WWW.GRADESETTER.COM

(1)	Permutation and combination (Contd)	✓
2)	Trigonometric ratio ~~Permutation and Combination (contd)~~	✓
3)	Trigonometric equation	✓
4)	Properties of triangle.	✓
5)	Radii of circle	✓
5)		
(6)	Inverse trigonometric functn	✓
(7)	mathematical Induction.	✓
8)		

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Combination of Alike objects

1.) No. of combinations of 'p' things alike of one kind; 'q' things alike of IInd kind; and 'r' things alike of third king. taking some of or all things at a time is

$$\boxed{(p+1)(q+1)(r+1)-1}$$

इसमें एक case ऐसा आयेगा। जिसमें कुछ भी नहीं होगा चूंकि कम से कम एक चाहिए इसलिए अनुपस्थित खाली वाले case को घटा देंगे।

(No. of ways to select at least one things)

○ ○ ○ ○ ○ ○

Note :→

B W

$(1+1)(1+1)-1 = 4-1 = 3$

B, W, BW, (—)

कोई नहीं इसे घटाना होगा

2) The No. of ~~formations~~ combination of 'p' things alike of one kinds; 'q' things alike of IInd kinds; 'r' things alike of IIIrd kind And Rest 't' different things taking some or all at a time is

$$\boxed{(p+1)(q+1)(r+1)2^t-1}$$

(No. of ways to select at least one things)

WWW.GRADESETTER.COM

$$\left\{ \overset{1}{O} \quad \overset{2}{O} \quad \overset{3}{O} \quad \overset{4}{O} \quad \overset{5}{O} \; \ldots \; t \right\}$$

$$\left({}^tC_1 + {}^tC_2 + {}^tC_3 \ldots + {}^tC_t + {}^tC_0 = 2^t \right)$$

(Example sheet-1) 18.

i) 5 apple, 6 mango, 11 banana

$$(\underset{1A}{1} + \underset{2A}{1} + \underset{3A}{1} + 1 + 1 + 1)\,(6+1)\;(11+1)$$

(यहाँ -1 नहीं होगा क्योंकि यह total माँगा रहा है खाली वाला भी Answer में आएगा)

ii) $(5+1)(6+1)(11+1) - \underset{\text{}}{1}$

↳ एक case ऐसा भी होगा जिसमें कोई भी नहीं होगा। उसे घटाया होगा।

iii) $\left(\underset{1A}{1} + \underset{2A}{1} + 1 + 1 + 1\right)\;(6)\;(11)$

iv) $\left(\underset{3A}{1} + \underset{4A}{1} + \underset{5A}{1}\right)\;(6+1)\;(9+1)$ → हो सकता है इता है।

GradeSetter

WWW.GRADESETTER.COM

No of Divisors! ⇒

$6 \Rightarrow 2^1 \times 3^1$

Divisors → $\underbrace{1,\overbrace{2,3}^{\text{Proper divisors}},6}_{\text{Divisors}}$

Let

$$\boxed{N = P_1^{\alpha_1} \cdot P_2^{\alpha_2} \cdots\cdots P_n^{\alpha_n}}$$

where

$P_1, P_2 \cdots P_n$ are distinct prime numbers

and

$\alpha_1, \alpha_2, \alpha_3 \cdots \alpha_n$ are Natural Numbers

i.e. $(\alpha_i \in N)$

1.) The total no. of divisors of ~~no.~~ number 'N' Including '1' and the no. itself is

$$\boxed{(\alpha_1+1)(\alpha_2+1) \cdots\cdots \alpha_n+1}$$

II) Total no. of proper divisors is

$$\boxed{(\alpha_1+1)(\alpha_2+1) \cdots (\alpha_n+1) - 2}$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

1st Choice

~~Q. Find the no. of proper~~ divisors of

(Example sheet-1)
19.

$N = 2^7 \cdot 3^5 \cdot 5^2 \cdot 7^3 \cdot 11^5$

i) $(7+1)(5+1)(2+1)(3+1)(5+1)$

ii) $(7+1)(5+1)(2+1)(3+1)(5+1) - 2$

iii) $(1+1+1+1+1+1+1)(5+1)(2+1)(3+1)(5+1)$

(one, 2) ~~two 2'~~

(कम से कम एक होना चाहिए) (दो तो होने चाहिए)

(सिर्फ दो का foundation है)

iv) $(2^0)(5+1)(2+1)(3+1)(5+1)$

दो तो आए ही नहीं (one)

एक आ जाए या दोनों आ जाए

या नहीं आए

v) (15)

$(7+1)(5)(2)(3+1)(5+1)$

एक 3 या पाँचों 3 आ जाए।

या एक 5 या दोनों 5 पाँच

(foundation है)

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

vi) $\underset{\downarrow}{(1} + 1 + 1 + 1 + 1)\ \underbrace{(5+1)(2+1)(3+1)(5+1)}$

three 2's
(कम से कम तीन हों)

(No boundation)

vii) $4x + 2$

~~कम~~ ही होने चाहिए

$2(2x+1)$

viii) $(2^0 + 2^1 - - 2^7)(3^0 + 3^1 + - - 3^5)(5^0 + 5^1 - - 5^2)$
$(7^0 + 7^1 - - 7^3)(11^0 + 11^1 - - 11^3)$

ix) $(2^1 - - - + 2^7)(3^0 + 3^1 + - - 3^5)(5^0 - - 5^2)$
$(7^0 - - - - - 7^3)(11^0 + 11^1 - - 11^5)$

x) $(2^0)(3^0 + 3^1 - - - 3^5)(5^0 + 5^1 - - 5^2)$
$(7^0 + 7^1 - - 7^3)$ ।

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Sum of divisors

Sum of divisors is given by

$$(P_1^0 + P_1^1 + P_1^2 + \dots P_1^{\alpha_1})(P_2^0 + P_2^1 + \dots P_2^{\alpha_2}) \dots$$

$$\dots (P_n^0 + P_n^1 + P_n^2 + \dots P_n^{\alpha_n})$$

eg: — $6 \Rightarrow 2^1 \times 3^1$

$$(2^0 + 2^1)(3^0 + 3^1)$$

$$(1+2)(1+3)$$

$\rightarrow 1 + 3 + 2 + 6$

WWW.GRADESETTER.COM

✗ Divisors into group

1:) The No. of ways in which (m+n) things can be divided into two groups containing 'm' and 'n' things is

$$\boxed{{}^{m+n}C_m \cdot {}^{n}C_n = \frac{\lfloor m+n}{\lfloor m \lfloor n}}$$

where {m ≠ n}

A B C D ⇒ 3 : 1

p

A B C	D
B C D	A
C D A	B
D A B	C

$$\frac{\lfloor 4}{\lfloor 3 \lfloor 1} = 4$$

so/

$$\boxed{{}^{m+n}C_m \cdot {}^{n}C_n = \frac{\lfloor m+n}{\lfloor m \lfloor n}}$$

Page No.
Date / /
1st Choice

2) The no. of ways in which '2n' different things can be divided into two groups each containing 'n' things is

$$\boxed{\frac{{}^{2n}C_n \cdot {}^{n}C_n}{\lfloor 2} = \frac{\lfloor 2n}{\lfloor 2 (\lfloor n)^2}}$$

$(m \neq n)$

A B C D

A B	C D
A C	B D
A D	B C
C D	A B
B D	A C
B C	A D

Sol

$$\frac{\lfloor 2n}{\lfloor 2 (\lfloor n)^2} = \frac{\lfloor 2\times 2}{\lfloor 2 (\lfloor 2)^2} = \frac{\lfloor 4}{\lfloor 2 \lfloor 2 \lfloor 2}$$

$$\Rightarrow \frac{\cancel{4}\times 3\times \cancel{2}\times \cancel{1}}{\cancel{2}\times\cancel{2}\times \cancel{2}} = 3$$

WWW.GRADESETTER.COM

3.) The no. of ways in which (m+n+r) things can be divided into three groups containing m, n and 'r' things respectively is

$$\boxed{{}^{m+n+r}C_m \cdot {}^{n+r}C_n \; {}^{r}C_r = \frac{\lfloor m+n+r}{\lfloor m \lfloor n \lfloor r}}$$

where $m \neq n \neq r$

4.) No. of ways in which three(3) n' things can be divided into 3 groups each containing n' things is

$$\boxed{\frac{{}^{3n}C_n \cdot {}^{2n}C_n \cdot {}^{n}C_n}{\lfloor 3} = \frac{\lfloor 3n}{(\lfloor n)^3 \lfloor 3}}$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

5.) The no. of ways in which 'mn' different things can be divided into 'm' different groups each containing 'n' things.

$$\boxed{\frac{mn}{\underbrace{\lfloor n \lfloor n \lfloor n \ldots}_{\text{'m times'}}} \times \frac{1}{\lfloor m} = \frac{\lfloor mn}{(\lfloor n)^m \lfloor m}}$$

Googlesheet-1

Q 25

$$\underbrace{\left[\frac{\lfloor 30}{(\lfloor 5)^6 \lfloor 6}\right]}_{\text{Division into groups}} \times \underbrace{\lfloor 6}_{\text{Distribution.}}$$

WWW.GRADESETTER.COM

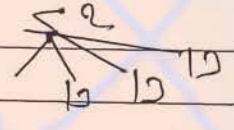
"Exemplar booklet-V"

24) (b) $\dfrac{\lfloor mn}{(\lfloor n)^m}$

Is the no. of ways to distribute mn different things to m persons equally.
which is always an Intege

so, $\lfloor mn$ is divisible is $(\lfloor n)^m$

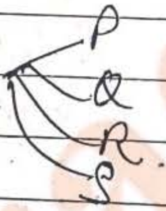
21) (i)



$\Rightarrow \dfrac{\lfloor 52}{\lfloor 13 \lfloor 13 \lfloor 13 \lfloor 13} \times \dfrac{1}{\lfloor 4}$ चूंकी 4 Group equal है इसलिए (सिर्फ Packet बनाए हैं।

$\Rightarrow \dfrac{\lfloor 52}{(\lfloor 13)^4 \lfloor 4}$

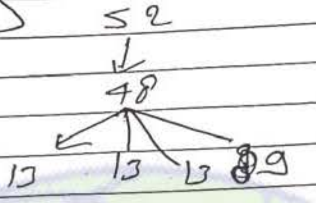
ii) $\dfrac{\lfloor 52}{(\lfloor 13)4} \times \dfrac{1}{\lfloor 4}$



$\underbrace{\left\{\dfrac{\lfloor 52}{(\lfloor 13)^4 \times \lfloor 4}\right\}}_{\text{Division into Groups}} \times \underbrace{\lfloor 4}_{\text{Distribution in 4 Players}}$

WWW.GRADESETTER.COM

iii)



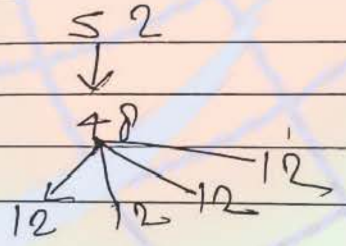
4K

$$\left\{\frac{\lfloor 48}{(\lfloor 13)^3\,\lfloor 9}\times\frac{1}{\underset{\text{for 3 equal group.}}{\lfloor 3}}\right\}\times\underset{\text{4 king}}{1}\times\lfloor 4$$

'4' ही प्रकार [illegible]

Distribution in '4' player.

iv)



4K

$$\left[\left(\frac{\lfloor 48}{(\lfloor 12)^4}\times\frac{1}{\lfloor 4}\right)\underset{\text{Because of '4' different king}}{\lfloor 4}\right]\times\lfloor 4$$

Distribution of '4' player

v)

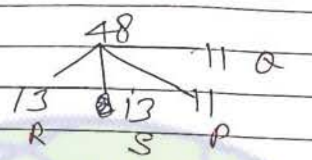
$$\cancel{\left[\left(\frac{\lfloor 44}{(\lfloor 11)^4}\times\frac{1}{\lfloor 4}\right)\lfloor 4\right]\lfloor 4}$$

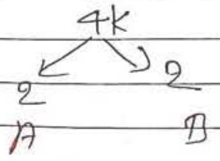
WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

(A.W) → D.P.P.S → 35/36

v)





$$\left[\frac{\lfloor 48}{(\lfloor 13)^2(\lfloor 11)^2\lfloor 2\lfloor 2}\right\}\left(\frac{\overset{A,B}{\lfloor 4}}{\lfloor 2\lfloor 2\lfloor 2}\right)\times\lfloor 2\Big]\lfloor 4$$

दो-दो का Pocket बनाया |

किसी को भी जा सकते हैं

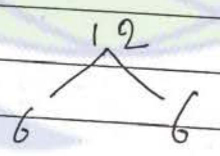
Distribute in '4' player

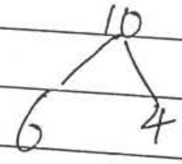
vi)

$$\left[\frac{\lfloor 48}{(\lfloor 13)^2(\lfloor 11)^2\lfloor 2\lfloor 2}\right\}\left(\frac{\lfloor 4}{\lfloor 2\lfloor 2\lfloor 2}\right)\times\lfloor 2\Big]\lfloor 2\lfloor 2$$

P, Q
Ramesh Suresh -

Example Sheet - 1
22.





— + 2Pen

$$\left(\frac{\lfloor 10}{\lfloor 6\lfloor 4}\right)\times 1\times\lfloor 2$$

दो pen रहेंगी वे पास जाएगा

→ Distribution of (Pen+book) in two Player.

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

~~25.~~
26.

$$\begin{tikzcd} & 7 \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \\ 1 & 2 & 4 \end{tikzcd}$$

At least "1" — 1

2 → महत्वपूर्ण नहीं है क्योंकि संख्या same हो जाएगी।

$$\left(\frac{7!}{1!\,2!\,4!}\right) \times 3!$$ → Distribution into three friends.

25. i)

$$\left[\frac{9!}{(3!)^3\,3!}\right] 3!$$

ii)

2	2	5
2	3	4
3	3	3

$$\left[\frac{9!}{(2!)^2\,5!} \times \frac{1}{2!} + \frac{9!}{2!\,3!\,4!} + \frac{9!}{(3!)^3\,3!}\right] \times 3!$$

Division into groups

Distribution into groups.

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Combination with repetition (multinomial theorem.)

~~1) coefficient of x^r in (1-x.~~

1) coefficient of "x^r" in "$(1-x)^{-n}$" is

$$\boxed{{}^{n+r-1}C_r}$$

Note :->

$\omega \quad \omega \quad \omega$
$O \quad O \quad O \qquad (x^0+x^1+x^2+x^3)$

$O \quad O \qquad (x^0+x^1+x^2)$
$B \quad B$

ω	B	
0	2	— B B
1	1	— ω B
2	0	— ω ω

1) The no. of combination of 'r' things out of 'n' things of which 'p' things are of one kind 'q' things are of second kind and "s" things are of third kind is equal to(=) coefficient of "x^r"

$$\text{in } (1+x+\cdots x^p)(1+x+x^2+\cdots x^q)$$

$$(1+x+\cdots\cdots x^r)$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM
1st Choice

3) The no. of combination of 'r' things out of 'n' things of which 'p' things are of one kind, 'q' things are of second ~~think~~ kind are rest (n-p-q) different things is

⇒ coefficient of x^r in $(1+x+\dots+x^p)$

$$(1+x+\dots+x^q)\underbrace{(1+x)(1+x)\dots(1+x)}_{n-p-q \text{ times.}}$$

that is coefficient of "x^r" in

$$(1+x+\dots x^p)(1+x+\dots x^q)(1+x)^{n-p-q}$$

4) The no. of combinations of 'r' things out of 'n' things of which 'p' things are of one kind 'q' things are of second kind and 'r' things are of third kind such that each things is taken "atleast one's" is equal to

⇒ coefficient of x^r in

$$(x+x^2+\dots+x^p)(x+x^2+\dots x^q)(x+x^2+\dots+x^r)$$

⇒ coefficient of x^{r-3} in

$$(1+x+\dots+x^{p-1})(1+x+\dots+x^{q-1})(1+x+\dots x^{r-1})$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

5:) The No. of ways in which 'r' Identical things can be distributed among 'n' persons when each person can get zero(0) or more things

⇒ coefficient of x^r in

$$\left(1+x+ \cdots +x^r\right)^n$$

which is equal to

⇒ coefficient of x^r in $\left(\frac{1-x^{r+1}}{1-x}\right)^n$ ①

⇒ Coefficient of x^r in $(1-x^{r+1})^n (1-x)^{-n}$

$$\boxed{\Rightarrow \quad {}^{n+r-1}C_r}$$

6:) The no. of non-negative Integral solution of this equation

$$x_1+2x_2+3x_3+ \cdots nx_n = m$$

is equals to

⇒ Coefficient of x^m in

$$(1-x)^{-1}(1-x^2)^{-1}(1-x^3)^{-1} \cdots (1-x^n)^{-1}$$

WWW.GRADESETTER.COM

Group sheet - 2

1.)

$\Rightarrow$ coefficient of x^{10} in

$(1+x+ \cdots\cdots +x^{10})(1+x+ \cdots\cdots x^{10})(1+x+ \cdots\cdots x^{10})$

$\Rightarrow (1+x+ \cdots\cdots x^{10})^3$

$\Rightarrow$ coefficient of x^{10} in $\left(\frac{1-x^{11}}{1-x}\right)^3$

$\Rightarrow$,, ,, ,, $(1-x^{11})^3(1-x)^{-3}$

कितना select करना है (r)

$\Rightarrow {}^{3+10-1}C_{10} = 66.$

(Correct Answer लिखें।)

शर्त (Given संख्या selection का eg(10) संख्या ज्यादा [illegible] संख्या equal हो तभी)

(No. of object to be selected eg. (10B, 10R, 10G) [illegible]=n)

यह (इसका trick → selection में हो ज्यादा $^{n+2}C_2$ → own trick → select) शर्त

Very short

2.) Coefficient of x^{13} in

$\Rightarrow (1+x+ \cdots\cdots x^{15})(1+x+ \cdots\cdots x^{15})(1+x+ \cdots\cdots x^{15})$

$\Rightarrow$ coefficient of x^{13} in $(1+x+ \cdots\cdots x^{15})^3$

$\Rightarrow$,, ,, ,, $\left(\frac{1-x^{16}}{1-x}\right)^3$

$\Rightarrow$,, ,, ,, $(1-x^{16})^3(1-x)^{-3}$

$\Rightarrow$ ~~${}^{3+13-1}C_{13}$~~ $\Rightarrow$ ~~${}^{15}C_{[illegible]}$~~ $\Rightarrow {}^{3+13-1}C_{13}$ Ans.

$^{27}C_{25}$

1st Choice

Page No.
Date / /

[illegible] गलती पकड़े |

Attention ([illegible])
(यदि [illegible] 18 [illegible] मांगे तो वह case और complex हो जाता है)

3) coefficient of x^{25} in

$\Rightarrow (1+x+ \text{---} x^{18})(1+x+ \text{--} x^{18})(1+x+ \text{--} x^{18})(1+x+ \text{--} x^{18})$

$\Rightarrow$ coefficient of x^{25} in $(1+x+ \text{--} +x^{18})^4$

$\Rightarrow$ 〃 〃 〃 $\left(\frac{1-x^{19}}{1-x}\right)^4$

$\Rightarrow$ 〃 〃 〃 $(1-x^{19})^4(1-x)^{-4}$

$\Rightarrow$ 〃 〃 〃 $({}^4C_0 - {}^4C_1 x^{19} + \text{--})(1-x)^{-4}$

$\Rightarrow$ coeffi of x^{25} in ${}^4C_0(1-x)^{-4}$ − coeff of x^6 in ${}^4C_1(1-x)^{-4}$

Attention

$\Rightarrow {}^{4+25-1}C_{25} - 4\left({}^{4+6-1}C_6\right)$

Q4) coefficient of x^{25} in

$(x+x^2+ \text{---} +x^{20})(x+x^2+ \text{---} x^{20})(x+x^2+ \text{---} x^{20})$

$\Rightarrow$ coefficient of x^{25-3} in

$\Rightarrow (1+x+ \text{---} +x^{20-1})(1+x+ \text{--} x^{20-1})(1+x+ \text{--} +x^{20-1})$

$\Rightarrow (1+x+ \text{---} x^{19})^3$

$\Rightarrow \left(\frac{1-x^{20}}{1-x}\right)^3$

WWW.GRADESETTER.COM

$$\Rightarrow - \text{———} (1-x^{20})^3 (1-x)^{-3}$$

$$\Rightarrow - \text{———} \left({}^3C_0 - {}^3C_1 x^{20} + {}^3C_2 x^{40} \text{———} \right) (1-x^{-3})$$

Attempt

$$\Rightarrow {}^3C_0 \underbrace{{}^{3+22-1}C_{22}}_{\text{Coefficient of } x^{22}} - {}^3C_1 \underbrace{{}^{3+2-1}C_2}_{\text{coeffi of } x^{22}}$$

Teacher answer :-

$\Rightarrow$ 20 R, 20 G, 20 W

Coeffi of x^{25} in

$$(x^1 + x^2 + \text{———} + x^{20})$$

$\Rightarrow$ coeffi of x^{22} in

$$(1 + x + \text{———} + x^{19})^3$$

$$\Rightarrow \text{———} (1-x^{20})^3 (1-x)^{-3}$$

$$\Rightarrow \text{———} \left({}^3C_0 - {}^3C_1 x^{20} + {}^3C_2 x^{40} \text{—} \right) (1-x^{-3})$$

$$\Rightarrow {}^3C_0 \underbrace{{}^{3+22-1}C_{22}}_{\text{coeffi of } x^{22}} - {}^3C_1 \underbrace{{}^{3+2-1}C_2}_{\text{Coeffi of } x^{22}}$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

9) coefficient of x^{10} in $(x^1 + \ldots x^6)^3$

$\Rightarrow$ - - - x^7 in $(1 + x + \ldots x^5)^3$

$\Rightarrow$ - - - - $(1-x^6)^3 (1-x)^{-3}$

$\Rightarrow$ - - $\left({}^3C_0 \, {}^{3+7-1}C_7 - {}^3C_1 \, {}^{3+1-1}C_1 \right)$

10) Coefficient of x^{150} in

$(1+x+ \ldots x^{100})^4$

$\Rightarrow$ Coeff. of x^{150} in $(1+x+ \ldots x^{100})^4$

$\Rightarrow$,, ,, ,, $\left(\frac{1-x^{101}}{1-x} \right)^4$

$\Rightarrow$,, ,, ,, $(1-x^{101})^4 (1-x)^{-4}$

~~$\Rightarrow$,, ,, ,, ${}^4C_0 \, {}^{4+150-1}C_{150} - {}^4C \, {}^{4+1-1}C$~~

$\Rightarrow$,, ,, ,, $\left(\overset{\boxed{150}}{{}^4C_0} - {}^3C_1 x^{101} \right) \overset{\boxed{49} \rightarrow 150}{} (1-x)^{-4}$

$\Rightarrow$,, ,, ,, $\left[{}^4C_0 \, {}^{4+150-1}C_{150} - {}^4C_1 \, {}^{4+49-1}C_{49} \right]$

$\Rightarrow$,, ,, ,, $\cancel{{}^{150}C_{150}} \left({}^{153}C_{150} - 4 \cdot {}^{52}C_{49} \right)$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Exam sheet. 2

11.)

(i) $3\,{}^{n}C_{n}$

ii) 1.

iii) Coefficient of x^n in

$$(1+x+x^2+\cdots\cdots+x^n)^3$$

$$\Rightarrow \ldots (1-x^{n+1})^3 (1-x)^{-3}$$

$$\Rightarrow {}^{3+n-1}C_{n} = {}^{n+2}C_{n}$$

$$\Rightarrow \frac{(n+2)(n+1)}{2}$$

iv)

$$(x^0+x^1+x^2+\cdots\cdots x^n)^2 (1+x)^n$$

$$\Rightarrow \text{coeff. of } x^n \text{ in } \left(\frac{1-x^{n+1}}{1-x}\right)^2 (1+x)^n$$

$$\Rightarrow \ldots\ldots\ldots (1-x^{n+1})^2 (1-x)^{-2} (1+x)^n$$

$$\Rightarrow \ldots\ldots (1+2x+3x^2+4x^3+\cdots\cdots)(1+x)^n$$

$$\Rightarrow 1\cdot{}^{n}C_{n} + 2\,{}^{n}C_{n-1} + 3\,{}^{n}C_{n-2} + \cdots\cdots\cdots + (n+1)\,{}^{n}C_{0}$$

$$\left(\because {}^{n}C_{r} = {}^{n}C_{n-r}\right)$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$\Rightarrow {}^nC_0 + 2\,{}^nC_1 + 3\cdot{}^nC_2 + \cdots + (n+1)\,{}^nC_n$

$\Rightarrow \sum (r+1)\,{}^nC_r$

$\Rightarrow \sum r\cdot{}^nC_r + \sum {}^nC_r$

$\Rightarrow n\cdot 2^{n-1} + 2^n$

v) Coeff. of x^n in $(1+x+\cdots+x^n)^1\,(1+x)^{2n}$ Ex-3 ⑭ que. no)

$\Rightarrow$ — — — — $(1-x^{n+1})(1-x)^{-1}(1+x)^{2n}$

$\Rightarrow$ — — — $(1+x+x^2+x^3+\cdots)(1+x)^{2n}$

$\Rightarrow 1\cdot{}^{2n}C_n + 1\cdot{}^{2n}C_{n-1} + 1\cdot{}^{2n}C_{n-2} + \cdots {}^{2n}C_0$

5) $x+y+z=15$

Theory

No. of ~~non-Integral~~ non-"ve" Integral solution of this equation
= No. of ways to distribute 15 Identical things among 3 person when each person can get Zero (0) or more thing.

$\boxed{{}^{n+r-1}C_r}$ so, ${}^{3+15-1}C_{15} = {}^{17}C_{15}$ Ans

WWW.GRADESETTER.COM

6.) $x+y+z+w=25$

$x, y, z, w \geq 1$

(+ve को non-negative Integral बनाने के लिए।

Let $x = x_1 + 1$
$y = y_1 + 1$
$z = z_1 + 1$
$w = w_1 + 1$

then, New equation will be

$x_1 + 1 + y_1 + 1 + z_1 + 1 + w_1 + 1 = 25$

$x_1 + y_1 + z_1 + w_1 = 21,$

$x_1, y_1, z_1, w_1 \geq 0$

${}^{4+21-1}C_{21}$

7.) ~~x+~~ $x + y + z + w = 25$

$x > 0$
$y > 0$
$z > 1$
$w > 2$

Let,

$y = y_1 + 1$
$z = 2 + z_1$
$w = 3 + w_1$

(Here we make non-negative

Then, New eqn will be

GradeSetter

WWW.GRADESETTER.COM

$^{3+18-1}C_{18} = \boxed{^{20}C_{18}}$

$$\Rightarrow \underbrace{x+y_1+z_1+w_1}_{\text{Non-negative}} = 10$$

$$^{4+10-1}C_{10}$$

8) $x+y+z \leq 18$ → This is start from '1'

so,

let $x = x_1+1$ etc.

$$\Rightarrow \boxed{\begin{aligned} x &= x_1+1 \\ y &= y_1+1 \\ z &= z_1+1 \end{aligned}}$$

$$x_1+y_1+z_1 \leq 15$$

so

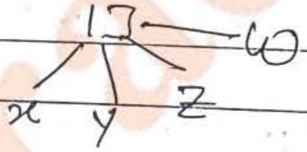
we adjust one non-negative Integer "t" to remove Inequality,

$$\underbrace{x_1+y_1+z_1+t}_{\text{Non-negative Integer}} = 15$$

one concept is in this question

$$^{4+15-1}C_{15}$$

13)



$$\underline{x+y+z+w} = 13$$

But all are ≥ 1 start from '1' so,

we let $x = x_1+1$ etc. or $\begin{aligned} x &= x_1+1 \\ y &= y_1+1 \\ z &= z_1+1 \\ w &= w_1+1 \end{aligned}$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Let $x = x_1 + 1$
$y = y_1 + 1$
$z = z_1 + 1$
$w = w_1 + 1$

$$\underbrace{x_1 + y_1 + z_1 + w_1}_{\text{non-negative}} = 9$$

$$^{4+9-1}C_9$$

12.)

i) No of ways to select 'r' consecutive things when 'n' things are arranged in a row is $(n-r+1)$

$$\text{so, } \underbrace{(25-10+1)}_{\text{ways of section of cosecutive things}} \underbrace{\lfloor 10}_{\text{10 आदमी को बैठाए।}}$$

ii)

$$x_1 + x_2 + \cdots\cdots x_{10} + x_{11} = 15$$

$x_1, x_{11} \geq 0$ (कितना भी भी बैठ सकता है)

$x_2, x_3 \cdots\cdots x_{10} \geq 1$ (दो के बिच का gaps)

Now.

WWW.GRADESETTER.COM

→ Let,

$x_2 = y_2 + 1$ etc.

$$\underbrace{x_1 + y_2 + y_3 + - - - y_{10} + x_{11}}_{\text{non-Negative}} = 15 - 9$$

$$\left({}^{11+6-1}C_6 \quad \underline{|10} \right)$$ Ans.

Q:-> Find the no. of Non-negative Integral solution of equation

$$x + y + z + 4t = 20$$

Ans. $x + y + z + 4t = 20$

$$0 \le t \le 5$$ $\boxed{0, 1, 2, 3, 4}$

1) $t = 0$

$x + y + z = 20$

${}^{3+20-1}C_{20}$

ii) $t = 1$

$x + y + z = 19$

${}^{3+19-1}C_{19}$

iii) $t = 2$

$x + y + z = 18$

WWW.GRADESETTER.COM

$^{3+18-1}C_8$

iv) $t = 3$

$x + y + z = 17$

iv)

v)

Alternate:-

coeff of x^{20} in

$(1-x)^{-1}(1-x)^{-1}(1-x)^{-1}(1-x^4)^{-1}$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$[10.9] =.$
$[\pi] = 3$

Exponent of Prime "p" in $\underline{|n}$:⇒

{ Let
$\underline{|n} = 2^{\alpha}.3^{\beta}.5^{\gamma}.7^{\delta} - - - -$
$= p_1^{\alpha}.p_2^{\beta}.p_3^{\gamma}.p_4^{\delta}$ }

$$\boxed{\text{Exponent}(\underline{|n}) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots\ldots}$$

where

([·] denotes greatest Integer function.)

17) i) $\underline{|50} = 2^{\alpha}.3^{\beta}.5^{\gamma}.7^{\delta} —$

$$E_2(\underline{|50}) = \left[\frac{50}{2}\right] + \left[\frac{50}{4}\right] + \left[\frac{50}{8}\right] + \left[\frac{50}{16}\right] + \left[\frac{50}{32}\right]$$

$\Rightarrow 25 + 12 + 6 + 3 + 1$
$\Rightarrow 47$

ii)

$$E_3(\underline{|50}) = \left[\frac{50}{3}\right] + \left[\frac{50}{9}\right] + \left[\frac{50}{27}\right]$$

$\Rightarrow 16 + 5 + 1$
$\Rightarrow 22$ Ans.

WWW.GRADESETTER.COM

iii) $\left[\frac{50}{5}\right] + \left[\frac{50}{25}\right] = 12$ $(\because 5 \times 2 = 10)$

$(5^2 \times 2^2 = 100)$

$2^{35} \times \underbrace{(5^{12} + 2^{12})}_{10^{12}} \times 3^{22}$ ↳ (limiting Reagent)

iv) $\left[\frac{50}{7}\right] + \left[\frac{50}{49}\right] = 8$

$2^{47} \cdot 7^{8}$

$2^{39} \cdot (2 \times 7)^{8} \times 3^{22}$ - - -

v) $2^{3} \cdot (4^{22} \times 3^{22})$ - - -

GradeSetter

1st Choice
WWW.GRADESETTER.COM

De-arrangements:-

1) If 'n' things are arranged in a row the no. of ways in which they can be arranged, so that "no one" of these occupies its original position is

$$\boxed{\lfloor n\left(1-\frac{1}{\lfloor 1}+\frac{1}{\lfloor 2}-\frac{1}{\lfloor 3}+\cdots\cdots(-1)^n\frac{1}{\lfloor n}\right)}$$

ii) If 'r' things go to wrong place out of 'n' things (that is "n-r" things go to ~~orig~~ original position) then the no. of ways is

$$\boxed{{}^nC_{n-r}\lfloor r\left(1-\frac{1}{\lfloor 1}+\frac{1}{\lfloor 2}-\frac{1}{\lfloor 3}\cdots\cdots(-1)^r\frac{1}{\lfloor r}\right)}$$

Sample sheet-1

20) i)

$$\lfloor 5\left(1-\frac{1}{\lfloor 1}+\frac{1}{\lfloor 2}-\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}-\frac{1}{\lfloor 5}\right)$$

ii)

$${}^5C_2\lfloor 3\left(1-\frac{1}{\lfloor 1}+\frac{1}{\lfloor 2}-\frac{1}{\lfloor 3}\right)$$

WWW.GRADESETTER.COM

Note! ⇒

Sum of the numbers that can be formed using n digits (repitition not allowed.

$a_1, a_2, \text{---} a_n$ is

equals to

$$\boxed{\Rightarrow \underline{\lfloor n-1} \left(a_1 + a_2 + \text{-----} + a_n\right)\left(\frac{10^n - 1}{9}\right)}$$

GradeSetter

WWW.GRADESETTER.COM

Ans

TRIGONOMETRIC RATIO'S

Trigonometry Ratios:→

Trigonometry is the branch of science in which we study about side and angle of the triangl

① Measurement of an angle

There are three system for measurement of an angle

① Degree System (°)

(2) grade system (g)

(3) Radian system ( )

or

Circular system (^{c})

① Degree system (°)

$$\text{One right angle} = 90^\circ$$

$$1^\circ = 60'$$

$$1' = 60''$$

(2) grade system (g)

$$\text{One right angle} = 100^g$$

$$1^g = 100`$$

$$1` = 100``$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Note: (1) One right angle = $90^\circ = 100^g$

$\boxed{1^\circ = \left(\frac{10}{9}\right)^g}$ or $\boxed{1^g = \left(\frac{9}{10}\right)^\circ}$

(2) $1^\circ > 1^g$

(3) Radian system

$\pi^c = 180^\circ$

one right angle $= 90^\circ = \frac{\pi^c}{2}$

$1^c = \left(\frac{180}{\pi}\right)^\circ$

$1^c = \left(\frac{180}{3.14}\right)^c$

$1^c \simeq 57^\circ 17' 35''$

$\boxed{1^c \simeq 57^\circ}$

Note →

① Relation $90^\circ = 100^g = \frac{\pi^c}{2}$

$$\frac{D}{90} = \frac{G}{100} = \frac{c}{\left(\pi/2\right)}$$

$$\frac{D}{90} = \frac{G}{100} = \frac{2c}{\pi}$$

WWW.GRADESETTER.COM

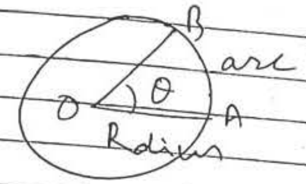
WWW.GRADESETTER.COM

$(1^c = 57^\circ)$

$= 100^g$

$\frac{9}{0})^\circ$

②



$$\text{angle } \theta = \frac{\text{Arc}}{\text{Radius}} = \left(\frac{AB}{OA}\right)^C$$

③ order

$$1^c > 1^\circ > 1^g$$

Q) If the three angle of the triangles are $\left(\frac{2}{3}x\right)^g$, $\left(\frac{3}{2}x\right)^\circ$ and $\left(\frac{\pi x}{75}\right)^C$ find all the angles in degree

Solution $\left(\frac{2}{3}x\right)^g + \left(\frac{3}{2}x\right)^\circ + \left(\frac{\pi x}{75}\right)^c = 180^\circ$

$$\left(\frac{2}{3}x \times \frac{9}{10}\right)^\circ + \left(\frac{3}{2}x\right)^\circ + \left(\frac{180x}{75}\right)^\circ = 180^\circ$$

$$\left(\frac{3}{5}x\right)^\circ + \left(\frac{3}{2}x\right)^\circ + \left(\frac{12}{5}x\right)^\circ = 180^\circ$$

$$x = 14$$

Ist $\left(\frac{3}{5}x\right)^\circ = 24^\circ$

IInd $\left(\frac{3}{2}x\right)^\circ \Rightarrow 60^\circ$

IIIrd $\left(\frac{180x}{75}\right)$

WWW.GRADESETTER.COM

Q) Which of the following statement is correct

(A) $\sin 1 < \sin 1^\circ$

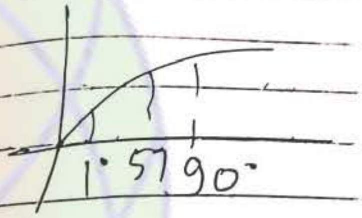
(B) $\sin 1 = \sin 1^\circ$

[✓ (C)] $\sin 1 > \sin 1^\circ$

(d) $\sin 1^\circ < \sin 1^g$

Ans – $\sin 1^c = \sin 57^\circ$

$\sin 57^\circ > \sin 1^\circ$



Q) The circular wire of diameter is 10 cm is cut and place along with the circumference of the circle whose diameter is 1 m. find the angle at the centre of the circle by the wire.

