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Page

1.) Progression ⇒ 1 to 154

2.) Point ⇒ 155 to 256

Remainder theorem →

when Polynomial $P(x)$ is divided by $(x-a)$ then $P(a)$ will be the remainder

Factor theorem

If $P(a)=0$ and $(x-a)$ is a factor of a then $(x-a)$ is a factor of $P(x)$

Q.) Find the remainder when $x^4+2x^3-3x^2+x-1$ is divided by $x+2$

Sol^n

$$x+2 \overline{)\, x^4+2x^3-3x^2+x-1\,} \left(x^3-3x+7 \right.$$

$$\underline{\cancel{x^4}+\cancel{2x^3}}$$
$$-\cancel{3x^2}+x-1$$
$$\underline{-\cancel{3x^2}-6x}$$
$$7x-1$$
$$\underline{\cancel{7x}+14}$$
$$-15$$

checque by remainder theorem

$x+2$

$P(-2)$ will be the remaind

$$P(-2) = (-2)^4+2(-2)^3-3(-2)^2-2-1$$
$$= -15$$

Q.) If $p(x) = ax^3 + bx^2 + 3x + 1$ is divisible by $(x-1)$ and $(x+1)$ then find the remainder when $p(x)$ is divided by $(x-2)$

Sol.

$x = -1$

$p(x) = ax^3 + bx^2 + 3x + 1$

$p(-1) = -a + b - 3 + 1$ —(I)

$= -a + b - 2$

$x - 1 = 0$

$x = 1$

$p(1) = ax^3 + bx^2 + 3x + 1$

$= a + b + 3 + 1$

$= a + b + 4$ —(II)

From eq (I) + eq (II)

$-a + b - 2 = 0$

$a + b + 4 = 0$

$2b = -2$

$b = -1$

$b = -1$

$a = -3$

$p(x) = -3x^3 - x^2 + 3x + 1$

$p(2) = -3 \times (2)^3 - (2)^2 + 3 \times 2 + 1$

$= -3 \times 8 - 4 + 6 + 1$

$= -24 - 4 + 6 + 1 \Rightarrow -21$

Arithematic Progression

* Sequence:→ It is set of numbers which follow a definite rule.

eg:→
$\{1, 2, 3, 4 \ldots\ldots\}$
$\{2, 4, 8, 16 \ldots\ldots\}$

* Series:→ By adding or substracting a terms of a sequence.

$1+2+3+4+\ldots\ldots$
$1-2+3-4+\ldots\ldots$

example:→ Q. Find first four terms of a sequence whose nth term is given by.

(i) $\frac{3+(-1)^n}{3^n}$ $(n \in N)$

(ii) $\frac{1}{n^2} \cdot \sin \frac{n\pi}{3}$ $(n \in N)$

(i) when $n=1$

$$\frac{3+(-1)^1}{3^1} = \frac{3-1}{3} = \frac{2}{3}$$

when $n=2$

$$\frac{3+(-1)^2}{3^2} = \frac{3+1}{9} = \frac{4}{9}$$

$\frac{27+3}{71}$

when $n=3$

$$\frac{3+(-1)^3}{3^3} = \frac{3-1}{27} = \frac{2}{27}$$

when $n=4$

$$\frac{3+(-1)^4}{3^4} = \frac{3+1}{71}$$

(ii) when $n = 1$

$$\frac{1}{1} \cdot \sin\frac{4\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

when $n = 2$

$$\frac{1}{2^2} \cdot \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{8}$$

when $n = 3$

$$\frac{1}{(3)^2} \cdot \sin\pi = 0$$

when $n = 4$ $\quad \frac{1}{(4)^2} \cdot \sin\frac{4\pi}{3} =$

Arithmetic Progression (A.P.) →

~~Def~~ If the difference between two conjugate terms is a constant ~~law then~~ place then it is Arithmetic progression :-

It's difference is called common difference

eg →

2, 4, 6, 8, 10 . . . .

other definition :-

If conjugate terms of a sequence increase or decrease by a constant number then it is called Arithmetic progression.

Let 'a' is the first term of an A.P and 'd' is a common difference then A.P will be

$a,\ a+d,\ a+2d,\ a+3d,\ a+4d\ .\ .\ .\ .\ .\ a+(n-1)d$

⇒ n^{th} term of an A.P

$$\boxed{T_n = a + (n-1)d}$$

Q.) If three no. a, b, c, are in A.P then

$$b - a = c - b$$

$$\boxed{2b = a + c}$$

ex→ Given :→

$T_{10} = 10$

$T_5 = 6$

find $T_6 = ?$

$T_{10} = a + (10-1)d = 10$

$= a + 9d = 10$ so, $a + 9d = 10$ —(I)

$T_5 = a + 4d = 6$

$a + 4d = 6$ —(II)

$a + 9d = 10$
$a + 4d = 6$
$- \quad - \quad -$

$5d = 4$

$d = \frac{4}{5}$

$a = 10 - 9 \times \frac{4}{5}$

$= \frac{50 - 36}{5}$

$= \frac{14}{5}$

$= a + (n-1)d$

$= \cancel{\frac{14}{5} + (6-1) \times \frac{4}{5}}$ ⇒ $\frac{14}{5} + 5 \times \frac{4}{5}$

$\cancel{= \frac{24+20}{5} = \frac{44}{5}}$ $= \frac{14}{5} + 4$

$= \frac{14+20}{5} = \frac{34}{5}$

Q→ which term is the first negative term in the sequence

2005, 2000, 1995

$a = 2005$

$T_n = 0$

$n = ?$

$d = -5$

$T_n = a + (n-1)d$

$0 = 2005 + (n-1) - 5$

$-2005 = -5n + 5$

$+\cancel{5}n = \cancel{+2010}$ 402

Here 402th term is zero

$n = 402+1$
$= 403$

so, (n+1)th term is negative
$(402+1) = 403$th term is negative

Teacher

$a = 2005$
$d = -5$

$T_n < 0$

$\Rightarrow a + (n-1)d < 0$

$\Rightarrow 2005 + (n-1)(-5) < 0$

$-5(n-1) < -2005$

$5(n-1) > 2005$

$n-1 > 401$

$n > 402$

$n = 403$

Q→ $T_p = q$

$T_q = p$

then find $T_r = ?$

$a + (p-1)d = q$

$a + (q-1)d = p$

$\Rightarrow (p-1)d - (q-1)d = q - p$

$\Rightarrow (p-1)d - (q-1)d = q-p$

$\Rightarrow pd - \cancel{d} - qd + \cancel{d} = q-p$

$\Rightarrow d\cancel{(p-q)} = -\cancel{(p-q)}$

$\Rightarrow d = -1$

$T_r = a + (r-1)(-1)$
$= a - r + 1$

Self Study :-

A sequence is called an arithmetic progression if the difference of a term and the previous term is always same. i.e.

$$\boxed{a_{n+1} - a_n = \text{constant} (=d) \text{ for all } n \in N}$$

\# Terms from the end :-

$\Rightarrow a_n - (n-1)d$
or $= l - (n-1)d$

Sum of A.P :→

Sum of the first 'n'-terms of an A.P :→

Let first term of an A.P is 'a', common difference is d, and no. of terms n.

A.P :→ a, a+d, a+2d ---- a+(n-1)d

Sum of n-terms

$$S_n = a + a+d + a+2d + \cdots a+(n-1)d$$

$$S_n = \{a+(n-1)d\} + \{a+(n-2)d\} \cdots a$$

$$2S_n = \{2a+(n-1)d\} + \{2a+(n-1)d\} + \cdots n(\text{time})$$

$$2S_n = n.\{2a+(n-1)d\}$$

$$\boxed{S_n = \frac{n}{2}\left(2a+(n-1)d\right)}$$

Also :→

$$S_n = \frac{n}{2}\{a + a + (n-1)d\}$$

$$= \frac{n}{2}(a+T_n)$$

$$\boxed{S_n = \frac{n}{2}(a+l)}$$

Q1.) Find the sum of all natural number divisible by 5 but less than 100.

5, 10, 15 95

$T_n = 95$, $a = 5$,

$a_n = a + (n-1)d$

$95 = 5 + (n-1)5$

$95 - 5 = (n-1)5$

$\cancel{90}^{18} = (n-1)\cancel{5}$

$(n-1) = 18$

$n = 19$

So,

$S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{19}{2}(2 \times 5 + (19-1) \times 5)$

$= \frac{19}{2} \times (10 + 90)$

$= \frac{19}{\cancel{2}} \times \cancel{100}^{50} = 950$

Q.) Find the minimum sum of AP

40, 38, 36 ....

$a = 40$,

$d = -2$

$n = 21$

$T_n = 0$

→ Properties of an A.P :→

1.) If each term of an A.P. is increased, decrease, multiply or divided by non-zero constant Then the resultant series will also be an A.P.

(2.) Sum of the term from equidistant from the begning and the end is constant and equal to sum of first and last term.

$$a,\ \underline{a+d},\ a+2d,\ a+3d \ldots a+(n-3)d,\ \underline{a+(n-2)d},\ a+(n-1)d$$

$$2a+d+(n-2)d$$

3.) In any A.P. each term (except the first term) is the A.M of the terms equidistant from it.

$$a,\ \boxed{a+d},\ \boxed{a+2d},\ \boxed{a+3d},\ \boxed{a+4d},\ a+5d,\ a+6d \ldots$$

$$\frac{a+a+4d}{2} = a+2d$$

$$\frac{a+d+a+3d}{2} = a+2d$$

$$\boxed{T_n = \frac{T_{n-k}+T_{n+k}}{2}}$$

4.) If no. of terms in an A.P is odd then the middle term is A.M of first and last term.

Q) If a, b, and c are in A.P. then show that
(1.) $b+c$, $c+a$, $a+b$ is also an A.P.

for ^{n}

$\cancel{\frac{b+c+c+a}{2} = \frac{a+b+c}{2}}$

we let AP then

$$2(c+a) = b+c+a+b$$

$$2c+2a = 2b+a+c$$

$$2b = a+c$$

Alternate method:-

$b+c,\ c+a,\ a+b \rightarrow A.P.$

Subtract :- $(a+b+c)$

$-a,\ -b,\ -c \rightarrow A.P.$

multiply by -1

$a, b, c \rightarrow A.P.$

Given ~~Also~~

$a, b, c \rightarrow A.P.$

multiply by -1

$-a, -b, -c \rightarrow A.P.$

Add $a+b+c$

$\Rightarrow b+c,\ c+a,\ a+b \rightarrow A.P$

(ii) $(b+c)^2 - a^2,\ (c+a)^2 - b^2,\ (a+b)^2 - c^2 \rightarrow A.P.$

$(b+c+a)(b+c-a),\ (c+a+b)(c+a-b),$

$(a+b+c)(a+b-c) \longrightarrow A.P.$

Divide by $(a+b+c)$

$b+c-a,\ c+a-b,\ a+b-c \rightarrow A.P$

Subtract $(a+b+c)$

$-2a, -2b, -2c \longrightarrow A.P$

Divide by -2

$a, b, c \longrightarrow A.P.$

~~2nd method~~ other method

$a, b, c \rightarrow A.P$

multiply by -2

$-2a, -2b, -2c \rightarrow A.P.$

Add $(a+b+c)$

$(b+c-a)$, $(c+a-b)$, $(a+b-c)$ $\rightarrow$ A.P.

multiply by $(a+b+c)$

Question:→ P, AP's are given 'P' containing 'n' terms of their first term are
1, 2, 3 ...... P respectively and

C. D are 1, 3, 5, 7,
then show that:
then find sum of all such A.P.

Soln> $S_1 = a = 1$
$d = 1$

$S_1 = \frac{n}{2}\{2\times1 + (n-1)2\} = \frac{n}{2}(n+1)$

$S_2 = a = 2$
$d = 3$

$S_2 = \frac{n}{2}\{2\times2 + (n-1)3\} = \frac{n}{2}(3n+1)$

$S_3 = \frac{n}{2}(5n+1)$

$S = S_1 + S_2 \text{ ---- } S_p$

$d = \frac{n}{2}(3n+1) - \frac{n}{2}(n+1)$

$d = n^2$

$a = \frac{n(n+1)}{2}$

No. of terms $= P$

$\frac{P}{2}\left\{2\times\frac{n(n+1)}{2} + (P-1)n^2\right\}$

$= \frac{nP}{2}(nP+1)$

$T_n = a+(n-1)d$
$= a+nd-d$

$T_n = nd + (a-d)$

$\boxed{T_n = An + B}$

nth term of an A.P. is linear function of n.

Also:

$S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow na + \frac{n(n-1)}{2} \cdot d$

$\Rightarrow \frac{n^2 d}{2} + n\left(a - \frac{d}{2}\right)$

$\boxed{S_n = Pn^2 + Qn}$

sum of nth term of an A.P. is a Quadratic function of n

Important:

$\boxed{T_n = S_n - S_{n-1}}$

$$\boxed{d = T_n - T_{n-1}}$$

Q.) If $S_n = 2n^2 + 3n$ then find T_{50}

Ans) $T_n = S_n - S_{n-1}$

$= 2n^2 - 3n - \{2(n-1)^2 + 3(n-1)\}$

$= \cancel{2n^2 + 3n}$

$T_n = 4n + 1$

$T_{50} = 4 \times 50 + 1 = 201$

Q.) Find A.P. If $S_n = 5n^2 + 2n + 4$

$T_n = S_n - S_{n-1}$

$T_n = 10n - 3$

$= 7, 17, 27 \ldots$

Q.) Sum of 'n' terms of two A.P's are in the ratio $7n + 1 : 4n + 27$ then find the ratio of their 11th term.

Sol)

$$\Rightarrow \frac{\frac{n}{2}\{2a_1 + (n-1)d_1\}}{\frac{n}{2}\{2a_2 + (n-1)d_2\}} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{7n+1}{4n+27}$$

$$\frac{(n-1)}{2} = 10 \Rightarrow n = 21$$

so, put $n = 21$

$$\frac{a_1 + 10d_1}{a_2 + 10d_2} = \left(\frac{148}{111}\right)$$

$$\frac{(T_{11})_1}{(T_{11})_2} = \frac{4}{3}$$

Q) Find values of x in the equation

$$\frac{x-1}{x} + \frac{x-2}{x} + \dots \frac{1}{x} = 3$$

$$a = \frac{x-1}{x}$$

when $n = 4$

$$d = \left(\frac{x-2}{x}\right) - \frac{x-2}{x}$$

$$\frac{n}{2} \; \frac{2}{x} \; \frac{x+1}{x}$$

$\Rightarrow$

Supposition of term in A.P :→

(1) Three terms in A.P :→

$$a-d,\ a,\ a+d$$

common diff "d"

(4) 4 terms in A.P. :-

$$(a-3d)\ (a-d)\ (a+d)\ (a+3d)$$

common difference "2d"

5) 5 terms in A.P. :-

$$(a-2d),\ (a-d),\ a,\ (a+d),\ (a+2d)$$

(6) odd term वाले के case में एक-एक बढ़ते हुए आगे बढ़े।

Q.) If sum of three no in A.P is 27 and sum their ~~so~~ squares is 293. Then find the numbers.

Ans)

$$a-\cancel{d}+a+a+\cancel{d}$$

$$3a = 27$$

$$a = 9$$

$$(a-d)^2 + a^2 + (a+d)^2 = 293$$

$$a^2+d^2-\cancel{2ad}+a^2+a^2+d^2+\cancel{2ad} = 293$$

$$3a^2 + 2d^2 = 293$$

$$3\times(9)^2 + 2\times d^2 = 293$$

$\frac{81\times 3}{243}$ $\frac{\begin{matrix}293\\243\end{matrix}}{050}$

$$3\times 81 + 2d^2 = 293$$

$$2d^2 = 293 - 243$$

$$\cancel{2}d^2 = \cancel{50}\,25$$

$$d = \pm 5.$$

So, Required three no. is

$(a-d), a, (a+d)$

$\Rightarrow (9-5), 9, (9+5)$

$\Rightarrow 4, 9, 14$ or $14, 9, 4$ Ans

Q.) Angles of a quadrilateral are in AP c.d. 10°. Find the angle

So $(a-3d)(a-d), (a+d)(a+3d)$

$$a - 3\cancel{d} + a - \cancel{d} + a + \cancel{d} + a + \cancel{3d} = 360^\circ$$

$$\cancel{4}a = \cancel{360}\;90$$

$$a = 90^\circ$$

$2d = 10$

$d = \frac{10}{2} = 5$

60
80
90
120
150

$\Rightarrow$ ~~$90-30, 90-10, 90+10, 90+30$~~

$\Rightarrow$ ~~$60, 80, 90, 120$~~

also, $2D = 10°$

$D = \frac{10}{2}$

$= 5°$

So,

$75°, 85°, 95°, 105°$

Q.) If First three of an increasing A.P are the roots of equation $4x^3 - 24x^2 + 23x + 18 = 0$ then,

Find the sum of n term of this A.P.

Sol^n

Arithmetic means

If a, and b are two number then arithmetic mean of two no. is given by $\frac{a+b}{2}$.

Arithmetic mean of 'n' ~~number~~ number $(a_1, a_2, a_3 \dots a_n)$

so, A.m of n number = $\left(\frac{a_1 + a_2 + a_3 \dots a_n}{n}\right)$

~~An arithmetic~~

'n' A.M. between two numbers.

Let a and b are two number and n A.m are Inserted between a and b.

Let ~~a~~ a, n are $a_1, a_2, a_3 \dots a_n$

so,

$a, A_1, A_2, A_3 \dots A_n, b$ are in A.P

No. of terms = $(n+2)$

$T_{n+2} = b$

so, $T_{n+2} = b = a + (n+2-1)d$

$b = a + (n+1)d$

$$\boxed{d = \frac{b-a}{n+1}}$$

$A_1 = a + d$

$A_2 = a + 2d$

$A_3 = a + 3d$

$\vdots$

$$\boxed{A_n = a + nd}$$

Q Insert 20 A.M between 2 and 86

$d = \frac{b-a}{n+1}$

$= \frac{86-2}{20+1} = 4$

$A_1 = 2+4 = 6$

$A_2 = 10$

$A_3 = 14$

Qn) n A.M are inserted between 20 and 80 such that ratio of first and last mean is 1:3 find n

$$d = \frac{b-a}{n+1}$$

$a_n = a + nd$

$80 = 20 + n\cancel{x}$

$$= \left(\frac{80-20}{n+1}\right)$$

$$= \left(\frac{60}{n+1}\right)$$

Given

$$\frac{A_1}{A_n} = \frac{1}{3}$$

$$\frac{a+d}{a+nd} = \frac{1}{3}$$

$$\frac{20 + \frac{60}{n+1}}{20 + \frac{n \cdot 60}{n+1}} = \frac{1}{3}$$

$$n = 11$$

Q-> ~~Sum of n A.M b/w two~~

Sum of n A.M b/w two numbers

$A_1 + A_2 + A_3 + \dots\dots\dots A_n$

$(a+d) + (a+2d) + \dots\dots\dots + (a+nd)$

$= na + d(1+2+\dots\dots n)$

$\Rightarrow na + d\left(\frac{n(n+1)}{2}\right)$

$\Rightarrow n\left(a + \frac{b-a}{n+1} \cdot \frac{n+1}{2}\right)$

$= n\left(a + \frac{b-a}{2}\right)$

$= n\left(\frac{a+b}{2}\right)$

Sum of n AM's b/w two no. is n times A.M of those two numbers

Q) Find the 101 A.M's between 1 and 99?

Given, $n = 101$, $a = 1$, $b = 99$

Sum $= 101\left(\frac{1+99}{2}\right)$

$= 101 \times \frac{\cancel{100}^{50}}{\cancel{2}}$

$= 5050$

Q) Sum of first 4 terms of an A.P is 56 and sum of last four terms is 112. If first term is 11 then find the no. of terms.

Ans:)

$S_4 = \frac{4}{2}(2\times 11 + (4-1)\times d)$

$\cancel{56}\,28 = \cancel{2}(22 + 3d)$

$28 = 22 + 3d$

$3d = 28 - 22$

$= 6$

$d = 2$

Sum of last n

$S_n = l - (n-1)d$

$= l - (4-1)d$

$= l - 3d$

Question based on Properties of A.P.

Q) If ~~these term are in A.P. then show that~~

$$\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$$

are in A.P. then show that

$$\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ are in A.P.}$$

Sol:- add '2',

$$\Rightarrow \frac{b+c-a}{a}+2, \frac{c+a-b}{b}+2, \frac{a+b-c}{c}+2 \rightarrow AP$$

$$\frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c} \rightarrow A.P$$

Divide by $(a+b+c)$

$$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \rightarrow A.P$$

Q) If a^2, b^2, c^2 are in A.P then show that

(I) $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b} \rightarrow A.P.$

(II) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b} \rightarrow A.P.$

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Some standard results of an A.P. :-

1.) Sum of first 'n' natural numbers.

$$\sum_{k=1}^{n} k = 1+2+3+ \cdots n$$

$$= \frac{n(n+1)}{2}$$

2.) Sum of squares of first 'n' natural numbers.

$$\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+ \cdots n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

(3.) Sum of cubes of first 'n' natural numbers.

$$\sum_{k=1}^{n} k^3 = 1^3+2^3+3^3+ \cdots n^3$$

$$= \left\{\frac{n(n+1)}{2}\right\}^2$$

4.) Sum of first n odd natural numbers

$$\sum_{k=1}^{n} (2k-1) = 1+3+5+7 \cdots +(2n-1)$$

$$= n^2$$

$\frac{36\times6}{216}$, $\frac{49\times7}{343}$ (6)

216
125
091

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Example:-

Find the sum of natural no. from 100 to 10000 which are of the form n^3 $(n \in N)$

~~125, 216, 343 ——~~

~~$a = 125$~~
~~$d = 91$~~ ~~$a_n = a + (n-1)d$~~
~~$T_n = 1000$~~

There no. lies between the square of

$S = 5^3, 6^3, \dots\dots 21^3$

$= (1^3 + 2^3 \dots 21^3) - \{1^3 + 2^3 + 3^3 + 4^3\}$

$\Rightarrow \left(\frac{21 \times 22}{2}\right)^2 - \left(\frac{4\times5}{2}\right)^2$

$= \left(\frac{43}{2}\right)^2 - \left(\frac{20}{2}\right)^2$

$= \left(\frac{43}{2}\right)^2 - (10)^2$

$= \cancel{\frac{43+10}{2}} \left(\frac{43}{2} + 10\right)\left(\frac{43}{2} - 10\right)$

$\left(\frac{43+20}{2}\right)\left(\frac{43-20}{2}\right)$

Q.) Show that $\sqrt{2}, \sqrt{3}, \sqrt{5}$ cannot be the terms of an A.P (not necessarily adjacent.)

Soln

$T_p = \sqrt{2} = a + (p-1)d$ — ①

$(p, q, r \in N)$ $T_q = \sqrt{3} = a + (q-1)d$ — ②

$T_r = \sqrt{5} = a + (r-1)d$ — ③

eq ② − eq ①

$\sqrt{3} - \sqrt{2} = (q-p)d$ — ④

Also eq ③ − eq ②

$\sqrt{5} - \sqrt{3} = (r-q)d$ — ⑤

Now $\frac{\text{eq } 4}{\text{eq } 5}$

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{5}-\sqrt{3}} = \left(\frac{q-p}{r-q}\right)$$

Irrational $\quad \left(\frac{p}{q}\right)$ Rational

Here Irrational $\neq$ Ration

So our supposition is wrong it is not in A.P.

Geometric Progression.

A sequence such that ratio of two conjugated terms is constant follow out.
This ratio is called common ratio of an A.P. and this is denoted by "r".

Let 'a' is the first term of the G.P. and 'r' is common ratio.
Then G.P. will be

'a' , 'r'

$a, ar^1, ar^2, ar^3 \ldots\ldots ar^{n-1}$

$$\boxed{T_n = ar^{n-1}}$$

$\Rightarrow$ n^{th} term of a G.P :

$$\boxed{T_n = ar^{n-1}}$$

~~Next~~ D.P.P. S $\Rightarrow$ 26

Q.1

D.P.P 2 → Progression.

example

$a = 2$

$r = 2$

$T_n = a \cdot r^{n-1}$

$= 2 \times 2^{n-1}$

$= 2^n$

(3)

$a = \frac{1}{4}$

$r = -\frac{1}{2} \div \frac{1}{4}$

$-\frac{1}{\cancel{2}} \times \frac{\cancel{4}^{2}}{1} = -2$

$T_{10} = \frac{1}{4} \cdot (-2)^{10-1}$

$= \frac{1}{4} \cdot (-2)^9$

$= \frac{1}{4} \times (-2)^9$

$= -128$ Ans

4.)

$a = 5,$

$r = \frac{n_2}{n_1} = \frac{10}{5} = 2$

$T_n = 5120$

$$5\overline{)5120}(1024$$

$$T_n = a \cdot r^{n-1}$$

$$= 5 \cdot 2^{n-1}$$

$$5120 = 5 \cdot 2^{n-1}$$

$$1024 = 2^{(n-1)}$$

$$2^{10} = 2^{(n-1)}$$

$$10 = n-1$$

$$\therefore n = 11$$

5.) $a = 3$

$r = \frac{6}{3} = 2$

$T_n = 3072$

$T_n = a \cdot r^{n-1}$

$\cancel{3072} = \cancel{3} \cdot 2^{n-1}$

$\Rightarrow 1024 = 2^{n-1}$

$2^{10} = 2^{n-1}$

$n-1 = 10$

$n = 11$

$$\boxed{n\text{th term from the end} = l\left(\frac{1}{r}\right)^{n-1}}$$

$a = 3,\ r = 2,\ l = 3072$

$\Rightarrow$ So,

4th term from end $\Rightarrow$

$\Rightarrow l\left(\frac{1}{r}\right)^{n-1}$

$\Rightarrow 3072 \times \left(\frac{1}{2}\right)^3$

$= 384$

6.) ~~GP~~

$T_3 = a r^{3-1}$

$= a r^2$

a, ar, ar^2, ar^3, ar^4, ~~...~~

$a^5 r^{10}$

$\Rightarrow (ar^2)^5$

$\Rightarrow 4^5 = 1024$

8.) $a = 1, r = 4, S_n = 5461, n = ?$

$S_n = \frac{a(r^n - 1)}{r-1} = \frac{(4^n - 1)}{3} = 5461$

$4^n - 1 = 5461 \times 3$

$4^n = 4^7$

$\therefore n = 7$

Sum of n terms of a G.P :->

$$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$$

$$rS_n = \quad + ar + ar^2 + \cdots\cdots + ar^{n-1} + ar^n$$

$$S_n - rS_n = a - a \cdot r^n$$

$$S_n(1-r) = a(1-r^n)$$

$$\boxed{S_n = \begin{cases} a\left(\frac{1-r^n}{1-r}\right) & r \neq 1 \\ a\left(\frac{r^n-1}{r-1}\right) & r > 1 \\ na & \end{cases}}$$

D.P.P.S sheet

(1) consider a G.P. 3, 6, 12, ... S_7

$a = 3,$

$r = \frac{6}{3} = 2$

$$S_n = \frac{a(r^n-1)}{r-1}, \quad r > 1$$

$$S_7 = \frac{3(2^7-1)}{2-1}$$

$$\Rightarrow \frac{3(128-1)}{3-1} = \frac{3\times 127}{2}$$

$$= 381$$

9.)

$a = 2,\ r = \frac{3}{2}$

$$S_n = \frac{2\left(\left(\frac{3}{2}\right)^n - 1\right)}{\frac{3}{2} - 1}$$

$$= \frac{2\left\{\left(\frac{3}{2}\right)^n - 1\right\}}{\frac{3-2}{2}}$$

$$= 2\left\{\left(\frac{3}{2}\right)^n - 1\right\} \times 2$$

$$\Rightarrow 4 \times \frac{3^n}{2^n} - 4$$

$$= 4\left(\frac{3}{2}\right)^n - 4$$

$$= 4\left(\left(\frac{3}{2}\right)^n - 1\right)$$

9) $\underset{G.P}{(2+4+8 \ldots n \text{ terms})} + \underset{A.P}{(3+6+9 \ldots n \text{ terms})}$

$$2\left(\frac{2^n-1}{2-1}\right) + \frac{n}{2}\left(2\times 3+(n-1)3\right)$$

13.)

$a = AR^{p-1}$ —(I)

$b = AR^{q-1}$ —(II)

$c = AR^{r-1}$ —(III)

$$\frac{eq(I)}{eq(II)} = \frac{a}{b} = R^{p-q}$$

$$\log\left(\frac{a}{b}\right) = (p-q)\log R$$

$$(p-q) = \frac{\log a - \log b}{\log R}$$

$$(p-q)\log c = \frac{\log c(\log a - \log b)}{\log R}$$

14.) $a = 1$

$T_3 + T_5 = 90$

$ar^2 + ar^4 = 90$

$ar^2(1+ar^2) = 90$

$r^2(1+r^2) = 90$

$r^4 + r^2 - 90 = 0$

$\Rightarrow (r^2 - 9)(r^2 + 10) = 0$

$r^2 = 9$ | $r^2 = -10$
$r = \pm 3$ | ✗

Q.) If the difference of 4th and first term of a G.P is ~~equal~~ 52 and sum of its first three ~~to~~ terms is 26 then find the sum of first six terms of a G.P.

Ans)

$T_4 - T_1 = 52$

$\Rightarrow ar^{n-1}$
$\Rightarrow ar^0$
$\Rightarrow a$

$ar^3 - a = 52$

$a(r^3 - 1) = 52$ ①

$a = \dfrac{52}{r^3 - 1}$

$S_3 = \dfrac{a(r^3 - 1)}{(r - 1)}$

$= \dfrac{52}{\cancel{r^3 - 1}} \left(\dfrac{\cancel{r^3 - 1}}{r - 1} \right)$

$\dfrac{52}{26}$
$\overline{78}$

$26 = \dfrac{52}{r - 1}$

$26(r - 1) = 52$

$26r - 26 = 52$

$26r = 52 + 26$

$\frac{26\times3}{78}$

$r = \frac{78}{26}$

$r = 3$

$a = \frac{52}{27-1}$

$= \frac{52}{26} = 2$

$a = 2$
$r = 3$

$\frac{6}{2} = 3$
$\frac{18}{6} = 3$

~~80~~, 2, 6, 18,

$S_6 = a\left(\frac{r^6-1}{r-1}\right)$

$= 2\left(\frac{3^6-1}{3-1}\right) = 728$ Ans

Q.) $9+99+999+ \dots +999+ \dots$ n times.

Ans

~~10 + 100 + 1000 + ... 10000 ... n times.~~

$a = 10$
$r = 10$

$\Rightarrow (10-1)+(100-1)+(1000-1) \dots + (10^n-1)$

$S_n = \Rightarrow (10-1)+(10^2-1)+(10^3-1) \dots (10^n-1)$

$\Rightarrow (10+10^2+ \dots 10^n) - n$

$\Rightarrow 10\left(\frac{10^n-1}{10-1}\right) - n$

$\Rightarrow \frac{10}{9}(10^n - 1) - n$

Example D.P.P.S $\Rightarrow$ sheet 1 -

20.) (i)

$\Rightarrow 5 + 55 + 555 + \cdots$

$\Rightarrow 5(1 + 11 + 111 + 1111 \cdots)$

$\Rightarrow \frac{5}{9}(9 + 99 + 999 + \cdots)$

$\Rightarrow \frac{5}{9}\left(\frac{10}{9}(10^n - 1) - n\right)$

$\Rightarrow$

(ii) $11 + 107 + 1005 + \cdots\ n$ times

$\Rightarrow (10 + 1) + (100 + 7) + (1000 + 5) + \cdots$

$\Rightarrow (10 + 1) + (10^2 + 7) + (10^3 + 5) + \cdots$

$\Rightarrow (10 + 10^2 + 10^3 + \cdots 10^n) + (1 + 7 + 5 + \cdots)$

$\Rightarrow 10\left(\frac{10^n - 1}{10 - 1}\right) + n^2$

$\boxed{b = \frac{a+c}{2}}$ for A.P $\boxed{b^2 = ac}$ For G.P

21.) T_2, T_3, T_6

$a+(n-1)d,$

$\Rightarrow a+d, \ a+2d, \ a+5d, \ \text{G.P}$ $\frac{a+2d}{a+d}$

$\Rightarrow$

$(a+2d)^2 = (a+d)(a+5d)$

$\cancel{a^2} + 4d^2 + 4ad = \cancel{a^2} + 5ad + ad + 5d^2$

$4d^2 - 5d^2 + 4ad - 5ad - ad = 0$

$-d^2 - 2ad = 0$

$d^2 + 2ad = 0$

$d(d+2a) = 0$

$d = 0$ $\quad$ $d + 2a = 0$

$\quad\quad\quad\; d = -2a$

a, a, a

$\boxed{r = 1}$ $\quad\quad\quad\quad$ $\boxed{r = 3}$

Sum of an Infinite terms

$$\Rightarrow \left(|r| < 1\right) \Leftarrow$$

$$S_n = a\left(\frac{1-r^n}{1-r}\right)$$

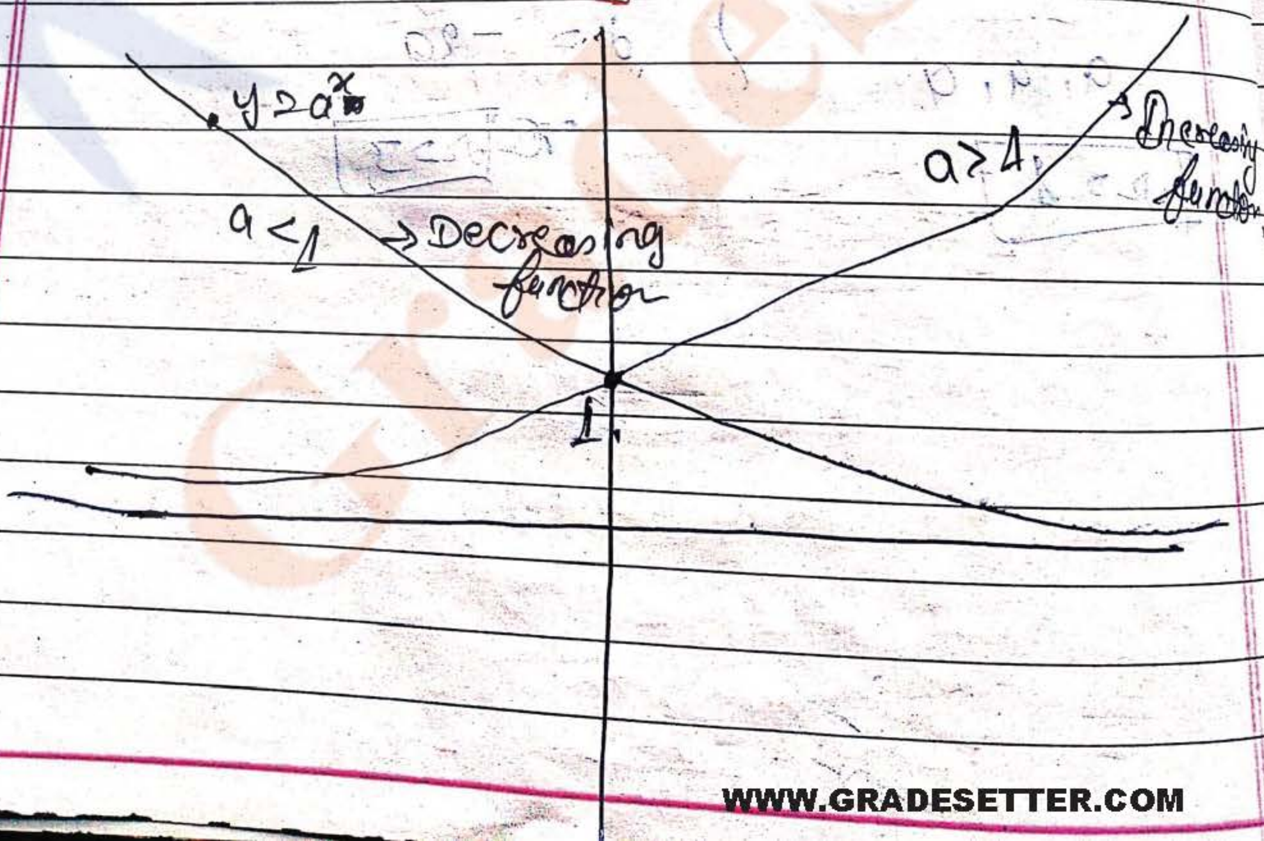
$$|r| < 1$$

$$n \to \infty$$

$$r^n \to 0$$

$$\Rightarrow S_\infty = a\left(\frac{1-0}{1-r}\right)$$

$$\boxed{S_\infty = \frac{a}{1-r}}$$



D.P.P.-3
D&I

10) $a = 1$

$r = \frac{1}{2}$

$S_\infty = \frac{1}{1-r}$

$= \frac{1}{1-\frac{1}{2}} \Rightarrow \frac{1}{\frac{2-1}{2}} = \frac{1}{\frac{1}{2}} = 2$

11) $\frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} \dots$ find S_∞

$a = \frac{1}{2}$

$r = \frac{1}{9} / \frac{1}{2}$

$r = \frac{1}{9} \times \frac{2}{1} = \frac{2}{9}$

$\therefore r = \frac{2}{9}$

$S_\infty = \frac{1}{1-\frac{2}{9}} = \frac{1}{\frac{9-2}{9}}$

$= \frac{1}{\frac{7}{9}} = \frac{9}{7}$

Seperate

$\Rightarrow \left(\frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} \dots\right) + \left(\frac{1}{3^2} + \frac{1}{3^4} + \dots\right)$

$\Rightarrow \left(\frac{\frac{1}{2}}{1-\frac{1}{2^2}}\right) + \left(\frac{1/9}{1-\frac{1}{9}}\right)$

Date: / /

$$\Rightarrow \left(\frac{\frac{1}{2}}{\frac{4-1}{4}}\right) + \left(\frac{1/9}{\frac{9-1}{9}}\right)$$

$$\Rightarrow \left(\frac{\frac{1}{2}}{\frac{3}{4}}\right) + \left(\frac{1/9}{\frac{8}{9}}\right)$$

$\frac{1}{2} \times \frac{9}{8}$

$$\Rightarrow \left(\frac{1}{\cancel{2}} \times \frac{\cancel{4}^{2}}{3}\right) + \frac{1}{\cancel{9}} \times \frac{\cancel{9}}{8}$$

$$\Rightarrow \frac{2}{3} + \frac{1}{8} = \frac{19}{24}$$

16.) or

18.) or

$S = 2$

$\frac{a}{1-r} = 1$ A/q

$a + ar = 1$

$\frac{1}{1-r} = 2$

$a(1+r) = 1$ — (1)

$1 = 1+r$

Now

$1 - r = 2$, $1 = 1$ A/q

$r = 0$

$a = ar + ar^2 + ar^3 + \ldots$

$\cancel{a} = \frac{\cancel{a}r}{1-r}$

$1 - r = r$

$2r = 1$

So, $r = \frac{1}{2}$ So, $a = \frac{2}{3}$

$\frac{2}{3}$,

16.) ans

$6^{1/2} \cdot 6^{1/4} \cdot 6^{1/8} \cdots \infty$

$6^{1/4} \div 6^{\frac{1}{2}}$

$6^{\frac{1}{4}-\frac{1}{2}}$

$6^{\frac{1-2}{4}}$

$6^{-1/4}$

$a = 6^{1/2}$

$r = \frac{6^{1/4}}{6^{1/2}}$

$\Rightarrow 6^{\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\cdots\right)}$

$\Rightarrow 6^{\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right)}$

$\Rightarrow 6^{1} = 6$

7. Supposition of terms in G.P. :→

1) 3 terms in G.P. :→ Attention these two

$\frac{a}{r}, a, ar$

common ratio 'r'

2) Four terms in G.P. Attention these two

$\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

common ratio (r^2)

3) 5 terms in G.P. :→

$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

common ratio 'r'

D.P.P. 8

Q.2.)

$\frac{a}{r} + a + ar \Rightarrow \frac{a + ar + ar^2}{r} = 19$

$\Rightarrow a\left(\frac{r^2 + r + 1}{r}\right) = 19$

$\Rightarrow$ from eq ①

$6\left(\frac{r^2 + r + 1}{r}\right) = 9$

$\Rightarrow 6r^2 + 6r + 6 - 9r = 0$

$r = \frac{2}{3}, \frac{3}{2}$

$$\frac{a}{\cancel{x}} \cdot a \cdot a\cancel{x} = 216$$

$$a^3 = 216$$

$$\boxed{a = 6} \longleftarrow eq(1)$$

23.)

$$\frac{a}{\cancel{r}} \cdot a \cdot a\cancel{r} = 216$$

$$a^3 = 216$$

$$\boxed{a = 6}$$

$$\Rightarrow \frac{a}{r} \cdot a + a \cdot ar + \frac{a}{\cancel{r}} \cdot a\cancel{r} = 156$$

$$\Rightarrow \frac{a^2}{r} + a^2 r + a^2 = 156$$

$$\Rightarrow a^2\left(\frac{1}{r} + r + 1\right) = 156$$

$$\Rightarrow 36(r^2 + r + 1) = 156r$$

$$\Rightarrow 36r^2 + 36r - 156r + 36 = 0$$

$$\Rightarrow 36r^2 - 120r + 36 = 0$$

$$4(9r^2 - 30r + 9) = 0$$

$$\Rightarrow 9r^2 - 30r + 9$$

$$\Rightarrow 3(3r^2 - 10r + 3) = 0 \quad \therefore r = \frac{1}{3}, 3$$

$36 \overline{)156} \; (6$
6

156
36
$\boxed{120}$

$3 \,|\, 81$
$3 \,|\, 27$
$3 \,|\, 9$
3

* Geometric mean (G.M.)

If 'a' and 'b' are two positive real numbers when G.M. of these numbers is

$$\boxed{(ab)^{1/2} = \sqrt{ab}}$$

If "$a_1, a_2, a_3, \ldots a_n$" are n positive real numbers then Geometric mean of those numbers is

~~$a_1, a_2, a_3, \ldots$~~

$$\boxed{(a_1 \cdot a_2 \cdot a_3 \cdot \ldots a_n)^{1/n}}$$

* Insertion of n G.M's b/w two numbers

1) If 'a' and 'b' are two numbers and 'n' G.M's

$$(G_1, G_2, G_3, \ldots G_n)$$

are Inserted between, ~~then~~,

Then

$a, G_1, G_2, G_3, \ldots G_n, b \rightarrow$ G.P

No. of terms = n+2

$T_{n+2} = b$

$\Rightarrow \quad b = a \cdot r^{(n+2-1)}$

$\frac{b}{a} = r^{n+1}$

$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$$\boxed{So,\ r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}}$$

$G_1 = a \cdot r^1$

$G_2 = a \cdot r^2$

$G_3 = a \cdot r^3$

$$\boxed{G_n = a \cdot r^n}$$

Example:->

Q8.)

$a = 5, \quad b = 160$

$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$= \left(\frac{160}{5}\right)^{\frac{1}{4+1}}$

$\Rightarrow \left(\frac{160}{5}\right)^{\frac{1}{5}}$

$\Rightarrow (32)^{\frac{1}{5}}$

$\boxed{r = 2}$

$G_3 = ar^3$

$\Rightarrow 5(2)^3 = 40$

Q.) If A.M of two numbers is 15 and G.M of these no. is 9. then find the number.

Sol ->

$\frac{a+b}{2} = 15$	$\sqrt{ab} = 9$
$a+b = 30$ —(1)	$ab = 81$ —(11)

$$a + \frac{81}{a} = 30$$

$$a^2 - 30a + 81 = 0$$

$$a = 27, 3$$

Hw

Q. If sum of two numbers is n-times their G.M then find the ratio of these numbers.

Ans $\frac{n+\sqrt{n^2-4}}{n-\sqrt{n^2-4}}$

Product of n G.M → b/w two numbers:→

Page No.: 58

$G_1 \cdot G_2 \cdot G_3 \cdot \text{———} G_n$

$\Rightarrow ar \cdot ar^2 \cdot ar^3 \cdot \ldots\ldots ar^n$

$\Rightarrow a^n \cdot r^{(1+2+3+ \text{———} +n)}$

$\Rightarrow a^n \cdot r^{\frac{n(n+1)}{2}}$

$\Rightarrow a^n \cdot \left\{\left(\frac{b}{a}\right)^{\frac{1}{n+1}}\right\}^{n\left(\frac{n+1}{2}\right)}$

$\Rightarrow a^n \cdot \left(\frac{b}{a}\right)^{n/2} \Rightarrow a^n \cdot \frac{b^{n/2}}{a^{n/2}}$

$\Rightarrow a^{n/2} \cdot b^{n/2} \Rightarrow (ab)^{n/2} \Rightarrow \boxed{(\sqrt{ab})^n}$

So Product of 'n' G.M between two number is nth power G.M of these number.

