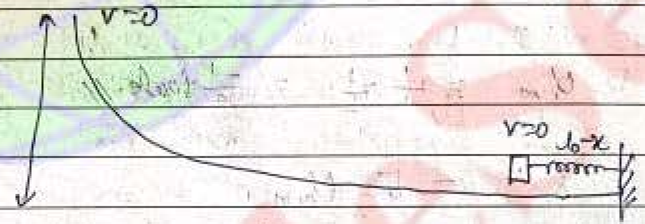
N (other), Fg, Fspring (Conservative)

$\boxed{W_N = 0}$

$\therefore \Delta E = 0$

or

$\Delta U_g = -mgh$



$\Delta U_g = -mgh$

$\Delta U_{spring} = \frac{kx^2}{2}$

$\Delta K = 0$

$\Delta U_g + \Delta U_{spring} + \Delta KE = 0$

$-mgh + \frac{kx^2}{2} + 0 = 0$

$x = \sqrt{\frac{2mgh}{k}}$

Note

① Deen

② From

Example

So/n

Note

① Decrease in K.E of the block = Increase in P.E of the spring

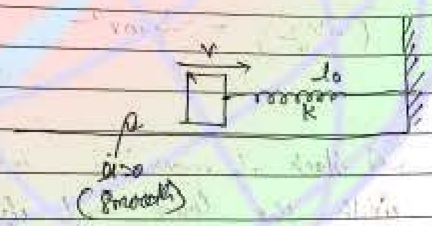
$$mgh = \frac{kx^2}{2}$$

② From work energy theorem.

$$W_N + W_g + W_{sp} = 0 - 0$$

$$0 + (mgh) + \left(\frac{-kx^2}{2}\right) = 0$$

Example



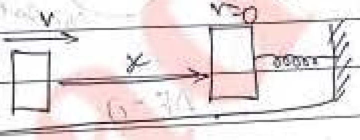
maximum Compression in the spring is —

Sol^n $\bigcirc N$ $\bigcirc F_g, F_{sp}$ and $W_N = 0$

other Cons.

$\Delta E = 0$

$\Delta U_g = 0$



$$\Delta U_{sp} = \frac{1}{2}kx^2$$

$$\Delta K = 0 - \frac{1}{2}mv^2$$

$$0 + \frac{1}{2}kx^2 - \frac{1}{2}mv^2 = 0$$

$$x = v\sqrt{\frac{m}{k}}$$

Note

① Decrease in K.E of the block = Increase in P.E of the spring

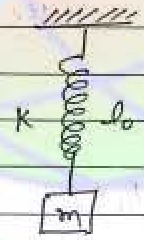
$$\frac{1}{2}mv^2 = \frac{kx^2}{2}$$

(ii) From work energy theorem

$$W_N + W_g + W_{sp} = 0 - \frac{1}{2}mv^2$$

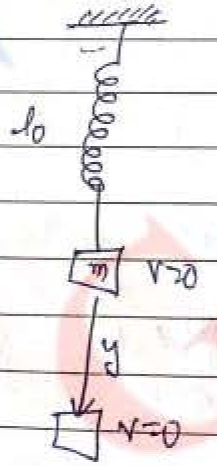
$$0 + 0 + \left(\frac{-kx^2}{2}\right) = -\frac{1}{2}mv^2$$

eg-2



A block of mass 'm' is attached with the lower end of the spring as shown in the fig. and release maximum extension in the spring will be —

Sol^n



$\boxed{F_g, F_{sp}}$ conservative

$$\Delta E = 0$$

$$\Delta U_g = -mgy$$

$$\Delta U_{sp} = \frac{ky^2}{2}$$

$$\Delta K.E = 0$$

$-mgy + \frac{ky^2}{2} + 0 = 0$

$\boxed{y = \frac{2mg}{k}}$

Note

① Decrease in gravitation potential energy = Increase in elastic potential energy