$2\pi r = 10\pi$

$$\theta = \left(\frac{10\pi}{50}\right) \left(\frac{\pi}{5}\right)^c = 36^\circ$$

WWW.GRADESETTER.COM

Basic trigonometric identity

① $\sin^2\theta + \cos^2\theta = 1$

② $1 + \tan^2\theta = \sec^2\theta$

$\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$$(\sec\theta \mp \tan\theta) = \frac{1}{(\sec\theta \pm \tan\theta)}$$

③ $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$$(\operatorname{cosec}\theta \mp \cot\theta) = \frac{1}{(\operatorname{cosec}\theta \pm \cot\theta)}$$

Q) If $\sin x + \sin^2 x = 1$, then find the value of $\cos^8 x + 2\cos^6 x + \cos^4 x$

$\sin x = 1 - \sin^2 x$

$\therefore \sin x = \cos^2 x$

$\cos^8 x + 2\cos^6 x + \cos^4 x$

$= \sin^4 x + 2\sin^3 x + \sin^2 x$

$= (\sin^2 x)^2 + 2(\sin^2 x)\sin x + (\sin x)^2$

$= (\sin^2 x + \sin x)^2$

$= 1^2 = 1$

WWW.GRADESETTER.COM

Q 2) If $(\sec\alpha - \tan\alpha)(\sec\beta - \tan\beta)(\sec\gamma - \tan\gamma) \cdot 0,$

$= (\sec\alpha + \tan\alpha)(\sec\beta + \tan\beta)(\sec\gamma + \tan\gamma)$

then each side will be (means find the value of right hand side or L.H.S

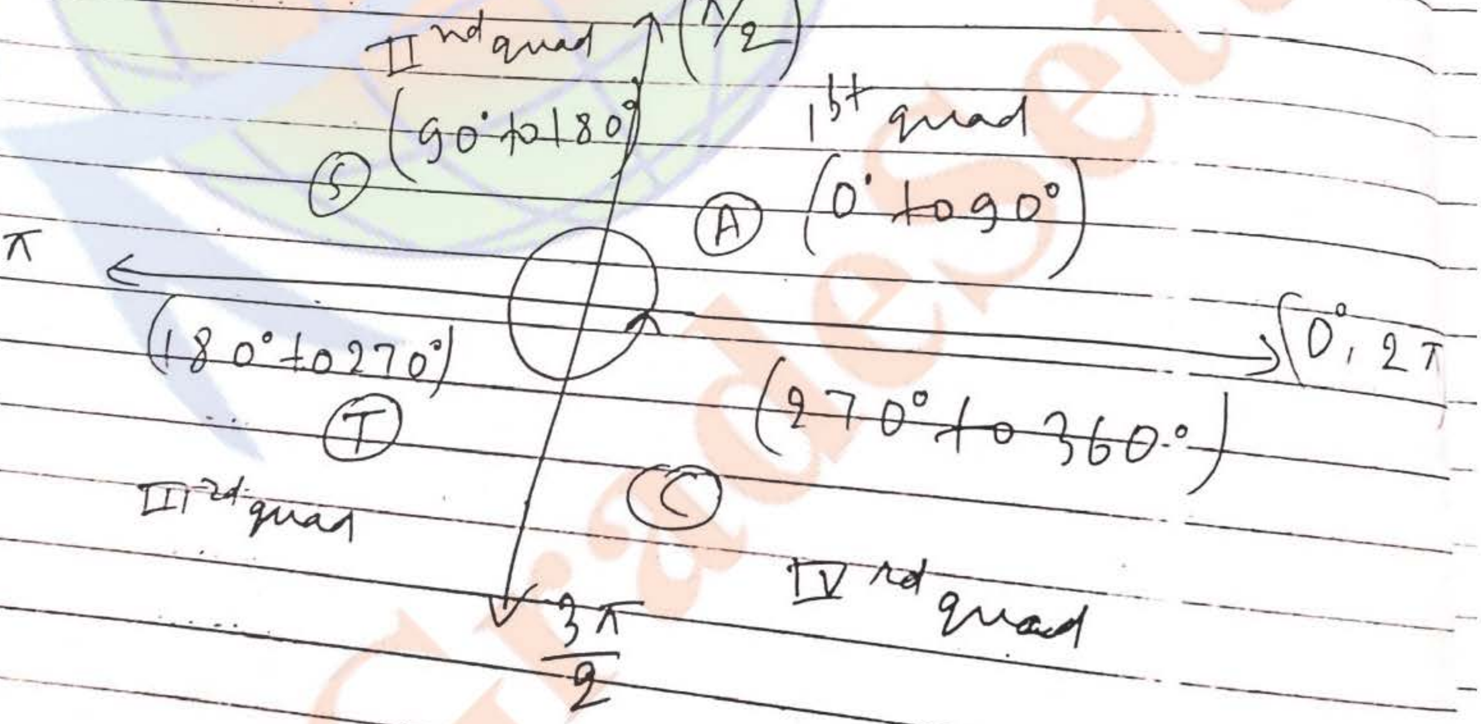
$(\sec\alpha + \tan\alpha)(\sec\beta + \tan\beta)(\sec\gamma + \tan\gamma)$

$= (\sec\alpha + \tan\alpha)(\sec\beta + \tan\beta)(\sec\gamma + \tan\gamma)$

$1 = (\sec\alpha + \tan\alpha)^2(\sec\beta + \tan\beta)^2(\sec\gamma + \tan\gamma)^2$

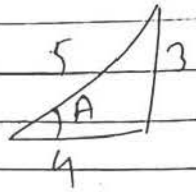
$\pm 1 = (\sec\alpha + \tan\alpha)(\sec\beta + \tan\beta)(\sec\gamma + \tan\gamma)$

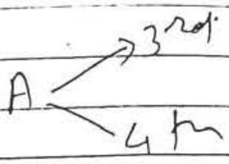
A, S, T, C Rule :→



Q) If $5 \sin A + 3 = 0$, find $\cos A$ and $\tan A$

$$\sin A = -\frac{3}{5}$$





	3^{rd} qu	4^{th} qu
$\cos A =$	$-\frac{4}{5}$	$\frac{4}{5}$
$\tan A$	$\frac{3}{4}$	$-\frac{3}{4}$

Q) What is the sign of the expression

$$\sin 2 \ \sin 3 \ \sin 5$$

$$= \sin 2^c \ \sin 3^c \ \sin 5^c$$

$$\left(1^c \simeq 57^\circ\right)$$

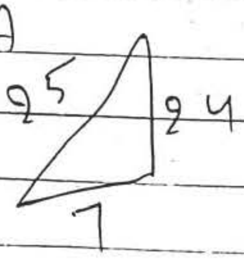
$$= \sin 114^\circ \ \sin 171^\circ \ \sin 295$$

$$= (+ve) \ (+ve) \ (-ve)$$

$$= (-ve)$$

Q) If $7 \tan A + 24 = 0$ where A lies b/w 90° to 18

find $3 \cos A - 4 \sin A + 5 \cot A$



$$\tan A = \frac{-24}{7}$$

$$\cot A = -\frac{7}{24}, \quad \cos A = -\frac{7}{25}, \quad \sin A = +\frac{24}{25}$$

WWW.GRADESETTER.COM

$\boxed{90° \leq \theta \leq 0}$

T. Ratios of allied angle

Q ~~Q~~)

I) according to the A S T C rule to d... the sign either (+) or (−) of the given Trigonometric function

II) 90° and 270° are the break line a... They convert + ~~sign in~~ sine into co... and cos convert in to ~~sign~~ sin and similarly change the other function

| sin ⟷ cos
| tan ⟷ cot
| ~~cosec~~
| sec ⟷ cosec

↓

90 break line 270

जब break line पर उल्टा

$\sin(90° - \theta) = \cos\theta$

$\sin(90° + \theta) = +\cos\theta$

$\sin(180° - \theta) = +\sin\theta$

$\tan(180° + \theta) = +\tan\theta$

$\cos(270° - \theta) = -\sin\theta$

$\tan(270° + \theta) = -\cot\theta$

$\text{cosec}(360° - \theta) = -\text{cosec}\,\theta$

$\sec(270° + \theta) = +\text{cosec}\,\theta$

Q1) If $\sin 135°$

$= \sin(90° + 45°)$ $\qquad \sin(180° - 45°)$

$= +\cos 45°$ $\qquad = +\sin 45°$

$= 1/\sqrt{2}$ $\qquad = 1/\sqrt{2}$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Note ① if $\sin(360°+\theta) = \sin\theta$

② if $\cos(360°+\theta) = \cos\theta$

③ if $\tan(360°+\theta) = \tan\theta$

Q) if $\sin 420°$

$= \sin(360 + 60°)$

$= \sin 60° = \frac{\sqrt{3}}{2}$

Q) find the value $\frac{\sin(315°)\tan(1110°)\cos(750°}{\sec(225°)}$

$= \frac{\sin(270+45°)\tan(3\times360°+30°)\cos(2\times360°}{\sec(180°+45°)}$

$= \frac{(-\cos 45°)(\tan 30°)(\cos 30°)}{-\sec 45°}$

$= \frac{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right)}{-\sqrt{2}}$

$\tan 88° = \tan(90° - 2°)$

$= \cot 2°$

$= \frac{1}{\tan 2°}$

Q) find the value of the expression

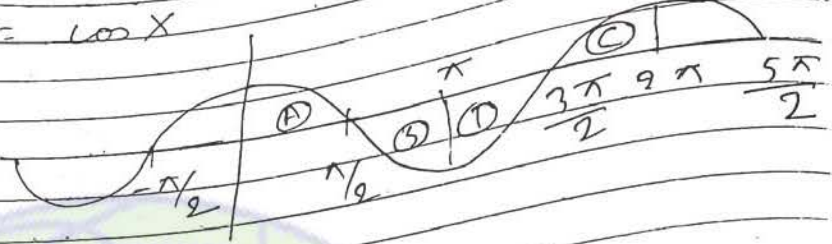
$e^{(\log\tan 1° + \log\tan 2° + \log\tan 3° + \text{--} \log\tan 8}$

$= e^{\log(\cancel{\tan 1°}\,\cancel{\tan 2°} \text{--} \tan 45° \text{--} \cancel{\tan 88°} \text{--} \tan 89°}$

$= e^{\log\tan 45°} = e^{\log 1} = e^{0} = 1$

WWW.GRADESETTER.COM

② $y = \cos x$



① Domain = R

② Range [-1,1]

③ This function is continuous and differentiable in R

④ ~~cos x~~ $-1 \le \cos x \le 1$

or $0 \le \cos^2 x \le 1$

⑤ Cos x is even function because $\cos(-x) = \cos$ similarly sec x is even function.

⑥ Period → 2π

⑦ Cos x is decreasing in 1st and 2nd quad by 1 to -1 and cos x is increasing ~~from~~ f in 3rd and 4th quad by -1 to 1

This function follow ASTC rule.

Q3) If ($\sqrt{3}$

find

Q 4)

Sol

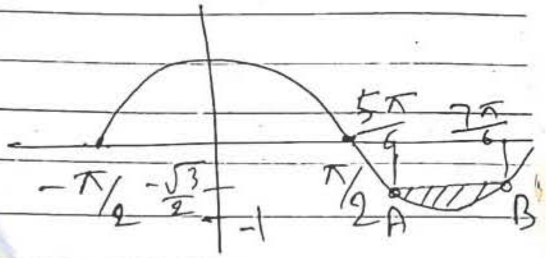
03) If $(\sqrt{3} + 2\cos x) < 0$, where $x \in [-\frac{\pi}{2}, \frac{3\pi}{2}]$

find thesolution set for x

$$\cos x < -\frac{\sqrt{3}}{2}$$

$x = \frac{5\pi}{6}$ | $x = \frac{\pi}{6}$

$x = \frac{7\pi}{6}$



$$x \in \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$$

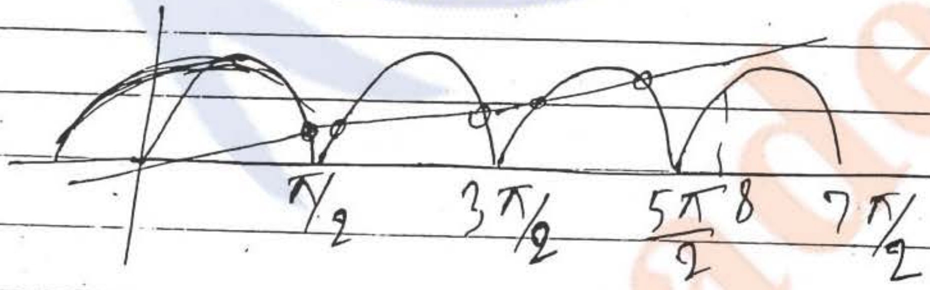
04) If $8|\cos x| = x$ find the no of solution

Solution $|\cos x| = \frac{x}{8}$

Let $y = |\cos x|$ — ①.

$y = \frac{x}{8}$ — ②

X	Y
0	0
8	1



No of solution = 5

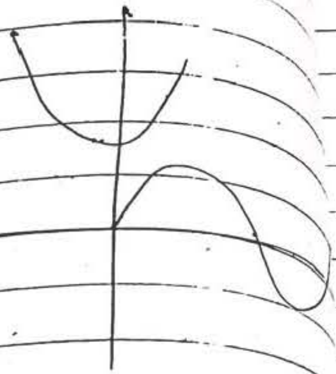
⑤ If $\sin x = x^2 + x + 1$ find no of solution

Sol. $\sin x = x^2 + X + 1$

$y = \sin x$ — (I)

$y = x^2 + x + 1$ — (II)

$y = (x+1)^2 + \frac{3}{4}$

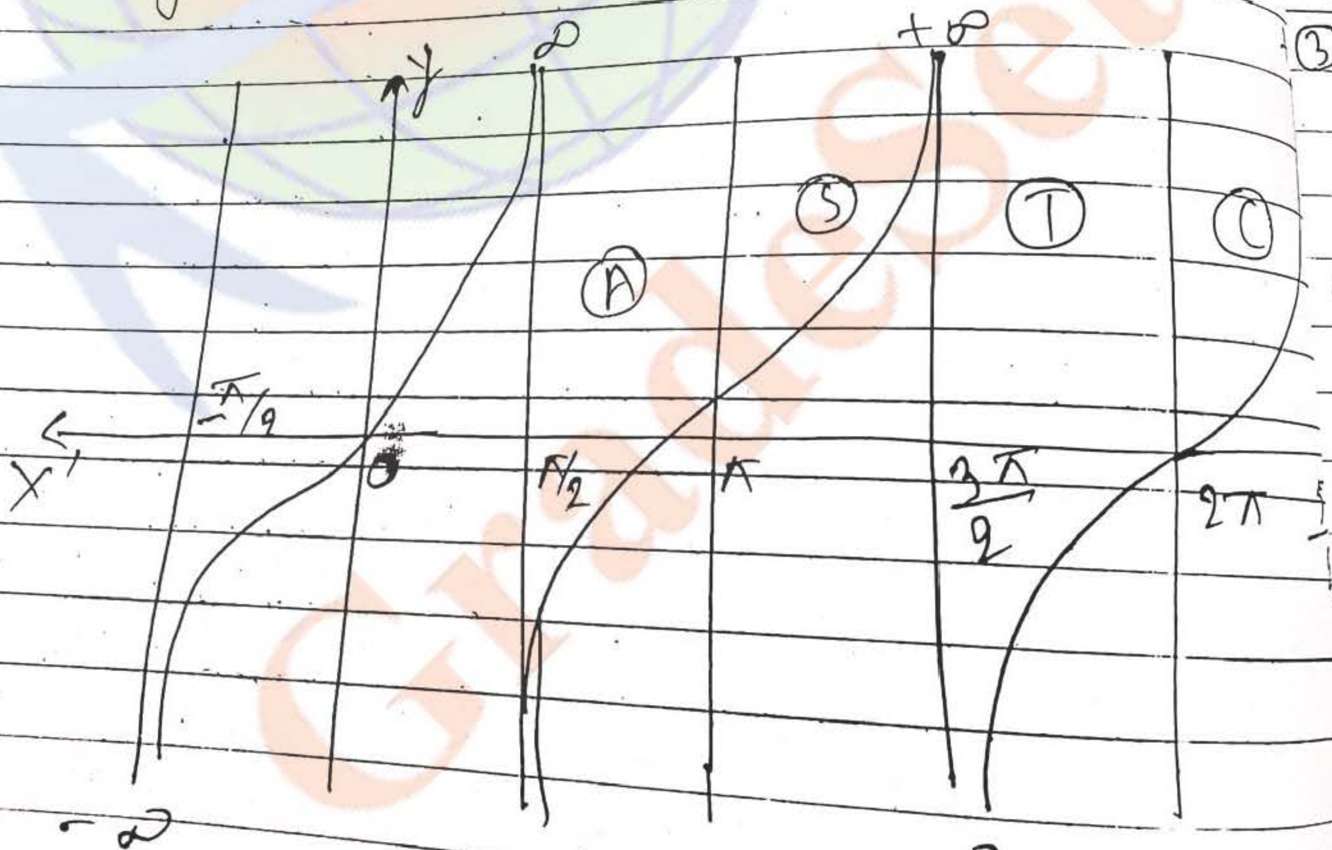


$\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$ Vertex → $\left(-\frac{1}{2}, \frac{3}{4}\right)$

~~No solution~~

No of solution = 0

③ $y = \tan x$



WWW.GRADESETTER.COM

~~$f(x) = x^2(x-1)(x+1)$~~ ~~$\begin{vmatrix} 1 & 1 \\ 2 & x-1 \\ 3 & x-2 \end{vmatrix}$~~

~~$R_1 \rightarrow R_1 + R_3 - 2R_2$~~

~~$f(x) = 0$~~

~~$\boxed{f(100) = 0}$~~

① Domain → $R-(2\pi+1)\pi/2$

② Range → R, or $(-\infty,\infty)$

③ This function is odd function symmetrical ~~lay~~ about origin
because $\tan(-x) = -\tan x$

④ This curve follow A S T C rule

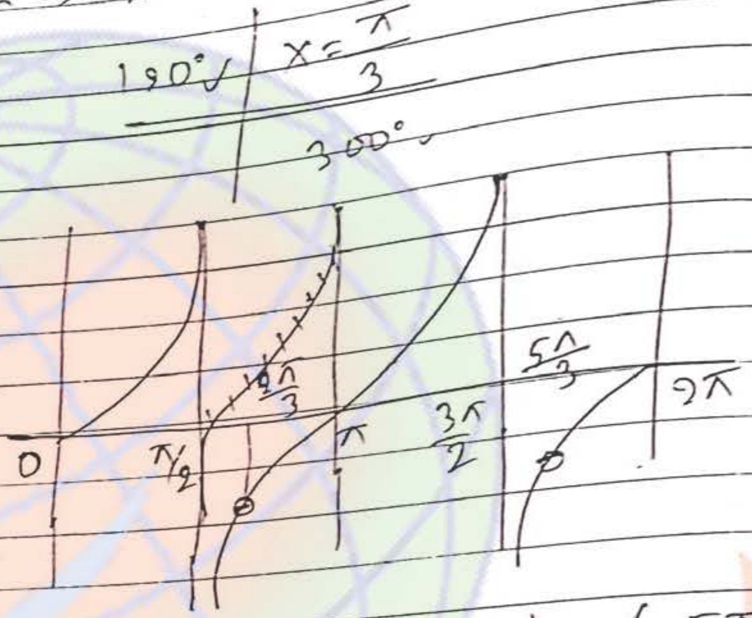
⑤ Period of this function π

⑥ This function is always increasing it means cot x is always decreasing function

WWW.GRADESETTER.COM

Q) If $(\tan x + \sqrt{3}) > 0$ where $x \in [0, 2\pi]$ find solution set for x

Solution → $\tan x > -\sqrt{3}$



$$x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{2\pi}{3}, \frac{3\pi}{2}\right) \cup \left(\frac{5\pi}{3}, 2\pi\right]$$

formulaes for sum and diff form for T. function

① $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

② $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

③ $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

π.

④ sin

= |sin

⑤ sin(

⑥ cos(

⑦ tan

tan

Not

1) sin

2) co

WWW.GRADESETTER.COM

$\prod \rightarrow$ continued Product series

④ $\sin(A+B+C) = \sin((A+B)+C)$

$= \sin(A+B)\cos C + \cos(A+B)\sin C.$

$= (\sin A \cos B + \cos A \sin B)\cos C + (\cos A \cos B - \sin A \sin B) s$

⑤ $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

⑥ $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

⑦ $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B\, ta}{1 - (\tan A \tan B + \tan B \tan C + \tan C \tan}$

$$\tan(A+B+C) = \left(\frac{\sum \tan A - \prod \tan A}{1 - \sum \tan A \tan B}\right)$$

Note →

① $\sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

② $\cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

③ $\sin 18^\circ = \dfrac{\sqrt{5}-1}{4} = \cos 72^\circ$

④ $\cos 36^\circ = \dfrac{\sqrt{5}+1}{4} = \sin 54^\circ$

WWW.GRADESETTER.COM

Q) If $A + B = \frac{\pi}{4}$, find $(1 + \tan A)(1 + \tan B)$

$\Rightarrow \tan(A+B) = \tan\frac{\pi}{4}$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$\tan A + \tan B = (1 - \tan A \tan B)$ — ①

$1 + (\tan A + \tan B) + \tan A \cdot \tan B$

$= 2$ By ①

II $A = 0, \quad B = 45°$

$(1 + \tan A)(1 + \tan B)$

$= (1 + 0)(1 + 1) = 2$

Q) find the value $\cos^2\left(\frac{4\pi}{15}\right) - \sin^2\left(\frac{\pi}{15}\right)$

$$\cos\left(\frac{4\pi}{15} + \frac{\pi}{15}\right)\cos\left(\frac{4\pi}{15} - \frac{\pi}{15}\right)$$

$$= \cos\frac{\pi}{3} \cdot \cos\frac{\pi}{5} = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{5}+1}{4}\right)$$

WWW.GRADESETTER.COM

$\tan B)$

Q) If $\frac{\tan^2 3\theta - \tan^2\theta}{1^2 - \tan^2 3\theta \tan^2\theta}$

$= \frac{(\tan 3\theta - \tan\theta)}{(1 + \tan 3\theta \tan\theta)} \frac{(\tan 3\theta + \tan\theta)}{(1 - \tan 3\theta \tan\theta)}$

$= \tan(3\theta - \theta) \tan(3\theta + \theta)$

$= \tan 2\theta \tan 4\theta$

Q) If $\sec^2\theta = \frac{4xy}{(x+y)^2}$ where $x \in R$, $y \in R$ is true if and only if

Ⓐ $x + y \neq 0$

Ⓑ $x \neq 0, y \neq 0$

Ⓒ $x = y$

ⓓ $x = y, x \neq 0$

$\cos^2\theta = \frac{(x+y)^2}{4xy}$ $(0 \leq \cos^2 x \leq 1)$

$0 \leq \frac{x+y}{4xy} \leq 1$

$0 \leq (x+y)^2 \leq 4xy$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$(x+y)^2 \le 4xy$

$(x^2+y^2-2xy) \le 0$

$(x-y)^2 \le 0 \quad \because (x-y)^2 \nless 0$

$(x-y)^2 = 0 \quad x \ne 0, y \ne 0$

$\boxed{x=y}$

II Q) $y|\cos x| = x$, where $x \in [0, 3\pi]$ find no. of solution

$|\cos x| = x$

$y = |\cos x|$ — ① $\boxed{y = x}$ — ⑪

Q) $\pi = 3.14$

$\frac{\pi}{2} = 1.57$

No of solution = 1

GradeSetter

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Formula to convert sum or diff into product (C and D)

1) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

2) $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

3) $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

4) $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

Q) find the value of $\frac{\cos 12 - \sin 12}{\cos 12^\circ + \sin 12^\circ} + \frac{\sin 147^\circ}{\cos 147^\circ}$

$$= \frac{\cos 12^\circ - \cos 78^\circ}{\cos 12^\circ + \cos 78^\circ} + \tan 147^\circ$$

$$= \frac{\cancel{2}\sin\left(\frac{12+78}{2}\right)\sin\left(\frac{78-12^\circ}{2}\right)}{\cancel{2}\cos\left(\frac{12+78}{2}\right)\cos\left(\frac{12-78}{2}\right)} + \tan(180^\circ-33$$

$$= \frac{\sin 45^\circ \sin 33^\circ}{\cos 45^\circ \cos(-33)} - \tan 33^\circ$$

$$= \tan 33^\circ - \tan 33^\circ = 0$$

WWW.GRADESETTER.COM

divide by cos 12°

Method II $\dfrac{① - \tan 12}{1 + ① \tan 12} + \tan 147° - \tan 33°$

$= \tan(45° - 12°) - \tan 33°$

$= 0$

[illegible]> If $b \sin\alpha = a \sin(\alpha + 2\beta)$ then $\left(\dfrac{a+b}{a-b}\right)$ is equal

Ⓐ $\dfrac{\tan B}{\tan(\alpha + B)}$ Ⓑ $\dfrac{-\tan B}{\tan(\alpha + B)}$

Ⓒ $\dfrac{-\cot B}{\cot(\alpha + B}$ Ⓓ $\dfrac{\cot B}{\cot(\alpha + B)}$

Solution $\dfrac{\sin\alpha}{\sin(\alpha + 2\beta)} = \dfrac{a}{b}$

$$\frac{\sin\alpha + \sin(\alpha + 2\beta)}{\sin\alpha - \sin(\alpha + 2\beta)} = \frac{a+b}{a-b}$$ (by using componendo and divid

$$\frac{2 \sin(\alpha + B) \cos(-\beta)}{2 \cos(\alpha + B)(\sin - \beta)} = \frac{a+b}{a-b}$$

$$\frac{-\cot B}{\cot(\alpha + B)} = \frac{a+b}{a-b}$$

M-2 Put $\alpha = 30,\ \beta = 30^\circ$

$$\frac{6}{2} = a \qquad \frac{\frac{6}{2}+6}{\frac{6}{2}-6} = -3$$

Put the value of angle in the given option

equal Q) If the expression $\frac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x}$

is equal to

[✓ (A)] $2\cos x$ (b) $\cos 2x$

(c) $1 + 2\cos x$ (d) $1 + 2\cos 2x$

Method-1

Put $x = 0^\circ$

$$\frac{1+6+15+10}{1+5+10} = \frac{32}{16} = 2$$

Put the value of angle in the given option

Method-2 $\frac{\cos 6x + \cos 4x + 5(\cos 4x + \cos 2x) + 10(\cos 2x + 1}{11}$

$$= \frac{2\cos 5x \cos x + 10\cos 3x \cos x + 20\cos^2 x}{11}$$

$$= 2\cos x\left(\frac{\cancel{11}}{\cancel{11}}\right) = 2\cos x$$

Formula to convert product into sum or difference (P to S formula)

① $2\sin A\cos B = \sin(A+B)+\sin(A-B)$

② $2\cos A\sin B = \sin(A+B)-\sin(A-B)$

③ $2\cos A\cos B = \cos(A+B)+\cos(A-B)$

④ $2\sin A\sin B = \cos(A-B)-\cos(A+B)$

Q) find the value of $\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

$$=\frac{1}{2}\left[\cos\frac{4\pi}{12}-\cos\frac{6\pi}{12}\right]$$

$$=\frac{1}{2}\left[\cos\frac{\pi}{3}-0\right]$$

$$=\frac{\sqrt{3}}{4}$$

good

Q) If the expression $\cos^2(A-B)+\cos^2 B-2\cos(A-B)\cos A\cos B$ is depend on

$= \cos^2(A-B)+\cos^2 B-\cos(A-B)\left(\cos(A+B)+\cos(A-B)\right)$

$= \cos^2(A-B)+\cos^2 B-\cos(A-B)\cos(A+B)-\cos^2($

$= \cos^2 B-\cos^2 A+\sin^2 B$

WWW.GRADESETTER.COM

Q6) If $T_n = \cos^n\theta + \sin^n\theta$, then find the value of $2T_6 - 3T_4 + 1$

Put $\theta = 0°$ or $90°$

$T_n = 1^n$, $T_n = 1$

$T_6 = 1$, $T_4 = 1$

$2T_6 - 3T_4 + 1$

$= 2(1) - 3(1) + 1 = 0$

Q7) find the value of

$$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\right.$$

$$= \left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$$

$$= \left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right) = \left(\sin^2\frac{\pi}{8}\right)\sin^2\frac{3\pi}{8}$$

$$= \frac{1}{4}\left(2\sin\frac{\pi}{8}\sin\frac{3\pi}{8}\right)^2$$

$$= \frac{1}{4}\left[\cos\frac{\pi}{4} - \cos\frac{\pi}{2}\right]^2$$

$$= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

Gradesetter

WWW.GRADESETTER.COM

9) If $\pi < \alpha < \frac{3\pi}{2}$ then find the value

$$\sqrt{4\sin^4\alpha + \sin^2 2\alpha} + 4\cos^2\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$$

$$= \sqrt{4\sin^4\alpha + (2\sin\alpha\cos\alpha)^2} + 2\left(2\cos^2\left(\frac{\pi}{4}\right.\right.$$

$$= 2|\sin\alpha|\left(\sqrt{\sin^2\alpha + \cos^2\alpha}\right) + 2\left(1 + \cos\left(\frac{\pi}{2} - \alpha\right)\right)$$

$$= -2\cancel{\sin\alpha} + 2 + 2\cancel{\sin\alpha}$$

$$= 2$$

(II) Triple angle convert into single ang

① $\sin 3A = 3\sin A - 4\sin^3 A$

$$\sin^3 A = \frac{3\sin A - \sin 3A}{4}$$

② $\cos 3A = 4\cos^3 A - 3\cos A$

$$\cos^3 A = \frac{3\cos A + \cos 3A}{3}$$

③ $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

***

10) fin

11)

WWW.GRADESETTER.COM

****

10) find the value of

$(32\cos^6 20° - 48\cos^4 20° + 18\cos^2 20 - 1)$

$= 2[(4\cos^3 20°)^2 - 2(4\cos^3 20°)(3\cos 20°) + (3\cos 20)^2$

$= 2[4\cos^3 20° - 3\cos 20°]^2 - 1$

$= 2[\cos 60°]^2 - 1$

$= \frac{1}{2} - 1 = \frac{-1}{2}$

11) If $\cos 5\theta = a\cos^5\theta + b\cos^3\theta + c\cos\theta$ then find the value of C

diff with respect to θ

$-5\sin 5\theta = -5a\cos^4\theta\sin\theta - 3b\cos^2\theta\sin\theta - c\sin$

$\theta = 90°$ put in the above eq and we get value of C

Gradesetter

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

V. Very important

Miscellaneous point

① · ~~sinθ / sin~~

① $\sin\theta \sin(60^\circ-\theta)\sin(60^\circ+\theta) = \frac{1}{4}\sin 3\theta$

② $\cos\theta \cos(60^\circ-\theta)\cos(60^\circ+\theta) = \frac{1}{4}\cos 3\theta$

③ $\tan\theta \cdot \tan(60^\circ-\theta)\tan(60^\circ+\theta) = \tan 3\theta$

2) find the value of $\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin$

$\sin 10^\circ \cdot \sin(60^\circ-10^\circ)\sin(60^\circ$

$= \frac{1}{4}\sin 30^\circ = \frac{1}{8}$

13) $\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ$ ⇒

$= \tan 6^\circ \tan(60^\circ+6^\circ)\left(\tan(60^\circ-18^\circ)\tan(60\right.$

$= \frac{\tan 6 \tan(60^\circ-6^\circ)\tan(60^\circ+6^\circ)}{\tan 54^\circ}\left(\tan(60-18^\circ)\tan(6\right.$

$= \frac{\tan 18^\circ \tan(60^\circ-18)\tan(60^\circ+18^\circ)}{\tan 54^\circ}$

$= \frac{\tan 54^\circ}{\tan 54^\circ} = 1$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Le-11to99. Le-2 1 T10

14) If $\tan\alpha$ = integral solution of the inequality $(4x^2-16x+15)<0$ and $\cos B$ is equal to slopes of the bisector of the first quadrant then find the value of $\sin(\alpha+B)\sin(\alpha-B)$

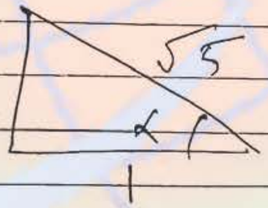
$(4x^2-16x+15)<0$

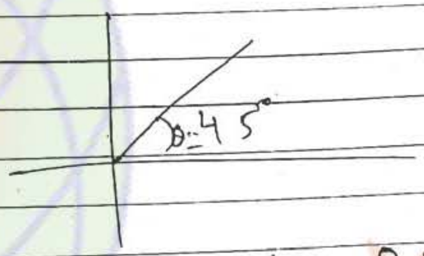
$(2x-3)(2x-5)<0$

$\frac{3}{2}<x<\frac{5}{2}$

$x=2$

$\tan\alpha=2$





Slope = $m=\tan\theta=1$

$\cos B=1$

$1-\cos^2B=\sin^2B$

$\Rightarrow \sin(\alpha+B)\sin(\alpha-B)$

$=\sin^2\alpha-\sin^2B$

$=\frac{4}{5}-0$

GradeSetter

WWW.GRADESETTER.COM

$\boxed{\cot\theta - \tan\theta = 2\cot 2\theta}$

④ $(\cot\theta - \tan\theta) = 2\cot 2\theta$

(1.1) find the value of $(\cot 10° - \tan 10°)\tan 20°$

$(\cot 10° - \tan 10°)\tan(180° + 20°)$

$= 2\cot 20° \tan(180° + 20°)$

$= 2\cot 20° (\tan 20°)$

$= 2 \frac{1}{\cancel{\tan 20°}} \times \cancel{\tan 20}$

② Sum of the series

1) $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha + 2\beta)$ - - - upto n

$$= \frac{\sin\left[\alpha + \left(\frac{n-1}{2}\right)\beta\right] \sin\frac{n\beta}{2}}{\sin\left(\frac{\beta}{2}\right)}$$

where α = initial angle

β = difference angle

n = no of terms

② $\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha + 2\beta +$ — upto n te

$$= \frac{\cos\left[\alpha + \left(\frac{n-1}{2}\right)\beta\right] \cdot \sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$$

9) find t

$\cos\frac{\pi}{11}$

α

= Go

⑥

WWW.GRADESETTER.COM

Q) find the value of the expression

$$\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}$$

$$\alpha=\frac{\pi}{11},\ \beta=\frac{2\pi}{11},\ n=5$$

~~$= \cos\left[\frac{\pi}{11}+\frac{4\pi}{11}\right]\sin \cos 5\pi$~~

$$= \frac{2\cos\left(\frac{5\pi}{11}\right)\sin\left(\frac{5\pi}{11}\right)}{2\sin\left(\frac{\pi}{11}\right)}$$

$$= \frac{\sin\left(\frac{10\pi}{11}\right)}{2\sin\left(\frac{\pi}{11}\right)} = \frac{\sin\left(\pi-\frac{\pi}{11}\right)}{2\sin\frac{\pi}{11}} = \frac{\cancel{\sin\frac{\pi}{11}}}{2\cancel{\sin\frac{\pi}{11}}}$$

$= \frac{1}{2}$ (Ans)

⑥ Product series :→

$\cos\alpha\,\cos(2\alpha)\,\cos(2^2\alpha)$ ---- upto n term

$$\boxed{= \frac{\sin 2^n\alpha}{2^n \sin\alpha}}$$

where α initial angle

n = No of terms.

Note - This formula is useful when all the angles are multiple of two.