Properti...

1.) If ... non-zer... be in G... a...

ak...

2.) If ca... powe... G.P.

(3.) ... from ... and ... and ...

4.) I... the...

Properties of G.P.:

1. If each term of a G.P. multiply or divided by non-zero constant then the resultant series will also be in G.P.

$a, ar, ar^2, ar^3 \ldots$

$ak, akr, akr^2 \ldots$

2. If each term of a G.P. is raised to the same power then the resultant series will also be in G.P.

3. In a G.P. product of the terms equidistance from the begining and the end is constant and it is equal to the product of first and last term.

$a, ar, ar^2, ar^3 \ldots ar^{n-3}, ar^{n-2}, ar^{n-1}$

$ar^2 \cdot ar^{n-3} = a^2 \cdot r^{n-1}$

4. In a G.P. any term (except the first term) is the G.M. of the terms equidistant from it.

$a, ar, ar^2, ar^3, ar^4, ar^5 \ldots$

$\sqrt{ar \cdot ar^5}$

$= \sqrt{a^2 r^6} = ar^3$

$$\boxed{T_n = \sqrt{T_{n-k} \cdot T_{n+k}}}$$

(iv). If no. of terms in a G.P is odd then the middle term is G.M of first and last term.

If $a_1, a_2 \text{---} a_n$ is a G.P with common ratio r_1 and $b_1, b_2 \text{---} b_n$ is also a G.P with common ratio r_2, then the sequence

$a_1 b_1;\ a_2 b_2;\ \text{---}\ a_n b_n$ then the sequence will be a G.P. then C.R r_1 and r_2.

$$r = \frac{a_2 b_2}{a_1 b_1} = \left(\frac{a_2}{a_1}\right)\left(\frac{b_2}{b_1}\right)$$

$$= r_1 \cdot r_2$$

If $a_1, a_2, a_3 \text{---} a_n$ are in G.P then $\log a_1, \log a_2 \cdots \text{---} \log a_n$ will lie on A.P.

So,

$$\boxed{r = r_1 r_2}$$

Very Important Concept of log

examples: If a, b, c are in G.P. then show that

$\log_a 10, \log_b 10, \log_c 10$ is ?

Sol→

$a, b, c \rightarrow$ G.P.

$\log_{10} a, \log_{10} b, \log_{10} c \rightarrow$ A.P.

$\frac{1}{\log_{10} a}, \frac{1}{\log_{10} b}, \frac{1}{\log_{10} c} \longrightarrow$ H.P.

for more see page → "113"

$\log_a 10, \log_b 10, \log_c 10 \longrightarrow$ H.P.

Note: → [G.P का यदि हम Log लेंगे तो वह A.P में बदल जाएगा] [और A.P का reverse H.P में बदल जाएगा]

~~Re-occuring~~

Recurring Decimal

eg → $0.\bar{3}$ is the form of $\frac{p}{q}$

0.3333

$0.3 + 0.03 + 0.003 + \ldots\ldots$

$\frac{3}{10}, \frac{3}{10^2}, \frac{3}{10^3} \ldots\ldots$

$\left(\frac{3/10}{1-\frac{1}{10}}\right) = \frac{3}{9} = \frac{1}{3}$

D. P. & Progression-1.

example

20.) Find

$0.\overline{125}$

$0.125\,125\,125\,125$

$\Rightarrow 0.125 + 0.000125 + 0.000000125$

$\Rightarrow \frac{125}{1000} + \frac{125}{10^6} + \frac{125}{10^9} \ldots$

$\Rightarrow \frac{125/1000}{1 - \frac{1}{1000}} = \frac{125}{999}$ Ans

Q.) $0.\overline{9} \Rightarrow 0.\overline{9}$

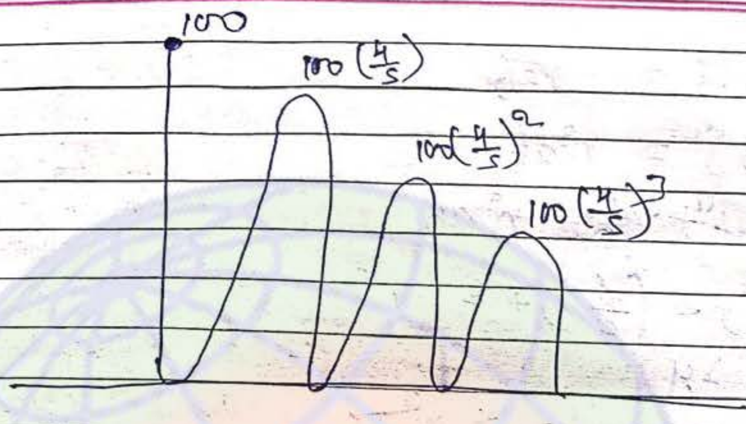
0.9999

$0.9 + 0.09 + 0.009 + 0.0009$

$\frac{9}{10} + \frac{9}{10^2} + \frac{9}{10^3} + \frac{9}{10^4} ---$

$\frac{9/10}{1 - \frac{1}{10}} \Rightarrow \frac{\frac{9}{10}}{\frac{10-1}{10}} \Rightarrow \frac{9}{9} \Rightarrow 1$

19.) a)



$$\Rightarrow 100 + 100\left(\frac{4}{5}\right)\times 2 + 100\left(\frac{4}{5}\right)^2\times 2 + 100\left(\frac{4}{5}\right)^3\times 2 \dots$$

$$\Rightarrow 100 + 2\times 100\left(\frac{4}{5} + \left(\frac{4}{5}\right)^2 + \dots\right)$$

$$\Rightarrow 100 + 2\times 100\left(\frac{4/5}{1-\frac{4}{5}}\right) = 900$$

21.)

$$\frac{a}{r}, a, ar, \dots \qquad (a-d), a, (a+d)$$

$$\frac{a}{6}, a, a\cdot 6 \dots (a-6)$$

G.P

$$a+6, \quad a-6, \quad a, \quad a+6$$

A.P

$$(a-6)^2 = a\cdot(a+6)$$

$$a^2 - 12a + 36 = a^2 + 6a$$

$$\boxed{a = 2}$$

8, -4, 2, 8

ex→ $a+b = n\sqrt{ab}$

$(a+b)^2 = n^2ab$

$a^2+b^2+2ab = n^2ab$

$\left(\frac{a}{b}\right)_{\to y} + \left(\frac{b}{a}\right) + 2 = n^2$

$y + \frac{1}{y} + 2 = n^2$

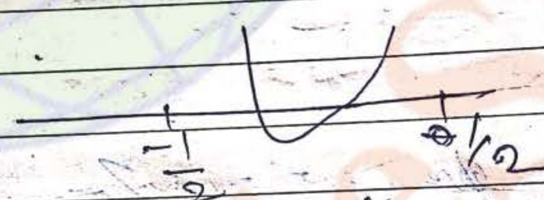
$y = 1$

ex→

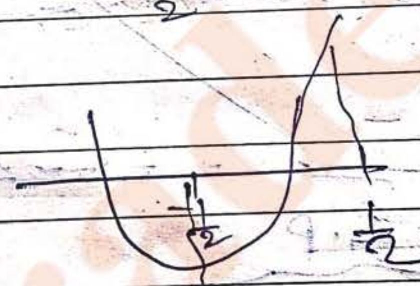
$\left(\frac{x}{1+x^2}\right)^2 - (m-3)\left(\frac{x}{1+x^2}\right) + m = 0$

$-\frac{1}{2} \le y \le \frac{1}{2}$

(I)



(II)



Exactly 1 root in $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Harmonic Progressions

~~Of the~~ sequence

It is a sequence reciprocal of whose terms are in A.P (Arithmetic Progression)

If 'a' is and 'd' are first term and common difference of the corresponding A.P then the H.P will be –

$$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots \frac{1}{a+(n-1)d}$$

n^{th} term of H.P = $\frac{1}{a+(n-1)d}$

$$\boxed{T_n = \frac{1}{a+(n-1)d}}$$

Note :→

(i) In any H.P No term can be zero.

(ii) If a, b, c are in H.P then,

$$\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \rightarrow A.P$$

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\frac{2}{b} = \frac{a+c}{ac} \Rightarrow \boxed{b = \frac{2ac}{a+c}}$$

34) Ans)

$$T_8 = \frac{1}{a+(8-1)d} = \frac{1}{a+7d} = \frac{1}{2}$$

$$T_{14} = \frac{1}{a+(14-1)d} = \frac{1}{a+13d}$$

$$T_{20} = \frac{1}{a+19d}$$

$$a+7d = 2 \quad \text{(i)}$$

$$a+13d = 3 \quad \text{(ii)}$$

$$\boxed{6d = 1 \qquad d = \frac{1}{6}}$$

$$a+7\times\frac{1}{6} = 2$$

$$a = 2-\frac{7}{6}$$

$$= \frac{12-7}{6} = \frac{5}{6}$$

$$T_{20} = \frac{1}{\frac{5}{6}+19\times\frac{1}{6}}$$

$$= \frac{1}{\frac{5}{6}+\frac{19}{6}} = \frac{1}{\frac{24}{6}} = \frac{1}{4}$$

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35.)

36.)

$$\frac{1}{a-d} + \frac{1}{a} + \frac{1}{a+d} = 37 \quad \text{—(I)}$$

$$\frac{1}{a-d} + \frac{1}{a} + \frac{1}{a+d} = 37$$

$$a - \cancel{d} + a + a + \cancel{d} = \frac{1}{37}$$

$$3a = \frac{1}{37} \quad \text{—(II)}$$

Now, $a = \frac{1}{3 \times 37}$

$$3a = \frac{1}{4}$$

$$a = \frac{1}{12} \quad \text{—(III)}$$

Now from eq (II) and eq (III)

$$3 \times \frac{1}{12}$$

36.)

$$\frac{1}{a-d} + \frac{1}{a} + \frac{1}{a+d} = 37 \quad \text{—(1)}$$

$$a-d + a + a + d = \frac{1}{4}$$

$$a = \frac{1}{12} \quad \text{—(2)}$$

$$\frac{1}{\frac{1}{12}-d} + \frac{1}{1/12} + \frac{1}{1/12+d} = 37$$

$$\frac{12}{1-12d} + 12 + \frac{12}{12d+1} = 37$$

$$d = \frac{1}{60}$$

example

$T_m = n \Rightarrow T_m$

$T_n = m$

Show

$$T_{m+n} = \left(\frac{mn}{m+n}\right)$$

$$T_m = \frac{1}{a+(m-1)d}$$

$$T_n = \frac{1}{a+(n-1)d}$$

T_{m+n}

$$\Rightarrow \frac{1}{a+(m-1)d} + \frac{1}{a+(n-1)d}$$

$$\Rightarrow \frac{a+nd-d+a+md-d}{(a+md-d)+(a+nd-d)}$$

$$\Rightarrow \frac{2a - 2d + d(m+n)}{a^2 + amd - ad +}$$

Example: $T_m = n$

$T_n = m$

Prove

$T_{m+n} = \left(\frac{mn}{m+n}\right)$

$a + (m-1)d = \frac{1}{n}$ — (i)

$a + (n-1)d = \frac{1}{m}$ — (ii)

From eq (i) and (ii)

$a = \frac{1}{mn}$

$d = \frac{1}{mn}$

$$T_{m+n} = \frac{1}{a + (m+n-1)d} = \frac{1}{\frac{1}{mn} + (m+n-1)\frac{1}{mn}}$$

$$= \left(\frac{mn}{1+m+n-1}\right) = \left(\frac{mn}{m+n}\right)$$

Harmonic mean :-

Harmonic mean of two no's 'a' and 'b' is given by $\boxed{\frac{2ab}{a+b}}$

$$\frac{2}{\frac{1}{a}+\frac{1}{b}} = \frac{2ab}{a+b}$$

Harmonic mean of 'n' non-zero numbers $(a_1, a_2, a_3 \ldots\ldots a_n)$ is given by

$$\frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\ldots\frac{1}{a_n}}$$

n H.m's b/w two numbers:

If n H.M b/w two

If 'n' H.m's $(H_1, H_2, H_3 \ldots\ldots H_n)$ are inserted between two no's 'a' and 'b' then

$a, H_1, H_2, H_3 \ldots H_n, b \rightarrow H.P$

$$\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \ldots \ldots \frac{1}{H_n}, \frac{1}{b} \longrightarrow A.P$$

no. of term $\Rightarrow n+2$

$$T_{n+2} = \frac{1}{b}$$

$$\therefore \frac{1}{b} = \frac{1}{a} + (n+2-1)d$$

$$\frac{1}{b} - \frac{1}{a} = (n-1)d$$

$$\boxed{d = \frac{\frac{1}{b} - \frac{1}{a}}{n+1}}$$

$$\frac{1}{H_1} = \frac{1}{a} + d$$

$$\frac{1}{H_2} = \frac{1}{a} + 2d$$

$$\boxed{\frac{1}{H_n} = \frac{1}{a} + nd}$$

Example:→ If 'H' is harmonic mean (H.M) between 'a' and 'b' then find the value of

$$\frac{H}{2a} + \frac{H}{2b}$$

Sol →

$$\Rightarrow \frac{2ab}{a+b} \times \frac{1}{2a} + \frac{2ab}{a+b} \times \frac{1}{2b}$$

$$\Rightarrow \frac{2ab}{(a+b)2a} + \frac{2ab}{(a+b)2b} \Rightarrow \frac{b}{a+b} + \frac{a}{a+b}$$

$$\Rightarrow \frac{a+b}{a+b} = 1, \quad \text{Ans} \Rightarrow 1$$

$$\Rightarrow \frac{2ab}{(a+b)} + \frac{a}{(a+b)}$$

$$\Rightarrow \frac{2ab+a}{(a+b)} \Rightarrow \frac{a(2b+a)}{a+b}$$

Q1 Insert 4 H.M between $\frac{2}{3}$ and $\frac{2}{13}$

$a = \frac{2}{3}$, $b = \frac{2}{13}$

$1 \div \frac{2}{13}$

$$d = \frac{\frac{13}{2} - \frac{3}{2}}{5}$$

$$d = \frac{\frac{13-3}{2}}{5} \Rightarrow \frac{10}{2} \times \frac{1}{5} = 1$$

$\frac{2}{3} + 1 = \frac{5}{3}$

$$\frac{1}{H_1} = \frac{1}{a} + n \cdot d$$

$$= \frac{3}{2} + 1 = \frac{5}{2}$$

$\frac{5}{3} +$

$$\text{so } H_1 = \frac{2}{5}$$

$$H_2 = \frac{2}{7}$$

$$H_3 = \frac{2}{9}$$

$$H_4 = \frac{2}{11}$$

Arithmetic Geometric Progression (AGP) :→

A sequence is said to be G.P if its each term is obtained by multiplying the corresponding terms of an A.P and a G.P

A.P :→ $a,\ a+d,\ a+2d,\ a+3d \ldots$

G.P → $b,\ br,\ br^2,\ br^3 \ldots$

A.G.P :→ $ab,\ (a+d)br,\ (a+2d)br^2,\ (a+3d)br^3$

इसी प्रकार के Sum की निश्चित रूप से प्रायः रखें।

[method → 1st] → "r" से multiply करके एक right करके Subtraction Rule.

Q.37:)

(1) 10, 20

$S = 1 + \frac{3}{4} + \frac{5}{4^2} + \frac{7}{4^3} + \frac{9}{4^4} \ldots$

(multiplying both side by common ratio of G.P is $\frac{1}{4}$ say)

$\frac{1}{4}S = \frac{1}{4} + \frac{3}{4^2} + \frac{5}{4^3} + \frac{7}{4^4} \ldots$

यहाँ पर एक स्टेप कैसे लिख सकते शुरू करें

$S - \frac{S}{4} = 1 + \left(\frac{2}{4} + \frac{2}{4^2} + \frac{2}{4^3} + \ldots\right)$

$\frac{3S}{4} = 1 + \left(\frac{2/4}{1-\frac{1}{4}}\right)$ Teache

$= 1 + \frac{2}{3}$

$\frac{3S}{4} = \frac{5}{3}$ So, $S = \frac{20}{9}$

Arithmetic Geometric Progression (AGP) :→

A sequence is said to be G.P. If its each term is obtained by multiplying the corresponding terms of an A.P and a G.P.

$A.P :\Rightarrow a, a+d, a+2d, a+3d \dots$

$G.P \rightarrow b, br, br^2, br^3 \dots$

$A.G.P :\Rightarrow ab, (a+d)br, (a+2d)br^2, (a+3d)br^3$

इसी प्रकार के Sum को निश्चित रूप से याद रखें।

Q. S.I.) [method → 1st] → "r" से multiply करके एक right करके Subtraction Rule.

(1) 10, 20

$$S = 1 + \frac{3}{4} + \frac{5}{4^2} + \frac{7}{4^3} + \frac{9}{4^4} \dots$$

(multiplying both side by common ratio of G.P. is $\frac{1}{4}$ only)

$$\frac{1}{4}S = \frac{1}{4} + \frac{3}{4^2} + \frac{5}{4^3} + \frac{7}{4^4} \dots$$

यहाँ पर एक स्टेप के बाद मिस्टेक शुरू करें।

$$S - \frac{S}{4} = 1 + \left(\frac{2}{4} + \frac{2}{4^2} + \frac{2}{4^3} + \dots\right)$$

Teacher

$$\frac{3S}{4} = 1 + \left(\frac{2/4}{1 - \frac{1}{4}}\right)$$

$$= 1 + \frac{2}{3}$$

$$\frac{3S}{4} = \frac{5}{3} \qquad \text{So,} \quad S = \frac{20}{9}$$

$\frac{4}{5}-1 = \frac{4-5}{5} = \frac{-1}{5}$

Q2.b) Ans

$S = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} \dots$

$\frac{1}{5}S = \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3}$

$S - \frac{1}{5}S = 1 + \left(\frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3}\right)$

$\frac{4S}{5} = 1 + \left(\frac{\frac{3}{5}}{1-\frac{1}{5}}\right)$

$\frac{4S}{5} = \frac{7}{4}$ $\qquad S = \frac{35}{16}$

$2n-1$

ii) $S = 1 + 2x + 3x^2 + 4x^3 + \dots, \; |x| < 1$

$xS = x + 2x^2 + 3x^3$

$S(1-x) = 1 + x + x^2 + x^3 \dots$

$S(1-x) = \left(\frac{1}{1-x}\right)$ — true, confused

$\boxed{S = \frac{1}{(1-x)^2}}$

example:-

$$S = 1 + 3x + 6x^2 + 10x^3 + \cdots$$

$$x \cdot S = 1x + 3x^2 + 6x^3$$

$$S(1-x) = 1 + 2x + 3x^2 + 4x^3$$

$$S(1-x) =$$

example:-

$$S = 1 + 3x + 6x^2 + 10x^3 + \cdots$$

$$xS = x + 3x^2 + 6x^3$$

$$S(1-x) = 1 + 2x + 3x^2 + 4x^3 \cdots$$

$$S(1-x) = \frac{1}{(1-x)^2}$$

$$\boxed{S = \frac{1}{(1-x)^3}}$$

ex:- $S = \frac{3}{19} + \frac{33}{19^2} + \frac{333}{19^3} \cdots$

$$\frac{1}{19} S = \frac{3}{19^2} + \frac{33}{19^3}$$

$$S - \frac{1}{19} S = \frac{3}{19} + \frac{30}{19^2} + \frac{300}{19^3}$$

$\frac{19}{54}$ Ans

$S\left(1-\frac{1}{19}\right) = \frac{3}{19} + \frac{30}{19^2} + \frac{300}{19^3}$

$S\left(\frac{19-1}{19}\right) = \frac{3}{19} + \frac{30}{19^2} + \frac{300}{19^3}$

$S\left(\frac{18}{19}\right) =$

method → 2nd

method - 2nd

$\Rightarrow \frac{3}{19}\left\{\frac{9}{19} + \frac{99}{19^2} + \frac{999}{19^3} \cdots\right\}$

$\Rightarrow \frac{3}{19}\left\{\frac{10-1}{19} + \frac{10^2-1}{19^2} + \frac{10^3-1}{19^3} \cdots\right\}$

$\Rightarrow \frac{3}{19}\left\{\frac{10}{19} - \frac{1}{19} + \frac{10^2}{19^2} - \frac{1}{19^2} + \frac{10^3}{19^3} - \frac{1}{19^3} \cdots\right\}$

$\Rightarrow \frac{3}{19}\left\{\frac{10}{19} + \left(\frac{10}{19}\right)^2 \cdots\right\} - \left[\frac{1}{19} + \frac{1}{19^2} \cdots\right\}$

$\Rightarrow \frac{3}{19}\left\{\frac{\frac{10}{19}}{1-\frac{10}{19}} - \frac{1/19}{1-\frac{1}{19}}\right\}$

5^{n} 5^{n-1} Page No.: 78

Sum of n-terms :-

Exercise 32

$S_n = 1 + \frac{4}{5} + \frac{7}{5^2} - - - n$ terms ;

$T_n = a+(n-1)d$
$\Rightarrow 1+(n-1)3$
$\Rightarrow 3n-2$

multiply each term by common ratio "5"

$S_n = \frac{1}{5^0} + \frac{4}{5} + \frac{7}{5^2} - - - \frac{3n-2}{5^{n-1}}$

$T_n = ar^{n-1}$
$\Rightarrow 5^{n-1}$

$\frac{1}{5}S_n = \frac{1}{5} + \frac{4}{5^2} \ldots + - + \frac{3n-2}{5^n}$

$S_n - \frac{S_n}{5} = 1 + \frac{3}{5} + \frac{3}{5^2} + - - - - \left(\frac{3n-2}{5^n}\right)$

$\frac{4}{5}S_n = 1 + \frac{3}{5}\left(\frac{1-\left(\frac{1}{5}\right)^{n-1}}{1-\frac{1}{5}}\right) - \frac{3n-2}{5^n}$

$\frac{4}{5}S_n = 1 + \frac{3}{4}\left(1 - \frac{1}{5^{n-1}}\right) - \left(\frac{3n-2}{5^n}\right)$

$= 1 + \frac{3}{4} \cdot \left(\frac{5^{n-1}-1}{5^{n-1}}\right) - \left(\frac{3n-2}{5^n}\right)$

$= \frac{4 \cdot 5^n + 3 \cdot 5\left(5^{n-1}-1\right) - 4(3n-2)}{4 \cdot 5^n}$

$= \frac{4 \cdot 5^n + 3 \cdot 5^n - 3 \cdot 5 - 12n + 8}{4 \cdot 5^n}$

$$S_n\left(\frac{4}{5}\right) = \frac{7\cdot 5^n - 12n - 7}{4\cdot 5^n}$$

$$S_n = \frac{5}{4}\cdot\left\{\frac{7\cdot 5^n - 12n - 7}{4\cdot 5^n}\right\}$$

$$S_n = \frac{5}{4}\left\{\frac{7}{4} - \frac{(12n+7)}{4\cdot 5^n}\right\}$$

As

$n \to \infty$

$$\frac{12n+7}{4\cdot 5^n} \to 0$$

so,

$$S_\infty = \frac{5}{4}\left(\frac{7}{4} - 0\right)$$

$$= \frac{35}{16}$$

Q.) Find the sum of 'n' terms of series $\frac{3}{5}, \frac{5}{15}, \frac{7}{45}, \frac{9}{135} + \dots$

Hence, or otherwise

find the sum of infinite terms of series.

Ans) $S_n = \frac{3}{5}$ $\qquad S_n = \frac{3}{25}\left(2 - \frac{n+2}{3^n}\right)$

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Diff.

(1) Q.)

$$(a^2+b^2+c^2)x^2 - 2(ab+bc+cd)x + (b^2+c^2+d^2) \le 0$$

$$(ax-b)^2 + (bx-c)^2 + (cx-d)^2 \le 0$$

$$ax-b=0 \Rightarrow x=\frac{b}{a}$$

$$bx-c=0 \Rightarrow x=\frac{c}{b}$$

$$cx-d=0 \Rightarrow x=\frac{d}{c}$$

back Q.)

Ans.)
$$S_n = \frac{3}{5}+\frac{5}{15}+\frac{7}{45}+\frac{7}{135}+ \dots + \left(\frac{2n+1}{5\times3^{n-1}}\right)$$

$$\frac{1}{3}S_n = \frac{3}{15}+\frac{5}{45}+\frac{7}{135}+ \dots + \left(\frac{2n+1}{5\times3^{n}}\right)$$

$$\frac{2}{3}S_n = \frac{3}{5}+\frac{2}{15}+\frac{2}{45}+ \dots - \frac{2n+1}{5\times3^{n}}$$

$$\frac{2}{3}S_n = \frac{3}{5}+\frac{2}{15}\left\{\frac{1-\left(\frac{1}{3}\right)^{n-1}}{1-\frac{1}{3}}\right\} - \left\{\frac{2n+1}{5\times3^{n}}\right\}$$

$$\Rightarrow \frac{3}{5} + \frac{1}{5}\left(\frac{3^{n-1}-1}{3^{n-1}}\right) - \left\{\frac{2n+1}{5\times 3^{n}}\right\}$$

[method 3rd]

* Use of Σ (sigma) Operator: → पहले n निकालें फिर r निकालें ||

1.) $\sum_{r=1}^{n} 1 = 1+1+1+1 - - - +1 = n$

2.) $\sum_{r=1}^{n} r = 1+2+3+ \cdots + n = \frac{n(n+1)}{2}$

3.) $\sum_{r=1}^{n} r^2 = 1^2+2^2+3^2 \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

4.) $\sum_{r=1}^{n} r^3 = 1^3+2^3+3^3 - - - + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$

5.) $\sum_{r=1}^{n} a \cdot T_r = a \sum_{r=1}^{n} T_r$ when a is constant.

eg →

$$\sum_{r=1}^{n} 2r = 2\sum_{r=1}^{n} r = 2\left\{\frac{n(n+1)}{2}\right\}$$

6) Distributive over addition and subtraction.

$$\sum_{r=1}^{n}(T_r \pm T'_r) = \sum_{r=1}^{n} T_r \pm \sum_{r=1}^{n} T'_r$$

7.) $$\sum_{r=1}^{n} T_r \cdot T'_r \neq \sum_{r=1}^{n} T_r \cdot \sum_{r=1}^{n} T'_r$$

(It is not true)

8.) $$\sum_{r=1}^{n}\left(\frac{T_r}{T'_r}\right) \neq \frac{\sum_{r=1}^{n} T_r}{\sum_{r=1}^{n} T'_r}$$

(It is not true

Note: It is not valid in multiplication and division

Example:-

$1\cdot2 + 2\cdot3 + 3\cdot4 + 4\cdot5 + \dots\dots\dots n$ terms

$T_r = r(r+1)$

$$S_n = \sum_{r=1}^{n} T_r$$

$$= \sum_{r=1}^{n} r(r+1)$$

$$= \sum_{r=1}^{n} (r^2+r)$$

$$= \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$\Rightarrow \frac{n(n+1)}{2}\left(\frac{2n+1}{3} + 1\right)$$

$$= \frac{n(n+1)(n+2)}{3}$$

Example:-

~~$T_n = 1, 6, 18, 40, 75, 126$~~

~~$= 1\cdot1 + 2\cdot3 + 1\cdot4 + 1$~~

$T_r =$

Notes $\rightarrow$ $\boxed{S_n = \sum_{n=1}^{n} T_n}$

9, 49, 121

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56)

(1) $1\cdot 2^2 + 2\cdot 3^2 + 3\cdot 4^2 + 4\cdot 5^2 + \dots$

$T_r = r(r+1)^2 = r(r^2+2r+1)$

$S_n = \sum_{r=1}^{n} T_r$

$= \sum (r^3 + 2r^2 + r)$

$= \sum r^3 + 2\sum r^2 + \sum r$

$= \left\{\frac{n(n+1)}{2}\right\}^2 + \frac{2\,n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

2) $3^2 + 7^2 + 11^2 + 15^2 \dots$

$T_r = (3 + (r-1)4)^2$

$= (3 + 4r - 4)^2$

$= (4r-1)^2$

$= (4r)^2 - 2\times 4r + 1$

$= 16r^2 - 8r + 1$

$S_n = \sum_{r=1}^{n} T_r$

$= 16\sum r^2 - 8\sum r + \sum 1$

$= 16\left(\frac{n(n+1)(2n+1)}{6}\right) - 8\times\frac{n(n+1)}{2} + 1$

$\Rightarrow 16 \{ n$

3.) $3\cdot 8 + 8\cdot 13 + 13\cdot 18 + \dots$

$T_r = r(r+5) \Rightarrow r^2 + 5r$

$S_n = \sum_{r=1}^{n} T_r$

$\Rightarrow \sum_{r=1}^{n} r^2 + 5r$

$\Rightarrow \sum r^2 + \sum 5r$

$\Rightarrow \frac{n(n+1)(2n+1)}{6} + 5\frac{n(n+1)}{2}$

Method of Difference :⇒

when the difference of successive terms of a series are in A.P and G.P

Step Ist :→

we note the n^{th} term of series by T_n and sum of series by S_n.

Step IInd :→

we ~~recorrite~~ write the given series in each term shifted by one place to the right.

Step IIIrd :→

Substract the above both two series to find T_n

use $S_n = \sum_{n=1}^{n} T_n$

example :→

$$\overbrace{3+8}^{5}+\overbrace{15}^{7}+\overbrace{24}^{9}+ \ldots \text{ n terms}$$

$$S_n = 3+8+15+24+\ldots + T_n$$

$$S_n = \quad 3+8+15+\ldots + T_{n-1} + T_n$$

$$- \quad\quad - \quad - \quad\quad - \quad\quad\quad - \quad\quad -$$

$$0 = \underbrace{3+5+7+9+\ldots}_{n \text{ terms}} + d - T_n$$

$$+d - T_n$$

$T_n = 3+5+7+ \cdots n$ terms

$= \frac{n}{2}\{2\times 3-(n-1)2\}$

~~$T_n = \frac{n}{2}\{6-2n\}$~~

So, $T_n = n(n+2)$

$S_n = \sum_{n=1}^{n} T_n = \sum_{n=1}^{n}(n^2+2n) = \sum n^2 + 2\sum n$

$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\times 2$

58.

(13) $4+7+12+19 \cdots$

$3, 5, 7$

$S_n = 4+7+12+19 \cdots$ n terms

$S_n = 4+7+12 \cdots T_{n-1} + T_n$

$0 = 4+3+5+7 \cdots - T_n$

$= 4 + \{\frac{n}{2}(2\times 3+(n-1)2)$

$= 4 + \{\frac{n}{2}(6+2n-2)$

$\Rightarrow 4 + \{\frac{n}{2}(4+2n)$ $\Rightarrow 4+n(2+n)$

$\Rightarrow 4 + (n(3+n))$ $\Rightarrow$

$\Rightarrow$

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Date:

$$T_n = 4 + \frac{n-1}{2}\left(2\times 3 + (n-1-1)2\right)$$

$$\Rightarrow 4 + (n-1)(3 + n - 2)$$

$$\Rightarrow 4 + n^2 - 1$$

$$\Rightarrow n^2 + 3$$

$$S_n = \frac{(2n+1)\, n(n+1)}{6} + 3n$$

14.)

1, 5, 13, 29

$$S_n = 1 + 5 + 13 + 29 + \cdots \text{ n term}$$

$$S_n = \quad 1 + 5 + 13 + \cdots T_{n-1} \text{th term}$$

$$0 = 1 + 4 + 8 + 16 \cdots - T_n$$

$$\Rightarrow \cancel{1 + 4 \times 2^{n-1}}$$

$$\cancel{1 + 4 \times 2^{n-1}}$$

$$\cancel{1 + 8^{n-1}}$$

Attention Directly लेना नहीं होगा देखें कैसे होगा।

$$\Rightarrow 1 + \frac{4(2^{n-1} - 1)}{2 - 1}$$

$$\Rightarrow 1 + 2^{n+1} - 4$$

$$T_n = 2^{n+1} - 3$$

$$S_n = \sum_{n=1}\left(2^{n+1} - 3\right)$$

$$\sum_{n=1}^{n} 2^{n+1} - 3 \sum_{n=1}^{n} 1$$

Attested

$$\Rightarrow \left(2^2 + 2^3 + \cdots 2^{n+1}\right) - 3n$$

$$\Rightarrow 4\left(\frac{2^n - 1}{2-1}\right) - 3n$$

40/15

5.)

(i) $1 + 6 + 18 + 40 + 75 + 126$

$S_n = 1 + 6 + 18 + 40 + 75 + 126 \cdots\cdots n^{th}$
$S_n = \quad 1 + 6 + 18 + 40 + \cdots\cdots T_{n-1} + T_n$

$0 = 1 + 5 + 12 + 22 \cdots\cdots - T_n$

then, Again we let

$S_n = 1 + 5 + 12 + 22 \cdots\cdots S_n$
$S_n = \quad 1 + 5 + 12 \cdots\cdots T_{n-1} + T_n$

$0 = 1 + 4 + 7 + 10 \cdots\cdots - T_n$

$= \frac{n}{2}(2a + (n-1)d$

$= \frac{n}{2}(2\times 1 + (n-1)3$

$\Rightarrow \frac{n}{2}(2 + 3n - 3)$

$\Rightarrow \frac{n}{2}(3n - 1)$

$\frac{2}{2}(6-1$

$\Rightarrow 5$

$\frac{1}{2}\times 2$

57.)
(i) $1+6+18+40+75+126 \cdots$

$S_n = 1+6+18+40+75+126 \cdots T_n$

$S_n = \quad 1+6+18+40+75 \cdots T_{n-1}+T_n$

$0 = 1+$

57.)
(ii)

$S_n = 1+2+5+12+25+46+ \cdots T_n$

$S_n = \quad 1+2+5+12+25 \cdots T_{n-1}+T_n$

$0 = 1+1+3+7+13+21+ \cdots - T_n$

$T_n = 1+1+3+7+13+21 \cdots + t_n$

$T_n = \quad 1+1+3+7+13 \cdots t_{n-1}+t_n$

$0 = 1+\underbrace{0+2+4+ \cdots}_{(n-1)\text{term}} - T_n$

$t_n = 1+\frac{n-1}{2}\{0+(n-1-1)2\}$

$= 1+(n-1)(n-2)$

$t_n = n^2 - 3n + 3$

so

$T_n = \sum_{n=1}^{n} t_n$

$= \sum (n^2 - 3n + 3)$

18.

$T_{m+1} = a + md$

$T_{n+1} = a + n \cdot d$

$T_{r+1} = a + rd$

$(a + nd)^2 = (a + md)(a + rd)$

$a^2 + 2and + d^2 n^2 = a^2 + ard + mda + mrd^2$

$2an + dn^2 = ar + ma + mrd$

$a(2n - r - m) = d(mr - n^2)$

$\frac{a}{d} = \left(\frac{mr - n^2}{2n - r - m}\right)$

$m, n, r \rightarrow H.P$

$n = \left(\frac{2mr}{m + r}\right)$

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$$mr = \frac{n(m+r)}{2}$$

So,

$$\frac{a}{d} = \frac{\frac{n(m+r)}{6} - n^2}{2n - r - n}$$

$$= -\frac{n}{2}$$

Method :-> 4th ⟶ T_n निकालो फिर S_n निकालो

Q) Find the sum of nth terms of the series

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots\ldots\ n \text{ terms}$$

$$T_r = \frac{1}{r(r+1)}$$

$$T_r = \left(\frac{1}{r}-\frac{1}{r+1}\right)$$

How Teach

$$\Rightarrow \quad S_n = \sum_{r=1}^{n} T_r$$

$$= \sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)$$

$$\Rightarrow \left(\frac{1}{1}-\cancel{\frac{1}{2}}\right)+\left(\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right)\ldots\ldots$$

$$\ldots\ldots+\left(\cancel{\frac{1}{n}}-\frac{1}{n+1}\right)$$

$$\Rightarrow \left(1-\frac{1}{n+1}\right)$$

$$\boxed{S_n \Rightarrow \left(\frac{n}{n+1}\right)}$$

$$S_n = 1-\frac{1}{n+1}$$

$n \to \infty$

$\frac{1}{n+1} \to 0$ so, $S_\infty = 1-0 = 1$

6) $\frac{1}{1\cdot4} + \frac{1}{4\cdot7} + \frac{1}{7\cdot10} + \cdots\cdots$

$T_r = \frac{1}{r(r+3)}$

$T_r = \left(\frac{1}{r} - \frac{1}{r+3}\right)$

$S_n = \sum_{r=1}^{n} T_r$

$= \sum_{r=1}^{n} \left(\frac{1}{r} - \frac{1}{r+3}\right)$

$\Rightarrow \left(\frac{1}{1} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{10}\right) \cdots + \left(\frac{1}{n} - \frac{1}{n+4}\right)$

$\Rightarrow \left(\frac{1}{1} - \frac{1}{n+4}\right)$

$= \frac{n+4-1}{n+4}$

$\boxed{T_r = \frac{1}{\{1+(r-1)3\}\{4+(r-1)3\}}}$

$= \frac{1}{(3r-2)(3r+1)}$

$$\Rightarrow \frac{1}{3}\left(\frac{1}{3r-2} - \frac{1}{3r+1}\right)$$

$$S_n = \sum_{r=1}^{n} T_r$$

$$\Rightarrow \frac{1}{3}\sum_{r=1}^{n}\left(\frac{1}{3r-2} - \frac{1}{3r+1}\right)$$

$$\Rightarrow \frac{1}{3}\left\{\left(\frac{1}{1} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{7}}\right) \cdots + \left(\cancel{\frac{1}{3n-2}} - \frac{1}{3n+1}\right)\right\}$$

$$\Rightarrow \frac{1}{3}\left(1 - \frac{1}{3n+1}\right) = \left(\frac{n}{3n+1}\right)$$

$$n \to \infty$$

$$\frac{1}{3n+1} \to 0$$

$$S_\infty = \frac{1}{3}\left(1 - \frac{3}{n+1}\right)$$

$$= \frac{1}{3}$$

Q2) Ans)

$$\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \cdots$$

$$T_r = \frac{3 + (r-1)2}{r^2 \cdot (r+1)^2}$$

$$T_r = \frac{2r+1}{r^2(r+1)^2}$$

$$\Rightarrow \left\{\frac{1}{r^2} - \frac{1}{(r+1)^2}\right\}$$

$$S_n = \sum_{r=1}^{n} T_r$$

$$= \sum_{r=1}^{n} \left(\frac{1}{r^2} - \frac{1}{(r+1)^2}\right)$$

$$= \left(\frac{1}{1} - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{3^2} - \frac{1}{4^2}\right) - - - \left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

$$S_n = 1 - \frac{1}{(n+1)^2} = \frac{n(n+2)}{(n+1)^2}$$

$\frac{1}{1}, \frac{9}{2}$ $\quad$ $\frac{1}{1}, \frac{3^2}{4}, 6^2$

9.) $$\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + \cdots$$

$$T_r = \frac{1^3 + 2^3 + 3^3 - - - r^3}{r^2}$$

$$= \frac{\left\{\frac{r(r+1)}{2}\right\}^2}{r^2}$$

$$T_r = \frac{1}{4}(r+1)^2$$

$\frac{27}{8}$ $\quad$ $\frac{9}{36}$

Note :→

$$\Rightarrow \frac{1}{4}\left(r^2+2r+1\right)$$

$$S_n = \sum \frac{1}{4}\left(r^2+2r+1\right)$$

$$= \frac{1}{4}\left\{\frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2} + n\right\}$$

Find the

(Type → 5th) → largest - smallest = ( ) से multiply divide करें

उपर वाले को largest - smallest से तोड़ दो ||

example

Find the sum of a series.

$$\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}\dots$$

$3-1=2$

$$\Rightarrow \frac{1}{2}\left\{\frac{2}{1\cdot2\cdot3}+\frac{2}{2\cdot3\cdot4}+\frac{2}{3\cdot4\cdot5}+\dots\frac{2}{n(n+1)(n+2)}\right\}$$

$$\Rightarrow \frac{1}{2}\left\{\frac{3-1}{1\cdot2\cdot3}+\frac{4-2}{2\cdot3\cdot4}+\frac{5-3}{3\cdot4\cdot5}+\dots\frac{(n+2)-n}{}\right\}$$

$$\Rightarrow \frac{1}{2}\left\{\left(\frac{1}{1\cdot2}-\cancel{\frac{1}{2\cdot3}}\right)+\left(\cancel{\frac{1}{2\cdot3}}-\cancel{\frac{1}{3\cdot4}}\right)+\left(\cancel{\frac{1}{3\cdot4}}-\cancel{\frac{1}{4\cdot5}}\right)\dots+\left(\cancel{\frac{1}{n(n+1)}}-\frac{1}{(n+1)(n+2)}\right)\right\}$$

HW

$$\Rightarrow \frac{1}{2}\left\{\frac{1}{2}-\frac{1}{(n+1)(n+2)}\right\}$$

Ques 5.

$$\frac{1}{1\cdot3\cdot5}+\frac{1}{3\cdot5\cdot7}+\frac{1}{5\cdot7\cdot9}\dots$$

$$\Rightarrow \frac{1}{4}\left\{\frac{4}{1\cdot3\cdot5}+\frac{4}{3\cdot5\cdot7}+\frac{4}{5\cdot7\cdot9}\dots+\cancel{\frac{4}{n(n+2)(n+4)}}\dots+\frac{4}{(2n-1)(2n+1)(2n+3)}\right\}$$

$$\Rightarrow \frac{1}{4}\left\{\frac{5-1}{1\cdot 3\cdot 5}+\frac{7-1}{3\cdot 5\cdot 7}+\cdots\frac{(2n+3)-(2n-1)}{(2n-1)(2n+1)(2n+3)}\right\}$$

$$\Rightarrow \frac{1}{4}\left\{\left(\frac{1}{1\cdot 3}-\cancel{\frac{1}{3\cdot 5}}\right)+\left(\cancel{\frac{1}{3\cdot 5}}-\cancel{\frac{1}{5\cdot 7}}\right)-\cdots+\left(\cancel{\frac{1}{(2n-1)(2n+1)}}-\frac{1}{(2n+1)(2n+3)}\right)\right\}$$

$$\Rightarrow \frac{1}{4}\left\{\frac{1}{1\cdot 3}-\frac{1}{(2n+1)(2n+3)}\right\}$$

$$\Rightarrow \frac{1}{4}\left\{\frac{1}{3}-\frac{1}{(2n+1)(2n+3)}\right\}$$

$$\frac{1}{1\cdot 2\cdot 3\cdot 4}+\frac{1}{2\cdot 3\cdot 4\cdot 5}+\frac{1}{3\cdot 4\cdot 5\cdot 6}+\cdots n\text{ terms}$$

Relation Among A.M, G.M and H.M

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39.)

Part 1st :-

If A, G, and H are A.M, G.M and H.M of two positive real numbers, then $\boxed{G^2 = A \cdot H}$

$$G^2 = (\sqrt{ab})^2 = ab$$

$$A = \frac{a+b}{2}$$

$$H = \frac{2ab}{a+b}$$

$$A \cdot H = ab$$

39.)

$H.M = 4$

A.M. G.M. $\rightarrow 2A + G^2 = 27$

$$\frac{2ab}{a+b} = 4 \qquad H = \frac{G^2}{A}$$

$$\frac{2ab}{a+b} = \frac{\cancel{a}b}{\cancel{a}}$$

$$2\left(\frac{a+b}{\cancel{2}}\right) + ab = 27$$

$$a+b + ab = 27 \qquad \frac{2ab}{a+b} = b = 4$$

$$b + ab = 27$$

39.) $H = 4$

$2A + G^2 = 27$

$2A + A \cdot H = 27$ — ①

$6A = 27$

$A = \frac{9}{2}$

$G^2 = A \cdot H$

$G^2 = \frac{9}{2} \times 4 = 18$

$ab = 18$ — ②

$\frac{a+b}{2} = \frac{9}{2}$

$a+b = 9$ — ③

$a + \frac{18}{a} = 9$

$\boxed{a = 3, 6}$

40) $A.M + H.M = 25$ $\qquad$ $ab = \cancel{1}2$

$G.M = 12$

$\frac{a+b}{2} + \frac{2ab}{a+b} = 25$

$\frac{(a+b)^2 + (2ab)^2}{(a+b)2} = 25$

A0) $A + H = 25$ —(I)

$G^2 = AH = 144$ —(II) $H = \frac{144}{A}$

$A + \frac{144}{A} = 25$

$A = 9, 16$

$a+b = 2A$

$a+b = 18, 32$

Every year IIT Question

Part 2 →

A.M, G.M inequality :→

$$\boxed{A.M \geq G.M \geq H.M}$$

Note :→

1) This Inequality is valid only for Positive real numbers.