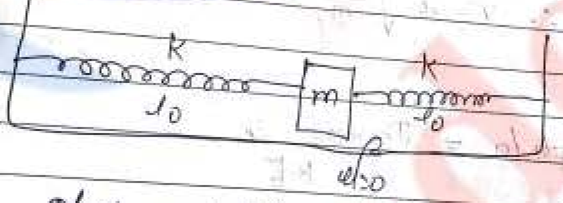
$mgy = \frac{1}{2}ky^2$

② From work energy theorem

$W_g + W_{sp} = 0 - 0$

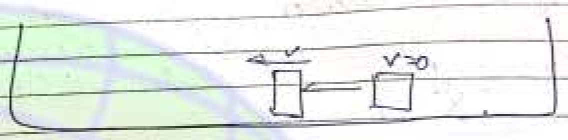
$mgh - \frac{ky^2}{2} = 0$

Eg)



Block is slightly displaced by x rightwards and released. Find its speed when it again comes to its original position. //

8/11



N, Fg, Fsp

other, conservative

$w_N = 0$

$\Delta E = 0$

$\Delta U_g = 0$

$\Delta U_{sp} = +2\left(\frac{kx^2}{2}\right)$

$\Delta KE = \frac{1}{2}mv^2$

$0 + 2\left(\frac{kx^2}{2}\right) + \frac{1}{2}mv^2 = 0$

$\therefore\ v = x\sqrt{\frac{2k}{m}}$

Note

① Decrease in Elastic PE = Increase in KE

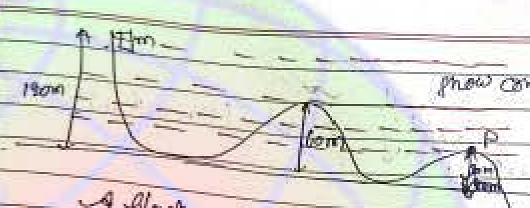
$$2\left(\frac{1}{2}kx^2\right) = \frac{1}{2}mv^2$$

② From W.E.T.J.y

$$w_N + w_g + w_{sp} = \frac{1}{2}mv^2 - 0$$

$$0 + 0 + \frac{2kx^2}{2} = \frac{1}{2}mv^2$$

eg.)



Show curved hill ($\mu=0$)

A block is released from the position shown in fig.

80/11 Find its speed when it passes through the point 'P'.

$\boxed{N}$ $\boxed{F_g}$

$W_N = 0$ Conse.

$\Delta E = 0$

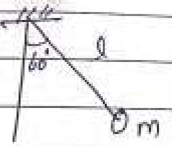
$$\begin{cases} \Delta U_g = -mgh \\ \Delta KE = \frac{1}{2}mv^2 \end{cases}$$

$-mgh + \frac{1}{2}mv^2 = 0$

$v = \sqrt{2hg}$

$\sqrt{2(80)(10)} = 40$

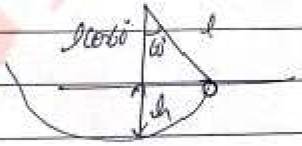
eg.)



(Simple Pendulum)

Pendulum is Released from the position shown in fig. speed of its bob when it pass through the lowest position is —

80/11



Power

① scalar quantity
② S.I unit = watt (W)

$$1\,W = 1\,\frac{J}{s}$$

$$1\text{ watt} = 1\cdot kg\cdot\frac{m^2}{s^3}$$

③ $[Power] = [M^1L^2T^{-3}]$

④ Average power - (P_{avg})

$$\left(\begin{array}{c}\text{Average power delivered}\\ \text{by a force}\end{array}\right) = \frac{\text{work done by the given force}}{(\text{time taken})}$$

Instantaneous power

Instantaneous power delivered by a force = Rate of doing work by the given force w.r.t. time

$$\boxed{P = \frac{dW}{dt}} = \frac{\vec{F}\cdot d\vec{s}}{dt}$$

$$\boxed{\vec{P} = \vec{F}\cdot\vec{v} = |\vec{F}|\,|\vec{v}|\cos\theta}$$

If $\theta = 90°$, $P = 0$
If $\theta = 0°$ $P = F.v$

if $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

and

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

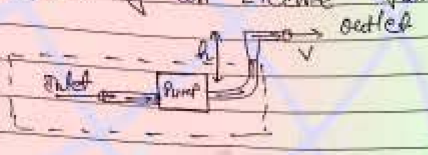
then

L-1 → 18 to 32
L-3 → 6, 7
1A → 8, 12, 17, 13
1B → 26, 7, 14

$P = \vec{F} \cdot \vec{v}$

$$\boxed{P = F_x v_x + F_y v_y + F_z v_z}$$

* Power delivered by an Electric pump



Note

$$\boxed{W_{total} = \Delta K.E + \Delta P.E}$$

suppose pump ejects water of mass "dm" in time "dt"

