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"Heat"

Q) ⟶ It is the form of energy Intransitic during the flow from high temp. body to low temp. body.

$$\boxed{W = J\,Q}$$

W → In joule
J → Conversion factor
Q → In calorie

$$\boxed{\begin{aligned}1\ cal &= 4.18\ J\\ 1\ cal &\approx 4.2\ J\end{aligned}}$$

IIT 2005 *

1 calorie: →
Definition :→ Amount of heat energy required to raise the temp. of 1 g of water (H_2O) through 1°C (14.5°C to 15.5°C) at 760 mm Hg.

$$\boxed{1°C\ (14.5°C \text{ to } 15.5°C) \text{ at } 760\ mmHg}$$

↳ Experimental find value.

⟹ specific heat of water = 1 cal/gm °C = $\frac{1\ cal}{g\ °C}$

Specific heat capacity or Specific Heat.

"(C)" or "S"

1 g + 1°c

$$\boxed{C \text{ or } S = \frac{Q}{m\Delta T}}$$

$m = 1g$, $\Delta T = 1°c$

$(C = Q)$

$$\boxed{Q = mC(\Delta T)}$$ → Tem change

→ c is constant.

or

$\boxed{Q = mS\Delta T}$

$\boxed{Q = mS\Delta T}$

$\boxed{Q = nC\Delta T}$ → No. of moles

Unit of 'c'

Unit of "c"

$$\frac{J}{kg \cdot K} \text{ or } \frac{cal}{g\,c°} \text{ or } \frac{K\,cal}{kg\,°c} \text{ or } \frac{J}{kg\,°c}$$

S.I Unit

$\boxed{Q = nC_m\Delta T}$

↓ molar heat capacity

* Specific heat capacity (c) ⇒

Amount of heat energy required to raise the temp. of 1.0g mass of sub. through 1°c

Specific heat of substance mainly depends on nature of material and state of substance. It very slightly depends on temperature

* If 'c' varies with Temp. 'T'

$$\boxed{Q = m\int_{T_i}^{T_f} c\,dT \Rightarrow \text{If } c = f(T)}$$

Thermal capacity (T_h)

↳ "m g" का $1°C$ से बढ़ाना

$$\boxed{T_h = mC} \quad \Delta T = 1°C$$

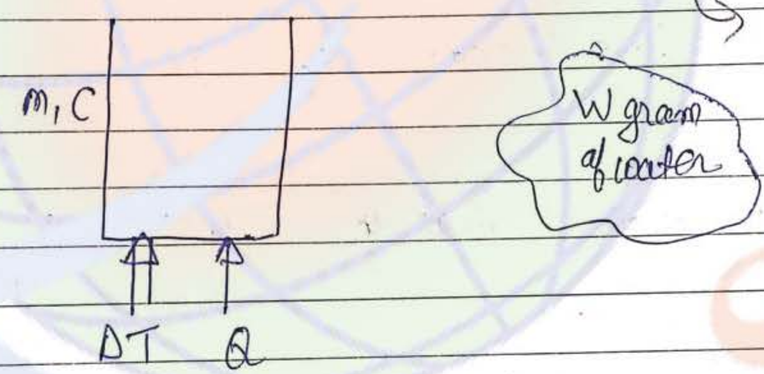
Amount of heat required to raise the temperature of m g of body through $1°C$.

$$\boxed{T_h = ms = mc = \frac{Q}{\Delta T} = W}$$

unit of (T_h) ⇒ $\frac{\text{Joule}}{°C}$ or $\frac{\text{cal}}{°C}$

Water equivalent (W) ⇒

↳ vessel (copper) का ही है।



$$Q = mC\Delta T = Q = W \times C_w \times \Delta T$$

$$mC\cancel{(\Delta T)} = W \times 1 \times \cancel{(\Delta T)}$$

$$\boxed{T_h = mC = W}$$

unit → J/K or JK^{-1}

Definition:→

Thermal capacity of a body of mass 'm' and specific heat 'c' is expressed in terms of mass of water (H_2O) i.e. → The mass of water having the same heat capacity as a given body is called the water equivalent of the body.

Latent heat (L) — phase chang

Amount of heat required to change the state of 1°g of substance at "constant temperature."

$$\boxed{Q = mL}$$

where m ⇒ mass of molten form or vaporised form or steam for.

$$\boxed{\text{Unit of "L"} \rightarrow \frac{cal}{g} \text{ or } \frac{J}{kg}}$$

$L_f \rightarrow$ Latent heat of fusion

$L_v \rightarrow$ Latent heat of vaporization

* $C_{ice} = 0.5 \frac{cal}{g°C}$: (Specific heat of ice → $0.5 \frac{cal}{g°C}$

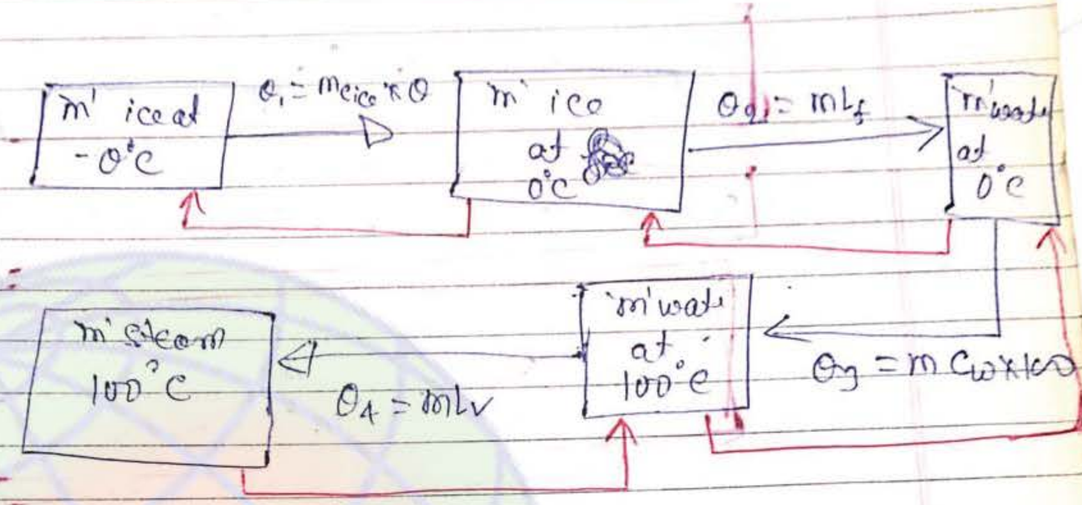
water $C_w = 1 \frac{cal}{g°C}$

$L_{ice} = 80 \frac{cal}{g}$ (latent heat of Ice → $80 \frac{cal}{g}$).

$L_{steam} = 536 \frac{cal}{g}$

$\approx 540 \frac{cal}{g}$

Example :->

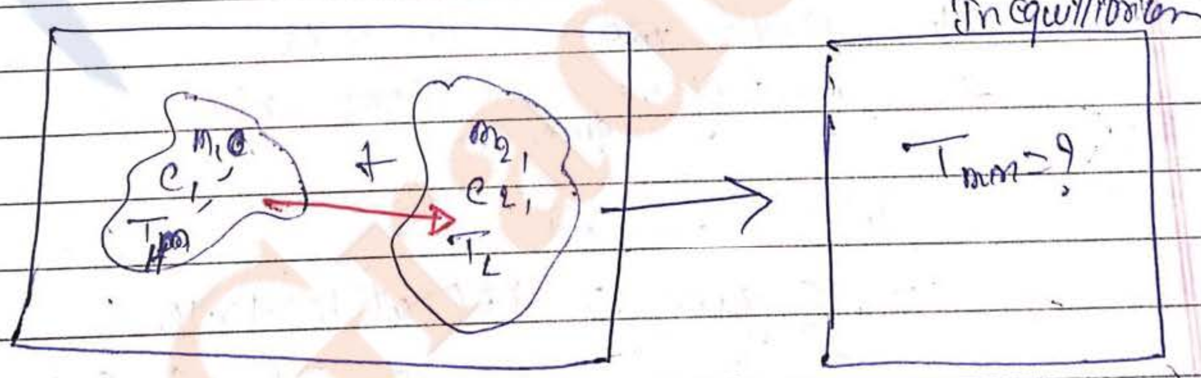


* Principle of Calorimetry :->

It is based on principal of conservation of energy, i.e. According to Principal of calorimetry ⇒ The total heat given by the hot objects equals the total heat received by the cold objects.

$$\boxed{\text{Heat given} = \text{Heat taken}}$$

In equillibrium



$$\left(T_L \leq T_{mix} \leq T_H \right)$$

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$$\boxed{Q_{given} = m_1 C_1 (T_{H} - T_{mix})}$$

$$\boxed{Q_{taken} = m_2 C_2 (T_{mix} - T_2)}$$

$$\boxed{T_{mix} = \frac{m_1 C_1 T_{H} + m_2 C_2 T_2}{m_1 C_1 + m_2 C_2}}$$

IIT screening.

Example:> 100 g of Ice at 0°C is mixed with 10 g of water at 10°C in a thermally Insulated vessel. Find the final temperature of mixture and find the contents of Ice and water in the mixture in equillibrium state

Ans:>

100 g ice at 0°C

For complete melting energy required $= 100 \times 80 = 8000$ cal (∵ phase change)

10°C ⟶ 0°C

$Q_{given} = 10 \times 1 \times 10$
$= 100$ cal $< Q$ required

So,

$$100 = m_{ice} \times 80$$

$$m_{ice} = \frac{10}{8} = \frac{5}{4} = \frac{5}{4} g$$

In mixture

Left Ice $\rightarrow \left(100-\frac{5}{4}\right)g$

water $\rightarrow \left(10+\frac{5}{4}\right)g$

$T_{mix} = 0^\circ C$

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Graph

$\boxed{2kg \text{ Ice at } -20^\circ C + 5 kg \text{ water at } 25^\circ}$

2kg Ice at $-20^\circ c$ mixed with 5kg water at $25^\circ C$ In a thermally Insulated vessel of negligible heat capacity. Find the final temp. of the mixture and content of water in the mixture.

$L_F = 80 \frac{kcal}{kg}$

$C_{ice} = 0.5 \frac{kcal}{kg^\circ c}$

$C_w = \frac{1 kcal}{kg^\circ C}$

Ans $-20^\circ \longrightarrow 0^\circ c$ (ice)

$Q = mc\Delta T$

$= 2 \times 0.5 \times 20$

$= 20$ Kcal

$0^\circ C$ ice $\longrightarrow 0^\circ$ water

$Q_2 = 2 \times 80 = 160$ K cal

1st steap & First find Heat taken or Heat given.

$20^\circ C \longrightarrow 0^\circ C$

$Q_{given} = 5 \times 1 \times 20$
$= 100$ Kcal

$T_{mix} = 0^\circ C$

$m_{water} = 6$ kg &.
(In mixture)

Example: Find the amount of steam (at 100°C) required to raise the temp. of 100g of water (H_2O) 24°C to 90°C

Ans) From 24°C to 90°C

$L_v = 540$ cal/g (Steam)

Heat taken by the water, $Q_1 = 100 \times 1 \times 66$
$= 6600$ cal —(1)

Heat given by Steam at 100°C (m gram)

$$Q_2 = \underbrace{mL_v}_{100^\circ C\ water} + \underbrace{m \times 1 \times 10}_{water\ at\ 90^\circ}$$

$= 540m + 10m$
$= 550m$ —(2)

From eq (1) and eq (2)
Heat taken = Heat given
$55\cancel{0}m = 660\cancel{0}$
$m = \frac{660}{55} = 12$ g

[H.CV - XII]

HT(1) (ण)

A calimeter of water equivalent 15 g contents 165 g of water at 25°C steam at 100°C is passed through calorimeter for some time. The final temperature of mixture is found to be 70°C and mass of the mixture is increased by 1.5 g

Find the latent heat of vaporisation (L_v):

Ans:⇒ Heat taken:→

$$Q_1 = \underbrace{m\,c \times S}_{\text{vassel}} + \underbrace{m_w \times C_w \times S}_{\text{water}}$$

∴ we take common "S"

$$\Rightarrow \cancel{S\,(m_c + m_w c_w)}$$

$$= (165 + 15) \times 1 \times 5$$

$$= 900\ \text{cal} \quad \text{--- (1)}$$

Heat given

$$Q_2 = \underbrace{1.5 \times L_v}_{100^\circ C} + \underbrace{1.5 \times 1 \times 70}_{\text{water}}$$

$$\Rightarrow 1.5\omega + 105$$

Now Heat taken = Heat given.

$$900 = 1.5\omega + 105$$

$$\omega = 530\ \frac{\text{cal}}{g}$$

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Example V.VI A lead bullet initially at temp. of 27°C and just completely penetrate into an obstacle melts. (~~just melt~~)

50% of mechanical energy is huge d in heating the bullet

as melting point of lead is 327°C

or specific heat of lead (c) is $0.03 \frac{cal}{g°C}$

Latent heat of fusion of lead (L_F) = $\frac{6 cal}{g}$

Find the speed of the bullet

m.p.t of lead = 327°C

$C_{Lead} = 0.03 \frac{cal}{g°C}$

$L_{F_{Lead}} = 6 \frac{cal}{g}$

Ans.) Here, kinetic energy is present in the form of heat energy.

$$\frac{1}{2}\left(\frac{1}{2}\cancel{m}v^2\right) = \cancel{m}C_{Lead} \times 300 + \cancel{m}L_F$$

$$\frac{v^2}{4} = \frac{0.03 \times 4.2 \times 10^3 J \times 300}{kg°C} + \frac{6 \times 4.2 \times 10^3 J}{kg}$$

$v = \boxed{}$ m/s

Example: 1 kg of Ice at 0°C is mixed with 1 kg of steam 100°C in a thermally insulated vessel of negligible heat capacity.
Find the final temperature of the mixture in thermal equilibrium and contents of mixture in the thermal equillibrium.

$L_f(\text{Ice}) = 3.36 \times 10^5 \frac{J}{kg}$

Ans:

$L_v = 2.26 \times 10^6 \frac{J}{kg}$

Ans:⇒

1 kg steam 100°C ⟶ water 100°C

$Q_{given} = 22.6 \times 10^5 J$

0°C water ⟶ 100°C water

$Q_w = 1 \times 4200 \times 100$
$= 4.2 \times 10^5 J$

+ 0°C Ice ⟶ 0°C

$Q_1 = 1 \times 3.36 \times 10^5 J$

$Q_1 + Q_w = (4.2 + 3.36) \times 10^5 J$

$= 7.56 \times 10^5 J$

Heat taken = Heat given

$7.56 \times 10^5 = m_{steam} \times 22.6 \times 10^5$

$m_{steam} = \frac{7.56}{22.6} = 0.335$ kg of

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In final mixture amount of water ⇒ 1 + 0.335 kg
⇒ 1.335 kg

Amount of steam = 1 − 0.335
= 0.665 kg.

Exa) A geaser of 2kW is used to heat the 2 kg of liquid from 10°C to 30°C. For a time 5'58".

Find the specific heat of liquid. If the 10% of electric energy is lost to the surrounding.

Ans

10°C ⟶ 30°C

$Q_{taken\ by\ liquid} = 2 \times C_l \times 20 J$

$\frac{90}{100}(2000 \times 5) J = 2 \times C_l \times 20 J$

$C_l = 225 \frac{J}{kg°C}$

Examples) A metalic cubical block of mass '2kg' is heated to 400°C. Now this metalic cube is placed on a large Ice block at 0°C.
Find the maximum amount of molten Ice.

$C_{metal} = 0.09 J/kg°C$

Ans ⇒

$Q_{given} = 2 \times 0.09 \times 400 J$

$C_F = 80 \frac{cal}{g}$

$Q_{taken} = M_{ice} \times 80 \times 4.2 \times 10^3 J$

$Q_{given} = Q_{taken}$

$0.18 \times 400 = m_{ice} \times 80 \times 4.2 \times 10^{3}$

$m_{ice} = \frac{0.18 \times 4}{42 \times 80}$ kg

$= \frac{0.18 \times 4 \times 1000}{42 \times 80}$ g

Question (concept ! → $\boxed{\text{kg} \cdot \text{m/s} \Rightarrow \text{Joule is called}}$

and

$\boxed{1 \text{ Cal} = 4.186 \text{ J}}$

eg:- what is the kinetic energy of a 10kg mass moving at a speed of 36 km/h^{-1} in calorie

Sol^n → The kinetic energy is $\Rightarrow \frac{1}{2} mv^2$

where $m = 10$ kg

$v \Rightarrow 36 \frac{km}{h} \times \frac{5}{18}$ (∵ convert m/s

$\Rightarrow 10$ m/s

Now

$K.E = \frac{1}{2} \times 10 \times (10)^2$

$= 500$ J

Now

$500 \text{J} = \frac{500}{4.186}$ Cal $\simeq 120$ Cal.

1st Choice

Mechanical equivalent of Heat :> (J)

Amount of mechanical work is needed to raise the temperature of 1g of water by 1°C.

$$\boxed{W = J H} \quad \text{or} \quad \boxed{W = J Q}$$

where :>
J > is called mechanical equivalent of heat

W > mechanical work (work done)
H > Heat

Note: >
i) If W and H are both measured in the same unit then J = 1.

ii) If W is measured in joule (work done by a force of 1N in displacing an object by 1 m in its direction) and
H in calorie (heat required to raise the temperature of 1g of water by 1°C) then 'J' is expressed in Joule per calorie.

Start

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Kinetic theory of gas (K.T.G.)

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Assumptions :⇒

1) The Force of Interaction b/w the gas molecules is negligible.

2.) The collision b/w the molecules and collision b/w the molecules and wall of container is perfectly elastic.

3.) Effect of gravity is negligible.

4.) Gas molecules follow the "newton's law of motion".

5.) ~~Gas~~ Contact time b/w wall of container with molecule is negligible.

Pressure exerted by gas :→

$$\boxed{P = \frac{1}{3}\rho\, v_{rms}^2}$$

where ρ → Density of gas

$$\rho = \frac{mN}{V}$$

N → Total no. of molecules

m → mass of a single molecule.

N → Total no. of molecule of gas.

V_{rms} → root mean square speed.

$V_{avg} = \langle v \rangle = \bar{v}$ → read as v-bar

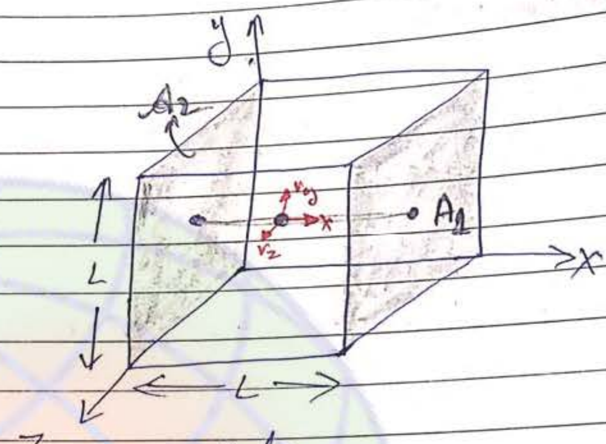
root of mean of square of speed

$V_{rms} = \sqrt{\overline{v^2}}$ → mean

$\overline{v^2} = V_{rms}^2$

mean square speed

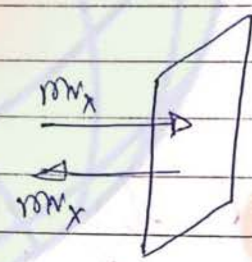
square root



cube.

$$V = L^3$$

$$\Delta t = \frac{2L}{v_x}$$



$$\Delta p = 2mv_x$$

$$F = \frac{\Delta p}{\Delta t} = \frac{\cancel{2}mv_x^2}{\cancel{2}L}$$

$$F_1 = \frac{mv_x^2}{L}$$

$$F = \frac{m}{L}\sum v_x^2$$

$$P = \frac{m}{L^3}\sum v_x^2$$

$$P = \frac{m}{V} N \frac{\sum v_x^2}{N}$$

$$\boxed{P = \left(\frac{mN}{V}\right) \overline{v_x^2}}$$ → सिर्फ एक direction में है

$$\boxed{P = \frac{1}{3} \rho \overline{v^2}}$$ → In case of may be three direction.

rms

$$\boxed{P = \frac{1}{3} \rho v_{rms}^2}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$\sum v^2 = \sum v_x^2 + \sum v_y^2 + \sum v_z^2$$

$$\boxed{v_{rms} = \sqrt{\frac{3P}{\rho}}}$$

$$\boxed{v_{rms} = \sqrt{\frac{3RT}{M}}}$$

where → M is molecular weight

$$\boxed{M = m N_A}$$

N_A → Avagadro No.

1.) ~~Average speed (V_{avg})~~

Average speed (V_{avg})

$$\Rightarrow \sqrt{\frac{8RT}{\pi m}}$$

$$= \sqrt{\frac{8k \cdot T}{\pi m}}$$

$k \rightarrow$ Boltzmann's constant

2.) **Most probable speed:**

$$V_{mps} = \sqrt{\frac{2RT}{m}} = \sqrt{\frac{2kT}{m}}$$

Eg.) Find the r.m.s of O_2 molecule of O_2 gas at 27°C

Ans.)

$V_{rms} = \sqrt{\frac{3RT}{m}}$

$V_{rms} = \sqrt{\frac{3 \times 300 \times 25}{10^{-3} \times 32}}$

$\Rightarrow \sqrt{18.7 \times 10^{4}}$

$\Rightarrow 500$ m/s

27 + 273 = 300

$R = \frac{25}{3}$ J/K

$$V_{rms} = \sqrt{\frac{3 \times \frac{25}{3} \times 300}{32 \times 10^{-3}}} \approx 500 \text{ m/s}$$

(attention please be careful.)