II) Equality when only if all the no. number are same.

eg → a, a

$A = \frac{a+a}{2} = a$

$G = \sqrt{a^2} = a$

$H = \frac{2a^2}{a+a} = a$

4.) $x + \frac{1}{x} \geq 2$ $\quad x \neq 0, x > 0$

$\Rightarrow \cancel{\frac{x^2+1}{x} - 2 \geq 0 \Rightarrow \frac{x^2+1-2x}{x} \geq 0}$

$\Rightarrow \cancel{\frac{(x-1)^2}{x} \geq 0}$

$$\frac{x+\frac{1}{x}}{2} \geq \sqrt{\left(x \cdot \frac{1}{x}\right)}$$

$$\frac{x+\frac{1}{x}}{2} \geq 1$$

$$x+\frac{1}{x} \geq 2$$

45) 1st $\Rightarrow$ $(b+c)(c+a)(a+b) > 8abc$

Ans: $\Rightarrow$ $AM \geq G.M$

$$\cancel{\frac{b+c}{2} \geq \sqrt{ac}}$$

$$\frac{b+c}{2} > \sqrt{bc}$$

$$(b+c) > 2\sqrt{bc} \quad \text{—(1)}$$

$$c+a > 2\sqrt{ca} \quad \text{—(2)}$$

$$a+b > 2\sqrt{ab} \quad \text{—(3)}$$

multiply

$(a+b)(b+c)(c+a) > 8\sqrt{ab \cdot bc \cdot ca}$

$> 8abc$

ii) $(a+b+c)\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right] > 9$ $\left[\frac{a+b+c}{abc}\right] > 9$

$\frac{a+b+c}{3} > \sqrt[3]{abc}$

$a+b+c > 3\sqrt[3]{abc}$ — ①

$\Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{1}{c} > 3\sqrt[3]{\frac{1}{abc}}$

$\frac{1}{a+b+c} > \cancel{\frac{1}{3\sqrt{\quad}}}$

$\frac{a+b+c}{2ac \times 2} > \cancel{\sqrt{\quad}}$

ii) Teache

$AM > H.M$

$\frac{a+b+c}{3} > \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

$\Rightarrow (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) > 9$

Sol) A.M > H.M

$$\frac{\left(\frac{a}{c}+\frac{b}{b}+\frac{c}{a}\right)}{3} > \frac{3}{\frac{a}{c}+\frac{b}{b}+\frac{c}{a}}$$

$$\left(\frac{a}{c}+\frac{b}{b}+\frac{c}{a}\right)\left(\frac{a}{c}+\frac{b}{b}+\frac{c}{a}\right) > 9 \text{ proof}$$

$$\frac{b+c}{a}+\frac{c+a}{b}+\frac{a+b}{c} > 6$$

$\Rightarrow$

$$\cancel{\frac{\frac{b+c}{a}+\frac{c+a}{b}+\frac{a+b}{c}}{3}}$$

$2\cancel{(}$

$$\frac{\frac{b}{a}+\frac{c}{a}+\frac{c}{b}+\frac{a}{b}+\frac{a}{c}+\frac{b}{c}}{6} > \left(\frac{b}{a}\cdot\frac{c}{a}\cdot\frac{c}{b}\cdot\frac{a}{b}\cdot\frac{a}{c}\cdot\frac{b}{c}\right)^{\frac{1}{6}}$$

$$\frac{(\quad)}{6} > 4$$

$$(\quad) > 6$$

52.)

iv) $\frac{2}{b+c} + \frac{2}{c+a} + \frac{2}{a+b} > \frac{9}{a+b+c}$

$A.M \geq H.M$

$\therefore \frac{\frac{2}{b+c} + \frac{2}{c+a} + \frac{2}{a+b}}{3} > \frac{3}{\frac{b+c}{2} + \frac{c+a}{2} + \frac{a+b}{2}}$

so, $\frac{\frac{2}{a+b} + \frac{2}{c+a} + \frac{2}{a+b}}{3} > \frac{9}{\frac{2(a+b+c)}{2}}$

and $\frac{2}{a+b} + \frac{2}{c+a} + \frac{2}{a+b} > \frac{9}{a+b+c}$

(42) $A.M \geq G.M$

$\frac{1+a^2}{2} \geq \sqrt{1.a^2}$

$1+a^2 \geq 2a$

$\frac{1+a^2}{a} \geq 2$ — (A)

$\frac{1+b^2}{b} \geq 2$ — multiply

$\frac{1+c^2}{c} \geq 2$

$\frac{1+d^2}{d} \geq 2$

52.) $\frac{(1+a_1+a_1^2)(1+a_2+a_2^2)\ldots\ldots(1+a_n+a_n^2)}{a_1a_2a_3\ldots a_n} \geq 3^n$

$(1+a_1+a_1^2) \geq 3\sqrt[3]{1\cdot a_1\cdot a^3}$

$1+a+a_1^2 \geq 3a$

44.) $1, a, a^2$

$\frac{x+y}{2} \geq \sqrt{xy}$

$\cancel{x+y}$ $x+y \geq 2\sqrt{xy}$ (1)

$y+z \geq 2\sqrt{yz}$ (2)

$x+z \geq 2\sqrt{xz}$ (3)

multiply

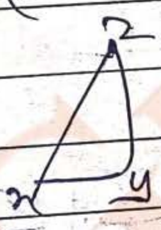
$(x+y)(y+z)(z+x) \geq 8xyz$

Given

$x+y+z=1$

$(1-z)(1-x)(1-y) \geq 8xyz$

47.)



$\frac{\cancel{y}+z+\cancel{x}+z+\cancel{x}-\cancel{y}}{2} \geq \sqrt{(x+z-y)(z+x-y)}$

$\frac{\cancel{2}z}{\cancel{2}} \geq \sqrt{(x+z-y)(z+x-y)}$ (1)

$$\Rightarrow \frac{(\cancel{z+x-y})(x+y-z)}{2} \geq \sqrt{(z+x-y)(x+y-z)}$$

$$\Rightarrow \frac{2x}{2} \geq \sqrt{(z+x-y)(x+y-z)} \quad \text{--- (ii)}$$

$$x \geq \sqrt{(z+x-y)(x+y-z)}$$

$$\frac{y+z-\cancel{x}+\cancel{x}+y-\cancel{z}}{2} \geq \sqrt{(y+z-x)(x+y-z)}$$

$$y \geq \sqrt{(y+z-x)(x+y-z)} \quad \text{--- (iii)}$$

$$xyz \geq \sqrt{(y+z-x)^2 (z+x-y)^2 (x+y-z)^2}$$

$$xyz \geq \sqrt{\{(y+z-x)(z+x-y)(x+y-z)\}^2}$$

$$xyz \geq (y+z-x)(z+x-y)(x+y-z)$$

Q55.

$a+b+c = 7$

$(a^2 b^2 c^2)$

$$\frac{\frac{a}{3}+\frac{a}{3}+\frac{a}{3}+\frac{b}{2}+\frac{b}{2}+\frac{c}{2}+\frac{c}{2}}{7} \geq \left(\frac{a}{3}\cdot\frac{a}{3}\cdot\frac{a}{3}\cdot\frac{b}{2}\cdot\frac{b}{2}\cdot\frac{c}{2}\cdot\frac{c}{2}\right)^{\frac{1}{7}}$$

$$\frac{3}{7} \geq \left(\frac{a^3 \cdot b^2 \cdot c^2}{3^3 \cdot 2^4}\right)^{\frac{1}{7}}$$

$$\frac{3^7}{7^7} \geq \frac{a^3 b^2 c^2}{3^3 \cdot 2^4}$$

$$\left(\frac{3^{10} \cdot 2^4}{7^7}\right) \geq a^3 \cdot b^2 \cdot c^2$$

4b) $2^n > 1 + n\sqrt{(2^{n-1})}$

5b)

8) $\frac{1^2}{1\cdot3} + \frac{2^2}{3\cdot5} + \frac{3^2}{5\cdot7} + \ldots$

$$T_r = \frac{r^2}{(2r-1)(2r+1)}$$

$$= \frac{1}{4}\left(\frac{(4r^2-1)+1}{4r^2-1}\right)$$

$$\Rightarrow \frac{1}{4}\left\{1 + \frac{1}{4r^2-1}\right\}$$

$$\Rightarrow \frac{1}{4}\left\{1 + \frac{1}{(2r-1)(2r+1)}\right\}$$

$$\Rightarrow \frac{1}{4}\left\{1 + \frac{1}{2}\left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)\right\}$$

$$S_n = \sum T_r$$

$$\Rightarrow \sum\left(\frac{1}{4}+\frac{1}{8}\left(\frac{1}{2r-1}-\frac{1}{2r+1}\right)\right)$$

$$\Rightarrow \frac{1}{4}\sum_{r=1}^{n} 1 + \frac{1}{8}\sum_{r=1}^{n}\left(\frac{1}{2r-1}-\frac{1}{2r+1}\right)$$

$$\Rightarrow \frac{1}{4}\cdot n + \frac{1}{8}\left[\left(\frac{1}{1}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}\right) \dots + \left(\cancel{\frac{1}{2n-1}}-\frac{1}{2n+1}\right)\right]$$

$$\Rightarrow \frac{n}{4}+\frac{1}{8}\left[1-\frac{1}{2n+1}\right]$$

12.) $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\dots$

$$T_r = \frac{r}{\cancel{r^4}\ \cancel{r^2}+r^4}$$

$$T_r = \frac{r}{1+r^2+r^4}$$

$$\Rightarrow \frac{r}{(1+2r^2+r^4)-r^2}$$

$$\Rightarrow \frac{r}{(r^2+1)^2-r^2}$$

$$\Rightarrow \frac{r}{(r^2-r+1)(r^2+r+1)}$$

$$T_r \Rightarrow \frac{1}{2}\left(\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right)$$

$$S_n = \frac{1}{2}\sum_{r=1}^{n} T_r$$

$$\Rightarrow \frac{1}{2}\sum_{r=1}^{n}\left(\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right)$$

$$\Rightarrow \frac{1}{2}\left(\left(\frac{1}{1}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{7}}\right)+ \dots \left(\cancel{\frac{1}{n^2-n+1}} - \frac{1}{n^2+n+1}\right)\right)$$

$$\Rightarrow \frac{1}{2}\left(1 - \frac{1}{n^2+n+1}\right)$$

Page - 61 का Concept :->

If a, b, c are in G.P then

(i) $\log_{10} a$, $\log_{10} b$, $\log_{10} c \rightarrow$ A.P (चूंकी G.P का log A.P में आयेगा।)

(ii) $\log_a 10$, $\log_b 10$, $\log_c 10 \rightarrow$ H.P (चूंकी A.P का reverse H.P में होगा है)

Soln → (1) a, ar, ar^2 → G.P is in the form of.

(1) a, b, c → G.P

then,

~~we take log,~~ 3:>

~~$\log_{10} a$, $\log_{10} b$, $\log_{10} c$~~

$\because$ G.P is in the form of a, ar, ar^2 so in the place of a, b, c we also take a, ar, ar^2

$a, b, c \rightarrow$ G.P

$a, ar, ar^2 \rightarrow$ G.P (form)

we take log

$\log_{10} a$, $\log_{10} b$, $\log_{10} c \rightarrow$ A.P ← So it is in A.P.

$\log a$, $\log ar$, $\log ar^2$

$\log a$, $\log a + \log r$, $\log a + 2\log r$ So,

a, $a+d$, $a+2d \rightarrow$ It is form of A.P

(ii) From ear(lier)

$a, b, c \rightarrow$ G.P

$\log_{10} a$, $\log_{10} b$, $\log_{10} c \rightarrow$ A.P

Now, Reverse of A.P is in H.P

So,

$\frac{1}{\log_{10} a}$, $\frac{1}{\log_{10} b}$, $\frac{1}{\log_{10} c} \rightarrow$ H.P

Hence, $\log_a 10$, $\log_b 10$, $\log_c 10 \rightarrow$ H.P (By the law of log).

3.)

a

$b = a + d$

$c = a + (n-1)d$

$\Rightarrow d = \frac{b-a}{2}$ — (1)

$\Rightarrow c-a = (n-1)d$

$\frac{c-a}{d} = n-1$

$n = 2\left(\frac{c-a}{b-a}\right) + 1$ — (2)

$S_n = \frac{n}{2}(a+c)$

$\Rightarrow \frac{1}{2}\left\{\frac{2(c-a)}{b-a} + 1\right\}(c+a)$

4.) $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$a_1 + (a_1+4d) + (a_1+9d) + (a_1+14d) + (a_1+19d) + a + 23d = 225$

$6a_1 + 69d = 225$

$2a + 23d = 75$ — (1)

$S_{24} = \frac{24}{2}\{2a_1 + (24-1)d\}$

$= 12 \times 75$

$= 900$

6.)

6.) $a = \frac{x+y}{2}$; $b = \frac{y+z}{2}$

$$\frac{a+b}{2} = ? \quad \frac{\frac{x+y}{2} + \frac{y+z}{2}}{2}$$

$$\Rightarrow \frac{x+2y+z}{4} \Rightarrow \frac{2y+2y}{4} = y$$

7.) $a = 1$,
$b = 1$

$d = \frac{b-a}{n+1}$

$$\frac{a_7}{A_{n-1}} = \frac{a+7d}{a+(n-1)d} = \frac{5}{9}$$

8.) $a^2(b+c)$, $b^2(c+a)$, $c^2(a+b)$

$$2b^2(c+a) = a^2(b+c) + c^2(a+b)$$

$$2b^2(c+a) = a^2b + a^2c + c^2a + c^2b$$

$$2b^2(c+a) = b(a^2+c^2) + ac(a+c)$$

$$2b^2(c+a) = b\left[(a+c)^2 - 2ac\right] + ac(a+c)$$

$$2b^2(a+c) + 2abc = b(a+c)^2 + ac(a+c)$$

$$2b\left[b(a+c) + ac\right] \quad (a+c)\left[b(a+c) + ac\right]$$

$\Rightarrow \{ \cancel{b(a+c)} + \cancel{ac} \} \ (2b-(a+c)) = 0$

$(ab+bc+ca)\ (2ba-c) = 0$

12) $\frac{1}{2\cdot4}+\frac{1}{4\cdot6}+\frac{1}{6\cdot8}+\frac{1}{8\cdot10}+\dots$ is

$\frac{1}{2}\left[\frac{4-2}{2\cdot4}+\frac{6-4}{4\cdot6}+\frac{8-6}{6\cdot8}+\dots\right]$

$\Rightarrow \frac{1}{2}\left\{\left(\frac{1}{2}-\cancel{\frac{1}{4}}\right)+\left(\cancel{\frac{1}{4}}-\cancel{\frac{1}{6}}\right)+\left(\cancel{\frac{1}{6}}-\cancel{\frac{1}{8}}\right)\dots\right.$

$\Rightarrow \frac{1}{2}\times\frac{1}{2} \Rightarrow \frac{1}{4}$

13) No. of terms $= 2n$

$a, ar, ar^2, ar^3 \dots$

$S = \frac{a(r^{2n}-1)}{r-1}$ — (1)

Take (2)

$a, ar^2, ar^4 \dots n$ terms

$S' = \frac{a((r^2)^n-1)}{r^2-1}$ — (2)

given,

$5S' = S$

14) $a_n = 96$

$a \cdot r^{n-1} = 96$

$3 \cdot r^{n-1} = 96$

$r^{n-1} = 32$ — (1)

$\Rightarrow r^n = 32r$

$S_n = 189 = a \cdot \left(\frac{r^n - 1}{r - 1}\right)$

19) $\frac{7}{9}\left(0.9 + 0.99 + \cdots + 10 \text{ terms}\right)$

$\Rightarrow \frac{7}{9}\left(\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \cdots \left(1 - \frac{1}{10^{10}}\right)\right)$

$\Rightarrow \frac{7}{9}\left\{10 - \left(\frac{1}{10} + \frac{1}{10^2} + \cdots \frac{1}{10^{10}}\right)\right\}$

$\Rightarrow \frac{7}{9}\left\{10 - \frac{1}{10}\left(\frac{1 - \left(\frac{1}{10}\right)^{10}}{1 - \frac{1}{10}}\right)\right\}$

$\Rightarrow \frac{7}{9} \cdot \left\{10 - \frac{1}{9} \cdot \left(1 - \frac{1}{10^{10}}\right)\right\}$

$\Rightarrow \frac{7}{9}\left\{\frac{90 - 1 + \frac{1}{10^{10}}}{9}\right\}$

$\Rightarrow \frac{7}{81}\left(89 + \frac{1}{10^{10}}\right)$

20.) $\sqrt{x}\left\{\sqrt{a}+\sqrt{x}+\sqrt{ab}+\sqrt{xy}+b\sqrt{a}+y\sqrt{x}\ldots\right\}$

$\Rightarrow \sqrt{x}\left\{\sqrt{a}+\sqrt{ab}+b\sqrt{a}\ldots\right)+\left(\sqrt{x}+\sqrt{xy}+y\sqrt{x}\ldots\right)$

$\Rightarrow \sqrt{x}\left(\frac{\sqrt{a}}{1-\sqrt{b}}+\frac{\sqrt{x}}{1-\sqrt{y}}\right)$

21.)

$\frac{2}{3}+\frac{8}{9}+\frac{26}{27}+\frac{80}{81}+\ldots$

$\Rightarrow \left(1-\frac{1}{3}\right)+\left(1-\frac{1}{3^2}\right)+\left(1-\frac{1}{3^3}\right)+\ldots$

$\Rightarrow n-\left(\frac{1}{3}+\frac{1}{3^2}+\ldots n \text{ terms}\right)$

$\Rightarrow n-\frac{1}{3}\left(\frac{1-\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}\right)$

22.)

$\frac{a+b}{2} \geq 2\sqrt{ab}$ Divide both side by $\sqrt{ab}$

$a+b \geq 4\sqrt{ab}$

$\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}} \geq 4$

y

$y+\frac{1}{y} \geq 4$

$y^2 \sim$

$y^2 = \sim$

24.)

a, G_1, G_2, b

$p = G_1 = ar$

$q = G_2 = ar^2$

$r = \left(\frac{b}{a}\right)^{\frac{1}{3}}$

$p = a \cdot r = a\left(\frac{b}{a}\right)^{\frac{1}{3}}$ — ①

$q = ar^2 = a\left(\frac{b}{a}\right)^{2/3}$ — ②

$\frac{p^2}{q} + \frac{q^2}{p}$

$\frac{p^3+q^3}{pq} = \frac{a^2b + ab^2}{a\cdot\left(\frac{b}{a}\right)^{\frac{1}{3}} \cdot a\left(\frac{b}{a}\right)^{2/3}}$

$= \frac{ab(a+b)}{a^2\left(\frac{b}{a}\right)}$

$\Rightarrow \frac{ab(a+b)}{a^2\left(\frac{b}{a}\right)}$

$\Rightarrow a+b$

$\Rightarrow 2A.$

26.) $a-d, a, a+d$

$a-d+a+a+d = 15$

$\Rightarrow a = 5$

$6-d, 9, 24+d$ — G.P

$$9^2 = (6-d)(24+d)$$

$$\Rightarrow (24\times 6 + 6d) - (24d - d^2)$$

$$\Rightarrow$$

29.) $x, y, z = G.P$

$$y^2 = xz \quad \text{---(1)}$$

$$x^2+y^2,\ xy+yz,\ y^2+z^2$$

$$x^2+xz,\ y(x+z),\ xz+z^2$$

$$x(x+z),\ y(x+z),\ z(x+z)$$

$$\Rightarrow G.P.$$

31.) $a, b, c \rightarrow G.P$

$$b^2 = ac$$

(A) $a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)$

$$\Rightarrow \left(\frac{b^2c^2}{a}+\frac{a^2c^2}{b}+\frac{a^2b^2}{c}\right)$$

$$\Rightarrow \left(\frac{a\cdot c\cdot c^2}{a}+\frac{b^4}{b}+\frac{a^2\cdot ac}{c}\right)$$

$$\Rightarrow c^3+b^3+a^3$$

33.)

$$S_1 = \frac{1\cdot 3}{2} + \frac{3\cdot 5}{2^2} + \frac{5\cdot 7}{2^3} + \frac{7\cdot 9}{2^4} + \ldots$$

$$-\frac{1}{2}S = \frac{1\cdot 3}{2^2} + \frac{3\cdot 5}{2^3} + \frac{5\cdot 7}{2^4}$$

$$\frac{S_1}{2} = \frac{1\cdot 3}{2} + \frac{3\cdot 4}{2^2} + \frac{5\cdot 4}{2^3} + \frac{7\cdot 4}{2^4} + \ldots$$

$$\frac{S_1}{2} = \frac{1\cdot 3}{2} + \frac{4}{2^2}\left(\frac{3}{4} + \frac{5}{2} + \frac{7}{2^2} + \ldots\ldots\right) \quad (1)$$

$$S_2 = \frac{3}{4} + \frac{5}{2} + \frac{7}{2^2} + \ldots$$

$$+\frac{S_2}{2} = \frac{3}{2} + \frac{5}{2^2} +$$

$$\frac{S_2}{2} = \frac{3}{4} + \left(\frac{2}{2} + \frac{2}{2^2} + \frac{2}{2^3} + \ldots\right)$$

35.) a, b

$$A.P \Rightarrow \frac{1}{a}, \frac{1}{b}$$

$$d = \left(\frac{1}{b} - \frac{1}{a}\right)$$

$$T_n = \frac{1}{\frac{1}{a} + (n-1)d}$$

37.)

$$2(y-a), \quad y-x, \quad y-z$$

H.M

$$2(y-a) = \frac{2(y-x)(y-z)}{y-x+y-z}$$

$$(y-a)(2y-x-z) = (y-x)(y-z)$$

$$2y^2 - yx - yz - 2ay + ax + az = y^2 - yz - xy + xz$$

$$y^2 - 2ay = xz - ax - az$$

$$y^2 - 2ay + a^2 = xz - ax - az + a^2$$

$$(y-a)^2 = x(z-a) - a(z-a)$$

$$(y-a)^2 = (x-a)(z-a)$$

40) $\lfloor n = n!$ (factorial)

= Product of first n natural number

$= n \cdot (n-1) \cdot (n-2) \text{ --- } 2 \cdot 1$

$\lfloor 3 = 3 \cdot 2 \cdot 1 = 6$

$\lfloor 5 = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$\Rightarrow$ Now,

$$A \cdot M \geq G \cdot M$$

$$\frac{1+2+3+ \text{ --- } + n}{n} \geq (1 \cdot 2 \cdot 3 \text{ --- } n)^{1/n}$$

$$\frac{n(n+1)}{2n} \geq (\lfloor n)\frac{1}{n}$$

$$\frac{n+1}{2} \geq (\lfloor n)^{1/n}$$

$$\left(\frac{n+1}{2}\right)^n \geq \lfloor n$$

$$\geq 1+2+ \cdots n$$

$$\frac{n+n+\cdots+n}{n} \geq \frac{1+2+\cdots n}{n} \geq (\lfloor n)^{1/n}$$

$$\frac{n \cdot n}{n} \geq (\lfloor n)^{1/n}$$

$$n \geq (a_n)^{1/n}$$

$$\boxed{n^n \geq a_n}$$

41.)

(A) $\left(\frac{a}{b}+\frac{b}{c}\right)\left(\frac{c}{d}+\frac{d}{e}\right) \leq 4\sqrt{\frac{a}{e}}$

$$\frac{\frac{a}{b}+\frac{b}{c}}{2} \geq \sqrt{\frac{a}{b} \cdot \frac{b}{c}}$$

$$\frac{a}{b}+\frac{b}{c} \geq 2\sqrt{\frac{a}{c}} \quad \text{--- (1)}$$

$$\frac{c}{d}+\frac{d}{e} \geq 2\sqrt{\frac{c}{d} \cdot \frac{d}{e}}$$

$$\frac{c}{d}+\frac{d}{e} \geq 2\sqrt{\frac{c}{e}} \quad \text{--- (2)}$$

multiples

$\left(\frac{a}{b}+\frac{b}{c}\right)\left(\frac{c}{d}+\frac{d}{e}\right) \geq 4\sqrt{\frac{a}{e}}$

$\boxed{n \geq 2}$

1.) $a = \frac{5}{2}$, $H = \frac{2ab}{a+b} = \frac{2 \times 2 \times 3}{5} = \frac{12}{5}$

$\frac{5}{2} + \frac{6 \times 5}{12} = 5$

2.) $S_n = an(n-1)$

$T_n = S_n - S_{n-1}$

$= an(n-1) - a(n-1)(n-2)$

$= a(n-1)\,(n-(n-2))$

$T_n = 2a(n-1)$

$\sum_{n=1}^{n} T_n^2 = \sum_{n=1} 4a^2(n-1)^2$

$= 4a^2 \sum_{n=1}^{n} (n-1)^2$

$= \frac{4a^2 \cdot (n-1)(n-1+1)(2(n-1)+1)}{6}$

3.)

3.) 2, 5, 8, 11 ——— 60 terms

3, 5, 7, 9, 11 ——— 50 terms

$\rightarrow T_{50} = 101$

$\rightarrow T_{60} = 179$

5, 11, 17, ——— n terms

$T_n = 101$

$= 5 + (n-1)6 = 101$

$\boxed{n = 17}$

4.)

$a+(p-1)d \quad + a+(q-1)d \quad , a+(r-1)d$

$$\frac{a+(q-1)d}{a+(p-1)d} = \frac{a+(r-1)d}{a+(q-1)d} = x \quad (1)$$

Note :-

$$\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d} = \frac{a-c}{b-d}$$

eg :-

$$\frac{1}{2} = \frac{2}{4} = \frac{1+2}{2+4} = \frac{1-2}{2-4}$$

$$\Rightarrow \frac{a+(q-1)d - \{a+(r-1)d\}}{a+(p-1)d - \{a+(q-1)d\}}$$

$$= \frac{(q-r)\cancel{d}}{(p-q)\cancel{d}}$$

5.) $ax^3 + bx^2 + cx + d = 0$ — roots A/r, A, Ar

$$\frac{A}{\cancel{r}} \cdot A \cdot A\cancel{r} = \frac{-d}{a}$$

$$A^3 = \frac{-d}{a} \quad -(1)$$

$$a \cdot A^3 + b \cdot A^2 + c \cdot A + d = 0$$

$$a\left(\frac{-d}{a}\right) + bA^2 + cA + d = 0$$

$$A(bA + c) = 0$$

$$\Rightarrow A = \frac{-c}{b} \quad -(2)$$

from 1st.

$$\left(\frac{-c}{b}\right)^3 = \frac{-d}{a}$$

$$\boxed{c^3 a = b^3 d}$$

6.)

$x^2 - 3x + p = 0$ — roots a, ar

$x^2 - 12x + q = 0$ — roots ar^2, ar^3

$$a + ar = 3$$

$$a(1+r) = 3 \quad (1)$$

$$ar^2 + ar^3 = 12$$

$$ar^2(1+r) = 12 \quad (2)$$

7.)

$$S_\infty - S_n < \frac{1}{1000}$$

$$\left(\frac{1}{1-\frac{1}{2}}\right) - 1 \cdot \left(\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}\right) < \frac{1}{1000}$$

$$2 - 2\left(1-\left(\frac{1}{2}\right)^n\right) < \frac{1}{1000}$$

$$\frac{2}{2^n} < \frac{1}{1000}$$

$$\frac{2^n}{2} > 1000$$

$$2^{n-1} = 1000$$

$$n-1 \geq 10$$

8) $2\left(\frac{a}{2}\right) = 5^{1+x} + 5^{1-x} + 25^{x} + 25^{-x}$

$a \geq 2\sqrt{5^{1+x} \cdot 5^{1-x}} + 2\sqrt{25^{x} \cdot 25^{-x}}$

$\because \left\{ \frac{a+b}{2} \geq \sqrt{ab} \quad a+b \geq 2\sqrt{ab} \right\}$

$\geq 10 + 2$

9) $b^2 = ac$

$ax^2 + 2bx + c = 0$

$ax^2 + 2\sqrt{ac}\,x + c = 0$

$\left(\sqrt{a}\,x + \sqrt{c}\right)^2 = 0$

$x = -\sqrt{\frac{c}{a}}$

$dx^2 + 2ex + f = 0 \quad - \sqrt{\frac{c}{a}}$

$d\left(-\sqrt{\frac{c}{a}}\right)^2 + 2e\left(-\sqrt{\frac{c}{a}}\right) + f = 0$

$\frac{dc}{a} + f = 2e\sqrt{\frac{c}{a}}$

Divided by "c"

$$\frac{d}{a} + \frac{f}{c} = 2e\sqrt{\frac{c}{ac^2}}$$

$$\frac{d}{a} + \frac{f}{c} = 2e\sqrt{\frac{c}{ac^2}}$$

$$= 2e\sqrt{\frac{1}{ac}}$$

$$= 2e\sqrt{\frac{1}{b^2}}$$

$$\frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$$

12) $a_1, a_2 \text{ ——— } a_n$

$$a, ar, ar^2, \text{ ——— }, ar^{n-1}$$

$$S = a\left(\frac{r^n - 1}{r - 1}\right) \text{ — } (1)$$

$$T = \frac{1}{a} \cdot \left(\frac{\left(\frac{1}{r}\right)^n - 1}{\left(\frac{1}{r}\right) - 1}\right)$$

$$= \frac{1}{a} \cdot \frac{r(1 - r^n)}{r^n(1 - r)}$$

$$T = \frac{1}{a} \cdot \frac{1}{r^{n-1}} \cdot \frac{(r^n - 1)}{r - 1} \text{ — } (2)$$

$$\frac{S}{T} = a^{2} \cdot r^{n-1} \quad \text{--- (3)}$$

$$P = a_1, a_2 \text{ --- } a_n$$

$$= a \cdot ar \cdot ar^2 \text{ --- } ar^{n-1}$$

$$= a^n \cdot r^{1+2 \text{ --- } +(n-1)}$$

$$P = a^n \cdot r^{\frac{n(n-1)}{2}}$$

$$P^2 = a^{2n} \cdot r^{n(n-1)}$$

$$= \left(a^2 \cdot r^{n-1}\right)^n$$

$$= \left(\frac{S}{T}\right)^n$$

14.) $$\sum_{r=1}^{n} \frac{1}{\sqrt{a+rx} + \sqrt{a+(r-1)x}}$$

Rationalise:

$$\sum \frac{\sqrt{a+rx} - \sqrt{a+(r-1)x}}{(a+rx) - (a+(r-1)x)}$$

$$= \frac{1}{x} \sum_{r=1}^{n} \left(\sqrt{a+rx} - \sqrt{a+(r-1)x}\right)$$

Date: / /

$$\frac{S}{T} = a^2 \cdot r^{n-1} \quad \text{--- (5)}$$

$$P = a_1 a_2 \text{ --- } a_n$$

$$= a \cdot ar \cdot ar^2 \text{ --- } ar^{n-1}$$

$$= a^n \cdot r^{1+2 \text{ --- } + (n-1)}$$

$$P = a^n \cdot r^{\frac{n(n-1)}{2}}$$

$$P^2 = a^{2n} \cdot r^{n(n-1)}$$

$$= \left(a^2 \cdot r^{n-1}\right)^n$$

$$= \left(\frac{S}{T}\right)^n$$

14.) $$\sum_{r=1}^{n} \frac{1}{\sqrt{a+rx} + \sqrt{a+(r-1)x}}$$

Rationalise:

$$\sum \frac{\sqrt{a+rx} - \sqrt{a+(r-1)x}}{(a+rx) - (a+(r-1)x)}$$

$$= \frac{1}{x} \sum_{r=1}^{n} \left(\sqrt{a+rx} - \sqrt{a+(r-1)x}\right)$$

17.) $$\sum_{k=1}^{n}\left(\sum_{m=1}^{k} m^2\right) = an^4+bn^3+cn^2+dn+e$$

$$\Rightarrow \sum_{k=1}^{n} \frac{k\cdot(k+1)(2k+1)}{6}$$

$$\Rightarrow \frac{1}{6}\sum_{k=1}^{n}\left(k^3+3k^2+k\right)$$

$$\Rightarrow \frac{1}{6}\left[\sum k^3+3\sum k^2+\sum k\right]$$

$$= \frac{1}{6}\left[\left(\frac{n(n+1)}{2}\right)^2 + \frac{3n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

19.) $$f(x) = \frac{1-x^{n+1}}{1-x}$$

$$= 1+x+x^2+ \text{---} + x$$

$$f(x) = 1+x+x^2+ \text{---}\ x^n$$

$$f'(x) = 1+2x+3x^2+4x^3+ \text{---} + n x^{n-1}$$

$$g(x) = 1-\frac{2}{x}+\frac{3}{x^2} + \text{---} (-1)^n\left(\frac{n+1}{x^n}\right) \text{the}$$

$g(x) = 1 - \frac{2}{x} + \frac{3}{x^2} - \frac{4}{x^3} + \text{———}$

S = Constant term in $f'(x) \cdot g(x)$

$S = 1^2 - 2^2 + 3^2 - 4^2 \text{———}$

I $n \in$ even

$S = 1^2 - 2^2 + 3^2 - 4^2 + \text{———} - n^2$

$S = (1^2 + 3^2 + 5^2 + \text{———}) - (2^2 + 4^2 + \text{—} n^2)$

$S = (1^2 + 2^2 + 3^2 + \dots + n^2) - 2(2^2 + 4^2 + \text{—} + n^2)$

$= \frac{n(n+1)(2n+1)}{6} - 2 \cdot 2^2 \left(1^2 + 2^2 + \text{—} \left(\frac{n}{2}\right)^2\right)$

$= \frac{n(n+1)(2n+1)}{6} - 2 \times 4 \left[\frac{\frac{n}{2}\left(\frac{n}{2}+1\right) \cdot 2\left(\frac{n}{2}\right)+1}{6}\right]$

$\Rightarrow \frac{n(n+1)}{6}\left[2n+1 - 4\left(\frac{n}{2}+1\right)\right.$

$\Rightarrow \frac{n(n+1)}{6}[1-4] = -\frac{n(n+1)}{2}$

Case II when $n \in$ odd.

$S = 1^2 - 2^2 + 3^2 \text{———} (n-1)^2 + n^2$

$$= \frac{(n-1)\cdot n}{2} + n^2$$

21) $\frac{5^{1+x} + 5^{1-x}}{2} = 13$

$5^x = y$

$\{x \mid x^2 - 1 = 0,\ x \in R\}$

crat

$\{1, -1\}$

21) $\frac{a+be^y}{a-be^y} = \frac{b+ce^y}{b-ce^y} = \frac{c+de^y}{c-de^y}$

$(a+be^y)(b-ce^y) = (a-be^y)(b+ce^y)$

$\cancel{ab} - ace^y + b^2e^y - bce^{2y} = \cancel{ab} + ace^y - b^2e^y - bce^{2y}$

$\cancel{2}b^2e^y = \cancel{2}ace^y$

$$\boxed{b^2 = ac}$$

28.) $\dfrac{x^{100}}{1+x+x^2+x^3+\cdots x^{200}}$

$\dfrac{1+x+x^2+x^3 \cdots + x^{200}}{201} \geq \left(1 \cdot x \cdot x^2 \cdots x^{200}\right)^{\frac{1}{201}}$

$\geq \left(x^{1+2+\cdots 200}\right)^{\frac{1}{200}}$

$\geq \left(x^{\frac{200 \cdot 201}{2}}\right)^{\frac{1}{201}}$

$\geq x^{100}$

$\dfrac{1}{201} \geq \dfrac{x^{100}}{1+x+\cdots x^{200}}$

30.) $\dfrac{a}{r}, a, ar$

i) $\dfrac{a}{r} + a > ar$

ii) $a + ar > \dfrac{a}{r}$

iii) $\dfrac{a}{r} + ar > a$

34.) $f(n+1) = \dfrac{2f(n)+1}{2}$

$f(n+1) = f(n) + \dfrac{1}{2}$

$f(n+1) - f(n) = \dfrac{1}{2}$

$F(101) - \cancel{F(100)} = \frac{1}{2}$

$F(100) - F(99)$

$F(99) - F(98) = \frac{1}{2}$

$|$

$\cancel{F(2)} - F(1) = \frac{1}{2}$

$F(101) - F(1) = \frac{1}{2} \times 100$

Ex 4

1.)

$S_n = \frac{1}{1^2} + \frac{1+2}{1^2+2^2} + \cdots$

$$T_r = \frac{1+2+\cdots+r}{1^2+2^2+\cdots+r^2}$$

$$= \frac{\frac{r(r+1)}{2}}{\left\{\frac{r(r+1)}{2}\right\}^2}$$

$$T_r = \frac{2}{r(r+1)}$$

$$T_r = 2\left(\frac{1}{r} - \frac{1}{r+1}\right)$$

$$S_n = \Sigma T_r$$

$$= 2\sum\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$= 2\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\ldots\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$S_n = 2\left(1-\frac{1}{n+1}\right)$$

$$S_\infty = 2$$

3) $S_9 > 200$

$$\frac{9}{2}\cdot(2a+8d) > 200$$

$$a+4d > \left(\frac{200}{9}\right)$$

$$(a+d)+3d > \frac{200}{9}$$

$$12+3d > \left(\frac{200}{9}\right)$$

$$3d > \left(\frac{92}{9}\right)$$

$$d > 3.4$$

Also,

$$S_9 < 200$$

$$d < 4.1$$

$$\Rightarrow \boxed{d = 4}$$

4) $(b-c)^2, (c-a)^2, (a-b)^2 \rightarrow A.P$

$$\frac{1}{b-c}, \frac{1}{c-a}, \frac{1}{a-b}$$

$$\Rightarrow \frac{2}{c-a} = \frac{1}{b-c} + \frac{1}{a-b}$$

$$\Rightarrow 2(b-c)(a-b) = (c-a)(a-c)$$

$$\Rightarrow 2(ab-b^2-ac+bc) = -(c-a)^2$$

$$-2ab+2b^2+2ac-2bc = (c-a)^2$$

$$(c-a)^2 - 2ab+2b^2+2ac-2bc = 2(c-a)^2$$

$$c^2+a^2-2ac-2ab+2b^2+2ac-2bc = 2(c-a)^2$$

$$(b^2-2ab+a^2)+(b^2-2bc+c^2) = 2(c-a)^2$$

$$(a-b)^2+(b-c)^2 = 2(c-a)^2$$

5. $(1), (3,5), (7,9,11), (13,15,17,19), (21 -$

$S = 1+3+7+13+21 \text{ ------ } + T_n$

$S = \quad 1+3+7+13 \text{ ------ } + T_n$

$0 = 1+2+4+6+ \text{ ------ } - T_n$

$\underbrace{\qquad\qquad\qquad}_{n \text{ term}}$

$T_n = 1 + (2 + 4 + \ldots (n-1) \text{ term})$

$T_n = 1 + \left(\frac{n-1}{2}\right) \cdot [4 + (n-2)2]$

$T_n = n^2 - n + 1$

$S_n = \frac{n}{2}[2 \times (n^2 - n + 1) + (n-1)^2]$

(iv) $S = 1 + \left(1 + \frac{1}{2}\right)\frac{1}{3} + \left(1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2\right)\left(\frac{1}{3}\right)^2 + \ldots \infty$

$\frac{1}{3}S = \frac{1}{3} + \left(1 + \frac{1}{2}\right)\left(\frac{1}{3}\right)^2 + \ldots$

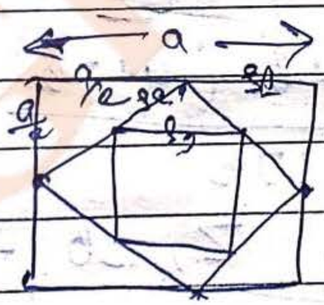
$\frac{2S}{3} = 1 + \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2^2} \cdot \frac{1}{3^2} + \frac{1}{2^3} \cdot \frac{1}{3^3} \ldots$

$= \left(\frac{1}{1 - \frac{1}{6}}\right)$

$\frac{2S}{3} = \frac{6}{5}$

$S = \left(\frac{3 \times 6}{5 \times 2}\right)$

9.)