GradeSetter

WWW.GRADESETTER.COM

Product of 5 terms

Q) find the value of $\prod_{r=1}^{5} \cos\left(\frac{2^r\pi}{33}\right)$

$$= \cos\left(\frac{2\pi}{33}\right)\cos\left(\frac{2^2\pi}{33}\right)\cos\left(\frac{2^3\pi}{33}\right)\cos\left(\frac{2^4\pi}{33}\right)\cos\left(\frac{2^5\pi}{33}\right)$$

$n = 5, \alpha = \frac{2\pi}{33}$

$$= \frac{\sin 2^5\left(\frac{2\pi}{33}\right)}{2^5 \sin\left(\frac{2\pi}{33}\right)} = \frac{\sin\frac{64\pi}{33}}{32\sin\frac{2\pi}{33}} = \frac{\sin\left(2\pi - \frac{2\pi}{33}\right)}{32\sin\frac{2\pi}{33}}$$

$$= \frac{-1}{32}$$

1) find the value of $\cos\frac{2\pi}{14}\cdot\cos\frac{4\pi}{14}\cos\frac{6\pi}{14}$

$$= \cos\frac{2\pi}{14}\cos\frac{4\pi}{14}\cos\left(\pi - \frac{8\pi}{14}\right)$$

$$= \cos\frac{2\pi}{14}\cos\frac{4\pi}{14}\left(-\cos\frac{8\pi}{14}\right)$$

$$= -\left[\cos\frac{2\pi}{14}\cdot\cos\frac{4\pi}{14}\cdot\cos\frac{8\pi}{14}\right]$$

$$= -\left[\cos\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}\right]$$

$$= -\frac{\sin 2^3\left(\frac{\pi}{7}\right)}{2^3 \sin\left(\frac{\pi}{7}\right)} = \frac{-\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}} = \frac{-\sin\left(\pi+\frac{\pi}{7}\right)}{8\sin\frac{\pi}{7}}$$

$$= \frac{\sin\frac{\pi}{7}}{8\sin\pi/7} = \frac{1}{8}$$

⇒ To find the min^m and max^m value of the expression :-

$$\boxed{\begin{aligned} -\sqrt{a^2+b^2} &\le (a\sin x \pm b\cos x) \le \sqrt{a^2+b^2} \\ -\sqrt{a^2+b^2} &\le (a\cos x \pm b\sin x) \le \sqrt{a^2+b^2} \end{aligned}}$$

Note – This formula is applicable only when both the terms are in linear power and both terms having same angle.

good IIT

Q) find the min^m and max^m value

$$27^{\cos x}\cdot 81^{\sin x}$$

$$= 3^{3\cos x}\cdot 3^{4\sin x}$$

$$= 3^{(3\cos x + 4\sin x)} \qquad (-5 \le 3\sin x + 4\cos x \le$$

$$= 3^5 \text{ or } 3^{-5}$$

$$= 243(\text{max}) \text{ or } \frac{1}{243}(\text{min})$$

GradeSetter

WWW.GRADESETTER.COM

2) find the range of the expression

$$\log_{\sqrt{5}} (\sqrt{2}(\sin x - \cos x) + 3)$$

$-\sqrt{2} \le (\sin x - \cos x$

for min	for max^m
$= \log_{\sqrt{5}} \sqrt{2}(-\sqrt{2}) + 3$	$\log_{\sqrt{5}} \sqrt{2}(\sqrt{2}) + 3$
$= \log_{\sqrt{5}} 1 = 0$	$= \log_{5^{1/2}} 5 = 2$

Range → [0, 2]

3 θ) $3 \cos X + 5 \sin\left(X - \frac{\pi}{6}\right)$ find min and max

$$= 3 \cos X + 5\left(\sin X \cos \frac{\pi}{6} - \sin \frac{\pi}{6} \cos X\right)$$

$$= 3 \cos X + \frac{5\sqrt{3}}{2} \sin x - \frac{5}{2} \cos X$$

$$= \frac{1}{2} \cos X + \frac{5\sqrt{3}}{2} \sin X$$

$$= \pm \sqrt{\frac{1}{4} + \frac{75}{4}} = \pm \sqrt{19}$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Q4) find the maxi^m and min value of the expression

$$\cos^2\theta + 2 + 3\sin^2\theta - 6\sin\theta\cos\theta$$

$$= (\cos^2\theta + \sin^2\theta) + 2 + 2\sin^2\theta - 3(2\sin\theta\cos\theta)$$

$$= 3 + (1 - \cos 2\theta) - 3\sin 2\theta$$

$$= 4 - [1\cdot\cos 2\theta + 3\sin 2\theta]$$

~~$= 4 - [\pm\sqrt{10}) \text{ to } \sqrt{10}$~~ $] = 4 - [-\sqrt{10} \text{ to } \sqrt{10}$

$$= (4 - \sqrt{10}) \text{ To } (4 + \sqrt{10})$$

Imp

Q5) If $A = \sin^2\theta + \cos^4\theta$ then

(a) $1 \leq A \leq 2$

(b) $0 \leq A \leq 1$

(c) ✓ $\frac{3}{4} \leq A \leq 1$

(d) $\frac{3}{4} \leq A \leq 1\frac{1}{4}$

$A = 1 - \cos^2\theta + \cos^4\theta$ (completing the square)

$$A = \left(\cos^2\theta - \tfrac{1}{2}\right)^2 + \frac{3}{4}$$

$$\frac{3}{4} \leq \left[\left(\cos^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4}\right] \leq \left[\left(1 - \frac{1}{2}\right)^2 + \frac{3}{4}\right]$$

$$\frac{3}{4} \leq A \leq 1$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

06) find the min value of

$(9\tan^2 x + 4\cot^2 x)$

M-1 if A.M ≥ G.M

$$\left(\frac{9\tan^2 x + 4\cot^2 x}{2}\right) \geq (9\tan^2 x \cdot 4\cot^2 x)^{\frac{1}{2}}$$

$$(9\tan^2 x + 4\cot^2 x) \geq 12$$

Method-2 $(3\tan x)^2 + (2\cot x)^2$

$$= \underbrace{(3\tan x - 2\cot x)^2}_{\downarrow} + 12$$

$$0 + 12 = 12$$

07) find max^m and min value

$$y = 12\sin x - 9\sin^2 x$$

$$y = -[9\sin^2 x - 12\sin x]$$

$$y = -[(3\sin x)^2 - 2(3\sin x)2 + 2^2 - 4]$$

$$y = 4 - [3\sin x - 2]^2$$

$$y = 4 - 0$$

$y = 4$ max

y = 4 –

y = 4.

y = –

08) If c

GradeSetter

WWW.GRADESETTER.COM

$y = 4 - [3\sin x - 2]^2$

$y = 4 - [3(1) - 2]^2$

$y = -2$ 1.(min)

½

08) If $\cos x = \frac{1}{4}$ find the value of

$$\sin\frac{x}{2} \cdot \sin\frac{5x}{2}$$

$$= \frac{1}{2}\left(2\sin\frac{x}{2}\sin\frac{5x}{2}\right)$$

$$= \frac{1}{2}\left[\cos\left(\frac{5\pi}{2} - \frac{x}{2}\right) - \cos\left(\frac{5x}{2} + \frac{x}{2}\right)\right]$$

$$= \frac{1}{2}\left[\cos 2x - \cos 3x\right]$$

$$= \frac{1}{2}\left[(2\cos^2 x - 1) - (4\cos^3 x - 3\cos x)\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{8} - 1\right) - \left(\frac{1}{16} - \frac{3}{4}\right)\right]$$

$$= \frac{1}{2}\left[\frac{-7}{8} - \left(\frac{1 - 12}{16}\right)\right]$$

$$= \frac{1}{2}\left[\frac{-14 + 11}{16}\right] = \frac{-3}{32}$$

GradeSetter

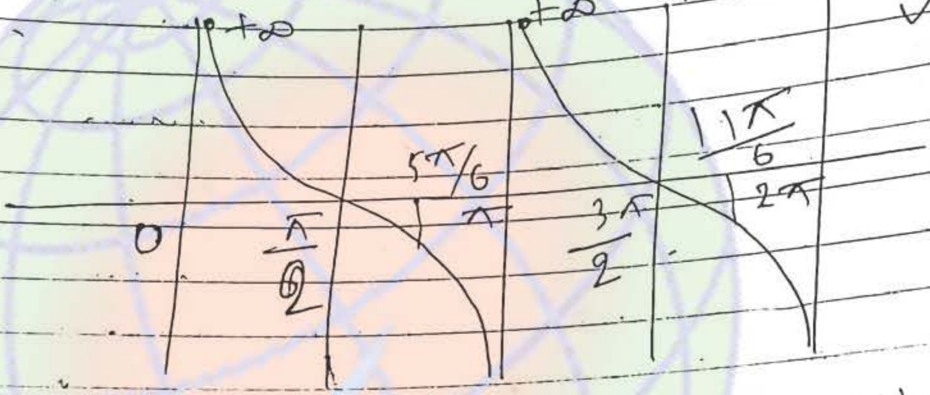
WWW.GRADESETTER.COM

09) If $(\cot x + \sqrt{5}) > 0$ where $x \in (0, 2\pi)$
find the solution set for x

$\cot x > -\sqrt{3}$

$\cancel{x} = \frac{5\pi}{6}$ ✓ | $x =$

$x \neq$ ✓



$x \in \left(0, \frac{5\pi}{6}\right) \cup \left(\pi, \frac{11\pi}{6}\right)$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Trigonometric equation

The eq^n involving trigonometric function of unknown angles is called trigonometric equation.

$$\underset{\text{Trigonometric function}}{\sin}\ \underset{\text{unknown Angle}}{\theta} = \frac{\sqrt{3}}{2}$$

1.) Principal Solution:—

The numerically least Angle satisfying the given equation.

$\sin\theta = \frac{1}{2}$ $\qquad$ $\theta = \frac{\pi}{6}$

$\sin\theta = -\frac{\sqrt{3}}{2}$ $\qquad$ $\theta = -\frac{\pi}{3}$

$\cos\theta = -\frac{1}{2}$ $\qquad$ $\theta =$

2.) General Solution! →

The solution consisting of all possible solutions of trigonometric equation is called general solution.

eg 1→

$$\sin\theta = 0 \Rightarrow, \theta = n\pi \quad (n \in I)$$

GradeSetter

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

1st Choice

Important Point:-

1) ~~con~~ Avoid Squaring:-

If squaring is done than check for "extra genius roots"

2) Never concell equal terms containing unknown on both side which are in product.

It may cause "root loss."

3.) check that denominator is not zero ~~and~~ at any ~~stad~~ stage while solving the equation.

II General Solution of Equation →

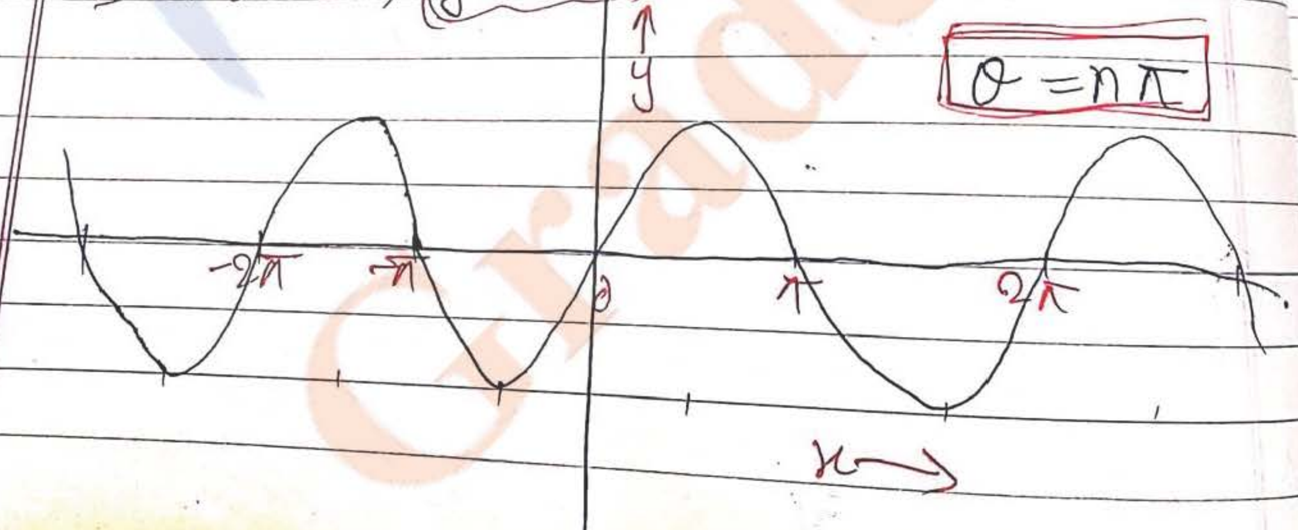
$\sin\theta = 0$

$\cos\theta = 0$

and $\tan\theta = 0$

a) $\sin\theta = 0$, $\boxed{y = \sin x}$

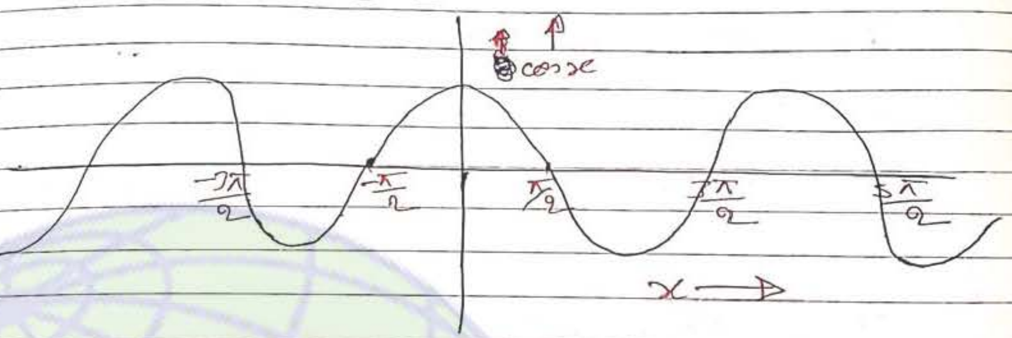
$\boxed{\theta = n\pi}$



WWW.GRADESETTER.COM

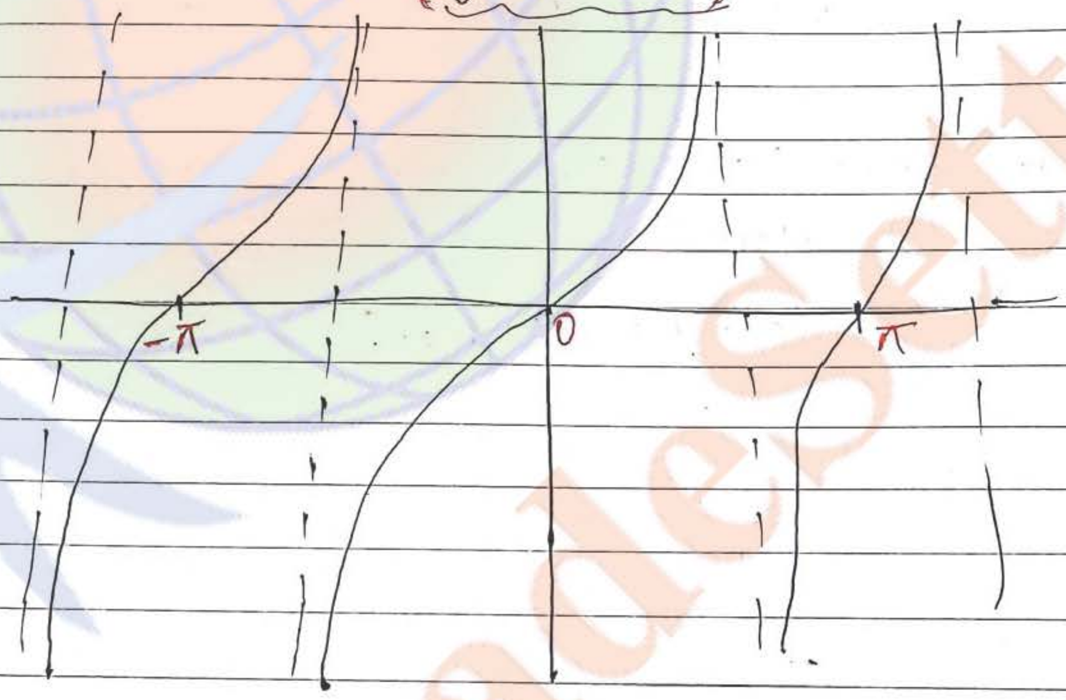
WWW.GRADESETTER.COM

b.) $\cos\theta = 0$, $(y = \cos x)$



$$\boxed{\theta = (2n+1)\pi/2 \quad (n \in I)}$$

c) $\tan\theta = 0$, $(y = \tan x)$



$$\boxed{(\theta = n\pi) \quad (n \in I)}$$

GradeSetter

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Q. Find the General Solution of the equation

(i) $\sin\frac{3\theta}{2} = 0$

(ii) $\cos 2\theta = 0$

(iii) $\tan(\theta - 5\pi) = 0$

Ans (i) $\frac{3\theta}{2} = n\pi$

$\theta = \frac{2}{3} n\pi$, $n \in I$

(ii) $2\theta = (2n+1)\frac{\pi}{2}$

$\theta = (2n-1)\frac{\pi}{4}$

(−ve Answer भी सही है concept to make odd)

(iii) $-\tan(5\pi - \theta) = 0$ (taking "−ve" term common)

$\tan\theta = 0$

$\theta = n\pi$, $n \in I$

WWW.GRADESETTER.COM

Page No.
Date / /

\# Solution of Equations when:-

$\sin\theta = \sin\alpha$
$\cos\theta = \cos\alpha$
$\tan\theta = \tan\alpha$

(a) If $\sin\theta = \sin\alpha$
then,

$$\boxed{\theta = n\pi + (-1)^n\alpha} \quad (n \in I)$$

Proof :→

$$\sin\theta = \sin\alpha$$
$$\sin\theta - \sin\alpha = 0$$
$$2\cos\frac{\theta+\alpha}{2}\sin\frac{\theta-\alpha}{2} = 0$$

$$\cos\frac{\theta+\alpha}{2} = 0$$

$$\frac{\theta+\alpha}{\cancel{2}} = (2m+1)\frac{\pi}{\cancel{2}}$$

$$\theta = (2n+1)\pi - \alpha$$

$$\sin\frac{\theta-\alpha}{2} = 0$$

$$\frac{\theta-\alpha}{2} = p\pi$$

$$\theta = 2p\pi + \alpha$$

$$\Rightarrow \boxed{\theta = n\pi + (-1)^n\alpha}$$

1st Choice

III

(b) If $\cos\theta = \cos\alpha$
than,

$$\boxed{\theta = 2n\pi \pm \alpha} \quad (n \in I)$$

(c) If $\tan\theta = \tan\alpha$
than

~~$\theta = 2n\pi \pm \alpha$~~

$$\boxed{\theta = n\pi + \alpha} \quad (n \in I)$$

Note !→ General solution for the equation
$\operatorname{cosec}\theta = \operatorname{cosec}\alpha$
is $\boxed{\theta = n\pi + (-1)^n \alpha}$ ~~cos~~ etc.
↳ same as $\sin\theta = \sin\alpha$

(III) General solution of Equations :->

$$\sin^2\theta = \sin^2\alpha$$
$$\cos^2\theta = \cos^2\alpha$$
$$\tan^2\theta = \tan^2\alpha$$

is

$$\boxed{\theta = n\pi \pm \alpha}$$

Q Find the General Solution of Equation

i) $\sec 2\theta = -\frac{2}{\sqrt{3}}$

ii) $\cos 3\theta = -\frac{1}{2}$

iii) $\tan\theta = 2$

iv) $7\cos^2\theta + 3\sin^2\theta = 4$

v) $\sin\left(\frac{\theta}{2} - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

vi) $\tan(a\theta + b) = \frac{1}{\sqrt{3}}$

vii) $\tan a\theta = \cot b\theta$

Ans 1) $\sec 2\theta = -\frac{2}{\sqrt{3}}$

$\sec 2\theta = \sec\left(\frac{5\pi}{6}\right)$

GradeSetter

WWW.GRADESETTER.COM

$$2\theta = 2n\pi \pm \frac{5\pi}{6}$$

$$\boxed{\theta = n\pi \pm \frac{5\pi}{12}}$$ A.

ii) $\cos 3\theta = -\frac{1}{2}$

$\cos 3\theta = \cos\frac{2\pi}{3}$

$3\theta = 2n\pi \pm \frac{2\pi}{3}$

$$\boxed{\theta = \frac{2n\pi}{3} \pm \frac{2\pi}{9}}$$

iii) $\tan\theta = 2$

$\tan\theta = \tan(\tan^{-1}2)$

$$\theta = n\pi + \tan^{-1}2$$

iv) $7\cos^2\theta + 3\sin^2\theta = 4$

$\cos^2\theta = \frac{1}{4}$

$\cos^2\theta = \left(\frac{1}{2}\right)^2$

$\cos^2\theta = \cos^2\frac{\pi}{3}$

$\theta = n\pi \pm \pi/3$

WWW.GRADESETTER.COM

v) $\sin\left(\frac{\theta}{2} - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$$\sin\left(\frac{\theta}{2} - \frac{\pi}{4}\right) = \sin\frac{\pi}{4}$$

$$\frac{\theta}{2} - \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$$

$$\theta = \left(2n\pi + \frac{\pi}{2}\right) + (-1)^n \cdot \frac{\pi}{2}$$

$(n \in I)$

vi) $\tan(a\theta + b) = \tan\frac{\pi}{6}$

$$a\theta + b = n\pi + \frac{\pi}{6}$$

$$\theta = \frac{1}{a}\left(n\pi + \frac{\pi}{6} - b\right)$$

vii) $\tan a\theta = \cot b\theta$

$$\tan a\theta = \tan\left(\frac{\pi}{2} - b\theta\right)$$

$$a\theta = n\pi + \frac{\pi}{2} - b\theta$$

$$\theta(a+b) = n\pi + \frac{\pi}{2}$$

$$\theta = \frac{1}{a+b}\left(n\pi + \frac{\pi}{2}\right)$$

GradeSetter

1st Choice

2.)

Different types of technique for solving equation

1.) Factorisation :→

1)eg) Find the general solution of eqn

$\Rightarrow (2\sin x - \cos x)(1+\cos x) = \sin^2 x$

$\Rightarrow 2\sin x + 2\sin x\cos x - \cos x - \cos^2 x = \sin^2 x$

$\Rightarrow 2\sin x + 2\sin x\cos x - \cos x - 1 = 0$

$\Rightarrow 2\sin x(1+\cos x) - (\cos x+1) = 0$

$(2\sin x - 1)(\cos x+1) = 0$

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$\sin x = \sin\frac{\pi}{6}$

$x = n\pi + (-1)^n\frac{\pi}{6}$

~~cosx~~ $\cos x + 1 = 0$

$\cos x = -1$

$\cos x = \cos\pi$

$x = 2n\pi \pm \pi$

whe $(n \in I)$

So,

$x = n\pi + (-1)^n \pi/6 \; ; \; 2n\pi \pm \pi$

$(n \in I)$

WWW.GRADESETTER.COM

2.) $\cot x - \cos x = 1 - \cos x \cdot \cot x$

Cot ?

$\frac{\cos x}{\sin x} \to 0$

$\cot x + \cos x \cdot \cot x - \cos x - 1 = 0$

$\cot x(1 + \cos x) - 1(\cos x + 1) = 0$

$(\cot x - 1)(1 + \cos x) = 0$

$(1 + \cos x) = 0$

$\cos x = -1$

$\cos x = \cos \pi$

$\boxed{x = 2n\pi \pm \pi}$

(Not defined)

$\to$ (✗)

$(\cot x - 1) = 0$

$\cot x = 1$

$\cot x = \cot \frac{\pi}{4}$

$\boxed{x = n\pi + \pi/4}$

(only defined)

$\to$ (✓)

so (Here, In this value of "x" ~~cos~~ cox is not defined.)

3.) $\cos^3 x + \cos^2 x - 4\cos^2 \frac{x}{2} = 0$

~~$(\cos 2x = 2\cos^2 x - 1)$~~

use

$\left(\cos x = 2\cos^2 \frac{x}{2} - 1\right)$

$\to$ $\cos^3 x + \cos^2 x - 2(1 + \cos x) = 0$

$\cos^2 x(\cos x + 1) - 2(1 + \cos x) = 0$

$(\cos x + 1)(\cos^2 x - 2) = 0$

$\cos x = -1$

$\cos x = \cos \pi$

$x = 2n\pi \pm \pi$

$\underbrace{\cos^2 x = 2}$

(✗)

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

1st Choice

Q4) If sin α, 1, ~~[illegible]~~ are in G.P then find General solution for "α!"

~~Ar.~~

~~Q If~~
4) If sinα, 1, Cos2α are in G.P then find General solution for "α":

Ans) 1 = sinα cos2α

1 = sin α(1−2sin²α)

2sin³α − sinα + 1 = 0

or ~~[illegible]~~ (sin α = −1)

Teacher ⇒ 2sin³α + 2sin²α − 2sin²α − 2sinα + sinα + 1 = 0

⇒ (2sin²α − 2sinα + 1)(sinα + 1) = 0

2sin²α − 2sinα + 1 = 0

Δ < 0

sinα + 1 = 0

sinα = −1

sinα = sin(−π/2)

$\alpha = n\pi + (-1)^n \cdot \left(-\frac{\pi}{2}\right)$

Student

Note :→ Important trick to solve this question

VV

Let sinα = t

then

$t^3 - t + 1 = 0$

t = −1 (इस vahe पर t zero होता है)

so (t+1) = 0

→ Now Put the value

$2t^2(t+1) - 2t(t+1) + t+1 = 0$

$(t+1)(2t^2 - 2t + 1) = 0$

Now, sin α + 1 = 0

so $\alpha = n\pi + (-1)^n\left(\frac{-\pi}{2}\right)$ S.

2sin²α − 2sinα+1

Δ < 0

Delta

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

5.) $\frac{1}{6}\sin\theta, \cos\theta, \tan\theta$ are in G.P than find General Solution for "θ".

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

17/12/22

1st Choice

Technique 2:→ Trigonometric equation which can be solved by reducing then in quadratic equation.

Que 1) find the General solution of the equation

$$\cot^2\theta + 3\operatorname{cosec}\theta + 3 = 0$$

Ans $\cot^2\theta + 3\operatorname{cosec}\theta + 3 = 0$

$\operatorname{cosec}^2\theta - 1 + 3\operatorname{cosec}\theta + 3 = 0$

$\operatorname{cosec}^2\theta + 3\operatorname{cosec}\theta + 2 = 0$

$(\operatorname{cosec}\theta + 1)(\operatorname{cosec}\theta + 2) = 0$

$\operatorname{cosec}\theta = -1$	$\operatorname{cosec}\theta = -2$
$\operatorname{cosec}\theta = \operatorname{cosec}\left(\frac{-\pi}{2}\right)$	$\operatorname{cosec}\theta = \operatorname{cosec}\left(-\frac{\pi}{6}\right)$
$\theta = n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$	$\theta = n\pi + (-1)^n\left(\frac{-\pi}{6}\right)$

$(n \in I)$

2) $5\tan^4 x - \sec^4 x = 29$

$\Rightarrow 5\tan^4 x - (1+\tan^2 x)^2 = 29$

$\Rightarrow (\tan^2 x - 3)(2\tan^2 x + 5) = 0$

$\Rightarrow \tan^2 x = 3$	$\tan^2 x = -5/2$
$\tan^2 x = \tan^2 \pi/3$	$(\times)$
$\boxed{x = n\pi \pm \pi/3}$	

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

17/12/2011

(Q) Find the General solution of equation $3\cos^2 x - 10\cos x + 3 = 0$, and also find no. of solution when $x \in [0, 4\pi]$

An $3\cos^2 x - 10\cos x + 3 = 0$

$(\cos x - 3)(3\cos x - 1) = 0$

$\underline{\cos x = 3}$

(✗)

$3\cos x - 1 = 0$

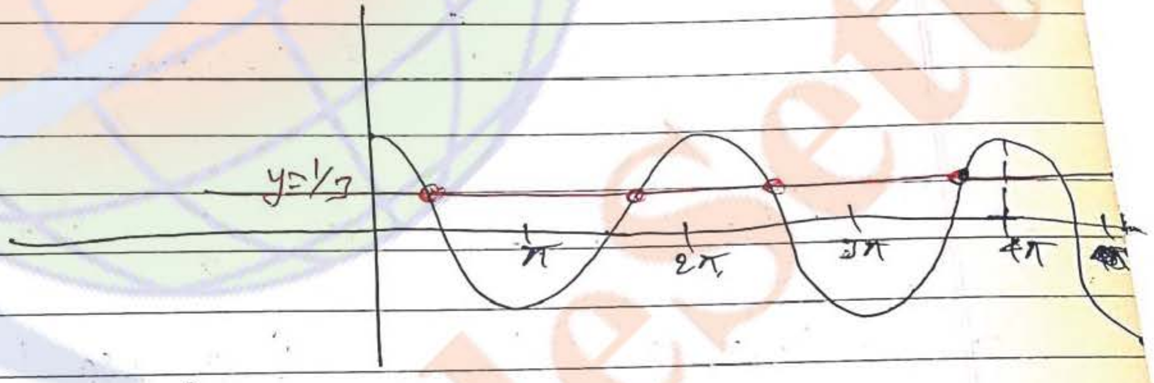
$\cos x = \frac{1}{3}$

$x = 2n\pi \pm \cos^{-1}\left(\frac{1}{3}\right)$

Graph:→

$\cos x = \frac{1}{3}$

$$\left[\begin{matrix} y = \cos x \\ y = \frac{1}{3} \end{matrix}\right]$$



So,

No. of solution = "4"

WWW.GRADESETTER.COM

Page No.
Date / /

Ex: Find the no. of Solutions of equation $10\sin x = x$

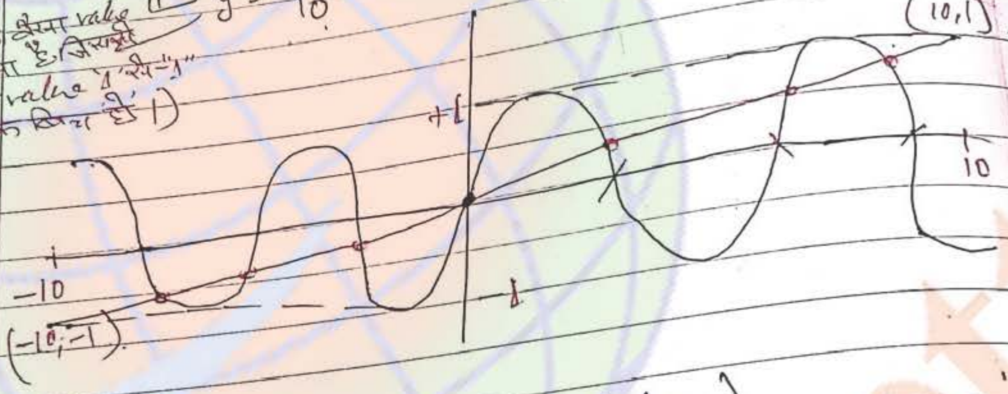
Ans $10\sin x = x$

$\sin x = \left(\frac{x}{10}\right)$

i) $y = \sin x$

ii) $y = \frac{x}{10}$

('x' का कैसा value रखा जाता है, जिससे 'y' का value '1' से "-1" के बीच होते।)



Ans ⇒ 7 (solution.)

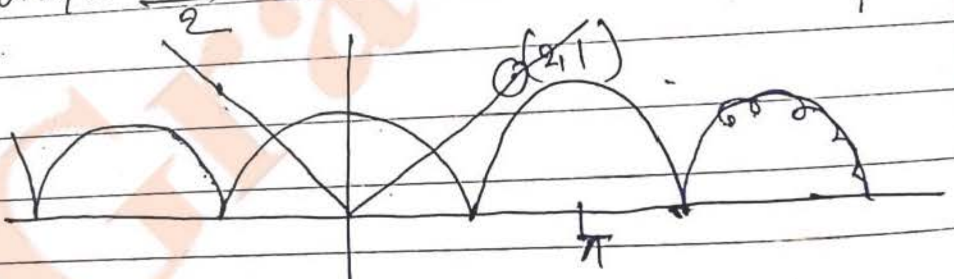
Ex Find the no. of solution of $2|\cos x| = |x|$

Ans. $2|\cos x| = |x|$

$|\cos x| = \frac{|x|}{2}$

i) $y = |\cos x|$

ii) $y = \frac{|x|}{2}$



Solution = 2 ✓

WWW.GRADESETTER.COM

Q) Find the sum of all solution's of equation

$\cos 4x + 6 = 7\cos 2x$ when $x \in [0, 314]$

Ans

$(\cos 2x = 2\cos^2 x - 1)$

$\cos 4x + 6 = 7\cos 2x$

$\Rightarrow 2\cos^2 2x - 1 + 6 = 7\cos 2x$

$\Rightarrow \underbrace{(2\cos 2x - 5)}_{\times} (\cos 2x - 1) = 0$

$\Rightarrow$ ~~$\cos 2x = 4$~~

so, $\cos 2x = 1$

$\cancel{2}x = \cancel{2}n\pi$

$x = n\pi$

$x = 0, \pi, 2\pi, 3\pi, \ldots\ldots 98\pi, 99\pi$

so,

$\pi + 2\pi + \ldots\ldots 99\pi$

$\frac{99 \times 100}{2}\pi$

नोट
$99 \times 3.14\ldots$
$100 \times 3.14 = 314.1\ldots$

I Tuse this concept क्योंकि 3.14 के बाद मे कोई नंबर आता है

Q) Solve $\cos 2\theta - (\sqrt{2}+1)\left(\cos\theta - \frac{1}{\sqrt{2}}\right) = 0$

Ans:- $(2\cos^2\theta - 1) - (\sqrt{2}+1)\left(\cos\theta - \frac{1}{\sqrt{2}}\right) = 0$

$\Rightarrow 2\left(\cos^2\theta - \frac{1}{2}\right) - (\sqrt{2}+1)\left(\cos\theta - \frac{1}{\sqrt{2}}\right) = 0$

$\Rightarrow 2\left(\cos\theta - \frac{1}{\sqrt{2}}\right)\left(\cos\theta + \frac{1}{\sqrt{2}}\right) - (\sqrt{2}+1)\left(\cos\theta - \frac{1}{\sqrt{2}}\right) = 0$

$\Rightarrow \left(\cos\theta - \frac{1}{\sqrt{2}}\right)\left[2\left(\cos\theta + \frac{1}{\sqrt{2}}\right) - (\sqrt{2}+1)\right] = 0$

WWW.GRADESETTER.COM

$$\left(\cos\theta - \frac{1}{\sqrt{2}}\right)\left(2\cos\theta - 1\right) = 0$$

$\cos\theta = \cos \pi/4$

$\theta = 2n\pi \pm \pi/4$

$\cos\theta = \cos \pi/3$

$\theta = 2n\pi \pm \pi/3$

GradeSetter

WWW.GRADESETTER.COM

(III) type →

Trigonometric equation which can be solved by transforming a sum or difference of trigonometric ratio into their Product.