$$dW_{pump} = \frac{1}{2}(dm)v^2 + dm.g.h$$

$$\boxed{P_{pump} = \frac{dW_{pump}}{dt} = \left(hg + \frac{v^2}{2}\right)\left(\frac{dm}{dt}\right)}$$

output power

where

$\frac{dm}{dt}$ = rate of discharge of water.

Note

$$\boxed{\underset{(efficiency)}{\% \eta} = \frac{\text{output power}}{\text{input power}} \times 100}$$

★ $(P_{out})_{pumps} = \left(hg + \frac{v^2}{2}\right) \cdot \frac{dm}{dt}$

⇒ Special case

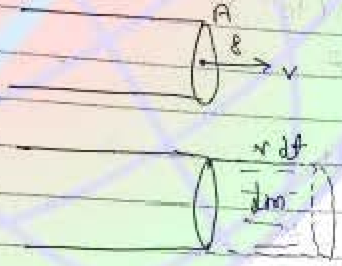
① if $v \simeq 0$

$P_{out} = hg\left(\frac{dm}{dt}\right)$

(ii) if $h \simeq 0$

$(P_{out})_{pump} = \frac{v^2}{2}\left(\frac{dm}{dt}\right)$

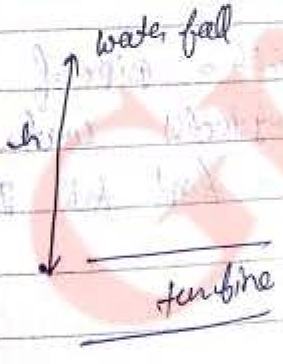
Note



$dm = (v\,dt\,A)\,\rho$

$$\boxed{\frac{dm}{dt} = A.\rho.v = \text{mass flow rate}}$$

★ Power delivered to hydro generator ⇒



Let water of mass dm falls down on the turbine in time dt

$$\boxed{\text{hydro Power delivered to the turbine} = \frac{dm.\,g.h}{dt} = hg.\frac{dm}{dt}}$$

% Efficiency of hydro generator $= \frac{\text{Electric power output}}{\text{Hydro power input}} \times 100$

Q) water falls down on a turbine from a height of 100m at ~~the real~~ rate of 10^4 kg/sec efficiency of ~~ammeter is~~ generator is 80% find its electrical power output

Sol^n

$P_{in} = hg\frac{dm}{dt}$

$= (100)(10)(10^4)$

$= 10^7$ watt

$\frac{P_o}{10^7} \times 100 = 80$

$P_o = 8 \times 10^6$ watt

$= 8MW.$

Note $1\,kwh = 10^3(watt)(3600\,s)$

$= 3.6 \times 10^6$ joule

Q) A pump is connected with a pipe of Area of crossection "A", pipe ejects water with vel. and its density is "ρ". find K.E Imparted to water per unit time.

Sol^n

10/4

$$dK = \frac{1}{2}(dm)(v^2)$$

$$\frac{dk}{dt} = \frac{1}{2}\times\frac{dm}{dt}\times v^2$$

$$\frac{dk}{dt} = \frac{1}{2}(A S v)\times v^2$$

$$\boxed{KE = \frac{1}{2} A S v^3}$$ Ans

Q) An automobile engine delive constant powe to a body which is initially at rest. body moves along straight line. Distane travelled by the body in time t is proportion to