$s_1 = a^2 = a^2$

$s_2 = \left(\frac{a}{\sqrt{2}}\right)^2 = \frac{a^2}{2}$

$s_3 = \left(\frac{a}{2}\right)^2 = \frac{a^2}{4}$

$s = \left(\frac{a^2}{1-\frac{1}{2}}\right)$

$= 2a^2$

10.) $a, x, y, z, b \rightarrow A.P$

$d = \left(\frac{b-a}{3+1}\right)$

$x = A_1 = a+d$

$x = a + \left(\frac{b-a}{4}\right) = \left(\frac{3a+b}{4}\right)$

$y = a+2d = \left(\frac{a+b}{2}\right)$

$z = a+3d = \left(\frac{a+3b}{4}\right)$

given

$xyz = \frac{15}{2}$

$\frac{(3a+b)(a+b)(a+3b)}{32} = \frac{15}{2}$ — (1)

11) $a, \underbrace{x, y, z}_{H.M}, b \rightarrow H.P$

$$d = \left(\frac{\frac{1}{b}-\frac{1}{a}}{4}\right) = \frac{1}{4}\left(\frac{1}{b}-\frac{1}{a}\right)$$

$$\frac{1}{H_1} = \frac{1}{x} = \frac{1}{a}+d$$

$$\Rightarrow \frac{1}{x} = \frac{1}{a}+\frac{1}{4}\left(\frac{1}{b}-\frac{1}{a}\right)$$

$$\frac{1}{x} = \left(\frac{3a+b}{4ab}\right)$$

$$x = \frac{4ab}{3a+b}$$

$$y = \frac{2ab}{a+b}$$

$$z = \frac{4ab}{a+3b}$$

$$xyz = \frac{18}{5}$$

$$\frac{32(ab)^3}{(3a+b)(a+b)(a+3b)} = \frac{18}{5} \quad (2)$$

$$(1) \times (2)$$

$$(ab)^3 = \frac{18}{5} \times \frac{15}{2}$$

$$(ab)^3 = 27$$

$$ab = 3$$

Passage-1
Q17

11.) $a, b, c \rightarrow H.P$

$b = \frac{2ac}{a+c}$

$\Rightarrow 1 + \frac{3}{2}\left(\frac{c}{a} + \frac{a}{c}\right) \geq 1 + \frac{3}{2} \times 2$

12(i) $\frac{1}{5} \sum r(r+1)(r+2)(r+3)\left[r+4-(r-1)\right]$

$\frac{1}{5}\left[\sum r(r+1)(r+2)(r+3)(r+4) - (r-1)r(r+1)(r+2)(r+3)\right.$

$r = 1, 2, 3, 4 \text{ ---- } n$

16) $a_r = \frac{r^5 + 6r^4 + 11r^3 + 6r^2 + 4r + 6}{r^4 + 6r^3 + 11r^2 + 6r}$

$\Rightarrow r + \frac{4r+6}{r(r^3 + 6r^2 + 11r + 6)}$

$\Rightarrow r + \frac{4(r+1)+2}{}$

$\Rightarrow r + \frac{4}{r(r+2)(r+3)} + \frac{2}{r(r+1)(r+2)(r+3)}$

$S_n = \sum a_r$

Passage-1

Q17)

$$\frac{p+q}{2} - \sqrt{pq} = \frac{1}{2}$$

$$\sqrt{pq} - \frac{2pq}{p+q} = \frac{1}{4}$$

$p+q = A$

$\sqrt{pq} = B$

$$\frac{A}{2} - B = \frac{1}{2} \quad -①$$

$$B - \frac{2B^2}{A} = \frac{1}{4} \quad -②$$

$A = 2$

$B = \frac{1}{2}$

$pq = B^2$

$$pq = \frac{1}{4}$$

18)

$x^2 - (p+q)x + pq = 0$

$$x^2 - 2x + \frac{1}{4} = 0$$

18)

$$f(x) = x^2 - 2x + \frac{1}{4}$$

$$= \frac{-D}{4a} \Rightarrow \frac{-(4 - 4\cdot\frac{1}{4})}{-4} = \frac{3}{4}$$

19) $(1-b)\,(1+2a)+(2a)^2+ \text{——} = 1-b^6$

$1+2a+4a^2+ \text{——} +32a^5 = \left(\frac{1-b^6}{1-b}\right)$

$= 1+b+b^2+b^3+b^4+b^5$

$b = 2a$

$\frac{b}{a} = 2$

Passage-2

$(4n+1)$ terms

G.P

$2n+1$ terms

A.P

First term of G.P $= a+2n\cdot d$

$(T_{n+1})_{A.P} = (T_{n+1})_{G.P}$

$a+n\cdot d = (a+2nd)\,R^n$

$a+2n = (a+4n)\cdot\left(\frac{1}{2}\right)^n$

$a+2n = \frac{a}{2^n} + \frac{4^n}{2^n}$

20) a, b, c, d divide

$\frac{1}{bcd} + \frac{1}{a}$

$b, c, d,$

6

Ex 2 4

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9) $a, b, c, d \rightarrow A.P$

Divide by $abcd$

$\frac{1}{bcd}, \frac{1}{acd}, \frac{1}{abd}, \frac{1}{abc} \rightarrow A.P$

$bcd, acd, abd, abc \Rightarrow H.P$

5) $\frac{a_1 + a_2 + a_3 - - - + 2a_n}{n} \geq (a_1 . a_2 . - - 2a_n)^{1/n}$

$\frac{(\qquad)}{n} \geq (2c)^{1/n}$

$(\qquad) \geq n.(2c)^{1/n}$

$a-d, a, a+d \rightarrow A.P$

$a-d + a + a + d = \frac{3}{2}$

$a = \frac{1}{2}$

Also,

$\left(\frac{1}{2} - d\right)^2, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2} + d\right)^2 \rightarrow G.P$

$\left(\left(\frac{1}{2}\right)^2\right)^2 = \left(\frac{1}{2} - d\right)^2 \cdot \left(\frac{1}{2} + d\right)^2$

$\pm \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2} - d\right)\left(\frac{1}{2} + d\right)$

7) a, A_1, A_2, b

(i) $d = \left(\frac{b-a}{n+1}\right)$

$A_1 = a + d$

$A_2 = a + 2d$

(ii) $G_1 G_2 = (\sqrt{ab})^2 = ab$ V.V.I

(iii) a, H_1, H_2, b

$d = \dfrac{\left(\frac{1}{b} - \frac{1}{a}\right)}{3}$

$\dfrac{1}{H_1} = \dfrac{1}{a} + d$

$\dfrac{1}{H_2} = \dfrac{1}{a} + 2d$

8.) $2b = a + c$ —— (1)

$a^2, b^2, c^2 \rightarrow$ H.P —

$b^2 = \dfrac{2a^2c^2}{a^2+c^2}$

$\left(\dfrac{a+c}{2}\right)^2 = \dfrac{2a^2c^2}{a^2+c^2}$

$(a+c)^2 (a^2+c^2) = 8a^2c^2$

$\left[(a^2+c^2) + 2ac\right](a^2+c^2) = 8a^2c^2$

$(a^2+c^2) + 2ac \cdot (a^2+c^2) = 8a^2c^2$

add both side a^2c^2

$(a^2+c^2) + 2ac(a^2+c^2) + a^2c^2 = 9a^2c^2$

$(a^2+c^2+ac)^2 = 9a^2c^2$

$a^2+c^2+ac = \pm 3ac$

$$a^2+c^2+ac = 3ac$$
$$(a-c)^2 = 0$$
$$a-c = 0$$
$$a = c$$

Put this in eq (1) then,
we get,
$$a = b = c$$

$$a^2+b^2+ac = -3ac$$
$$a^2+c^2+2ac = -2ac$$
$$(a+c)^2 = -2ac$$
$$4b^2 = -2ac$$
$$b^2 = \frac{-ac}{2}$$
$$b^2 = a \cdot \left(\frac{-c}{2}\right)$$

9.) $$\frac{x}{1-r} = 5$$
$$x = 5 - 5r$$
$$5r = 5 - x$$
$$r = \left(\frac{5-x}{5}\right)$$
$$-1 < r < 1$$
$$-1 < \frac{5-x}{5} < 1$$
$$-5 < 5-x < 5$$
$$-10 < -x < 0$$
$$10 > x > 0$$

10.) $$(1+a)(1+b)(1+c) = 1+a+b+c+ab+bc+ca+abc$$

$$A.M \geq G.M$$
$$\frac{a+b+c+ab+bc+ca+abc}{7} \geq (a^4b^4c^4)^{1/7}$$

$$\Rightarrow \frac{1+a+b+c+ab+bc+ac+abc}{7} \geq (a^4b^4c^4)^{1/7}$$
$$\Rightarrow \frac{(1+a)(1+b)(1+c)}{7} \geq (a^4b^4c^4)^{1/7}$$

$$a^2+c^2+ac = 3ac$$
$$(a-c)^2 = 0$$
$$a-c=0$$
$$a=c$$

Put this in eq ① then, we get,
$$a=b=c$$

$$a^2+b^2+ac = -3ac$$
$$a^2+c^2+2ac = -2ac$$
$$(a+c)^2 = -2ac$$
$$4b^2 = -2ac$$
$$b^2 = \frac{-ac}{2}$$
$$b^2 = a\cdot\left(\frac{-c}{2}\right)$$

9.)
$$\frac{x}{1-r} = 5$$
$$x = 5-5r$$
$$5r = 5-x$$
$$r = \left(\frac{5-x}{5}\right)$$
$$-1<r<1$$
$$-1<\frac{5-x}{5}<1$$
$$-5<5-x<5$$
$$-10<-x<0$$
$$10>x>0$$

10.)
$$(1+a)(1+b)(1+c) = 1+a+b+c+ab+bc+ca+abc$$

$A\cdot M \geq G\cdot M$
$$\frac{a+b+c+ab+bc+ca+abc}{7} \geq (a^4b^4c^4)^{1/7}$$

$$\Rightarrow \frac{1+a+b+c+ab+bc+ac+abc}{7} \geq (a^4b^4c^4)^{1/7}$$
$$\Rightarrow \frac{(1+a)(1+b)(1+c)}{7} \geq (a^4b^4c^4)^{1/7}$$

11) $(\alpha^2+\beta^2) = (\alpha+\beta)(\alpha^3+\beta^3)$

$\alpha^4+\beta^4+2\alpha^2\beta^2 = \alpha^4 + \beta^4 + \alpha\beta^3 + \beta\alpha^3$

$\alpha\beta(\alpha^2+\beta^2 - 2\alpha\beta) = 0$

$\alpha\beta[(\alpha+\beta)^2 - 4\alpha\beta] = 0$

$\frac{c}{a}\cdot\left(\frac{b^2}{a^2} - \frac{4c}{a}\right) = 0$

$c(b^2-4ac) = 0$

$c \cdot D = 0$

12) $a_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \text{---}$

$a_n = \frac{3}{4}\cdot\left(\frac{1-\left(\frac{-3}{4}\right)^n}{1+\left(\frac{3}{4}\right)}\right)$

$= \frac{3}{7}\cdot\left(1-\left(\frac{-3}{4}\right)^n\right)$

$a_n = \frac{3}{7} - \frac{3}{7}\cdot\left(\frac{-3}{4}\right)^n$

$b_n = 1-a_n$

$= 1-\left(\frac{3}{7} - \frac{3}{7}\left(\frac{-3}{4}\right)^n\right)$

$b_n = \frac{4}{7} + \frac{3}{7}\left(\frac{-3}{4}\right)^n$

$b_n > a_n$

$$\frac{4}{7} + \frac{3}{7}\left(\frac{-3}{4}\right)^n > \frac{3}{7} - \frac{3}{7}\left(\frac{-3}{4}\right)^n$$

$$\frac{1}{7} > \frac{-6}{7}\cdot\left(\frac{-3}{4}\right)^n$$

$$1 > \frac{-(2\times 3)\,(-3)^n}{4^n}$$

$$\frac{4^n}{2} > (-3)\,(-3)^n$$

$$2^{2n-1} > (-3)^{n+1}$$

This inequality should be satisfied by

$n = 1$ (✗)
$n = 2$ (✓)
$n = 3$ (✗)
$n = 4$ (✓)
$n = 5$ (✗)
$n = 6$ (✓)
$n = 7$ (✓)
$n = 8$ (✓)
$n = 9$ (✓)

Passage-III

$$V_r = \frac{r}{2}\left[2r + (r-1)(2r-1)\right]$$

17.) ΣV_r

$= \Sigma$

Passage-III

$A_1 H_1 = G_1$, $2ab$

20) $A_n = \frac{A_{n-1} + H_{n-1}}{2}$

$G_n^2 = A_{n-1} \cdot H_{n-1}$

$H_n = \frac{2 A_{n-1} \cdot H_{n-1}}{A_{n-1} + H_{n-1}}$

$G_2^2 = A_1 H_1 \ (n = 2)$

$G_3^2 = A_2 H_2$

$= \left(\frac{A_1 + H_1}{2}\right)\left(\frac{2 A_1 \cdot H_1}{A_1 + H_1}\right)$

$G_3^2 = A_1 H_1$

$= G_2^2$

$A_2 = \frac{A_1 + H_1}{2} < \frac{A_1 + A_1}{2}$

$\Rightarrow A_2 < A_1$

21) $a_k = 2a_{k-1} - a_{k-2}$

$a_{k-1} = \left(\frac{a_k + a_{k-2}}{2}\right)$

22) a, $a+d$, $a+2d$ ---- $a+10d$

$$\Rightarrow \frac{a^2 + (a+d)^2 + (a+2d)^2 - \ldots (a+10d)^2}{11} = 90$$

$$\Rightarrow \frac{11 \cdot a^2 + d^2(1^2+2^2 \ldots +10^2) + 2ad(1+2+\ldots 10)}{11}$$

$$\cancel{\boxed{s = ut + \frac{1}{2}at^2}} = 90$$

$d = 7$

27.) $\frac{S_m}{S_n}$ $a_1 = 3$, $m = 5n$

$$\frac{\frac{n}{2}\left[2a_1 + (m-1)d\right]}{\frac{n}{2}\left(2a_1 + (n-1)d\right)} = \frac{5n}{n}\left[\frac{6 + (5n-1)d}{6 + (n-1)d}\right]$$

$$\Rightarrow 5\left[\frac{(6-d) + 5nd}{(6-d) + nd}\right]$$

24.)

$$\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^1}{8} \geq (\ldots)^{\frac{1}{8}}$$

4.) $a_1, a_2, a_3, \ldots a_n \rightarrow$ G.P

$a, ar, ar^2, \ldots ar^{n-1}$

$G_1 = a$

$G_2 = (a_1 \cdot a_2)^{1/2}$

$G_k = (a_1 \cdot a_2 \ldots a_k)^{1/k}$

$= (a \cdot ar \cdot ar^2 \ldots ar^{k-1})^{1/k}$

$= \left[a^k \cdot r^{(1+2+\ldots k-1)}\right]^{1/k}$

$G_k = \left[a^k \cdot r^{\frac{k(k-1)}{2}}\right]^{1/k}$

$G_k = a \cdot r^{\frac{k-1}{2}}$

$A_k = \left(\dfrac{a_1 + a_2 + \ldots a_k}{k}\right)$

$H_k = \dfrac{k}{\frac{1}{a_1} + \frac{1}{a_2} + \ldots \frac{1}{a_k}}$

$$\boxed{A_k H_k = G_k^2}$$

Course : IIT-F

DISTANCE R

Q.1 (a) Dra
(b) Dr

Q.2 A po
mov

Q.3 Let

Q.4 Th

Q.5 F

Q.6

Q.7

Q.8

SE

Point

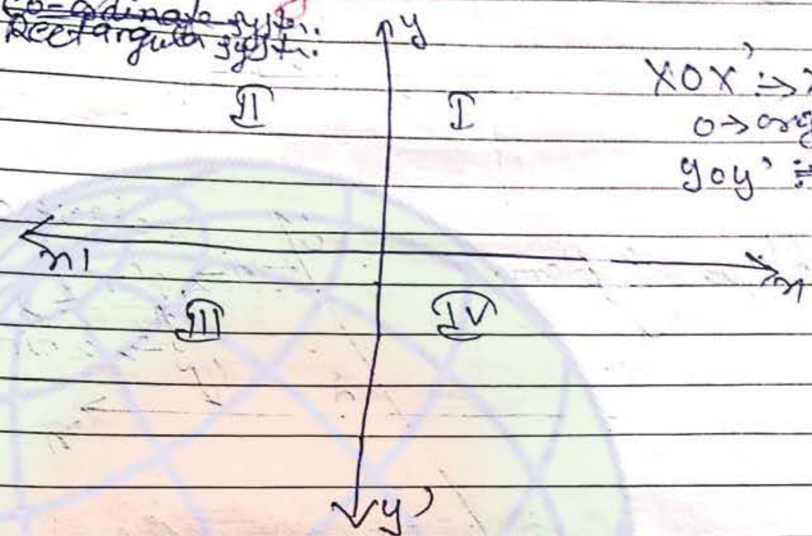
Co-ordinate system: →

(I) ~~Co-ordinate syst~~ Rectangular syst:

$XOX' \Rightarrow x$-axis

$O \rightarrow$ origin

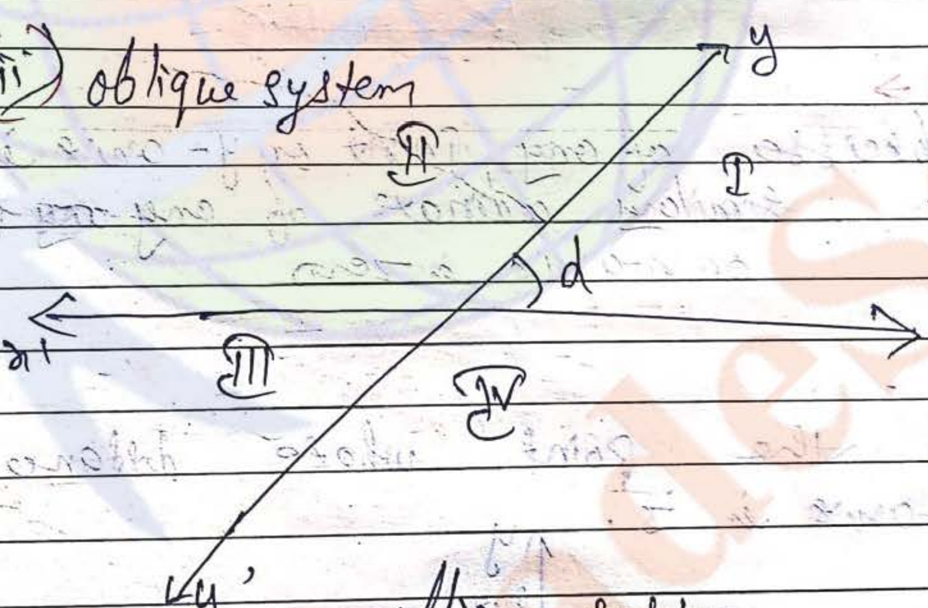
$YOY' \Rightarrow y$-axis



Rectangular system

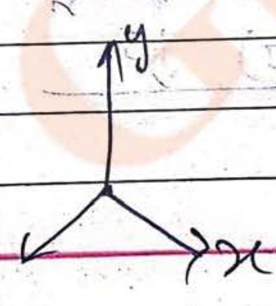
(ii) oblique system

$\boxed{\alpha \neq 90^\circ}$



oblique system

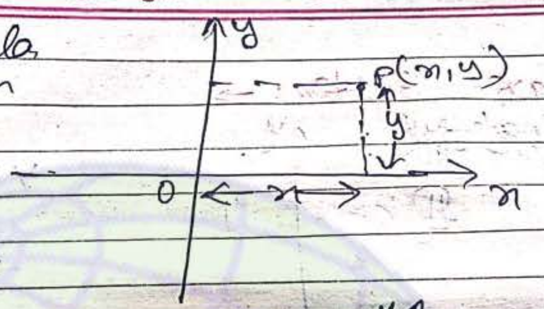
⇒



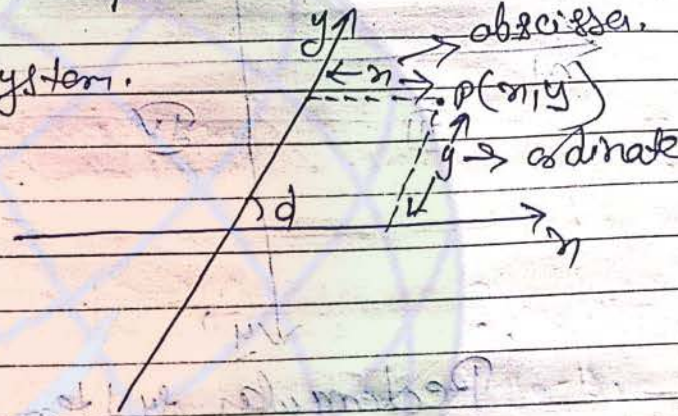
(Right hand thumb rule)

Cartesian Co-ordinates :→

1) Rectangular System



II) Oblique System.

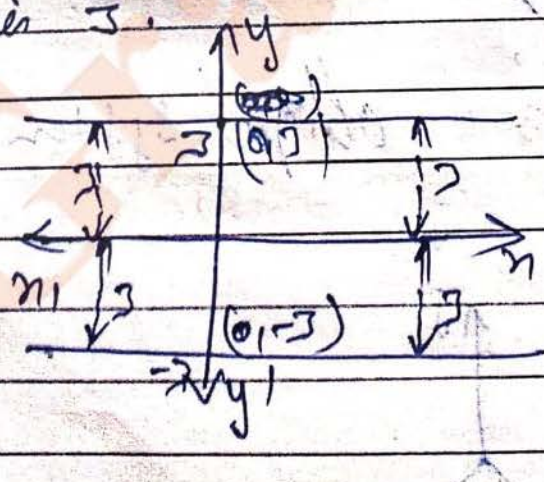


Example:→

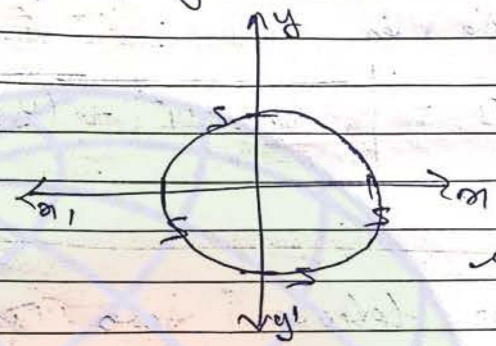
Note:→

(1) abscissa of any point on y-axis is zero similarly ordinate of any point on x-axis is zero

Q→ Draw the point whose distance from x-axis is 3.



Q) Find the area generated by the points whose distance from origin is 5.



$r = 5$

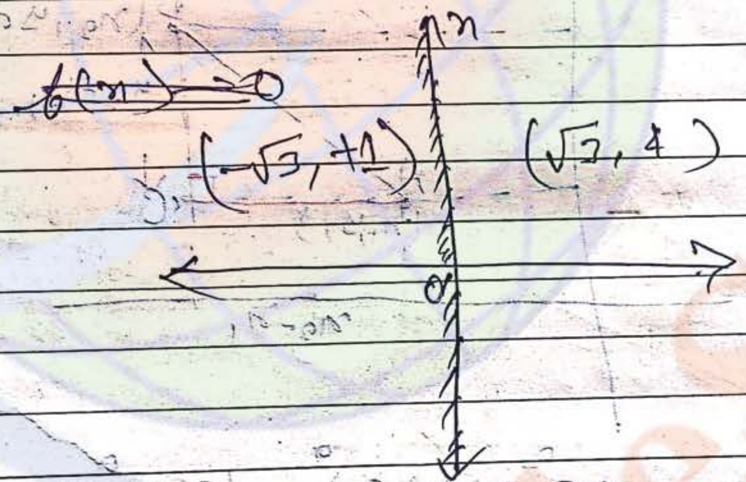
Area for ⊙ $\Rightarrow \pi r^2$

$\Rightarrow 3.14 \times 25$

$\Rightarrow 25\pi$

Q) Find the Image of point $(\sqrt{3}, 4)$ with respect to the mirror $x = 0$

Here axis is mirror where $x=0$



27/8/

Distance formula :→

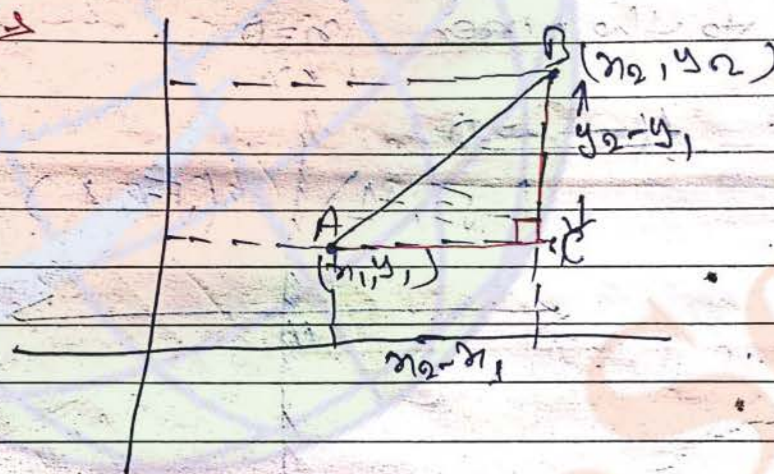
Distance between the point $A(x_1, y_1)$, and $B(x_2, y_2)$ is

$$AB = \left| \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \right|$$

Note :→

Distance between two points is always non-negative.

Proof :→



$$AB^2 = (AC)^2 + (BC)^2$$
$$= (x_2-x_1)^2 + (y_2-y_1)^2$$

$$\therefore AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Q) show that distance of point $(a\cos\alpha, a\sin\alpha)$ from origin is Independent of "α".

$$\Rightarrow \sqrt{a^2\cos^2\alpha + a^2\sin^2\alpha}$$

$$\Rightarrow a$$

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Date: / /

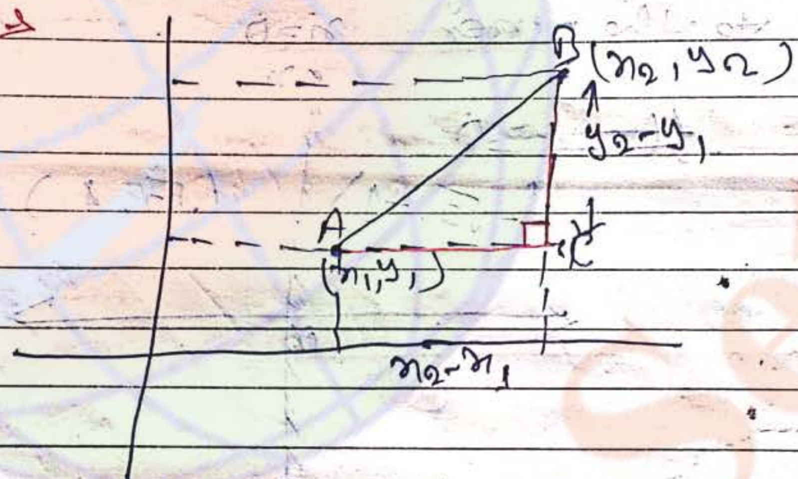
Distance formula :→

Distance between the point $A(x_1, y_1)$, and $B(x_2, y_2)$ is

$$AB = \left| \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \right|$$

Note :→
Distance between two points is always non-negative.

Proof :→



$$AB^2 = (AC)^2 + (BC)^2$$
$$= (x_2-x_1)^2 + (y_2-y_1)^2$$

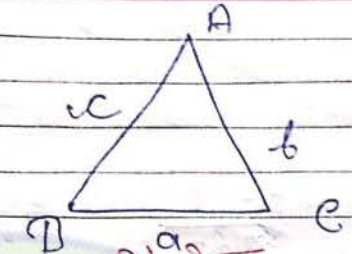
$$\therefore AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Q) show that distance of point $(a\cos\alpha, a\sin\alpha)$ from origin is Independent of "α"!

$$\Rightarrow \sqrt{a^2\cos^2\alpha + a^2\sin^2\alpha}$$

$$\Rightarrow a$$

Cosine formula :→



minus में होगा

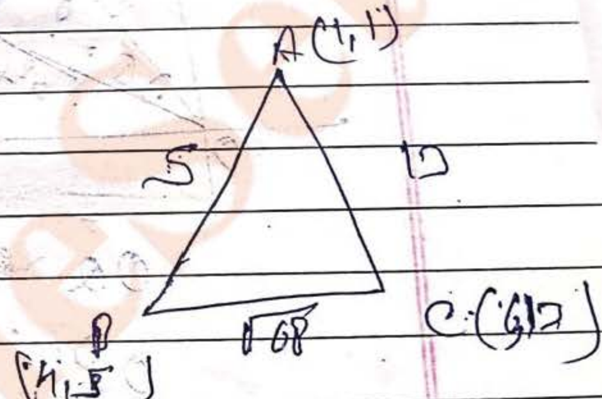
i) $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

उपस्थित होंगे →

ii) $\cos B = \dfrac{c^2 + a^2 - b^2}{2ca}$

iii) $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Q:→ vertices of a triangle A, B, C are A(1, 1), B(4, 5), C(6, 7) are given then find cos A



$AB = \sqrt{3^2 + 4^2}$

$\Rightarrow \sqrt{9 + 16}$

$\Rightarrow 5$

$\frac{8}{4}$
$\frac{4}{2}$

~~$BC = \sqrt{(2)^2 + (2)^2}$~~

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~~$\Rightarrow 2\sqrt{2}$~~

$BC = \sqrt{(2)^2 + (7)^2}$

~~$AC = \sqrt{\ldots}$~~ $= \sqrt{4 + 64}$

~~$\Rightarrow \ldots$~~

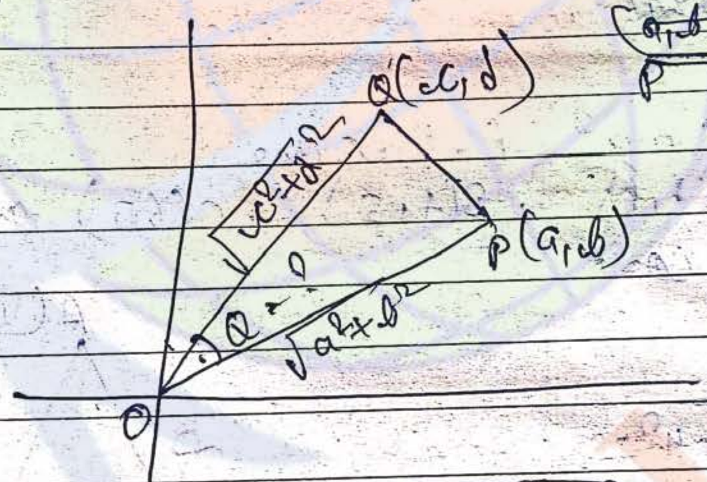
$\Rightarrow \sqrt{68}$

AC =

$$\cos A = \frac{13^2 + 5^2 - 68}{2 \times 65} = \frac{63}{65}$$

Q1 If co-ordinates of points P and Q are (a, b), (c, d), find the angle subtended by line segment PQ at origin.

Soln ⇒



P(a, b)
Q(c, d)

$OQ = \sqrt{c^2 + d^2}$

$OP = \sqrt{a^2 + b^2}$

$QP = \sqrt{(c-a)^2 + (d-b)^2}$

We know that

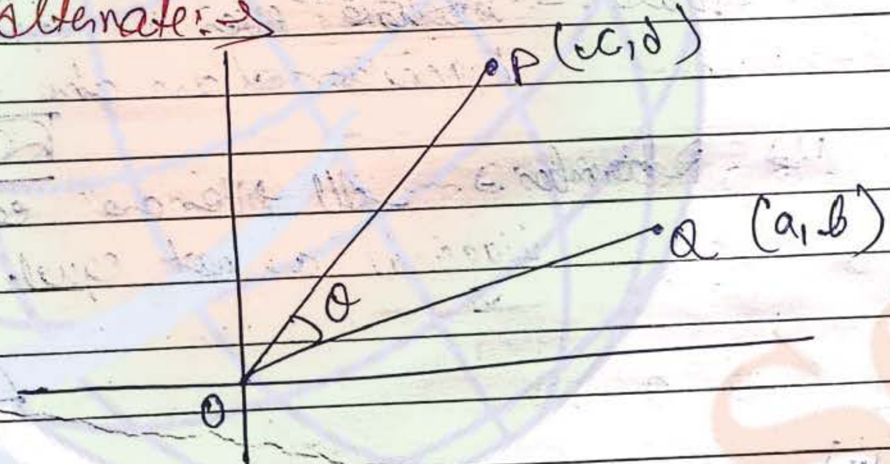
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{OP^2 + OQ^2 - PQ^2}{2OP \cdot OQ}$$

$$\Rightarrow \frac{(a^2+b^2) + (c^2+d^2) - \{(a-c)^2 + (b-d)^2\}}{2\sqrt{a^2+b^2}\ \sqrt{c^2+d^2}}$$

$$\cos\theta = \frac{ac + bd}{\sqrt{a^2+b^2}\sqrt{c^2+d^2}}$$

alternate:→



$$\vec{OP} = a\vec{i} + b\vec{j}$$

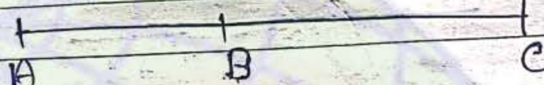
$$OQ = c\vec{i} + d\vec{j}$$

$$\cos\theta = \frac{\vec{OP} \cdot \vec{OQ}}{|\vec{OP}| \cdot |\vec{OQ}|}$$

$$\Rightarrow \frac{ac + bd}{\sqrt{a^2+b^2}\sqrt{c^2+d^2}}$$

Use of distance formula:

i) If three points (A, B, C) are collinear then
$$|AB \pm BC| = AC$$



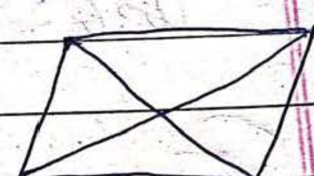
ii) In order to prove that given figure is

(i) Square: → All sides are equal and diagonals are also equal

ii) Rhombus: → All sides are equal but diagonal are not equal.

iii) Rectangle: → Opp sides are equal and diagonals are also equal

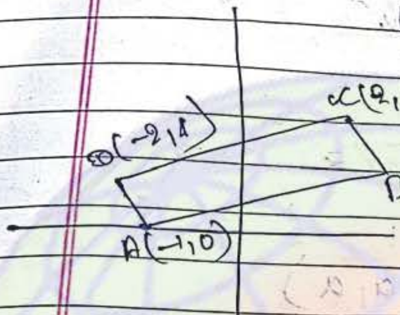
IMP iv) Parallelogram: → Opp. sides are equal but diagonals are not equal

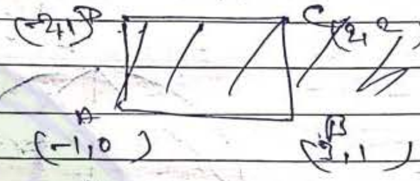


v) Right angle triangle

$$H^2 = P^2 + B^2$$

Q→ Find the nature of quadrilateral forme by $(-1, 0)$, $(3, 1)$ $(2, 2)$, $(-2, 1)$.





$$AB = \sqrt{(-1+3)^2 + (0-1)^2}$$

$$\rightarrow \sqrt{(2)^2 + (1)^2} \rightarrow \sqrt{5}$$

$$BC = \sqrt{(1)^2 + (1)^2} \rightarrow \sqrt{2}$$

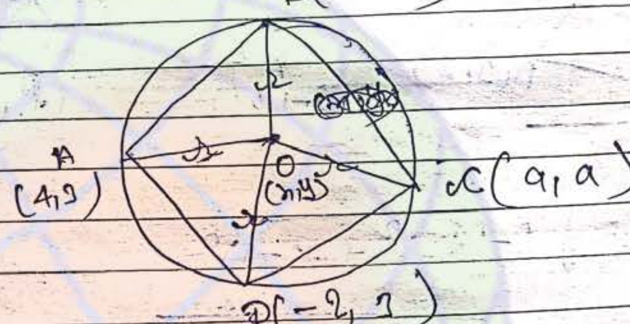
$$CD = \sqrt{(2-2)^2 + (2-1)^2}$$

$$\rightarrow \sqrt{(1)^2} \rightarrow 1$$

$$AD = \sqrt{(-3)^2 + (1)^2}$$

$$\rightarrow \sqrt{9+1} = \sqrt{10}$$

5) a) Find the value of 'a' for which A, B, C, D is a cyclic quadrilateral.



b) If Co-ordinates of P, Q and S are

$P\left(at^2, 2at\right)$

$Q\left(\frac{a}{t^2}, -2at\right)$

$S\left(a, 0\right)$

Then show that $\frac{1}{SP} + \frac{1}{SQ}$ is Independent of 't'.

Solution→

$$OA^2 = OB^2$$

$$(x-4)^2 + (y-3)^2 = (x-6) + (y+1)^2 \quad \text{①}$$

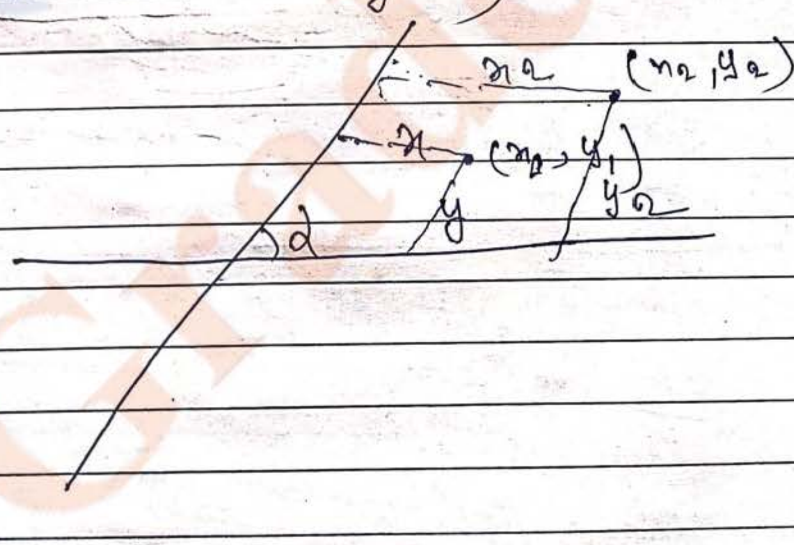
$$OA^2 = OD^2$$

$$(x-4)^2 + (y-3)^2 = (x+2)^2 + (y-3)^2$$

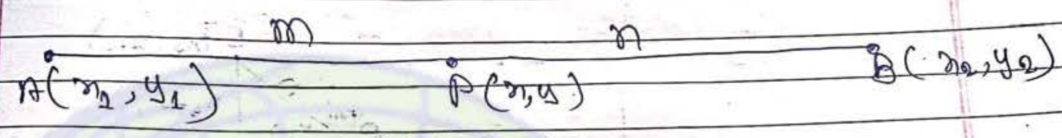
$x = 8$
$y = 1$
$OA^2 = OC^2$

Q.) Find distance between the points (x_1, y_1) and (x_2, y_2) when the co-ordinate axes are at angle "α" (oblique co-ordinate system.)

use cosine formula



Section formula:→



Co-ordinates of the point which divides the line segment joining the points A(x_1, y_1) and B(x_2, y_2) in the ratio $m:n$ internally.

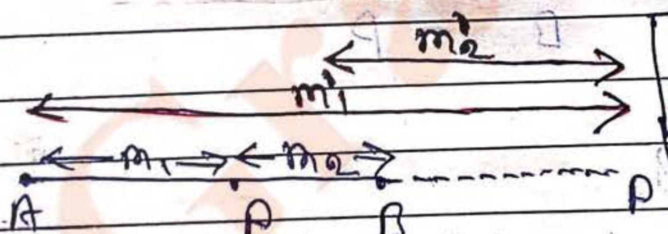
$$P\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Note: याद रखें:→

ii) For External division

$$P\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$$

Note:→



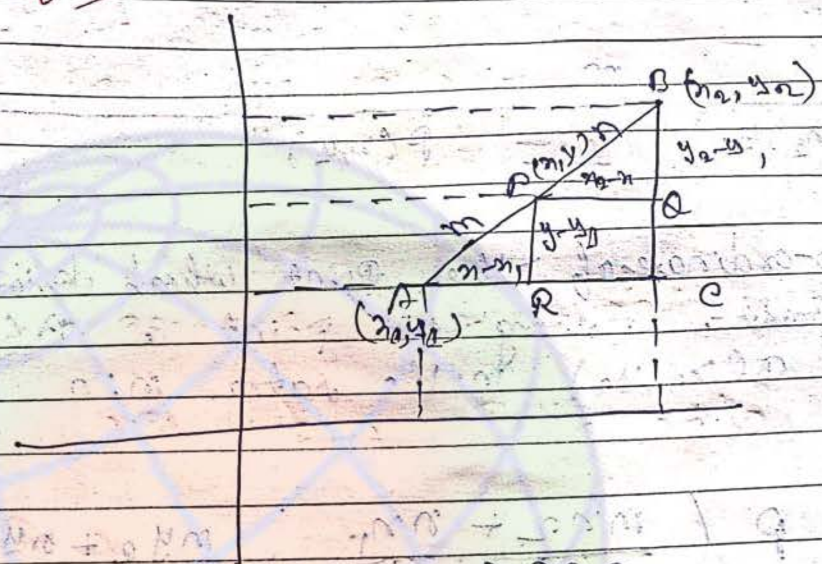
External division

$$\frac{PA}{PB}=\frac{m_1'}{m_2'}$$

Internal division

$$\frac{PA}{PB}=\frac{m_1}{m_2}$$

Proof :→



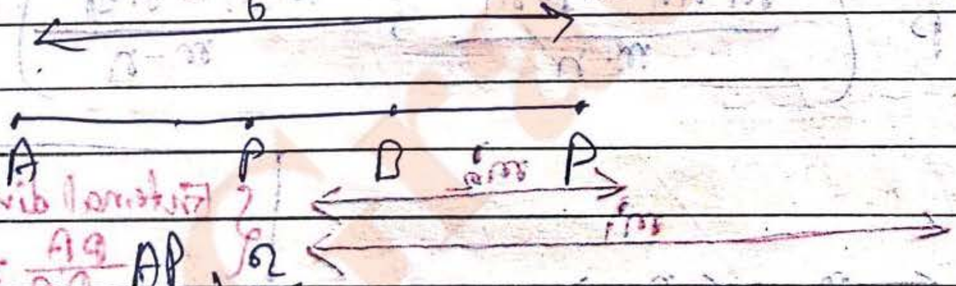
$\Delta APR \sim \Delta PBQ$

$$\frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y} = \frac{m}{n}$$

$$x = \left(\frac{m x_2 + n x_1}{m+n}\right)$$

$$y = \left(\frac{m y_2 + n y_1}{m+n}\right)$$

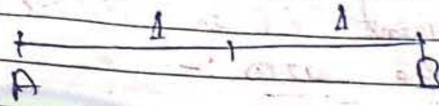
&



$$\frac{AP}{PB} = \frac{2}{1}$$

$$\frac{AP}{PB} = \frac{4}{2} = \frac{2}{1}$$

Co-ordinate of mid-point of line segment AB is



$$P\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

1) If the ratio is negative then the point is corresponding to the external division.

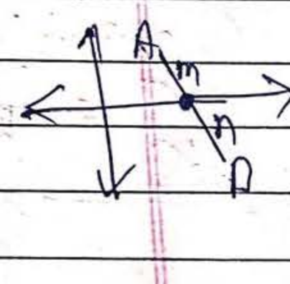
ii) To find the ratio we use $\lambda : 1$, so the co-ordinates of P are :-

$$P\left(\frac{\lambda x_2 + x_1}{\lambda+1}, \frac{\lambda y_2 + y_1}{\lambda+1}\right)$$

☆

(1) If line segment AB is divided by x-axis in the ratio :-

$$\lambda = \left(\frac{-y_1}{y_2}\right)$$



ii) If line segment AB is divided by y-axis ~~the~~ in the ratio :-

" $\lambda = \left(\frac{-x_1}{x_2}\right)$ "

iii) Line segment AB is divided by the line $ax+by+c=0$, In the ratio

$$\Rightarrow -\left(\frac{ax_1+by_1+c}{ax_2+by_2+c}\right)$$

Q) Find the ratio with which line joining the points (1, 2), and (2, −1) is divided –
1) x-axis
ii) y-axis

1) when x-axis

$$\lambda = \frac{-y_1}{y_2} \Rightarrow \frac{+2}{+1} = 2$$

$\Rightarrow$ (2, 0)

ii) y-axis

$$\left(\frac{-x_1}{x_2}\right) \Rightarrow \frac{-1}{2} = \frac{-1}{2}$$

19)

A(2,1) —— $3\sqrt{2}$ —— B(5,4) —— $\sqrt{2}$ —— C(6,5)

$AB = \sqrt{(2-5)^2 + (1-4)^2}$

$\Rightarrow \sqrt{(3)^2 + (3)^2}$

$= \sqrt{9+9}$

$\Rightarrow \sqrt{18}$

$\Rightarrow 3\sqrt{2}$

$3|18$, $3|9$, 3

$BC \Rightarrow \sqrt{(1)^2 + (1)^2}$

$= \sqrt{2}$

$AC = \sqrt{(4)^2 + (4)^2}$

$= \sqrt{16+16}$

$\Rightarrow \sqrt{32}$

$\Rightarrow 4\sqrt{2}$

$2|32$, $2|16$, $2|8$, $2|4$, 2

Here, $AC = AB + BC$

20)

A(6,3) —— 3 —— | —— 2 —— B(-4,5)

$$(0,3) \qquad (4,5)$$

Internally

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$\Rightarrow \frac{-12 + 12}{5} = \frac{0}{5} = 0$$

$$y = \frac{15 + 6}{5} = \frac{21}{5}$$

$$(0, 5)$$

Externally :→

$$x = \frac{-12 - 12}{1} = \frac{-24}{1}$$

$$y = \frac{15 - 6}{1} = \frac{9}{1}$$

$$(-24, 9)$$

$$\left(\frac{-24}{5}, \frac{9}{5}\right)$$

21.)

$$(1, -2) \quad (x_1, y_1) \quad (x_2, y_2) \quad (-3, 4)$$

$$x_1 = \frac{-3 + 2}{3} \Rightarrow \frac{-1}{3}$$

$$y_1 = \frac{4 - 4}{0} = 0$$

$$\left(\frac{-1}{3}, 0\right)$$

$x_2 = \frac{-6+4}{3} = \frac{-5}{3}$

$y_2 = \frac{8+(-2)}{3} = \frac{8-2}{3} = \frac{6}{3} = 2$

$\left(\frac{-5}{3}, 2\right)$

22.)