Ex $\cos 3x + \sin 2x - \sin 4x = 0$

$$\boxed{(\sin C - \sin D) = 2\cos\frac{C+D}{2} \times \sin\frac{C-D}{2}}$$

$\Rightarrow \cos 3x + 2\cos 3x \cdot \sin(-x) = 0$
$\Rightarrow \cos 3x\,(1 - 2\sin x) = 0$

$\cos 3x = 0$	$\sin x = \frac{1}{2}$
$3x = 2n\pi \pm \pi/2$	$x = n\pi + (-1)^n\left(\frac{\pi}{6}\right)$

Ex $\sin 7\theta = \sin 3\theta + \sin\theta$

$\Rightarrow \sin 7\theta - \sin\theta = \sin 3\theta$
$\Rightarrow 2\cos 4\theta \cdot \sin 3\theta = \sin 3\theta$
$\Rightarrow \sin 3\theta\,(2\cos 4\theta - 1) = 0$

$\sin 3\theta = 0$	$2\cos 4\theta = 1$
$3\theta = n\pi$	$4\theta = 2n\pi \pm \pi/3$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Q.) $5\sin x + 6\sin 2x + 5\sin 3x + \sin 4x = 0$

$$\boxed{\sin C + \sin D \Rightarrow 2\sin\frac{C+D}{2}\cdot\cos\frac{C-D}{2}}$$

Ans $5(\sin x + \sin 3x) + 6\sin 2x + \sin 4x = 0$

$5(\sin x + \sin 3x) + 6\sin 2x + 2\sin 2x\cos 2x = 0$

$\Rightarrow 5\cdot 2\sin 2x\cdot\cos x + 6\sin 2x + 2\sin 2x\cos 2x = 0$

$\Rightarrow 2\sin 2x(5\cos x + 3 + \cos 2x) = 0$

$\Rightarrow 2\sin 2x(2\cos^2 x + 5\cos x + 2) = 0$

$\Rightarrow 2\sin 2x(\cos x + 2)(2\cos x + 1) = 0$

$(\times)$

$\sin 2x = 0$ | $\cos x = -1/2$

$2x = n\pi$ | $x = 2n\pi \pm \left(2\pi/3\right)$

Q.) Find the solutions of equation $\sin\theta + \sin 5\theta = \sin 3\theta$ when $0 \le \theta \le \pi$)

Ans $\sin\theta + \sin 5\theta = \sin 3\theta$

$\Rightarrow 2\sin 3\theta\cos 2\theta - \sin 3\theta = 0$

$\Rightarrow \sin 3\theta(2\cos 2\theta - 1) = 0$

$\sin 3\theta = 0$ | $2\cos 2\theta = 1$

$3\theta = n\pi$ | $2\theta = 2n\pi \pm \pi/3$

$\theta = \frac{n\pi}{3}$

Sol. $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$ and $\frac{\pi}{6}, \frac{5\pi}{6}$

WWW.GRADESETTER.COM

So Ans

$$\theta = \underbrace{0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi}, \underbrace{\frac{\pi}{6}, \frac{5\pi}{6}}$$

Q $\cos\theta - \sin 3\theta = \cos 2\theta$

$$\boxed{\cos C - \cos D = 2\sin\frac{C+D}{2} \cdot \sin\frac{D-C}{2}}$$

$$\boxed{\sin 3\theta = 3\sin\theta - 4\sin^3\theta}$$

$$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$$

$\Rightarrow \cos\theta - \cos 2\theta - \sin 3\theta = 0$ (altenate)

$\Rightarrow 2\sin\frac{3\theta}{2} \cdot \sin\frac{\theta}{2} - 2\sin\frac{3\theta}{2} \cdot \cos\frac{3\theta}{2} = 0$

$\Rightarrow 2\sin\frac{3\theta}{2}\left(\sin\frac{\theta}{2} - \cos\frac{3\theta}{2}\right) = 0$

$\Rightarrow 2\sin\frac{3\theta}{2} = 0$

$\frac{3\theta}{2} = n\pi$

$$\boxed{\theta = \frac{2n\pi}{3}}$$

$\cos\frac{3\theta}{4} = \sin\frac{\theta}{2}$

$\cos\frac{3\theta}{2} = \cos\left(\pi/2 - \frac{\theta}{2}\right)$

$\frac{3\theta}{2} = 2n\pi \pm \left(\frac{\pi}{2} - \frac{\theta}{2}\right)$

~~$\frac{3\theta}{2} = 2n\pi \pm$~~

Q) Solve the eqn $\cos^2 x + \cos^2 2x + \cos^2 3x + \cos^2 4x = 2$

$$\boxed{\cos C + \cos D = 2\cos\frac{C+D}{2} \cdot \cos\frac{C-D}{2}}$$

Ans: C

$$\boxed{\begin{gathered}\cos 2x = 2\cos^2 x - 1 \\ \cos^2 x = \left(\frac{1+\cos 2x}{2}\right)\end{gathered}}$$

1st Choice

~~$\Rightarrow \left(\frac{1+\cos 2x}{2}\right) + ($~~

$\Rightarrow \left(\frac{1+\cos 2x}{2}\right) + \left(\frac{1+\cos 4x}{2}\right) + \left(\frac{\cos 6x+1}{2}\right) + \left(\frac{1+\cos 8x}{2}\right)$

$\Rightarrow (\cos 2x + \cos 8x) + (\cos 4x + \cos 6x) = 0$

$\Rightarrow 2\cos 5x \cdot \cos 3x + $ ~~$2\cos 5x$~~ $2\cos 5x \cos x = 0$

$\Rightarrow 2\cos 5x[\cos 3x + \cos x] = 0$

$\Rightarrow 2\cos 5x\,(2\cos 2x \cdot \cos x) = 0$

$\cos x = 0$	$\cos 2x = 0$	$\cos 5x = 0$
$x = (2n+1)\pi/2$	$2x = (2n+1)\pi/2$ $x = (2n+1)\frac{\pi}{4}$	$5x = (2n+1)\pi/2$ $x = (2n+1)\frac{\pi}{10}$

GradeSetter

WWW.GRADESETTER.COM

~~type~~ (IV)

Trigonometric Equation which can be solved by transforming a product of trigonometric ratio into their sum or Difference:→

Q. $\sin 5x \cdot \cos 3x = \sin 6x \cdot \cos 2x$

Ans

$$\boxed{\sin(A+B)+\sin(A-B)=2\sin A\cdot\cos B}$$

⇒ ~~Sin (5x + 2x) + Sin (3x~~

$\Rightarrow \cancel{\sin 8x} + \sin 2x = \cancel{\sin 8x} + \sin 4x$

$\Rightarrow \sin 4x - \sin 2x = 0$

$\Rightarrow 2\sin 2x \cdot \cos 2x - \sin 2x = 0$

$\sin 2x(2\cos 2x - 1) = 0$

Q. $\sin 3\theta = 4\sin\theta \cdot \sin 2\theta \cdot \sin 4\theta$

$$\boxed{2\sin A\cdot\sin B=\cos(A-B)-\cos(A+B)}$$

Ans $\sin 3\theta = 2[\cos\theta - \cos 3\theta]\sin 4\theta$

$\Rightarrow \sin 3\theta = 2\sin 4\theta\cos\theta - 2\sin 4\theta\cdot\cos 3\theta$

$\Rightarrow \cancel{\sin 3\theta} = (\sin 5\theta + \cancel{\sin 3\theta}) - (\sin 7\theta + \sin\theta)$

$\Rightarrow \sin\theta + (\sin 7\theta - \sin 5\theta) = 0$

$\Rightarrow \sin\theta + 2\cos 6\theta \cdot \sin\theta = 0$

$\Rightarrow \sin\theta(1 + 2\cos 6\theta) = 0$

$\sin\theta = 0$	$\cos 6\theta = 1/2$
$\theta = n\pi$	$6\theta = 2n\pi \pm \frac{2\pi}{3}$

GradeSetter

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Page No.
Date / /

Q:> Find the solution of equation:-

$\cos\theta \cdot \cos 2\theta \cdot \cos 3\theta = \frac{1}{4}$ θ lying in the Interval $(0<\theta<\pi)$

Ans $2\left[2\cos\theta \cdot \cos 3\theta\right] \cos 2\theta = 1.$

$\Rightarrow 2\left[\cos 4\theta + \cos 2\theta\right] \cos 2\theta = 1.$

$\Rightarrow 2\cos 4\theta \cdot \cos 2\theta + (2\cos^2 2\theta - 1) = 0$

$\Rightarrow 2\cos 4\theta \cdot \cos 2\theta + \cos 4\theta = 0$

$\cos 4\theta\,(2\cos 2\theta + 1) = 0$

$4\theta = 2n\pi \pm \pi/2$	$2\theta = 2n\pi \pm \frac{2\pi}{3}$
$\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$	$\frac{\pi}{3}, \frac{2\pi}{3}$

Q:> $\tan\theta \tan\left(\frac{\pi}{3}+\theta\right) \tan\left(\frac{\pi}{3}-\theta\right) = \frac{1}{\sqrt{3}}$

$\tan 3\theta = \frac{1}{\sqrt{3}}$

$\because \tan\theta \cdot (\tan 60-\theta) \tan(60$
$\Rightarrow \tan 3$

$3\theta = n\pi + \frac{\pi}{6}$

$\theta = \frac{n\pi}{3} + \frac{\pi}{18}$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

the Interval
$0<\theta<\pi$)

Q:) Solve the equation:-

$$\sin 3\alpha = 4\sin\alpha \cdot \sin(x+\alpha)\,(\sin x-\alpha)$$

(given $\sin\alpha \neq 0$)

) $\tan(60+\theta)$
$\Rightarrow \tan 3\theta$

GradeSetter

WWW.GRADESETTER.COM

1st Choice

V) Trigonometric equation of the form $\boxed{a \sin x + b \cos x = c}$:⇒

Let

$$\left.\begin{aligned} a &= r\sin\alpha \\ b &= r\cos\alpha \end{aligned}\right\}$$

$$\boxed{r^2 = a^2 + b^2}$$

$$\boxed{\tan\alpha = \frac{a}{b}}$$ → (sin की coefficient) (cos की coefficient)

$\alpha = \tan^{-1}\frac{a}{b}$

so,

$$r[\sin\alpha \sin x + \cos\alpha \cos x] = c$$

$$\cos(x-\alpha) = \frac{c}{r}$$

$$x - \alpha = 2n\pi \pm \cos^{-1}\left(\frac{c}{r}\right)$$

so,

$$\boxed{x = 2n\pi + \alpha \pm \cos^{-1}\left(\frac{c}{r}\right)}$$

→ (This is not formula)

Q. Solve the equation $1\sin x + 1\cos x = \sqrt{2}$

A. $\left.\begin{array}{l} 1 = r\sin\alpha \\ 1 = r\cos\alpha \end{array}\right\}$ $r = \sqrt{2}$, $\tan\alpha = 1$

$\Rightarrow r\left[\cos x\cdot\cos\alpha + \sin x\sin\alpha\right] = \sqrt{2}$

$\Rightarrow \sqrt{2}\cos(x-\alpha) = \sqrt{2}$

$\cos\left(x-\frac{\pi}{4}\right) = 1$

$x - \frac{\pi}{4} = 2n\pi$

$$\boxed{x = 2n\pi + \frac{\pi}{4}}$$

Q. Solve the equation $\sqrt{3}\cos x + \sin x = 2$

A. $\sqrt{3} = r\cos\alpha$

$1 = r\sin\alpha$

$r^2 = a^2 + b^2$

$= (\sqrt{3})^2 + 1$

$= 4$

$r = 2$ and $\tan\alpha = \sqrt{3}$

(original eq^n [illegible] भाग [illegible]

$\Rightarrow \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = \frac{2}{2}$

value

$\Rightarrow \cos\frac{\pi}{6}\cdot\cos x + \sin\frac{\pi}{6}\cdot\sin x = 1.$

WWW.GRADESETTER.COM

$\frac{7}{6}$

$$~~\cos~~$$
$$\cos\left(x-\frac{\pi}{6}\right)=1$$

$$\left\{\begin{matrix}\cos\theta = 1\\ (\alpha)\end{matrix}\right.$$

Go-1

$$x-\frac{\pi}{6}=2n\pi$$

$$\boxed{x=2n\pi+\frac{\pi}{6}}$$

Q6 Solve the equation $3\cos x+4\sin x=5$

$$3=r\cos\alpha$$
$$4=r\sin\alpha$$

$$r^2=(3)^2+(4)^2$$
$$\Rightarrow 5$$

$$\tan\alpha=\frac{3}{4}$$

$$\frac{3}{5}\cos x+\frac{4}{5}\sin x=1$$

$$\cos\alpha\cos x+\sin\alpha\sin x=1$$

$\alpha = \cos^{-1}$ 5

(8)

$$\cos(x-\alpha)=1$$

$$(x-\alpha)=2n\pi$$

$$\boxed{x=2n\pi+\alpha}$$

where

$$\tan\alpha=\frac{4}{3}$$

WWW.GRADESETTER.COM

Q-1 $\sin 2x + \cos 2x = 1$

$1 = r \sin 2x$

$1 = r \cos 2x$

$r = \sqrt{2}$, $\tan \alpha = 1$.

~~$\sin 2x + \cos 2x = 1$~~

~~$r (\sin \alpha \sin$~~

$$\Rightarrow \frac{1}{\sqrt{2}} \sin 2x + \frac{1}{\sqrt{2}} \cos 2x = \frac{1}{\sqrt{2}}$$

$$\sin \frac{\pi}{4} . \sin 2x + \cos \frac{\pi}{4} \cos 2x = \frac{1}{\sqrt{2}}$$

Attention $\boxed{\sqrt{2}}$

$$\cos\left(2x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$2x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$$

$$2x = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$$

Note: ~~2~~

Note:→

The equation

$a\sin x + b\cos x = c$

has a solution only if $\boxed{|c| \le \sqrt{a^2+b^2}}$

Q) Find the no. of Integral values of 'k' for which the equation $3\cos x + \sin x = k$ is solvable (solve करने योग्य हो')

$\because \sqrt{a^2+b^2} = \sqrt{1}$

As

$$-\sqrt{10} \le k \le +\sqrt{10}$$

$$-3, -2, -1, 0, 1, 2, 3$$

WWW.GRADESETTER.COM

$\pm\sqrt{10}$

<VI>

*. Solution when two trigonometric equation are given

* Periodic function:- ~~If $f(x) = f(x)+T$~~

If $f(x) = f(x+T)$

$\forall\, x \in D_f$ (Domain of x)

Then

$f(x)$ is called periodic function with period "T"

eg! → $\sin(x+2\pi) = \sin x$

$\forall\, x \in R$

So, $\sin x$ is a periodic function with period "2π".

1) If period of function "$f(x) \to$ "T""

Then period of $\boxed{f(ax) = \frac{T}{|a|}}$

eg ~~$\sin 2x$~~

Period of $\sin 2x = \left(\frac{2\pi}{2}\right) = \pi$.

2.) If period of function $f(x)$ is "T_1" and period of function $g(x)$ is "T_2"

Then,

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

1st Choice

Common period of f(x) and g(x) is

$$\boxed{L.C.M(T_1, T_2)}$$

$$\boxed{L.C.M\ of \left(\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\right) = \frac{L.C.M(a,c,e)}{H.C.F(b,d,f)}}$$

Steps:->

1) Find the least non-negative common solution of both the equation.

2.) multiply the common period to 'n' and add it to the answer in step "1".

Notes:-> Periods of Some Important functi

1) sinx, cosx, secx, cosecx → "2π"

2.) tanx, cotx → "π"

3.) Period of
sin²x, cos²x, tan²x, cot²x, sec²x and cose
is "π"

WWW.GRADESETTER.COM

Eg!- Find the Common solution of Equations $\sin x = \frac{1}{2}$; $\tan x = \frac{-1}{\sqrt{3}}$

Ans 1 step!— $\sin x = \frac{1}{2}$

$x = n\pi + (-1)^n \frac{\pi}{6}$

n	x
0	$\pi/6$
1	$5\pi/6$

$\tan x = \frac{-1}{\sqrt{3}}$

$x = n\pi + -\frac{\pi}{6}$

n	x
0	$-\pi/6$
1	$5\pi/6$

So, least non-negative com. solution $= \frac{5\pi}{6}$

ii) Step!—

$T_1 = 2\pi$

$T_2 = \pi$

Common period $= \text{L.C.m}(2\pi, \pi)$

$= 2\pi$

n' से multiply

So,

General Solution :-

$$\boxed{x = 2n\pi + \frac{5\pi}{6}}$$

→ least common non-negative Solution

WWW.GRADESETTER.COM

0, 30, 45, 60, 90, 120, 150, 180

Page No.
Date / /

1st Choice

Eg:) $\sin 3x + \cos 2x = -2$

Ans

$\sin 3x = -1$	$\cos 2x = -1$
$x = n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$	$x = 2n\pi \pm \pi$

A:) $2n\pi + \pi/2$

(Sol)

Ex:) $\cos\theta + \cos 2\theta + \cos 3\theta = 3$

1st

$\cos\theta = 1$	$\cos 2\theta = 1$	$\cos 3\theta = 1$
$\theta = 2n\pi$	$2\theta = 2n\pi$	$3\theta = 2n\pi$
$n = 0,\ 0$	0	0

IInd :-

$$\text{L.C.M of } \left(2\pi, \frac{2\pi}{2}, \frac{2\pi}{3}\right)$$

$$= \frac{\text{L.C.M}\,(2\pi, 2\pi, 2\pi)}{\text{H.C.F}\,(1,2,3)} = 2\pi$$

$$\Rightarrow \boxed{\theta = 2n\pi + 0}$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Q1) $\sin x\cdot\left(\cos\frac{x}{4}-2\sin x\right)+\left(1+\sin\frac{x}{4}-2\cos x\right)\cdot\cos x=0$

1st step! –

$\Rightarrow\left(\sin x\cos\frac{x}{4}+\cos x\cdot\sin\frac{x}{4}\right)-2\left(\sin^2x+\cos^2x\right)+\cos x=0$

$\Rightarrow \sin\frac{5\pi}{4}+\cos x=2$

$\sin\frac{5x}{4}=1$

$\frac{5x}{4}=n\pi\pm(-1)^n\pi/2$

x	n
$2\pi/5$	0
$6\pi/5$	1
$\boxed{2\pi}$	2

$\cos x=1$

$x=2n\pi$

x	n
0	0
$\boxed{2\pi}$	4

IInd period,

L.C.M $\left(\frac{2\pi\times4}{5},2\pi\right)$

$$\boxed{x=8n\pi+2\pi}$$

Q) $2\cot^2x+2\sqrt{3}\cot x+4\operatorname{cosec}x+8=0$

Ans – $\underline{\cot^2x}+\left(\operatorname{cosec}^2x-1\right)+\underline{2\sqrt{3}\cot x}+4\operatorname{cosec}x+8=0$

$\Rightarrow \cot^2x+2\sqrt{3}\cot x+\operatorname{cosec}^2x+4\operatorname{cosec}x+7=0$

$\Rightarrow\left(\cot x+2\sqrt{3}\cot x+3\right)+\left(\operatorname{cosec}^2x+4\operatorname{cosec}x+4\right)=0$

$\Rightarrow\left(\cot x+\sqrt{3}\right)^2+\left(\operatorname{cosec}x+2\right)^2=0$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

0, 30, 45, 60, 90, 120, 15

$\cot x = -\sqrt{3}$

$x = n\pi + -\left(\frac{\pi}{6}\right)$

$-\pi/6$

$5\pi/6$

$11\pi/6$

$\operatorname{cosec} x = -2$

$x = n\pi + (-1)^n\left(\pi/6\right)$

$-\pi/6$

$11\pi/6$

IInd Step:-

Common period $= L.C.M(\pi, 2\pi)$
$= 2\pi$

$$\boxed{x = 2n\pi + 11\pi/6}$$

WWW.GRADESETTER.COM

Use of boundness of trigonometric function

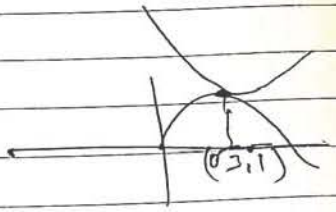
Ex Solve the equation $\sin\frac{\pi x}{2\sqrt{3}} = (x^2 - 2\sqrt{3}x + 4)$

$$\underset{\le 1}{\sin\frac{\pi x}{2\sqrt{3}}} = \underset{\ge 1}{(x-\sqrt{3})^2 + 1}$$

$\sin\frac{\pi(\sqrt{3})}{2\sqrt{3}} = 1$ | $\boxed{x=\sqrt{3}}$

so

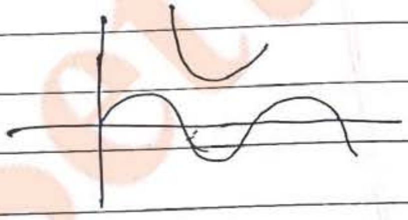
$\boxed{x=\sqrt{3}}$ Ans



Ex — Find the solution of equation $\sin x = (x^2 - 2\sqrt{3}x + 4)$

$\sin x = (x-\sqrt{3})^2 + 1$

$\sin(\sqrt{3}) \neq 1$ | $x=\sqrt{3}$



Ex Find the ordered pair (x,t) satisfying the equation.

$\sin t = x^2 - 2\sqrt{3}x + 4$

Ans

$$\underset{\le 1}{\sin t} = \underset{\ge 1}{(x-\sqrt{3})^2 + 1}$$

Now
$\sin t = 1$ | $x=\sqrt{3}$

$t = n\pi + (-1)^n \frac{\pi}{2}$

(x, t)

$\left(\sqrt{3};\ n\pi + (-1)^n \frac{\pi}{2}\right)$

Q solve the equation $\cot^2(x+y) + \tan^2(x+y) + y^2 + 2y$

Ans

GradeSetter

$+y^2+2y-1=0$

Trigonometric equation of the form
"$P(\sin x \pm \cos x, \ \sin x \cdot \cos x) = 0$"

Hint:—

Let $(\sin x + \cos x = t)$

$\sin^2 x + \cos^2 x + 2\sin x \cos x = t^2$

$1 + 2\sin x \cos x = t^2$

$\left(\sin x \cos x = \left(\frac{t^2-1}{2}\right)\right)$

Eg:- Show the equation $\sin x + \cos x = 1 + \sin x \cdot \cos x$

Ans:-
$\sin x + \cos x = t$
$\sin x \cos x = \frac{t^2-1}{2}$

$t = 1 + \left(\frac{t^2-1}{2}\right) \Rightarrow \boxed{t = 1}$

$\Rightarrow \sin x + \cos x = 1$

$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}}$

$\cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

WWW.GRADESETTER.COM

Eg-4) ~~sinx + cosx =~~ $\sin x + \cos x - 2\sqrt{2}\sin x \cos x = 0$

8) →

GradeSetter

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Mis cellaneous :⇒

1.) $5\sin^2 x - 7\sin x\cos x + 16\cos^2 x = 4$

Ans Divide by $\cos^2 x$

$\Rightarrow 5\tan^2 x - 7\tan x + 16 = 4\sec^2 x$

$\Rightarrow 5\tan^2 x - 7\tan x + 16 = 4(1 + \tan^2 x)$

$\tan^2 - 7\tan x + 12 = 0$

$(\tan x - 3)(\tan x - 4) = 0$

$\tan x = 3$ | $\tan x = 4$

$x = n\pi + \tan^{-1} 3$ | $x = n\pi + \tan^{-1} 4$

GradeSetter

WWW.GRADESETTER.COM

1st Choice

Page No.
Date / /

Trigonometric Inequality :-

1.) Find the solution of Inequality

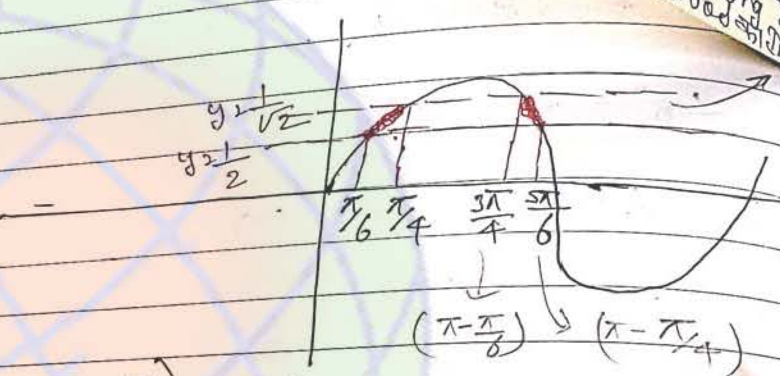
$$\frac{1}{2} \le \sin\theta \le \frac{1}{\sqrt{2}}$$

$y = \frac{1}{\sqrt{2}}$

$y = \frac{1}{2}$

Note :- कोई भी Interval ले सकते same meaning है π ले सकते (Period की Interval ले)

(Sine की Graph दो Interval में देखें)



$$\left(\frac{\pi}{6} \le \theta \le \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4} \le \theta \le \frac{5\pi}{6}\right)$$

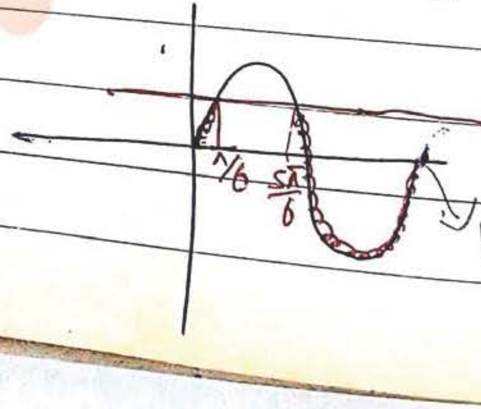
• To make this equation General we add "2nπ".

$$\left\{2n\pi + \frac{\pi}{6} \le \theta \le 2n\pi + \frac{\pi}{4}\right\} \cup \left\{2n\pi + \frac{3\pi}{4} \le \theta \le 2n\pi + \frac{5\pi}{6}\right\}$$

2.) $\sin\theta \le \frac{1}{2}$

$$\text{so } \left(0 \le \theta < \frac{\pi}{6},\ \frac{5\pi}{6} \le \theta < 2\pi\right).$$

$2n\pi + 0 \le \theta \le 2n$

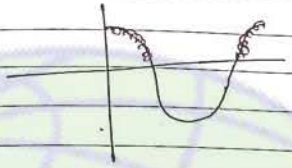


(2π) का Period पर ही रहेगा

$\frac{\pi}{2}+\frac{\pi}{6} = \frac{7\pi+\pi}{6} = \frac{4\pi}{3}$ $\frac{4\pi}{3}$

(Generally "+ve" side को लेते हैं।

Q $0 < \cos < 4$



$0 < \theta < \frac{\pi}{2}$ $\frac{3\pi}{2} < \theta < 2\pi$

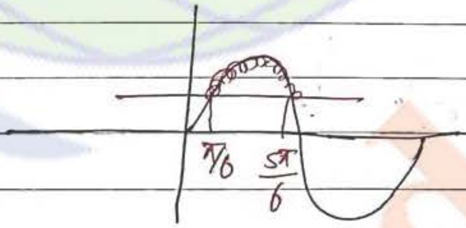
so,

$$\left(2n\pi + 0 < \theta < 2n\pi + \frac{\pi}{2} , \ \frac{3\pi}{2} + 2n\pi < \theta < 2n\pi + \frac{\pi}{2} \right)$$

Q) $4\sin^2\theta - 8\sin\theta + 3 \leq 0$

$$\Rightarrow (2\sin\theta - 1)\underbrace{(2\sin\theta - 3)}_{\ominus} \leq 0$$

$$\Rightarrow 2\sin\theta - 1 \geq 0 \Rightarrow \sin\theta \geq \frac{1}{2}$$



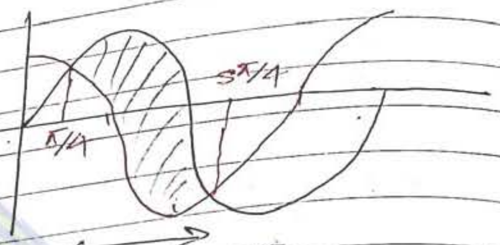
$$\frac{\pi}{6} \leq \theta \leq \frac{5\pi}{6}$$

Q) ~~$\sin x > \cos x$~~

WWW.GRADESETTER.COM

mod का Interval "π"

[9 i) $\sin x > \cos x$

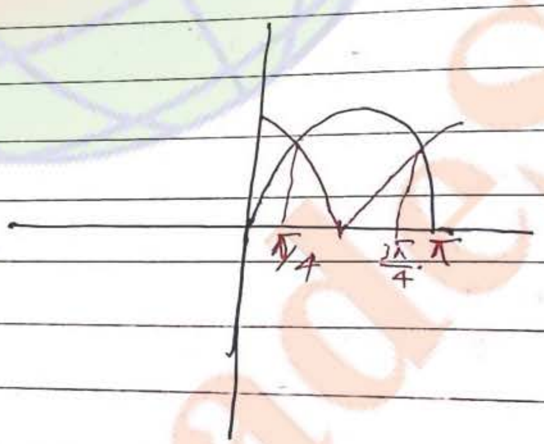


$\frac{\pi}{4} < x < \frac{5\pi}{4}$

ii) $\cos x > \sin x$ (Ans)

$\left(0 < \theta < \frac{\pi}{4}, \ \frac{5\pi}{4} < \theta < 2\pi\right)$

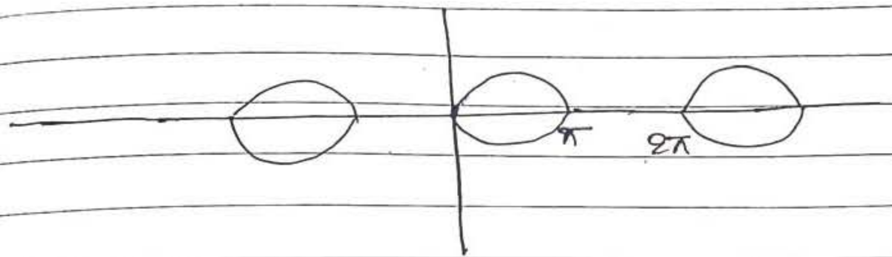
[10 $|\sin x| < |\cos x|$



$0 < \theta < \pi/4 \ , \ \frac{3\pi}{4} < \theta < \pi$

WWW.GRADESETTER.COM

(x) $|y| = \sin x$



GradeSetter

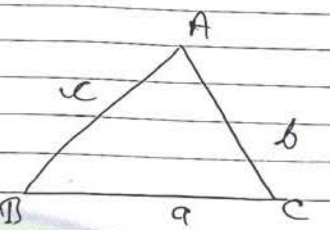
WWW.GRADESETTER.COM

22/12/2011

1st Choice

Properties of Triangle

Page No.
Date / /



A, B, C
a, b, c } 6 elements.

* Condition :-

i) $A + B + C = \pi$

ii) $a + b > c$

$b + c > a$

$c + a > b$

* Sine Rule :⇒

In a ΔABC :-

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R}}$$

where R → circumradius of triangle ABC

or

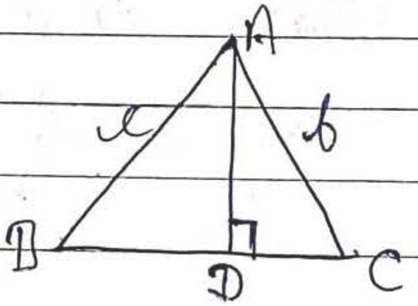
$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R}$$

Proof :-

From ΔABD :⇒

$AD = AB \sin B$

$AD = c \sin B$ —(1)



WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

now,
In ΔADC
$AD = AC \sin C$
$AD = b \sin C$ —②

~~$\Rightarrow c \sin B = b \sin C$~~
~~$\Rightarrow c \sin B \sin C$~~
$\Rightarrow c \sin B = b \sin C$

$$\Rightarrow \frac{\sin B}{b} = \frac{\sin C}{c}$$

Ex In a ΔABC, show that ~~(b−c)sin~~

$$(b-c)\sin A + (c-a)\sin B + (a-b)\sin C = 0$$

Ans — ~~$\frac{a}{\sin A}$ $b\sin A - a\sin A + c\sin B - a\sin B +$~~
~~$a\sin C - b\sin C = 0$~~

$\sin A = k \cdot a$
$\sin B = k \cdot b$
$\sin C = k \cdot c$

Here, we let $\frac{1}{2R} = k$

Now

$$(b-c)ak + (c-a)bk + (a-b)ck = 0$$

Ex) Prove that In a ΔABC

$$\frac{a+b}{c} = \frac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}}$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

1st Choice

An L.H.S

$$\frac{k\sin A + k\sin B}{k\sin C}$$

Here we let 2R = k

$\frac{a}{\sin A} = 2R = k$

a = k sin A

$$\Rightarrow \frac{2\sin\frac{A+B}{2}\cdot\cos\left(\frac{A-B}{2}\right)}{\sin C}$$

$$\Rightarrow \frac{\cancel{2}\sin\left(\frac{\pi}{2}-\frac{C}{2}\right)\cos\frac{A-B}{2}}{\cancel{2}\sin C/2\,\cancel{\cos C/2}}$$ 4 St

$$= \frac{\cos\left(\frac{A-B}{2}\right)}{\sin C/2}$$

Q Show that $a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0$

Ans⇒ Let 2R = k, so, a = k sin A

$\Rightarrow k[\sin A\sin(B-C) + \sin B\cdot\sin(C-A) + \sin C\sin(A-B)]$

$\Rightarrow k[\sin(B+C)(\sin B-C) + \sin(C+A)(\sin C-A) + \sin(A+B)\sin(A-B)]$

$\Rightarrow k[\sin^2 B - \sin^2 C) + (\sin^2 C - \sin^2 A) + \sin^2 A - \sin^2 B)] = 0$

Pr

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Page No.
Date / /

~~ABC of~~

Q) If $\angle A, B, C$ of a "Δ" are in A.P, then findthe angles,

$$\frac{b}{c} = \frac{\sqrt{3}}{2},$$

$\because \left(B = \frac{A+C}{2}\right)$

Ans:-

$$2B = A + C$$

$$3B = A + B + C = 180^\circ$$

$$B = 60^\circ$$

$$\frac{\sin 60^\circ}{b} = \frac{\sin C}{c}$$

$$\frac{\sqrt{3}}{2} = \left(\frac{b}{c}\right)\sin C$$

$$\sin C = 1$$

$$\boxed{C = 90}$$

HW

(Q1) In a ΔABC show that

$$a\sin\left(\frac{A}{2} + B\right) = (b + c)\sin\frac{A}{2}$$

(Q2) In a ΔABC the median, Internal angle bisector and the Altitude drawn from "a" divides the angle at A into four equal parts, then findthe angles.

Q:- $A+B+C=\pi$

(Q.1)

An $a\sin($

$\Rightarrow \sin$

RHS:-

(Q2)

An

WWW.GRADESETTER.COM

If A+B+C=π then sin A = sin(B+C)
sin B = sin(A+C)
sin C = sin(A+B)

Q.1)

Ans $a\sin\left(\frac{A}{2}+B\right)=(b+c)\sin\frac{A}{2}$

$\Rightarrow \frac{\sin\left(\frac{A}{2}+B\right)}{\sin A/2}=\left(\frac{b+c}{a}\right)$

RHS :-

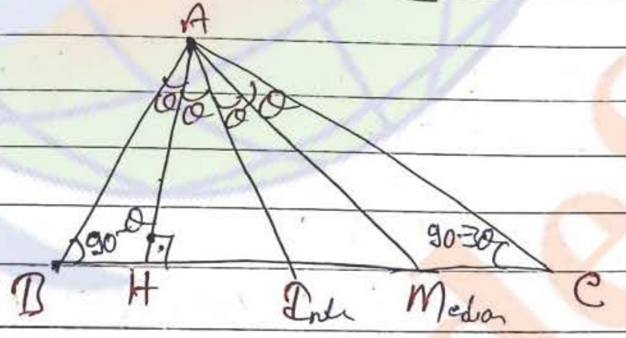
$\frac{\sin B+\sin C}{\sin A}$

$\Rightarrow \frac{\sin B+\sin(A+B)}{\sin A}$

$\Rightarrow \frac{2\sin\left(\frac{A+2B}{2}\right)\cdot\cancel{\cos A/2}}{2\sin A/2\cdot\cancel{\cos A/2}}$

Q2)

Ans



CM = BM ∵ median of Δ.

ΔABM :-

$\frac{AM}{\sin(90-\theta)}=\frac{BM}{\sin 3\theta}$ —— ①

In ΔAMC

$\frac{AM}{\sin(90-3\theta)}=\frac{CM}{\sin\theta}$ —— ②

1st Choice

now eq①/eq②

$$\frac{\sin(90-3\theta)}{\sin(90-\theta)} = \frac{\sin\theta}{\sin 3\theta}$$

$2\cos 3\theta . \sin 3\theta = 2\sin\theta . \cos\theta$

$\Rightarrow \sin 6\theta = \sin 2\theta$

$\Rightarrow \sin 6\theta = \sin(\pi - 2\theta)$

$\Rightarrow 6\theta = \pi - 2\theta$

$\Rightarrow \theta = \pi/8$

$\angle A = \pi/2$

⑥ In a ΔABC If $a\cos A = b\cos B$ then show that ΔABC is either Isoceles or right angle.

Ans. $k\sin A \cdot \cos A = k\sin B \cos B$

$\sin 2A = \sin 2B$

i) $\cancel{2}A = \cancel{2}B$

$A = B$

(Δ is Isoscele)

ii) $2A = \pi - 2B$ $\because \sin(\pi-\theta) = \sin\theta$

$A+B = \pi/2$

$\boxed{C = \pi/2}$

$\therefore$ Δ is Right angle

WWW.GRADESETTER.COM

1) Cosine Rule:—

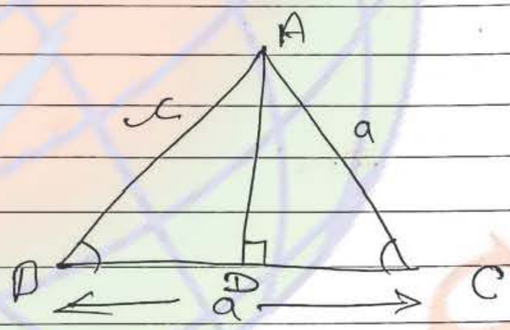
In a ΔABC:—

i) $\boxed{\cos A = \frac{b^2+c^2-a^2}{2bc}}$

ii) $\boxed{\cos B = \frac{a^2+c^2-b^2}{2ac}}$

iii) $\boxed{\cos C = \frac{a^2+b^2-c^2}{2ab}}$

Proof:—



In ΔABD

$$AB^2 = BD^2 + AD^2$$

$$= (BC - CD)^2 + AD^2$$

$$AB^2 = BC^2 + (CD^2 + AD^2) - 2BCCD$$

$$AB^2 = BC^2 + AC^2 - 2BC \cdot AC\cos C$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\boxed{\cos C = \frac{a^2+b^2-c^2}{2ab}}$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Q. In a ΔABC If $a=13$, $b=8$, $c=7$ then find $\sin A$.