11 Tough

Example. D

$T\,K$: O_2 → $V_{rms} = V$

$T\,K$: O → $V_{rms} = ?$

$V_{rms} \Rightarrow \sqrt{\dfrac{3KT}{m}}$

for O $= \sqrt{2}\,V$

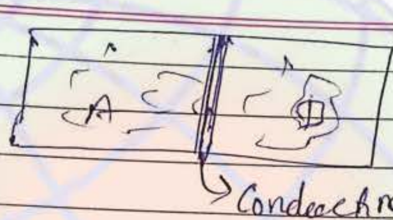
Example 2)

A: O_2 gas, $T\,K$ → $V_{rms} = V_1$

B: N_2 gas, $T\,K$ → $V_{rms_{N_2}} = V_2$

C: $(O_2 + N_2)$, $T\,K$ → $V_{rms_{O_2}} = ?$ $\quad \sqrt{\dfrac{3RT}{M_{O_2}}} = V_1$

Q.1)



Diff. Ideal gas are filled in chamber 'A' and 'B'.

rms speed of molecules of gas 'A' = Avg. of speed of molecules of gas 'B'

Find the ratio of mass of molecule of 'A' and molecule of gas B.

$$\frac{m_A}{m_B} = ?$$

$$\sqrt{\frac{3kT}{m_A}} = \sqrt{\frac{8kT}{\pi m_B}}$$

$$\frac{m_A}{m_B} = \frac{3\pi}{8}$$

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Degree of freedom (f)

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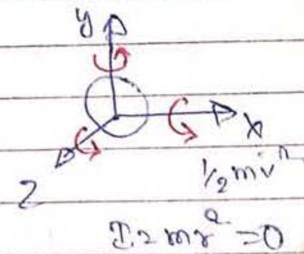
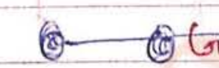
It is the no of Independent ways in which the system can have energy

1) ⇒ For monoatomic molecule :→



$$\boxed{f = 3_{\text{translational}}}$$

2) ⇒ For Diatomic molecule :→

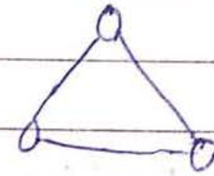


$$f = 3_{\text{translation}} + 2_{\text{rot}}$$

$$\boxed{f = 5}$$ (If the vibration is neglected.

× $$\boxed{f = 7}$$ (If the vibration is considered

3.) ⇒ For triatomic molecules :→ (Non-linear.)



$$f = 3_{\text{tran}} + 3_{\text{rot}}$$

$$\boxed{f = 6}$$

1st Choice

K.E. of Gas (K)

→ kinetic energy

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$$K = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \text{-----} + \frac{1}{2}mv_N^2$$

$$K = \frac{1}{2}mN\frac{(v_1^2 + v_2^2 + \text{-----} v_N^2)}{N}$$

$$\boxed{K = \frac{1}{2}(mN)v_{rms}^2}$$ → kinetic equation

Kinetic energy of a single molecule of gas →

Average transiational K.E. of a single molecule of gas → $\boxed{K_1 = \frac{1}{2}mv_{rms}^2}$

$$K_1 = \frac{1}{2}\cancel{m}\frac{3kT}{\cancel{m}}$$

$$\boxed{K_1 = \frac{3}{2}kT}$$ → Rotation is not Included only trational E.K.

Note :→ Avg. traiational K.E. of a single molecule of gas "only depends on absolute temperature of gas." and "Independent of nature of gas".

Imp

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Kinetic energy of gas per unit ~~vols~~ volume(V) of a gas: →

$$E = \frac{K}{V}$$

$$E = \frac{1}{2} \frac{mN}{V} \cdot V_{rms}^2$$

$$E = \frac{1}{2} \rho V_{rms}^2 \quad \text{--- (1)}$$

$$P = \frac{1}{3} \rho V_{rms}^2 \quad \text{--- (2)}$$

$$\boxed{P = \frac{2}{3} E}$$

Law of Equipartition of energy :⇒

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[Average K.E. of a single molecule of a gas is equally distributed among its various degrees of freedom.]

Average K.E. of corresponding to each degree of freedom is

$$\boxed{= \frac{1}{2} kT}$$

Note :⇒
Internal energy of 'n' moles of a "gas" (In which a molecule has 'f' degrees of freedom)

$$U_{in} = \frac{1}{2} f k N_A RT \quad \Big| \quad R = k N_A$$

$$\boxed{U_{in} = \frac{1}{2} f \cdot nRT}$$

⇒ For the given gas :-

$$(f, n) \rightarrow \text{Constant}$$

$$U_{in} \propto T$$

$$\Delta U_{in} = n \frac{fR}{2} (\Delta T)$$

$$\Delta U_{in} = n C_V (\Delta T)$$

$$\boxed{C_V = \frac{fR}{2}}$$

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1st Choice

Thermodynamics

* 1st law of Thermodynamics! ⇒

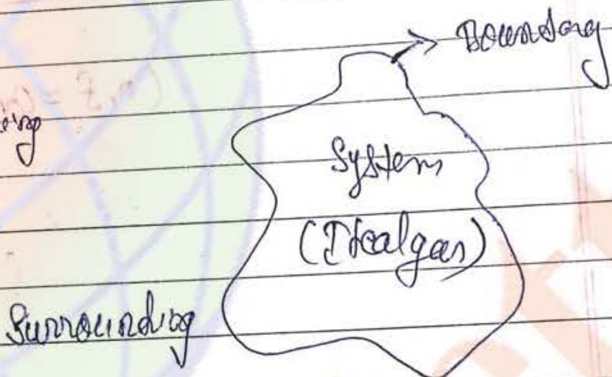
↳ 1) It is based on principle of conservation of energy.

$$\boxed{Q = \Delta U + W}$$

सिर्फ यह chem. से उल्टा होगा

where :⇒

Q ⇒ Heat exchange between system (gas) and surrounding



Note:⇒

Heat वाला chemistry से Same है सिर्फ work done वाला उल्टा है।

i) $Q = +ive$ ⇒ Heat is absorbed by the gas or Heat is taken by the gas

ii) $Q = -ive$ ⇒ Heat is released / given by the gas.

"Q ⇒ Path dependent"

⇒ $\Delta U = (U_f - U_i)$

↳ change in Internal Energy of gas.

f : degree of freedom

we know that
$$U = \frac{1}{2} f n R T$$

$$\Delta U = \left(\frac{f n R}{2}\right)(\Delta T)$$

$$\boxed{\Delta U = n C_V \Delta T}$$ where $\left(C_V = \frac{fR}{2}\right)$

→ For the given gas. (n, f = constant) → V.V.I

→ It is true for all type of process (Adiabatic, Polytropic, etc.)

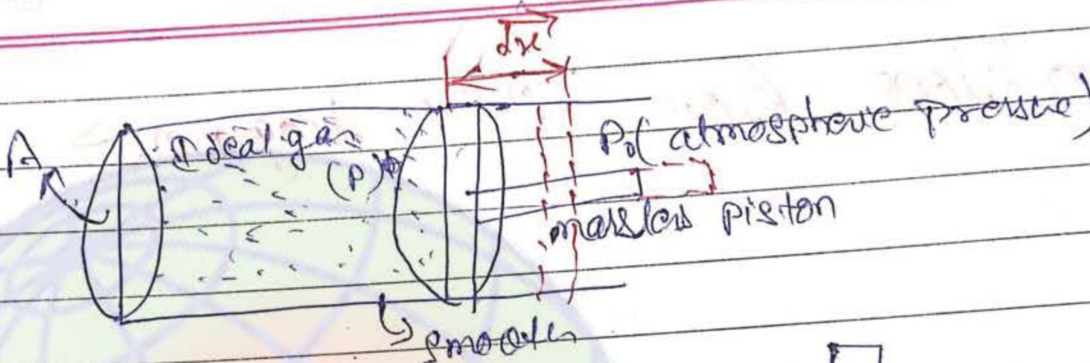
* change in Internal energy of the gas for the given Initial and final state of gas is Independent of Path. It depends on Initial and final position only.

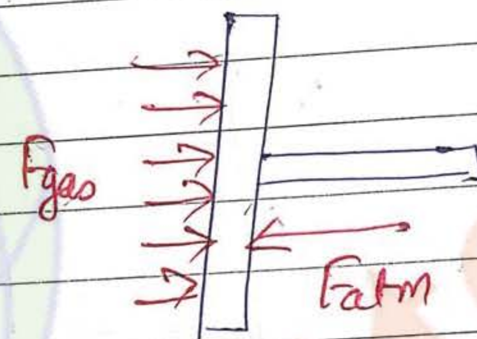
* workdone ⇒

1) W = +ive ⇒ work is done by the gas. (In expansion Process)

11) W = −ive ⇒ work is done on the gas. (In compression Process)

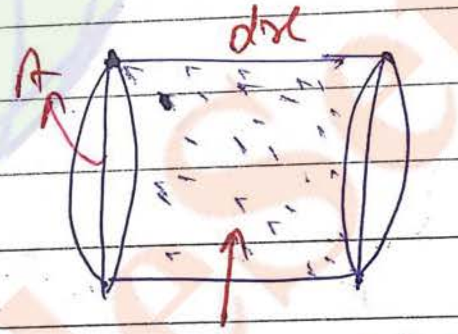
→ It is Path dependent.





$$W = \int dW = \int F_{gas}\, dx$$

$$= \int PA\, dx$$



$dV = A dx$

$$\boxed{W = \int_{V_i}^{V_f} P\, dV}$$

→ Area bounded by P-V graph on volume axis.

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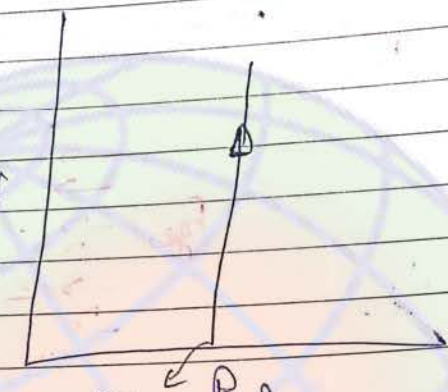
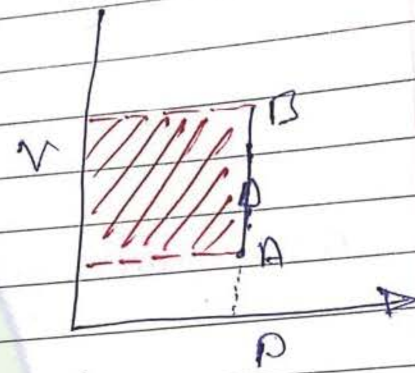
* Workdone from P–V diagram:→

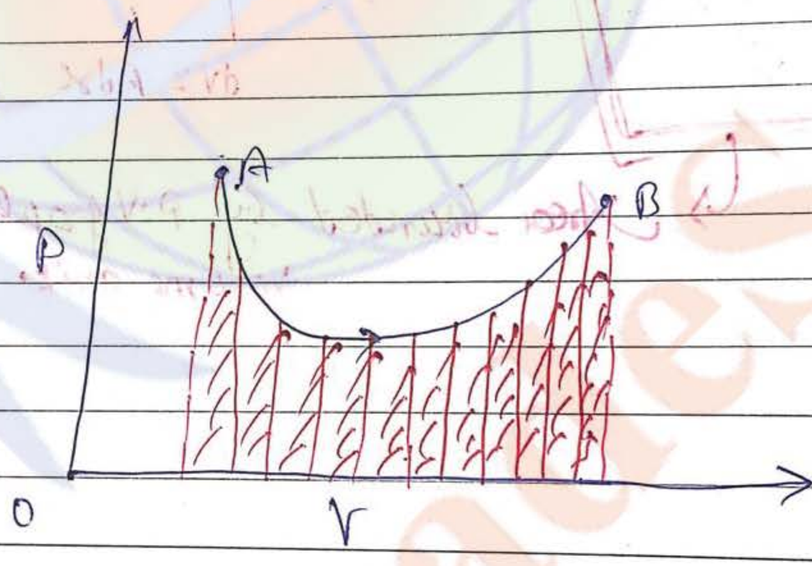
eg:→





इतना नीचे से,
कम Possible है इसलिए
यह लिखना सम्भवतः
गलत हो सकता है।

eg 2.)



During of A to B workdone by the gas.

$W_{A\to B} \Rightarrow$ Continuously Increase.

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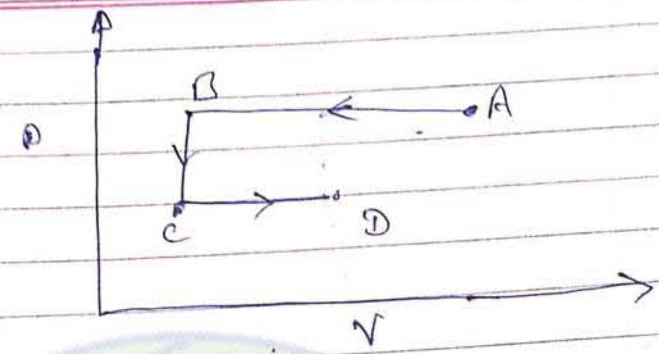
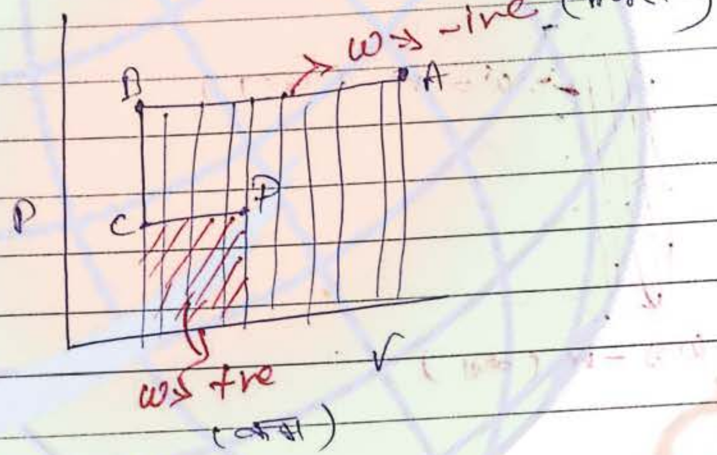
eg 7.)

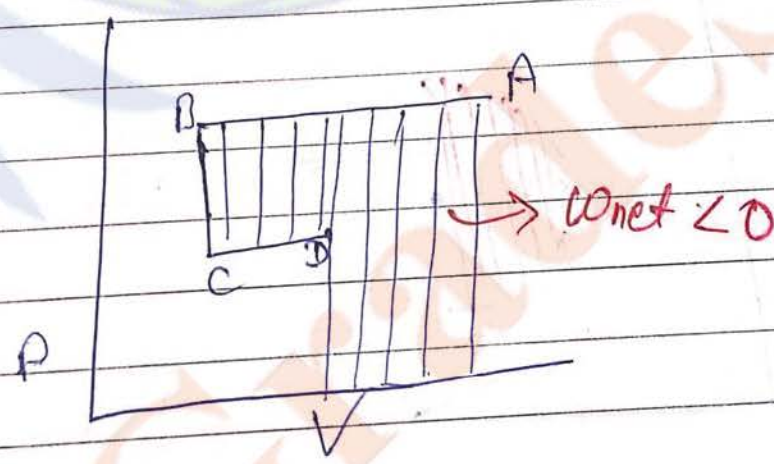


Net 'w' by gas during this process.

Ans)



So,



Note! → ~~The workdone~~ +ve and (−ive) sign of workdone is determined by 'V'

$V \uparrow \rightarrow$ workdone +ve

$V \downarrow \rightarrow$ workdone −ve

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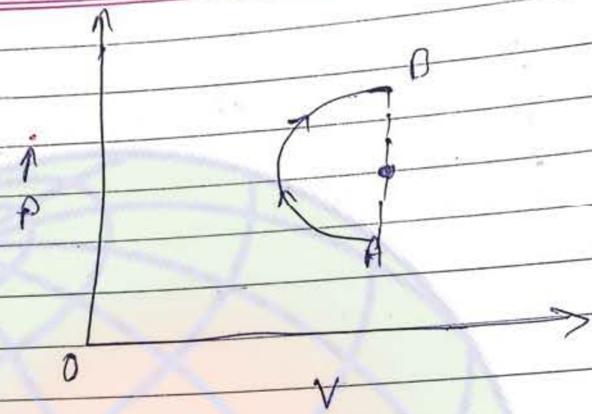
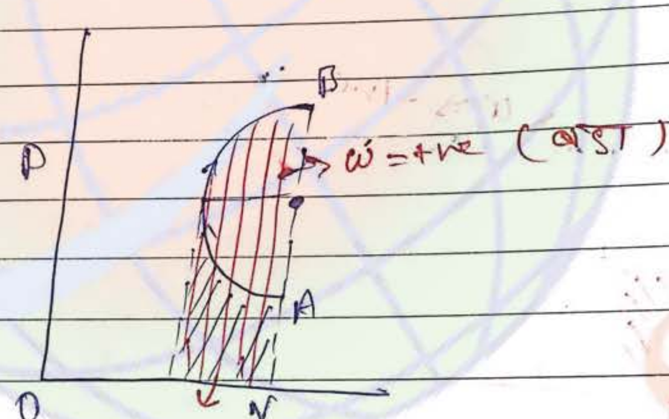
eg:)



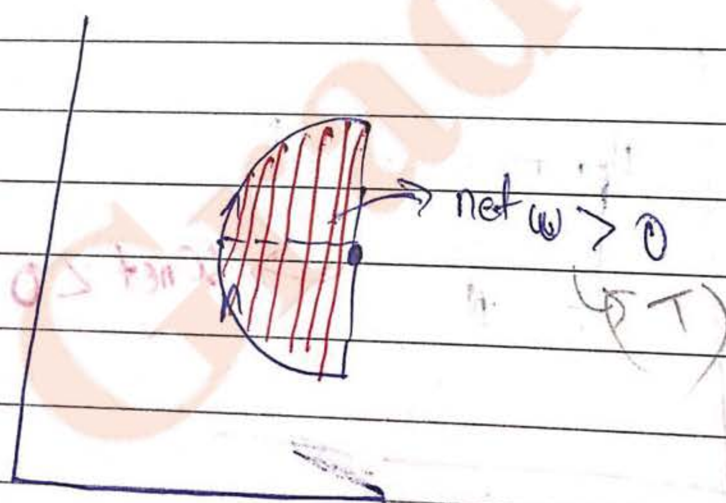
Find net workdone.

Ans:)



W = -ve (कम) है इसलिए घट जाएगा।

So,



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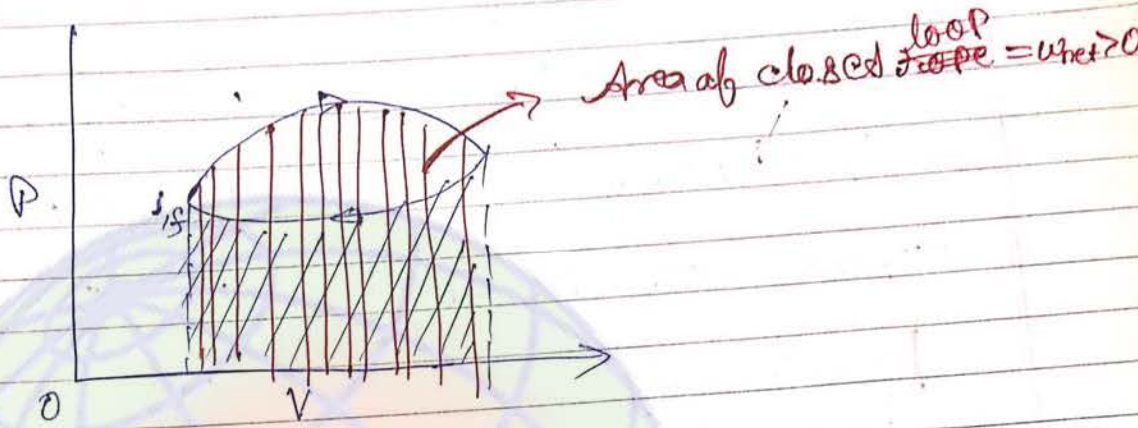
eg :->



Cv (molar Specific heat at constant volume)

$$C_V = \frac{Q_V}{n(\Delta T)}$$

$$\Delta U = \boxed{Q_V = n C_V (\Delta T)}$$

Amount of heat enchange at constant volume.

from first law

$Q_V = \Delta U + \cancel{W}$ 0

Cp (molar specific heat at constant pressure)

$$\boxed{Q_P = n C_P (\Delta T)}$$

~~Proof :-> At constant pressure~~ (P)

$Q_P =$

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Proof:
At constant (P)

$Q_p = \Delta U + W$

$\cancel{n} C_p \cancel{\Delta T} = \cancel{n} C_v \cancel{\Delta T} + \cancel{n} R \cancel{\Delta T}$

$\boxed{C_p - C_v = R}$

$W = P\int_{V_i}^{V_f} dV$

$W = P(\Delta V)$

$PV = nRT$

$P\Delta V = nR(\Delta T)$

$\gamma = \frac{C_p}{C_v}$

$\gamma = \frac{\cancel{R}\left(\frac{f}{2}+1\right)}{\frac{f\cancel{R}}{2}}$

$\boxed{C_v = \frac{fR}{2}}$

$C_p = \frac{fR}{2} + R$

$= R\left(\frac{f}{2}+1\right)$

$\boxed{\gamma = \left(1+\frac{2}{f}\right)}$

$\boxed{\gamma = \frac{C_p}{C_v} = \left(1+\frac{2}{f}\right)}$

$C_v = \frac{fR}{2}$

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1st Choice

☆ Different Process In thermodynamics :-

1.) Isobaric Process :⇒

$(P = \text{constant})$

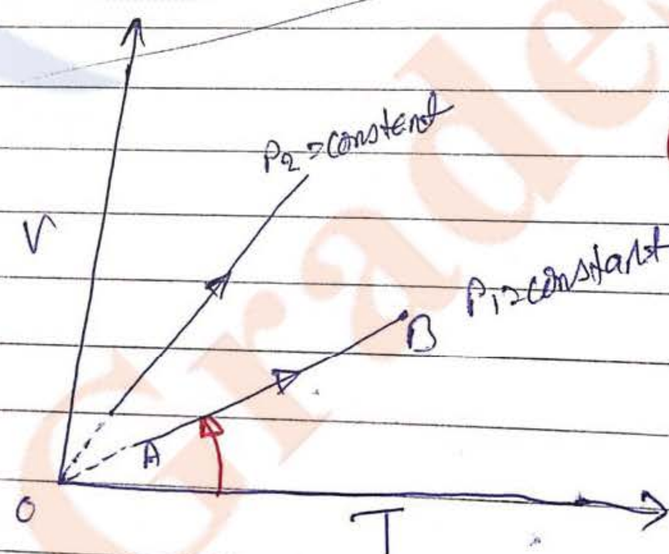
* $PV = nRT$

$V \propto T$

$\left(\frac{V}{T} = \text{constant}\right)$

* $V = \frac{nR}{P} T$

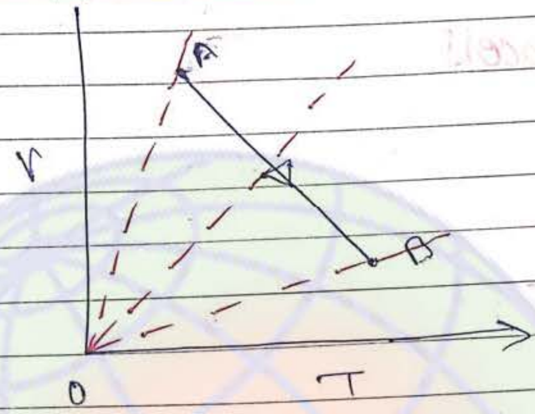
$\boxed{\text{Slope of } V-T \text{ graph} = \tan\theta \propto \frac{1}{P}}$