$\lambda \qquad 4$

$(4,4) \qquad (-1,-1) \qquad (7,7)$

$\cancel{-1 = \frac{7m+4n}{m+n}}$

$\left(\frac{7\lambda+4}{\lambda+4}, \frac{7\lambda+4}{\lambda+4}\right)$

$\cancel{-1 = \frac{4n+7m}{m+n}}$

$\frac{7\lambda+4}{\lambda+4} = -1$

$\lambda = \frac{-5}{8}$ (Indicate Extend division,

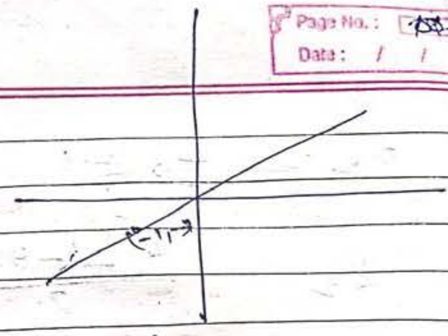
$\Rightarrow 5 : 8$ (Internally division

$\lambda \qquad 4$

$(4,4) \qquad (-1,-2) \qquad (7,7)$

$\left(\frac{7\lambda+4}{\lambda+4}, \frac{7\lambda+4}{\lambda+4}\right)$

$$\frac{7\lambda+4}{\lambda+1} = -1$$



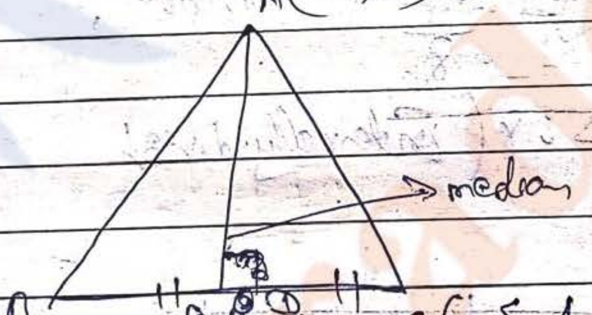
$$\lambda = \frac{-5}{8}$$

Same

$\Rightarrow 5:8$ (externally divide)

$$\frac{7\lambda+4}{\lambda+1} = -2$$

17.) A(-1, 7)

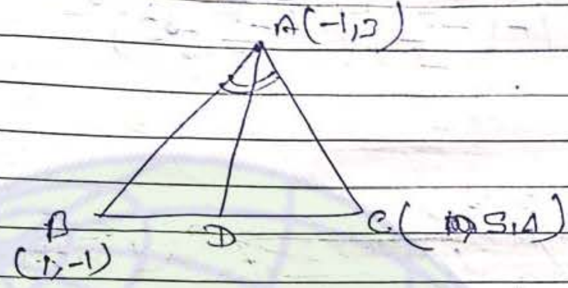


$$x_1 = \frac{1+5}{2} = \frac{16}{2} = 7$$

$$y_1 = \frac{-1+1}{0} = 0$$

so, D (7, 0)

AD = S



Note:→

(i) $\boxed{\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}}$

$AB = \sqrt{(1+1)^2 + (-1-3)^2}$

$\Rightarrow \sqrt{4+16}$

$\Rightarrow \sqrt{20}$

$\Rightarrow 2\sqrt{5}$

2 | 20
2 | 10
5

$AC = \sqrt{36+4}$

$\Rightarrow \sqrt{40}$

$\Rightarrow 2\sqrt{10}$

2 | 40
2 | 20
2 | 10
5

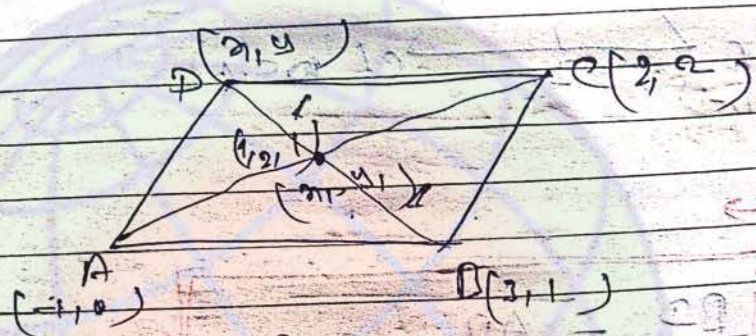
$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC} = \frac{1}{2}$; 1 : 2

$D\left(\frac{5+2}{1+2}, \frac{1-2}{1+2}\right)$

$\Rightarrow D\left(\frac{7}{3}, \frac{-1}{3}\right)$

$$AD = \sqrt{\left(-1-\frac{-7}{3}\right)^2 + \left(\frac{-1}{3} - 3\right)^2}$$

10) Ans



$\frac{x_1+x_2}{2}$

$= \frac{-1+2}{2}$

$= \frac{1}{2}$

$x_1 = \frac{1}{2}$

$y = \frac{2}{2} = 1$

$(0.5, 1)$

$\frac{x+x_2}{2} \Rightarrow \frac{x+3}{2} = \frac{1}{2}$

$x+3 = \frac{2}{2}$

$x = 1-3 \Rightarrow x = -2$

$\frac{1+y}{2} = 1$

$(-2, 1)$

$1+y = 2$

$y = 2-1$

$= 1$

Example sheet :-

Q.)

A B C

$(2,1)$ $B(5,4)$ $(6,5)$

$AB = \sqrt{(5-2)^2+(4-1)^2}$

$\Rightarrow \sqrt{(3)^2+(3)^2}$

$\Rightarrow \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$BC \Rightarrow \sqrt{(6-5)^2+(5-4)^2}$

$\Rightarrow \sqrt{(1)^2+(1)^2} \Rightarrow \sqrt{2}$

$AC \Rightarrow \sqrt{(6-2)^2+(5-1)^2}$

$\Rightarrow \sqrt{(4)^2+(4)^2}$

$\Rightarrow \sqrt{16+16}$

$\Rightarrow 4\sqrt{2}$

$AB+BC \Rightarrow 3\sqrt{2}+\sqrt{2} \Rightarrow 4\sqrt{2}$

So $AB+BC = AC$ (It's collinear)

10)
IV.

A ——— K ——— P ——— L ——— B

A $(1,3)$ P $(3,5)$ B $(7,9)$

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{K x_2 + x_1}{K+1}$$

$$3 = \frac{7K + 1}{K+1}$$

$$3(K+1) = 7K+1$$

$$3K + 3 = 7K + 1$$

$$3K - 7K = 1 - 3$$

$$\cancel{-}4K = \cancel{-}2$$

$$K = \frac{2}{4} \Rightarrow 1:2 \quad \text{Ans}$$

But

$$5 = \frac{9K+3}{K+1}$$

$$\Rightarrow 5K + 5 = 9K + 3$$

$$\Rightarrow 5K - 9K = 3 - 5$$

$$\cancel{-}4K = \cancel{-}2$$

$$K = \frac{1}{2} \Rightarrow 1:2$$

दोनों case में same ही answer आ रहा है कोई किसी भी से ratio निकाले

12) (i) Divided by x-axis $(x, 0)$

K : 1

$(1, 2)$ — $(x, 0)$ — $(2, -1)$

$0 = \frac{2k+1}{k+1}$ $\qquad$ $0 = \frac{-k+2}{k+1}$

$2k+1 = 2k+1$ $\qquad$ $\cancel{k+1} = -k+2$

$k - 2k = 1 - 1$ $\qquad$ $+k = 2$

$-k = 0$ $\qquad$ $k = 2$

$k = 0$ $\qquad$ So, ratio → $2:1$

For another

$x = \frac{2k+1}{k+1}$

(ii) Divided by y-axis

$(0, y)$

K : 1

$A(1, 2)$ — $(0, y)$ — $(2, -1)$

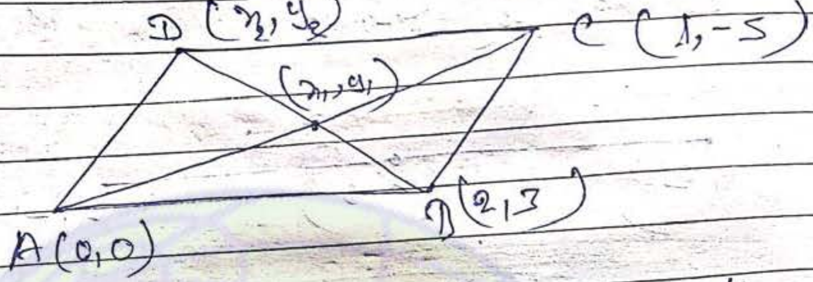
~~for for x~~ for x

$0 = \frac{2k+1}{k+1}$

$2k = -1$

$k = \frac{-1}{2}$

14. (a)



From line segment: → "AC"

$$x = \frac{0+1}{2} = \frac{1}{2}$$

$$y = \frac{0-5}{2} = \frac{-5}{2}$$

Now: →

$$x_1 = \frac{2+x_2}{2}$$

$$\frac{1}{\cancel{2}} = \frac{2+x_2}{\cancel{2}}$$

$$x_2 = -2$$

Again

$$\frac{-5}{\cancel{2}} = \frac{3+y_2}{\cancel{2}}$$

So fourth vertex (-2, -8)

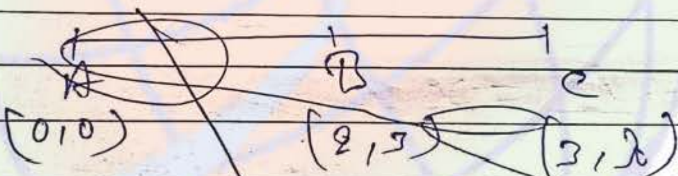
$$-5-3 = y_2$$

$$-8 = y_2$$

$$y_2 = -8$$

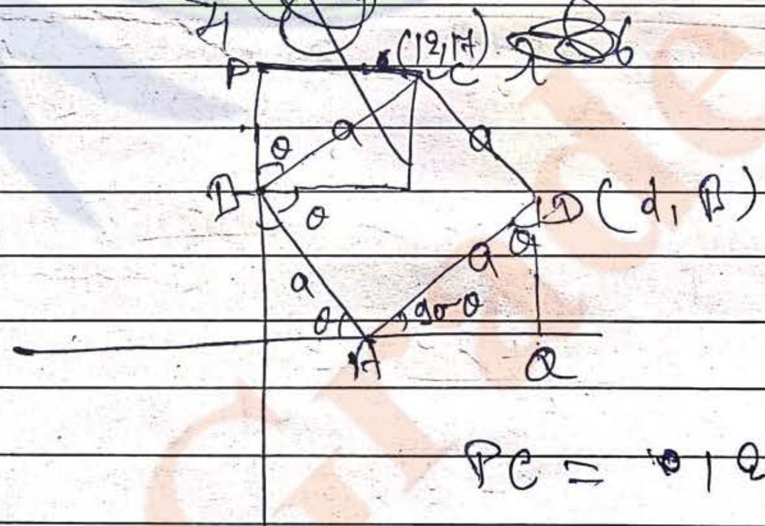
(6)

(9)



$2 = \frac{0+x}{2}$ $\quad \frac{\lambda+0}{2} = 3$

10.)



$PC = a \mid Q = a \sin\theta$ (1)

$y = OB + BP$

$17 = a\sin\theta + a\cos\theta$ — (2)

$d = OQ = OA + AQ$
$= a\cos\theta + a\sin\theta$

$d = 17$

$P = a\cos\theta = 5$

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Date:

Area of Triangle

Area of a triangle with vertex at $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by

$$\text{Area of } (\Delta ABC) = A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

modulus

$$= \frac{1}{2}\left[x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2)\right]$$

$$A = \left| \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] \right|$$

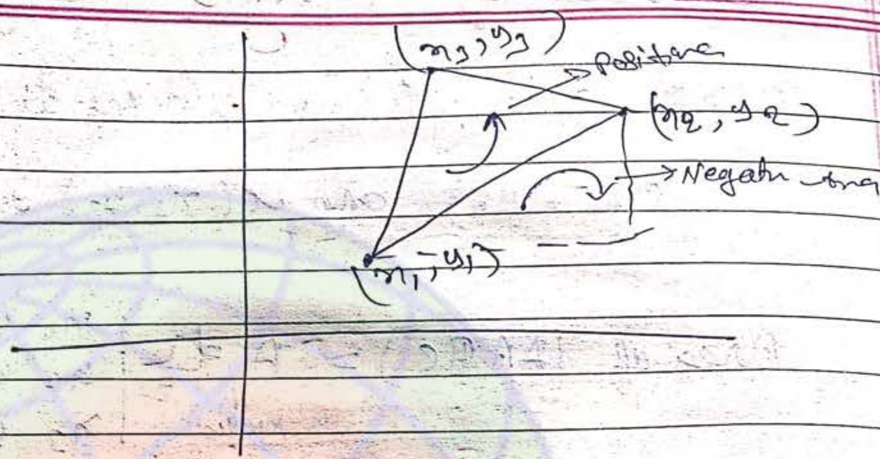
Note:→

(1) If Area of ΔABC are zero, then points A, B, C are collinear.

So,

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

(2) Area obtained will be positive if the points are taken in anticlockwise sense.

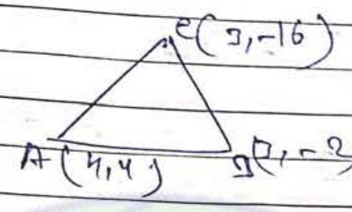


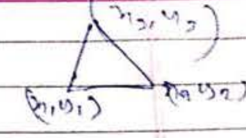
Determinant :→

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} =$$

$$a_1(b_2c_3 - c_2b_3) - a_2(b_1c_3 - c_1b_3) + a_3(b_1c_2 - c_1b_2)$$

25)





Ar Δ ABC $= \left| \frac{1}{2}\left(4(-2+16) + 3(-16-4) + 3(4+2)\right) \right|$

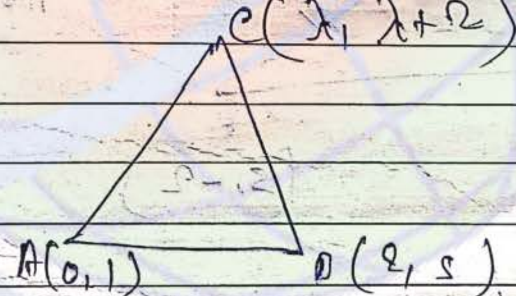
$\Rightarrow \left| \frac{1}{2}\left(4 \times 14 + 3 \times -20 + 3 \times 8\right) \right|$

$\Rightarrow \frac{1}{2} \times (56 - 60 + 24)$

$$\begin{array}{r} 24 \\ 32 \\ \hline 56 \end{array}$$

$\Rightarrow \frac{1}{2} \times \cancel{14} \Rightarrow 7$

27)



$\Rightarrow \left| \frac{1}{2} \begin{vmatrix} 0 & 1 & 1 \\ 2 & 5 & 1 \\ \lambda & \lambda+2 & 1 \end{vmatrix} \right| = 3$

$\Rightarrow \frac{1}{2} \left| 0 + 2((\lambda+2)-1) + \lambda(1-5) \right|$

$\Rightarrow \frac{1}{2} \left| 2\lambda + 2 + \lambda - 5\lambda \right|$

$\Rightarrow \left| 3\lambda - 5\lambda + 2 \right|$

$\Rightarrow \left| -2\lambda + 2 \right|$

$\Rightarrow \left| -\lambda + 2 \right|$

$$\left|\frac{1}{2}(2+2\lambda-4\lambda)\right| = 3$$

$$|1-\lambda| = 3$$

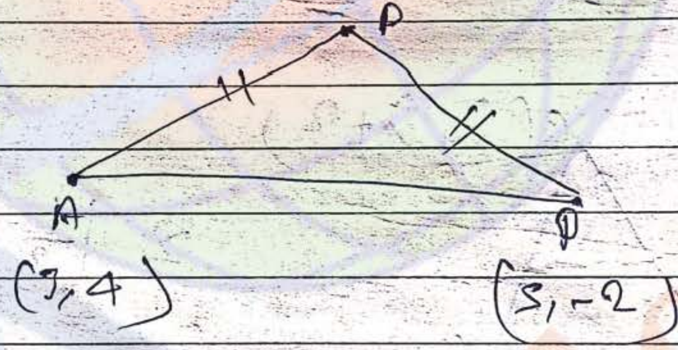
$$1-\lambda = \pm 3$$

$$1-\lambda = 3, -3$$

$$\boxed{\lambda = -2, 4}$$

26.

1.



As PA = PB (isosceles)

$$\left|\frac{1}{2}\left(2+2\lambda-4\lambda\right)\right| = 3$$

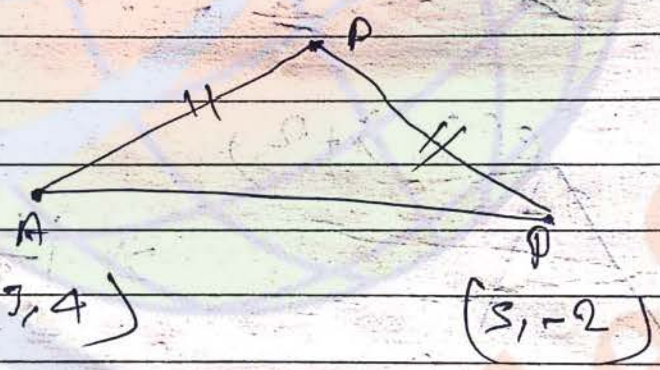
$$|1-\lambda| = 3$$

$$1-\lambda = \pm 3$$

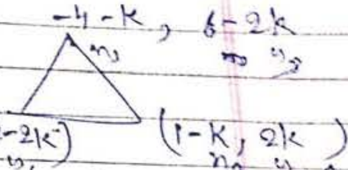
$$1-\lambda = 3, -3$$

$$\boxed{\lambda = -2, 4}$$

1.)



Ar PAB = 10 sq unit



26) Ar $\Delta ABC = \frac{1}{2}\left| k\left(2k-(6-2k)\right) + 1-k\left(6-2k-(2-2k)\right) + -4-k\left(2-2k-(2k)\right)\right| = 0$

$\Rightarrow \frac{1}{2}\left| k\left(\cancel{2k}-6+\cancel{2k}\right) + 1-k\left(6-\cancel{2k}-2+\cancel{2k}\right) + -4-k\left(2-2k-2k\right)\right|$

$\Rightarrow \frac{1}{2}\left| k(-6) + 1-k(8) + (-4-k)(2-4k)\right|$

$\Rightarrow \frac{1}{2}\left| -6k + 8 - 8k + (-8 + 16k - 2k + 8k^2)\right|$

$\Rightarrow \frac{1}{2}\left| 8 - 2k + 8 + 22k\right|$

$\Rightarrow \frac{1}{2}\left| \cancel{16} + \cancel{20}k\right|$ (8, 10)

$\Rightarrow 8 + 10k = 0$

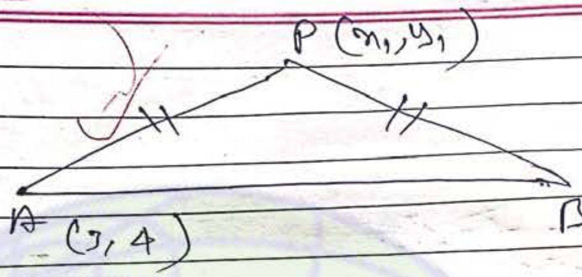
$10k = -8$

$k = -\frac{8}{10}$

$k = -\frac{4}{5}$

Ans $k = -1, \frac{1}{2}$

21)



Ar $\Delta PAB = 10$ sq unit

29.) m_1 and m_2 are root of

$$px^2 + 2qx + 1 = 0$$

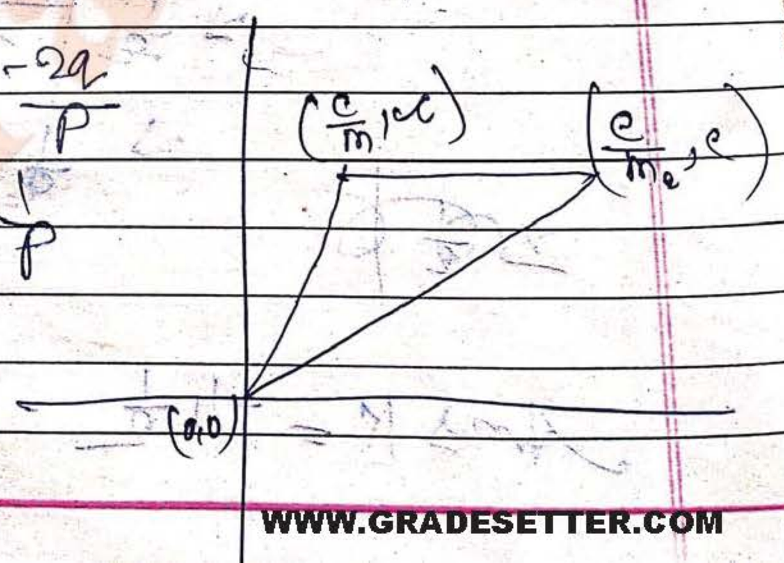
$$(0,0),\ \left(\frac{c}{m_1}, c\right),\ \left(\frac{c}{m_2}, c\right)$$

$$\boxed{c^2\sqrt{q^2 - p}}$$

$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \frac{c}{m_1} & c & 1 \\ \frac{c}{m_2} & c & 1 \end{vmatrix}$$

$$m_1 + m_2 = \frac{-2q}{p}$$

$$m_1 m_2 = \frac{1}{p}$$

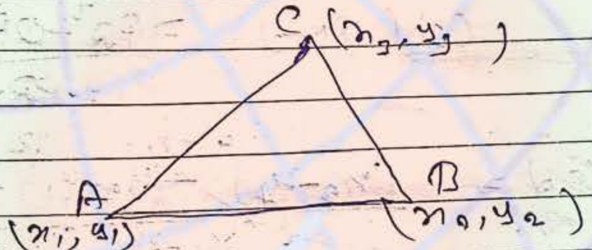


Area of triangle ⇒ "Δ"

$\Rightarrow \frac{1}{2} \times \text{base} \times \text{height}$

$\Rightarrow \frac{1}{2} \times c \times \left| \frac{c}{m_2} - \frac{c}{m_1} \right|$

$\Rightarrow \frac{1}{2} c^2 \left| \frac{\sqrt{(m_1+m_2)^2 - 4m_1m_2}}{m_1 m_2} \right|$



Let $x_i, y_i \in Q$

Ar $\Delta ABC\ (\Delta) = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$

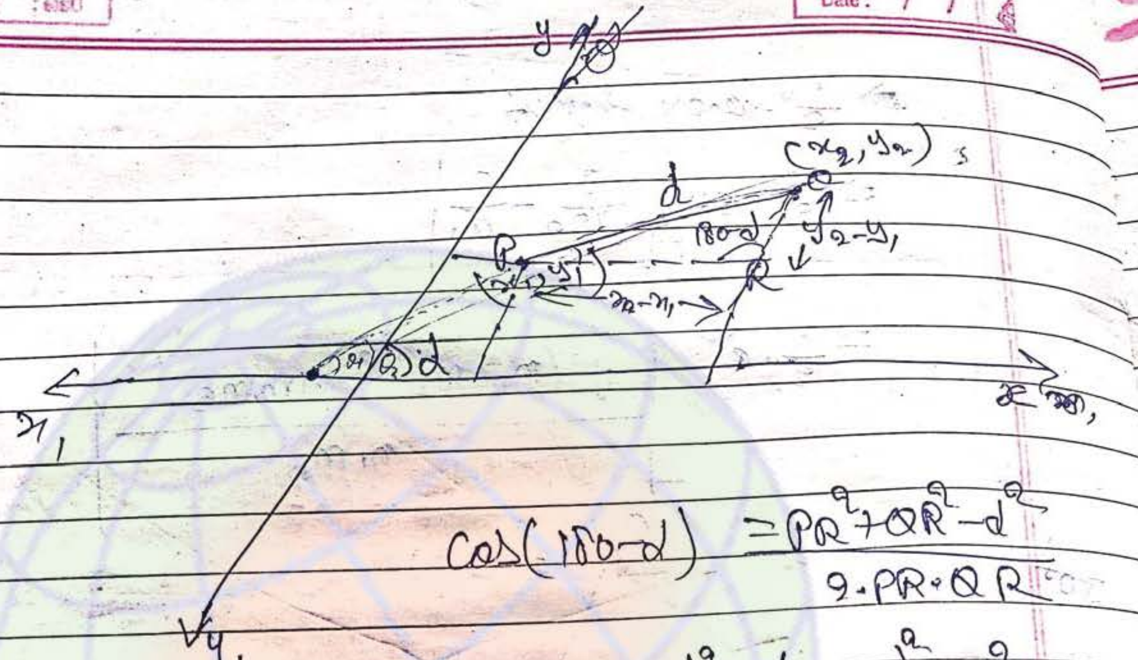
LHS $\Delta \in Q$ (Ration)

Also,

$\Delta = \frac{\sqrt{3}}{4} a^2$

$= \frac{\sqrt{3}}{4} \left[(x_1 - x_2)^2 + (y_1 - y_2)^2 \right]$

$\Delta \notin Q$ Irration



$$\cos(180-\alpha) = \frac{PR^2 + QR^2 - d^2}{2 \cdot PR \cdot QR}$$

$$-\cos\alpha = \frac{(x_2-x_1)^2 + (y_2-y_1)^2 - d^2}{2(x_2-x_1)(y_2-y_1)}$$

$$d^2 = (x_2-x_1)^2 + (y_2-y_1)^2 + 2(x_2-x_1)(y_2-y_1)\cos\alpha$$

$$\boxed{d(PQ) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + 2(x_2-x_1)(y_2-y_1)\cos\alpha}}$$

V-1

Area of Polygon

Area of Polygon with vertices

$(x_1,y_1), (x_2,y_2), (x_3,y_3) \ldots\ldots (x_n,y_n)$

(taken in order)

Then the area is given by

x-axis के point ऊपर

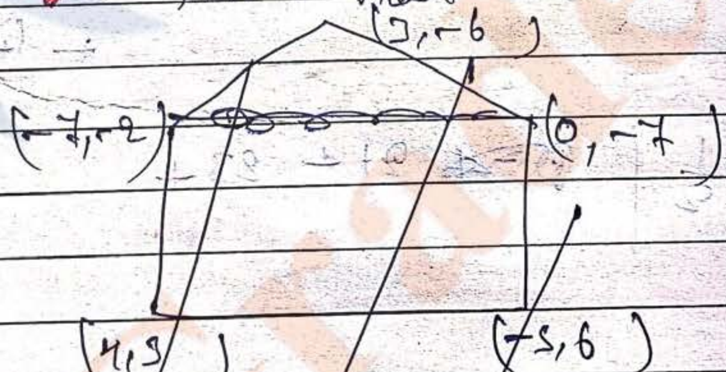
$$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & \cdots & x_n & x_1 \\ y_1 & y_2 & y_3 & y_4 & \cdots & y_n & y_1 \end{vmatrix}$$

y-axis के point नीचे

$$\boxed{\frac{1}{2}\left[(x_1y_2 - y_1x_2) + (x_2y_3 - y_2x_3) + \cdots + (x_ny_1 - y_nx_1)\right]}$$

Note: ⇒ Points must be taken in order.

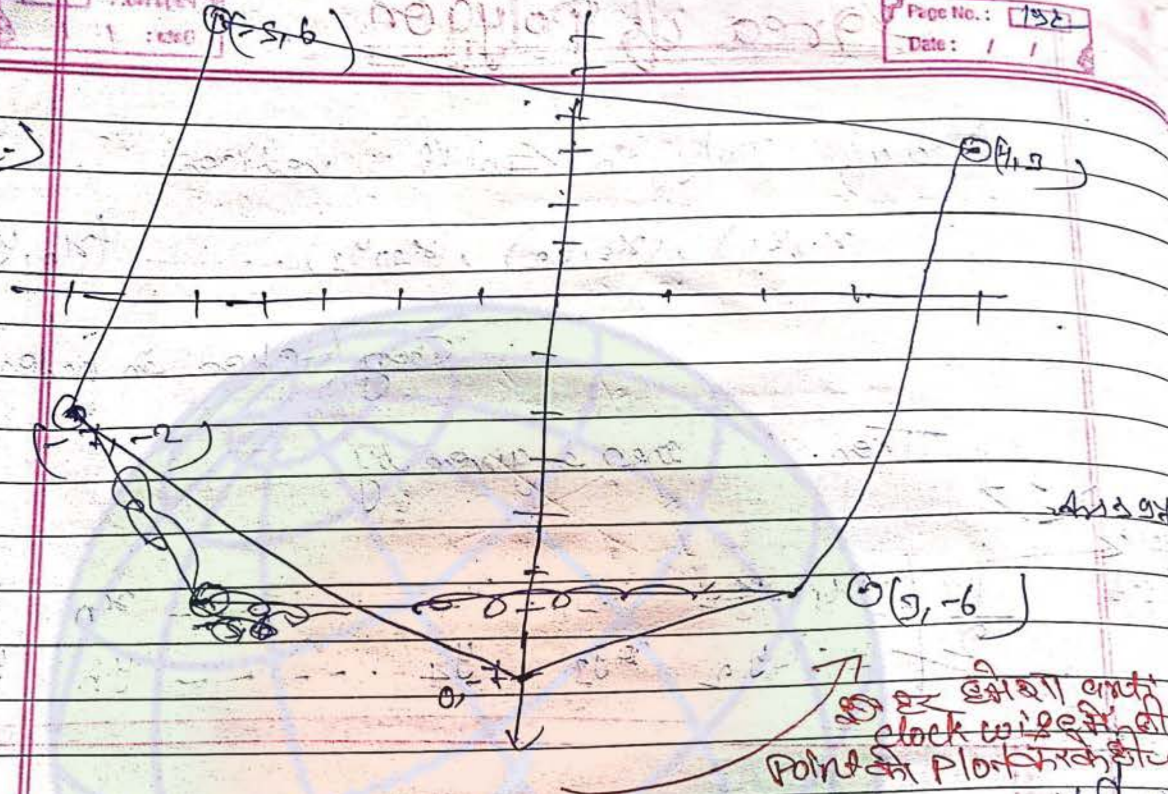
Q.)



$$\text{Ar polygon} = \frac{1}{2}\begin{vmatrix} 4 & -5 & 0 & 3 & -7 \\ 3 & 6 & -7 & -6 & -2 \end{vmatrix}$$

$$= \frac{1}{2}\left|(24+15) + (+35+0) + (0-21) + (-6-42)\right|$$

Area of polygon



हर हमेशा वर्तो clock wise में ही Point को Plot करके ही Area निकालें, अन्यथा गलत हो जाएगा (→ again)

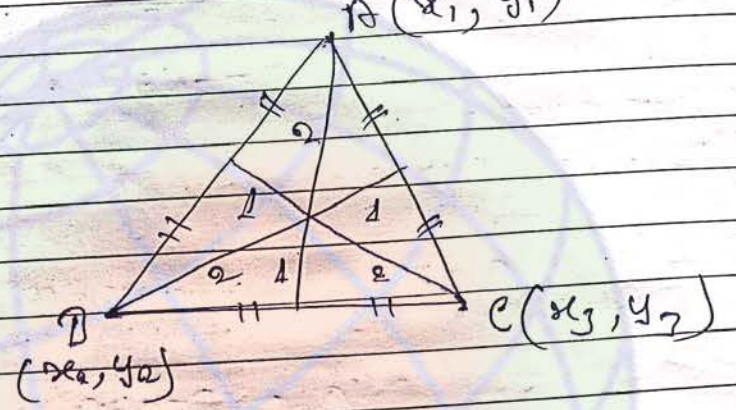
$$Ar \Rightarrow \frac{1}{2}\begin{vmatrix} 0 & 5 & 4 & -5 & -7 & 0 \\ -7 & -6 & 3 & 6 & -2 & 7 \end{vmatrix}$$

$$Ar \Rightarrow \frac{1}{2}\left|(-0+21)+(9+24)+(24+30)+(10+42)+(+0-49)\right|$$

$$\Rightarrow \frac{1}{2}\left|\ 21+23+\right.$$

Centres of triangle

i) Centroid (G) :⇒ It is the point of Intersection of medians of a triangle

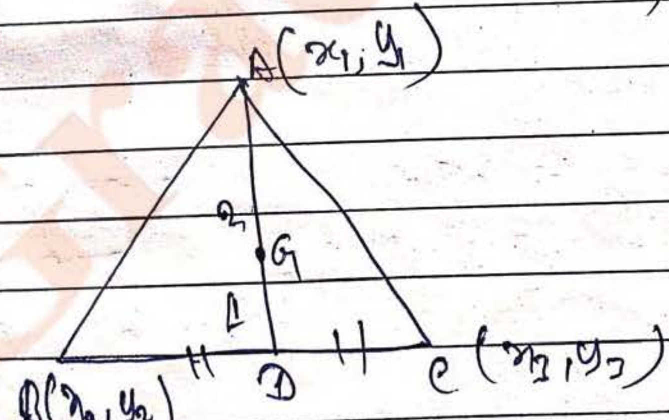


* Centroid Divides the median in 2 : 1

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle then the co-ordinates of its centroid are

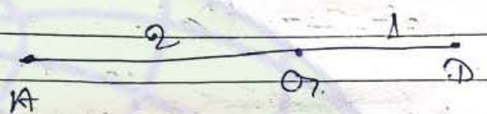
$$G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Proof :⇒



Co-ordinate of D:→

$$\Rightarrow \left(\frac{x_2+x_3}{2} , \frac{y_2+y_3}{2} \right)$$



$$G = \left(\frac{2\left(\frac{x_2+x_3}{2}\right)+x_1}{2+1} , \frac{2\left(\frac{y_2+y_3}{2}\right)+y_1}{2+1} \right)$$

$$\Rightarrow \left(\frac{x_1+x_2+x_3}{3} , \frac{y_1+y_2+y_3}{3} \right)$$

Example: vertices of a triangle are $(1, a)$, $(2, b)$, $(c^2, -3)$ where $a, b, c \in R$

then show that

i) Centroid cannot lie on y-axis

ii) Find the condition such that centroid lies on x-axis

Sol→

$$x = \frac{1+2+c^2}{3} , y = \frac{a+b-3}{3}$$

$$\Rightarrow \frac{3+c^2}{3} , y = \frac{a+b-3}{3}$$

$$\Rightarrow \left(\frac{3+c^2}{3} , \frac{a+b-3}{3} \right)$$

i) On y-axis $x=0$

$$\frac{7+c^2}{3}=0$$

$$c^2=-7 \quad (\times)$$

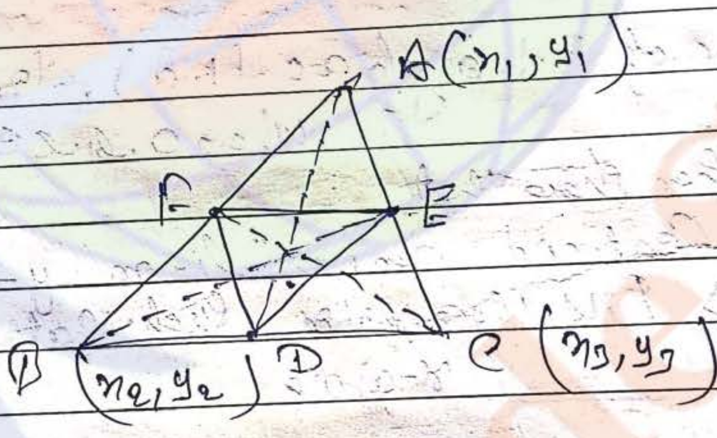
So centroid can't be on x-axis

ii) On x-axis $y=0$

$$\frac{a+b-7}{3}=0$$

$$a+b=7$$

2.)



D, E, and F are mid-point. Show that centroid of triang ABC and DEF are same (co-incide)

Soln

$$D = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$

$$E = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$$

$$F = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

centroid of $\triangle DEF$

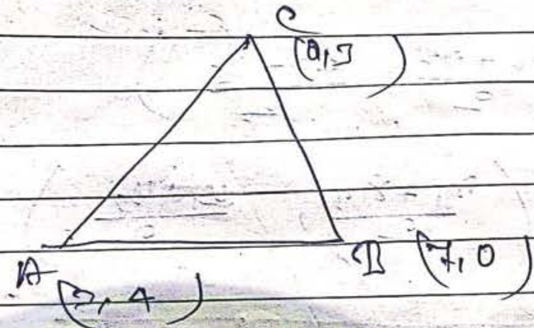
$$x \Rightarrow \left(\frac{\frac{x_2+x_3}{2} + \frac{x_1+x_3}{2} + \frac{x_1+x_2}{2}}{3}\right)$$

$$\Rightarrow \left(\frac{x_1+x_2+x_3}{3}\right)$$

Now

$y =$

32.



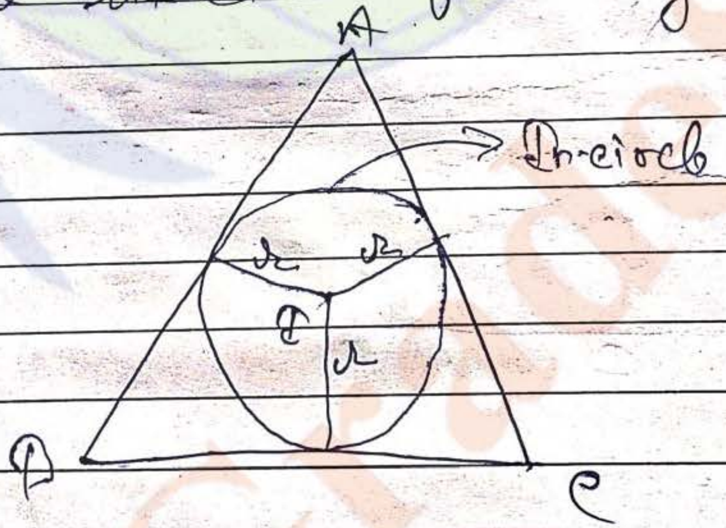
centre :→

$\frac{3+7+0}{3} = \frac{10}{3}$

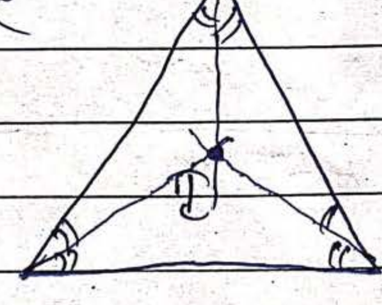
$\frac{4+0+3}{3} = \frac{7}{3}$

Incentre :→

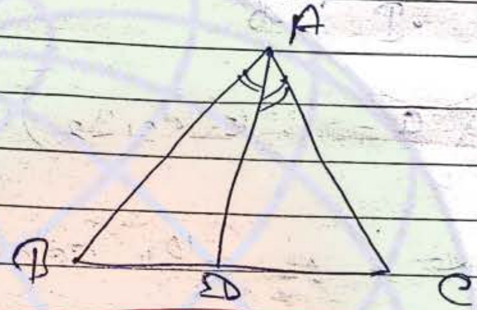
It is the point of Intersection Internal angle bisectors of a triangle



I → Incentre.

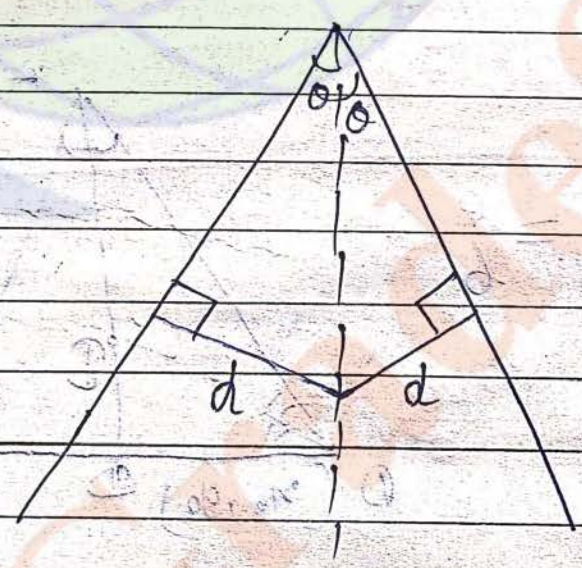


(i) Internal angle bisector divides the opposite side in the ratio of remaining side.

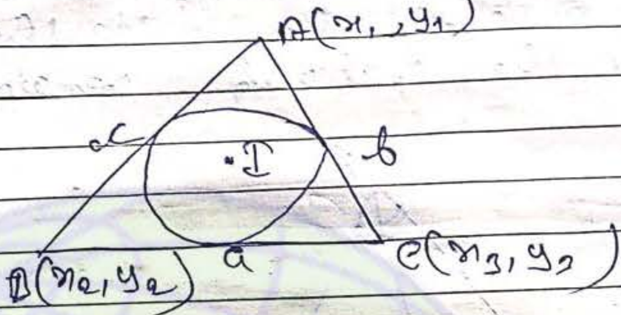


$$\boxed{\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}}$$

(ii) Any point on the angle bisector is equidistant from the sides.



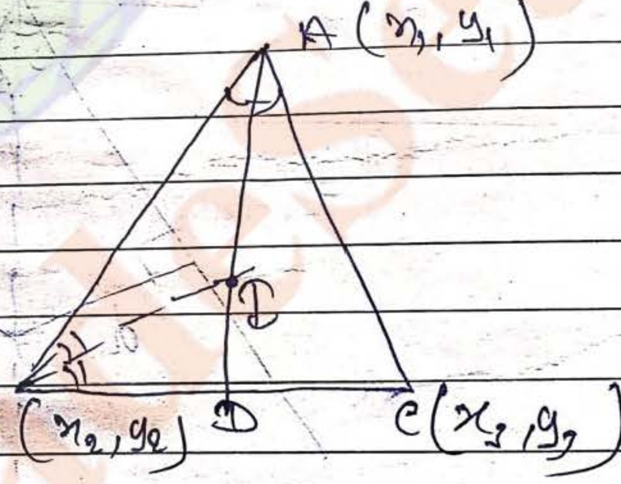
Co-ordinates :→



Co-ordinates of the centre are

$$\boxed{I\left(\frac{ax_1+bx_2+cx_3}{a+b+c},\ \frac{ay_1+by_2+cy_3}{a+b+c}\right)}$$

Proof :→



$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$$

$$D\left(\frac{cx_3+bx_2}{c+b},\ \frac{cy_3+by_2}{c+b}\right)$$

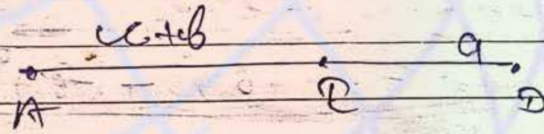
ΔABD

$$\frac{AI}{ID} = \frac{AB}{BD}$$

$$= \frac{c}{\left(\frac{c}{c+b} \times a\right)}$$

$$\frac{AI}{ID} = \frac{c+b}{a}$$

To



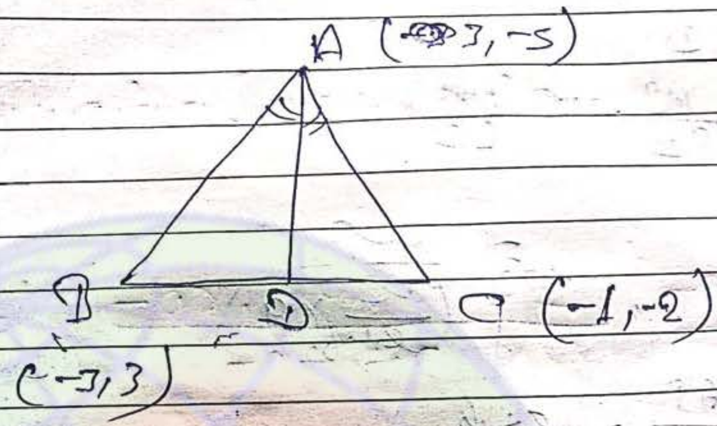
$$\Rightarrow \left(\frac{(c+b)\left(\frac{cx_3+bx_2}{c+b}\right)+ax_1}{c+b+a}, \frac{(c+b)\left(\frac{cy_3+by_2}{c+b}\right)+ay_1}{c+b+a} \right)$$

$$\Rightarrow \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c} \right)$$

Examples

Page No.: 90

Q-36.)



$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$AB = \frac{\sqrt{(6)^2 + (-8)^2}}{\sqrt{(4)^2 + (-3)^2}} = \frac{10}{5} = \frac{2}{1}$$

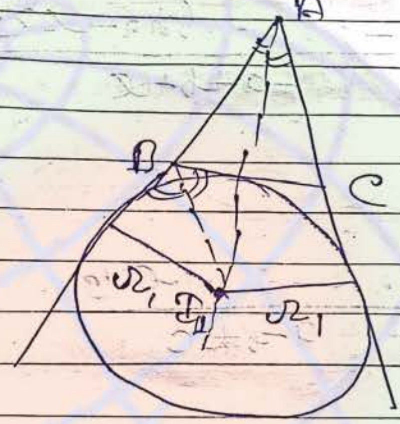
$$D\left(\frac{-5}{3}, \frac{-1}{3}\right)$$

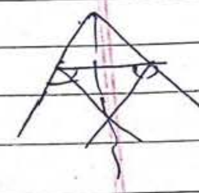
$$AD = \sqrt{\left(3+\frac{5}{3}\right)^2 + \left(-5+\frac{1}{3}\right)^2}$$

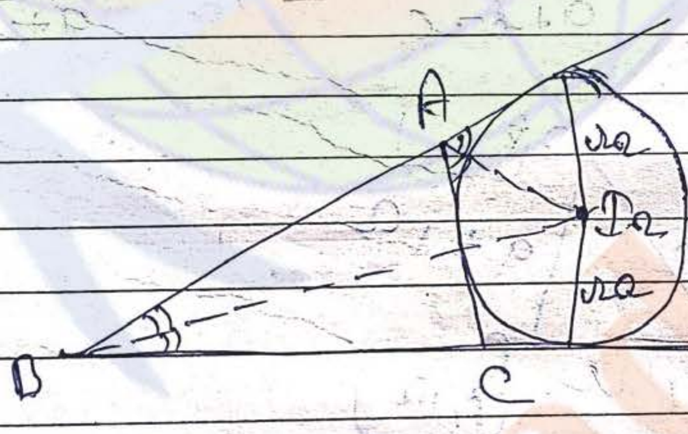
$$= \frac{14}{3}\sqrt{2}$$

Ex-Centres: →

The Point of Intersection of one Internal angle bisector and one external angle bisector of a triangle is called Ex-centre.