An $\cos A = -\frac{1}{2}$

so, $\sin A = \sqrt{3}/2$

Q. If $a : b : C = 2 : \sqrt{6} : \sqrt{3}+1$ then find the angle.

A $$\underset{\substack{\downarrow\\45^\circ}}{\frac{\sin A}{\left(\frac{2}{2\sqrt{2}}\right)}} = \underset{\substack{\downarrow\\60^\circ}}{\frac{\sin B}{\left(\frac{\sqrt{6}}{2\sqrt{2}}\right)}} = \underset{\substack{\downarrow\\75^\circ}}{\frac{\sin C}{\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}}$$

or

$$\underset{\substack{\downarrow\\45^\circ}}{\frac{a}{\left(\frac{2}{2\sqrt{2}}\right)}} = \underset{\substack{\downarrow\\60^\circ}}{\frac{b}{\left(\frac{\sqrt{6}}{2\sqrt{2}}\right)}} = \underset{\substack{\downarrow\\75^\circ}}{\frac{c}{\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}}$$

Q In a ΔABC If $\angle A = 60^\circ$ then find the value of $\left(1+\frac{a}{c}+\frac{b}{c}\right)\left(1+\frac{c}{b}-\frac{a}{b}\right)$

A

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$$\left(\frac{a+c+b}{c}\right)\left(\frac{b+c-a}{b}\right)$$

$$\Rightarrow \frac{(b+c)^2-a^2}{bc}$$

$$\Rightarrow 2\left(\frac{b^2+c^2-a^2}{2bc}\right)+2$$

$$\Rightarrow 2\cos 60^\circ + 2 = 3$$

Q. If $a = x^2+x+1$, $b = 2x+1$, and $c = x^2-1$ then find $\angle A$.

Ans:

GradeSetter

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

1st Choice

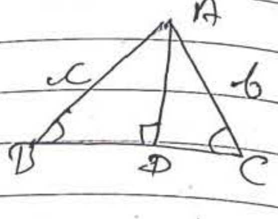
III) Projection formula ⇒

i) $a = b\cos C + c\cos B$

ii) $b = c\cos A + a\cos C$

iii) $c = \underset{\text{small}}{a}\cos \underset{\text{capital}}{B} + \underset{\text{small}}{b}\cos \underset{\text{capital}}{A}$

Proof ⇒



$BC = BD + CD$
$= AB\cos B + AC\cos C$

$$\boxed{a = c\cos C + b\cos c}$$

Q) Show that $(b+c)\cos A + (c+a)\cos B + (a+b)\cos C = a+b+c$ Q2)

Ans ⇒ $\Rightarrow b\cos A + c\cos A + c\cos B + a\cos B + a\cos C + b\cos C$

$\Rightarrow \underbrace{b\cos C + c\cos B}_{a} + c\underbrace{\cos A + a\cos}_{b} C + \underbrace{a\cos B + b\cos A}_{c}$

$\Rightarrow a+b+c$

Q) Show that $2\left(b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2}\right) = a+b+c$

L.H.S

$\Rightarrow b(1+\cos C) + c(a+\cos B)$

$\Rightarrow b+c+(b\cos C + c\cos B)$

$\Rightarrow b+c+a$

WWW.GRADESETTER.COM

Page No.
Date / /

Q.) $\frac{a\sec A+b\sec B}{\tan A+\tan B}=\left(\frac{a}{\sin A}\right)$

Ans:- $\frac{\frac{a}{\cos A}+\frac{b}{\cos B}}{\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}} \Rightarrow \frac{a\cos B+b\cos A}{\sin(A+B)}$

$\Rightarrow \frac{c}{\sin C}=\frac{a}{\sin A}$.

Q.1) If $a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=\frac{3b}{2}$

then show that a, b, c are in A.P

Ans.)

Q2.)

If Perimeter of ΔADC is 20cm then, find the length of tangent AE

WWW.GRADESETTER.COM
1st Choice

(6) $\frac{\cos B}{\cos C} = \frac{c - b\cos A}{b - c\cos A}$

~~cos B~~ R.H.S ⇒ $\frac{a\cos B + \cancel{b\cos A} - \cancel{b\cos A}}{\cancel{c\cos A} + a\cos C - \cancel{c\cos A}}$

$= \frac{\cancel{a}\cos B}{\cancel{a}\cos C}$

⇒ $\frac{\cos B}{\cos C}$ Done

GradeSetter
WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

IV) Napier Analogy (Tangent Rule) :-

In a ΔABC :-

i) $$\boxed{\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cdot\cot\frac{A}{2}}$$

or

$$\boxed{\frac{\tan\frac{B-C}{2}}{\tan\frac{B+C}{2}} = \frac{b-c}{b+c}}$$

ii) $$\boxed{\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cdot\cot\frac{B}{2}}$$

iii) $$\boxed{\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}}$$

Proof:-

R.H.S $\Rightarrow \frac{b-c}{b+c}$

$\Rightarrow \frac{k\sin B - k\sin C}{k\sin B + k\sin C}$ (∵ By using Sine Rule)

$\Rightarrow \frac{\sin B - \sin C}{\sin B + \sin C}$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$$\Rightarrow \frac{2\cos\frac{B+C}{2}\cdot\sin\frac{B-C}{2}}{2\cdot\sin\frac{B+C}{2}\cos\left(\frac{B-C}{2}\right)}$$

$$\Rightarrow \tan\left(\frac{B-C}{2}\right)\cdot\cot\left(\frac{B+C}{2}\right)$$

$$\Rightarrow \tan\frac{B-C}{2}\cdot\cot\left(\frac{\pi}{2}-\frac{A}{2}\right)$$

$$\frac{b-c}{b+c}=\tan\frac{B-C}{2}\cdot\tan\frac{A}{2}$$

(iv) Find the unknown elements of ΔABC in which

$a = \sqrt{3}+1$
$b = \sqrt{3}-1$
$c = 60$

Ans: $\tan\frac{A-B}{2} = \frac{2}{2\sqrt{3}}\cot 30^\circ$

$$\frac{A-B}{2} = 45^\circ$$

$A-B = 90^\circ$ — ①
$A+B = 120$ — ②

$A = 105^\circ,\ B = 15^\circ$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

(iv) In a ΔABC If $b=3$, $c=5$ and $\cos(B-C)$
then find the value $\tan\frac{A}{2}$.

Ans ⇒ $\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

('c' Angle 'b' angle से बड़ा है so,)

$$= \sqrt{\frac{1-\frac{7}{25}}{1+\frac{7}{25}}}$$

(5)

$$\tan\frac{C-B}{2} = 3/4$$

Ans $= \frac{1}{3}$

(v) In a ΔABC If $a=6$, $b=3$, $\cos(A-B) = \frac{4}{5}$
then show that ABC is right angle triangle

Ans ⇒ $\tan\frac{A-B}{2} = \sqrt{\frac{1-4/5}{1+4/5}} = \frac{1}{3}$ (Sol)

$$\tan\frac{A-B}{2} = \frac{6-3}{6+3}\cot\frac{C}{2}$$

$$\frac{1}{3} = \frac{1}{3}\cot\frac{C}{2}$$

$$\frac{C}{2} = 45^\circ$$

$$C = 90^\circ$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

(x) Prove that $a(b\cos C - c\cos B) = b^2 - c^2$

L.H.S.

Ans $\Rightarrow \underline{(b\cos C + c\cos B)}\ \underline{(b\cos C - c\cos B)}$

$\Rightarrow (b\cos C)^2 - (c\cos B)^2$

$\Rightarrow b^2\cos^2 C - c^2\cos^2 B$

$\Rightarrow b^2(1-\sin^2 C) - c^2(1-\sin^2 B)$

$\Rightarrow b^2 - c^2 + \underbrace{c^2\sin^2 B - b^2\sin^2 C}_{=0 \text{ (sin formula)}}$

$\Rightarrow b^2 - c^2$

(XV) Half Angle formulae

(Given
$(2s = a+b+c)$

(A)

$$\boxed{\text{I) } \sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}}$$

$$\boxed{\text{II) } \sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ac}}}$$

$$\boxed{\text{III) } \sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}}$$

Proof :-

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$1 - 2\sin A/2 = \frac{b^2+c^2-a^2}{2bc}$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$$\sin^2\frac{A}{2} = \frac{2bc - b^2 - c^2 + a^2}{4bc}$$

$$= \frac{a^2 - (b-c)^2}{4bc}$$

$$= \frac{(a+b-c)(a-b+c)}{4bc}$$

$$\sin^2 A/2 \rightarrow \frac{(2s-c-c)(2s-b-b)}{4bc}$$

Now:-

i) $\boxed{\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}}$

ii) $\boxed{\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}}$

iii) $\boxed{\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}}$

GradeSetter

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Ex:- $2\left[a\sin^2\frac{C}{2}+c\sin^2\frac{A}{2}\right]=a+c-b$

Ans:- $2\left[\frac{\cancel{a}(s-b)(s-a)}{\cancel{a}b}+\frac{(s-b)(s-c)}{b\cancel{c}}\cdot\cancel{c}\right]$

$\Rightarrow \frac{2(s-b)}{b}\left[s-a+s-c\right]$

$\Rightarrow \frac{(2s-2b)}{\cancel{b}}\left[a+b+c-\cancel{a}-\cancel{c}\right)$

$\Rightarrow a+b+c-2b$

$\Rightarrow \cancel{a+b}\,a+c-b$

Alternate.

$a\left(2\sin^2 C/2\right)+c\left(2\sin^2 A/2\right)$

$\Rightarrow a(1-\cos C)+c(1-\cos A)$

$\Rightarrow a+c-(a\cos C-c\cos A)$

$\Rightarrow a+c-b$

Ex→ If a, b, c are in A.P then find the value of $\frac{\sin B/2}{\sin A/2\sin C/2}$

Ans. $\left(\frac{2b}{2s-2b}\right)\Rightarrow\frac{2b}{a+b+c-2b}\Rightarrow\frac{2b}{\cancel{b}+\cancel{2b}-\cancel{2b}}=2$ Ans.

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Q) If a, b, c are in A.P then show that

$$a\cos^2\frac{C}{2} + c\cos^2 A/2 = \frac{3b}{2}$$

Ans. —

GradeSetter

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

(VII)

1) $\tan\frac{A}{2} = \frac{\sin A/2}{\cos A/2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

Now:-

$\Rightarrow \sqrt{\frac{(s-b)(s-c)\,s\cdot(s-a)}{\{s(s-a)\}^2}}$

$\Rightarrow \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s(s-a)}$

$\because$ Area of $(\Delta) = \sqrt{s(s-a)(s-b)(s-c)}$ → By Heron's formula

$$\boxed{\tan\frac{A}{2} = \frac{\Delta}{s(s-a)}}$$

ii) $\tan\frac{B}{2} = \frac{\Delta}{s(s-b)} = \sqrt{\frac{(s-a)(s-c)}{s(s-B)}}$

iii) $\tan\frac{C}{2} = \frac{\Delta}{s(s-c)} = \sqrt{\frac{(s-b)(s-a)}{s(s-c)}}$

(x) If a, b, c are in A.P. then show that $\cot A/2$, $\cot\frac{B}{2}$ and $\cot\frac{C}{2}$ are also in A.P.

Ans:- $2\cot\frac{C}{2} = \cot\frac{A}{2} + \cot\frac{C}{2}$

as $\frac{\Delta}{s(\sim)}$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

1st Choice

Q) If ~~B~~ a, b, c are in A.P then find the val
of $\tan\frac{A}{2} \cdot \tan\frac{C}{2}$

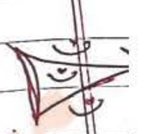
Ans) $2b = a+c$

$$\Rightarrow \sqrt{\frac{(s-b)\cancel{(s-c)}}{s\cancel{(s-a)}} \cdot \frac{\cancel{(s-a)}(s-b)}{s\cancel{(s-c)}}}$$

$$\Rightarrow \sqrt{\frac{(s-b)^2}{s^2}}$$

$$\Rightarrow \frac{(s-b)}{s} \Rightarrow \frac{2s-2b}{2s}$$

$$\Rightarrow \frac{a+\cancel{b}+c-2b}{a+b+c} = \frac{\cancel{b}}{3\cancel{b}} = \frac{1}{3}$$



Q) Prove that $(a+b+c)\left(\tan\frac{A}{2} + \tan\frac{B}{2}\right) = 2c\cot\frac{C}{2}$

L.H.S $\quad 2s\left(\frac{\Delta}{s(s-a)} + \frac{\Delta}{s(s-b)}\right)$

$$\Rightarrow 2s\left(\frac{2s-a-b}{(s-a)(s-b)}\right)$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

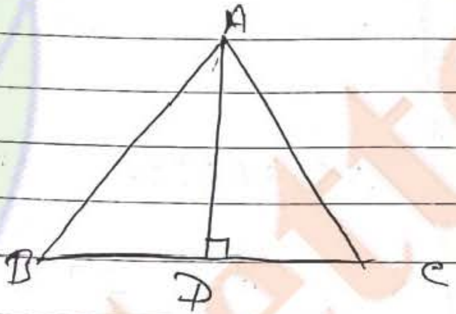
$$\Rightarrow 2c\left(\frac{\Delta}{(s-a)(s-b)}\right)$$

$$\Rightarrow 2c\,\frac{\sqrt{s(s-a)(s-b)(s-c)}}{(s-a)(s-b)}$$

$$\Rightarrow 2c\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$$

$$\Rightarrow 2c \cot \frac{C}{2}$$

Area of Triangle



Ar $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times BC \times AD$$

$$\Rightarrow \frac{1}{2} \times BC \times AB \sin B$$

$$\boxed{\begin{aligned} \Delta &= \frac{1}{2} a \cdot c \sin B \\ \Delta &= \frac{1}{2} ab \sin C \\ \Delta &= \frac{1}{2} \times bc \cdot \sin A \end{aligned}}$$

Now :—

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

1st Choice

Page No.
Date / /

$\Delta = \frac{ac}{2} \sin B$

$\Rightarrow ac\left(\sin \frac{B}{2} \cdot \cos \frac{B}{2}\right)$

$= \cancel{ac} \sqrt{\frac{(s-a)(s-c)}{\cancel{ac}}} \times \sqrt{\frac{s(s-b)}{\cancel{ac}}}$

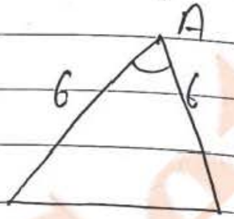
so

Ar $\Delta \Rightarrow \sqrt{s(s-a)(s-b)(s-c)}$

so

$$\boxed{\text{Ar } \Delta = \sqrt{s(s-a)(s-b)(s-c)}}$$

Q → In an Isosceles ~~triangle~~ 'Δ' length of equal side is 6cm and Area is 9cm² then find the angle.



Ans →

$\frac{1}{2}(6)(6) \sin A = 9$

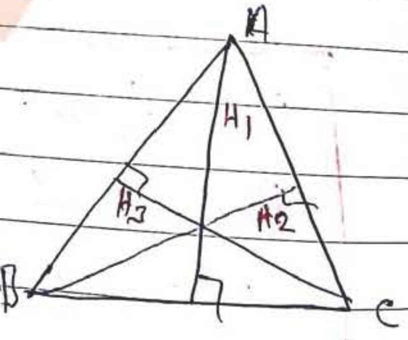
$A = 30$

$B = C = 75°$

Q If the sides of a Δ are in A.P then show that Altitudes w.r.t. their side are in H.P

Ans.

$\Delta = \frac{1}{2} \times a \times H_1$



$a = \frac{2\Delta}{H_1}$

$b = \frac{2\Delta}{H_2}$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$c = \frac{2\Delta}{H_3}$

Given $2b = a+c$

$$2\left(\frac{2\Delta}{H_2}\right) = \frac{2\Delta}{H_1} + \frac{2\Delta}{H_3}$$

$$\frac{2}{H_2} = \frac{1}{H_1} + \frac{1}{H_3}$$

Q6. In a ΔABC $\sin A \cdot \sin B \cdot \sin C = 2:3:4$ and $b = 4$ then find find Area and perimeter of triangle.

Ans
$$\frac{\sin A}{\frac{4}{3}(2)} = \frac{\sin B}{\frac{4}{3}(3)} = \frac{\sin C}{\left(\frac{4}{3}\right)4}$$

$a = 8/3$

$b = 4$

$c = 16/3$

Ar $\Delta = \sqrt{\frac{80}{3}}$

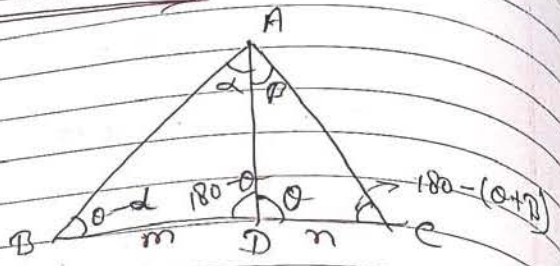
$P = 12$

GradeSetter

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

M-n Problem



$$\frac{BD}{DC} = \left(\frac{m}{n}\right)$$

$$\boxed{(i)\ (m+n)\cot\theta = m\cot\alpha - n\cot\beta}$$

$$\boxed{(ii)\ (m+n)\cot\theta = n\cot B - m\cot C}$$

Proof:-

ΔABD

$$\frac{AD}{\sin(\theta-\alpha)} = \frac{BD}{\sin\alpha} \quad ①$$

Now, In ΔACD

$$\frac{AD}{\sin(180-(\beta+\theta))} = \frac{CD}{\sin\beta} \quad ②$$

Divide

$$\frac{\sin(\beta+\theta)}{\sin(\theta-\alpha)} = \frac{BD}{CD}\cdot\frac{\sin\beta}{\sin\alpha}$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$$\Rightarrow \frac{\sin(\beta+\theta)}{\sin(\theta-\alpha)} = \frac{m}{n}\cdot\left(\frac{\sin\beta}{\sin\alpha}\right)$$

$$\Rightarrow n\sin\alpha\cdot\sin(\beta+\theta) = m\sin\beta\cdot\sin(\theta-\alpha)$$

⇒ ~~$n\sin\alpha\,[\sin\beta\cos\beta+\cos\beta\sin\theta)$~~

$$\Rightarrow n\sin\alpha\,[(\sin\beta\cos\theta+\cos\beta\cdot\sin\theta)] = m\sin\beta\,[(\sin\theta\cos\alpha - \cos\theta\sin\alpha)]$$

$$\Rightarrow (m+n)\sin\alpha\cos\beta\cos\theta = m\sin\beta\cdot\sin\theta\cos\alpha - n\sin\alpha\sin\theta\cos\beta$$

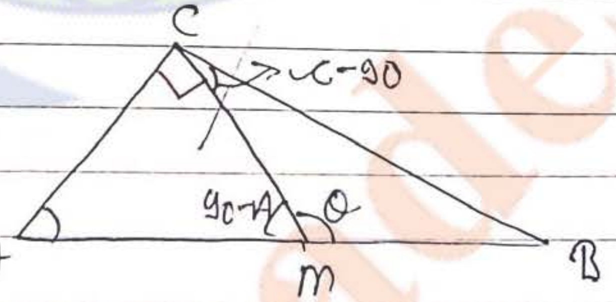
Divide by $\sin\alpha\sin\beta\sin\theta$

$$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$$

median from the vertex "C" on opposite side is perpendicular to AC then ~~prove~~ prove that

$$2\tan A + \tan C = 0$$

Ans



By m-n therom.

$$2\cot\theta = 1\cdot\cot 90 - 1\cdot\cot(c-90)$$

WWW.GRADESETTER.COM

$2\cot(90+A) = +\cot(90-C)$

$-2\tan A = \tan C$

$2\tan A + \tan C = 0$

WWW.GRADESETTER.COM

1st Choice

Solution of Triangle

Page No.
Date / /

1) when three sides a, b, c are given :⇒

Given
a, b, c

method

i) $\cos(A) = \frac{b^2+c^2-a^2}{2bc}$

$\frac{\sin A}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

ii) $A(\Delta) = \sqrt{S(S-a)(S-b)(S-c)}$

use

$\frac{1}{2}ab\sin(C) = \Delta$

iii) $\sin\frac{(A)}{2} = \sqrt{\frac{(S-b)(S-C)}{bc}}$

— —

2) when two angles and one side is given —

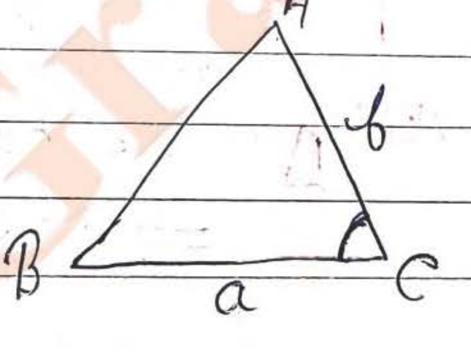
Given
a, A, B

method :—

$C = \pi - (A+B)$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{C}$

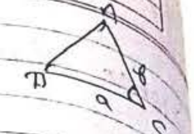
3) when two Sides and Including Angle is given;



WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

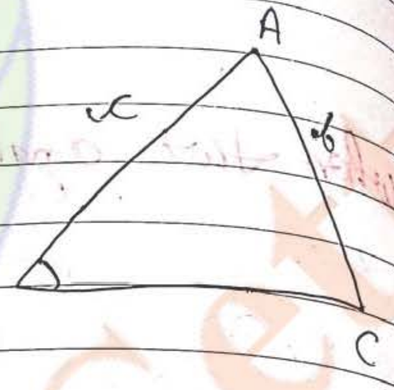
Given	method
a, b, ∠C	$\cos C = \frac{a^2+b^2-\boxed{c^2}}{2ab}$
	$\frac{\sin \boxed{A}}{a} = \frac{\sin \boxed{B}}{b} = \frac{\sin C}{c}$
	ii) $\tan \frac{\boxed{A-B}}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$
	$A+B = \pi - c$



Ambiguous case (Doubtful case) (संदेहास्पद)

When two sides and one opposite Angle is given! →

Given	method
b, c, B	



$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\boxed{\sin C = \frac{c \sin B}{b}}$$

Notes → i) If $b < c \sin B$

$$\Rightarrow \sin C > 1$$

⇒ No any triangle is form

WWW.GRADESETTER.COM

ii) $b = c\sin B$

$\sin C = 1 \Rightarrow C = 90°$
(only Right angle triangle is form.

iii) $b > c\sin B$

$\sin C < 1$
so, two values of C are possible
However
Both the values are not always exceptable.

eg:-
If $b > c$ ∵ $(B > C)$
then the obtuse angle of "C" is not exceptable because $b > C$.

a) If $b < C$ and $\angle B$ is acute $(B < C)$
so two values of "C" are possible one acute and one obtuse hence two triangle are possible.

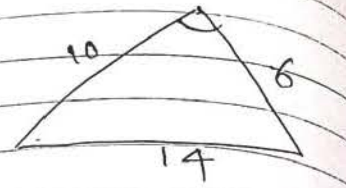
(b) If B is obtuse
i) $b > c$
so only value of 'C' is possible (acute) so, only one triangle is form

ii) $b \leq c$ → No triangle is possible.

GradeSetter

WWW.GRADESETTER.COM

Ex If sideof triangle are 6, 10, and 14 find the largest angle.



Ans

$$\cos A = \frac{36+100-196}{2\times6\times10} = \frac{-1}{2}$$

$$A = 120^\circ$$

Ex Two sides of a triangle are roots of equation $x^2 - 2\sqrt{3}x + 2 = 0$ and angle b/w then is $\pi/3$ find the perimeter of triangle

$\pi/3$

Ans. $x^2 - 2\sqrt{3}x + 2 = 0 \begin{matrix} \nearrow a \\ \searrow b \end{matrix}$

$$\cos \pi/3 = \frac{a^2+b^2-c^2}{2ab}$$

$$\frac{1}{2} = \frac{(a+b)^2 - 2ab - c^2}{2ab}$$

$$\frac{1}{2} = \frac{(2\sqrt{7})^2 - 4 - c^2}{4}$$

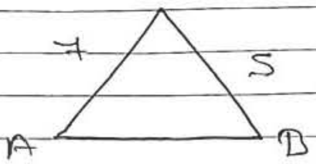
$$\boxed{c = \sqrt{6}}$$

$$(a+b) + c = 2\sqrt{3} + \sqrt{6}$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Ex Find the solution of triangle in which
(i) $a = 5$, $b = 7$, $\sin A = 3/4$ ..



Ans $\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{3/4}{5} = \frac{\sin B}{7}$.

$\Rightarrow \sin B = \frac{21}{20} \} (X)$

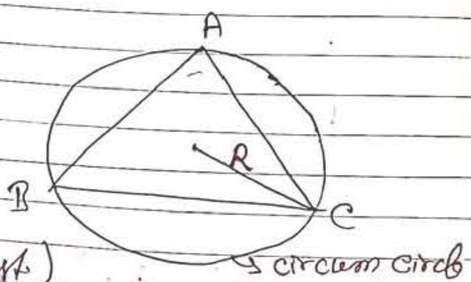
(ii) $a = 2$, $c = \sqrt{3} + 1$, $A = 45°$

WWW.GRADESETTER.COM

Radii of circle

1) Circum radius (R) ! ⇒

~~Formula :→~~



i) Formula :- 1st :→ (method 1st)

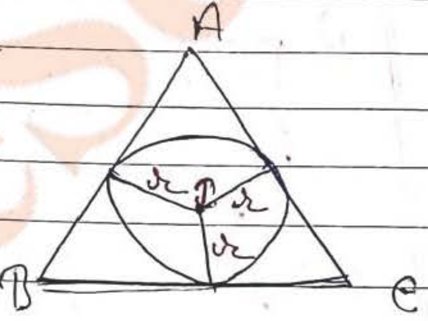
$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R}}$$

ii) (Formula) method 2nd :—

$$\boxed{R = \frac{abc}{4\Delta}}$$

Δ = Area of ΔABC

2) In-radius (r) ⇒



i)

$$\boxed{r = \left(\frac{\Delta}{s}\right)}$$

where

s → semiperimeter $\left(\frac{a+b+c}{2}\right)$

ii)

$$\boxed{r = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2} = (s-c)\tan\frac{C}{2}}$$

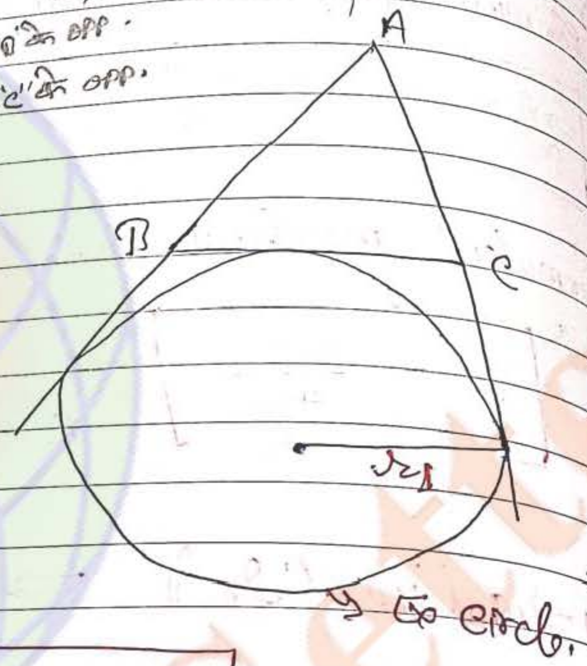
WWW.GRADESETTER.COM

iii)
$$\boxed{r = 4R\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}}$$

2) Ex-radii $(r_1) \rightarrow$ A के opp. $\Rightarrow$

$(r_2) \rightarrow$ 'B' के opp.

$(r_3) \rightarrow$ 'C' के opp.



i)
$$\boxed{r_1 = \left(\frac{\Delta}{s-a}\right) = s\tan\frac{A}{2}}$$

ii)
$$\boxed{r_2 = \left(\frac{\Delta}{s-b}\right) = s.\tan\frac{B}{2}}$$

iii)
$$\boxed{r_3 = \left(\frac{\Delta}{s-c}\right) = s\tan\frac{C}{2}}$$

WWW.GRADESETTER.COM

1st Choice

capital S ⇒ Area of triangle
Small s ⇒ semi perimeter of triangle

Ex: $r \cdot r_1 r_2 r_3 = S^2$

where S ⇒ Area of triangle.

Ans: $\frac{\Delta}{s} \cdot \frac{\Delta}{s-a} \cdot \frac{\Delta}{s-b} \cdot \frac{\Delta}{s-c}$

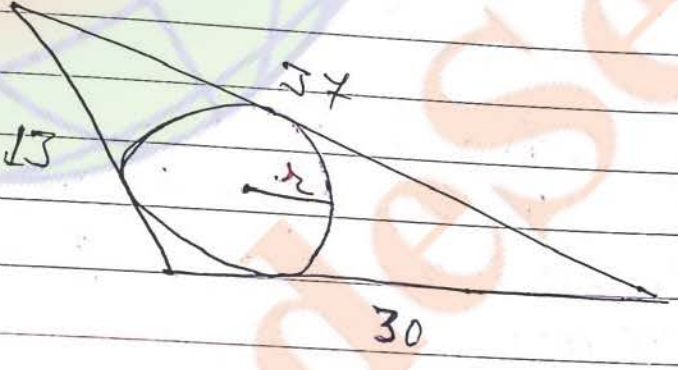
$\Rightarrow \frac{\Delta^4}{\Delta^2} = \Delta^2$ Ans

Ex: Prove that $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$

Ans: $\frac{(s-a)}{\Delta} + \frac{s-b}{\Delta} + \frac{s-c}{\Delta}$

$\Rightarrow \frac{3s-(a+b+c)}{\Delta} = \frac{3s-2s}{\Delta} = \left(\frac{s}{\Delta}\right) = \frac{1}{r}$

Ex:



Ans: $r = \frac{\Delta}{s} = \frac{9}{2}$. Ans

WWW.GRADESETTER.COM

Q. Prove that $r_2r_3 + r_3r_1 + r_1r_2 = s^2$

Ans. $\frac{\Delta}{(s-b)} \cdot \frac{\Delta}{(s-c)} + \frac{\Delta^2}{(s-c)(s-a)} + \frac{\Delta^2}{(s-a)(s-b)}$

$$\Delta^2\left[\frac{3s-(a+b+c)}{(s-a)(s-b)(s-c)}\right]$$

$$\Rightarrow \frac{\cancel{\Delta^2}\cdot s^2}{\cancel{s(s-a)(s-b)(s-c)}} = s^2$$

Q. $\sin A + \sin B + \sin C = \left(\frac{s}{R}\right)$

Ans. $\frac{a}{2R} + \frac{b}{2R} + \frac{c}{2R}$

$$\Rightarrow \frac{a+b+c}{2R} = \frac{\cancel{2}s}{\cancel{2}R} = \frac{s}{R}$$

Q. If the Area of a triangle is 96 and radii of In-~~ex~~ circle (ex-circle) are 8, 12 and 24 then find the Perimeter of a triangle.

Ans. $s-a = \frac{\Delta}{r_1}$

So,

$s-a = \frac{96}{8} = 12$ —(i)

WWW.GRADESETTER.COM

$$s-b = \frac{\Delta}{r_2} = 8 \qquad \text{—(ii)}$$

$$s-c = \frac{\Delta}{r_3} = 4 \qquad \text{—(iii)}$$

Add eq (i) + eq (ii) + eq (iii)

$$3s - (a+b+c) = 24$$

$$s = 24$$

$$2s = 48$$

[6] $$\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c} - \frac{1}{s} = \frac{4R}{\Delta}$$

Ans $$\Rightarrow \frac{2s-(a+b)}{(s-a)(s-b)} + \frac{s-(s-c)}{s(s-c)}$$

$$\Rightarrow c\left[\frac{s(s-c) + (s-b)(s-a)}{s(s-a)(s-b)(s-c)}\right]$$

$$\Rightarrow c\left[\frac{s^2 - cs + s^2 - s(a+b) + ab}{\Delta^2}\right]$$

$$\Rightarrow c\left[\frac{\cancel{2s^2} - \cancel{cs} - s(\cancel{2s} - \cancel{c}) + ab}{\Delta^2}\right]$$

$$\Rightarrow \left(\frac{abc}{\Delta}\right) \cdot \frac{1}{\Delta} = \frac{4R}{\Delta}$$

GradeSetter

WWW.GRADESETTER.COM

Q. Prove that $\frac{r_1 - r}{a} + \frac{r_2 - r}{b} = \frac{c}{r_3}$

Ans $\frac{\frac{\Delta}{s-a} - \frac{\Delta}{s}}{a} + \frac{\frac{\Delta}{s-b} - \frac{\Delta}{s}}{b}$

$\Rightarrow \frac{\Delta}{s}\left[\left(\frac{1}{(s-a)} + \frac{1}{(s-b)}\right)\right]$

$\Rightarrow \frac{\Delta}{s}\left[\frac{2s - a - b}{(s-a)(s-b)}\right]$

$\Rightarrow c \cdot \frac{\Delta(s-c)}{s(s-a)(s-b)(s-c)}$

$\Rightarrow \frac{c \cdot \Delta(s-c)}{\Delta^2}$

$\Rightarrow \frac{c}{r_3}$

Q. In a ΔABC, If $a=13$, $b=14$, $c=15$ then find its circum Radius.

Ans $S = \frac{13+14+15}{2}$ 2 2.

Ar

Ans $\left(\frac{65}{8}\right)$

Q) If

Ans:-

WWW.GRADESETTER.COM

(x) If $\left(1-\frac{r_1}{r_2}\right)\left(1-\frac{r_1}{r_3}\right)=2$

then prove that triangle is right angle

Ans:-

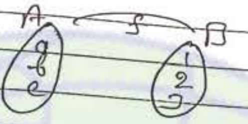
1st Choice

Inverse Trigonometric function

Page No.
Date / /

✗ Inverse function:⇒

$$f : A \rightarrow B$$



$f(a) = 1$ $f(b) = 2$ $f(c) = 3$

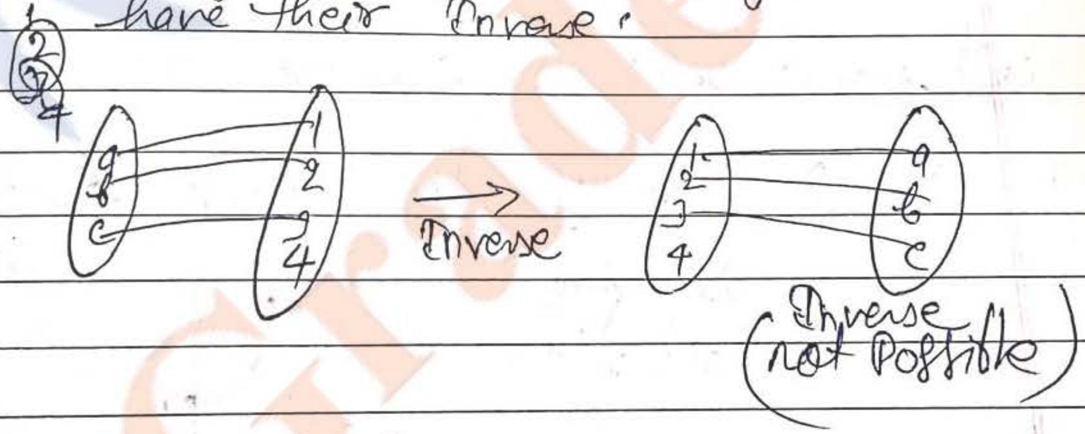
$$g : B \rightarrow A$$

$g(1) = a$, $g(2) = b$, $g(3) = c$

⊛ Let $f(x) = y$ is a function and 'g' is a function such that $g(y) = x$.
Then "g" is called Inverse of (f)

$$\boxed{g(x) = f^{-1}(x)}$$

Note:⇒ only one-one and onto function can have their Inverse.