$(P_1 > P_2)$

(Iso bars always appears to through origin.)

eg:>



$(P_B > P_A)$

* workdone in Isobaric Process.

$$W = \int_{V_i}^{V_f} P\,dV \longrightarrow P = f(V)$$

$$\boxed{W = P(V_f - V_i)}$$

$PV_i = nRT_i$

$PV_f = nRT_f$

$$\boxed{W = nR(T_f - T_i)}$$

$$T_f - T_i = \Delta T$$

* $Q_p = \Delta U + W$

$$\boxed{Q_p = nC_p \Delta T}$$

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2.) Isochoric Process

($V = constant$)

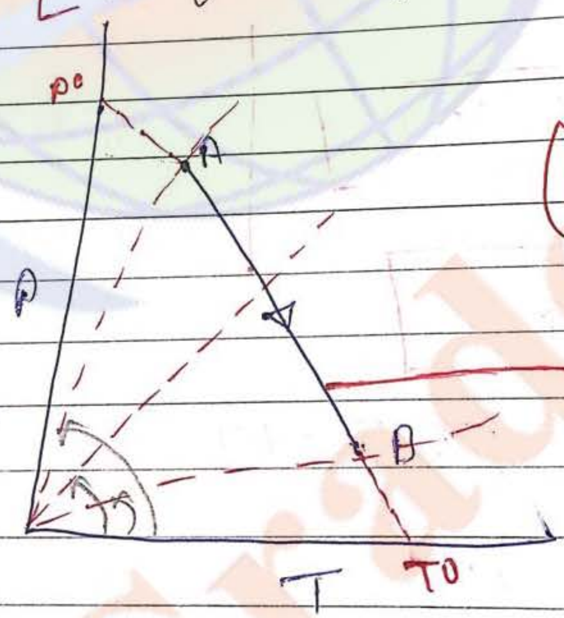
$PV = nRT$

$P \propto T$

$\frac{P}{T} = constant$

$\boxed{\text{Slope of } P\text{—}T \text{ graph} = \tan\theta \propto \frac{1}{V}}$

Atlest example eg! →



$(V_B > V_A)$

$P = -mT + P_0$

$P = -m\frac{PV}{nR} + P_0$

$PV = nRT$

$T = \frac{PV}{nR}$

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$\boxed{W = 0}$

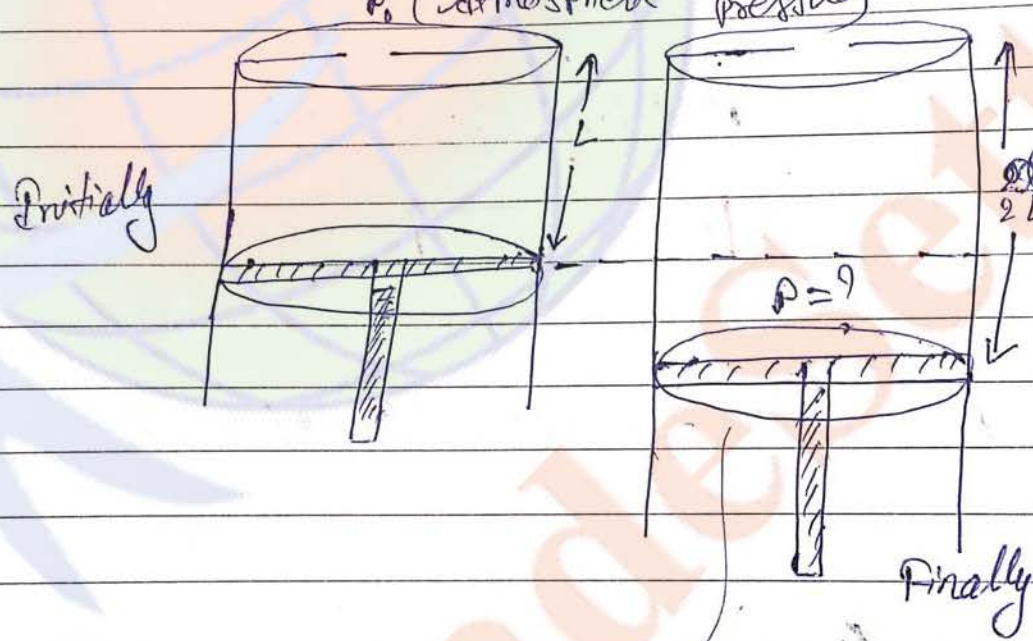
$\boxed{Q_v = \Delta U} = n C_v \Delta T$

$Q_v = +ive$, $\Delta U = +ive$

Temp → Increase.

eg :->

P₀ (Atmospheric pressure)



Ans $P = P^0$

(Pressure is Constant

1st Choice

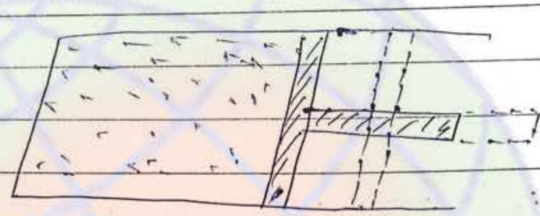
(Practical Isothermal)

3.) Isothermal Process :→

- It is slow process
- container is perfectly conduction

$(T = constant)$

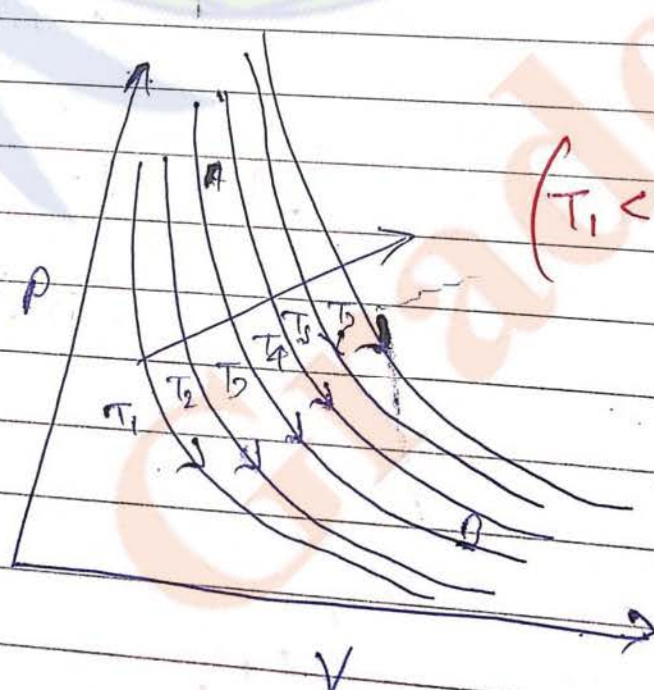
[illegible]



$PV = nRT = constant$

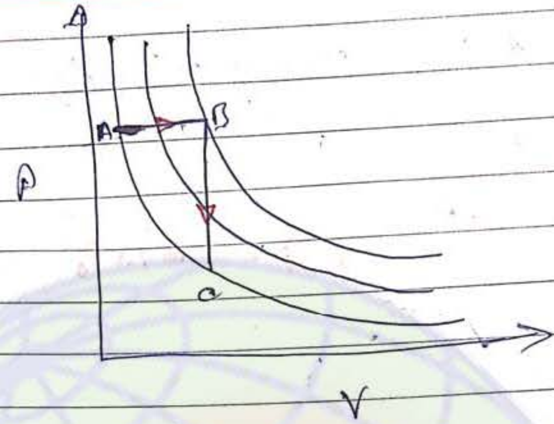
$\boxed{PV = constant}$

$P_1 V_1 = P_2 V_2 = \text{------} \quad constant$



$(T_1 < T_2 < T_3 < T_4 < T_5 < T_6)$

eg :>



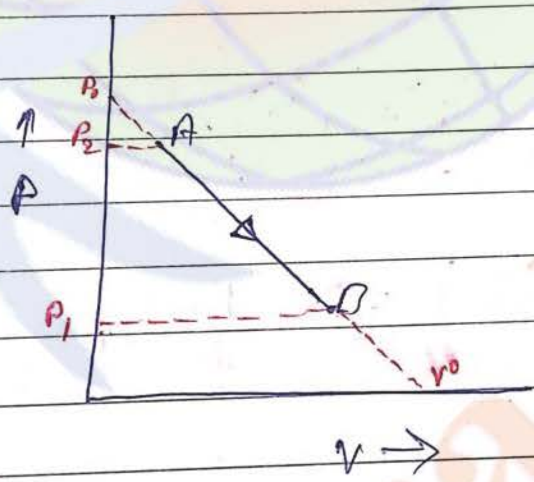
if A → B

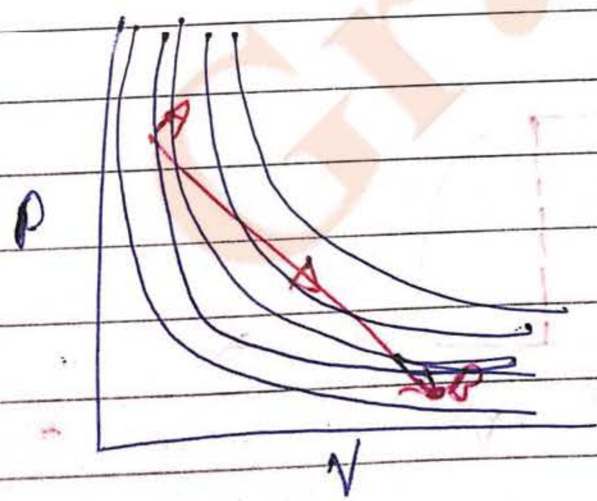
$\uparrow V \propto T \uparrow$

if B → C

$\downarrow P \propto T \downarrow$

eg :>



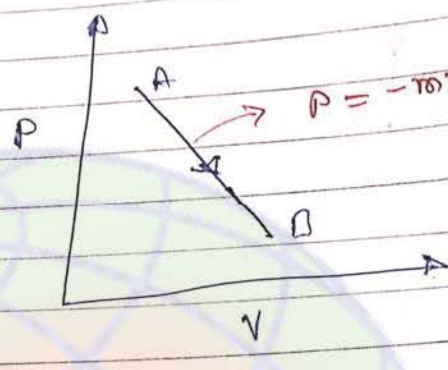


for A → B

↳ Temp. first increases, reach at its max. value and then decreases

And also draw T-V graph by using P-V graph.

$P = \frac{nRT}{V}$



So, $P = -mV + P_0$ | $P = \frac{nRT}{V}$

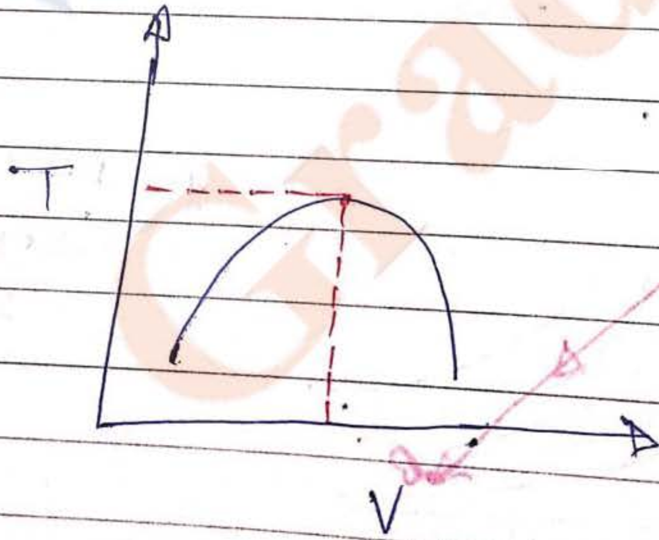
$$\frac{nRT}{V} = -mV + P_0$$

$$nRT = -mV^2 + P_0 V$$

Eqⁿ of Parabola

$$T = -\frac{m}{nR}V^2 + \frac{P_0}{nR}V$$

For Temp (T) To be max $\Rightarrow \frac{dT}{dV} = 0$



* workdone by gas in Isothermal process

$$W = \int_{V_i}^{V_f} P\,dV \qquad P = \frac{nRT}{V}$$

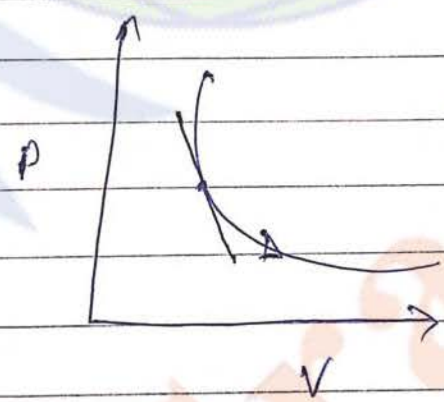
$$W = nRT \int_{V_i}^{V_f} \frac{dV}{V}$$

$$\boxed{W = nRT \ln\left(\frac{V_f}{V_i}\right)} \qquad P_iV_i = P_fV_f$$

$$\boxed{W = nRT \ln\left(\frac{P_i}{P_f}\right)}$$

$$Q = W \qquad (\because \Delta U = 0)$$

*



$$\boxed{\text{Slope of Isothermal graph} = \frac{dP}{dV}}$$

$PV = \text{constant}$

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$$P \frac{dV}{dV} + V \frac{dP}{dV} = 0$$

$$\boxed{\left(\frac{dP}{dV}\right)_{isothermal} = \frac{-P}{V}}$$

* Bulk modulus (B)

modulus of elasticity for Isothermal → $\boxed{B_{isothermal} = -V\left(\frac{dP}{dV}\right) = P}$

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4.) Adiabatic Process :→

→ Insulated system

$$\boxed{Q = 0}$$

$$\boxed{W = -\Delta U}$$

Allen to physics में इसका उल्टा है।

$\because \cancel{Q}^{\,0} = -\Delta U + W$

$$\boxed{W = -nC_v\Delta T}$$

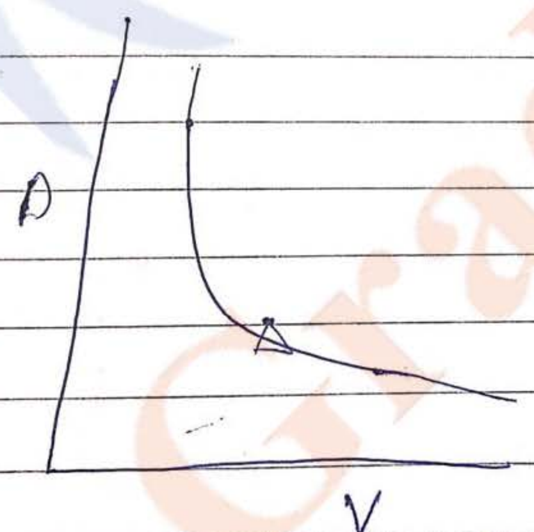
$$\boxed{W = \frac{nR}{(1-\gamma)}(\Delta T)}$$

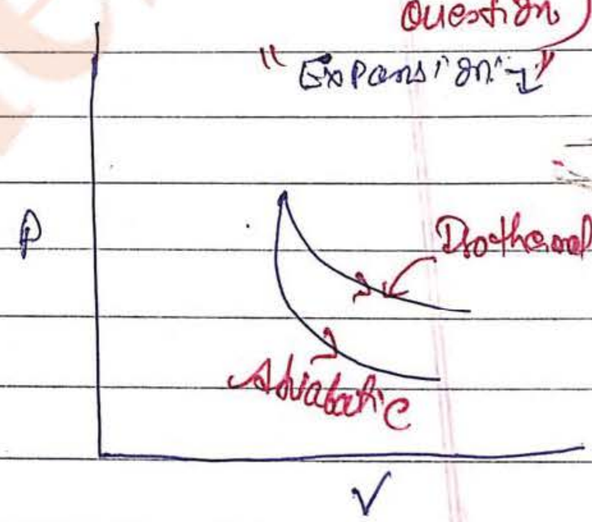
* Process equation :→

$\Rightarrow \quad PV^{\gamma} = \text{constant}$

$\Rightarrow \quad TV^{(\gamma-1)} = \text{Constant}$ → (IIT ask more and more question)

"Expansion" ↓





Slope of Adiabatic curve :⇒

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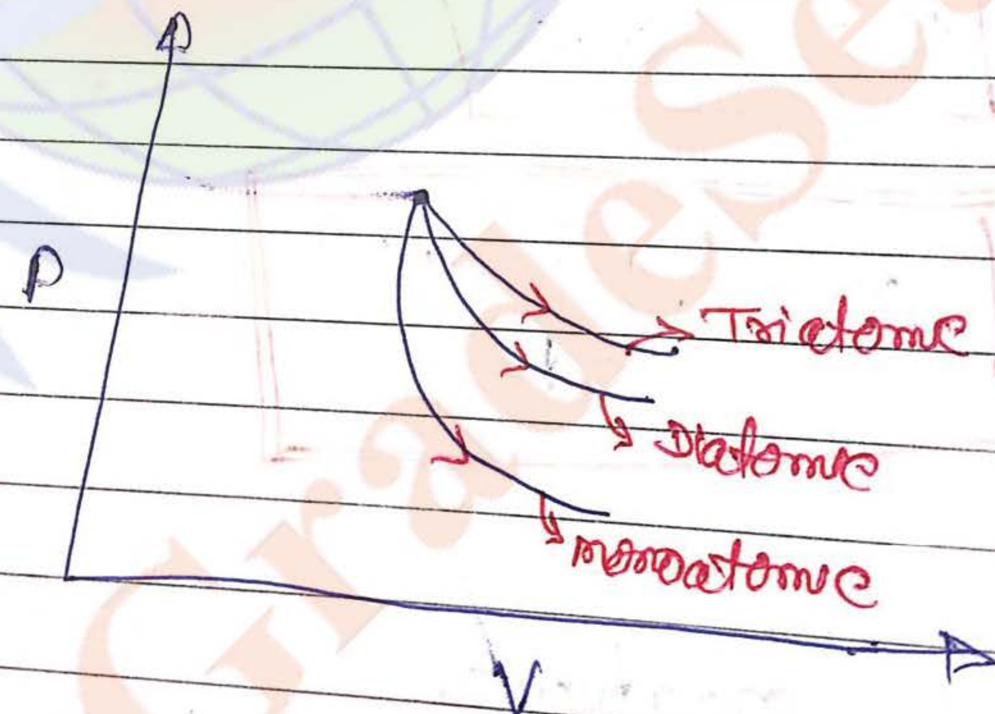
$$\boxed{\left(\frac{dP}{dV}\right)_{Adiabatic} = -\gamma\left(\frac{P}{V}\right)}$$

$$\Rightarrow \boxed{\left(\frac{dP}{dV}\right)_{Adiabatic} = \gamma\left(\frac{dP}{dV}\right)_{Isothermal}}$$



5.) Adiabatic elasticity :⇒

$$\boxed{B_{Adiabatic} = \gamma P}$$

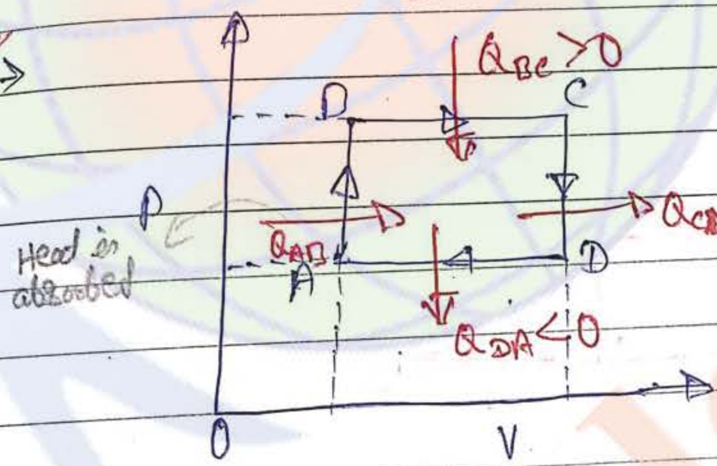
6.) Cyclic Process :⇒

For complete cycle,

$$\boxed{(\Delta U = 0)}$$

$$\boxed{(Q_{Net} = W_{Net})}$$

eg. :⇒



A → B	C → D	B → C	D → A
$W_{AB} = 0$	$W_{CD} = 0$	P ⇒ constant	↓ $V \propto T$ ↓
↑ $P \propto T$ ↑	↓ $P \propto T$ ↓	↑ $V \propto T$ ↑	$\Delta U < 0$
$\Delta U_{AB} > 0$	$\Delta U < 0$	$\Delta U > 0$	$W_{DA} < 0$
$Q_{AB} > 0$	$Q_{CD} < 0$	$Q_{BC} > 0$	$Q_{DA} < 0$
		$W_{BC} > 0$	

$Q_{net} = W_{net} > 0$

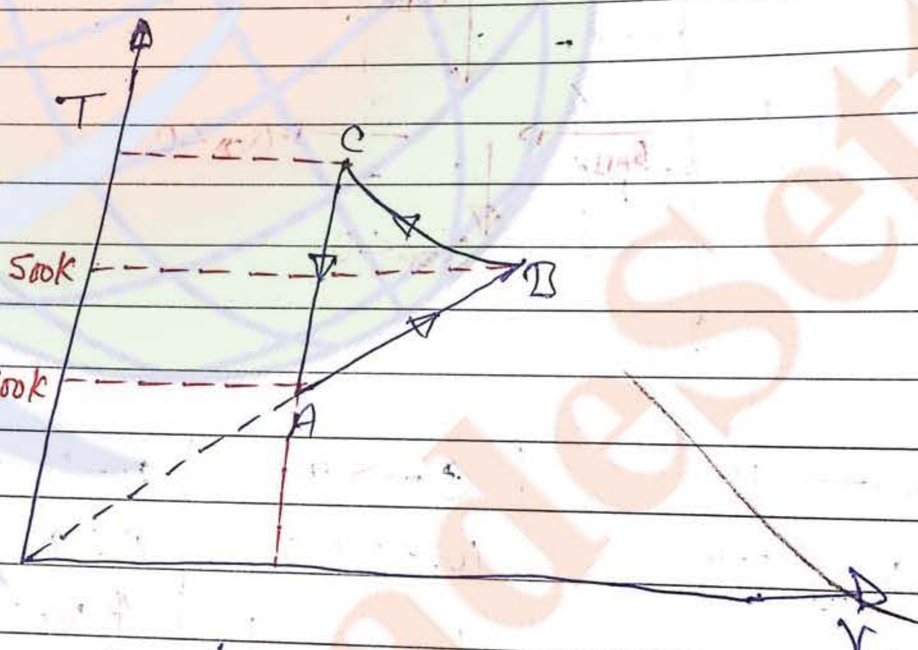
* Efficiency of cyclic Process :→ (η)

$$\eta = \frac{W_{net}}{Q_{+ive}}$$ → only absorbed heat

$$\boxed{\eta = \frac{W_{net}}{Q_{+ive}} = \frac{\text{Net work done}}{\text{Heat absorbed}}}$$

$$\boxed{\%\eta = \frac{W_{net}}{Q_{+ive}} \times 100}$$

question → eg:→



Two moles of the monoatomic Ideal gas is taken through a process ABCA as shown in the figure.
For the complete cyclic process ABCA 1200 J is withdrawn from the gas.
1) Find the work done by the gas during

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Process B to C.