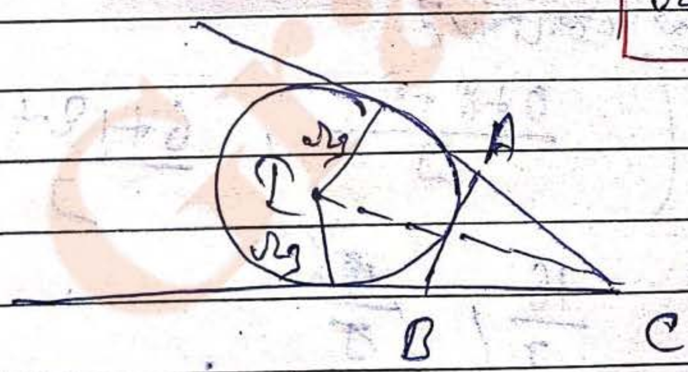






$r \to$ In-radius

r_1, r_2, r_3 :- Ex-radii



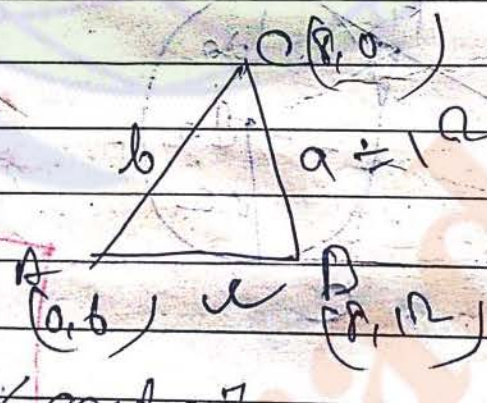
Co-ordinate of ex-centre.

$$I_1 = \left(\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c} \right)$$

$$I_2 = \left(\frac{ax_1-bx_2+cx_3}{a-b+c}, \frac{ay_1-by_2+cy_3}{a-b+c} \right)$$

$$I_2 = \left(\frac{ax_1+bx_2-cx_3}{a+b-c}, \frac{ay_1+by_2-cy_3}{a+b-c} \right)$$

4)



1) centroid

$$C\left(\frac{0+8+8}{3}, \frac{6+12+0}{3} \right)$$

$$\left(\frac{16}{3}, \cancel{\frac{18}{3}}\ 6 \right)$$

$$\Rightarrow \left(\frac{16}{3}, 6 \right)$$

(4)

In centre:

$$a = BC = \sqrt{(0)^2 + (12)^2}$$
$$= 12$$

$$\Rightarrow \frac{ax_1 + bx_2 + cx_3}{a+b+c}$$

$$\Rightarrow b = AC = \sqrt{(8)^2 + (6)^2}$$
$$= \sqrt{64+36}$$
$$= 10$$

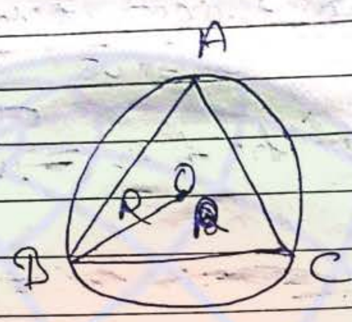
$$c = \sqrt{(64) + (6)^2}$$
$$= 10$$

$$I \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

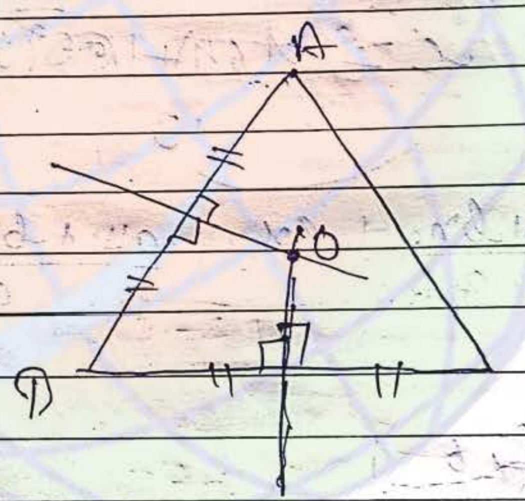
$$= (3, 6)$$

Circum Centre: →

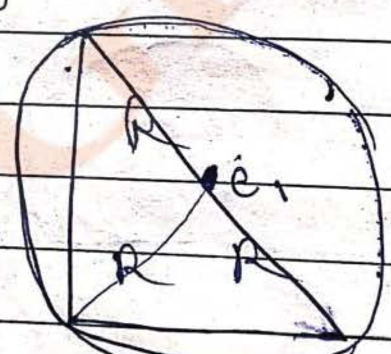
It is the point of Intersection of perpendicular bisector of a triangle.



radius
'R' की सहायता से point पहचानिए

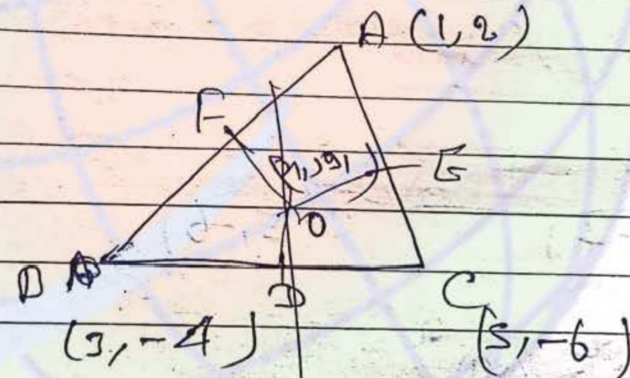


Note: →
Mid-point of hypotenuse is equidistant from all the vertices. So it is the circumcentre of right angle triangle.



Proof :-

44.)



$F = \left(\frac{1+3}{2}, \frac{2-4}{2}\right)$, $E = \left(\frac{6}{2}, \frac{-4}{2}\right)$

$\Rightarrow (2, -1)$ $\quad = (3, -2)$

$D = \left(\frac{8}{2}, \frac{-10}{2}\right)$

$= (4, -5)$

$FO = \sqrt{(x_1 - 2)^2 + (y_1 + 1)^2}$

$\Rightarrow \sqrt{(x_1)^2 + 4 - 4x + y^2 + 1 + 2y}$

(x, y) $(3, -2)$

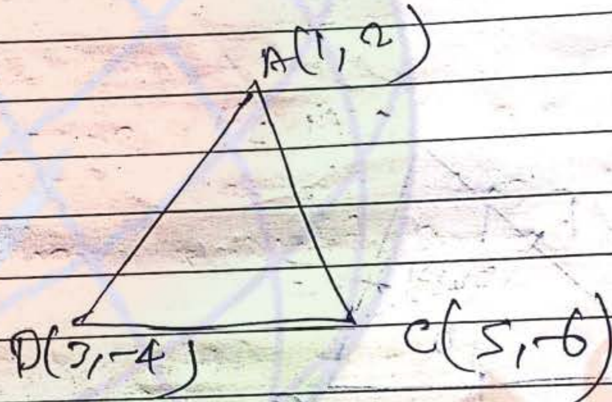
$EO = (x-3)^2 + (y+2)^2$

$(x-3)^2 + (y+2)^2 = (x, -2)^2 + (y+1)^2$

$\cancel{x^2} + 9 - 6x + \cancel{y^2} + 4 + 4y = \cancel{x^2} + 4 - 24x + \cancel{y^2} + 1 + 2y$

$13 - 6x + 4y =$

9.)



$OA^2 = OD^2$

$(x-1)^2 + (y-2)^2 = (x-3)$

Q. If α, β, γ are roots of equation $x^3 - 3px^2 + 3qx - 1 = 0$ then find the centroid of triangle with vertices

$$\left(\alpha, \frac{1}{\alpha}\right), \left(\beta, \frac{1}{\beta}\right), \left(\gamma, \frac{1}{\gamma}\right)$$

Soln

$$G\left(\frac{\alpha+\beta+\gamma}{3}, \frac{\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}}{3}\right)$$

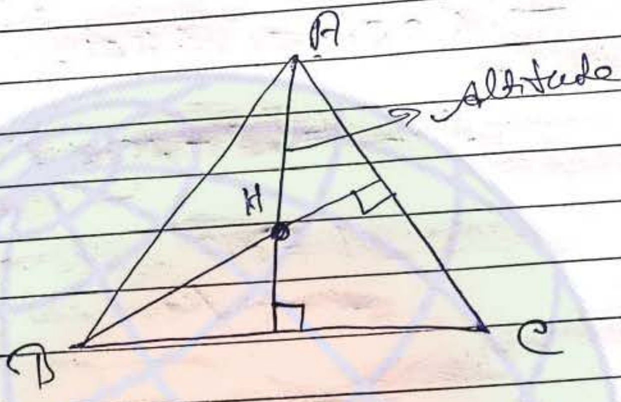
$$G\left(\frac{3p}{3}, \frac{\alpha\beta+\beta\gamma+\gamma\alpha}{3\alpha\beta\gamma}\right)$$

$$G\left(p, \frac{3q}{3}\right)$$

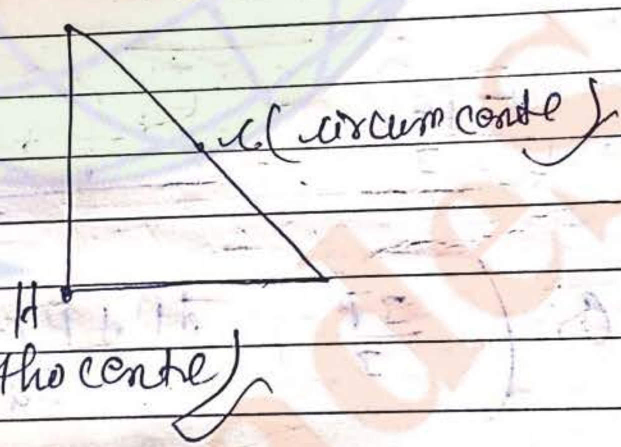
$$G(p, q)$$

Ortho centre (H) →

It is the point of Intersection of Altitudes of a triangle.



Note: →
orthocentre centre of a Right angle triangle lies at the vertex where where the right angle is formed.

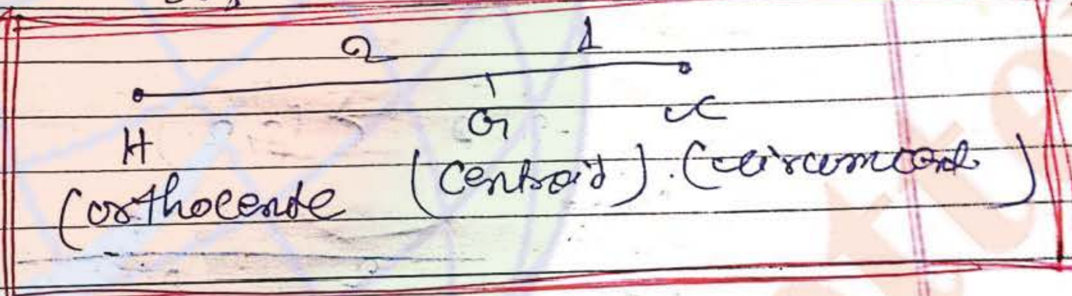


Important points related to Points

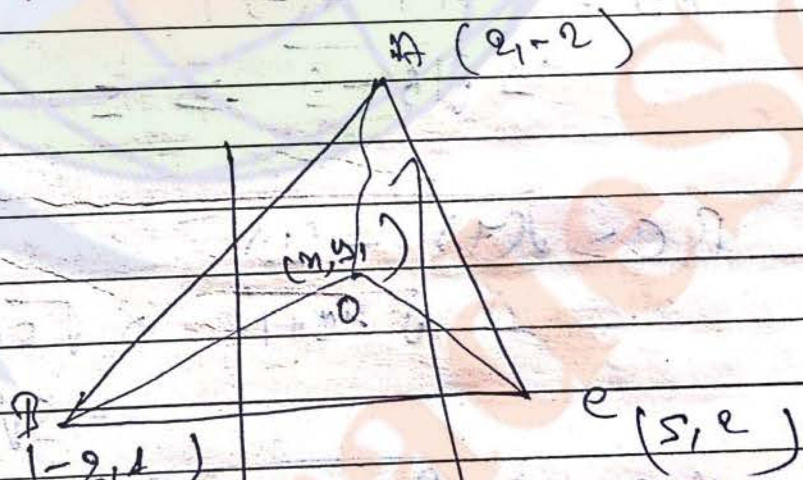
1.) In an equilateral triangle centroid, In-centre, circumcentre, and orthocentre all coincide at the same point,

2.) Euler line :→

In any triangle ~~centr~~ orthocentre, centroid, and circumcentre are always collinear and centroid divides the line joining orthocentre and circumcentre in 2 : 1



Q.7.)



$$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$\Rightarrow \left(\frac{\cancel{-2}+5+\cancel{2}}{3}, \frac{1+\cancel{2}+\cancel{-2}}{3}\right)$$

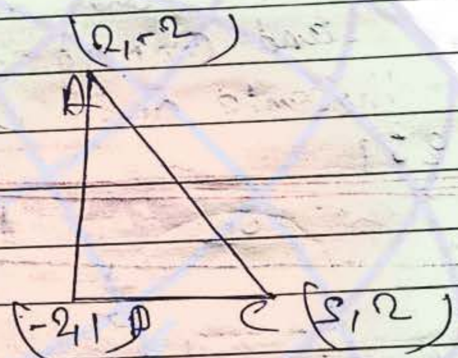
$\left(\frac{5}{3}, \frac{1}{3}\right)$

$C = (OA)^2 = (OC)^2$

$(x_1 - 2)^2 + y_1 + 2$

$(2, -2)$

17)



$AB = \sqrt{(4)^2 + (-3)^2}$

$\Rightarrow \sqrt{16 + 9} \Rightarrow 5$

$BC = \sqrt{(7)^2 + (1)^2}$

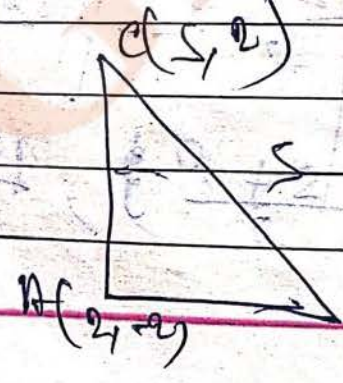
$\Rightarrow \sqrt{49 + 1} = \sqrt{50}$

$= 5\sqrt{2}$

$AC = \sqrt{(-3)^2 + (+4)^2}$

$\Rightarrow 5$

So

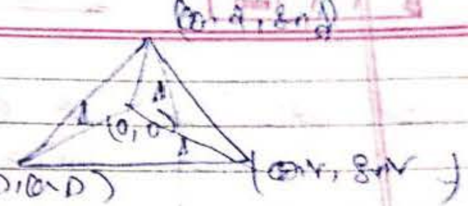


$H = (2, -2)$

$C\left(\frac{7}{2}, \frac{3}{2}\right)$

213

10.)



$AB = \sqrt{\cos^2\alpha + \sin^2\alpha}$ $(\cos\alpha, \sin\alpha)$

$= 1$

$AB = BC = AC$

G of $\triangle$ $= \left(\frac{\cos\alpha + \cos\beta + \cos\gamma}{3}, \frac{\sin\alpha + \sin\beta + \sin\gamma}{3}\right)$

$\overset{2}{H \text{———} G} \overset{1}{\text{———}} C(0,0)$

$G = \left(\frac{0+x}{3}, \frac{0+y}{3}\right)$

$G = \left(\frac{x}{3}, \frac{y}{3}\right)$

So, $\frac{x}{3} = \frac{\Sigma \cos\alpha}{3}$

$x = \Sigma \cos\alpha$

$y = \Sigma \sin\alpha$

$H = (\Sigma \cos\alpha, \Sigma \sin\alpha)$

Locus

$\sqrt{h^2+k^2} = 1$

It is ~~so~~ the curve, traced by point which moves under given geometrical condition

For example:→
Locus of a point whose distance from origin is always 1.
Is a circle with centre at origin and radius "1."

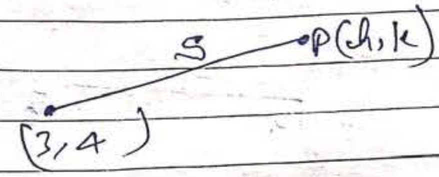
Steps:→
1) Assume the co-ordinates of point whose locus is to be found as "h, k"

2) write the ~~gir~~ given Geometrical condition In mathematical form Involving "h and k"

3) Eliminate the ~~varies~~ variable(s) If any

4) Replace $h \to x$ and $k \to y$
The equation so obtained is locus of point which moves under given condition.

48)



$$\sqrt{(h-3)^2+(k-4)^2} = 5$$

$$h^2-6h+9+k^2-8k+16 = 25$$

$$h^2+k^2-6h-8k=0$$

Locus is $x^2+y^2-6x-8y=0$

It is a equational circle with centre (3, 4) radius with 5 cm

49) $(h-1)^2+(k-2)^2 = (h-3)^2+(k-4)^2$

Ans $x+y=5$

57)

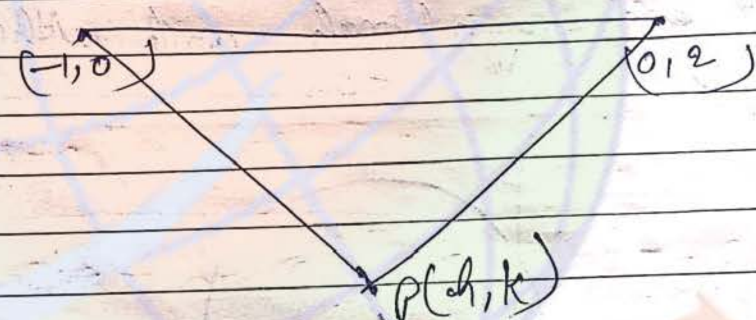
(-1,0) (0,2)

$(h+1)^2 + (k-0)^2 = 9\{(h+0)^2 + (k-2)^2\}$

$h^2+1+2h+k^2 = 9\{h^2+k^2+4-4k\}$

$h^2-9h^2+k^2-9k^2$

57)

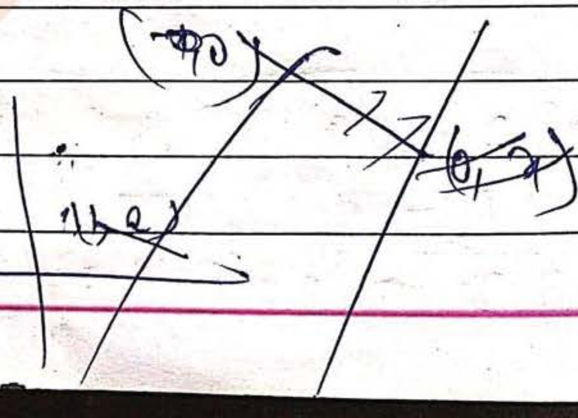


$\sqrt{(h+1)^2+(k)^2} = 3\sqrt{h^2+(k-2)^2}$

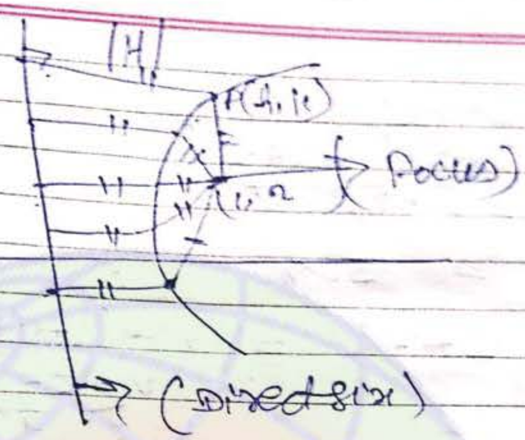
$h^2+2h+k^2 = 9(h^2+k^2-4k+4)$

$8(x^2+y^2) - 2x - 36y + 35 = 0$

58)



58)



$$\sqrt{(h-1)^2+(k-2)^2} = |h|$$

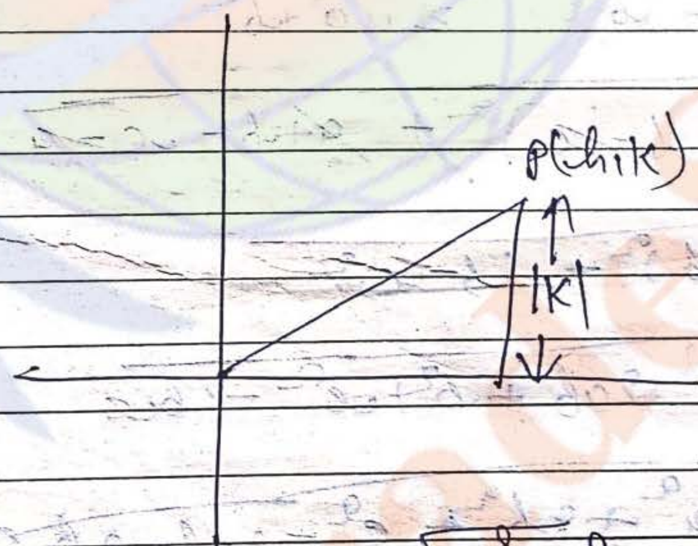
$$(h-1)^2 + (k-2)^2 = h^2$$

$h \to x$
$k \to y$

Locus

$$y^2 - 4y - 2x + 5 = 0$$

59)



$$\frac{1}{2}\sqrt{h^2+k^2} = |k|$$

Ans) $x^2 = 3y^2$

S.L. Loni Co-ordinate >(ai-1

1.)

$\sqrt{(5-2)^2+(7-3)^2}$

$\Rightarrow \sqrt{(3)^2+(4)^2}$

$\Rightarrow 5$

2.) $\sqrt{(-1-4)^2+(5+7)^2} \Rightarrow \sqrt{(-5)^2+(12)^2}$

$\Rightarrow \sqrt{25+144}$

$\Rightarrow \sqrt{169}$

$\Rightarrow 13\,cm$

3.) $\sqrt{(-6+3)^2+(7+2)^2} \Rightarrow \sqrt{(-3)^2+(9)^2}$

$\Rightarrow \sqrt{9+81}$

$\Rightarrow \sqrt{90}$

$3\sqrt{10}$

4.) $\sqrt{(0-a)^2+(b-0)^2} \Rightarrow \sqrt{a^2+b^2}$

5.) $\sqrt{(\cancel{c}+a-b-\cancel{c})^2+\cancel{a}+b-c-\cancel{a}}$

$\sqrt{(a-b)^2+(b-c)^2}$

$=\sqrt{a^2+b^2-2ab+b^2+c^2-2bc}$

$\Rightarrow \sqrt{a^2+2b^2+c^2-2ab-2bc}$

6.) $\sqrt{(a\cos\beta-a\cos\alpha)^2+(a\sin\beta-a\sin\alpha)^2}$

$\Rightarrow \sqrt{a^2\cos^2\beta+a^2\cos^2\alpha-2a\cos\beta\cdot a\cos\alpha+a^2\sin^2\beta+a^2\sin^2\alpha-2a\sin\beta\cdot a\sin\alpha}$

$\Rightarrow \sqrt{1+1-2a^2\cos\alpha\cos\beta-2a^2\sin\beta\sin\alpha}$

$\sqrt{2 - 2a^2\cos\beta\cos\alpha - 2a^2\sin\beta\sin\alpha}$

7c) $\sqrt{(am_2^2 - am_1^2)^2 + (2am_2 - 2am_1)^2}$

$\Rightarrow \sqrt{(am_2^2)^2 + (am_1^2)^2 - 2am_2^2 \cdot am_1^2 + (2am_2)^2 + (2am_1)^2 - 2am_2 \cdot 2am_1}$

8)

$(1,-3)$ ——— $(-2,1)$

$\sqrt{(-2-1)^2 + (1+3)^2} \Rightarrow \sqrt{(-3)^2 + (4)^2}$

$\Rightarrow \sqrt{9+16}$

$\Rightarrow 5$ units

9)

$(x,2)$ ——— $(3,4)$

$8 = \sqrt{(3-x)^2 + (4-2)^2}$

$8 = \sqrt{\cancel{\quad}}^2$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$(3-x)^2 + (4-2)^2 = 64$

$3^2 + x^2 - 6x + 4 = 64$

$x^2 - 6x + 13 - 64 = 0$

$x^2 - 6x - 51 = 0$

$3 \,|\, 51$
$\quad 17$

$\Rightarrow b^2 - 4ac$

$\Rightarrow 36 - 4 \times 1 \times -5$

$\Rightarrow$

8b) $a = 1,\ b = -6,\ c = -5$

(1) $a=1,\ b=-6,\ c=-51$

$\Delta(D) = b^2-4ac$
$= (-6)^2 - 4\times1\times(-51)$
$\Rightarrow 36 + 204$
$\Rightarrow 240$

$2\,|\,240$
$2\,|\,120$
$2\,|\,60$
$2\,|\,30$
$3\,|\,15$
5

$4\sqrt{15}$

$x = \frac{-b\pm\sqrt{D}}{2a} \Rightarrow \frac{-6\pm4\sqrt{15}}{2\times1}$

$\Rightarrow \frac{\cancel{2}(3\pm2\sqrt{15})}{\cancel{2}}$

$\Rightarrow (3\pm2\sqrt{15})$

Teach

10.)

A $(2,-3)$ — 10 — B $(10, x)$

Prove $x \Rightarrow 3$, or -9

$10 = \sqrt{(10-2)^2+(x+3)^2}$

$\Rightarrow \sqrt{(8)^2 + x^2+9+6x}$

$\Rightarrow \sqrt{64+x^2+9+6x}$

$\frac{64}{\frac{+9}{73}}$

$10 \Rightarrow \sqrt{x^2+6x+73}$

$\therefore x^2+6x+73 = (10)^2$

$x^2+6x-27=0$

$3\,|\,27$
$3\,|\,9$
3

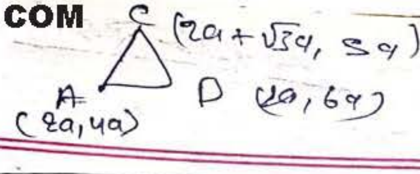
$\Rightarrow x^2+9x-3x-27=0$

$\Rightarrow x(x+9)-3(x+9)=0$ So $x=3,-9$

11.) AD=

12.)

21.)



11.) $AB = \sqrt{(2a-2a)^2 + (6a-4a)^2}$

$\Rightarrow \sqrt{(2a)^2}$

$\Rightarrow \sqrt{2^2a^2} = 2a$

$BC = \sqrt{(2a+\sqrt{3}a-2a)^2 + (5a-6a)^2}$

$\Rightarrow \sqrt{3a^2 + 1a^2} \Rightarrow \sqrt{4a^2}$
$\Rightarrow 2a$

$AC = \sqrt{(2a+\sqrt{3}-2a)^2 + (5a-4a)^2}$

$\Rightarrow \sqrt{3a^2+1a^2} = \sqrt{4a^2} = 2a$

So, each three is equilateral triangle.

12.)

21.) $(a+b, a-b)$ — a : b — $(a-b, a+b)$

Internal point

$$\left(\frac{(a-b)a + b(a+b)}{a+b}, \frac{a(a+b) + b(a-b)}{a+b}\right)$$

$$\left(\frac{a^2 - \cancel{ab} + \cancel{ab} + b^2}{a+b}, \frac{a^2+ab+ab-b^2}{a+b}\right)$$

Angula point

$$\left(\frac{a^2+b^2}{a+b}, \frac{a^2-b^2+2ab}{a+b}\right)$$

Point Internallesss

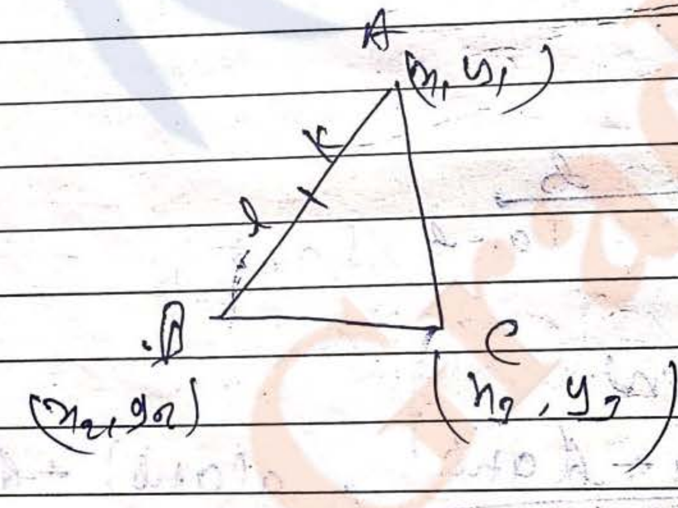
$(a+b)\ (a-b)$ $\qquad$ $(a-b)\ (a+b)$

Internalpoint $\left(\frac{a(a-b)-b(a+b)}{a-b}, \frac{a(a+b)-b(a-b)}{a-b}\right)$

$$\Rightarrow \left(\frac{a^2-ab-ab-b^2}{a-b}, \frac{a^2+ab-ab-b^2}{a-b}\right)$$

$$\left(\frac{a^2-b^2-2ab}{a-b}, \frac{a^2-b^2}{a-b}\right)$$

22.



22, 24, 25, 26, 27 → 21 → Cheque

23.) (2, 3) (m, y) (3, 4)

$x = \frac{2+3}{2}$, $y = \frac{3+4}{2}$

$2x = 5$ $2y = 7$

$x = \frac{5}{2}$ $y = \frac{7}{2}$

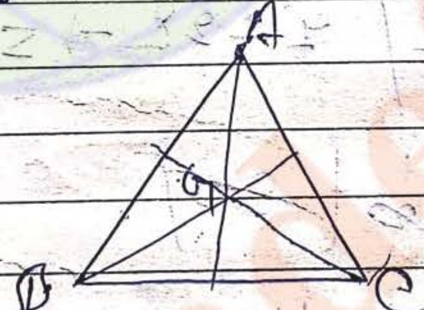
$x-y = \frac{5}{2} - \frac{7}{2}$

$x-y = \frac{5-7}{2}$

$x-y = \frac{-2}{2}$

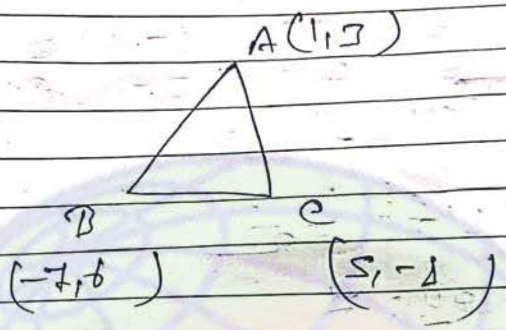
$x-y+1 = 0$

24.)



Ex > 2 (S.L. LONI Co-ordinates) 224

1.)



Method Ist :->

Area of $\Delta ABC = \frac{1}{2}\left| x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) \right|$

$= \frac{1}{2}\left| 1(6+1) + (-7)(-1-3) + 5(3-6) \right|$

$\Rightarrow \frac{1}{2}\left| 7+28-15 \right|$

$\Rightarrow \frac{1}{2}\left| 20 \right|$

$= 10$

Method IInd :-> (By Polygon rule)

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -7 & 5 & 1 \\ 3 & 6 & -1 & 3 \end{bmatrix}$$

Area of triangle $= \frac{1}{2}\left|(x_1 y_2 - y_1 x_2) + (x_2 y_3 - y_2 x_3) + (x_3 y_1 - y_3 x_1)\right|$

$\Rightarrow \frac{1}{2}\left|(6+21) + (7-30) + (15+1)\right|$

$\Rightarrow \frac{1}{2}(27 - 23 + 16)$

$\Rightarrow \frac{1}{2}(20)$

$= 10$

$27 + 15 = 42$, $42 - 22 = 20$

2)

0	3	-8	0
4	6	-2	4

Area $= \frac{1}{2}\left|(0-12) + (-6+48) + (-32-0)\right|$

$= \frac{1}{2}\left|-12+42-32\right|$

$\Rightarrow \frac{1}{2}\left|-42+42\right|$

$\Rightarrow \cancel{\frac{1}{2} \times 2} = 21$

$32 + 22 = 54$, $32 + 12 = 44$

3)

5	-9	-3	5
2	-3	-5	2

Area $= \frac{1}{2}\left((15+18) + (45+9) + (-6+25)\right)$

$= \frac{1}{2}(38 + 54 + 19)$

3.) $\begin{matrix} 5 & -9 & -3 & 5 \\ -2 & -3 & -5 & 2 \end{matrix}$

Area of triangle $= \frac{1}{2}\left|(-15+18)+(45-9)+(-6+25)\right|$

$\Rightarrow \frac{1}{2}\left|3+36+19\right|$

$\Rightarrow \frac{1}{2}\left|50\right|$

$\frac{1}{2}\times 50 = 29$

4.) $\begin{matrix} a & a & -a & a \\ b+c & b-c & c & b+c \end{matrix}$

Area $= \frac{1}{2}\left|\{a(b-c) - a(b+c)\} + \{ac-(-a)(b-c)\} + \{(-a)(b+c) - ac\}\right|$

$\Rightarrow \frac{1}{2}\left|\{ab-ac-ab-ac\} + \{ac+ab-ac\} + \{-ab-ac-ac\}\right.$

$\Rightarrow \frac{1}{2}\left| -ac - ac + \cancel{ab} - \cancel{ab} - ac - ac \right|$

$\Rightarrow \frac{1}{2}\left| -4ac \right|$

$\Rightarrow \frac{1}{2} \times 4ac$

$\Rightarrow 2ac$

5.) a

6.) $a\cos\phi_1 \quad a\cos\phi_2 \quad a\cos\phi_3 \quad a\cos\phi_1$

$b\sin\phi_1 \quad b\sin\phi_2 \quad b\sin\phi_3 \quad b\sin\phi_1$

Area $= \frac{1}{2}\left| (a\cos\phi_1 \cdot b\sin\phi_2 - b\sin\phi_1 \cdot a\cos\phi_2) + (a\cos\phi_2 \cdot b\sin\phi_3 - b\sin\phi_2 \cdot a\cos\phi_3) \right.$

$\left. a\cos\phi_3 \cdot b\sin\phi_1 - b\sin\phi_3 \cdot a\cos\phi_1) \right|$

$\frac{1}{2}\Big|$

7, 8, 9

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Date: / /

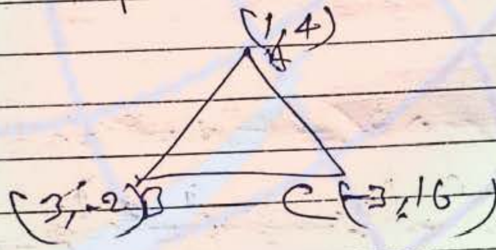
7.) $am_1^2, \quad am_2^2 \quad am_3^2 \quad am_1^2$

$2am_1 \quad 2am_2 \quad 2am_3 \quad 2am_1$

Area $= \frac{1}{2} \Big| \left(am_1^2 \cdot 2am_2 - 2am_1 \cdot am_2^2\right) + \left(am_2^2 \cdot 2am_3 - 2am_2 \cdot am_3^2\right) + \left(am_3^2 \cdot 2am_1 - 2am_3 \cdot am_1^2\right) \Big|$

$\Rightarrow \frac{1}{2} \Big| am_1^2 \cdot 2am_2$

11.)



$$\begin{matrix} 1 & -3 & -3 & -1 \\ 4 & -2 & 16 & 4 \end{matrix}$$

Area $\Delta = \frac{1}{2} \left| (-2-12) + (48-6) + (-12-16) \right|$

$= \frac{1}{2} \left| -14 + 42 - 28 \right|$

$= \frac{1}{2} |0|$

$= 0$

Here Area of Δ is zero so all three point lie in straight line.

11) $\frac{-1}{2}$

3

6.)

11)

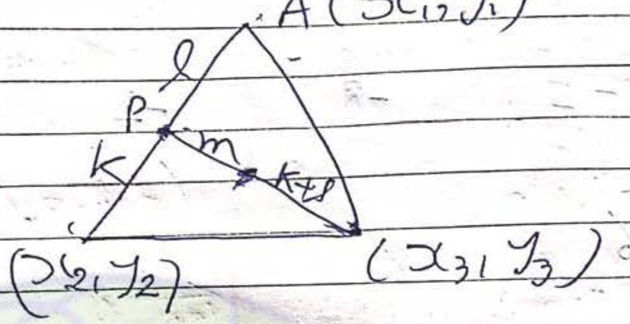
$\frac{-1}{2}$ -5 -8 $\frac{-1}{2}$

3 6 8 8

6. $\sqrt{(a\cos\alpha - a\cos\beta)^2 + (a\sin\alpha - a\sin\beta)^2}$

$\sqrt{a^2[\cos\alpha - \cos\beta]^2 + a^2[\sin\alpha - \sin\beta]^2}$

$a\sqrt{\left(2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}\right)^2 + \left(2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right)^2}$



x_1, y

Example sheet

Q.

Doubt:

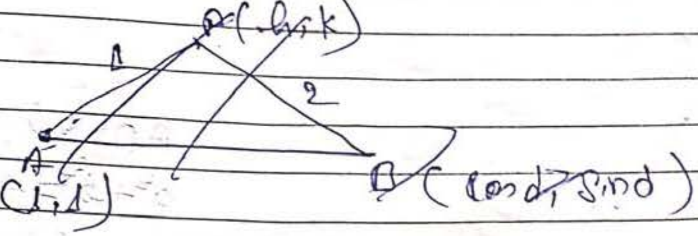
1.) $\Rightarrow \sqrt{(h-ae)^2+k^2} = 2a - \sqrt{(h+ae)^2+k^2}$

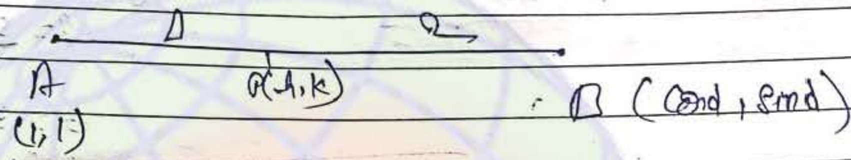
$\Rightarrow \cancel{(h-ae)^2+k^2 = 4a^2}$

Q.) A point moves so that the sum of its distance from $(ae, 0)$ and $(-ae, 0)$ is $2a$, prove that the equation to its locus is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ where, } b^2=a^2(1-e^2)$$

50)





$$h = \frac{\cos\alpha + 2}{3} \Rightarrow 3h = \cos\alpha + 2$$

$$k = \frac{\sin\alpha + 2}{3} \Rightarrow 3k = \sin\alpha + 2$$

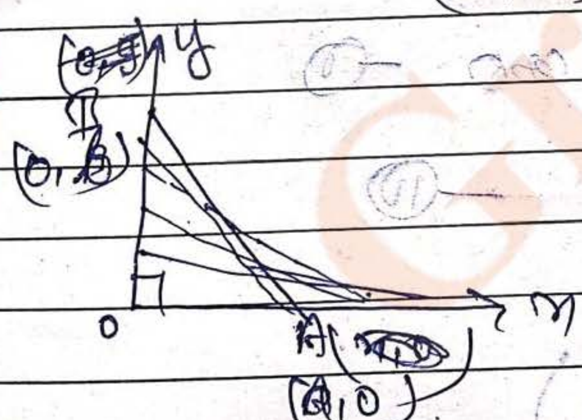
$$\Rightarrow \cos\alpha = (3h - 2)$$
$$\sin\alpha = (3k - 2)$$

$$\cos^2\alpha + \sin^2\alpha = 1$$

$$(3h-2)^2 + (3k-2)^2 = 1$$

Locus $\Rightarrow (3x-2)^2 + (3y-2)^2 = 1.$

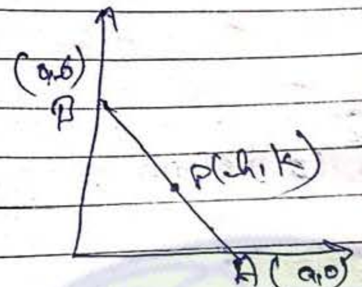
55)



$$OA^2 + OB^2 = AB^2$$

$\Rightarrow$

S.S.)



$AB = 2l$

$a^2 + b^2 = 4l^2$ — (1)

By mid point

$h = \frac{a}{2}$

$a = 2h$ — (2)

$k = \frac{b}{2}$

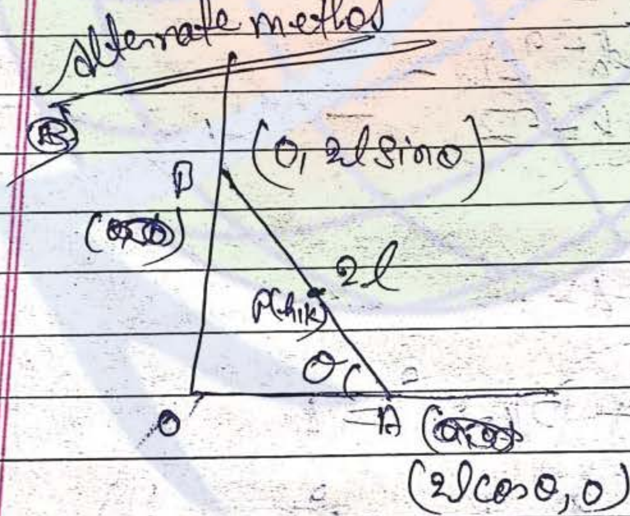
$b = 2k$ — (3)

$(2h)^2 + (2k)^2 = 4l^2$

$h^2 + k^2 = l^2$

Locus = $\boxed{x^2 + y^2 = l^2}$

Alternate method



$h = \frac{2l\cos\theta}{2} = l\cos\theta$ — (1)

$k = l\sin\theta$ — (11)

$(1)^2 + (2)^2$

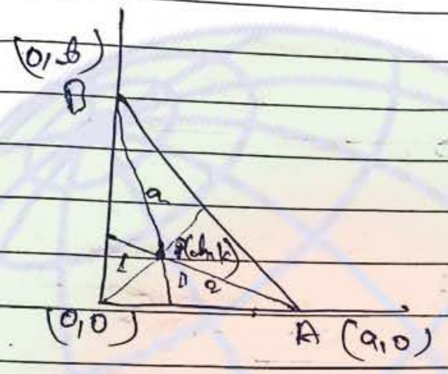
$h^2 + k^2 = l^2(\cos^2\theta + \sin^2\theta)$

$h^2+k^2=l^2$

Locus

$$\boxed{x^2+y^2=l^2}$$

ii)



Alternate method

$h=\frac{2l\cos\theta}{3}$

$k=\frac{2l\sin\theta}{3}$

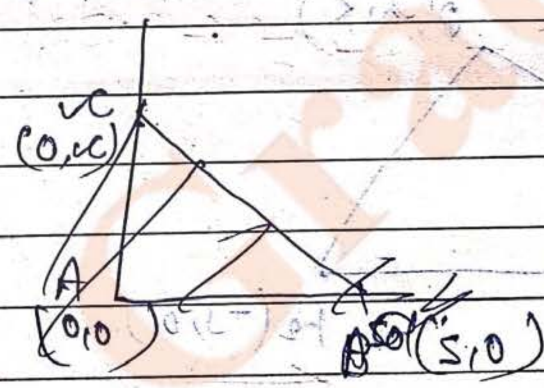
$h^2+k^2=\frac{4l^2}{9}$

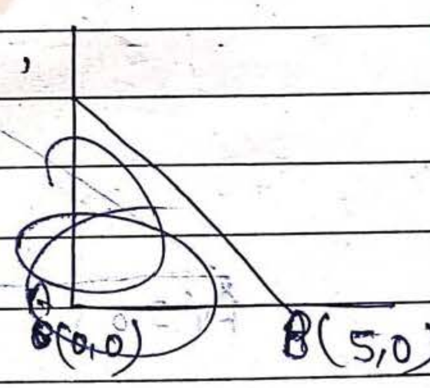
$h=\frac{0+a+0}{3}$

~~$3h=a$~~ $h=\frac{a}{2}$

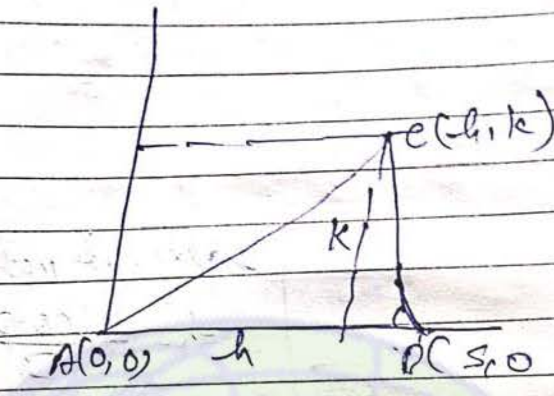
$k=\frac{b}{2}$

54)





51.)



$$\tan A + \tan B = \lambda$$

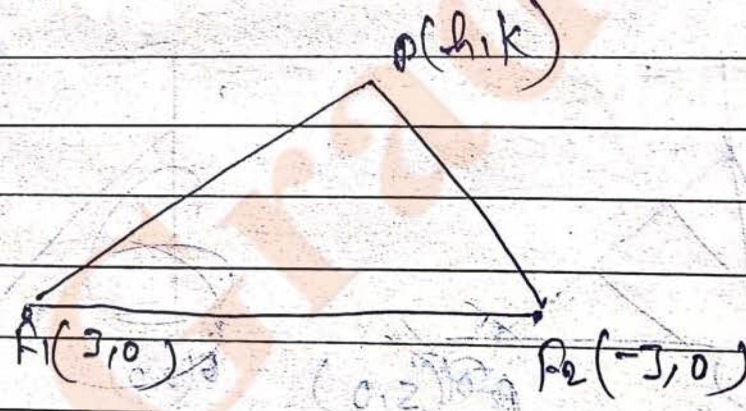
$$\frac{k}{h} + \frac{k}{5-h} = \lambda$$

$$k \cdot \left(\frac{5-h+h}{h(5-h)} \right) = \lambda$$

$$5k = h(5-h)\lambda$$

Locus $5y = x(5-x)\lambda$

53.)



$$PP_1 + PP_2 = 10$$

$\sqrt{(h-3)^2+(k)^2} + \sqrt{(h+3)^2+(k)^2} = 10$

$(h-3)^2 + k^2 + (h+3)^2 + k^2 + 2\sqrt{[(x-3)^2+k^2][(h+3)^2+k^2]} = 10$

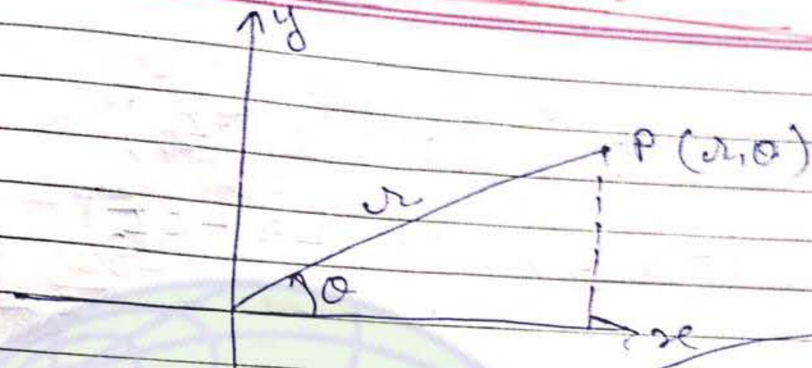
Ans $\Rightarrow \dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$

$\sqrt{(h-3)^2+(k)^2} = 10 - \sqrt{(h+3)^2+(k)^2}$

$(h-3)^2 + (k)^2 = \left(10 - \sqrt{(h+3)^2+(k)^2}\right)^2$

$h^2 + 9$

Polar Co-ordinate system



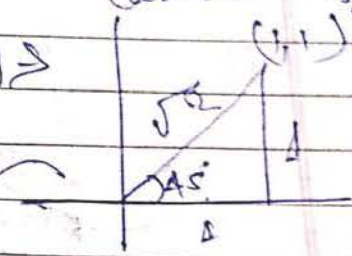
$r \rightarrow$ radius vector
$\theta \rightarrow$ vectorial angle.