WWW.GRADESETTER.COM

Ex. Let $f(x) = x+1$, $f : R \to R$

Let $f^{-1}(x) = y$

$x = f(y)$

$x = y+1$

$y = (x-1)$

$$\boxed{f^{-1}(x) = x-1}$$

Ex Find the Inverse function of $f(x) = x^3-1$

Ans Let $f^{-1}(x) = y$

$x = f(y)$

$x = y^3-1$

$y = \sqrt[3]{x+1}$

$f^{-1}(x) = \sqrt[3]{x+1}$

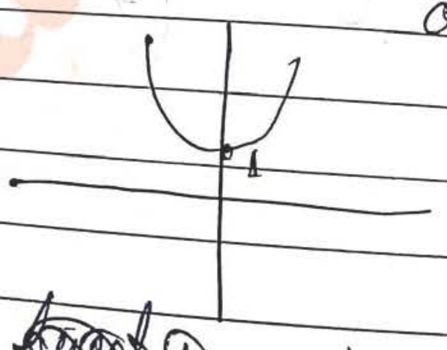
$= (1+x)^{1/3}$

Ex $f(x) = x^2+1$ $f : R \to R$

A Let $\cancel{f^{-1}(x) = y}$ First check function is defined or not

$\cancel{x = f(y)}$

$x = x^2+1$



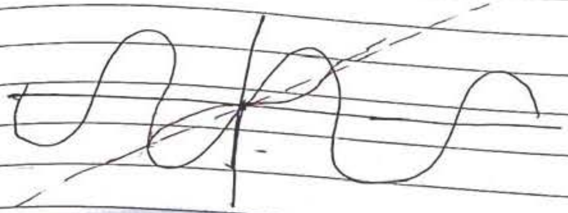
$[1, \infty)$

Inverse is not possible

WWW.GRADESETTER.COM

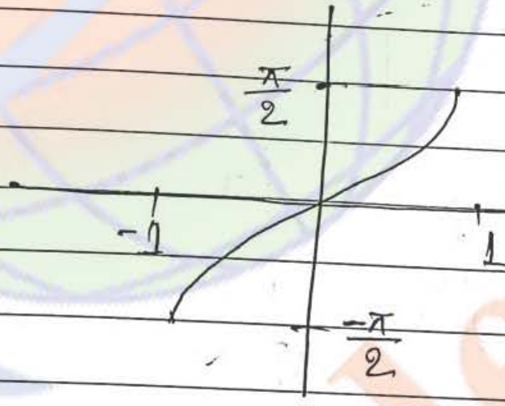
WWW.GRADESETTER.COM

$\boxed{x}$ $y = \sin x$



Graph of Inverse Trigonometric functions

i) $y = \sin^{-1} x$ (angle for which $\overset{\text{value of}}{\sin}$ becomes x)



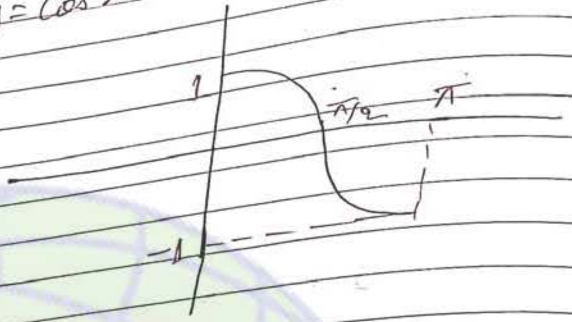
Domain $\in [-1, 1]$

Range $\in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right)$

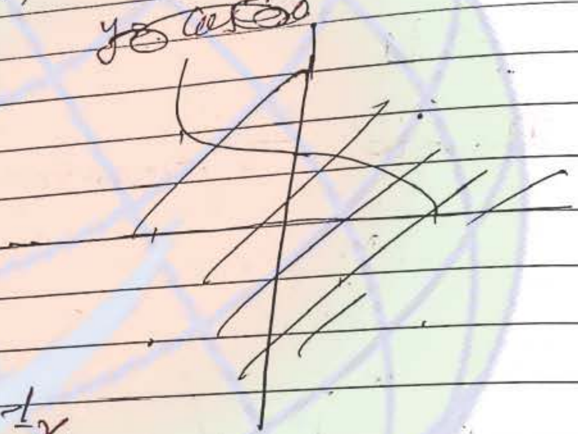
WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

ii) y = Cos x

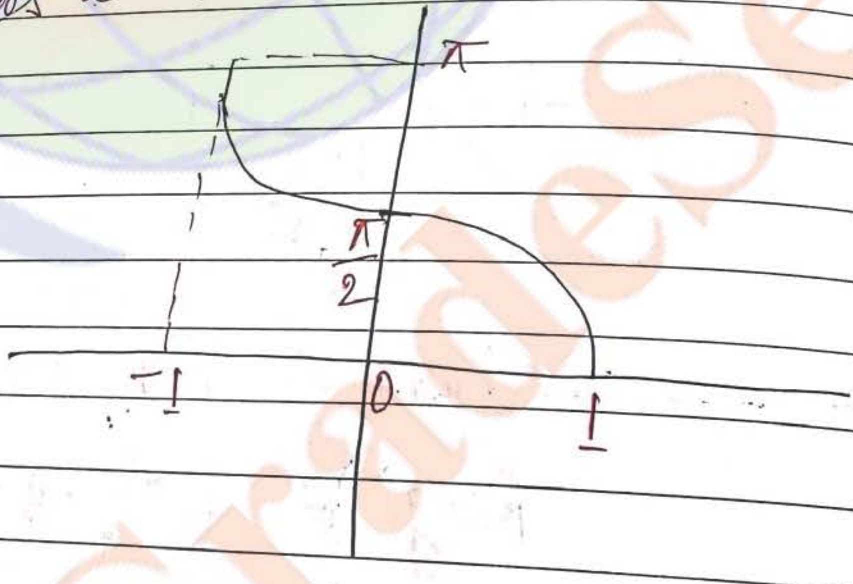


So,



So,

$y = \cos^{-1} x$



Domain ∈ [-1, 1]
Range ∈ [0, π]

7/1/2018

1st Choice

iii) y = t

iv)

GradeSetter

WWW.GRADESETTER.COM

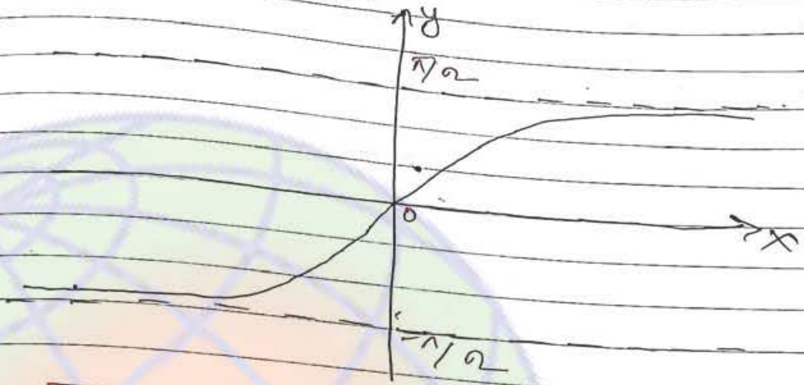
WWW.GRADESETTER.COM

7/1/2019.

1st Choice

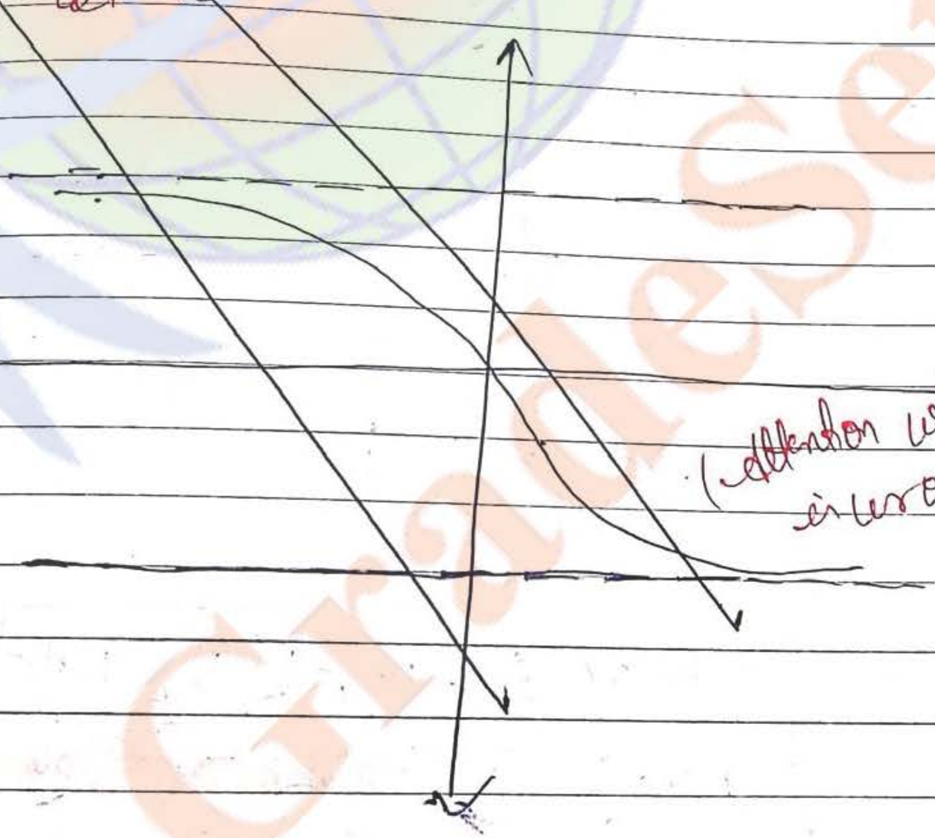
Page No.
Date / /

iii) $y = \tan^{-1} x$



$$\boxed{\begin{array}{l}\text{Domain} \in R \\ \text{Range} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\end{array}}$$

iv) $y = \cot^{-1} x$

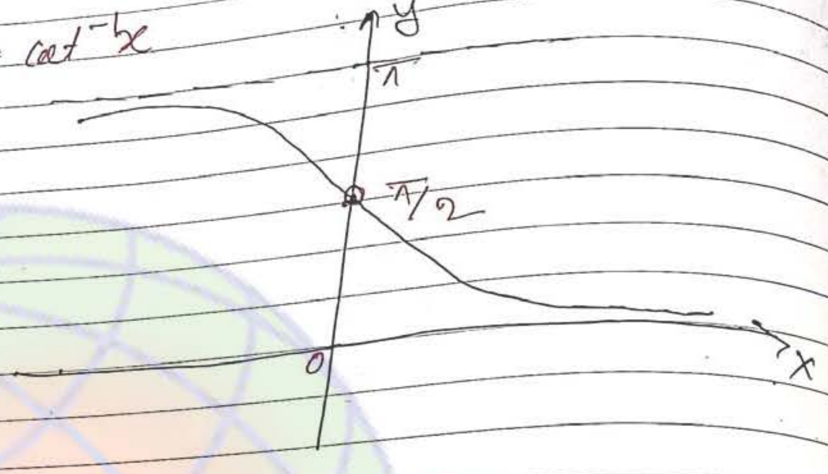


(attention why this is wrong.

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

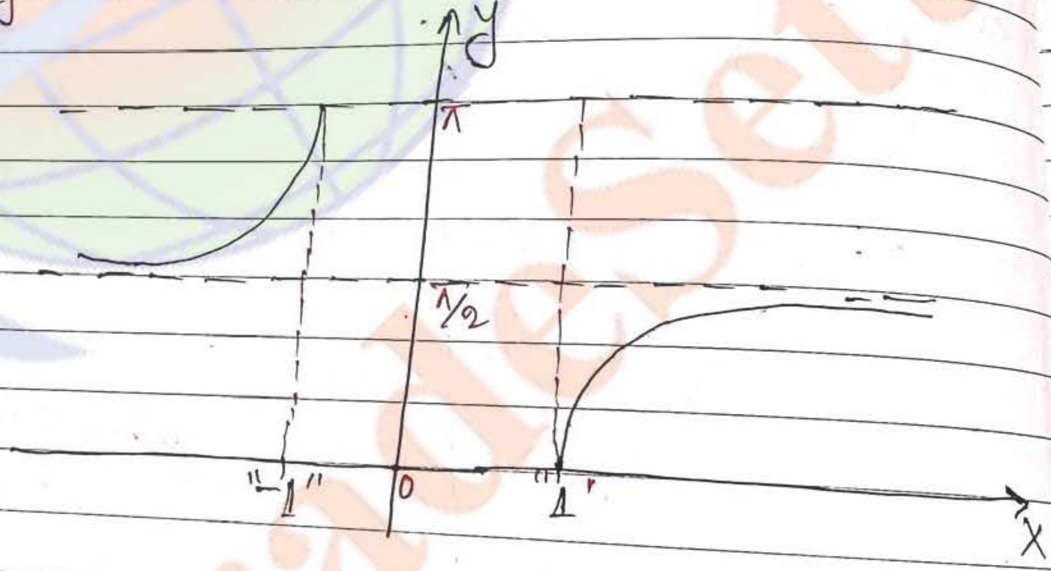
iv) $y = \cot^{-1}x$



Domain ∈ R

Range ∈ (0, π)

v) $y = \sec^{-1}x$



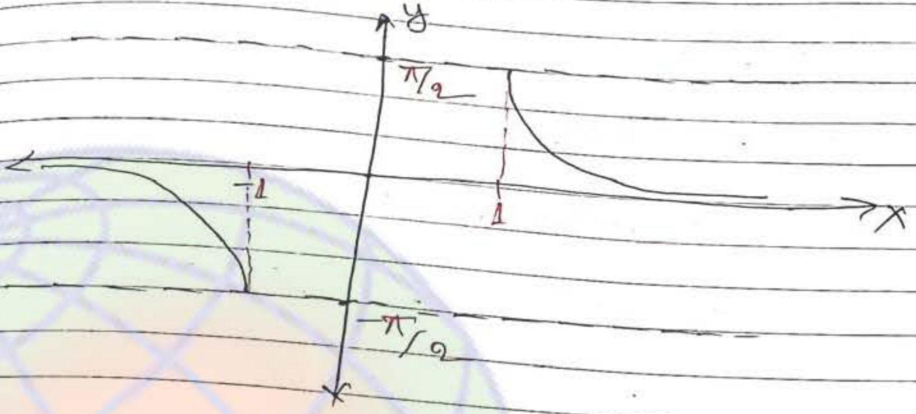
Domain ∈ $(-\infty, -1] \cup [1, \infty)$

Range ∈ $[0, \pi] - \{\pi/2\}$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

(v) $y = \operatorname{cosec}^{-1} x$



Domain $\in (-\infty, -1] \cup [1, \infty)$

Range $\in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Note:->

Function	Domain	Range
$\sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}x$	R	$(0, \pi)$
$\sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\operatorname{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

0, 30, 45, 60, 90, 120, 135, 150,

1.) $\tan^{-1}(-1) = -\frac{\pi}{4}$

2.) $\cot^{-1}(-1) = \frac{3\pi}{4}$

3.) $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

4.) $\sin\left[\tan^{-1}(-\sqrt{3}) + \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right]$

$\cancel{\sin\left[\frac{2\pi}{3} + \frac{2\pi}{3}\right]} \Rightarrow \cancel{\sin\left[\frac{4\pi}{3}\right]}$

$\cancel{\Rightarrow 2}$

$\sin\left[-\frac{\pi}{3} + \frac{5\pi}{6}\right]$

$\sin\frac{\pi}{2} = 1$

5.) $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right)$

$\sin\left(\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right)$ $\Rightarrow \cancel{\sin\frac{\pi}{6}}$

$\Rightarrow$

$\sin\frac{\pi}{2} = 1.$

6.) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$\tan^{-1}(-1)$

$\Rightarrow -\frac{\pi}{4}$

7.) $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) - \tan^{-1}(-\sqrt{3}) + \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$-\frac{\pi}{4} + \frac{3\pi}{4} - \left(-\frac{\pi}{3}\right) + \frac{2\pi}{3}$

(5.) Find th

1) f(x

(10) f

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Q.) Find the domain of following functions-

1) $f(x) = \sin^{-1}(2x^2-1)$

$-1 \le 2x^2-1 \le 1$

Add 1.

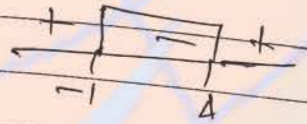
$\underbrace{0 \le 2x^2}_{\text{Always true.}} \le 2$

so, $2x^2 \le 2$

$x^2 \le 1$

$x^2-1 \le 0$

$(x-1)(x+1) \le 0$

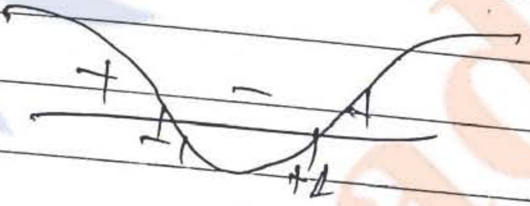


so, $x \in [-1, 1]$

(ii) $f(x) = \tan^{-1}(\sqrt{x^2-1})$

$x^2-1 \ge 0$

$(x-1)(x+1) \ge 0$



$(-\infty, -1] \cup [1, \infty)$

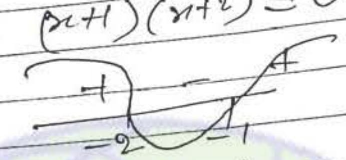
WWW.GRADESETTER.COM

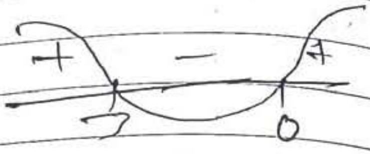
WWW.GRADESETTER.COM

ii) $f(x) = \sec^{-1}(x^2+3x+1)$

A. $x^2+3x+1 \le -1$ | $x^2+3x+1 \geq 1$

$(x+1)(x+2) \le 0$ | $x(x+3) \geq 0$





$[-2, -1]$

$(-\infty, -3] \cup [0, \infty)$

$$x \in (-\infty, -3] \cup [-2, -1] \cup [0, \infty)$$

iii) $f(x) = \cos^{-1}\left(\frac{x^2}{x^2+1}\right)$

~~$-1 \le \frac{x^2}{x^2+1} \le 1$~~ $0 < \frac{x^2}{x^2+1} < 1$

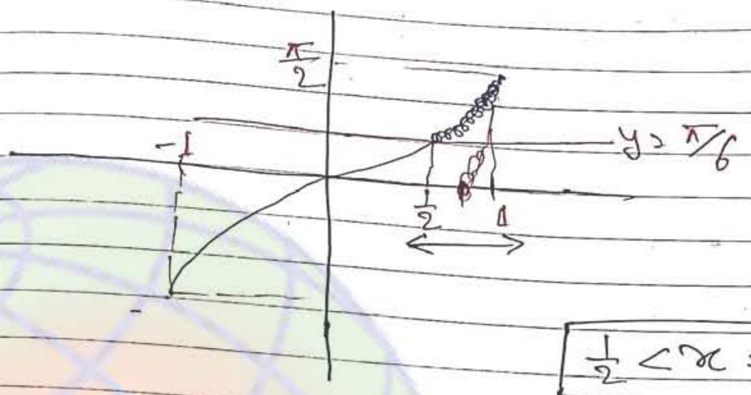
(~~चूँकी [illegible]~~ ~~eslyab~~ This is always +ve ones)

$(x \in R)$

WWW.GRADESETTER.COM

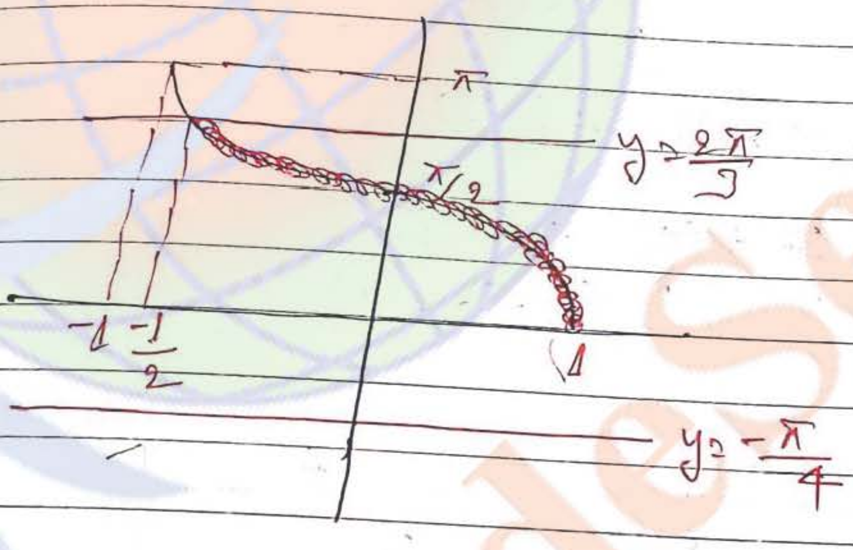
Solve the following Inequalities

i) $\sin^{-1}x > \frac{\pi}{6}$



$$\boxed{\frac{1}{2} < x \le 1}$$

ii) $\frac{-\pi}{4} < \cos^{-1}x < \frac{2\pi}{3}$



$$\boxed{-\frac{1}{2} < x \le 1}$$

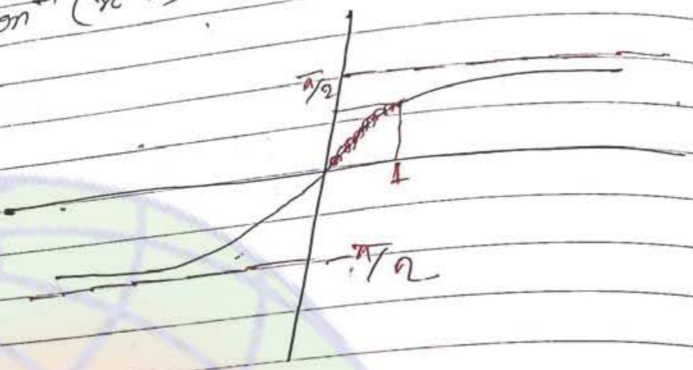
GradeSetter

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

V.V.I ~~S.~~ $\log_{\frac{\pi}{4}}\left(\tan^{-1}(x-1)\right) \geq 1$ (Attention)

$\tan^{-1}(x-1) \leq \frac{\pi}{4}$



~~नहीं~~ नहीं log defined नहीं होगा।

$0 < x-1 \leq 1$

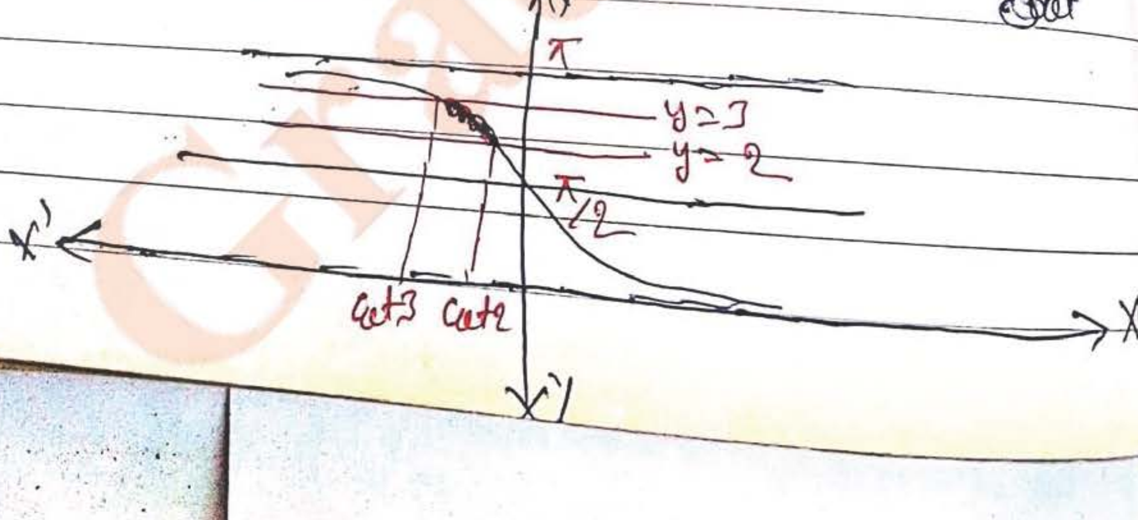
$\boxed{1 < x \leq 2}$ Attention.

Ex.-> $(\cot^{-1}x)^2 - 5\cot^{-1}x + 6 < 0$

~~$(\sin^{-1}x)^2 - 3(\sin^{-1}x) + 2 \leq 0$~~

Ans:-> $(\cot^{-1}x - 2)(\cot^{-1}x - 3) < 0$

$2 < \cot^{-1}x < 3$



WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

so,

$\cot 3 < x < \cot 2$ Ans.

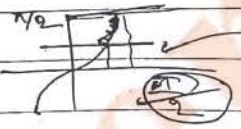
(x) $(\sin^{-1}x)^2 - 3(\sin^{-1}x) + 2 \le 0$

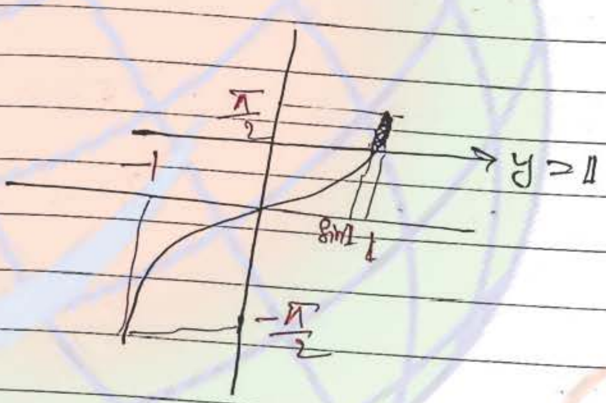
$(\sin^{-1}x - 1)\cdot\underbrace{(\sin^{-1}x - 2)}_{e} \le 0$

$\{\because \sin x \le \frac{\pi}{2}\}$

$\Rightarrow \sin^{-1}x - 1 \geq 0$

$\Rightarrow \sin^{-1}x \geq 1$





$\sin 1 \le x \le 1$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

1st choice

Properties of Inverse Trigonometric func (I.T.F)

1st Properties → Rule → पहले sin रहेगी तो Domain हे देखेंगे।

1) $\sin[\sin^{-1}x] = x$ if $x \in [-1, 1]$

2) $\cos[\cos^{-1}x] = x$ if $x \in [-1, 1]$

3) $\tan[\tan^{-1}x] = x$ if $x \in R$

4) $\cot[\cot^{-1}x] = x$ if $x \in R$

5) $\sec[\sec^{-1}x] = x$ if $x \in (-\infty, -1] \cup [1, \infty)$

6) $\operatorname{cosec}[\operatorname{cosec}^{-1}x] = x$ if $x \in (-\infty, -1] \cup [1, \infty)$

Proof :-

~~$\sin x = \theta$~~

~~$x = \sin^{-1}\theta$~~

~~$\boxed{x = \sin^{-1}(\sin x)}$~~

$y = \sin[\sin^{-1}x] = x$

$\sin^{-1}x = \theta$

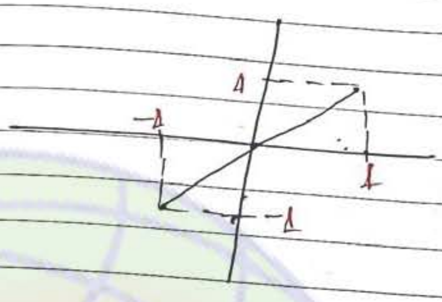
$x = \sin\theta$

$x = \sin[\sin^{-1}x]$

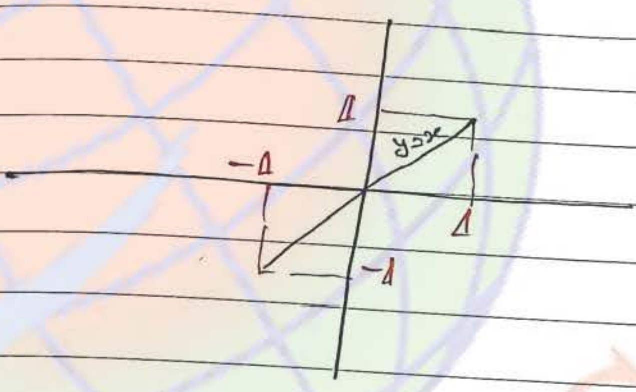
WWW.GRADESETTER.COM

Graphs:-

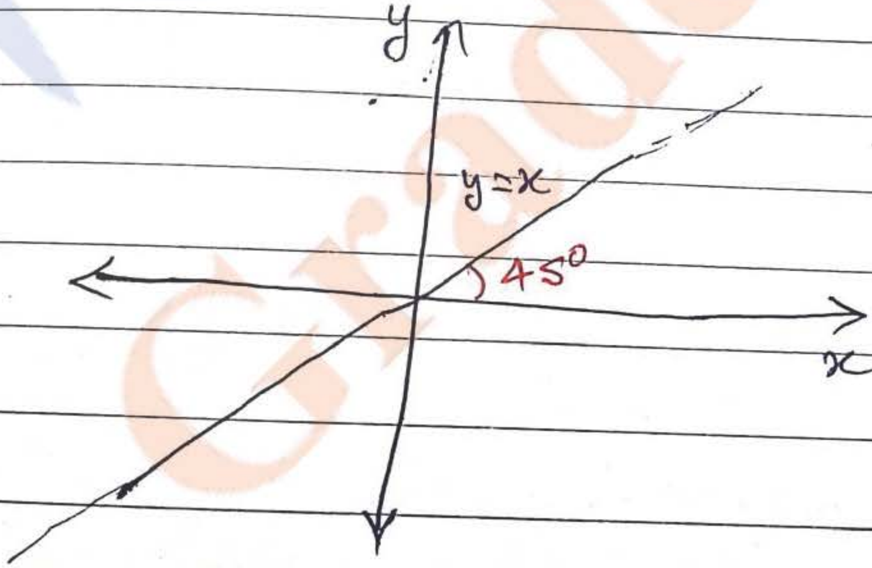
1) $y = \sin[\sin^{-1}x]$
$y = x$ $\quad x \in [-1, 1]$



$y = \cos[\cos^{-1}x]$
$y = x$ $\quad x \in [-1, 1]$



2) $y = \tan[\tan^{-1}x] = x$
$y = \cot[\cot^{-1}x] = x$ $\Big\}$ $x \in R$



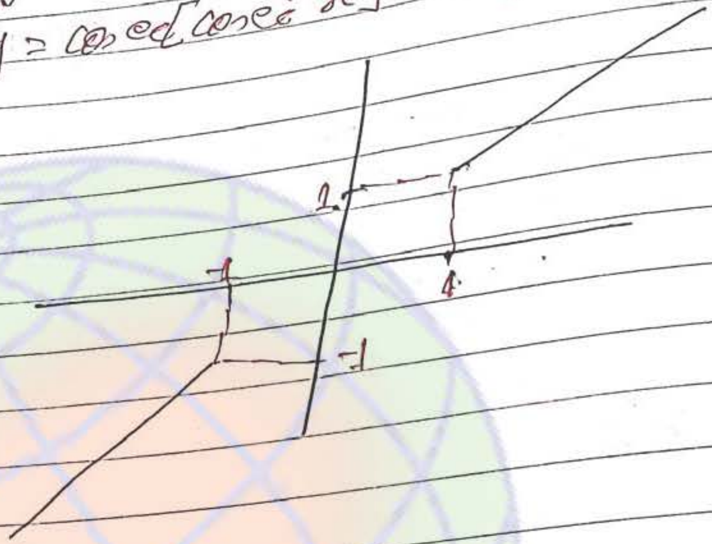
WWW.GRADESETTER.COM

1st Choice

Date / /

111) $y = \sec[\sec^{-1}x] = x$ $\Big\}$ $x \in (-\infty, -1) \cup [1, \infty)$

$y = \operatorname{cosec}[\operatorname{cosec}^{-1}x] = x$



WWW.GRADESETTER.COM

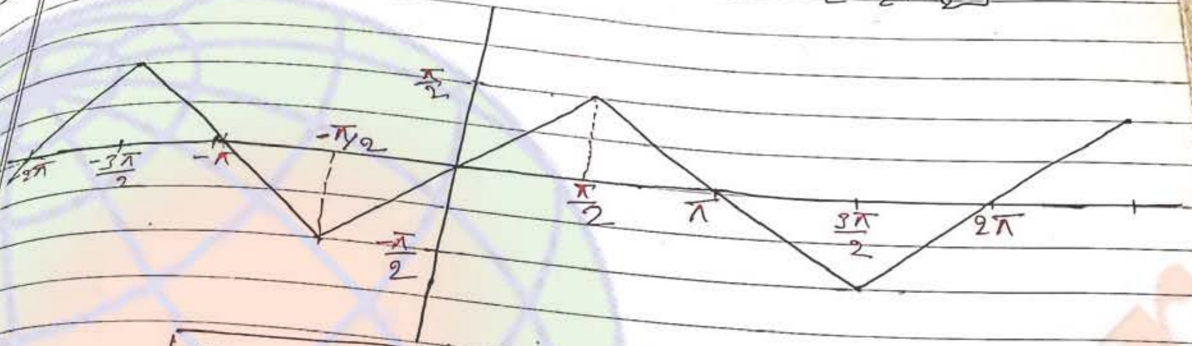
WWW.GRADESETTER.COM

$-\infty,-1) \cup [1,\infty)$

IInd Properties

1.) $y = \sin^{-1}(\sin x)$

$= x$ if $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$



Domain ∈ R
Range ∈ $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Proof:-

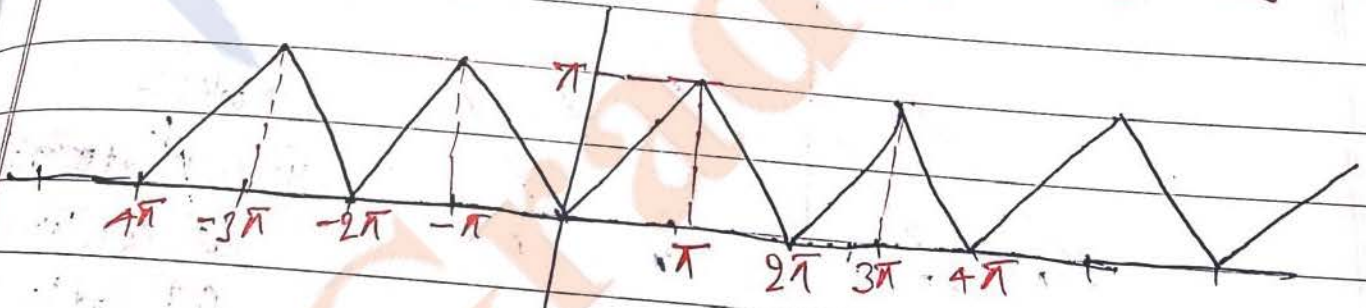
$\sin x = \theta$

$x = \sin^{-1}\theta$

$x = \sin^{-1}(\sin x)$

2.) $y = \cos^{-1}\cancel{x}[\cos x]$

$= x$ if $x \in [0, \pi]$



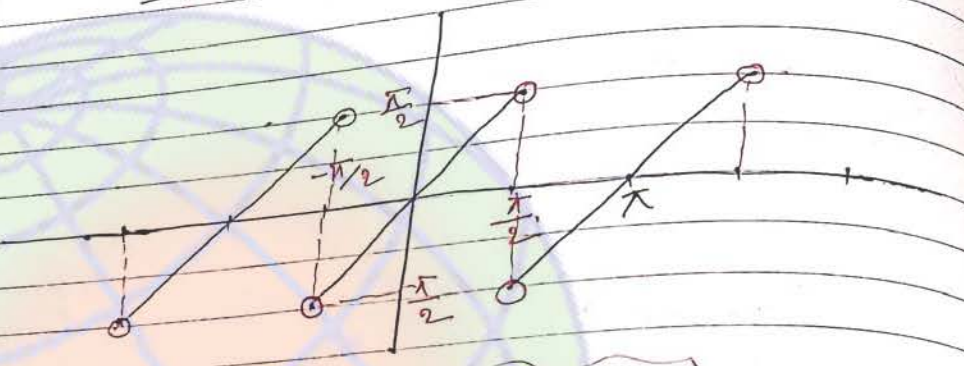
Domain ∈ R
Range ∈ $[0, \pi]$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Note → $\cos^{-1}(\cos 181)$
$\Rightarrow \cos^{-1}(\cos(360-179))$
$\Rightarrow \cos^{-1}\cos 179$
$= 179$

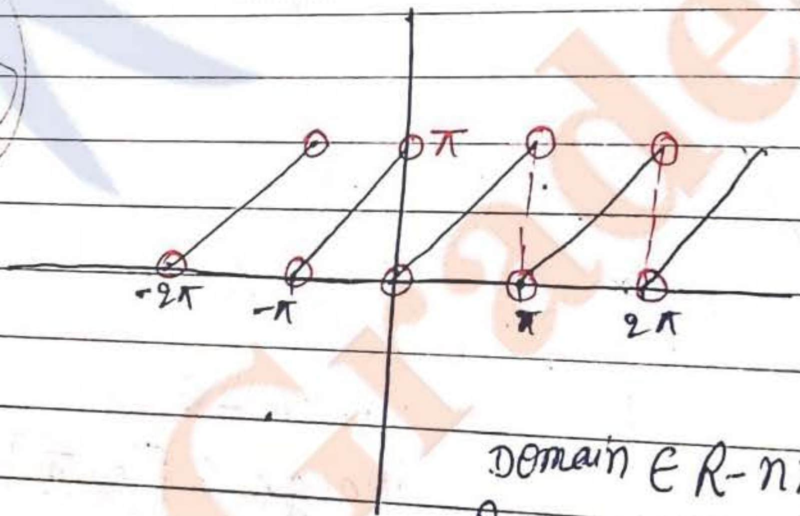
3.) $y = \tan^{-1}(\tan x)$ if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ open
$= x$



Domain $\in R - (2n+1)\frac{\pi}{2}$
Range $\in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Notes →
$y = \tan^{-1} \tan 91$
$= \tan^{-1} \tan(180-89)$
$= \tan^{-1}(-\tan 89)$
$= \tan^{-1} \tan(-89)$
$= -89$

4.) $y = \cot^{-1}(\cot x)$
$= x$ if $x \in (0, \pi)$



Notes →
$y = \cot^{-1}(\cot 181)$
$\cot^{-1} \cot(180+1)$
$\cot^{-1} \cot 1 = 1$

Domain $\in R - n\pi$
Range $\in (0, \pi)$

5.) $y =$

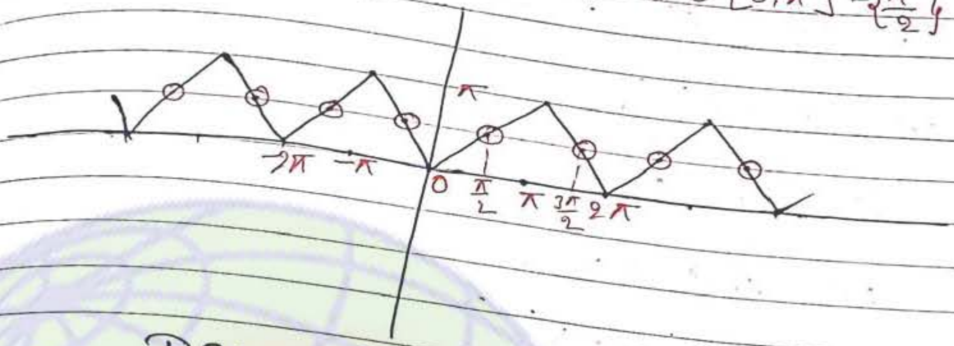
6.)