Ans ~~ABCD~~ $A \rightarrow B \rightarrow C \rightarrow D$

$$\Delta U = 0$$

$$Q_{net} = W_{net}$$

$$-1200 = W_{AB} + W_{BC} + \cancel{W_{CA}}^{0}$$

?

$A \rightarrow B$

$$V \propto T$$

$$P = \text{constant}$$

$$W_{AB} = nR(\Delta T)$$

$$= 2 \times 8.3 \times \cancel{8000}(500-300)$$

$$\Rightarrow 3320\,J$$

Physics में ऐसा जबकि chemistry में '–' होता है

so,

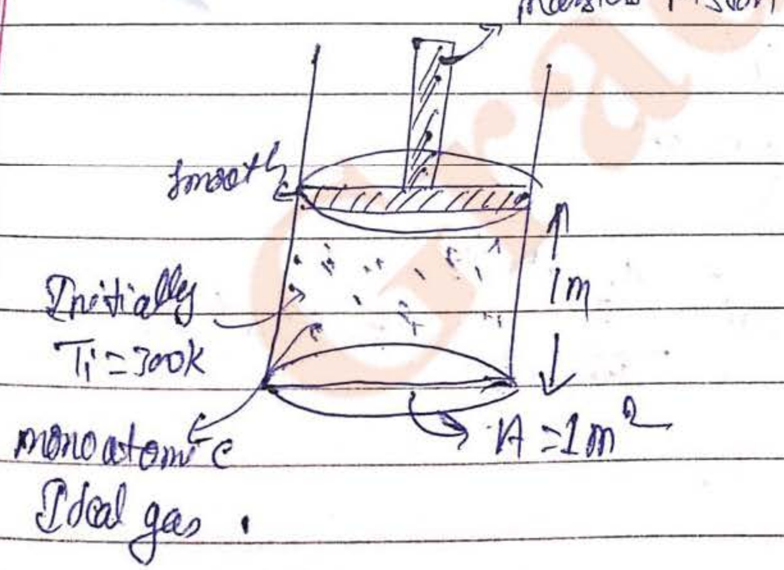
$$-1200 = W_{AB} + W_{BC}$$

$$-1200 = 3320\,J + W_{BC}$$

$$\therefore W_{BC} = -4520\,J$$

IIT 2004

Q



Gases heated to 400
at constant Presr
1) Find the final ~~Temp. of~~
position of the piston
the bottom of the cylind

Volume → Area of base × height

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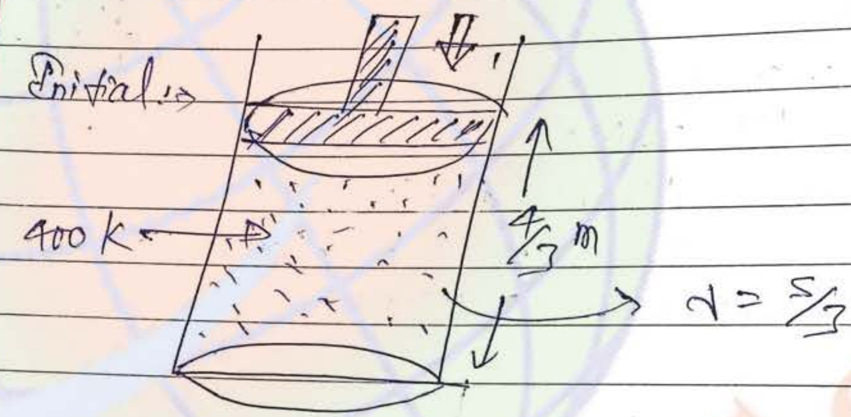
$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{A \cdot 1}{300} = \frac{A \times h}{400}$$

$$h = \frac{4}{3} m$$

eg: 3)



The piston is moved to initial ~~condition~~ position of ~~previous~~ previous question ~~with~~ at a height of 1 m without any loss of heat

1) Find the final temp. of the gas when the piston is at height of 1m from the bottom

Ans)

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1}$$

$$= 400\left(\frac{\frac{4}{3}\times 1}{1\times 1}\right)^{\frac{5}{3}-1} = 400\left(\frac{4}{3}\right)^{2/3}$$

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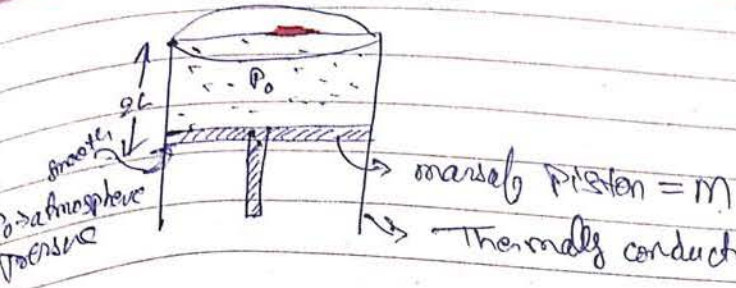
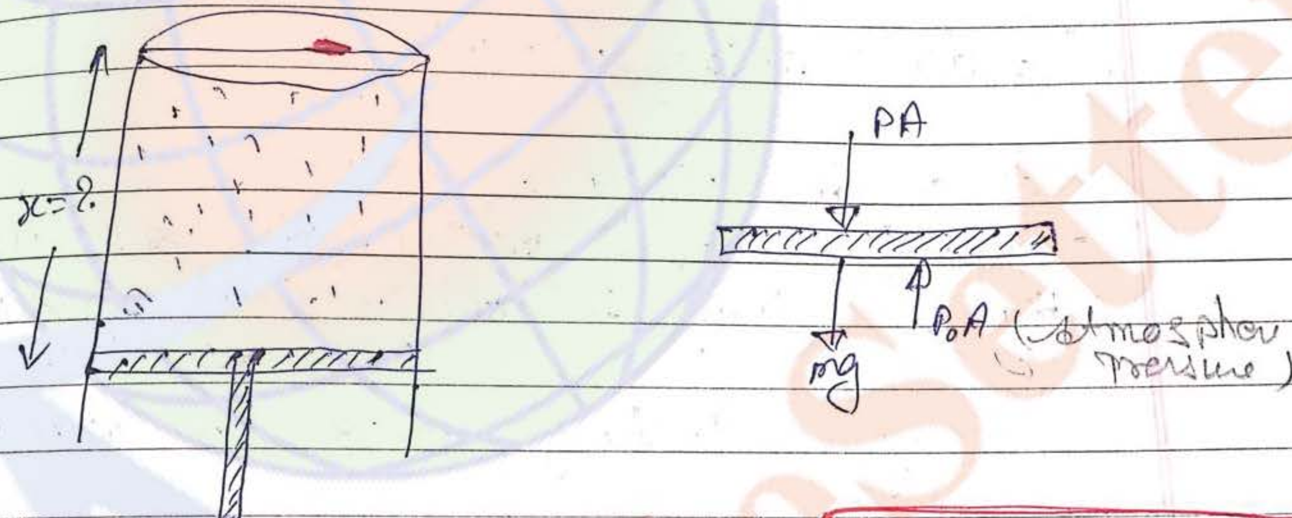
1.) 

The hole is filled at position 2L and then piston is released.
The piston is stage in equillibrium at new position.
i) Find the Equillibrium (new) position of the system ~~at~~ ~~in~~ (Initial position) from the top



$$\boxed{\text{Pressure} = \frac{\text{Force}}{\text{Area}}}$$
$$\boxed{\text{volume} = \text{Area of base} \times \text{height}}$$

In eq^m

$$P_A + mg = P_0A$$

$$P = P_0 - \frac{mg}{\pi R^2}$$

Here T = constant

$$P_1V_1 = P_2V_2$$

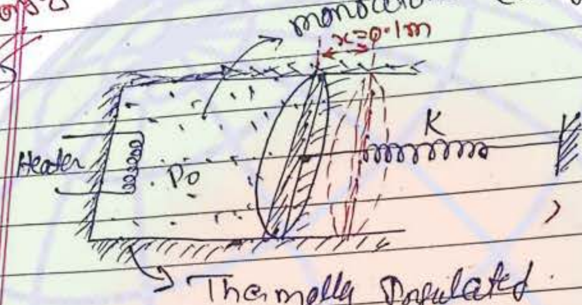
$$\Rightarrow P_0(2L \times A) = \left(P_0 - \frac{mg}{\pi R^2}\right)(A \cdot x)$$

so,

$$x = \frac{P_0(2L)}{\left(P_0 - \frac{mg}{\pi R^2}\right)}$$

monoatomic Ideal gas ($\gamma = 5/3$)

Diagram of gas



Initial volume of gas = V_i = 2.4×10^{-3} m^3

$A = 8 \times 10^{-3}$ m^2
Area of cross section.

$K = 8000 \frac{N}{m}$

Atomic Pressure $(P_0) = 10^5 N/m^2$

Piston and Spring are massless
Thermal capacity of coil of heater is negligible
Heat loss through the coil of heater is negligible.

Spring is Initially at relaxed position.

(Initial Temp of gas = 300K)

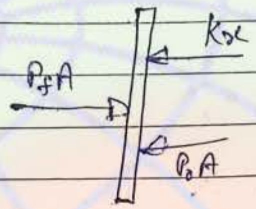
Gas is heated by heater and the piston is slowly moved through distance of 0.1 m

i) Find the final Temp. of the gas
ii) The heat supplied by the heater to the gas.

P_0

Ans:⇒ ~~$P_f V_f$~~ $\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}$

~~$V_f = A \cdot x + V_i$~~



$P_f A = Kx + P_0 A$

$P_f = \left(\frac{Kx}{A} + P_0\right)$

$P_f = \left(\frac{8000 \times 0.1}{8 \times 10^{-3}} + 10^{5}\right)$

$= 2P_0$

$V_f = V_i + A \cdot x$

$V_f = 2.4 \times 10^{-3} + 8 \times 10^{-3} \times 0.1$

$= 3.2 \times 10^{-3}\ m^3$

so,

$\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}$

$\frac{P_0 \times 2.4 \times 10^{-3}}{300} = \frac{2P_0 \times 3.2 \times 10^{-3}}{T_f}$

$T_f = 800K$

ii) $\Delta U = nC_v \Delta T$

$= n\frac{3R}{2} \times 500$

$= 750(nR)$

$$\frac{P_i V_i}{T_i} = nR = \frac{10^5 \times 2.4 \times 10^{-3}}{300}$$

$$=$$

$P_i = P_0$, $P_f = 2P_0$

$T_i = 300K$, $T_f = ?$

$V_i = 2.4 \times 10^{-3} m^3$, $V_f = 3.2 \times 10^{-3} m^3$

$$nR = \frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f} \quad \text{—(1)}$$

$$T_f = \frac{P_f V_f T_i}{P_i V_i}$$

$$T_f = 800K$$

$$Q = \Delta U + W$$

From work energy theorem,

$$\boxed{W_{spring} + W_{atm} + W_{gas} = 0}$$

$$-\frac{1}{2}kx^2 - (P_0 A)x + W_{gas} = 0$$

$$W_{gas} = P_0 A \cdot x + \frac{1}{2}kx^2$$

$$= 10^5 \times 8 \times 10^{-3} \times 0.1 + \frac{1}{2} \times 8000 \times (0.1)^2$$

$$= 120 J$$

$$\Delta U = \frac{n3R}{2}(500)$$

$$= \frac{P_1 V_1}{T_1} \times \frac{3}{2} \times 500 \quad (\text{from eq. 1})$$

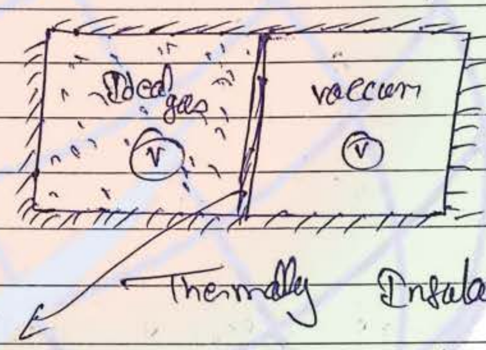
$$= \frac{10^5 \times 2.4 \times 10^{-3}}{300} \times \frac{3}{2} \times 500$$

$$= 600\,J$$

$$Q = 600 + 120 = 720\,J$$

IIT 2008
[Q.13]

Free expansion Process.



If the gas expands against vaccum.

$$\left\{\begin{array}{l} W_{gas} = 0 \\ Q = 0 \\ \Delta U = 0 \end{array}\right\} \Rightarrow$$

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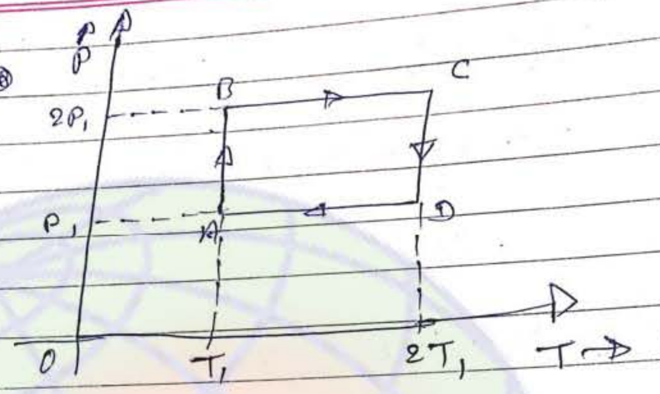
1st Choice

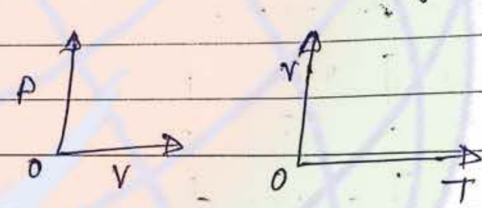
Ex :-



2 moles Ideal monoatomic gas

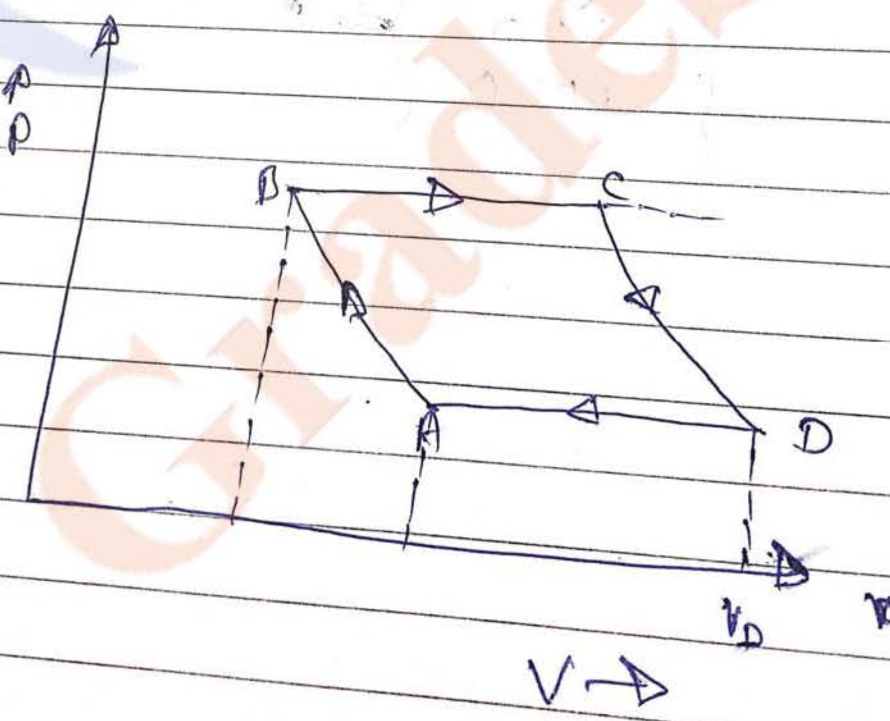
$T_1 = 300K$

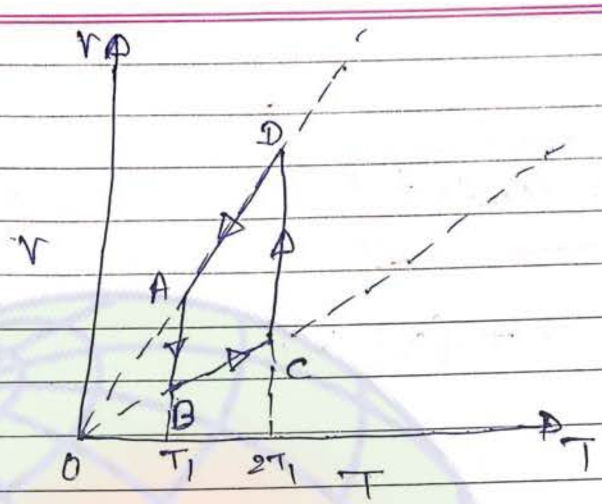
i) Draw P-V and V-T graph



ii) Find the heat absorbed ~~(Released)~~ or release by the gas and workdone by the gas in the process ABCD.

Ans)





ii) $A \to B$

$\Delta U_{AB} = 0$

$Q_{AB} = W_{AB} = nRT \ln \frac{V_B}{V_A} = 2R\,300 \ln \frac{1}{2} < 0$ (Released)

$B \to C$

$W_{BC} = nR(\Delta T)$
$= 2R \times 300$
$\Rightarrow 600R > 0$

$(\Delta U)_{BC} = nC_v \Delta T$
$= 2 \times \frac{3R}{2} \times 300$

$Q_{BC} = nC_p \Delta T$
$= 2 \times \frac{5R}{2} \times 300$
$= 1500R$

$\Delta U_{CD} = 0$

$W_{CD} = 2 \times R \times 600 \ln \frac{V_D}{V_C}$

$= 1200R \;, \ln 2 > 0$

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1st Choice

Polytropic Process

$$C = \frac{Q}{n\Delta T}$$

$$\boxed{Q = nC\Delta T}$$

$$dQ = nC(dT)$$

$$\boxed{PV^{x} = \text{constant}}$$

$$x \neq (0, 1, \gamma)$$

जब कुछ और C_V नहीं रहे तब इसमें 'C' से बदली होते हैं।

$$\boxed{C = C_V + \frac{R}{(1-x)}}$$

Molar specific heat for Polytropic Process.

$$\boxed{Q = nC\Delta T}$$

for all process.

इसमें सिर्फ 'C' आएगा

$$\Delta U = nC_V\Delta T$$

$$Q = \Delta U + W$$

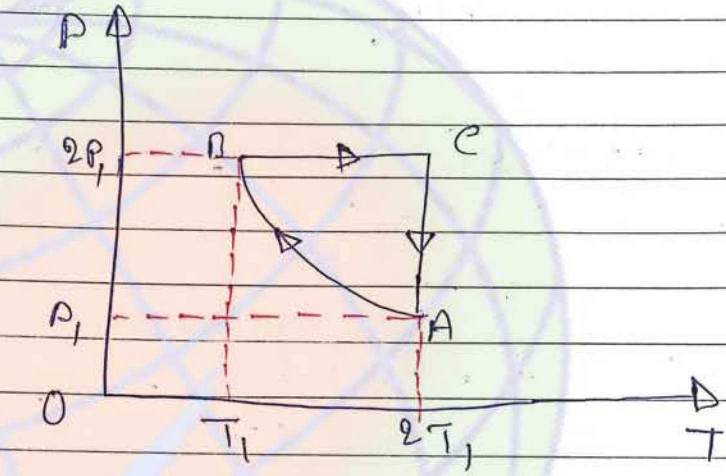
$$W = Q - \Delta U$$

$$= nC\Delta T - nC_V\Delta T$$

$$= n(\Delta T)(C - C_V)$$

Imp. $\boxed{W = \dfrac{nR(\Delta T)}{1-x}}$

Ex:- ⟹



$T_1 = 300K$
$n = 2$ mole
Ideal monoatomic gas.
$A \to B$
$PT =$ constant

Two moles of monoatomic Ideal gas is taken through cyclic process ABCA.
For A to B process gas follows ~~also~~ PT = constant

1) Find the workdone by gas during process A to B.

ii) Heat released and absorbed by gas during process AB, BC and CA
And ~~also~~ also find workdone during B to C.