- The pair of real numbers (r, θ) is the polar co-ordinate of point P.

Possible value of θ lies in $\boxed{-\pi < \theta \leq \pi}$

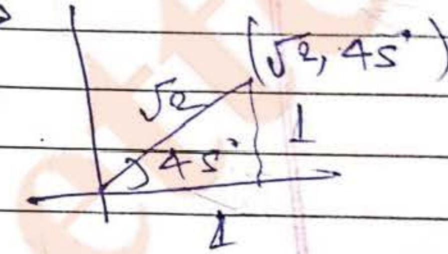
eg →

Cartesian Co-ordinate system

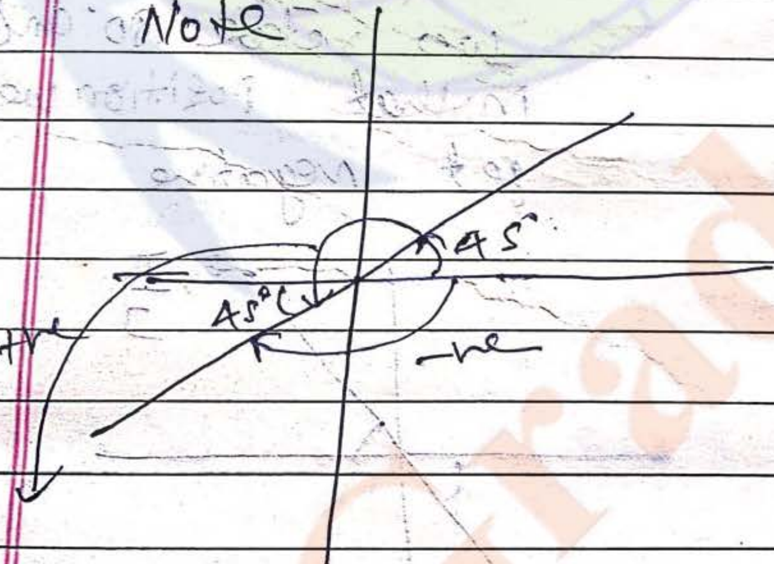


convert into

Polar system



Note



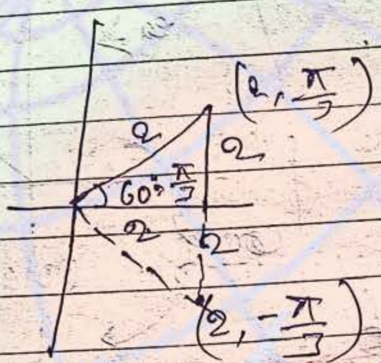
Example:→

1st mark

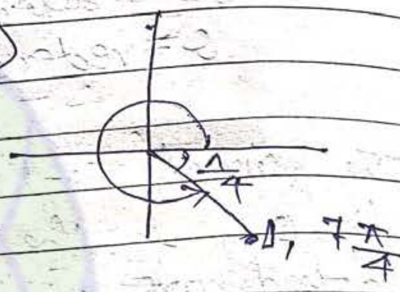
(i) $\left(2, \pm \frac{\pi}{3}\right)$ (ii) $\left(1, \frac{7\pi}{4}\right)$

(iii) $\left(-2, \frac{\pi}{3}\right)$ (iv) $\left(-2, -\frac{5\pi}{3}\right)$

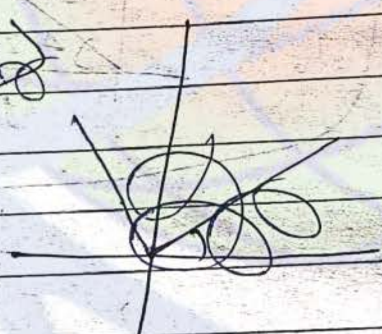
(i)



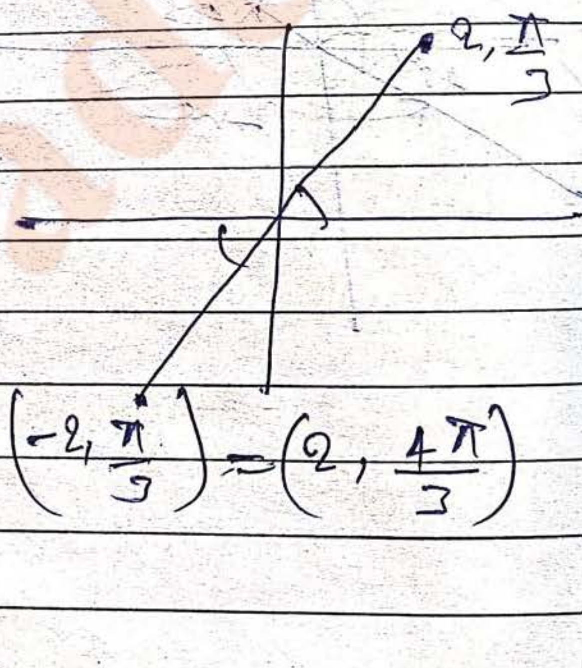
(ii)



(iii)

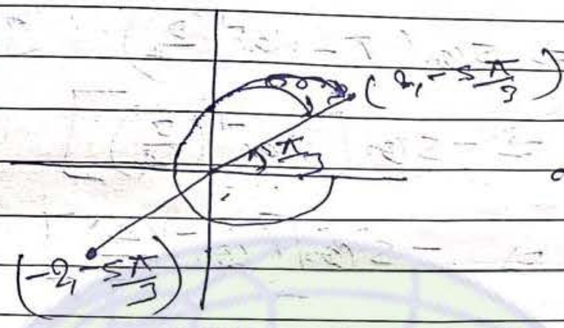


(iii) Firstly we mark $\left(2, \frac{\pi}{3}\right)$ then we rotate 180° and in that position we get negative

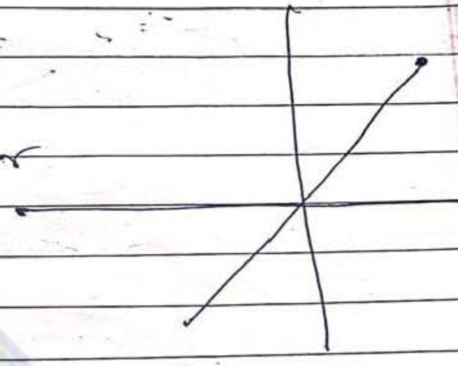


$$\left(-2, \frac{\pi}{3}\right) = \left(2, \frac{4\pi}{3}\right)$$

iv)



or



or

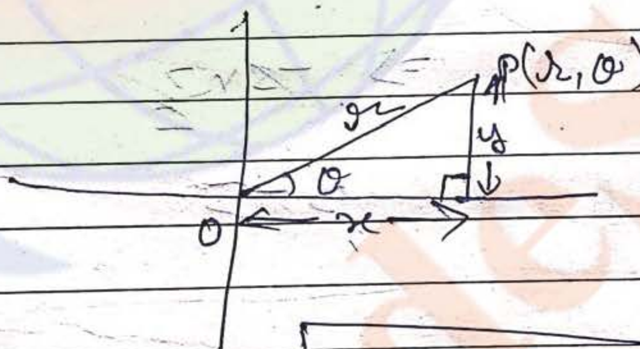
$2, \frac{4\pi}{3}$

or

$\left(2, -\frac{2\pi}{3}\right)$

* Relation between polar and cartesian co-ordinates

(1.)) Polar → cartesian co-ordinate



$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

46.)

$r = 5$

$\theta = \tan^{-1}\left(\frac{3}{4}\right) - \pi$

$x = r\cos\theta$

$= 5 \cdot \cos\left(\tan^{-1}\left(\frac{3}{4}\right) - \pi\right)$

$\sin[\sin^{-1}(x)] = x$

$\cos[(\cos^{-1}(x))] = x$

Same as all other T-ratio

$\Rightarrow 5\cos\left(\pi - \tan^{-1}\left(\frac{3}{4}\right)\right)$

((2))

$\Rightarrow -5\cos\left[\tan^{-1}\left(\frac{3}{4}\right)\right]$

$\Rightarrow -5\cos\left[\left(\cos^{-1}\left(\frac{4}{5}\right)\right)\right]$

$\Rightarrow -5 \times \frac{4}{5}$

$\Rightarrow -4$

$y = 5\sin\left(\tan^{-1}\left(\frac{3}{4}\right) - \pi\right)$

$\Rightarrow -5\sin\left(\pi - \tan^{-1}\left(\frac{3}{4}\right)\right)$

$\Rightarrow -5 \cdot \sin\left(\tan^{-1}\left(\frac{3}{4}\right)\right)$

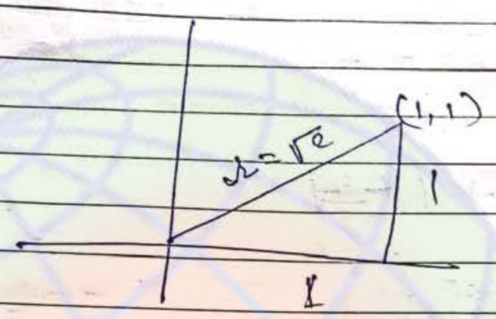
$\Rightarrow -5\sin\left(\sin^{-1}\left(\frac{3}{5}\right)\right)$

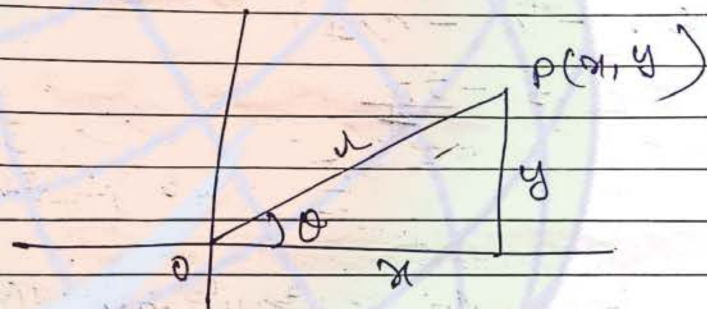
$\Rightarrow -\cancel{5} \times \frac{3}{\cancel{5}}$

$\Rightarrow -3$

1(2))

⇒ Cartesion → Polar





$$r = \sqrt{x^2+y^2}$$

$$\tan\theta = \frac{y}{x}$$ (we not use directly)

Let α

$$\alpha = \tan^{-1}\left|\frac{y}{x}\right|$$

	I	II	III	IV
θ	α	$180-\alpha$	$-(180-\alpha)$	$-\alpha$

45) $(-2, -2)$

$r = \sqrt{(-2)^2 + (-2)^2}$

$= 2\sqrt{2}$

$\alpha = \tan^{-1}\left(\left|\frac{-2}{-2}\right|\right)$

$= \tan^{-1}(1)$

$\theta = -(\pi - \pi/4)$

$= \frac{-3\pi}{4}$

$(r, \theta) = \left(2\sqrt{2}, \frac{-3\pi}{4}\right)$

Q) Change the equation $x^2+y^2=ax$ in polar form.

Sol $x^2+y^2=ax$

$r^2 = a(r\cos\theta)$

$\boxed{r = a\cos\theta}$

47) $r^2 = a^2\cos 2\theta$ in cartesian form.

$r^2 = a^2\cos 2\theta$

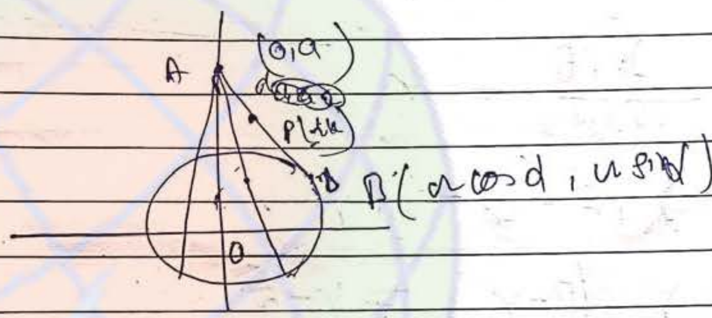
$= a^2(\cos^2\theta - \sin^2\theta)$

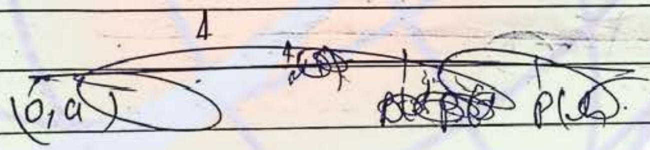
$r^4 = a^2(\cos^2\theta - \sin^2\theta)$

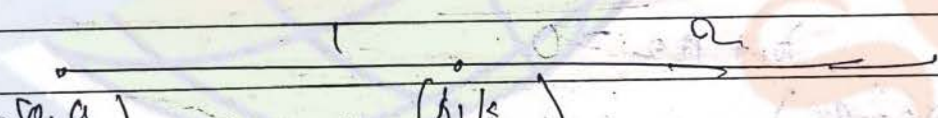
$r^4 = a^2[(r\cos\theta)^2 - (r\sin\theta)^2]$

$$\boxed{(x^2+y^2)^2 = a^2(x^2-y^2)}$$

S(o) $x^2+y^2 = r^2$







$h = \frac{r\cos\alpha}{3}$

$r\cos\alpha = 3h$ — (1)

$k = \frac{r\sin\alpha + 2a}{3}$

$r\sin\alpha = 3k - 2a$ — (2)

$(1)^2 + (2)^2$

$r^2(\cos^2\alpha + \sin^2\alpha) = 9h^2 + (3k-2a)^2$

Locus :→ $9x^2 + (3y-2a)^2 = r^2$

18.

$$A(\alpha,0) \xrightarrow{\lambda_1 \ : \ 1} C(\gamma,0) \quad B(\beta,0)$$

$$C\left(\frac{\lambda_1 \beta + \alpha}{\lambda_1 + 1}, 0\right)$$

$$\frac{\lambda_1 \beta}{\lambda_1 + 1} = \gamma$$

$$\lambda_1 \beta + \alpha = \lambda_1 \gamma + \gamma$$

$$\lambda_1 (\beta - \gamma) = \gamma - \alpha$$

$$\lambda_1 = \left(\frac{\gamma - \alpha}{\beta - \gamma}\right) \text{ (1)}$$

Given

$$\lambda_1 + \lambda_2 = 0$$

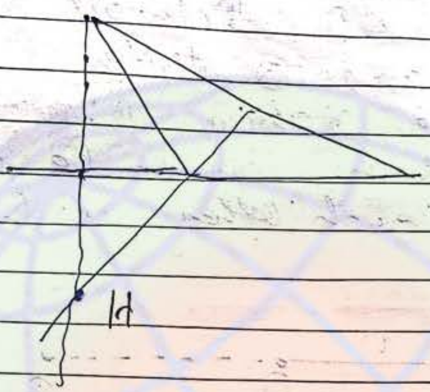
$$\left(\frac{\gamma - \alpha}{\beta - \gamma}\right) + \left(\frac{\delta - \gamma}{\beta - \delta}\right) = 0$$

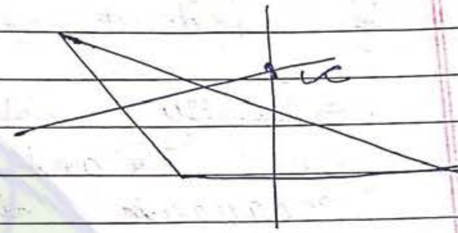
$$(\gamma - \alpha)(\beta - \delta) + (\beta - \gamma)(\delta - \alpha) = 0$$

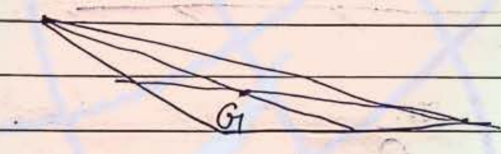
$$(\alpha + \beta)(\gamma + \delta) = 2(\alpha\beta + \delta\gamma)$$

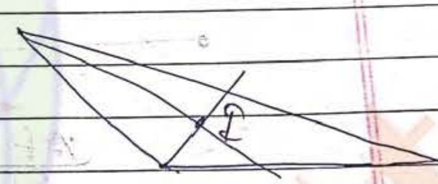
$$\left(\frac{-2b_1}{a_1}\right) \cdot \left(\frac{-2b_2}{a_2}\right) = 2\left(\frac{c_1}{a_1} + \frac{c_2}{a_2}\right)$$

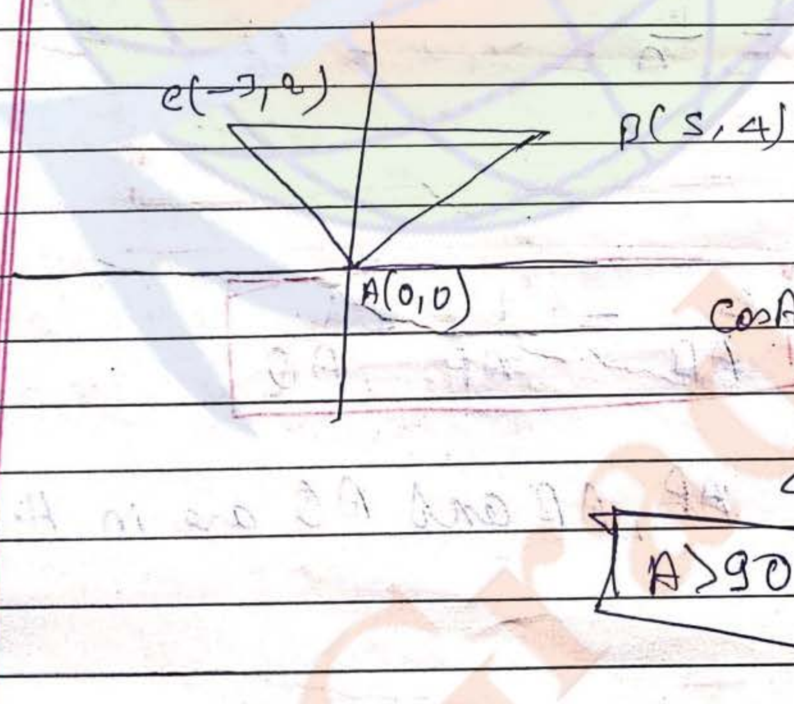
A(0,1), B(4,4), C(3,5)











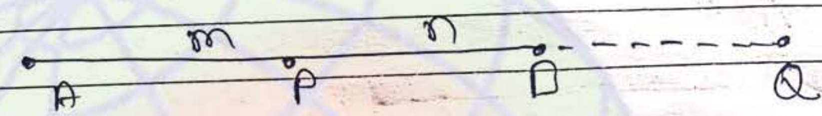
$$\cos A = \frac{AB^2 + AC^2 - BC^2}{2\,AB\,AC}$$

< 0

$$\boxed{A > 90}$$

Harmonic Conjugate

If 'P' divides AB Internall in the ratio m:n and Q divides AB externally in the ration m:n then Points P and Q are said to be harmonic congugate of each other with respect to AB.



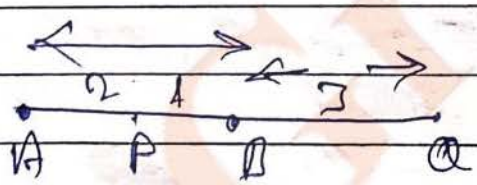
$$\frac{AP}{PB} = \frac{m}{n}$$

$$\frac{AQ}{BQ} = \frac{m}{n}$$

mathomally $\boxed{\frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}}$

so AP, AB and AQ are in H.P

Example



$$\frac{AP}{PB} = \frac{2}{4}$$

$$\frac{AQ}{BQ} = \frac{6}{3} = \frac{2}{1}$$

$$\frac{2}{AB} = \frac{2}{3}$$

$$\frac{1}{AP} = \frac{1}{2}$$

$$\frac{1}{AQ} = \frac{1}{6}$$

so, $\frac{1}{AP} + \frac{1}{AQ} = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

Q19.

$$\boxed{\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)}$$

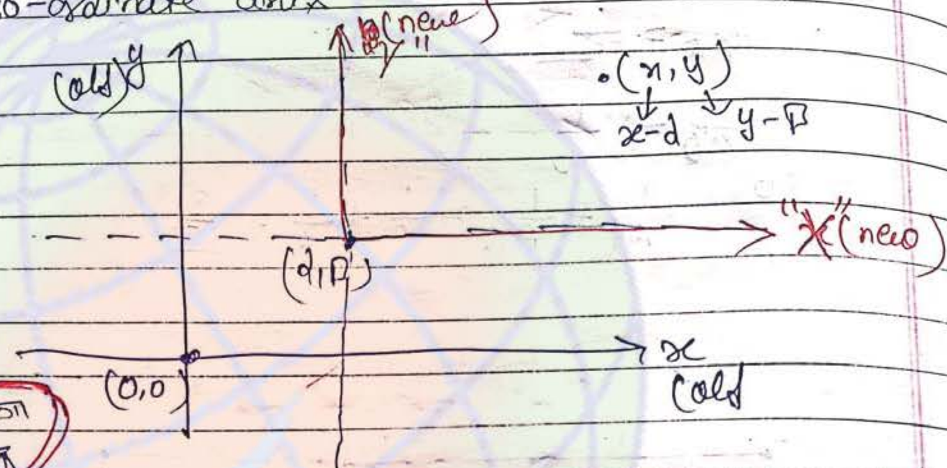
$$Q\left(\frac{7-2}{1-2}, \frac{9-6}{1-2}\right)$$

$$Q(-5, -3)$$

Transformation of Axes :⇒

1.) Parallel transformation :⇒

The origin is being shifted to the point (α, β) without any rotation of co-ordinate axis



बड़े letter new होगा
small letter old होगा

$$\boxed{\begin{array}{l|l} X = x - \alpha & x, y \to \text{old} \\ Y = y - \beta & X, Y \to \text{New.} \end{array}}$$

N.V (new)

Q) To what point should the origin be shifted so that co-ordinates of point $(-1, 8)$ become $(2, 6)$

Ans :→ $(-1, 8)$
$(2, 6)$

$X = 2$, $Y = 6$ | $x = -1$, $y = 8$

$X = x - \alpha$
$2 = -1 - \alpha$
$\alpha = -3$

$6 = 8 - \beta$
$\beta = 2$

so $(\alpha, \beta) = (-3, 2)$

$\boxed{old = new - d}$ $\boxed{capital \to New}$

Q) Find the equation of curve $2x^2 + y^2 - 3x + 5y - 8 = 0$ when origin is shifted to $(1, 2)$

Ans:⇒

$$X = x + 1 \Rightarrow x = (x - 1)$$

$$Y = y - 2 \quad y = Y + 2$$

Eq^n in new system:–

$$2(x-1)^2 + (y+2)^2 - 3(x-1) + 5(y+2) - 8 = 0$$

Q) The equation of curve when origin is to $(4, 5)$ is $x^2 + y^2 = 36$. Then find the equation refer to the original axes.

$X = x + 1$

$4 - 1 = x$

$x = 3$

$$\Rightarrow X^2 + Y^2 = 36$$

$$X = x - 4$$

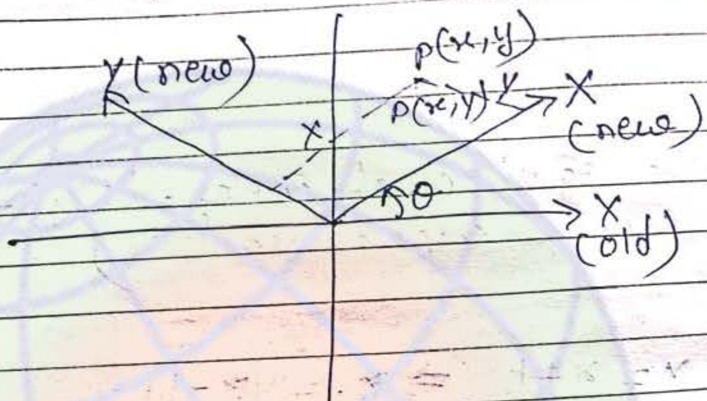
$$Y = y - 5$$

eq in old system

$$(x-4)^2 + (y-5)^2 = 36$$

Page No.: 9

ii) Rotation of axes :- (without shifting the origin)



If the co-ordinate axes rotated through an angle 'θ' in anticlockwise direction then the new co-ordinates will be :-

	old x	old y
new X	$\cos\theta$	$\sin\theta$
new Y	$-\sin\theta$	$\cos\theta$

Trick to learn

$$X = x\cos\theta + y\sin\theta$$
$$Y = -x\sin\theta + y\cos\theta$$
new

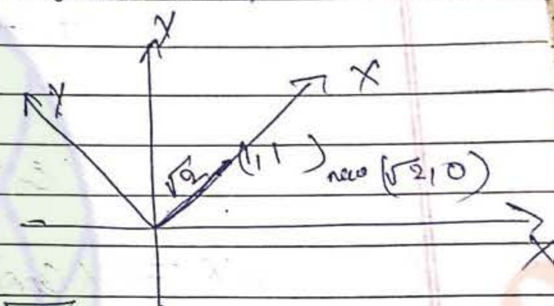
$$x = X\cos\theta - Y\sin\theta$$
$$y = X\sin\theta + Y\cos\theta$$
old

Example

Find the new co-ordinates of point (1,1) when co-ordinate axes are rotated through an angle 45°

(i) In Anticlockwise direction
(ii) In clockwise direction.

Solution: ⇒ $x = 1$
$y = 1$
$\theta = 45°$



$X = x\cos\theta + y\sin\theta$
$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$

$Y = -\sin\theta + y\cos\theta$
$= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$
$= 0$

$(\sqrt{2}, 0)$

In clockwise
$(0, \sqrt{2})$

Q) To what angle should the axes be rotated so that $9x^2 - 2\sqrt{3}xy + 7y^2 = 10$ in old system changes to $3x^2 + 5y^2 = 9$ in new system.

Q-) What will this equation $x^2 - y^2 = a^2$ become in the co-ordinate axis are rotated through angle 45° in clock wise direction.

Solution (Page-252)

Ans:⇒

	x	y
X	$\cos\theta$	$\sin\theta$
Y	$-\sin\theta$	$\cos\theta$

$$\theta = -45^\circ$$

$$x = X\cos\theta - Y\sin\theta$$
$$= X\cos(-45^\circ) - Y\sin(-45^\circ)$$

$$x = \left(\frac{X+Y}{\sqrt{2}}\right)$$

$$y = X\sin\theta + Y\cos\theta$$
$$= X\sin(-45^\circ) + Y(\cos(-45^\circ))$$

$$y = \left(\frac{Y-X}{\sqrt{2}}\right)$$

$$\left(\frac{X+Y}{\sqrt{2}}\right)^2 - \left(\frac{Y-X}{\sqrt{2}}\right)^2 = a^2$$

$$(X+Y)^2 - (Y-X)^2 = 2a^2$$

$$\boxed{XY = \frac{a^2}{2}}$$

sol

Solution (Page 257)

$$9x^2 - 2\sqrt{3}xy + 7y^2 = 10$$

$$\Rightarrow 3X^2 + 5Y^2 = 5 \quad \text{--- (1)}$$

$$\theta$$

$$9(X\cos\theta - Y\sin\theta)^2 - 2\sqrt{3}(X\cos\theta - Y\sin\theta)$$

$$(X\sin\theta + Y\cos\theta) + 7(X\sin\theta + Y\cos\theta)^2 = 10 \quad \text{--- (2)}$$

From (2)

coefficient of $XY = 0$

$$9(-2\sin\theta\cos\theta) - 2\sqrt{3}(\cos^2\theta - \sin^2\theta) + 7(2\sin\theta\cos\theta) = 0$$

$$-2\sin 2\theta - 2\sqrt{3}\cos 2\theta = 0$$

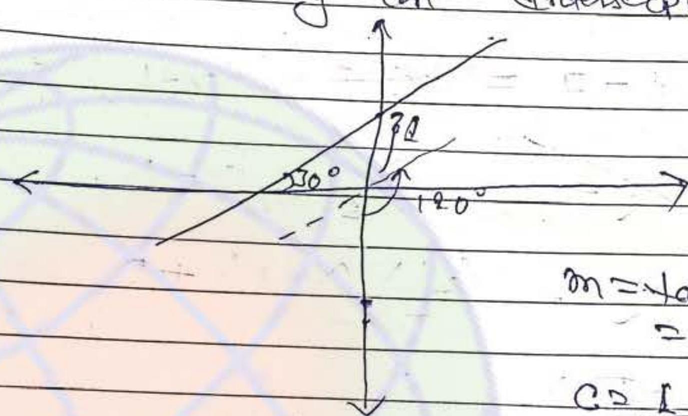
$$\tan 2\theta = \sqrt{3}$$

$$2\theta = 120^\circ$$

$$\theta = 60^\circ$$

Q5) Find the equation of straight line making an angle 120° with negative y-axis in anticlock course direction and having an intercept of length 4.

Ans:



$m = \tan 30°$
$= \frac{1}{\sqrt{3}}$

$c = 4$

Equation ⇒ $\boxed{y = \frac{1}{\sqrt{3}} x + 4}$

2) Slope point form:⇒

Equation of straight line passing through the points (x_1, y_1) and having slope 'm' is

$$\boxed{y - y_1 = m(x - x_1)}$$

3) Two point form:⇒

Equation of straight line passing through the points ~~x₁ and~~ (x_1, y_1) and (x_2, y_2) is given by

$$\boxed{y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)}$$

Put the value of m.

Q Find the equation of line, joining the points $\underset{x_1\ y_1}{(-1,3)}$, $\underset{x_2\ y_2}{(4,2)}$

Sol

$$\Rightarrow y-3 = \left(\frac{2-3}{4+1}\right)(x+1)$$

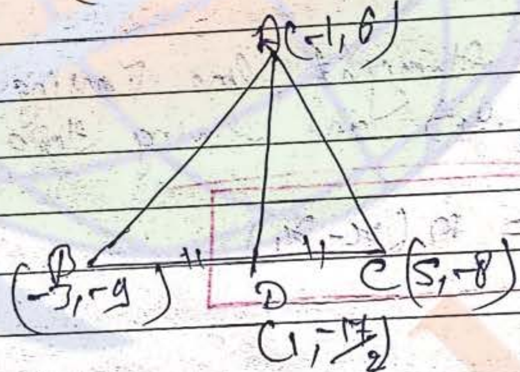
$$\Rightarrow y-3 = \left(\frac{-1}{5}\right)(x+1)$$

$$\Rightarrow y = \left(\frac{-1}{5}\right)(x+1)+3$$

$$\boxed{x+5y=14}$$

Q Find the equation of linea slop of D' with the vertices $(-1,6)$, $(-3,-9)$, $(5,-8)$

Sol

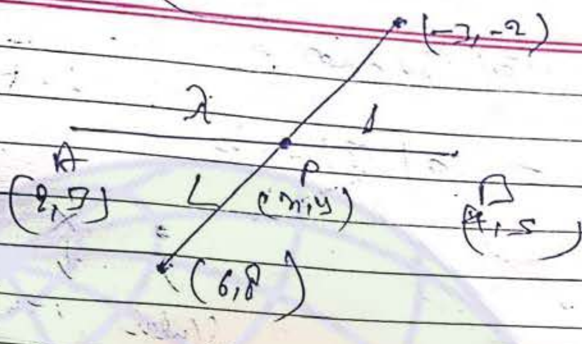


$$y-6 = \left(\frac{\frac{-17}{2}-6}{1+1}\right)(x+4)$$

Q Find the ratio in which the line segment the Joining the points (2,3) and (4,5) which divides by the line joining (6,8), (-3,-2)

(Negative shows External divide)

Sol/



$$x = \frac{4\lambda + 2}{\lambda + 1}, \quad y = \frac{5\lambda + 3}{\lambda + 1}$$

Line :⇒

$$L :\Rightarrow \quad y - 8 = \left(\frac{-2-8}{-3-6}\right)(x - 6)$$

$$9y - 72 = 10x - 60$$

$$10x + 12 = 9y \quad \text{—— (1)}$$

$$P\left(\frac{4\lambda+2}{\lambda+1}, \frac{5\lambda+3}{\lambda+1}\right)$$

P lies on line L :⇒

$$10\left(\frac{4\lambda+2}{\lambda+1}\right) + 12 = 9\left(\frac{5\lambda+3}{\lambda+1}\right)$$

$$\lambda = -5/7$$

(external.)

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$

$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

$\cdot$ → Scalar $\times$ → vector

$\cdot$ ~~Scalar~~

$\times$ ~~vector product~~

$\hat{i}, \hat{j}, \hat{k}$ $\hat{i}\cdot\hat{i} = 1$

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

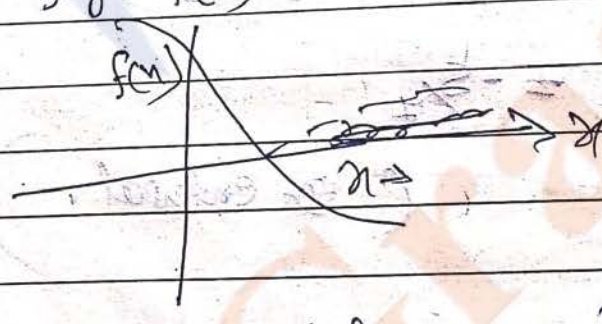
$\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$

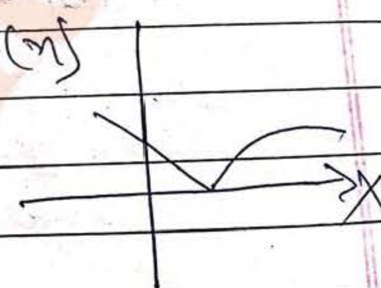
$\vec{a}\cdot\vec{b} = 2 - 2 - 3 = -3$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & -1 & -1 \end{vmatrix}$$

D.P.P. 5 ⇒ 19

Q) $y = f(x)$

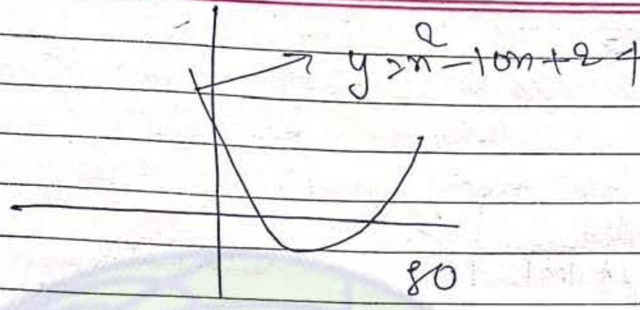


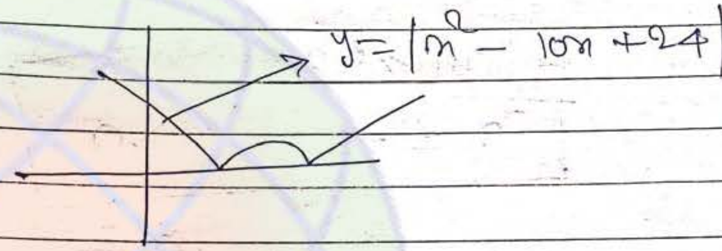


$f(x) = |x^2 - 10x + 24|$

$y = x^2 - 10x + 24$

Page No. :
Date : / /





6.) $\dfrac{x^2 - 7|x| + 10}{(x-3)^2} < 0$

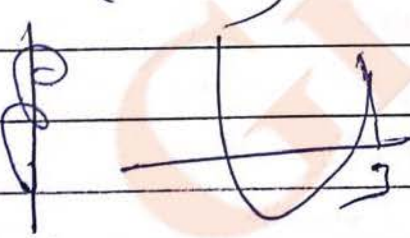
Case 1st! ⇒ $x < 0$ Case 2nd $x \geq 0$

$\dfrac{x^2 + 7x + 10}{(x-3)^2} < 0$

$\dfrac{(x+2)(x+5)}{(x-3)^2} < 0$

$(-5, -2)$

7.)



(i) $D \geq 0$

(ii) $1 \cdot f(3) > 0$

iii) $\frac{-b}{2a} \leq 3$

10) $\left|1 - \frac{|x|}{1+|x|}\right| \geq \frac{1}{2}$

$$\boxed{\begin{array}{c} |x| \geq a \\ x \leq -a \text{ or } x \geq a \end{array}}$$

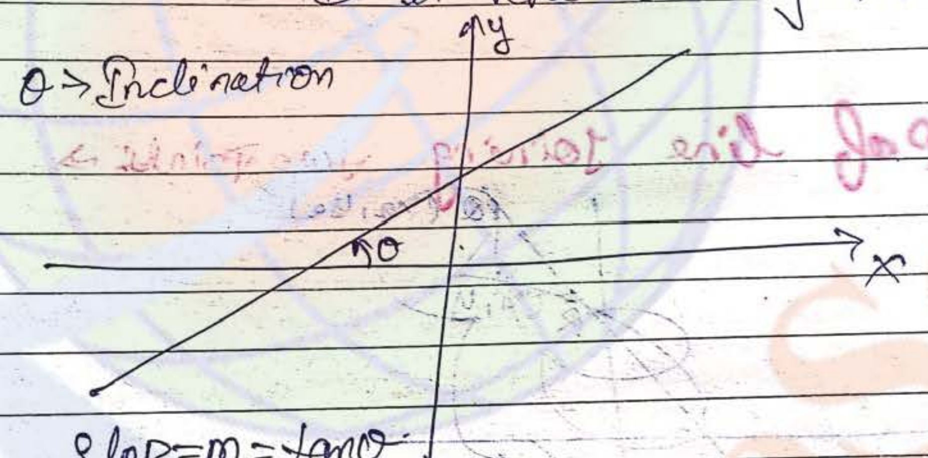
$1 - \frac{\lvert x\rvert}{1+\lvert x\rvert} \leq -\frac{1}{2}$	or $1 - \frac{\lvert x\rvert}{1+\lvert x\rvert} \geq \frac{1}{2}$
$\frac{3}{2} \leq \frac{\lvert x\rvert}{1+\lvert x\rvert}$	$\frac{1}{2} \geq \frac{\lvert x\rvert}{1+\lvert x\rvert}$
$3 + 3\lvert x\rvert \leq 2\lvert x\rvert$	$1 + \lvert x\rvert \geq 2\lvert x\rvert$
$\lvert x\rvert \leq -3$	$\lvert x\rvert \leq 1$
✗	

$$\boxed{-1 \leq x \leq 1}$$

Cartesian → Straight line

It is a curve such that the line segment joining any two point on it completly lies on it.

Slope of straight line :→
It is a tangent of an angle made by the line with positive direction of x-axis measured in anticlockwise direction.
It is represented by 'm'.