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

5.) $y = \sec^{-1}(\sec x)$
$= x$

if $x \in [0,\pi] - \{\frac{\pi}{2}\}$

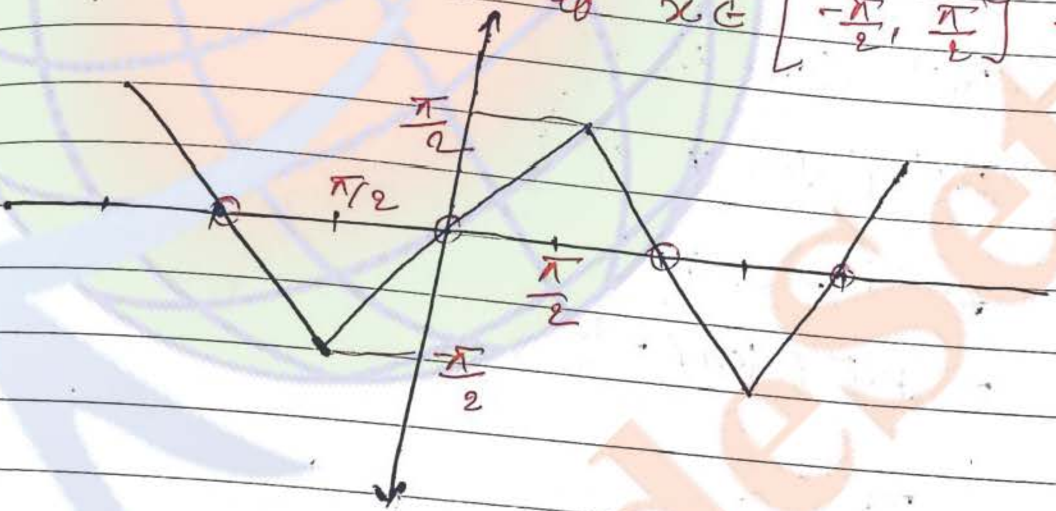


Domain ∈ $R - (2n+1)\frac{\pi}{2}$

Range ∈ $[0,\pi] - \{\frac{\pi}{2}\}$

6.) $y = \mathrm{cosec}^{-1}(\mathrm{cosec}\, x)$
$= x$

if $x \in [\frac{-\pi}{2}, \frac{\pi}{2}] - \{0\}$



Domain ∈ $R - n\pi$

Range ∈ $[\frac{-\pi}{2}, \frac{\pi}{2}] - \{0\}$

WWW.GRADESETTER.COM

0, 30, 45, 60, 90, 120, 135 150, 180

i) $\sin^{-1}(\sin\frac{\pi}{4}) = \frac{\pi}{4}$

2) $\cos^{-1}(\cos(\frac{2\pi}{3})) = \frac{2\pi}{3}$

3) $\sin^{-1}(\sin(\frac{2\pi}{3}))$

$\sin^{-1}(\sin(\pi-\frac{\pi}{3}))$

$\sin^{-1}(\sin\frac{\pi}{3}) = \frac{\pi}{3}$

iv) $\cos^{-1}(\cos(\frac{7\pi}{6}))$

$\Rightarrow \cos^{-1}(\cos 2\pi-\frac{5\pi}{6})$

$\Rightarrow \cos^{-1}(\cos(\frac{5\pi}{6}) = \frac{5\pi}{6}$ &.

v) $\tan^{-1}(\tan(\frac{2\pi}{3}))$ $(\pi-\frac{\pi}{3})$

$\Rightarrow \tan^{-1}(\tan(\pi-\frac{\pi}{3})$

$\Rightarrow \tan^{-1}(-\tan\frac{\pi}{3})$

$= \tan^{-1}(\tan(-\frac{\pi}{3})$

$\Rightarrow -\frac{\pi}{3}$

v) $\sin^{-1}(\sin 1) = 1$

vi) $\sin^{-1}(\sin 2)$

$\Rightarrow \sin^{-1}(\sin(\pi-2))$

$\Rightarrow \pi-2$

1st Choice

vii) $\sin^{-1}$ (sin

6. $\sin^{-1}$ (

(8) $\sin^{-1}$ (

8 $\sin^{-1}$ (

$\Rightarrow$ 3

(9) Cos

$\Rightarrow$ Cos

$\Rightarrow$ (4

(10) Cot

$\Rightarrow$ (

$\Rightarrow$

WWW.GRADESETTER.COM

viii) $\sin^{-1}(\sin 5)$

S. $\sin^{-1}(\sin(5-2\pi)) = 5-2\pi$

[illegible] $\sin^{-1}(\sin 10)$

S $\sin^{-1}(\sin 3\pi - 10)$

$\Rightarrow 3\pi - 10$

S.

$\Rightarrow \sin(3\pi-10)$
$\Rightarrow \sin 10$

$\Rightarrow 9.42 - 10$
$\Rightarrow 0.58$

$\Rightarrow \sin(10-3\pi)$
$\Rightarrow -\sin(3\pi-10)$
$\Rightarrow \boxed{-\sin 10}$

original in sin10 but the obtain −sin10 so not possible

[illegible] $\cos^{-1}(\cos 10)$

$\Rightarrow \cos^{-1}(4\pi-10)$

$\Rightarrow (4\pi-10)$ S.

[illegible] $\cot^{-1}(\cot 4)$

$\Rightarrow \cot^{-1}(\cot(4-\pi))$

$\Rightarrow 4-\pi$

$2\pi-$
6.28
$(\pi-4)$
$(4-3\pi)$

check:⇒
$\cot(4-\pi)$
$\Rightarrow -\cot(\pi-4)$
$\Rightarrow -(-\cot 4)$
$\Rightarrow \cot 4$

WWW.GRADESETTER.COM

2π

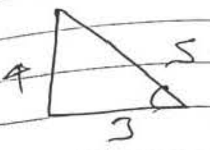
6.18

Q→ $\tan^{-1}(-\tan(-6)$

$\Rightarrow \tan^{-1}(\tan(2\pi-6)$

$\Rightarrow 2\pi-6$

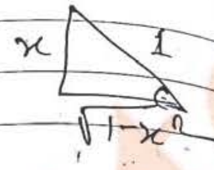
Ex→ $\sin(\cos^{-1}(\frac{3}{5}))$



$\Rightarrow \sin[\sin^{-1}(\frac{4}{5})]$

$\Rightarrow \frac{4}{5}$ Ans.

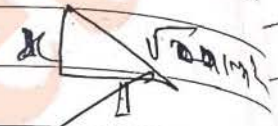
Ex→ $\cos(\sin^{-1}x)$



$\Rightarrow \cos(\cos^{-1}\sqrt{1-x^2})$

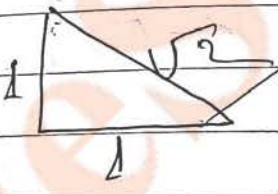
$\Rightarrow \sqrt{1-x^2}$

Ex→ $\cos\tan^{-1}\sin\cot^{-1}x$



Sol.

$\Rightarrow \cos[\cos^{-1}\frac{1}{\sqrt{2}}]\sin\cot^{-1}(\frac{x}{1})$

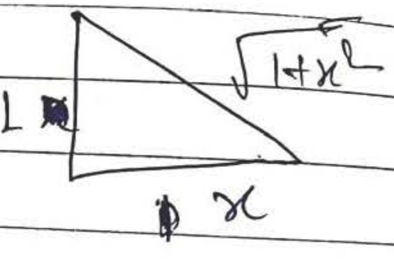


$\frac{1}{\sqrt{2}}$ $\frac{1}{4}$ $\sqrt{1}$

$\Rightarrow$

$\cos\tan^{-1}\sin\cot^{-1}x$

$\cos\tan^{-1}\sin\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$



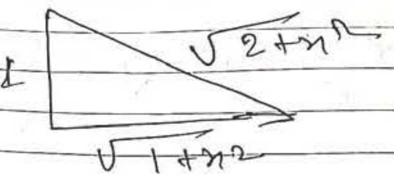
$\cos\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

WWW.GRADESETTER.COM

6.18

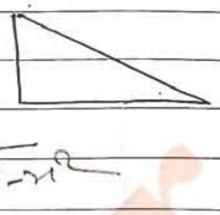
$\cos\cos^{-1}\left(\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\right)$

$= \sqrt{\frac{1+x^2}{2+x^2}}$



(ii) $\operatorname{cosec}.\tan^{-1}\cos\cot^{-1}.\sec\sin^{-1}x$

$\operatorname{cosec}.\tan^{-1}$



WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Property III

i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$; $x \in [-1, 1]$

ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$; $x \in R$

iii) $\sec^{-1}x + \csc^{-1}x = \frac{\pi}{2}$; $x \in (-\infty, -1] \cup [$

Property IV ⇒

{ याद रखें, STC Rule
S → sin, T → tan, C → cosec

i) $\sin^{-1}(-x) = -\sin^{-1}x$; $x \in [-1, 1]$

ii) $\cos^{-1}(-x) = \pi - \cos^{-1}x$; $x \in [-1, 1]$

iii) $\tan^{-1}(-x) = -\tan^{-1}x$; $x \in R$

iv) $\cot^{-1}(-x) = \pi - \cot^{-1}x$; $x \in R$

v) $\sec^{-1}(-x) = \pi - \sec^{-1}x$; $x \in (-\infty, -1] \cup [1, \infty)$

vi) $\csc^{-1}(-x) = -\csc^{-1}x$; $x \in (-\infty, -1] \cup [1, \infty)$

WWW.GRADESETTER.COM

Q) Solve the following equation:—

1.) $5\tan^{-1}x + 3\cot^{-1}x = 2\pi$

Sol. $5\tan^{-1} + 3\left(\frac{\pi}{2} - \tan^{-1}x\right) = 2\pi$

$2\tan^{-1}x = 2\pi - \frac{3\pi}{2}$

$\tan^{-1}x = \frac{\pi}{4}$

$x = \tan\frac{\pi}{4}$ (multiply both side by ~~tan~~ ~~tan~~ tan)

$x = 1.$

ii) $4\sin^{-1}x + \cos^{-1}x = \pi$

S $4\sin^{-1}x + \frac{\pi}{2} - \sin^{-1}x = \pi$

$\sin^{-1}x = \frac{\pi}{6}$

$x = \frac{1}{2}$

iii) $\sin\left[\frac{\pi}{2} + \cos^{-1}x\right] = \frac{1}{5}$

Sol. ~~$\sin\left[\frac{\pi}{2} + \frac{\pi}{2} - \sin^{-1}x\right] = \frac{1}{5}$~~

~~$\sin\left[\pi - \sin^{-1}x\right]$~~

$\because \sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$

So,

$\cos(\cos^{-1}x) = \frac{1}{5}$

$x = \frac{1}{5}$

1st Choice

$$(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$$

1) $(\tan^{-1}x + \cot^{-1}x)^2 - 2\tan^{-1}x\cot^{-1}x = \frac{5\pi^2}{8}$

~~Ans~~ $(\tan^{-1}x + \cot^{-1}x)^2 - 2\tan^{-1}x\left(\frac{\pi}{2} - \tan^{-1}x\right) = \frac{5\pi^2}{8}$

$$\left(\frac{\pi}{2}\right)^2 - 2\tan^{-1}x\left(\frac{\pi}{2} - \tan^{-1}x\right) = \frac{5\pi^2}{8}$$

$$\tan^{-1}x = \frac{3\pi}{4}\ (\times)\quad \frac{-\pi}{4}\ \checkmark$$

$$\tan^{-1}x = \frac{-\pi}{4}$$

$$\boxed{x = -1}\ \&$$

[Q.] Find the max. and min. value of

$$f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$$

Ans $f(x) = \underbrace{\sin^{-1}x + \cos^{-1}x}_{x \in [-1, 1]} + \tan^{-1}x$

$$f(x) = \frac{\pi}{2} + \tan^{-1}x$$

maximum $\frac{\pi}{2} + \tan^{-1}(1)$

$$\frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

minimum $\frac{\pi}{2} + \tan^{-1}(-1)$

$$\Rightarrow \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{\pi}{4}$$

Q) Find the max. and min. value of $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$

Ans:- $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$

$$\Rightarrow \frac{\pi}{2}\left[\left(\frac{\pi}{2}\right)^2 - 3\sin^{-1}x\cdot\cos^{-1}x\right]$$

$$\Rightarrow \frac{3\pi}{2}\left[\frac{\pi^2}{12} - \sin^{-1}\left(\frac{\pi}{2} - \sin^{-1}x\right)\right]$$

$$\Rightarrow \frac{3\pi}{2}\left[(\sin^{-1}x)^2 - \frac{\pi}{2}\sin^{-1}x + \frac{\pi^2}{12}\right]$$

$$\Rightarrow \frac{3\pi}{2}\left[\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{12} - \frac{\pi^2}{16}\right]$$

$$\Rightarrow \frac{3\pi}{2}\left[\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{48}\right]$$

min! →

$$\Rightarrow \frac{3\pi}{2}\left[0 + \frac{\pi^2}{48}\right]$$

max! →

$$\Rightarrow \frac{3\pi}{2}\left[\left(\frac{-\pi}{2} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{48}\right]$$

WWW.GRADESETTER.COM

Properties V

1) $\sin^{-1}\left(\frac{1}{x}\right) = \text{cosec}^{-1}x$ If $x \in (-\infty, -1] \cup [1, \infty)$

2) $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}x$, if $x \in (-\infty, -1] \cup [1, \infty)$

III) $\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1}x & \text{if } x > 0 \\ -\pi + \cot^{-1}x & \text{if } x < 0 \end{cases}$

IX) $\tan^{-1}\frac{2x}{x^2-1} + \cot^{-1}\frac{x^2-1}{2x} = \frac{2\pi}{3}$, if $x > 1$

S: $\tan^{-1}\frac{2x}{x^2-1} + \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$

$\tan^{-1}\frac{2x}{x^2-1} = \frac{\pi}{3}$

$\frac{2x}{x^2-1} = \sqrt{3}$

$x = \sqrt{3}, \boxed{-\frac{1}{\sqrt{3}}}$

$\boxed{\times}$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

(E) Find the values of:—

i) $\tan\left(\cot^{-1}\left(-\frac{2}{3}\right)\right)$

$\Rightarrow \tan\left(\pi - \cot^{-1}\left(\frac{2}{3}\right)\right)$

$\Rightarrow -\tan \cot^{-1}\left(\frac{2}{3}\right)$

$\Rightarrow -\tan\left(\tan^{-1}\left(\frac{3}{2}\right)\right)$

$\Rightarrow -\frac{3}{2}$

ii) $\tan^{-1} \cot\left(-\frac{1}{4}\right)$

iii) $\cos^{-1}(\sin(-5))$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Property (VI)

1) $$\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right) & \text{if } xy<1 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) & \text{if } x>0, y \\ & xy>1 \\ -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) & \text{if } x<0, \\ & y<0, \\ & xy>1 \end{cases}$$

Proof -

$$\begin{cases} \tan^{-1}x = A, \\ x = \tan A \end{cases} \quad \begin{array}{l} \tan^{-1}y = B \\ y = \tan B \end{array}$$

$$\tan(A+B) = \left(\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}\right)$$

$$\tan(A+B) = \left(\frac{x+y}{1-xy}\right) \quad \text{—— ①}$$

(I) $x>0, \; y>0, \; xy<1$

$$0 < \tan^{-1}x < \frac{\pi}{2}$$

$$0 < A < \pi/2$$

$$0 < B < \pi/2$$

$$0 < A+B < \pi$$

i) RHS

$$\tan(A+B) > 0$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$xy < 1$

if $x>0, y>0$
$xy>1$

$x<0,$
$y<0$
$xy>1$

$\Rightarrow 0 < A+B < \frac{\pi}{2}$

$A+B = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(ii) $x>0, \quad y>0, \quad xy>1$

$0 < A+B < \pi$

(ii) from eq① RHS.

$\tan(A+B) < 0$

$\frac{\pi}{2} < A+B < \pi$

$A+B-\pi = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$A+B = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(ii)
$$\tan^{-1}x - \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x-y}{1+xy}\right) & \text{if } xy > -1 \\ \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right) & \text{if } x>0, y<0, xy<-1 \\ -\pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right) & \text{if } x<0, y>0, xy<-1. \end{cases}$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Find the value's of the following:-

Ex.)

1) $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}$

Sol: $\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}}\right)$

$\Rightarrow \tan^{-1}(1)$

$\Rightarrow \frac{\pi}{4}$

Ex.) $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}$

Ans:- $\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{10}}\right)+\tan^{-1}\frac{1}{8}$

$\Rightarrow \tan^{-1}\frac{7}{9}+\tan^{-1}\frac{1}{8} = \tan^{-1}\left(\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{9}\times\frac{1}{8}}\right)$

$\Rightarrow \tan(1) \Rightarrow \frac{\pi}{4}$

Ex.) $\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}-\tan^{-1}\frac{8}{19}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{9}{20}}\right)-\tan^{-1}\frac{8}{19}$

$\Rightarrow \tan^{-1}\frac{27}{11}-\tan^{-1}\frac{8}{19}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{27}{11}-\frac{8}{19}}{1+\frac{27}{11}\times\frac{8}{19}}\right) \Rightarrow \tan^{-1}(1) = \frac{\pi}{4}$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

wing:—

(a) $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8}$

Ans = $\frac{\pi}{4}$

Ans $\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{35}}\right) + \tan^{-1}\frac{\left(\frac{1}{3}+\frac{1}{8}\right)}{\left(1-\frac{1}{24}\right)}$

$\Rightarrow \tan^{-1}\left(\frac{12}{34}\right) + \tan^{-1}\left(\frac{11}{23}\right)$

$\Rightarrow \tan^{-1}\frac{\left(\frac{12}{34}+\frac{11}{23}\right)}{\left(1-\frac{12}{34}\times\frac{11}{23}\right)}$

$\Rightarrow \tan^{-1}\left(\frac{\cancel{650}}{\cancel{650}}\right)$

$\Rightarrow \tan^{-1}(1)$

$\Rightarrow \frac{\pi}{4}$ Ans

(a) $\tan^{-1}2 + \tan^{-1}3$

A $\Rightarrow \pi + \tan^{-1}\left(\frac{2+3}{1-6}\right)$

$\Rightarrow \pi + \tan^{-1}(-1)$

$\Rightarrow \pi - \frac{\pi}{4}$

$\Rightarrow \frac{3\pi}{4}$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Ex) $\tan^{-1}4 + \tan^{-1}5 = \cot^{-1}x$

$\Rightarrow \pi + \tan^{-1}\left(\frac{4+5}{1-20}\right) = \cot^{-1}x$

$\Rightarrow \pi - \cot^{-1}x = -\tan^{-1}\left(\frac{9}{-19}\right)$

$\Rightarrow \cot^{-1}(-x) = \cot^{-1}\left(\frac{19}{9}\right)$

$-x = \frac{19}{9}$

Problem based on Series $\Rightarrow$

Ex) ~~find~~ find the value of $\tan^{-1}\frac{x}{1+(2x)(x)} + \tan^{-1}\frac{x}{1+(3x)(2x)}$ $+ \tan^{-1}\frac{x}{1+(4x)(3x)} + \text{-----} n \text{ terms}$

Ans) $\tan^{-1}\frac{2x-x}{1+(2x)(x)} + \tan^{-1}\frac{3x-2x}{1+(3x)(2x)} + \text{-----} n \text{ terms}$

~~Here~~ Now, we apply $\tan A - \tan B = \frac{\tan A - \tan B}{1+\tan A\cdot\tan B}$

$\Rightarrow \left(\cancel{\tan^{-1}2x} - \tan^{-1}x\right) + \left(\cancel{\tan^{-1}3x} - \cancel{\tan^{-1}2x}\right) + \cancel{\ } \text{---} + \text{---} \cancel{\ } + \left(\tan^{-1}(n+1)x - \cancel{\tan^{-1}nx}\right)$

$\Rightarrow \tan^{-1}(n+1)x - \tan^{-1}x$

WWW.GRADESETTER.COM

Page No.
Date / /

Q. $S = \tan^{-1}\frac{2}{1^2} + \tan^{-1}\frac{2}{2^2} + \tan^{-1}\frac{2}{3^2} + - - - - - n\text{ terms}.$
Hence find the sum of infinite terms of the series.

Sol. $T_r = \tan^{-1}\frac{2}{1+r^2-1}$

$= \tan^{-1}\left\{\frac{(r+1)-(r-1)}{1+(r+1)(r-1)}\right\}$

$T_r = \left\{\tan^{-1}(r+1) - \tan^{-1}(r-1)\right\}$ → (Solve)

So,
$\Rightarrow$ ~~ket tan⁻¹~~
$\left\{\tan^{-1}(2) - \tan(0)\right\} + \left\{\tan^{-1}(3) - \tan^{-1}(1)\right\}$
$+ \left\{\tan^{-1}(4) - \tan^{-1}(2)\right\} + \left\{\tan^{-1}(5) - \tan^{-1}(3)\right\}$

WWW.GRADESETTER.COM

1st Choice

$\boxed{10}$) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \tan^{-1}\frac{1}{21} + \text{------} \text{ n terms}$

Hence, find the sum of Infinite terms of this series

Ans ⇒ $\frac{\pi}{4}$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

n terms

ons of

x ⇒ $\frac{\pi}{4}$

Property VII ⇒

1) $$\cos^{-1}x+\cos^{-1}y = \begin{cases} \cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right) \\ \quad \text{if } -1\le x,\ y\le 1, \text{ and } x+y\ge 0 \\ 2\pi-\cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right) \\ \quad \text{if } -1\le x,\ y\le 1 \text{ and } x+y<0 \end{cases}$$

Q) Solve the equation $\cos^{-1}x\sqrt{3}+\cos^{-1}x = \frac{\pi}{2}$

$\cos^{-1}\left(x^2\sqrt{3}-\sqrt{1-3x^2}\sqrt{1-x^2}\right) = \frac{\pi}{2}$

$\Rightarrow x^2\sqrt{3}-\sqrt{1-3x^2}\sqrt{1-x^2} = 0$

$\Rightarrow 3x^4-(1-3x^2)(1-x^2) = 0$

$x = \pm\frac{1}{2}$

$\boxed{x = \frac{1}{2}}$ ✓ Ans.

Q) $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z = \pi$ Then Prove that $x^2+y^2+z^2+2xyz = 1$

$\cos^{-1}x+\cos^{-1}y = \pi-\cos^{-1}z$

$\cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right) = \cos^{-1}(-z)$

WWW.GRADESETTER.COM

⇒ ~~$xy + z$~~ =

$$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$$

$$\sqrt{1-x^2}\sqrt{1-y^2}$$

$$\Rightarrow (xy - z) = \sqrt{1-x^2}\sqrt{1-y^2}$$

$$\Rightarrow x^2y^2 + 2xyz + z^2 = (1-x^2)(1-y^2)$$

$$\Rightarrow x^2+y^2+z^2+2xyz = 1 .$$

ख

प्र– [illegible]

WWW.GRADESETTER.COM

Property VIII

i) $\sin^{-1}x+\sin^{-1}y = \begin{cases} \sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right) & \text{if } -1\le x,y\le 1 \text{ and } x^2+y^2\le 1 \\ & \text{or if } xy<0,\ x^2+y^2>1 \\ \pi-\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right) & \text{if } 0<x,y\le 1 \\ & \text{and } x^2+y^2>1 \\ -\pi-\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right) & \text{if } -1\le x,y<0 \\ & \text{and } x^2+y^2>1 \end{cases}$

1) Find the values of $\sin^{-1}\frac{3}{5}+\tan^{-1}\frac{15}{8}+\sin^{-1}\frac{84}{85}$

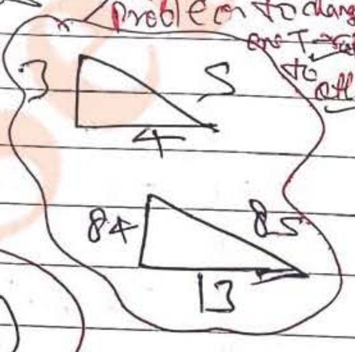
Ans $\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{15}{8}\right)+\sin^{-1}\left(\frac{84}{85}\right)$.

$\Rightarrow \pi+\tan^{-1}\left(\frac{\frac{3}{4}+\frac{15}{8}}{1-\frac{3}{4}\cdot\frac{15}{8}}\right)+\tan^{-1}\left(\frac{84}{13}\right)$

$\Rightarrow$ ~~$\pi \pm \tan^{-1} 6+$~~ $\pi+\tan^{-1}\left(\frac{-84}{13}\right)+\tan^{-1}\left(\frac{84}{13}\right)$

$\Rightarrow \pi$

use this concept to solve many problem to change one T-ratio to other



है +ve का
Sum का जब Sum करते
जब "-ve" नहीं हो सकता
[illegible] होगा|

WWW.GRADESETTER.COM

1st Choice

[Ex] Find $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{4}{5}-\tan^{-1}\frac{63}{16}$

S. $\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{4}{3}-\tan^{-1}\frac{63}{16}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{9}}\right)-\tan^{-1}\left(\frac{63}{16}\right)$

$\Rightarrow \cancel{\tan^{-1}\left(\frac{63}{16}\right)-\tan^{-1}\left(\frac{63}{16}\right)}$

$\Rightarrow 0$

(यहाँ हम [illegible] इन सबका Ratio को $\tan^{-1}$ में [illegible] तो easy हो [illegible] इसलिए change [illegible] formula Apply करें)

[Ex] Solve for x:-

1) $\sin^{-1}6x+\sin^{-1}6\sqrt{3}x=-\frac{\pi}{2}$

$\Rightarrow \sin^{-1}6\sqrt{3}x=-\left[\frac{\pi}{2}+\sin^{-1}6x\right]$

$\Rightarrow -\sin 6\sqrt{3}x=\left[\sin^{-1}1+\sin^{-1}6x\right]$

$\Rightarrow \sin^{-1}(-6\sqrt{3}x)=\sin^{-1}\left[\sqrt{1-36x^2}+0\right]$

$\Rightarrow -6\sqrt{3}x=\sqrt{1-36x^2}$

$\Rightarrow 36\times 3x^2+36x^2=1$

$x=\pm\frac{1}{12}$

$x=\frac{-1}{12}$, $x=\frac{+1}{12}$ ✗

Here we not change, the given trigo-ratio other because to solving in trigono T-ratio is easy than any other T-ratio (यह स्वयं अनुमान से)

(जो "-ve" का sum [illegible] है इसलिए [illegible] "+ve" का sum "-ve" [illegible] संभव नहीं है)

WWW.GRADESETTER.COM

1st Choice

Page No.
Date / /

Ratio की
change
Apply

ii) $\sin^{-1}x + \sin^{-1}2x = \pi/3$

Ans.) $\sin^{-1}2x = \frac{\pi}{3} - \sin^{-1}x$

$\Rightarrow \sin^{-1}2x = \left[-\left(\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right] - \sin^{-1}x$ ~~Ans → -ve reject only +ve value~~

$\Rightarrow \sin^{-1}2x = -\left[\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \sin^{-1}x\right]$ ~~Ans → $\frac{1}{2}\sqrt{\frac{3}{7}}$~~

$\because \left[-\left(-\frac{\pi}{2}\right)\right] = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(-2x) = \sin^{-1}\left[\frac{-\sqrt{3}}{2}\left(\sqrt{1-x^2} + x\sqrt{1-\frac{3}{4}}\right)\right]$

$\Rightarrow$ ~~$\sin^{-1}(-2x) = \sin^{-1}$~~ $\Rightarrow -2x = \frac{-\sqrt{3}}{2}\left(\sqrt{1-x^2}\right) + x\sqrt{\frac{1}{4}}$

$\Rightarrow -4x = -\sqrt{3}\left(\sqrt{1-x^2}\right) + x$

$\Rightarrow -5x = (-\sqrt{3})\sqrt{1-x^2}$

squaring both side

$25x^2 = 3(1-x^2)$

$28x^2 = 3$

$x^2 = \frac{3}{28}$

$x = \pm\frac{1}{2}\sqrt{\frac{3}{7}}$

so

$x = +\frac{1}{2}\sqrt{\frac{3}{7}}$ ✓, $\boxed{x = -\frac{1}{2}\sqrt{\frac{3}{7}}}$ ✗

x का value put करने पर हमें "-ve" value आएगा ~~कास~~ लेकिन हमें "+ve" value $\left(\frac{\pi}{3}\right)$ चाहिए इसलिए ऐसा value लेंगे जिससे की हमारा Answer satisfy हो।

(हम Direct $\sin^{-1}x - \sin^{-1}y$ का formula लगाकर भी बना सकते है।)

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Properties IX

i) $$2\tan^{-1}x = \begin{cases} \tan^{-1}\frac{2x}{1-x^2} & \text{if } -1<x<1 \\ \pi+\tan^{-1}\frac{2x}{1-x^2} & \text{if } x>1 \\ -\pi+\tan^{-1}\frac{2x}{1-x^2} & \text{if } x<-1 \end{cases}$$

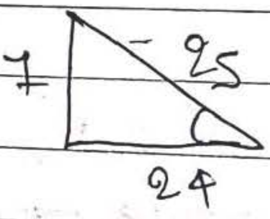
ii) $$3\tan^{-1}x = \begin{cases} \tan^{-1}\frac{3x-x^3}{1-3x^2} & \text{if } \frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}} \\ \pi+\tan^{-1}\frac{3x-x^3}{1-3x^2} & \text{if } x>\frac{1}{\sqrt{3}} \\ -\pi+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) & \text{if } x<\frac{-1}{\sqrt{3}} \end{cases}$$

Q) Find the values of following :—

i) $\cos\left(2\tan^{-1}\frac{1}{7}\right)$

$\Rightarrow \cos\left[\tan^{-1}\left(\frac{2/7}{1-\frac{1}{49}}\right)\right]$

$\Rightarrow \cos\left[\tan^{-1}\left(\frac{7}{24}\right)\right]$



$\Rightarrow \cos\left(\cos^{-1}\frac{24}{25}\right) \Rightarrow \frac{24}{25}$ A

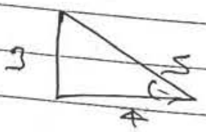
WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$\sin\left(2\tan^{-1}\frac{1}{3}\right)$

$\Rightarrow \sin\left(\tan^{-1}\left(\frac{2/3}{1-\frac{1}{9}}\right)\right)$

$\Rightarrow \sin\tan^{-1}\left(\frac{3}{4}\right)$



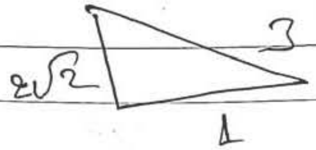
$\Rightarrow \sin\sin^{-1}\frac{3}{5}$

$\Rightarrow \frac{3}{5}$ Ans.

$\cos\left(\tan^{-1}2\sqrt{2}\right)$

$\Rightarrow$ ~~$\cos\left(\pi + \frac{2\times2\sqrt{2}}{1-8}\right)$~~

$\Rightarrow \cos\frac{1}{3}$



$\sin\left(2\tan^{-1}2\right) \rightarrow$ मद the

$\Rightarrow \sin\left(\pi + \tan^{-1}\frac{4}{(1-4)}\right) \rightarrow$ मद -ve इसलिए π add कीजिए

$\Rightarrow \sin\left(\pi - \tan^{-1}\frac{4}{3}\right)$

$\Rightarrow$ ~~$\sin\left(\tan^{-1}\frac{4}{3}\right)$~~ $\Rightarrow \sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)$

$\Rightarrow \frac{4}{5}$

$\sin\left(4\tan^{-1}\left(\frac{1}{3}\right)\right)$

$\Rightarrow \sin\left(2\tan^{-1}\left(\frac{2/3}{1-\frac{1}{9}}\right)\right)$

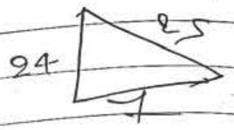
WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$\Rightarrow \sin\left(2\tan^{-1}\left(\frac{3}{4}\right)\right)$

$\Rightarrow \sin \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\frac{9}{16}}\right)$

$\Rightarrow \sin \tan^{-1}\left(\frac{24}{7}\right)$



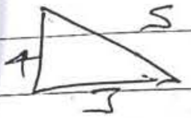
$\Rightarrow \sin \frac{24}{25}$ Ans.

(v) $\sin\left(\frac{1}{2}\underbrace{\tan^{-1}\frac{4}{3}}_{\theta}\right)$

Do one time must to find mean concept.

Let $\tan^{-1}\frac{4}{3} = \theta$

$\tan\theta = \frac{4}{3}$



$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$

$= \sqrt{\frac{1-3/5}{2}}$

$\Rightarrow \frac{1}{\sqrt{5}}$

(vi) $\tan\left(\frac{1}{2}\underbrace{\cos^{-1}\frac{\sqrt{5}}{3}}_{\theta}\right)$

Let

$\cos\theta = \frac{\sqrt{5}}{3}$

$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$$2\sqrt{\frac{1-\frac{5}{3}}{1+\frac{\sqrt{5}}{3}}} \Rightarrow \sqrt{\frac{3-\sqrt{5}}{1+\sqrt{5}}}$$

Rationalise.
and get answer.

$$2\sin^{-1}\left(\frac{2x}{1+x^2}\right) - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \frac{\pi}{3}$$

Let $x = \tan\theta$

$$2\sin^{-1}(\sin 2\theta) - \cos^{-1}(\cos 2\theta) = \frac{\pi}{3}$$

$$4\theta - 2\theta = \pi/3$$

$$\theta = \pi/6$$

$$\tan^{-1}x = \pi/6$$

$$\boxed{x = \frac{1}{\sqrt{3}}}$$ Ans.

Properties ⊗

1) ~~tan~~ $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

if $x>0,\ y>0$ or $xy<1$

(और जो other case Property-VII में देखे)

~~$\tan^{-1}x +$~~

2.) $$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$$

GradeSetter

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Imp

Note: ⇒ The above Result can be generalized as follows :⇒

$$\boxed{\tan^{-1}x_1+\tan^{-1}x_2+\tan^{-1}x_3+\cdots\cdots+\tan^{-1}x_n = \tan^{-1}\left[\frac{S_1-S_3+S_5+\cdots\cdots}{1-S_2+S_4-S_6+\cdots}\right]}$$

where S_k denotes the sum of the product $x_1, x_2, \cdots\cdots x_n$ taken "K" at a time

eg:⇒ Find the sum of $\sum_{i=1}^{4}\tan^{-1}x_i$.