Ans) $PT =$ constant

$P\left(\frac{PV}{nR}\right) =$ constant

$P^2 V =$ constant

$$P^2 V = \text{constant}$$

$$P \propto \frac{1}{\sqrt{V}}$$

$$P^1 V^{1/2} = \text{constant}$$

$$W = K\int_{v_i}^{v_f} \frac{dv}{v^{1/2}}$$

$$x = \frac{1}{2}$$

$$W = \frac{nR(-300)}{\left(1-\frac{1}{2}\right)} = -1200R$$

11) $$C = C_V + \frac{R}{1-\frac{1}{2}} = \frac{3R}{2} + 2R = \frac{7R}{2}$$

$$Q_{AB} = nC(-300)$$

$$= \cancel{2} \times \frac{7R}{\cancel{2}} \times (-300)$$

$$= -2100R$$

$$Q_{BC} = nC_p \Delta T$$

$$= \cancel{2} \times \frac{5R}{\cancel{2}} \times (600-300)$$

$$= 1500R$$

$$Q_{CA} = W = nRT \ln\left(\frac{V_f}{V_i}\right)$$

$$= nR(600) \ln \frac{P_i}{P_f}$$

$$= 2 \times R \times 600 \ln \frac{2\cancel{P_i}}{\cancel{P_i}}$$

$$= 1200R \ln 2$$

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$$W_{B\to C} = nR(300)$$

Q) One mole of monoatomic Ideal gas undergoes through a partical process In which temp. of gas changes with volume of gas ~~acco~~ 'V' acc^n to given relation $\boxed{T = T_0 + \alpha V}$

where, "T_0, α" → constant.

i) Find the molar specific heat of gas as a function of volume 'V' of gas for the given process in terms of C_p

ii) In the given process the gas is expanded from Initial ~~value~~ volume V_0 to final volume $2V$. Find the heat absorbed by the gas for the given expansion process.

Ans.

$$\boxed{dQ = n\,C\,dT}$$

$$\boxed{C = \frac{dQ}{n\,dT}}$$

$$\boxed{\begin{aligned} dQ &= dU + dW \\ dQ &= nC_v dT + P dV \end{aligned}}$$

→ for small variation

$$C = \frac{nC_v dT + P dV}{n\,dT}$$

Imp

for all process

$$\boxed{C = C_V + \left(\frac{P}{n}\right)\left(\frac{dV}{dT}\right)}$$

molar specific heat for all type of process

$$T = T_0 + \alpha V$$

Diff w.r.t "T"

$$1 = 0 + \alpha\left(\frac{dV}{dT}\right)$$

$$\frac{dV}{dT} = \frac{1}{\alpha}$$

$$PV = nRT$$

$$\frac{P}{n} = \frac{RT}{n}$$

$$= \frac{R}{V}\left(T_0 + \alpha V\right)$$

$$\frac{P}{n} = \frac{RT_0}{V} + R\alpha$$

So,

$$C = C_V + \left(\frac{RT_0}{V} + R\alpha\right)\frac{1}{\alpha}$$

$$C = C_V + \frac{RT_0}{V\alpha} + R$$

$$\boxed{C = C_p + \frac{RT_0}{V\alpha}}$$

A.

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$$V = \frac{T - T_0}{\alpha}$$

i) $V \rightarrow 2V$

$$Q = n\int_{T_i}^{T_f} c\,dT$$

$$T_i = T_0 + \alpha V$$

$$T_f = T_0 + \alpha 2V.$$

$$\Delta T = (T_f - T_i) = \alpha V$$

ii) $Q = \Delta U + \Delta W$

$$\Delta U = nC_v\,(\Delta T)$$
$$= 1 \times C_v\,(\alpha V)$$

$$W = \int_V^{2V} p\,dv$$

$$\therefore \frac{P}{\underbrace{n}_{=1}} = \frac{RT_0}{V} + R\alpha$$

$$= \int_V^{2V}\left(\frac{RT_0}{V} + R\alpha\right)dv.$$

$$= RT_0 \ln\left(\frac{2\cancel{V}}{\cancel{V}}\right) + R\alpha V$$

1st Choice

Q:) For one mole monoatomic Ideal gas Pressure changes with volume. acc^n to given Relation $P = \left(\frac{\alpha}{V} + \beta\right)$

where (α and β) → constant

The gas is expanded from Initial volume "V" to final volume "3V"

1) Find the workdone by the gas during this expansion process.

Ans:)

$$W = \int_{V}^{3V} \left(\frac{\alpha}{V} + \beta\right) dV$$

$$= \left[\alpha \ln \frac{3V}{V} + \beta(3V - V)\right]$$

$$= \alpha \ln(3) + 2\beta V$$

Q.) Two moles of a monoatomic Ideal gas undergoes through a process $V^2 T$ = constant

If Initial tem. of gas is 400K and the gas is expanded from $V_i = V$ to $V_f = 2V$

1) find the workdone by the gas during this expansion and the heat absorbed by the gas in this expansion.

As $V^2T = \text{constant}$

$$V^2\frac{PV}{nR} = \text{constant}$$

$PV = nRT$

$T = \frac{PV}{nR}$

$$PV^3 = \text{constant}$$

$$x = 3$$

and, ~~$W = 2R(\Delta T)\gamma$~~

$$T \propto \frac{1}{V^2}$$

$T_i = 400$

$T_f = 100K$

So,

$$W = \frac{2R(\Delta T)}{(1-x)}$$

$$= \frac{2R(-300)}{1-x}$$

$$= \frac{-600R}{(1-3)} = 300R$$

$$C = C_V + \frac{R}{1-3}$$

$$= \frac{3R}{2} - \frac{R}{2} = R$$

$Q = nC\Delta T$

1st Choice

Heat Transfer

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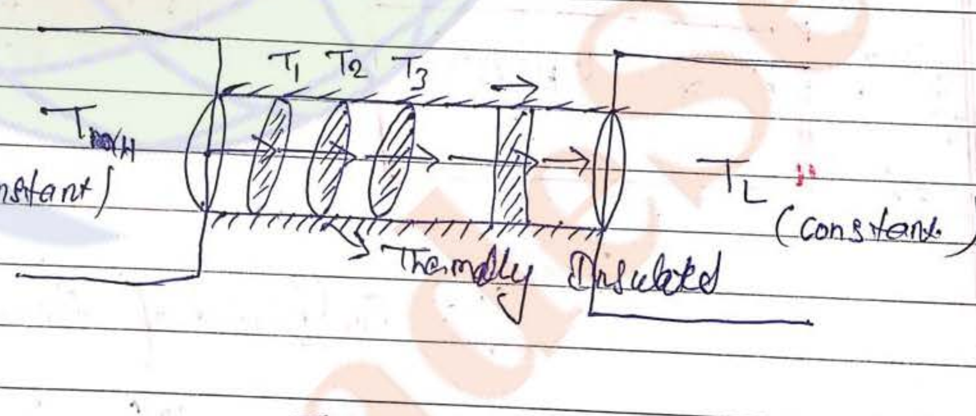
- (i) By ~~cond~~ conduction → (solid)
- (ii) By convection → (liquid, gas)
- (iii) By Radiation ✓ → (No medium is required)

* ~~Cod~~ Conduction :→ In case of conduction heat is transfered from one place at higher temp. ~~to~~ to another place at low temp. through molecule collision with actual moment of molecules inside the material.

* Steady State :⇒



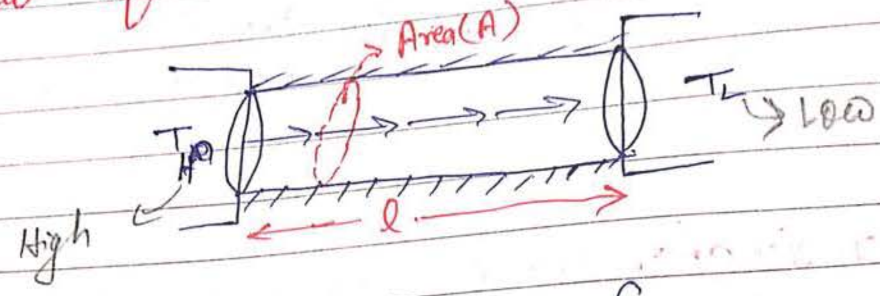
$$T_1 > T_2 > T_3 > \ldots\ldots$$

~~Temp~~ At the steady state temp. of different cross-section of the rode does not ~~vary~~ varies with time.

But temp. of different cross-section of rode are different.

(But varies with space) x, y, z

* Law of Conduction :⇒



$$H \propto A$$
$$\propto \Delta T$$
$$\propto \frac{1}{l}$$

$\Delta T = (T_H - T_L)$

$$H \propto \frac{A \Delta T}{l}$$

Rate of Heat Transfer or Heat current → $\boxed{H \propto \frac{KA(\Delta T)}{l}}$

$\left(H = \frac{\Delta Q}{\Delta t}\right) = \frac{Q}{t}$

where

K = Coefficient of thermal conductivity

→ Measures (gives) the heat conducting ability of materials mainly depends on the nature of matter.

Unit → $\frac{W}{mK}$

W → watt

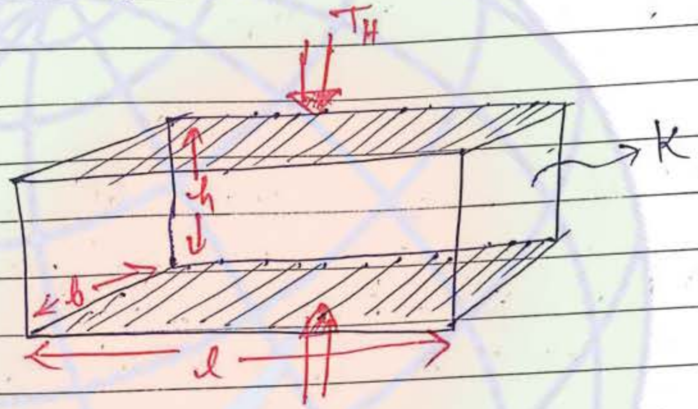
m → meter

K → kelvin

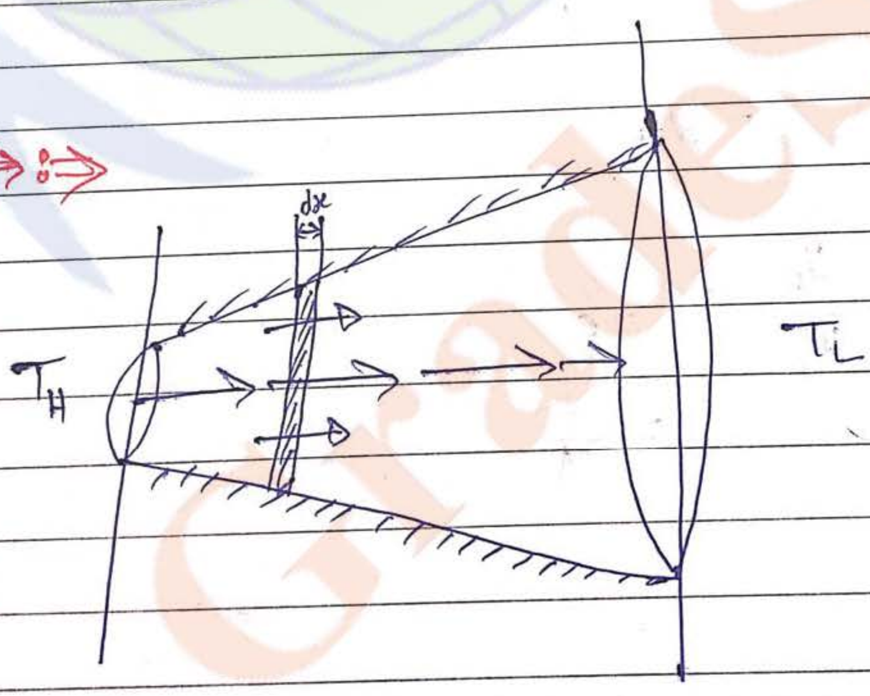
l ⇒ length of material along which the heat follows.

A ⇒ Area of cross-section of the rode perpendicular ($\perp$) to the direction of heat follows.

eg :⇒



$$H = \frac{K(bl)(T_H - T_L)}{h}$$



$$\boxed{H = -KA\frac{dT}{dx}}$$

 velocity radiant Distance

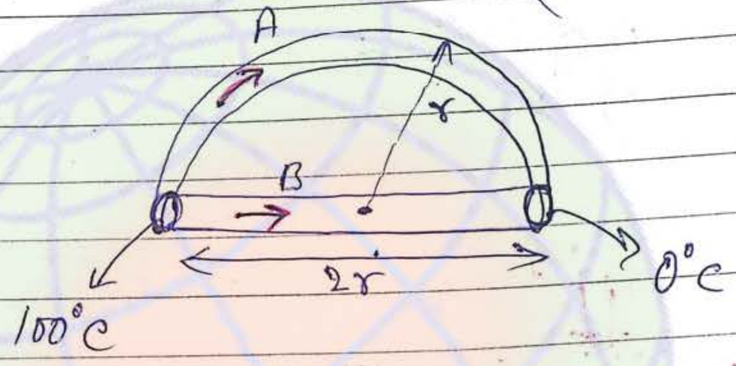
1st Choice

$$\left(\frac{-dT}{dx}\right) = \text{Temperature gradient}$$

(change in temperature per unit distance)

(minus sign indicates that $\left(\frac{dT}{dx}\right)$ is negative along the direction of heat follows.

Ex! ⇒



Both rods are made of same material and are having same area of cross section
1) Find the ratio of rate of heat flow through rods 'A' and rods 'B' in the same time.

Ans:

$$H = \frac{kA(\Delta T)}{l}$$

$$H \propto \frac{1}{l}$$ (Alternative)

$$\frac{H_A}{H_B} = \frac{2r}{\pi r}$$

$$= \frac{2}{\pi}$$

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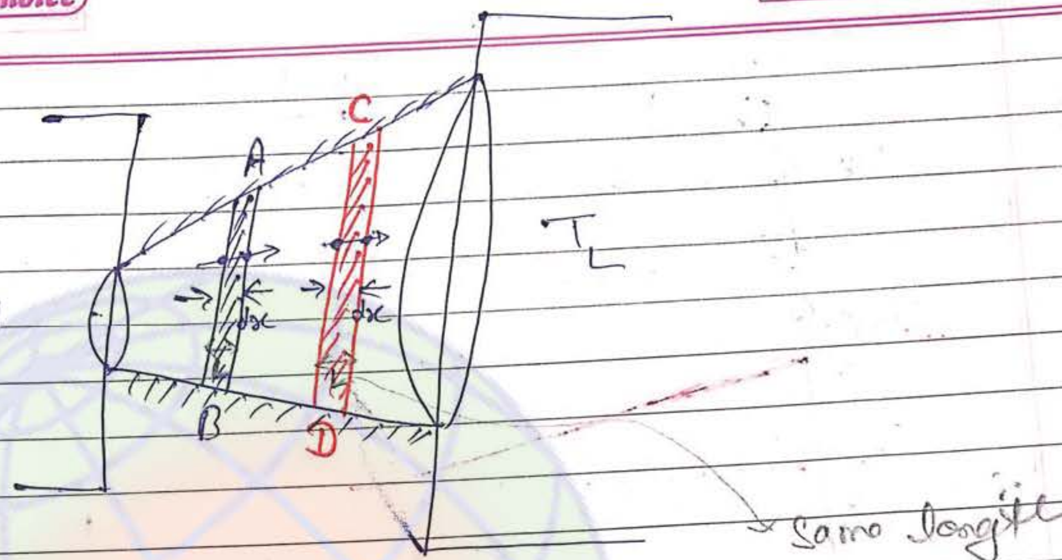
Ex ⇒



Correct option is

1) $(\Delta T)_{AB} > (\Delta T)_{CD}$

(II) $(\Delta T)_{AB} = (\Delta T)_{CD}$

(III) $(\Delta T)_{AB} < (\Delta T)_{CD}$

(IV) None

Ans) $$H = \frac{KA(\Delta T)}{l}$$ Diff. ...

$$\Delta T \propto \frac{1}{A}$$

(1) $(\Delta T)_{AB} > (\Delta T)_{CD}$ Ans.

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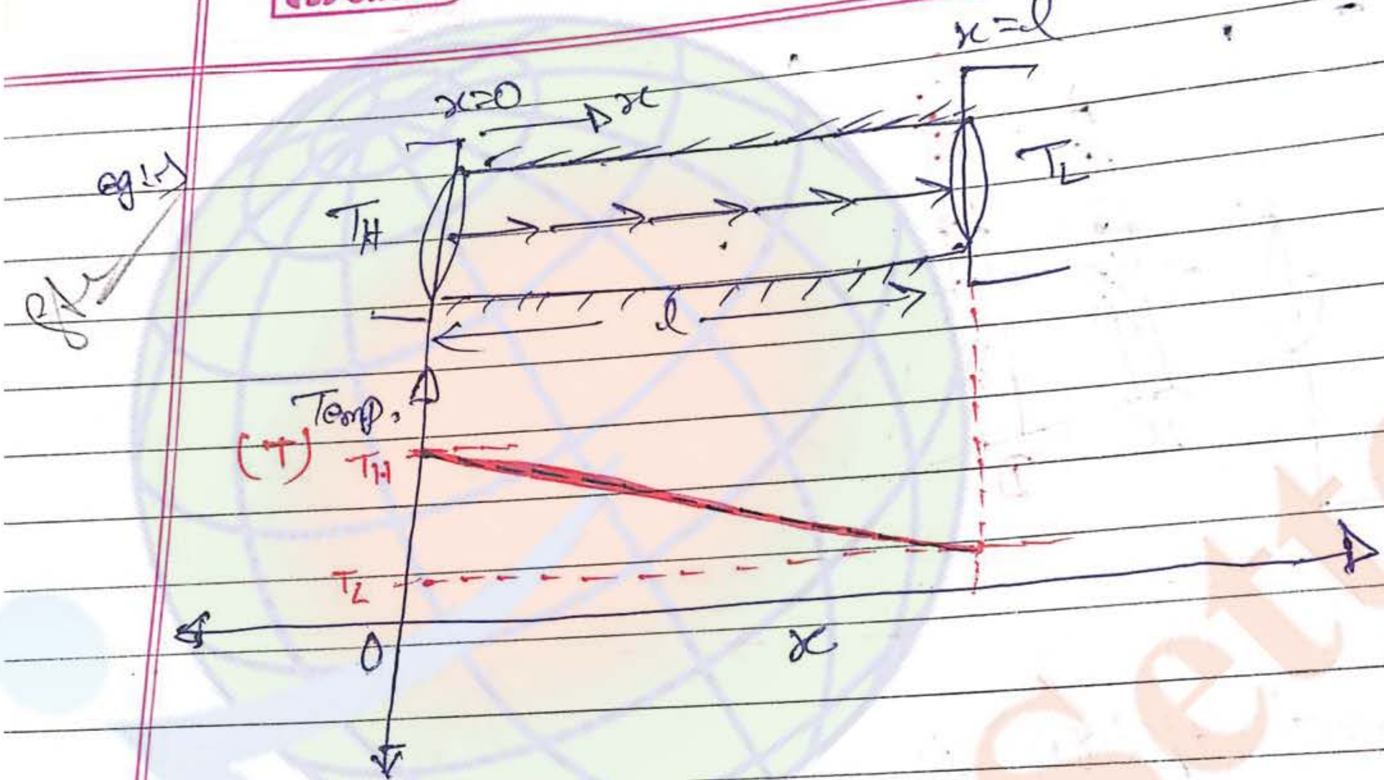


$$\mathrm{H} = -KA\left(\frac{dT}{dx}\right)$$

constant

$$\frac{dT}{dx} = -ve \text{ (constant)}$$

$\boxed{V = I R}$

$H = \cancel{KA\,(\Delta T)}$

Heat transfer

$$H = KA\frac{\Delta T}{l}$$

$$H = \frac{\Delta T}{l/KA}$$

"Heat current" → $\boxed{H = \frac{\Delta T}{R}}$ ⇒ $\boxed{I = \frac{\Delta V}{R}}$

where R → Thermal Resistance

Thermal resistance ← $\boxed{R = \frac{l}{KA}}$

Thermal conductivity

Temp difference → ΔT

Current electricity

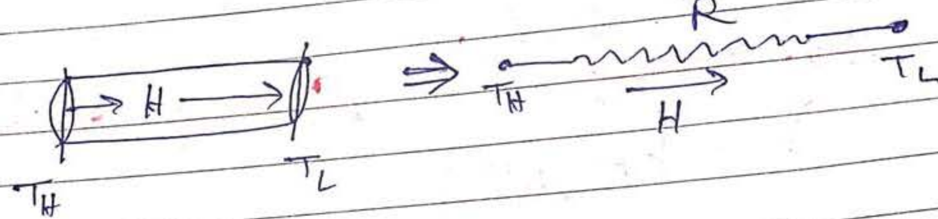
$$\boxed{R = \frac{\rho l}{A} = \frac{l}{\sigma A}}$$

where $\sigma = \frac{1}{\rho}$

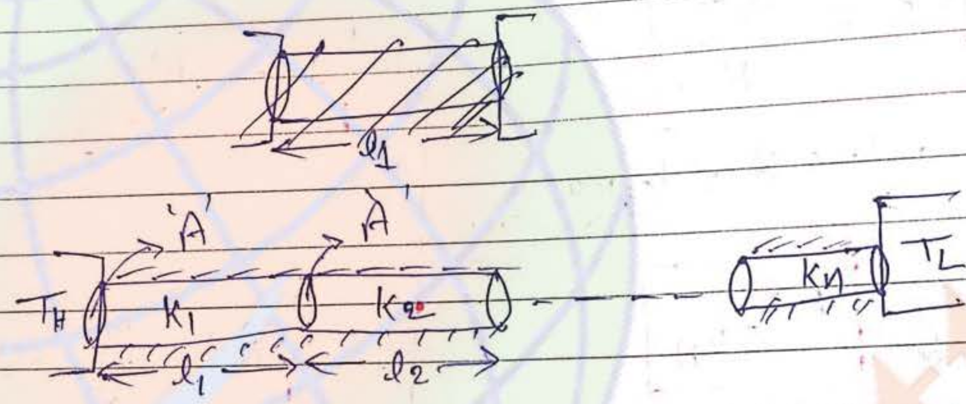
ΔV ⇒ Potential difference.