Slop = m = tan θ

θ lies between 0 to π

$$\boxed{0 \le \theta < \pi}$$

→ $\theta \in [0, \pi)$

(ii) Slope of the line parallel to y-axis is not defined

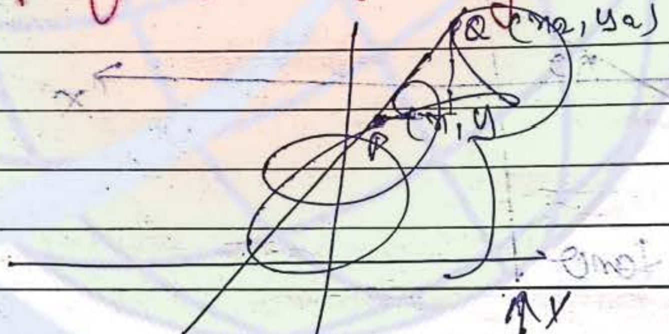
iii) If points A, B, C are collinear then slope of AB

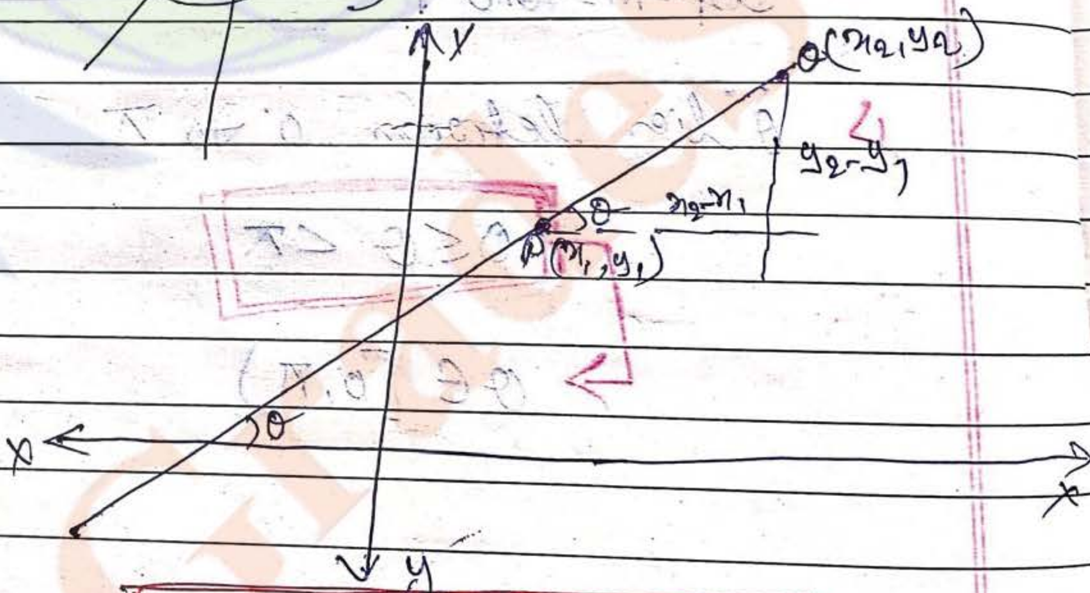
Slope of AB = Slope of BC

= Slope of AC

(Prove collinear)

Slope of line joining two points ⇒





$$\boxed{\text{So, } m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}}$$

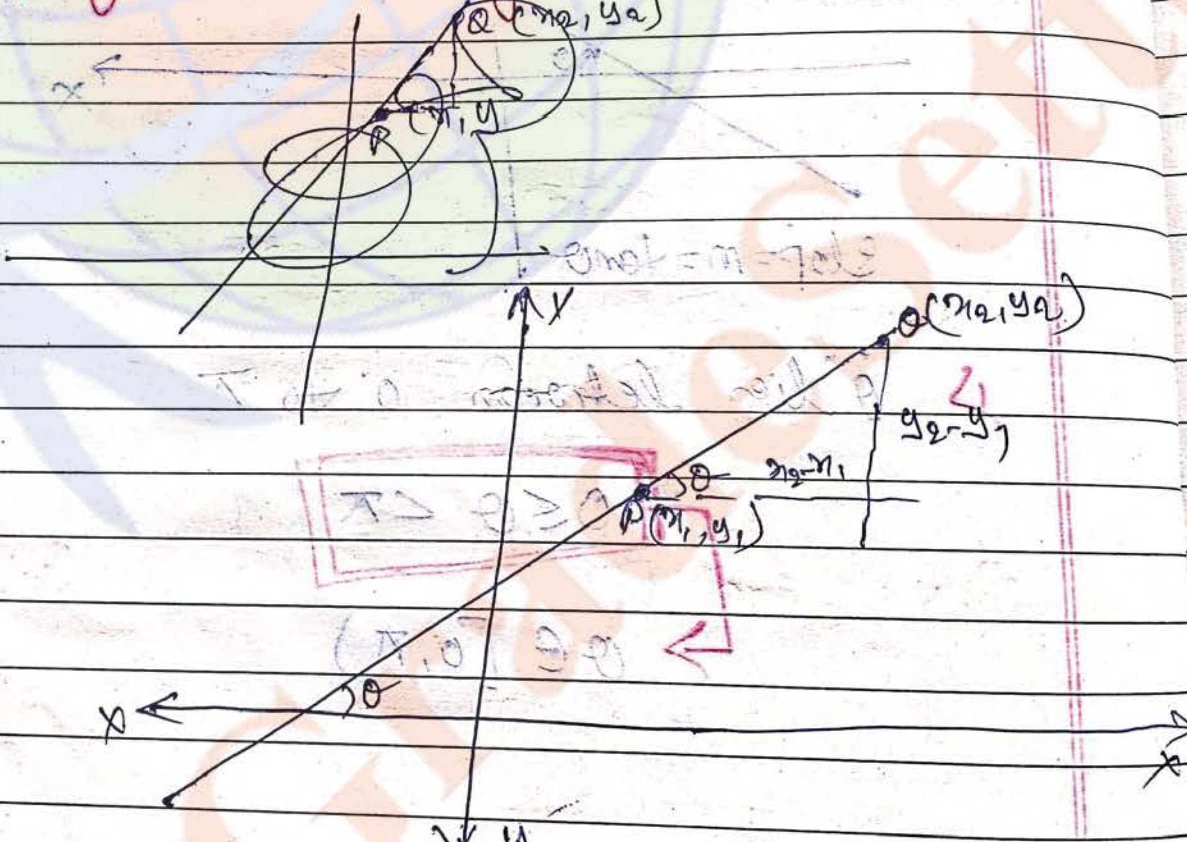
(ii) Slope of the line parallel to y-axis is not defined

iii) If points A, B, C are collinear then slope of AB

Slope of AB = Slope of BC (Prove Collinear)

= Slope of AC

Slope of line joining two points ⇒



So, $\boxed{m = \tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}}$

Q-) Find the slope of the line passing through the line

(i) (1,6), (-4,2)

(ii) (5,9), (2,9)

(i)

$\tan\theta = \frac{-4-1}{-4-6} = \frac{-5}{-10} = \frac{4}{5}$

(ii) $\tan\theta = 0$

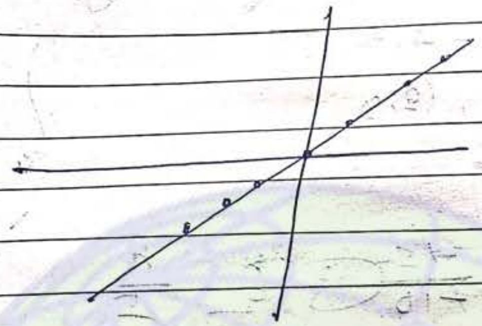
Q-) Show that point (1,1), (4,3) and (7,5) are collinear

A B C

(1,1) (4,3) (7,5)

(X) Equation of Straight line :⇒

$$y = x$$



It is a linear eq in 'x' and 'y' which is satisfied by each and every point on the line.

Equation of Any Straight line can be expressed by $ax + by + c = 0$. (where a and b are not zero simultaneously)

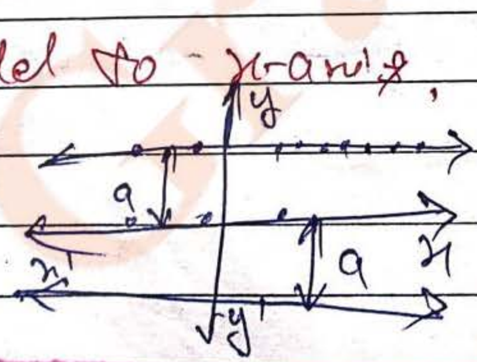
$$ax + by + c = 0$$
$$\Downarrow$$
$$\left(\frac{a}{c}\right)x + \left(\frac{b}{c}\right)y + 1 = 0$$

$$\lambda_1 x + \lambda_2 y + 1 = 0$$

Straight line parallel to coordinate axes

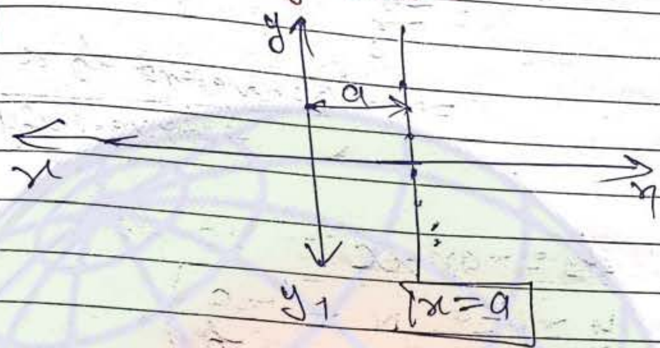
(1) Parallel to x-axis.

$y = a$



$y = -a$

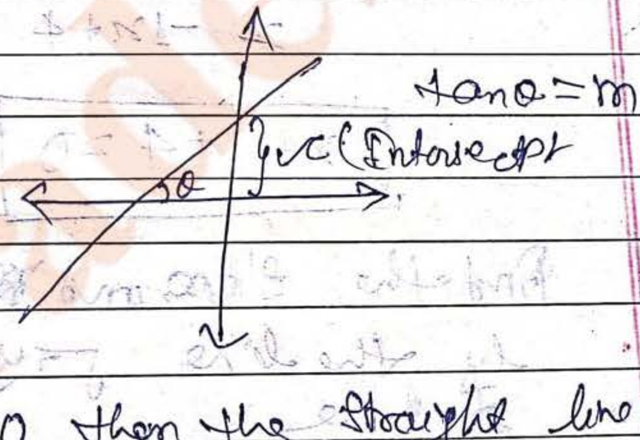
2) Parallel to y-axis:→



Note:⇒
Equation of x-axis is $y = 0$ and eq: of y-axis is $x = 0$.

Equation of straight line in different forms:⇒

1) Slope Intercept form:→
Equation of the straight line with slope 'm' and which cuts an intercept of 'c' on y-axis is $\boxed{y = mx + c}$



Note:⇒ i) If $m=0$ then the straight line is parallel to x-axis.
~~ii)~~ $m=0$
iii) If $c=0$ then the line passes through the origin.

iii) Slope of line is $ax+by+c=0$

$$m = \frac{-a}{b}$$

$$= -\left(\frac{\text{coefficient of } x}{\text{coefficient of } y}\right)$$

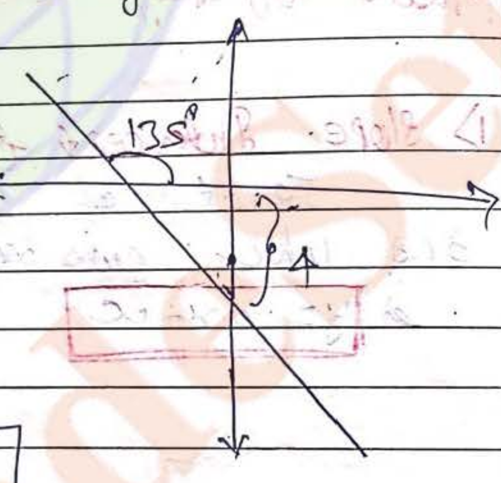
$$by = -ax - c$$

$$y = \left(\frac{-a}{b}\right)x + \left(\frac{-c}{b}\right)$$

$$y = mx + c'$$

Example: Find the equation of straight line which slope is -1 and and cutting of intersect of 4 units on negative direction on y-axis.

Ans:



$m = -1$

$c = -4$

$y = mx + c$

$\Rightarrow -1x + 4$

$\Rightarrow -1x + 4$

$$\boxed{x + y + 4 = 0}$$

Exam: Find the slope and intersect on y-axis by the line $\sqrt{3}y + x = \sqrt{3}$. And draw the line.

$$\sqrt{3}y = \sqrt{3} - x$$

$$y = \frac{\sqrt{3}}{\sqrt{3}} - \frac{x}{\sqrt{3}}$$

$y = mx + c$

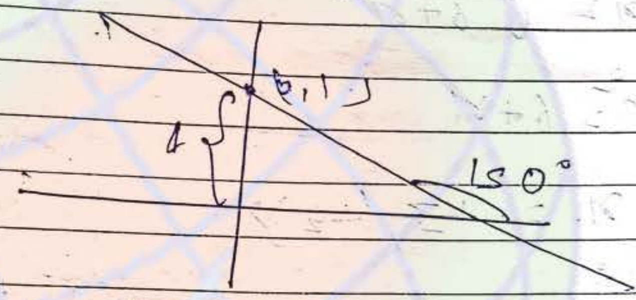
$m = 1$

$$y = \cancel{0} + 1 - \frac{x}{\sqrt{3}}$$

$$y = \left(-\frac{1}{\sqrt{3}}x\right) + 1 \qquad \because y = mx + c$$

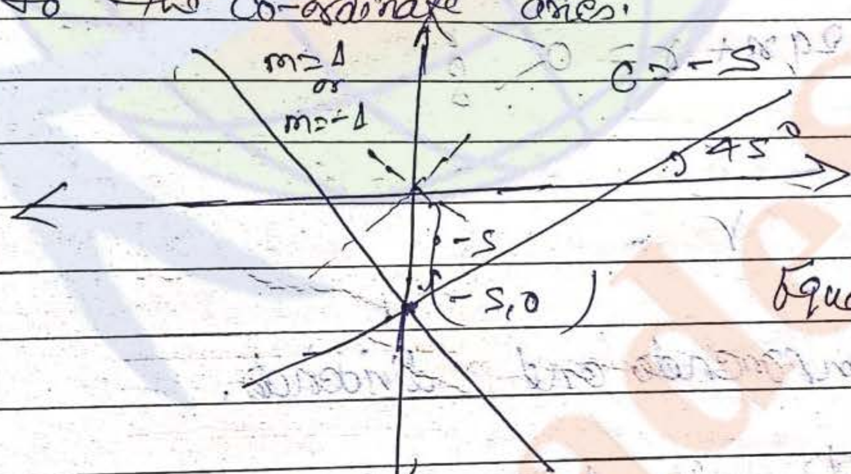
$$m = -\frac{1}{\sqrt{3}}$$

$$c = 1$$



Q) Find the equation of straight line cutting off an intercept of −5 on y-axis and equally inclined to the co-ordinate axes.

Sol



Equation (i) $y = x - 5$

(ii) $y = -x - 5$

Study material Sheet — Read Quadratic — Ex 2

(a)

$x = \sqrt{6+\sqrt{6+\sqrt{6}}}$ then

$x = \sqrt{6+x}$

$x^2 = 6+x$

$x = +3, \cancel{(-2)}$

(b) $ax^2+2bx+c=0 \quad \alpha, \beta$

$px^2+2qx+r=0 \quad \gamma, \delta$

$\frac{\beta}{\alpha} = \frac{\delta}{\gamma}$

use componendo and dividendo,

$\cancel{\frac{\beta+\alpha}{\alpha-}} \quad \frac{\beta+\alpha}{\beta-\alpha} = \frac{\delta+\gamma}{\delta-\gamma}$

$\frac{(\beta-\alpha)^2}{(\beta-\alpha)^2} = \frac{(\delta+\gamma)^2}{(\delta-\gamma)^2}$

$\frac{(\beta+\alpha)^2}{(\beta+\alpha)^2-4\alpha\beta} = \frac{(\delta+\gamma)^2}{(\delta+\gamma)^2-4\delta\gamma}$

$$\frac{\left(-2b/a\right)^2}{\left(\frac{-2b}{a}\right)^2 - 4 \cdot \frac{c}{a}} = \frac{\left(-2q/p\right)^2}{\left(\frac{-2q}{p}\right)^2 - 4\frac{r}{p}}$$

5.)

$$x^3 - x - 1 = 0 \begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix} \quad \text{—} (1)$$

$$\sum \frac{1+\alpha}{1-\alpha}$$

$$p = \frac{1+\alpha}{1-\alpha}, \quad q = \frac{1+\beta}{1-\beta}, \quad r = \frac{1+\gamma}{1-\gamma}$$

$$p = \frac{1+\alpha}{1-\alpha}$$

$$p - p\alpha = 1 + \alpha \Rightarrow \alpha(1+p) = p - 1$$

$$\alpha = \frac{p-1}{p+1} \quad \text{—} (2)$$

α is a root of eq (1)

$$\left(\frac{p-1}{p+1}\right)^3 - \left(\frac{p-1}{p+1}\right) - 1 = 0$$

$$(p-1)^3 - (p-1)(p+1)^2 - (p+1)^3 = 0$$

$$(x-1)^3 - (x-1)(x+1)^2 - (x+1)^3 = 0 \begin{matrix} p \\ q \\ r \end{matrix}$$

$$p+q+r = \frac{(-1)\text{ coeff of } x^{3-1}}{\text{coeff of } x^3}$$

$\boxed{3>2 \text{ than } \frac{1}{3}<\frac{1}{2}}$

6.) $f(x)=Ax^2-Bx+C$

$$B=(\sin\theta+\operatorname{cosec}\theta)^2+(\cos\theta+\sec\theta)^2-(\tan^2\theta+\cot^2\theta)$$

$$2(\sin^2\theta+\operatorname{cosec}^2\theta+2)+(\cos^2\theta+\sec^2\theta+2)-(\tan^2\theta+\cot^2\theta)$$

$$\Rightarrow 1+1+1+2+2=7$$

7.) $\alpha-\beta \le 4$

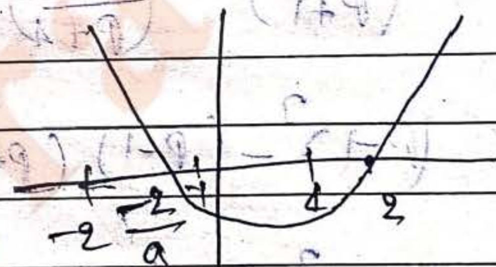
$$(\alpha-\beta)^2\le 16$$

$$(\alpha+\beta)^2-4\alpha\beta\le 16$$

8.) $x=p,q,0$ — (कमती की [illegible])

9.) $f(x)=ax^2+2x(1-a)-4$

$$\Rightarrow (x-2)(ax+2)$$



$$-2\le \frac{-2}{a}<-1$$

$$2\ge \frac{2}{a}>1$$

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$$\Rightarrow \frac{1}{2} \le \frac{a}{2} < 1 \Rightarrow 1 \le a < 2$$

11.) Let P is a common root :-

$$\alpha P^2 - 7P - 2 = 0$$

$$-2P^2 + 7P + \alpha = 0$$

$$\frac{P^2}{7\alpha + 14} = \frac{P}{4 - \alpha^2} = \frac{1}{7\alpha + 14}$$

$$P = \frac{7\alpha + 14}{4 - \alpha^2} \quad — (2) \qquad P = \frac{4 - \alpha^2}{7\alpha + 14} \quad — (1)$$

$$P = \frac{7\alpha + 14}{4 - \alpha^2} \quad — (2)$$

(1) - (2)

12) $(x^2+1)^2 = x(3x^2+4x+3)$

$= x^4 + 2x^2 + 1 = 3x^3 + 4x^2 + 3x$

$$x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$$

Let

$$x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$$

$$(x^2 + Px + 1)(x^2 + qx + 1)$$

$$= x^4 + qx^3 + x^2 + Px^3 + Pqx^2 + Px + x^2 + qx + 1$$

$$= x^4 + x^3(P+q) + x^2(2+Pq) + x(P+q) + 1$$

Compute coefficient

$(P+q) = -3$,

$2 + Pq = -2$

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14.) $(15+4\sqrt{14})^t + (15-4\sqrt{14})^t = 30$

$y + \frac{1}{y} = 30$

$y^2 - 30y + 1 = 0$

$(15+4\sqrt{14})^t + \left(15-4\sqrt{14} \times \frac{15+4\sqrt{14}}{15+4\sqrt{14}}\right)^t = 30$

$(15+4\sqrt{14})^t + \left(\frac{1}{15+4\sqrt{14}}\right)^t = 30$

$y + \frac{1}{y} = 30$

$y^2 - 30y + 1 = 0$

$y = 15 \pm 4\sqrt{14}$

$(15-4\sqrt{14})^t = 15+4\sqrt{14}$ | $(15+4\sqrt{14})^t = (15-4\sqrt{14})$

$t = 1$ | $t = -1$

15.) $\alpha + 2\beta = 0$

$(\alpha+\beta) + \beta = 0$

$\alpha + \beta = 0$

$\boxed{\beta = -p}$

$(-p)^2 - p(-p) + q = 0$

14) $(15+4\sqrt{14})^t + (15-4\sqrt{14})^t = 30$

$y + \frac{1}{y} = 30$

$y^2 - 30y + 1 = 0$

$(15+4\sqrt{14})^t + \left(15-4\sqrt{14} \times \frac{15+4\sqrt{14}}{15+4\sqrt{14}}\right)^t = 30$

$(15+4\sqrt{14})^t + \left(\frac{1}{15+4\sqrt{14}}\right)^t = 30$

$y + \frac{1}{y} = 30$

$y^2 - 30y + 1 = 0$

$y = 15 \pm 4\sqrt{14}$

$(15-4\sqrt{14})^t = 15+4\sqrt{14}$

$t = 1$

$(15+4\sqrt{14})^t = (15-4\sqrt{14})$

$t = -1$

15) $\alpha + 2\beta = 0$

$(\alpha+\beta) + \beta = 0$

$\alpha + \beta = 0$

$\boxed{\beta = -p}$

$(-p)^2 - p(-p) + q = 0$

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19.) $\sqrt{9\sqrt{a\cdot\sqrt{a}}} = y$

$\sqrt{a\cdot y} = y$

$ay = y^2$

$\boxed{y = 9}$

$\sqrt{x\sqrt{x\sqrt{x}}} = x$

$\left(\sqrt{49 + 20\sqrt{6}}\right)^6 + \left(5 - 2\sqrt{6}\right)^{x^2 + x - 3 - x} = 0$

$\left[\sqrt{(5+2\sqrt{6})^2}\right]^{9} + \left[5-2\sqrt{6}\right]^{x^2-3} = 10$

$\Rightarrow (5+2\sqrt{6})^9 + (5-2\sqrt{6})^9 = 10$

$\Rightarrow \frac{(5+2\sqrt{6})^9}{y} + \left(\frac{1}{5+2\sqrt{6}}\right)^9 = 10$

$a = +1, -1$

$x^2 - 3 = 1$

$x = -2, 2$

20.) ans:- $x^2 - bx + c = 0$ α β

$\alpha = \sqrt{7-4\sqrt{3}}$

$= \sqrt{4 - 2\cdot 2\cdot\sqrt{3} + 3}$

$= \sqrt{(2-\sqrt{3})^2}$

$\alpha = 2-\sqrt{3}$
$\beta = 2+\sqrt{3}$

$\alpha+\beta = 4 = b$
$\alpha\beta = 1 = c$

21.) $(a+2)x^2 + bx + c = 0$

$2x^2 + 3x + 4 = 0$ — $a+ib$, $a-ib$

$\frac{a+2}{2} = \frac{b}{3} = \frac{c}{4}$

$\left.\begin{matrix} c=4 \\ b=3 \\ a=0 \end{matrix}\right\} \times$ $\left.\begin{matrix} c=8 \\ b=6 \\ a=2 \end{matrix}\right\} \checkmark$

28.) $x^2 + ax + b = 0$ $a, b \in I$

$x = \left(\frac{-a \pm \sqrt{a^2-4b}}{2}\right)$

(1) $a \leftrightarrow$ odd

$= \frac{-odd \pm \sqrt{(odd)^2 - even}}{2}$

$= \frac{-odd \pm odd}{2}$

case $a \leftrightarrow$ even

$= \frac{-Even \pm \sqrt{(even)^2 - even}}{2}$

$= \frac{even}{2} \in I$.

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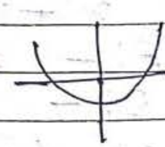
31.) $f(0) \Rightarrow \underset{\ominus}{a}\ \underset{\oplus}{b}\ \underset{\oplus}{c}$



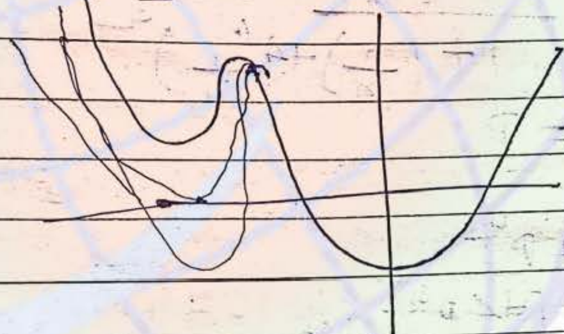
32.)

$$(a_1, a_2, a_3, a_4, a_5) \equiv (1, -4, 6, 7, -10)$$

$$f(x) = a_1x^4 + a_2x^3 + a_3x^2 + a_4 + a_5$$

$$f(1) = a_1 + a_2 + a_3 + a_4 + a_5 = 0$$

$$f(0) = a_5 = -10 < 0$$



34.) $f(x) = 4x^3 + 3x^2 - 6x + 1$

$$f'(x) = 12x^2 + 6x - 6$$

$$f'(x) = 0$$

$$x = -1, \tfrac{1}{2}$$

$f(0) > 0 \quad f\left(\frac{1}{2}\right) < 0$

$f(1) > 0$

Quadratic

$\boxed{E x > 3}$

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3.) $x^2 + px + q = 0$

$A :- \; 2 \times 6 = q$

$B :- \; -9 + 7 = -p$

equation :- $x^2 + 7x + 12 = 0$

5.)

4.) $E = \dfrac{x^2 + y^2}{x^2 + xy + 4y^2}$

6.)

Divide by x^2

$$\frac{1 + \left(\frac{y}{x}\right)^2}{1 + \left(\frac{y}{x}\right) + 4\left(\frac{y}{x}\right)^2}$$

let $\left(\frac{y}{x}\right) = P$

$$E = \frac{1 + P^2}{1 + P + 4P^2}$$

$E + EP + 4EP^2 = 1 + P^2$

$P^2(4E - 1) + EP + (E - 1) = 0$

$\Delta \geq 0 \quad [\because P \in R]$

$x^2 - 5x + 6 \leq 0$

$(x-2)(x-3) \leq 0$

$+ \quad 2 \quad - \quad 3 \quad +$

$E^2 - 4(4E - 1)(E - 1) \geq 0$

$-15E^2 + 20E - 4 \geq 0$

$15E^2 - 20E + 4 \leq 0$ m M

$+ \quad m \quad - \quad M \quad +$

Avg $\dfrac{m + M}{2} = \dfrac{\left(\frac{20}{15}\right)}{2} = \dfrac{2}{3}$

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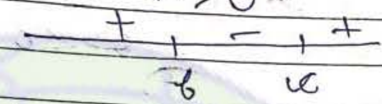
5.) $\frac{x^2 - bc}{2x - b - c} = y$

$\Delta \geq 0$



6.)

$\frac{ax^2 + 3x - 4}{a + 3x - 4x^2} = y$

$\Rightarrow ax^2 + 3x - 4 = ay + 3xy - 4x^2 y$

$\Rightarrow x^2(a + 4y) + 3x(1 - y) - (ay + 4) = 0$

$\Delta \geq 0$

$\Rightarrow 9(1 - y)^2 + 4(a + 4y)(ay + 4) \geq 0$

$\Rightarrow y^2(9 + 16a) + 2(2a^2 + 23)y + (9 + 16a) \geq 0$

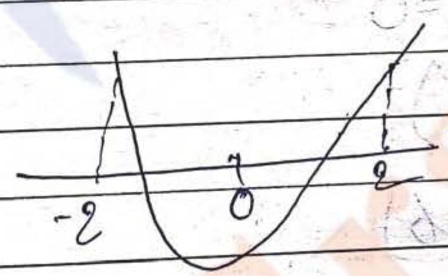
(a) coeff of $y^2 > 0$

(b) $\Delta \leq 0$

10.)

$f(x) = ax^2 + bx + c$

case I $\boxed{a > 0}$



$f(2) > 0$

$4a + 2b + c > 0$

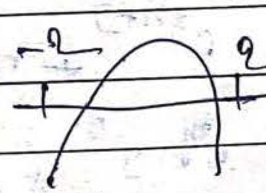
$1 + \frac{c}{4a} + \frac{b}{2a} > 0$ — (1)

$f(-2) > 0$

$1 + \frac{c}{4a} - \frac{b}{2a} > 0$ — (2)

$\Rightarrow 1 + \frac{c}{4a} - \left|\frac{b}{2a}\right| > 0$

Case II $\boxed{a < 0}$



$f(2) < 0$

$f(-2) < 0$

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(12.) $f(x) = \dfrac{ax^2 + 2(a+1)x + 9a + 4}{\underbrace{x^2 - 8x + 32}_{D<0 \;\; (+)}}$

$\Rightarrow ax^2 + 2(a+1)x + 9a + 4 < 0 \quad \forall x \in R$

(i) $a < 0$

(ii) $\Delta < 0$

13.)

$x^2 - ax + b = 0 \; \begin{matrix} \alpha \\ \beta \end{matrix}$

$x(x^2 - px + q) = 0$

(i) $x = 0$ not a common root

(ii) $x^2 - px + q = 0 \; \begin{matrix} \alpha \\ \beta \end{matrix}$

$\alpha + \alpha = p$

$\alpha\alpha = p$ — (1)

Also :- $\alpha^2 = q$ — (2)

$\alpha^2 - a\alpha + b = 0$

$q - \dfrac{ap}{2} + b = 0$

$\dfrac{ap}{2} = (q + b)$

$ap = 2(q + b)$

18.)

$2^{2x+3} + 3^{2x+1} = 10 \cdot 6^x$

$2^{2x} \cdot 3^3 + 3^{2x} \cdot 3^1 = 10(2 \times 3)^x$

$8 \cdot 2^{2x} + 3 \cdot 3^{2x} = 10 \cdot 2^x \cdot 3^x$

$$8\cdot\frac{2^{2x}}{2^x\cdot 3^x}+3\cdot\frac{3^{2x}}{2^x\cdot 3^x}=10$$

$$8\cdot\left(\frac{2^x}{3^x}\right)+3\left(\frac{3^x}{2^x}\right)=0$$

$$\underbrace{8\left(\frac{2}{3}\right)^x}_{y}+3\left(\frac{3}{2}\right)^x=10$$

$$8y+\frac{3}{y}=10$$

19.)

$$\left(\frac{x}{1+x^2}\right)^2-(m-3)\left(\frac{x}{1+x^2}\right)+m=0$$

$$\frac{x}{1+x^2}\to y$$

(a) $D\geq 0$

(b) $1\cdot f\left(\frac{1}{2}\right)>0$

(c) $1\cdot f\left(-\frac{1}{2}\right)>0$

(d) $-\frac{1}{2}<\frac{-b}{2a}<\frac{1}{2}$

$\to y^2-(m-3)y+m=0$

$$\frac{x}{1+x^2}=y$$

$$x=y+yx^2$$

$$yx^2-x+y=10\Rightarrow\boxed{D\geq 0}$$

$$1-4y^2\geq 0$$

$$4y^2-1\leq 0$$

$$(2y-1)(2y+1)\leq 0$$

$+\quad -\frac{1}{2}\quad\quad \frac{1}{2}\quad +$

$$y\in\left[-\frac{1}{2},\frac{1}{2}\right]$$

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21.) Let α be a common root

$$\alpha^2 + a\alpha + 12 = 0 \quad \text{---(I)}$$
$$\alpha^2 + b\alpha + 15 = 0 \quad \text{---(II)}$$
$$\alpha^2 + (a+b)\alpha + 36 = 0 \quad \text{---(III)}$$

eq (I) - eq (II)

$$\alpha(a-b) = 3 \quad \text{---(4)}$$

eq (III) - eq (II)

$$a\alpha = -21 \quad \text{---(5)}$$

eq (III) - eq (I)

$$b\alpha = -24 \quad \text{---(6)}$$

so,
$$\frac{a\alpha}{b\alpha} = \frac{-21}{-24}$$

$$\frac{a}{b} = \frac{7}{8} \quad \text{---(1)}$$

$$\frac{a-b}{a} = \frac{3}{-21} \quad \text{---(2)}$$

22.)

$$x^4 - 18x^3 + kx^2 + 200x - 1984 = 0 \quad \alpha, \beta, \gamma, \delta$$

$$\alpha\beta = -32 \quad \text{---(I)}$$

$$(\alpha\beta)\gamma\delta = \frac{-1984}{1}$$

$$\gamma\delta = 62 \quad \text{---(2)}$$

$$\underbrace{\alpha + \beta}_{} + \underbrace{\gamma + \delta}_{} = 18$$

$$A + B = 18 \quad \text{---(I)}$$

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$\alpha\beta\gamma + \beta\gamma\delta + \gamma\delta\alpha + \delta\alpha\beta = -200$

$\alpha\beta(\gamma+\delta) + \gamma\delta(\beta+\alpha) = -200$

$-32\cdot B + 62\cdot A = -200$ — (3)

$31A - 16B = -100$ — (4)

$K = \alpha\beta + \beta\gamma + \gamma\delta + \delta\alpha + \beta\delta + \gamma\alpha$

$= -32 + 62 + \beta(\gamma+\delta) + \alpha(\gamma+\delta)$

$K = 30 + (\alpha+\beta)(\gamma+\delta)$

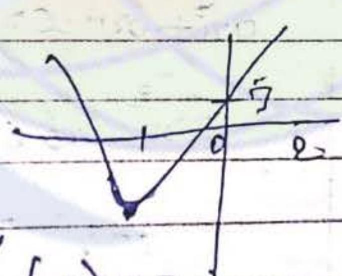
$= 30 + AB$

$A = 4$

$B = 14$

Passage-1

(24) $f(x) = 4x^2 - 4ax + a^2 - 2a + 2$



$f(0) \geq 3$

Passage-2

$f_1(x) = a_1x^2 + b_1x + c_1$ (α, β), $f_2(x) = a_2x^2 + b_2x + c_2$ (γ, δ)

$\alpha + \beta = \gamma + \delta$

$\frac{-b_1}{a_1} = \frac{-b_2}{a_2}$ — (1)

$\frac{\alpha+\beta+\gamma+\delta}{4} = 1$

$\frac{-b_1}{a_1} - \frac{b_2}{a_2} = 4$

$\frac{-2b_1}{a_1} = 4$

$2a_1 + b_1 = 0$

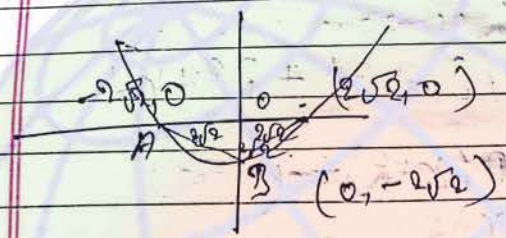
Sheet

M 2 4

28.) $\frac{-b}{2a} = \frac{-b_2}{2a_2}$

Passage ii

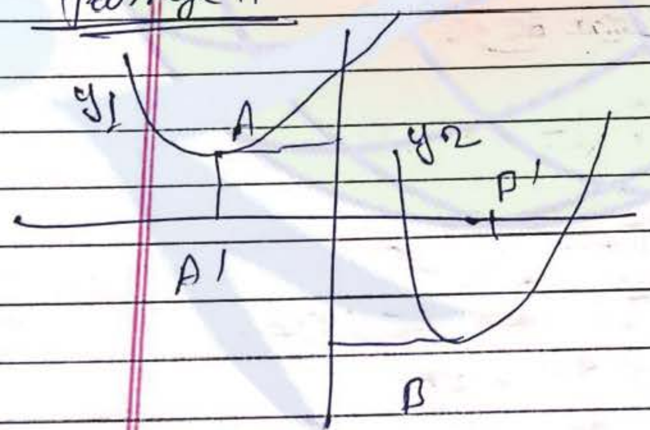
$\frac{-b}{2a} = \frac{-b_2}{2a_2}$



32) $-2\sqrt{2} < \frac{k}{2} < 2\sqrt{2}$

$-4\sqrt{2} < k < 4\sqrt{2}$

Passage H



$y_1 = x^2 + 2ax + b$

$y_2 = cx^2 + 2dx + 1$

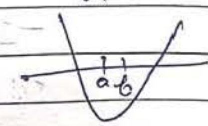
(i) $\frac{-2a}{2} + \left(\frac{-2d}{2c}\right) = 0$

ii $\frac{-D}{4a} + \frac{-D_2}{4a_2} = 0$

3) $f(x) = (x-a)(x-b) - 1$

$f(a) = -1$

$f(b) = -1$



6.)

4.) $ax^2 + bx + c = 0$ ⟨ α, β

$Ax^2 + Bx + C = 0$ ⟨ $\alpha + \delta = p$, $\beta + \delta = q$

$(\alpha - \beta)^2 = (p - q)^2$

$(\alpha + \beta)^2 = (p + q)$

$(\alpha + \beta)^2 - 4\alpha\beta = (p+q)^2 - 4pq$

$\left(\frac{-b}{2a}\right)^2 - \frac{4c}{a} = \left(\frac{-B}{2A}\right)^2 - \frac{4C}{A}$

7

5.)

$ax^2 + bx + c = 0$ ⟨ α, β

$a^3x^2 + abcx + c^3 = 0$ ⟨ p, q

$p + q = \frac{-abc}{a^3}$

$= -\left(\frac{b}{a}\right)\left(\frac{c}{a}\right)$

$(p+q) = (\alpha+\beta)(\alpha\beta)$

$pq = \frac{c^3}{a^3} = \left(\frac{c}{a}\right)^3$

$pq = \alpha^3\beta^3$ — ②

from ① & ②

$p + \frac{\alpha^3\beta^3}{p} = \alpha\beta(\alpha+\beta)$

$\Rightarrow P^2 - \alpha\beta(\alpha+\beta)P + \alpha^3\beta^3 = 0$

$\Rightarrow (P - \alpha^2\beta)(P - \alpha\beta^2) = 0$

6.

$x^2 - x + P = 0 \; \begin{matrix} a \\ ar \end{matrix}$

$x^2 - 4x + q = 0 \; \begin{matrix} ar^2 \\ ar^3 \end{matrix}$

$a + ar = 1$ — (1)

$ar^2 + ar^3 = 4$ — (2)

7.

$x^2 + (a-b)x + (1-a-b) = 0$

$\Delta > 0 \quad \forall\, b \in R$

$(a-b)^2 - 4 \cdot 1 \cdot (1-a-b) > 0$

$a^2 + b^2 - 2ab - 4 + 4a + 4b > 0$

$b^2 - 2b(2-a) + (a^2 + 4a - 4) > 0$

$\forall\, b \in R$

$D < 0$

$4(2-a)^2 - 4(a^2 + 4a - 4) < 0$

$4(4 - 4a + a^2 - a^2 - 4a + 4) < 0$

11.

$x^2 - 10cx - 11d = 0 \; \begin{matrix} a \\ b \end{matrix}$

$x^2 - 10ax - 11b = 0 \; \begin{matrix} c \\ d \end{matrix}$

$a + b = 10c$ — (1)

$c + d = 10a$ — (2)

$a + b + c + d = 10(a + c)$ — (3)

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$(1) - (2)$

$(a-c) + (b-d) = 10(c-a)$

$b-d = 11(c-a) \quad -(4)$

15 >

$a^2 - 10ac - 11d = 0$

$c^2 - 10ac - 11b = 0$

Substract

$a^2 - c^2 + 11(b-d) = 0$

$a^2 - c^2 = 11(d-b)$

$(a+c)(a-c) = 11(d-b)$

$(a+c)(c-a) = 11(b-d)$

From (4) we

$(a+c)(c-a) = 11 \cdot 11(c-a)$

$a+c = 121$

from (1)

$a+b+c+d = 10(121)$

12 >

$x^2 - px + r = 0 \quad \alpha, \beta$

$x^2 - qx + r = 0 \quad \alpha/2, 2\beta$

$\alpha + \beta = p \quad -(1)$

$\frac{\alpha}{2} + 2\beta = q \quad (2)$

$\alpha = ?$

$\beta = ?$

$r = \alpha \cdot \beta$

15.) $\alpha+\beta = -p$

$\alpha^3+\beta^3 = q$

$\gamma = \frac{\alpha}{\beta}, \ \delta = \frac{\beta}{\alpha}$

$\gamma+\delta = \frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)$

$\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$ — (1)

$\alpha^3+\beta^3 = q$

$(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = q$

$(-p)^3 - 3\alpha\beta(-p) = q$

$3\alpha\beta p = p^3+q$

$\alpha\beta = \left(\frac{p^3+q}{3p}\right)$ — (2)

16.) $x^2-6x-2=0 <^{\alpha}_{\beta}$

$a_n = \alpha^n - \beta^n$

$\frac{a_{10}-2a_8}{2a_9}$

$\Rightarrow \frac{(\alpha^{10}-\beta^{10}) - 2(\alpha^8-\beta^8)}{2a_9}$

$\Rightarrow \frac{\alpha^{10}-2\alpha^8-\beta^{10}+2\beta^8}{2a_9}$

$\alpha^2-6\alpha-2=0$

$\alpha^2-2=6\alpha$

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$$\Rightarrow \frac{\alpha^8(\alpha^2-a)-\beta^8(\beta^2-a)}{2a^9}$$

$$\Rightarrow \frac{\alpha^8(6\alpha)-\beta^8(6\beta)}{2a^9}$$

$$\Rightarrow \frac{6[\alpha^9-\beta^9]}{2a^9}$$

$$\Rightarrow 3\ \frac{a^9}{a^9} \Rightarrow 3$$

17.

$\alpha \rightarrow$ common root

$$\alpha^2+b\alpha-1=0 \quad (1)$$

$$\alpha^2+\alpha+b=0 \quad (2)$$

$$\Rightarrow \alpha(b-1)-1-b=0$$

$$\alpha=\left(\frac{b+1}{b-1}\right) \quad (3)$$

$\Rightarrow$ from (1)

$$\Rightarrow \left(\frac{b+1}{b-1}\right)^2+b\left(\frac{b+1}{b-1}\right)-1=0$$

$$b^2+3=0$$

$$\Rightarrow \frac{(b+1)^2+b(b+1)(b-1)-(b-1)^2}{(b-1)^2}=0$$

$$\Rightarrow (b+1)^2-(b-1)^2+b(b+1)(b-1)=0$$

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$\Rightarrow (2b)(2) + b(b-1) > 0$

$b(b^2+3) > 0$

$b > 0$

$b^2 = -3$

$b = \pm i\sqrt{3}$

13.) (i) $f(x) = \dfrac{x^2-6x+5}{x^2-5x+6}$

(i) $f(x) > 0$

$\dfrac{x^2-6x+5}{x^2-5x+6} > 0$

$\dfrac{(x-1)(x-5)}{(x-2)(x-3)} > 0$

+ | − | + | − | +
1, 2, 3, 5

(ii) $f(x) < 1$

$\dfrac{x^2-6x+5}{x^2-5x+6} < 1$

$\Rightarrow \dfrac{\cancel{x^2}-6x+5-\cancel{x^2}+5x-6}{x^2-5x+6} < 0$

$\Rightarrow \dfrac{-(x+1)}{(x-3)(x-2)} < 0$

$\Rightarrow \dfrac{(x+1)}{(x-2)(x-3)} > 0$

− | + | − | +
1, 2, 3

5.) $3^{2x^2} - 2\cdot 3^{x^2+x+6} + 3^{2(x+6)} = 0$

$(3^{x^2})^2 - 2\cdot 3^{x^2}\cdot 3^{x+6} + (3^{x+6})^2 = 0$

$(3^{x^2} - 3^{x+6})^2 = 0$

$3^{x^2} - 3^{x+6} = 0$

$3^{x^2} = 3^{x+6}$ $\quad \therefore x^2 = x+6$

Greatest Integer less than or equal to x

$a, A_1, A_2 \ldots A_n, b$

$b - A_n = d$

$A_n = b - d$

$\frac{AP}{PB} = \frac{2}{1}$

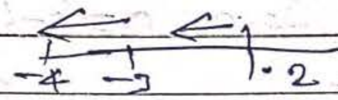
$\frac{AP}{BP} = \frac{6}{3} = \frac{2}{1}$

6) Concept –

Note ⇒ G.I.F

$[x] = G.I.F$

$[1.2] = 1$



$[\pi] = 3$

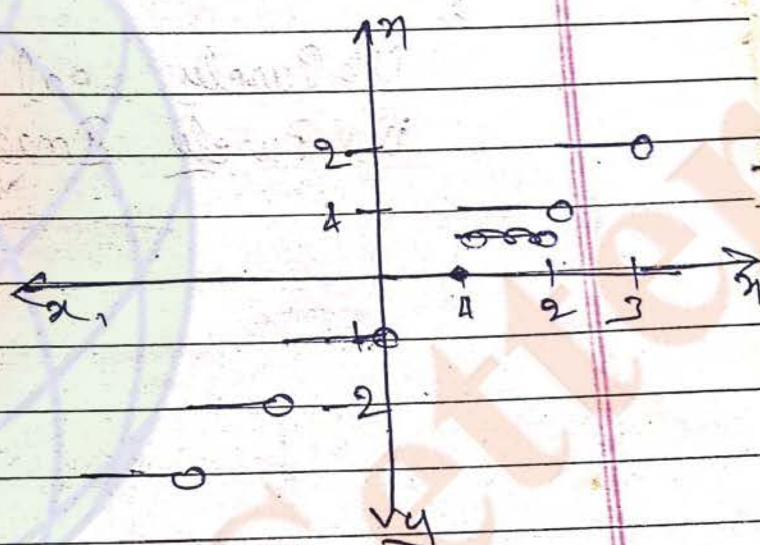
$[-\pi] = -4$ $\quad [1] = 1$

$[x] = [\pi]$

$[x] = 3$

$[3, 4)$

$y = [x]$



7) $\left|\frac{x}{|x|}\right| = \frac{x}{|x|}$

$\frac{x}{|x|} > 0$

$x \in R^+$