Ans:⇒

$$\sum_{i=1}^{4}\tan^{-1}x_i = \tan^{-1}x_1+\tan^{-1}x_2+\tan^{-1}x_3+\tan^{-1}x_4$$

So, By formula

Here four term's is present so, we apply above formula.

$$\tan^{-1}x_1+\tan^{-1}x_2+\tan^{-1}x_3+\tan^{-1}x_4 = \tan^{-1}\left[\frac{S_1-S_3}{1-S_2+S_4}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{\left(\begin{array}{c}\text{Sum of the product of}\\ x_1+x_2+x_3+x_4 \text{ taken}\\ \text{one at a time}\end{array}\right)-\left(\begin{array}{c}\text{Sum of the produ}\\ x_1+x_2+x_3+x_4\\ \text{"3" at a time}\end{array}\right)}{1-\left(\begin{array}{c}\text{Sum of the product}\\ x_1+x_2+x_3+x_4 \text{ taken}\\ \text{"2" at a time}\end{array}\right)+\left(\begin{array}{c}\text{Sum of the P}\\ x_1+x_2+x_3+x_4\\ \text{"4" at a tim}\end{array}\right)}\right]$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$$\Rightarrow \tan^{-1}\left[\frac{(x_1+x_2+x_3+x_4) - \{x_1x_2+x_2x_3+x_3x_4+x_1x_4\}}{1-\{x_1x_2x_3+x_2x_3x_4+x_1x_2x_4+x_3x_4x_1\}+\{x_1x_2x_3x_4\}}\right]$$

$$= \tan^{-1}\left(\frac{\sum x_1 - \sum x_1x_2x_4}{1-\sum x_1x_2 + x_1x_2x_3x_4}\right)$$

Ans

(x) If x_1, x_2, x_3, x_4 are roots of the equation $x^4 - x^3\sin 2\beta + x^2\cos 2\beta - x\cos\beta - \sin\beta = 0$ then

$$\sum_{i=1}^{4} \tan^{-1} x_i = ?$$

GradeSetter

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Trigonometric Ration

1st Choice

Regd No.
Date / /

Angle → Ratio ↓	0°	30° or $\frac{\pi}{6}$	45° or $\frac{\pi}{4}$	60° or $\frac{\pi}{3}$	90° or $\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\csc\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

ALL STUDENT TO CAREER POINT

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

Page No.
Date / /

$90°$ or $\frac{\pi}{2}$	$120°$ or $\frac{2\pi}{3}$	$135°$ or $\frac{3\pi}{4}$	$150°$ or $\frac{5\pi}{6}$	$180°$ or π
1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
	-2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	-1
	$-\frac{1}{\sqrt{3}}$	-1	$\sqrt{3}$	∞

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

1st Choice

Mathematical Induction

- (i) Establish Identities
- (ii) Divisible Problems
- (iii) Recursion type relation

Page No.
Date / /

1st step:— verification step:—

1st of all verify that P(1) is True.

P(1) is True ✓

IInd step:—

Assume that P(m) is true and from it prove that P(m+1) is also true. That is show that

$$P(m) \Longrightarrow P(m+1)$$

Q) Using Induction prove that

$$1+2+3+ \cdots +n = \frac{n(n+1)}{2}$$

~~1st step~~

Ans Let

$$P(n) = 1+1+ \cdots + n = \frac{n(n+1)}{2}$$

Put n=1

$$1 = \frac{1(1+1)}{\cancel{2}}$$

$$1 = 1$$ ✓

Hence our supposition is true for P(1).

Assume that P(m) is true i.e

$$1+2+3+ \cdots + m = \frac{m(m+1)}{2}$$ ✓

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$$\Rightarrow 1+2+3+ \cdots m+(m+1) = \frac{m(m+1)}{2} + (m+1)$$

$$\Rightarrow P(m+1) = (m+1)\left(\frac{m}{2}+1\right)$$

$$= \frac{(m+1)(m+2)}{2}$$

So, using Induction formula is true for every n' belongs to n.

Q.⟶ using Induction show that

$$1^2+2^2+3^2+ \cdots\cdots +n^2 = \frac{n(n+1)(2n+1)}{6}$$

A Let

i) $P(n) = 1^2+2^2+3^2+ \cdots +n^2 = \frac{n(n+1)(2n+1)}{6}$

Put, $n=1$

$$1 = \frac{1(1+1)(2\times1+1)}{6}$$

$$= \frac{1\times\cancel{2\times3}}{\cancel{6}}$$

$$= 1$$

L.H.S = R.H.S

Hence our supposition is true for $n=1$

ii) Assume it is true for $n = m$

$$1+2^2+ \cdots\cdots +m^2 = \frac{m(m+1)(2m+1)}{6}$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

+ (m+1)

$\Rightarrow 1^2 + 2^2 + \dots + m^2 + (m+1)^2 = \frac{m(m+1)(2m+1)}{6} + (m+1)^2$

$P(m+1) = \frac{m+1}{6}\left[m(2m+1) + 6(m+1)\right]$

$= \frac{(m+1)(m+2)(2m+3)}{6}$

$= P(m+1)$

This formula is true for P(m+1)
So, this is true for all natural number,

$1^3 + 2^3 + \dots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$

Let $P(n) = 1^3 + 2^3 + \dots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$

Put, n = 1

$1 = \left(\frac{1(2)}{2}\right)^2$

$= 1$

L.H.S = R.H.S Proof

Hence Our supposition is true for n = 1.

ii) Assume it is true for n = m

$1^3 + 2^3 + \dots + m^3 = \left\{\frac{m(m+1)}{2}\right\}^2$

$\Rightarrow 1^2 + 2^2 + \dots + m^2 + (m+1)$

WWW.GRADESETTER.COM

1st Choice
WWW.GRADESETTER.COM

(vi) $1.3 + 2.3^2 + 3.3^2 + \text{----} n.3^n = \frac{(2n-1)3^{n+1}+3}{4}$

Ans i) Let

$$p(n) = 1.3 + 2.3^2 + 3.3^2 + \text{----} n.3^n = \frac{(2n-1)3^{n+1}+3}{4}$$ (vi)

Put n=1

$$1.3^1 = \frac{(2-1)3^2+3}{4}$$

$$\Rightarrow \frac{9+3}{4}$$

$$\Rightarrow \frac{\cancel{12}}{\cancel{4}} 3$$

$$3 = 3$$

LHS = RHS ~~Proof~~

Hence our supposition is true ~~for~~ for n = 1.

ii) Assume it is true ~~for~~ for n = m

$$1.3 + 2.3^2 + \text{----} m.3^m = \frac{(2m-1)3^{m+1}+3}{4}$$

$$\Rightarrow 1.3 + 2.3^2 + \text{----} + m.3^m + (m+1)3^{m+1} = \frac{(2m-1)3^{m+1}+3}{4} + (m+1)3^{m+1}$$

$$\Rightarrow \frac{(2m-1)3^{m+1}+3+4(m+1).3^{m+1}}{4}$$

$$\Rightarrow \frac{3^{m+1}(6m+3)+3}{4}$$

GradeSetter

WWW.GRADESETTER.COM

$$\Rightarrow \frac{3^{m+2}(2m+1)+3}{4} = P(m+1)$$

This formula is true for P(m+1)

So, this is true for all natural number.

(v) $$\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \dots\dots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

GradeSetter

1st Choice
WWW.GRADESETTER.COM

Type IInd ⇒

Prove that $11^{n+2} + 12^{2n+1}$ ($n \in N$) is always divisible by 133

(Put n = 1)

i) step 11^3+12^3

$\Rightarrow 1331 + 1798$

$\Rightarrow 3059$

$\Rightarrow 23 \times 133$

ii) step

Assume

$P(m) = 11^{m+2} + 12^{2m+1}$ is divisible by 133

$P(m) = 133k$ (where k is an Integer.)

So,

$P(m+1) = 11^{m+3} + 12^{2m+1}$

$$\begin{array}{r|l} & 11 \\ \hline 11^{m+2}+12^{2m+1} & \cancel{11^{m+3}} + 12^{2m+3} \\ & \cancel{11^{m+3}} + 11 \cdot 12^{2m+1} \\ & - \qquad - \\ \hline & 12^{2m+1}\left(12^2-11\right) \end{array}$$

Dividend = quotient × diviser + remainder $133 \times 12^{2m+1}$

$\Rightarrow 11^{m+3} + 12^{2m+3} = 11\left(11^{m+2} + 12^{2m+1}\right) + 133 \cdot 12^{2m+1}$

$\Rightarrow P(m+1) = 11P(m) + 133 \cdot 12^{2m+1}$

$\Rightarrow 133\left[11k + 12^{2m+1}\right]$

(So

Ar

Page No.
Date / /

Q) $5^{2n+1} + 3^{n+2} \cdot 2^{n-1}$ is divisible by 19 for $\forall\, n \in N$

A) Step Ist I—

Let $P(n) = 5^{2n+1} + 3^{n+2} \cdot 2^{n-1}$

for $n = 1$

$P(1) = 5^3 + 3^3 \cdot 2^0$ $\qquad 19\overline{)152}(8$

$\Rightarrow 125 + 27$

$\Rightarrow 152$

$\Rightarrow 19 \times 8$

(Divisible by 19) S.

Step IInd: ⇒

Assume

$P(m) = 5^{2m+1} + 3^{m+2} \cdot 2^{m-1}$ is divisible by 19

$P(m) = 19K$, where "K" is an Integer.

so/

$P(m+1) = 5^{2m+3} + 3^{m+3} \cdot 2^{m}$

Now →

$5^{2m+1} + 3^{m+2} \cdot 2^{m-1} \overline{)\, 5^{2m+3} + 3^{m+3} \cdot 2^{m}\,}(5^2$

$\cancel{5^{2m+3}} + 5^2 \cdot 3^{m+2} \cdot 2^{m-1}$

$- \qquad -$

$\Rightarrow 3^{m+3} \cdot 2^{m} - 5^2 \cdot 3^{m+2} \cdot 2^{m-1}$

$\Rightarrow 3^{m+3} \cdot 2^{m} \left(3 - \frac{5^2}{2}\right)$

$\Rightarrow 3^{m+3} \cdot 2^{m} \left(\frac{19}{2}\right)$

$\Rightarrow \frac{3^{m+3} \cdot 2^{m}}{2} (19)$

$\Rightarrow 3^{m+3} \cdot 2^{m-1} \times (19)$

WWW.GRADESETTER.COM

Now we know that

Dividend = quotient × divisor + Remainder.

$$5^{2m+3}+3^{m+3}\cdot 2^{m} = 25\left(5^{2m+1}+3^{m+2}\cdot 2^{m-1}\right)+19\times 3^{m+3}\cdot 2^{m-1}$$

$$p(m+1) = 25(Pm)+19\times 3^{m+3}\cdot 2^{m-1}$$

$$\Rightarrow 25\times 19k+19\times 3^{m+3}\cdot 2^{m-1}$$

$$\Rightarrow 19\left[25k+3^{m+3}\cdot 2^{m-1}\right]$$

Hence P(m+1) is divisible by 19.

This shows that the result is true for n = m+1.

Hence by the principle of mathematical Induction, the result is true for all n ∈ N.

WWW.GRADESETTER.COM

1st Choice

★ Principle of mathematical Induction (M.I)

(1) First Principle of M.I :-
↳ To Prove a Proposition by M.I.

Step 1st :-
If P(n) depends on "n" then, (n = a natural number)
We will verify
for n = 1

Step 2nd :- (Induction step)
We will Assume that P(n) is true for n = k and
then will use it to prove for n = k+1

Step 3rd :-
(Generalisation step)
combining above two steps.

(2) Second Principle of M.I

If P(n) is Statement depending on n ∈ N but (n ≥ K) and
K is some Integer.
(यहाँ "n" "1" नहीं है।)

Step 1st :-
verify for n = K
P(n) to be verified for n = K

Step 2nd :- Assume P(n) to be true for n = m ≥ K

Step 3rd :- Using step 2nd to Prove for n = m+1
Then it is true for all "n".

WWW.GRADESETTER.COM

Tricks for solving objective problems :—

(1) The biggest no. which divides $P(n) = 3^{2n} - 2n - 1$

(A) 1. (B) 2 (C) 4 (D) 8

(✗) (✗) (✗)

Trick -
$P(1) = 6$, $P(2) = 3^4 - 4 - 1 = 76$ ✓ , $P(3) = 3^6 - 6 - 1 = 722$

↳ Note :- (n>1) से जो नंबर obtained होगा वह सभी तो नंबर divisible होना चाहिए। इसलिए option देखकर select करें ।।

(2) for S_n i.e. sum of n-terms :-
$1 \cdot 3^2 + 2 \cdot 5^2 + 3 \cdot 7^2 + \cdots$

(A) $\frac{n}{6}(n+1)(6n^2 + 14n + 7)$

(B) $\frac{n}{6}(n+1)(2n+1)(3n+1)$

(C) $4n^3 + 4n^2 + n$

(D) None

Trick :-

$n = 1$, $S_1 = 9$, (A) 9
$\boxed{(B)}$ ✗ ✗
(C) 9
(D)

$n = 2$, $S_2 = 9 + 50$
$= 59$
(A) 59 Ans
(C) 50 ✗
(D)

WWW.GRADESETTER.COM

Page No.
Date / /

(3) $\frac{1^3+2^3+3^3+ - - - +12^3}{1^2+2^2+3^2+ - - - +12^2}$ is equal to

(I) $\frac{234}{25}$ (II) $\frac{243}{35}$ (III) $\frac{263}{27}$ (IV) None of the

Trick) —

↳ अगर n-वाला term नहीं दिया हुआ रहे तो खुद से n-वाला term मान ले और फिर उसी n की सहायता से question को solve कर देवें

$$\frac{\left(\frac{n(n+1)}{2}\right)^2}{\frac{n(n+1)(2n+1)}{6}} = \frac{n(n+1)6}{4(2n+1)} = \frac{\cancel{12}^{3} \times 13 \times 6}{\cancel{4}(25)} = \frac{234}{25}$$ A

1st Choice

Revision class

D.P.P. 3 ⇒ 78

Page No.
Date / /

A) i) A B C

ii) $A \xleftrightarrow[6]{} B \xrightarrow[4]{} C \xrightarrow[4]{} B \xrightarrow[6]{} A$

2 576

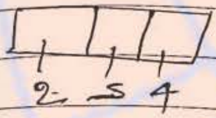
iii) $A \xrightarrow[6]{} B \xrightarrow[4]{} C \xrightarrow[3]{} B \xrightarrow[5]{} A$

④ i)

6	5	4

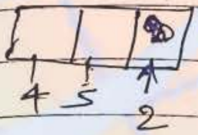
=120 Teacher

ii)



⇒ 40

iii)



⇒ 40

iv)

4	5	4

⇒ 80

v)

4	5	6

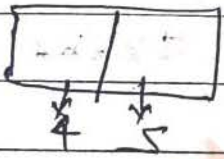
=100

vi)

4	5	1

⇒ 20

x)



1,3,5,7,9 → odd

~~0,2,4,6,8~~ ⇒

WWW.GRADESETTER.COM

~~4)~~

7 8 9

4.) 1 → 1000

(law of addition)

i) 9 9 8 ⇒ 648

ii) 9 9 ⇒ 81

iii) 9 ⇒ 9

738

जो आगया है इसकि वही नहीं आएगा.

2.) 1, 3, 5, 7, 9

5 4 4 4 4 4 4 4 4 4

$\Rightarrow 5 \times 4^9$

14) 9 9 8

8128
648

i) Total

9 10 10 ⇒ 900

∴ Repition is allowed.

~~9×10×4~~

ii) ~~7~~ Not Included

{0, 1, 2, 3, 4, 5, 6, 8, 9}

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

8 9 9 > 648

so, At least one 4 "

(i) - eq(10)

i) 1/3

3 3 2 1 2 1/3

ii) 26 25 9 8 7

26 25

10 9 8

2) i) 7 6 5 4 =

ii) 21 20 19 18

2 3 4 5 =120

21

H S T R Y

iii) 4 6 5 5 4

4 4 5 5 ⇒ 400 Ans.

iv) 2 6 5 4

→ vowel → I, O

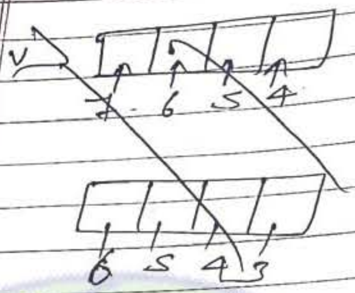
⇒ 240

~~4026~~ 240

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

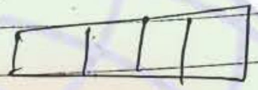
v)



24084
1680

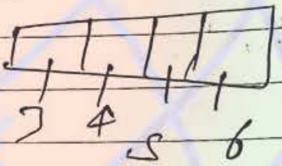
24069
620

vi) Total



→ 840

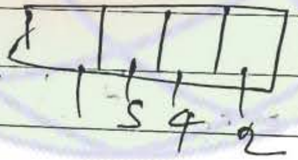
without y



= 360

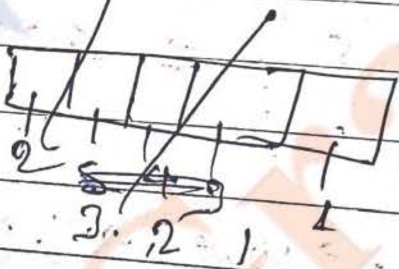
840
360
480

vii)

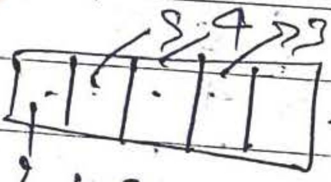


~~2)~~

AKSHI



A, I



2 120

~~9)~~

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

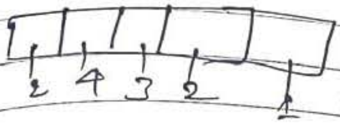
~~21067~~
1680

240~~67~~
620

A,I K S/d

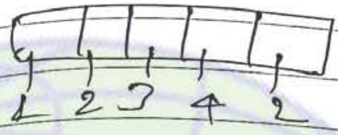
Q1)

i)



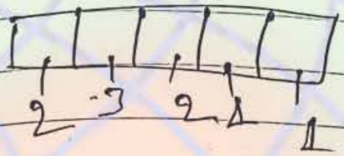
⇒ 48

ii)



⇒ 48

iii)



⇒ 12

so,

96 − 12 ⇒ 84

2) VARUN
ANRUV

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

D. P.P.S → 39

1.)

Total 900

without an 1, 3, 5, 7, 9

5 5 5 =125

80, 900−125 = ✓

2.) Total

i) 9 10 10 10 ⇒ ~~3024~~ 9000

9876 7267 504+66 8024 816

ii) without Repetition

9 9 8 7 = 4536

① − ca ⑩ = ✓

3.) 3

2 2 2 2 2

$^7C_2 \cdot 2^5$

5.) 5 | 7 | → 5

8 9 5

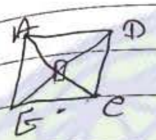
WWW.GRADESETTER.COM

$\underline{|1} + \underline{|2} + — + \underline{|x} = y^2$

1) $\underline{|1} = 1^2$ ✓

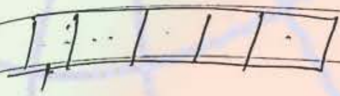
ii) $\underline{|1} + \underline{|2} = 3$ (✗)

iii)

5)


$^5C_3 - 2 = 8$ ✓

ABC, और EDC से नहीं Δ की form

10)


1) 1, 2, 3, 4, 5 → 15

5 4 3 2 1 = 120

ii) 0, 1, 2, 4, 5

4 4 3 2 1 = 96

Total 120 + 96 = 216

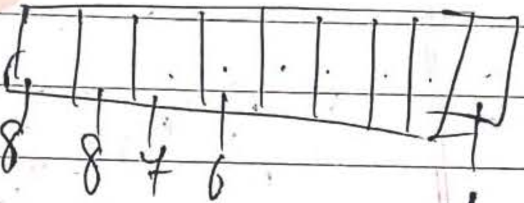
11) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

1) 1, 2, 3, 4, 5, 6, 7, 8, 9

9 8 7

$\Rightarrow \underline{|9}$

ii) 0, 1, 2, 3, 4, 5, 6, 7, 8



8 8 7 6 1

$8\underline{|8}$

Total $= \underline{|9} + 8\underline{|8}$

$= 9\underline{|8} + 8\underline{|8} = 17\underline{|8}$

WWW.GRADESETTER.COM

12) i) | | | 0 |

9 8 = 72

ii) | | | 5 |

8 8 ⇒ 64

iii) | | 0 |

9 = 9

iv) | | 5 |

8 = 8

v) | 5 | = 1

154

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

13) ~~| 9 | | |~~

~~9 8 = 72~~

| 8 | | |

8 7 = 56

17) i) | 1 | 0 | 0 | $= 1^2$

ii) | 2 | 0, 1 | 0, 1 |

2 2 $= 2^2$

iii) | 3 | 0, 1, 2 | 0, 1, 2 |

3 3 $= 3^2$

$\Rightarrow 9^2$ So, $n = 9$ = $n(n+1)(2n+1)$

WWW.GRADESETTER.COM

Page No.
Date / /

14) VALEDICTORY

VLDCTRY (AEIO)

$\lfloor 8 \, \lfloor 4$

15) $^{10}C_5 \, \lfloor 5$

14) Y G D R

$\odot \; {}^{6}C_4 \, \lfloor 4$

4 3 3 3 3 3

4×3^5

14) $^{9}C_2(1) \; ^{7}C_2(1) \; ^{5}C_2(1) \, \lfloor 3$

$P_c 9$

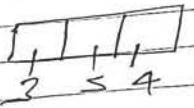
GradeSetter

WWW.GRADESETTER.COM

①.P.P.S ⇒ 40

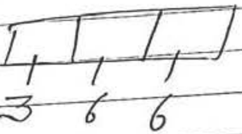
3, ~~5~~, ~~8~~, ~~15~~

11.) 1)



⇒ 60

11)



⇒ 36×3

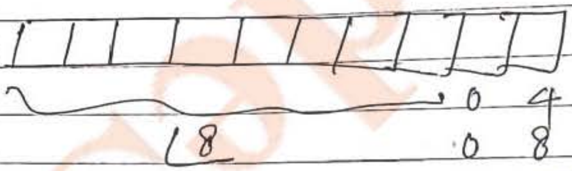
⇒ 108−1 = 107

⇓

400

नहीं आ रहा

10.) i)

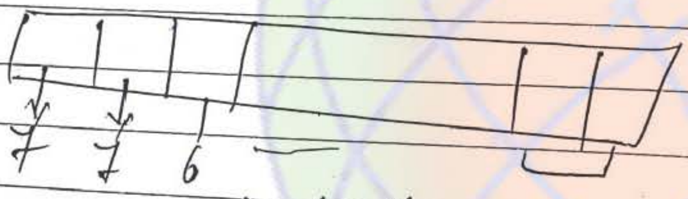


20
40
60
80

04
20
40
04
60
06

6×L8

ii)



12, 16, 24, 28, 32, 36, 48, 52, 56, 64, 68, 72, 76, 84, 92, 96,

(7 L6) 16

Total ⇒ 6L8 + 16×7 L7

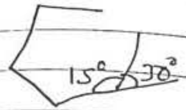
= 6L8 + 14L8

⇒ 20L8 Ay

WWW.GRADESETTER.COM

4) ${}^{18}C_3 - {}^{7}C_3 - {}^{7}C_3 - {}^{7}C_3$

6)

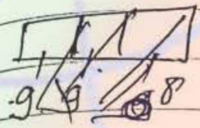


$n(30) = 360$

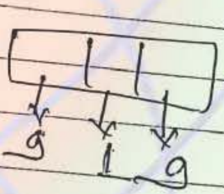
$\boxed{n = 12}$

${}^{12}C_2 - \underset{\text{side}}{\underset{\downarrow}{12}} \Rightarrow \text{Diagonal}$

8)

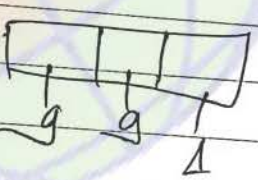


a a b $\Rightarrow$



$\Rightarrow 81$

b a a $\Rightarrow$



$\Rightarrow 81$

162

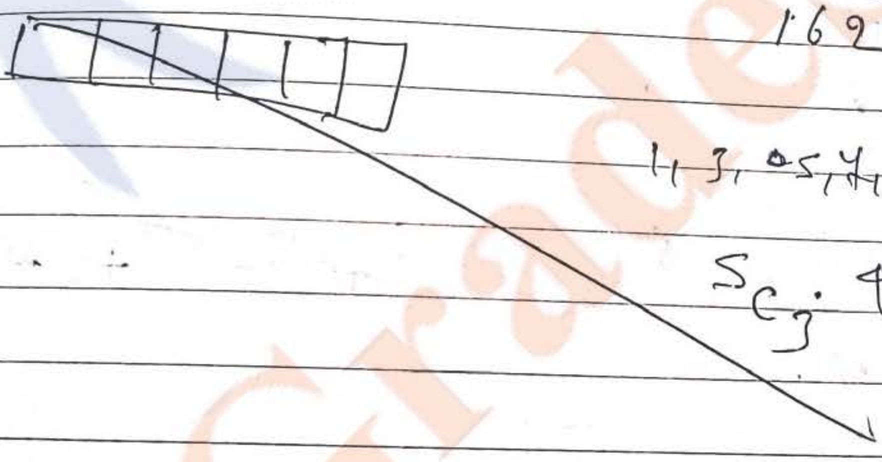
81×9

729

81×8

648

9)



1, 3, 5, 7, 9   2 4 6 8,

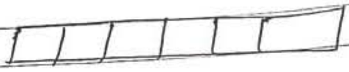
${}^5C_3 \cdot {}^4C_3$

[illegible]

$\Rightarrow \boxed{40}$

WWW.GRADESETTER.COM

9)



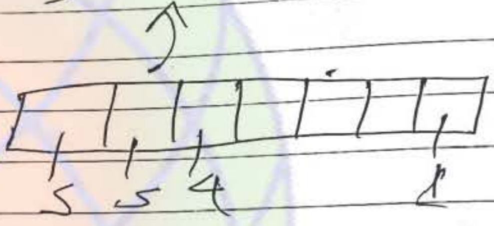
Even 0, 2, 4, 6, 8 odd 1, 3, 5, 7, 9

i) ~~0~~

$^4C_3 \cdot {}^5C_3 \lfloor 6$

ii) ~~odd~~ "0" include 0 ✓

$^4C_2 \cdot {}^5C_3 \cdot 5 \lfloor 5$

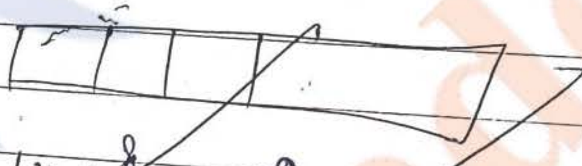


3)

$^5C_3 \cdot {}^8C_7 + {}^5C_4 \cdot {}^8C_6 + {}^5C_5 \cdot {}^8C_5 = 276$

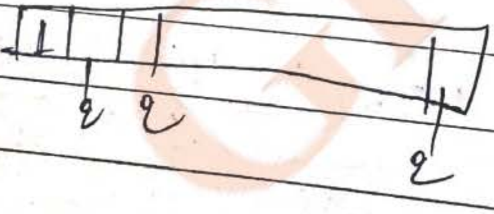
~~7)~~

~~2×10^8~~



$1 \times 2^8 + 2^8 + 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1$

7) 9 digit no.



$\Rightarrow 2^8 \times 1$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

odd

1, 3, 5, 7, 9

ii) 8 digit

$\Rightarrow 2^8$

$\rightarrow 2^7$

$\rightarrow 1$

2^1

$2\left(\frac{2^8-1}{2-1}\right) + 2^8$

14) ABC ABD AB~~CD~~

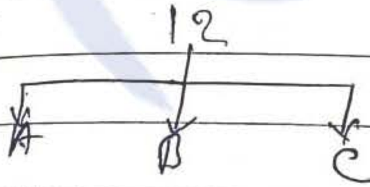
$^5C_2 + {}^5C_2 + {}^5C_3$

C~~AB~~ D~~AB~~ ~~AB~~ ~~CD~~

$^5C_4 + {}^5C_4 + {}^5C_5$

$\Rightarrow 41$

15)



4 4 4

1 1 3

$^4C_1 \cdot {}^4C_1 \cdot {}^4C_3 \times 3$

ii) 2 2 1

$^4C_2 \cdot {}^4C_2 \cdot {}^4C_1 \times 3$

Total ① + ⑪

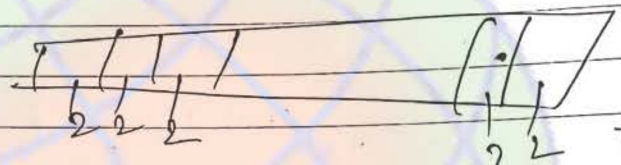
WWW.GRADESETTER.COM

[illegible])

A	P	B
4	5	6
	1	1
1	1	2
1	2	2
2	2	2

17) $\left({}^4C_1 + {}^4C_2\right)\left({}^5C_1 + {}^5C_2\right)\left({}^6C_1 + {}^6C_2\right)$

8)



$\Rightarrow 2^n - 2 = 510$

18) $\sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^{n+1}C_r} \Rightarrow \frac{{}^nC_0}{{}^{n+1}C_0} + \frac{{}^nC_1}{{}^{n+1}C_1} + \frac{{}^nC_2}{{}^{n+1}C_2} \cdots$

$$\sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}}$$

$$\sum \frac{{}^nC_r}{{}^{n+1}C_{r+1}}$$

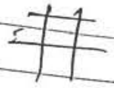
GradeSetter

WWW.GRADESETTER.COM

9.) i) [illegible]  $r = {}^9C_2 \cdot {}^9C_2$

$= (36)^2$

ii) square

$(n-r+1)$ 

$1\times1 = (9-2+1)(9-2+1) = 8^2$

$2\times2 = (9-3+1)(9-3+1) = 7^2$ 

⋮

$8\times8 = \cancel{800} = 1^2$

~~Number~~

Total square $= 1^2 + 2^2 + \text{---} + 8^2$

$= \frac{\overset{4}{\cancel{8}}(\overset{3}{\cancel{9}})(17)}{\cancel{6}} = 12\times17$

$\frac{s}{r} = \frac{12\times17}{(36)^2} = \frac{17}{36\times7} = \left(\frac{17}{108}\right)$ Ans

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM
1st Choice

16) $5\ 4, 3, 0, 9$ → मान लिये

$9, 5, 4, 3, 0$

but

$0\ 3\ 4\ 5\ 9$ — (X)

$m = {}^{10}C_5$

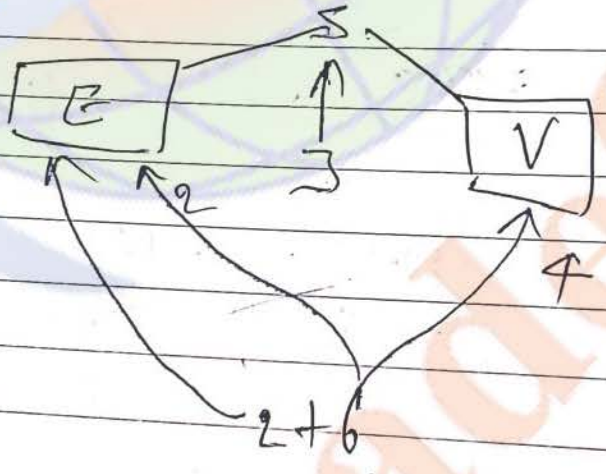
$n = {}^{9}C_5$

$m - n = {}^{10}C_5 - {}^{9}C_5$

$\frac{10}{5} \cdot {}^{9}C_4 - {}^{9}C_5$

$2\,{}^{9}C_5 - {}^{9}C_5 = {}^{9}C_5$ A.

17)



$m+n$ → m, n } two group

$$\frac{\lfloor m+n}{\lfloor m \lfloor n}$$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

1st Choice

Page No.
Date / /

$$^7C_1\left(\frac{\lfloor 6}{\lfloor 2\lfloor 2}\right)\left(\lfloor 4\cdot\lfloor 4\right)$$

D.P.P. 3 ⇒ 41

1) ~~$^8C_2$ ^8C A B~~ $\Rightarrow {}^7C_2 \lfloor 5 \times 2$

AB

$\left({}^7C_2 (1) \lfloor 5 \times \lfloor 2\right)$

2) $^{25}C_5 - {}^{24}C_4$

~~$^nC_r = {}^{n+1}C_r$ $r+1$~~

⇒

3) $^{25}C_5 - {}^{24}C_4$

$^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$^nC_r = {}^{n+1}C_r - {}^nC_{r-1}$

$$\boxed{{}^{24}C_5 = {}^{25}C_5 - {}^{24}C_4}$$

~~$^{10}C_4 + {}^9C_4 + {}^9C_4$~~

7) 9 + (A, B)

B ~~A~~ A ~~B~~

$^9C_5 + {}^9C_4 + {}^9C_4$

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

$= 3 \cdot {}^{9}C_{4}$

$\Rightarrow 3 \; \frac{9 \times \cancel{8} \times \cancel{7} \times \cancel{6}}{\cancel{6} \times \cancel{4}}$

$\Rightarrow 21 \times 18$

$= 378$ Ans.

Alt

7) ${}^{11}C_{5} - {}^{9}C_{3} \Rightarrow$ ✓

$\frac{5 \times \lfloor 4}{\lfloor 4 \times \lfloor 1}$

$5 \times \lfloor 4$

~~8) 444 ${}^{5}C_{4}$~~

8) R L

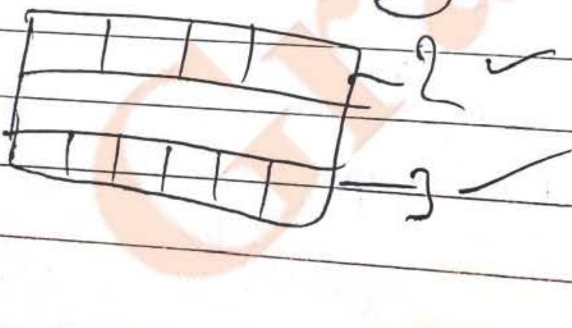
S S

$\Rightarrow {}^{5}C_{4} \cdot \cancel{{}^{5}C_{0}} + \cancel{{}^{5}C_{3}} \cdot {}^{5}C_{1} + \cancel{{}^{5}C_{2}} \cdot {}^{5}C_{2} + \cancel{{}^{5}C_{1} \cdot {}^{5}C_{3}} + {}^{5}C_{0}\,{}^{5}C_{4}$

$\Rightarrow {}^{5}C_{4} + {}^{5}C_{3} \cdot {}^{2}C_{1} + {}^{5}C_{2} \cdot {}^{3}C_{2} + {}^{5}C_{1} \cdot {}^{4}C_{3} + {}^{5}C_{0} \cdot {}^{5}C_{4}$

$\Rightarrow 80$ Ans

9)



$({}^{6}C_{3}\,\lfloor 3)({}^{4}C_{2}\,\lfloor 2)\lfloor 5$

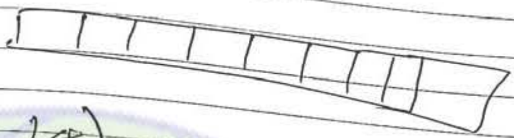
Ans

WWW.GRADESETTER.COM

WWW.GRADESETTER.COM

10) AUROBIND

(A O I U)



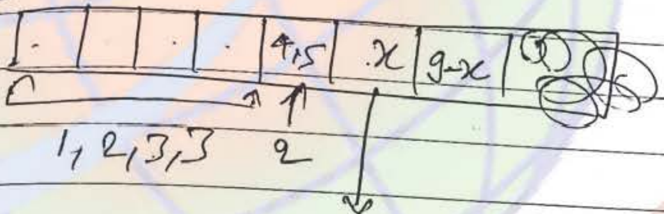
8

$\left({}^{8}C_{4}\,\underline{|4}\right)$

11) LL LC CC

${}^{9}C_{2}(1) + 9\cancel{(2)} + \cancel{{}^{9}C_{2}}\ \cancel{{}^{9}C_{2}}\ {}^{9}C_{1}\ {}^{9}C_{1}(2) + {}^{9}C_{2}(2)$

19)



(0,9),(1,8),(2,7),(3,6),(4,5)(5,4)
(6,3),(7,2),(8,1)(9,0)

$\Rightarrow \left(\frac{\underline{|4}}{\cancel{\underline{|2}}}\right)\cancel{\times 2}\times 10$

$\Rightarrow$ 240

WWW.GRADESETTER.COM