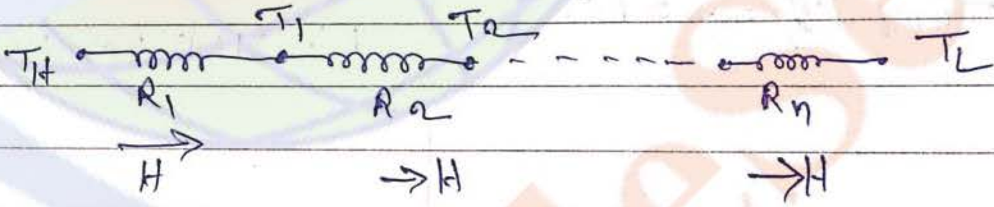
So.

$$\boxed{H = \frac{dQ}{dt} = \frac{KA\Delta T}{l} = \frac{\Delta T}{R}}$$



1) Series Combination of rods:⇒





$$H = \frac{T_H - T_1}{R_1} = \frac{T_1 - T_2}{R_2} = \text{-------}$$

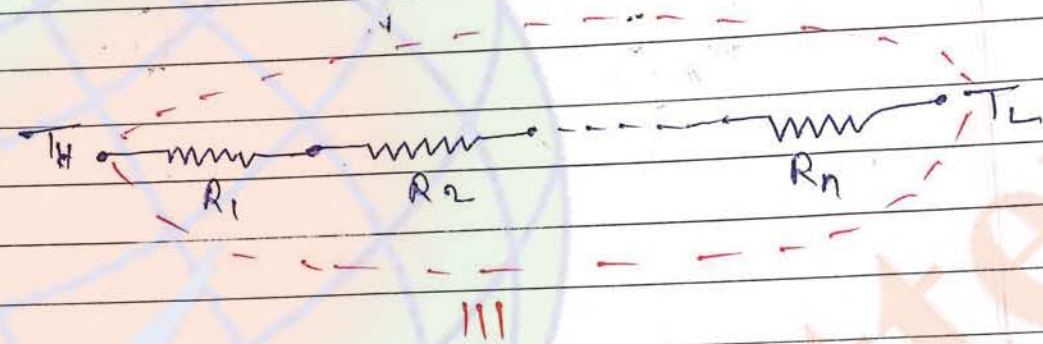
$$R_1 = \frac{l_1}{K_1 A}$$

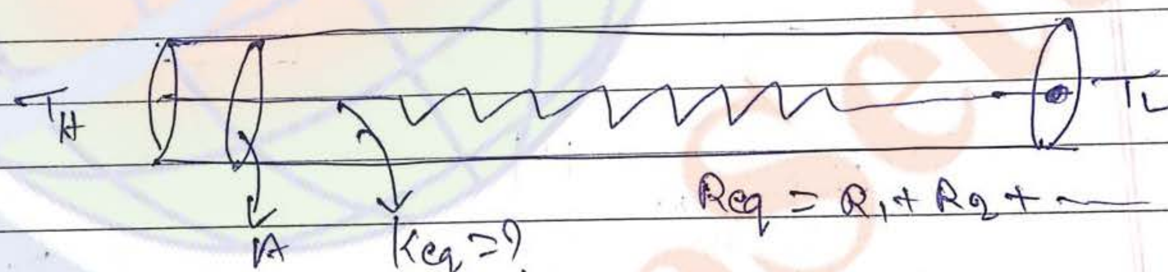
$$R_2 = \frac{l_2}{K_2 A}$$

$$H = \frac{T_H - T_L}{R_{equivalent}}$$

$$R_{equivalent} = R_1 + R_2 + \text{-----}$$

Equivalent thermal conductivity Series Combination (K_{eq})





$R_{eq} = R_1 + R_2 + \ldots$

$$R_{eq} = R_1 + R_2 + \ldots\ldots$$

$$\frac{l_1 + l_2 + \ldots}{K_{eq}\cancel{A}} = \frac{l_1}{K_1\cancel{A}} + \frac{l_2}{K_2\cancel{A}} + \ldots$$

$$\boxed{K_{eq} = \frac{(l_1 + l_2 + \ldots)}{\left(\frac{l_1}{K_1} + \frac{l_2}{K_2} + \ldots\right)}}$$

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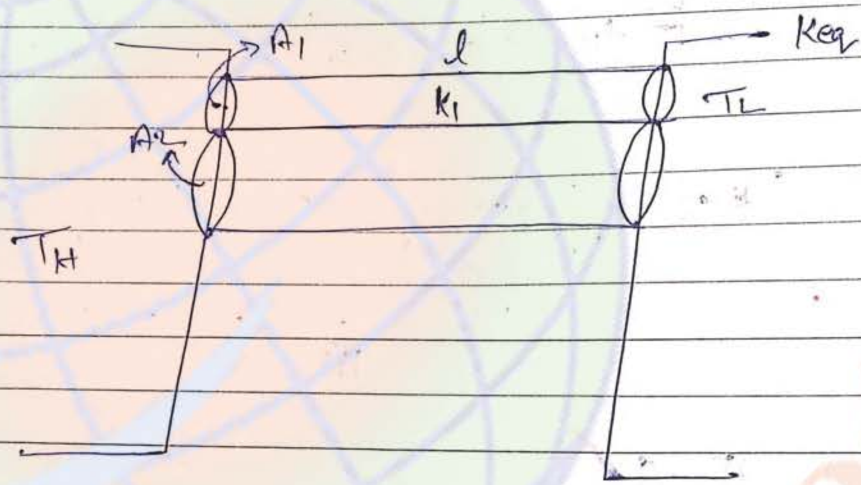
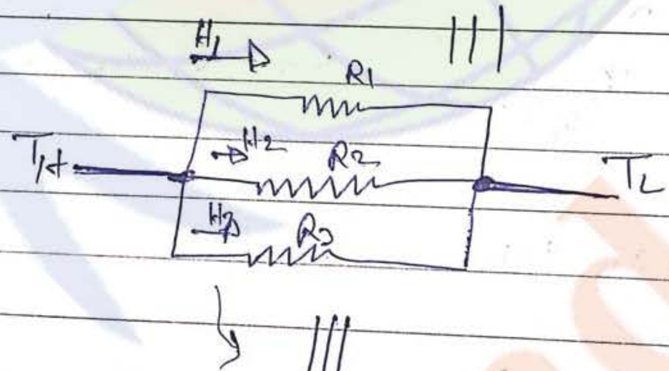
Parallel Combination of Rod :- $\Rightarrow$

Equivalent thermal conductivity in ~~Series~~ Parallel comb^n.





$$R_1 = \frac{l}{K_1 A_1}$$

$$R_2 = \frac{l_2}{K_2 A_2}$$

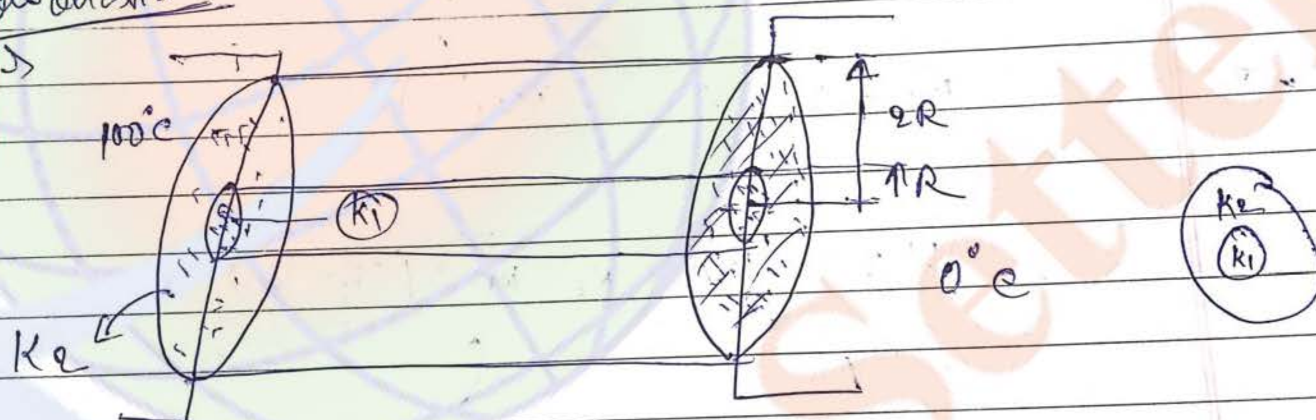
$$R_{eq} = \frac{l}{K_{eq}(A_1 + A_2 + \ldots)}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

$$\frac{K_{eq}(A_1 + A_2 + \cdots)}{l} = \frac{K_1 A_1}{l} + \frac{K_2 A_2}{l} + \cdots$$

$$\boxed{K_{eq} = \frac{(K_1 A_1 + K_2 A_2 + \cdots)}{(A_1 + A_2 + \cdots)}}$$

IIT old question
(Q.1)



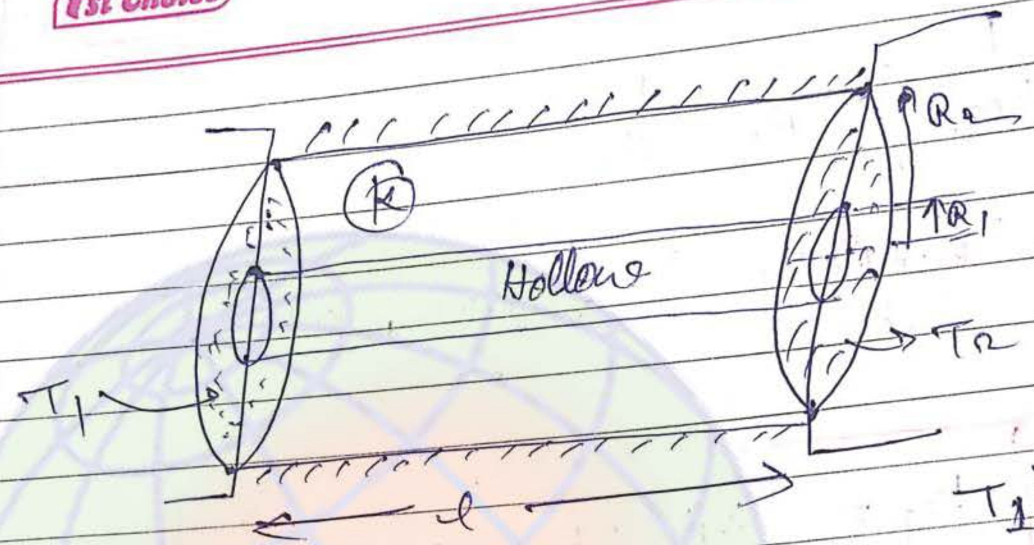
Find the equivalent thermal conductivity of combined rod

Ans $$K_{eq} = \left(\frac{K_1 A_1 + K_2 A_2 + \cdots}{A_1 + A_2 + \cdots}\right)$$

$$\Rightarrow \frac{K_1 \cancel{\pi} R^2 + K_2 3 \cancel{\pi} R^2}{\cancel{\pi} R^2 + 3 \cancel{\pi} R^2}$$

$$\Rightarrow \left(\frac{K_1 + 3K_2}{4}\right)$$

Ex:-



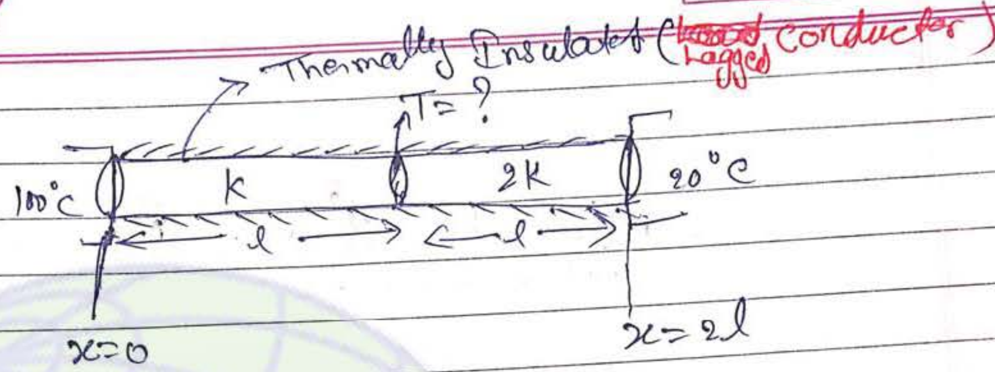
$T_1 > T_2$

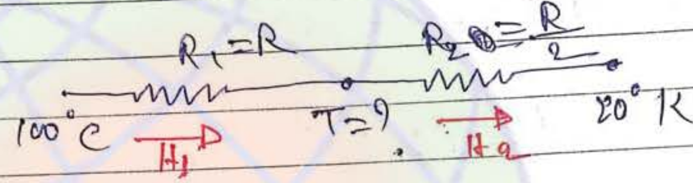
Find the rate of heat flow through the cylinder

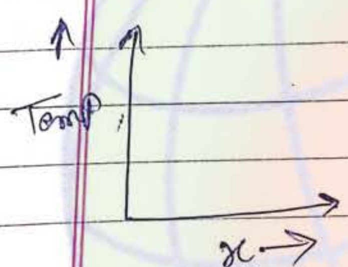
Ans: $\text{Area} = (\pi R_2^2 - \pi R_1^2)$

$$H = \frac{K\pi(R_2^2 - R_1^2)(T_1 - T_2)}{l}$$

Sol:







$$R_1 = \frac{l}{KA} = R$$

$$R_2 = \frac{l}{2KA} = \frac{R}{2}$$

$$H = \frac{\Delta T}{R}$$

$$\frac{100-T}{R} = \frac{T-20}{R/2}$$

$$100 - T = 2(T-20)$$

$$3T = 140$$

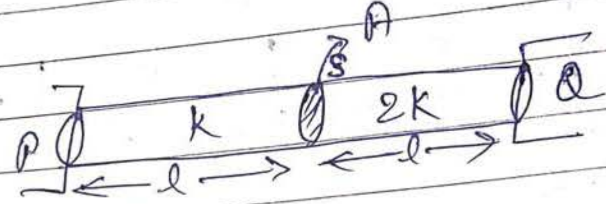
$$T = \frac{140}{3}\ ^\circ C$$

check:

$$H = \frac{KA\,\Delta T}{l}$$

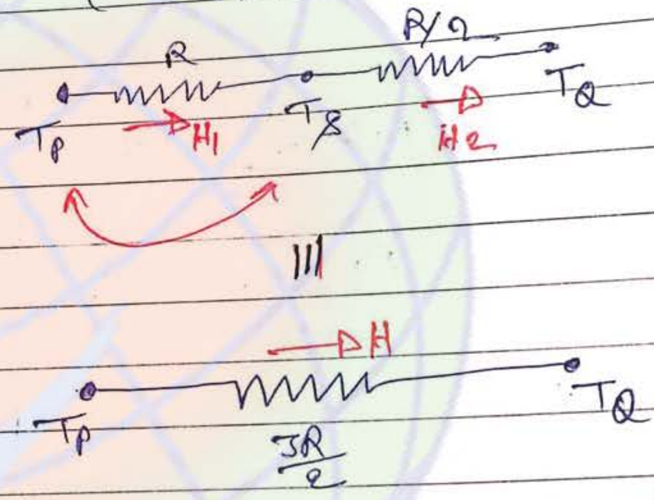
$$\Delta T \propto \frac{1}{K}$$

Q!5



$$(\Delta T)_{PQ} = 36^\circ C$$

$$(\Delta T)_{PS} = ?$$



$$H = H_1 = H_2$$

$$\frac{(\Delta T)_{PQ}}{(3R/2)} = \frac{(\Delta T)_{PS}}{R} = \frac{(\Delta T)_{SQ}}{R/2}$$

$$(\Delta T)_{PS} = (\Delta T)_{PQ}\,\frac{2\cancel{R}}{3\cancel{R}}$$

$$= 36 \times \frac{2}{3}$$

$$= 24^\circ$$

1st Choice

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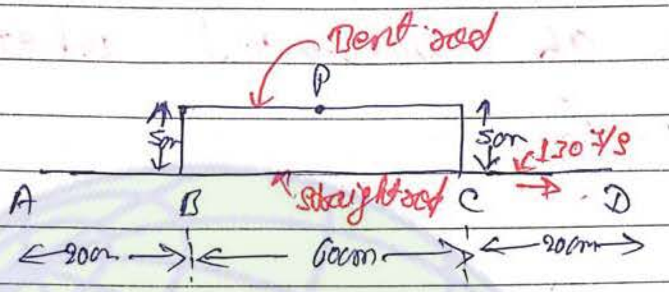
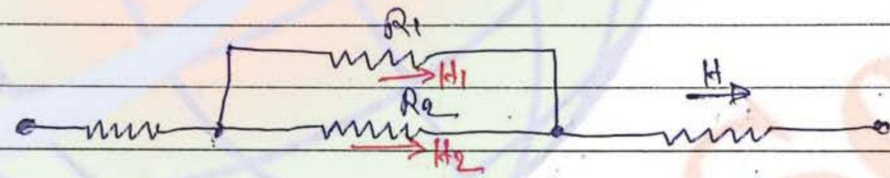
Ex! ⇒



(Rods are made of same material and of same Area of cross section.)

Rate of heat flow in rod CD is 130 J/s find the rate of heat flow through the bent rod (BPC).



$R_1 = 70cm$

$R_2 = 60cm$

$R = \frac{l}{KA}$

$R \propto l$

$\frac{H_1}{H_2} = \frac{R_2}{R_1} = \frac{60}{70}$ — (1)

$H = \frac{\Delta T}{R}$

$H_1 + H_2 = 130$ — (2)

$H \propto \frac{1}{R}$

$H_1 = 60 J/s$

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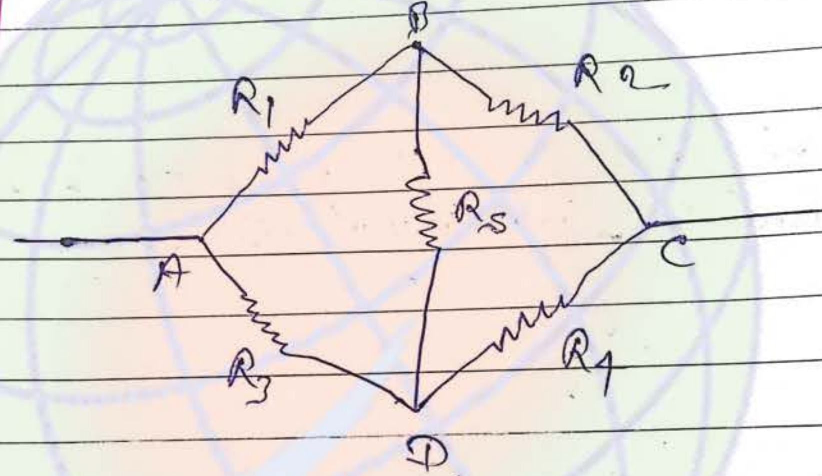
1st Choice

प्रश्न 8. Condition of balanced wheat stone Bridge:⇒



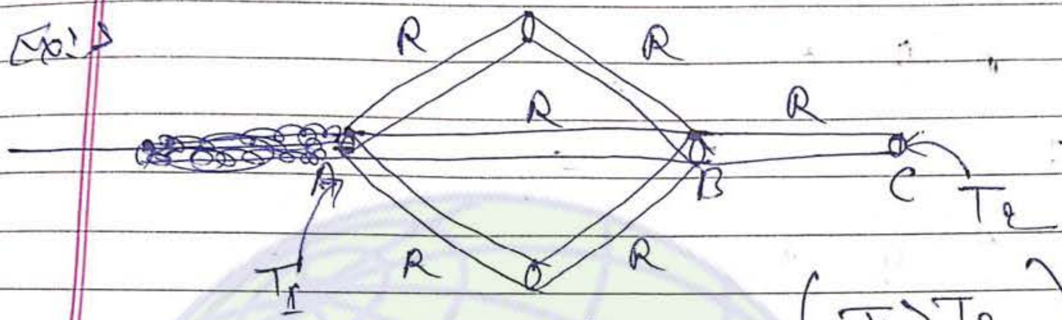
If $\frac{R_1}{R_3} = \frac{R_2}{R_4}$

$\frac{R_1}{R_2} = \frac{R_3}{R_4}$

Then R_5 can be removed ($T_B = T_C$)

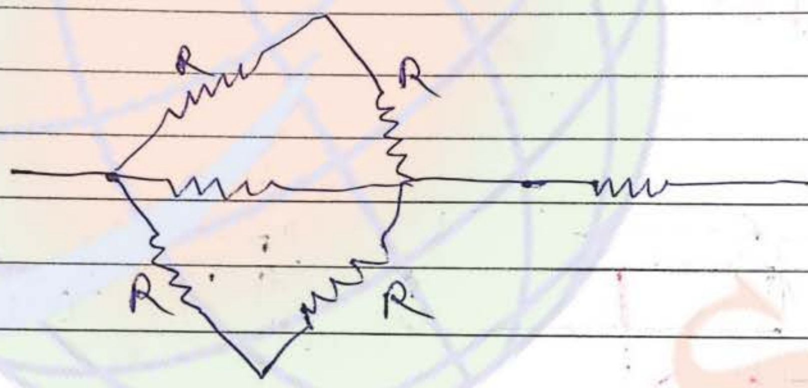
No heat current will flow R_5.