(A) t^2 (B) $t^{3/2}$

(C) $t^{2/3}$ (D) N.O.T

10/4

$$P = F\cdot v$$

$$F\cdot v = P$$

$$\left(m\cdot\frac{dv}{dt}\right)\cdot v = P$$

$$\int_0^v v\,dv = \frac{P}{m}\int_0^t dt$$

$$\frac{v^2}{} = \frac{P}{m}t$$

$\cdot\cdot T_1$

$$\cancel{v = \frac{2P}{m}\sqrt{t}}$$

$$\cancel{\frac{ds}{dt} = \frac{2P}{m}\sqrt{t}}$$

$$T_1 \int v = \frac{2P}{m}\sqrt{t}$$

$$\frac{ds}{dt} = \frac{2P}{m}\sqrt{t}$$

$$\int_0^s ds = \frac{2P}{m}\int_0^t \sqrt{t}\,dt$$

$$\cancel{\sqrt{t}^{3/2}} \quad s \propto t^{3/2}$$

Q 3) A particle initially at rest is set in motion with uniform acc^n and acquires velocity "v_1" at time "t_1"

(i) Find power delivered to the particle at any "t"

(ii) Avg. power delivered to the particle in time t is —

Sol^n ① $t = 0$ $\quad t_1$

$v = 0$ $\quad v_1$

$P = F.V$

$P = (ma)(at)$

but

$$a = \frac{v_1 - 0}{t_1}$$

$$P = m\left(\frac{v_1}{t_1}\right)^2 . t$$

Date / /
Page

(ii) P_{avg} = [illegible]

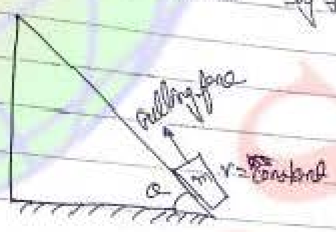
$P_{avg} = \frac{\text{total work done } (W)}{\text{total time } (t)}$

$= \frac{\frac{1}{2}mv^2}{t}$

$= \frac{1}{2} m \frac{a^2t^2}{t}$

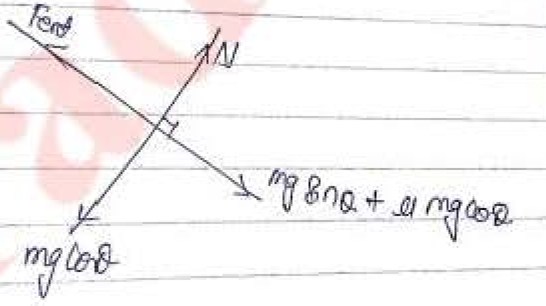
$= \frac{ma^2t}{2}$ where $a = \frac{v_1}{t_1}$

Q4) A block of mass 'm' is being pulling up to the rough inclined with constant velocity 'v'. find the power delivered to the block by the external agent



motion
'v' a ain
any time
in

Soln Power delivered by the external force = $F_{ext} \cdot v$



$F_{ext} = mg\sin\theta + \mu mg\cos\theta$

$P = (mg\sin\theta + \mu mg\cos\theta) \cdot v$

motion

b 'v' a ..

any tim

(ii) ~~Pavg = (a [illegible]) / t (constant)~~

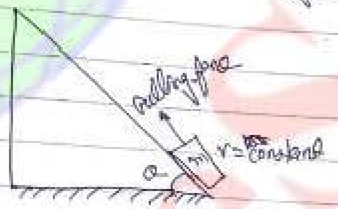
$$P_{avg} = \frac{\text{total work done}(W)}{\text{total time}(t)}$$

$$= \frac{\frac{1}{2}mv^2}{t}$$

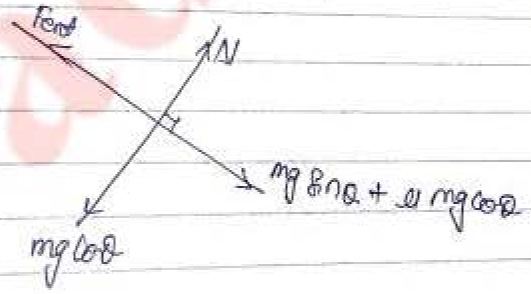
$$= \frac{1}{2} m \frac{a^2t^2}{t}$$

$$= \frac{ma^2t}{2} \quad \text{where } a = \frac{v_1}{t_1}$$

Q) A block of mass 'm' is being pulling up to the rough Inclined with constant velocity 'v'. find the Power delived to the block by the external agent



Sol: Power delived to the external force = Fext · V



$$F_{ext} = mg\sin\theta + \mu mg\cos\theta$$

$$P = (mg\sin\theta + \mu mg\cos\theta) \cdot V$$