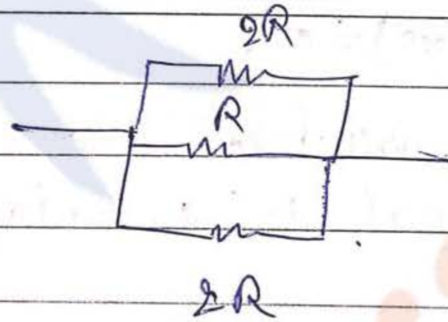
Ex:)



$(T_1 > T_2)$

1) Find the rate of heat flow through rod BC



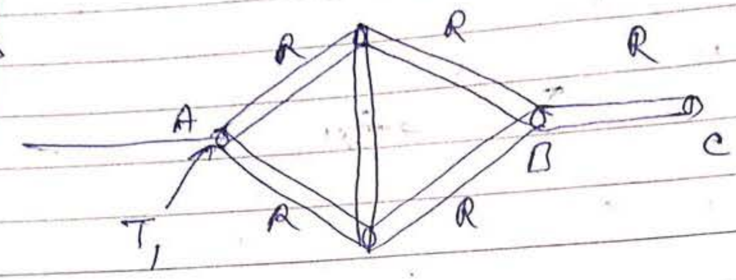


$= \frac{3R}{2}$

$\cancel{\frac{3R}{2}}$

$$H = \frac{(\Delta T)_{eq}}{\left(\frac{3R}{2}\right)_{equival}}$$

Sol.→

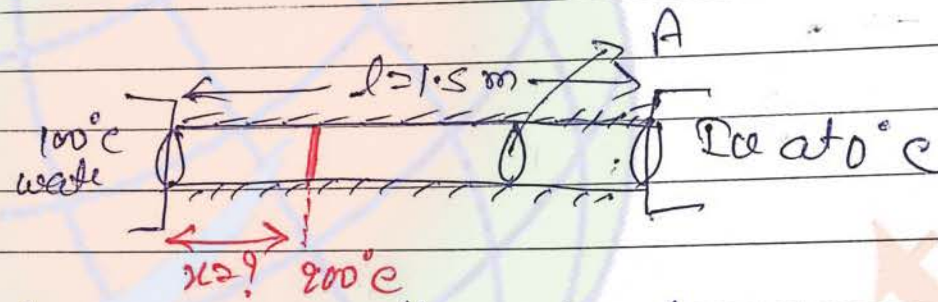


$(T_1 > T_2)$

$$H = \frac{(T_1 - T_2)}{2R}$$

Based on IIT 2009

Q.1.→ conduction + calorimetry→



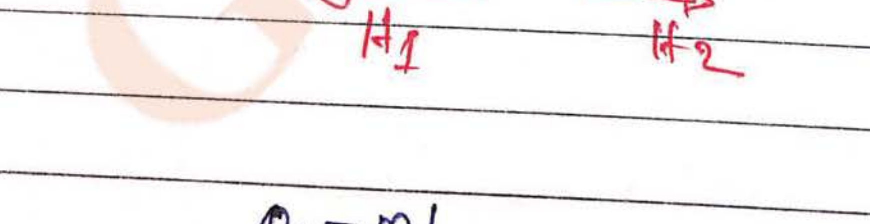
In what distance heat inflow (200°) that:

It is given that amount of water vaporizes per unit time is equal to amount of melting Ice per unit time

$L_f = 80\ cal/g$

$L_v = 540\ cal/g$

A.



$$Q_2 = mL_f$$

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Rate of heat flow (H)

$$H_2 = \frac{dQ_2}{dt} = L_f\left(\frac{dm}{dt}\right) \quad \text{—— (I)}$$

$$H_1 = \frac{dQ_1}{dt} = L_v\left(\frac{dm}{dt}\right) \quad \text{—— (II)}$$

→ Equal

eq (I) ÷ eq (II)

$$\frac{H_2}{H_1} = \frac{L_f}{L_v}$$

$$H_2 = \frac{200-0}{R_2}$$

$$H_1 = \frac{200-100}{R_1} = \frac{100}{R_1}$$

$$\frac{200}{R_2} \times \frac{R_1}{100} = \frac{L_f}{L_v}$$

$$\frac{2R_1}{R_2} = \frac{L_f}{L_v}$$

$$\frac{2R_1}{R_2} = \frac{80}{540} = \frac{4}{54}$$

$$R = \frac{l}{KA}$$

$$R \propto l$$

$$\frac{x}{(1.5-x)} = \frac{4}{54}$$

$$6 - 4x = 54x$$

$$x = \frac{6}{58}\ m$$

$$= 0.1084\ m$$

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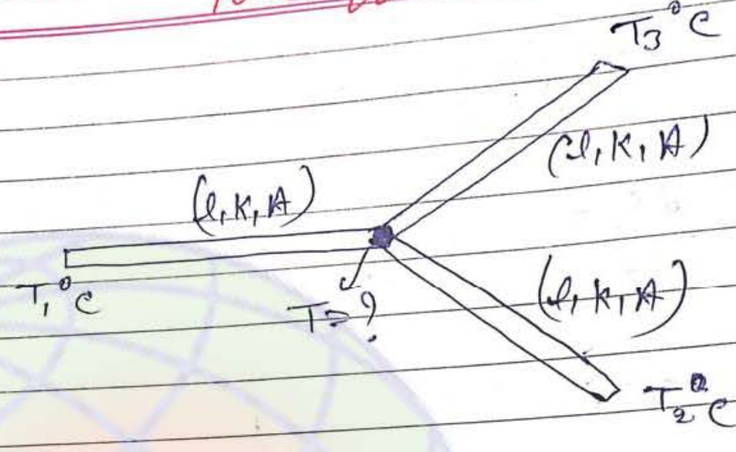
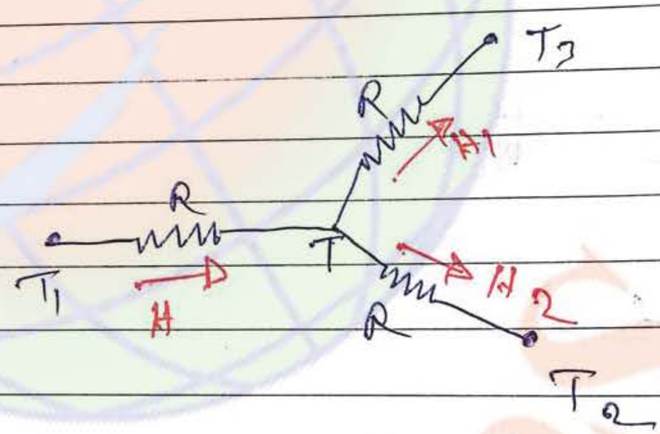
1st Choice

Problem based on Kirchoff's law:-

IIT question



Find the temperature of junction



$$H = H_1 + H_2$$

$$\frac{T_1 - T}{\cancel{R}} = \frac{T - T_3}{\cancel{R}} + \frac{T - T_2}{\cancel{R}}$$

$$\boxed{T = \frac{T_1 + T_2 + T_3}{3}}$$

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Application :→

→ 3-Dimensional Heat flow

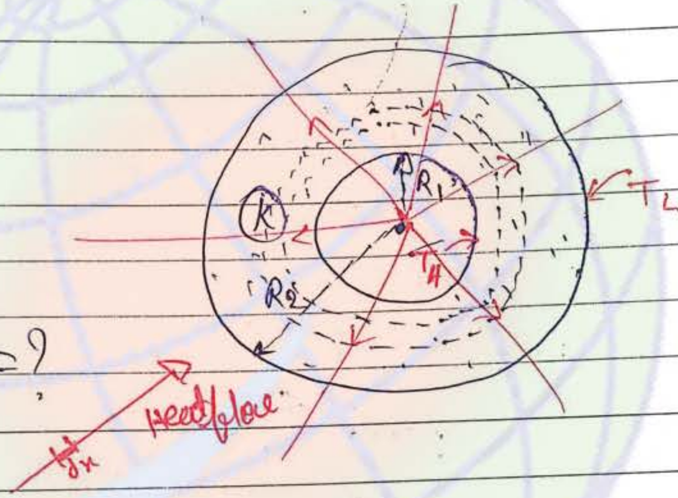
~~Radial heat flow through~~ a spher

Radial heat flow in case of spherical conductors

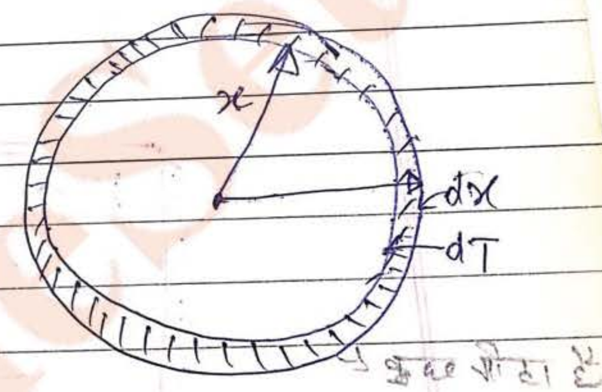
T_H → higher Temp

T_L → Low temp.

$H = ?$



$$H = -KA\frac{dT}{dx}$$



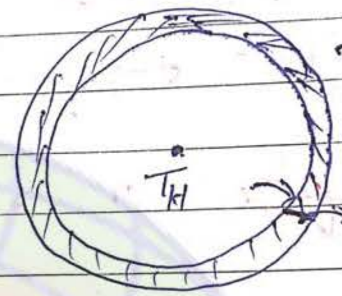
$$A = 4\pi r^2$$

$$H = -K(4\pi r^2)\frac{dT}{dx}$$

$$H\int_{x=R_1}^{x=R_2}\frac{dx}{x^2} = -(4\pi K)\int_{T_H}^{T_L} dT$$

$$\boxed{H = \frac{4\pi K R_1 R_2 (\Delta T)}{(R_2 - R_1)}}$$

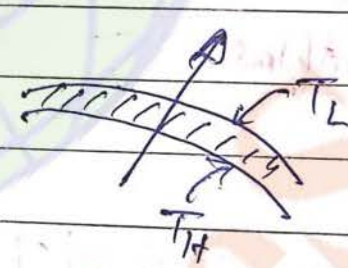
for thick :→



$\rightarrow x$ = thickness

$R_1 \approx R_2$

Appr $\left[\begin{array}{l} R_2 - R_1 = x \\ R_1 \approx R_2 = R \end{array}\right.$



$$\boxed{\begin{aligned} H &\Rightarrow \frac{KA\,\Delta T}{l} \\ &\Rightarrow \frac{K(4\pi R^2)\,\Delta T}{x} \end{aligned}}$$

so,

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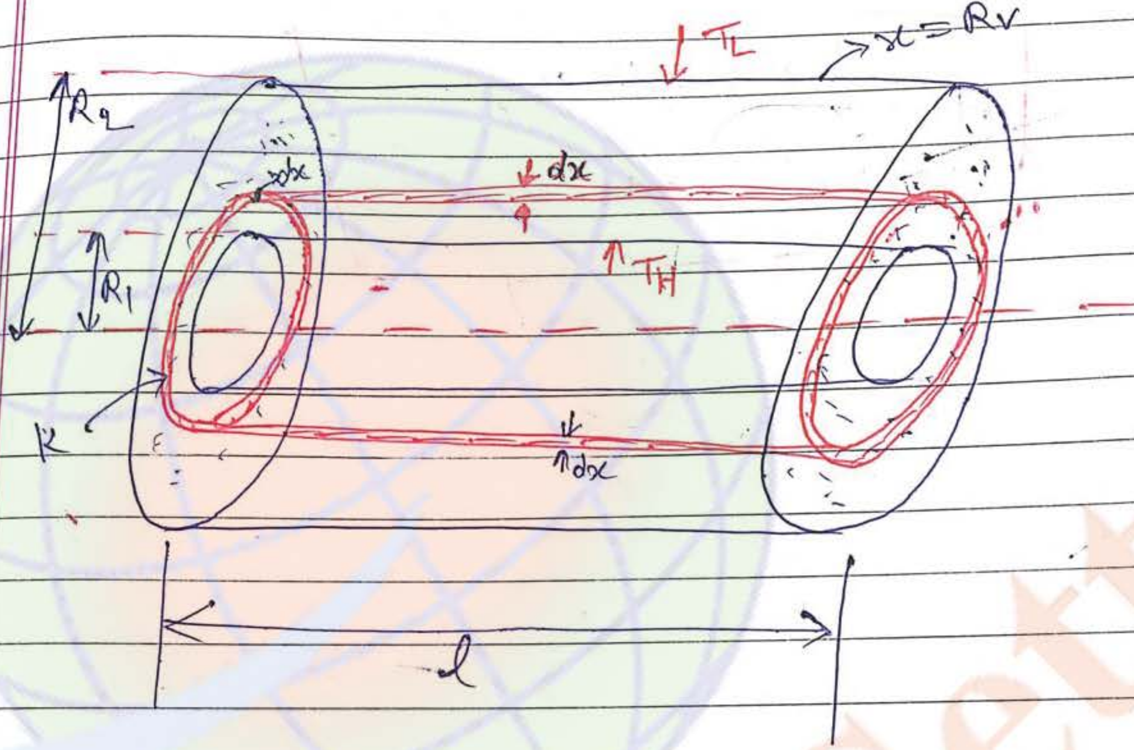
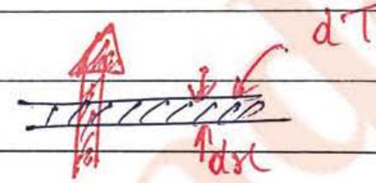
Radial heat flow through hollow cylinder.



$$H = -KA \frac{dT}{dx}$$



$$H = -K(2\pi x l)\frac{dT}{dx}$$

$$H\int_{x=R_1}^{x=R_2} \frac{dx}{x} = -(2\pi K l)\int_{T_H}^{T_L} dT$$

1st Choice

$$H \ln\left(\frac{R_2}{R_1}\right) = (2\pi K l)(T_H - T_L)$$

$$\boxed{H = \frac{(2\pi K l)(T_H - T_L)}{\ln\left(\frac{R_2}{R_1}\right)}}$$

Board + IIT

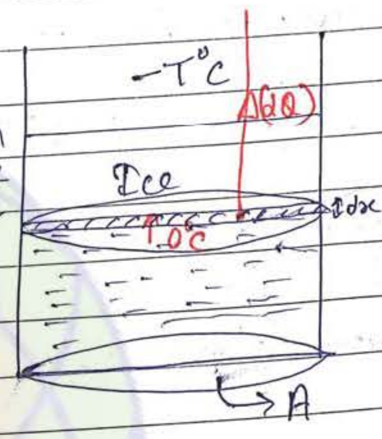
Density = $\frac{\text{mass}}{\text{volume}}$

Growth of Ice on Pond / lake

Heat released by water :→

$$dQ = m L_f$$

$$= A \cdot dx \, \rho L_f \quad \text{— (1)}$$



$$H = \frac{dQ}{dt} = kA\frac{(0-(-T))}{x}$$

$$dQ = \frac{kA\,T\,dt}{x} \quad \text{— (2)}$$

$$eq(1) = eq(2)$$

$$\cancel{A}\,dx\,\rho L_f = \frac{k\cancel{A}\,T\,dt}{x}$$

$$\frac{\rho L_f}{kT}\int_{x_1}^{x_2} x\,dx = \int_0^t dt$$

Imp

$$\boxed{t = \left(\frac{\rho L_f}{kT}\right)\left(x_2^2 - x_1^2\right)}$$

$$\boxed{t \propto \left(x_2^2 - x_1^2\right)}$$

If $t = 0$, $x_1 = 0$
$t = t$, $x_2 = x$

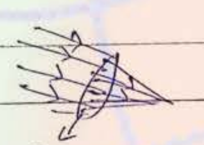
ρL = constant

Radiation

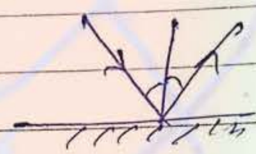
1) In case of radiation Heat is transfered from one place at high temperature to ~~lower~~ another place at lower temperature through electromagnetic waves (radiation)

2.) It is the fastest mode of heat transfer

3.) Heat radiation (thermal radiation) follows the law of Reflection, Refraction and other laws followed by light waves



Convex lens

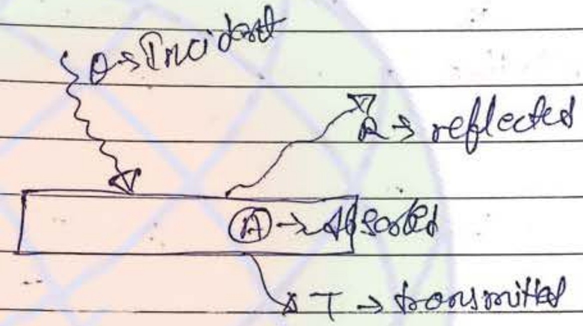


* Prevost theory of heat exchange :⇒

i) All the bodies at all the temperature (except ~~200~~ 0°K because 0°K is ~~not achieved~~ not achieved) and all the times emmite ~~e~~ e / absorbe heat radiation (or thermal radiation / ~~the~~ radiant energy /)

ii) Energy emmited by the body per unit time depends on absolute temperate of the surface of the body ~~at~~ and nature of surface of the body and its surface area.

iii) Energy absorbed (Power absorbed) by the body per unit time depends on the absolute temperature of ~~surro~~ surrounding, nature of surface of body and its surface area.



$$\frac{Q}{Q} = \frac{R + A + T}{Q}$$

$$\boxed{1 = \underbrace{\left(\frac{R}{Q}\right)}_{\text{Reflecting Power}} + \underbrace{\left(\frac{A}{Q}\right)}_{\text{Absorbing Power}} + \underbrace{\left(\frac{T}{Q}\right)}_{\text{Transmitting Power}}}$$

Absorbed ~~heat~~ Power:→

$$a = \frac{A}{Q}$$

absorptive Power

It is defined as the ~~abs~~ ratio of absorbed heat radiation and the total heat radiation at the given time

$$\boxed{a_{max} = 1} \Rightarrow A = Q$$

1st Choice

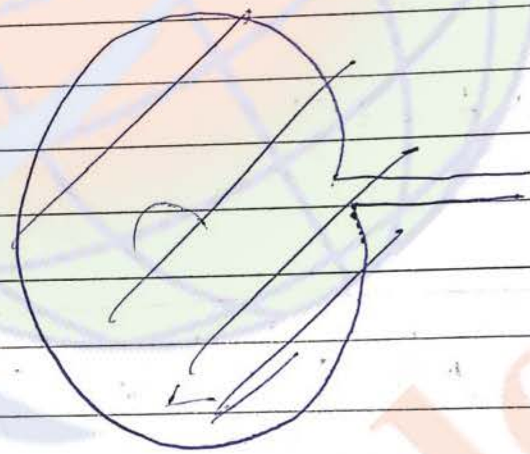
~~Black body :⇒~~

* Perfectly Black body :⇒ which absorbs all the thermal radiation incident over the surface.

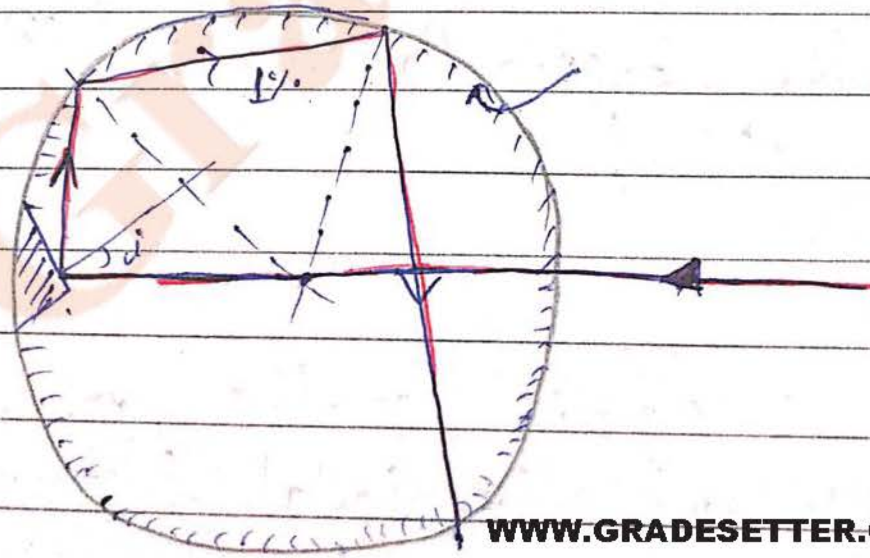
or

which absorbs the thermal radiation of all the wavelengths

$$\boxed{a_{max} = 1 \Rightarrow A = Q}$$



* Ferry's black body :⇒



Emissing Power :→

It is defined as energy emmited per unit time per unit Area by a body at the given temperature

$$E_p = \frac{P}{A \cdot t}$$

$$\boxed{P = E_p \cdot A \cdot t}$$

$$e_p = \frac{P}{A}$$

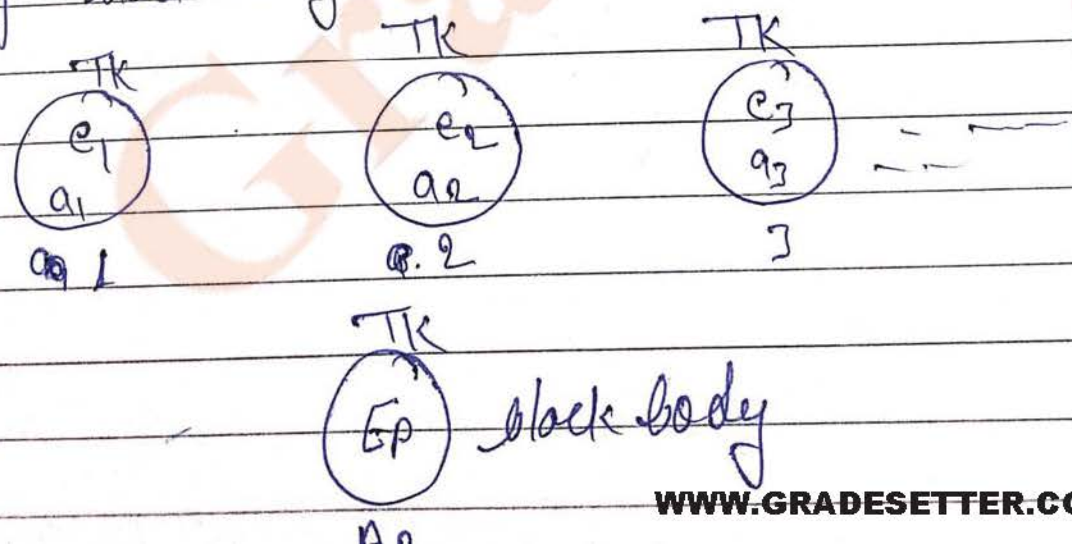
$$\boxed{P = e_p \cdot A}$$

Power emmited

As Area.

Kirchoff's law :→

The ratio of emmising power and absorping power of all the bodies at the same temperature are equal and it is equal to ratio of emmising power and absorping power of a perfectly black body at the same temperature.



$$\frac{e_1}{a_1} = \frac{e_2}{a_2} = \ldots\ldots$$

$$= \left(\frac{E_P}{A_P}\right)_{\text{black body}}$$

$$\downarrow = 1$$

∴ A good absorber of heat radiation is also a good ~~abs~~ emmetter of heat radiation.

Conduction law :-

Power $P = \frac{H}{A} = \frac{KA\Delta T}{d}$

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Stefan- Boltzmann law :→

Thermal radiation (thermal radiant energy) emmited per unit time per unit surface area of the perfectly black body is directly proportional to 4th power of absolute temperature of surface of black body.

$$E_p \propto T^4$$

sigma

$$\boxed{E_p = \sigma T^4}$$

Emissive power of perfectly black body.

σ → Stefan's constant

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$$

$$\boxed{P_{black} = \sigma A T^4}$$

VV Note :→

If a body is not a perfectly black body

$$\boxed{E_p = e \sigma T^4}$$

$\because \sigma T^4 = E$

emissive power

$e_p \Rightarrow$ emissive power of a body.

$E_p \Rightarrow$ emissive power of a black body.

$e_p = e E_p$

$$\boxed{e = \frac{e_p}{E_p}}$$ At same temperature

where

$e \Rightarrow$ emissivity $\Rightarrow$ It depends on smoothness and roughness of surface of body.

Perfect black body $e = 1$

$$\boxed{0 \le e \le 1}$$

$e = 1 \Rightarrow$ for perfectly black body

$e = 0 \Rightarrow$ for perfectly reflecting body.

$$\frac{e_p}{a_p} = E_p$$

$$\boxed{\frac{e_p}{E_p} = a_p = e}$$

$a_p \Rightarrow$ Absorptive power.

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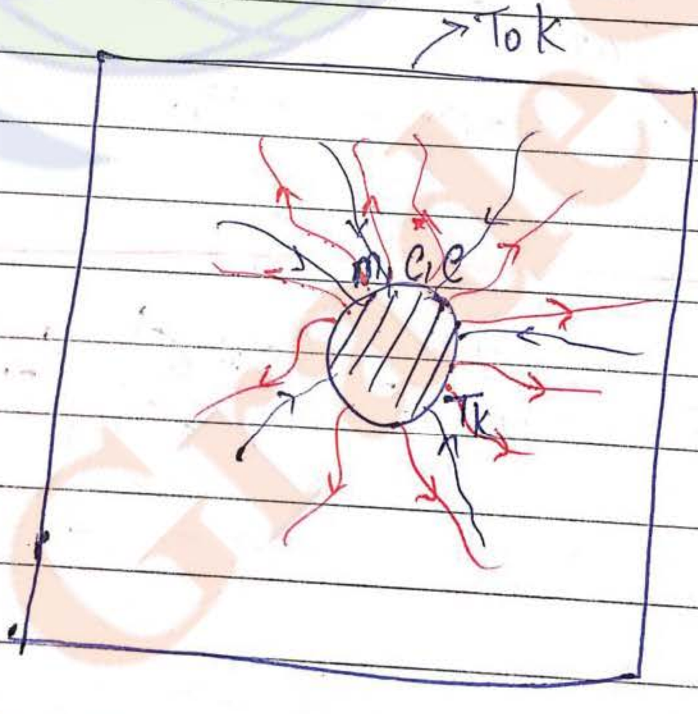
$$\boxed{P_{emission} = e A \sigma T^4}$$

Power emitted by a body ⟹ Rate of emission.

$$\boxed{P_{(absorption)} = e A \sigma T_0^4}$$

Rate of absorption

T_0 = Temp. of surrounding in "K".



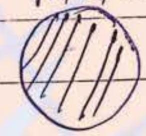
$(T > T_0)$

Cooling by Radiation:-

Stefan's law of cooling:-

$$\boxed{H = (P_{net})_{emission} = eA\sigma(T^4 - T_0^4)} \quad \text{--- (1)}$$

m, c, e



$$\frac{dQ}{dt} = mc\left(\frac{-dT}{dt}\right) \quad \text{--- (2)}$$

eq (1) = eq (2)

$$\boxed{R_c = \left(\frac{-dT}{dt}\right) = \left(\frac{eA\sigma}{mc}\right)(T^4 - T_0^4)}$$

Rate of Cooling

"Temp" Always in Kelvin

1st Choice

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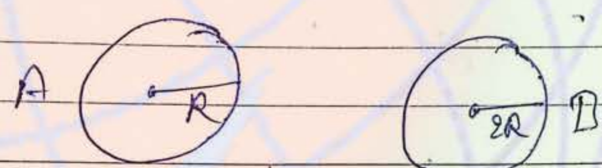
Q Two Spheres 'A' and 'B' made of same material and having same surface finish are kept in the same surrounding.

Temp. of both sphers are same, and greater than temperature of ~~surr~~ surrounding.

1) Find the ratio of rate of cooling of sphere 'A' and Rate of cooling of sphere 'B'



$$R_C = \frac{-dT}{dt} = \left(\frac{eA\sigma}{mc}\right)\left(T^4 - T_0^4\right)$$

$$R_C \propto \frac{A}{m}$$

$$A = 4\pi R^2$$

$$m = \frac{4}{3}\pi R^3 \times \rho$$

$$\boxed{R_C \propto \frac{1}{R}} \rightarrow \text{Radi}$$

Rate of culling in Inversly Proportional to the radia

$$\frac{R_{C,A}}{R_{C,B}} = \frac{R_B}{R_A} = \frac{2\cancel{R}}{\cancel{R}} = 2$$

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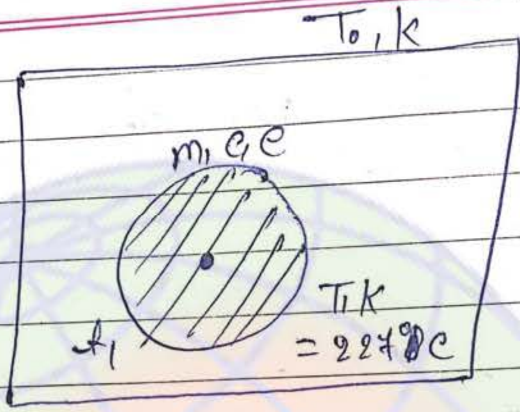
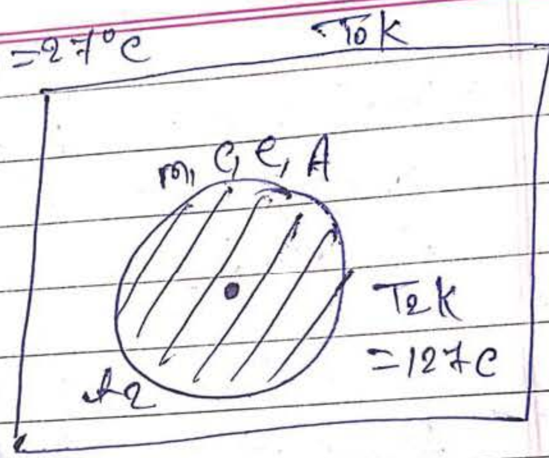
1st Choice

$T_0 = 27°C$

Ex!-





$T_2 > T_0$

R_{C_1} $T_1 > T_0$

R_{C_2}

$$\frac{R_{C_1}}{R_{C_2}} \Rightarrow \frac{(T_1^4 - T_0^4)}{(T_2^4 - T_0^4)}$$

$$\Rightarrow \frac{(5\cancel{00})^4 - (3\cancel{00})^4}{(4\cancel{00})^4 - (3\cancel{00})^4}$$

$T_1 = 227 + 273 = 500K$

$T_2 = 127 + 273 = 400K$

$$\Rightarrow \frac{625 - 81}{256 - 81}$$

$T_0 = 27 + 273 = 300K$

$\Rightarrow$

If $x <<< 1$

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Newton's law of cooling :⇒

(निउटन शीतलन नियम)

If the temp. of body is slightly different from the temp. of its surrounding,

The rate of cooling is directly proportional to Temperature difference between the system and surrounding.

$$\boxed{R_c \propto (T - T_0)}$$

$$T = T_0 + \Delta T$$ (If temp. difference is very small b/w system and surrounding)

$$\Delta T = (T - T_0)$$

$$R_c = -\frac{dT}{dt} = \left(\frac{ea\sigma}{mc}\right)\boxed{(T^4 - T_0^4)} \quad \text{--- (1)}$$

(dT → Temperature; dt → time)

$$T = T_0 + \Delta T$$
$$\Delta T = T - T_0$$

$(1+x)^n = 1 + nx$
If $x <<< 1$
so,
$\frac{\Delta T}{T_0} <<<< 1$

Concept
Required

$$T^4 - T_0^4 = (T_0 + \Delta T)^4 - T_0^4$$

$$= T_0^4\left[\left(1 + \left(\frac{\Delta T}{T_0}\right)\right)^4 - 1\right]$$

$$\Rightarrow T_0^4\left[1 + 4\left(\frac{\Delta T}{T_0}\right) - 1\right]$$

$$T^4 - T_0^4 = (4T_0^3)(\Delta T)$$

$$R_e = \frac{-dT}{dt} = \boxed{\left(\frac{ea\sigma\, 4 T_0^3}{mc}\right)} \Delta T$$

$$\boxed{R_e = \frac{-dT}{dt} = \alpha\,(\Delta T)}$$

$$\boxed{R_e = \frac{-dT}{dt} = \alpha\,(T - T_0)}$$

$(\because T = T_0 + \Delta T)$

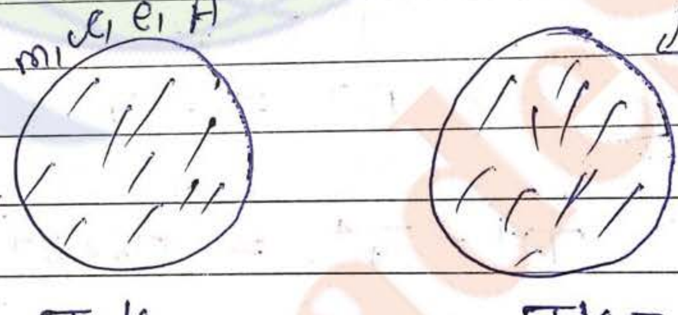
$\Rightarrow \Delta T = T - T_0$

where
$$\alpha = \left(\frac{eA\sigma\, 4T_0^3}{mc}\right) = \text{constant for a given body in the given surrounding}$$

⇒ Application of Newton law of cooling:

At $t = 0$ → m, c, e, A

at $t = t$



T_i K $\quad$ T K = ?

where T_0 → constant temp. of surrounding

$T_i > T_0$ $\quad$ $T > T_0$

$$\frac{-dT}{dt} = \alpha\,(T - T_0)$$

$$\int_{T_i}^{T} \frac{dT}{(T-T_0)} = -\alpha \int_{t=0}^{t} dt = -\alpha t$$

$$\left[\ln(T-T_0)\right]_{T_i}^{T} = -\alpha t$$

$$\ln \boxed{\frac{T-T_0}{T_i-T_0}} = -\alpha t$$

$$\frac{T-T_0}{T_i-T_0} = e^{-\alpha t}$$

$$T-T_0 = (T_i-T_0)\, e^{-\alpha t}$$

Impotant

$$\boxed{T = T_0 + (T_i + T_0)\, e^{-\alpha t}}$$

after long time	
$t = \infty$	$t = 0$
$T = T_0$	$T = T_i$
final temp. of body → Temp. of surrounding	Fi. temp. of body → Initial temperature

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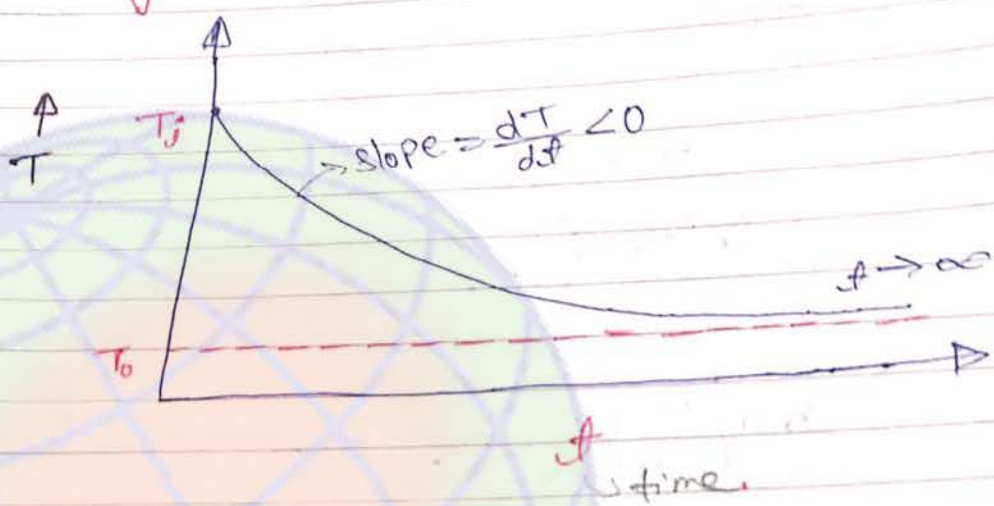
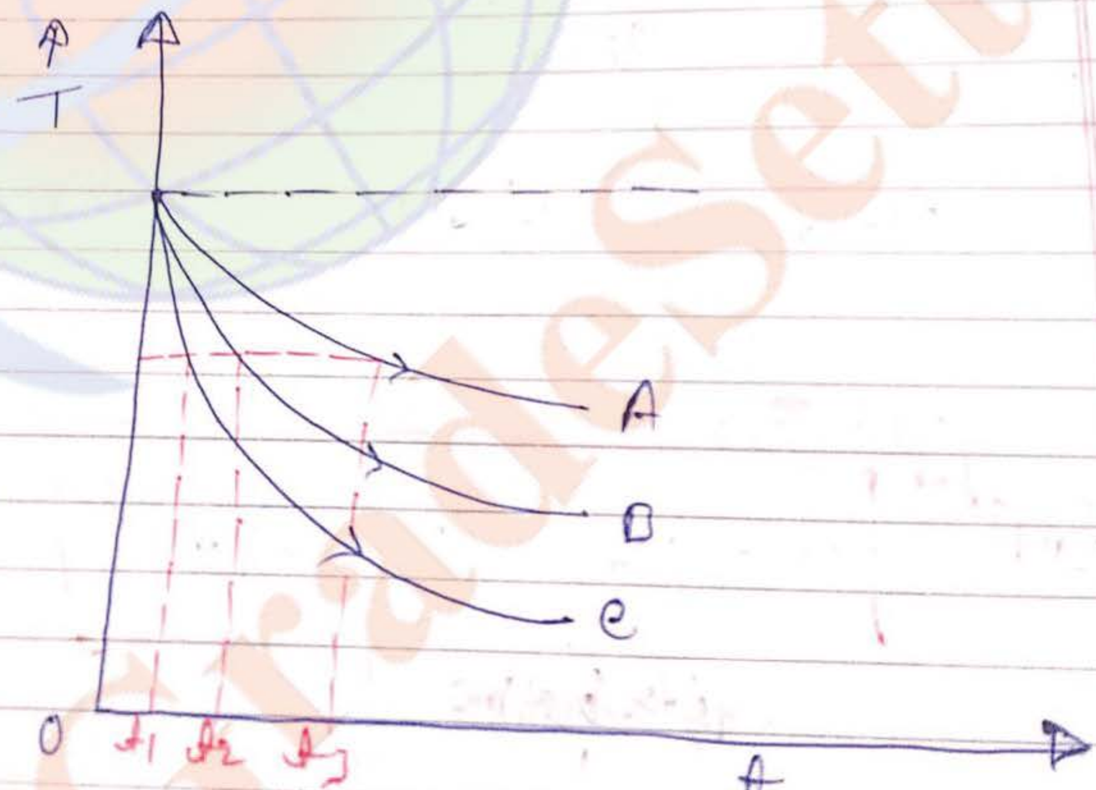
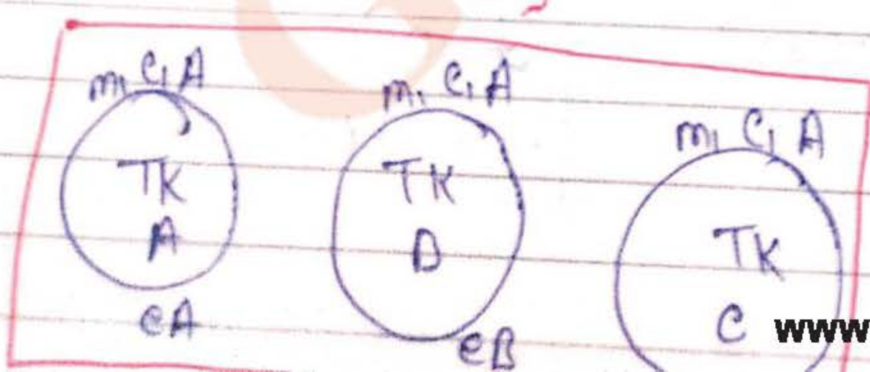
1st Choice

Cooling curve —



Example :→





which is correct :->

1) $e_A > e_B > e_C$

~~ii)~~ $e_A < e_B < e_C$

(iii) $e_A > e_C > e_B$

iv) $e_B > e_C > e_A$

Ans :->

$$\uparrow R_C \propto e \uparrow \propto a \uparrow$$

(ii) $e_A < e_B < e_C$

$$\therefore R_C = \frac{-dT}{dt} = \frac{ea\sigma}{mc}(T^4 - T_0^4)$$

Note :->
(सबसे नीचे वाला black body होगा यहाँ black bod "C" है।

black body सबसे जल्दी गर्म होगा और सबसे जल्दी ठंडा भी होगा।)

Note + Note :

If a body cools from θ_1°C to θ_2°C in time t,

$$\boxed{\frac{\theta_1 - \theta_2}{t} = \alpha\left(\theta_{average} - \theta_0\right)}$$

where

$$\theta_{avg} = \frac{\theta_1 + \theta_2}{2}$$

θ_0 = Temp. of Surrounding.

Q. Ex. Q :-> If a body cools from 70°C to 60°C in time 5 minutes, what is the time taken from 60°C to 50°C. Temp. of Surrounding is 20°. Find time taken.

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1st Choice

$$\Rightarrow \frac{\theta_1 - \theta_2}{t} = \alpha\left(\theta_{avg} - \theta_0\right)$$

$$\Rightarrow \frac{10}{5} = \alpha(45) \quad \text{—(I)}$$

$$\frac{10}{t} = \alpha(35) \quad \text{—(II)}$$

$$t = 6.4 \text{ min}$$

अनुमान
$$\downarrow R_c = \frac{\Delta T}{\Delta t}$$

Ex) A blackbody is at Temp of 400 K and the temp. of surrounding is 300 K. At time t = 0 ~~temp. of body = 400 K~~. Assume the temp. of surrounding is constant.
And body follows the Newton's law of cooling.
Find the time taken to fall the temp. from 400 to ~~300 K~~ 350 K.
$\alpha \rightarrow$ Const of proportionality.

Ans

$$\frac{-dT}{dt} = \alpha\left(T - T_0\right)$$

$$\ln\left(\frac{T-T_0}{T_i-T_0}\right) = -\alpha t$$

$$\ln\left(\frac{T_i-T_0}{T-T_0}\right) = \alpha t$$

Important

$$\boxed{t = \frac{1}{\alpha}\ln\left(\frac{T_i-T_0}{T-T_0}\right)}$$

IIT का asked Question

so, $T_i = 400\,K$
$T = 350\,K$
$T_0 = 300\,K$

$$t = \frac{1}{\alpha}\ln\left(\frac{400-300}{350-300}\right)$$

$$= \frac{1}{\alpha}\ln\left(\frac{100}{50}\right)$$

$$= \frac{1}{\alpha}\ln 2$$

[Q] A copper sphere is ~~...~~ suspended in a room and the temp. of sphere is maintained at constant temp of 400 K by using electrical heater of 200 W.
If the copper sphere is painted black then it is maintained at the same constant temp of 400 K in the same environment by using electric heater of 300 W.
1) Find the emissivity of copper sphere ~~surface~~ ~~and~~ the temp. of room is 300 K.

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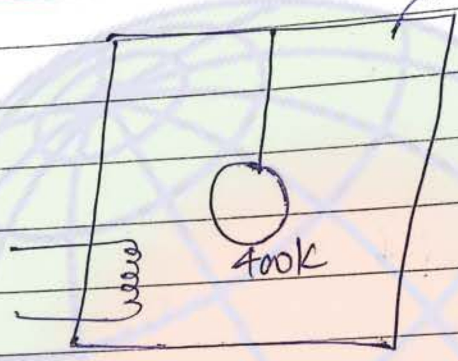
$\rightarrow T_0 = 300K$

An



$2\cancel{00} = (P_{net})_{emission} = eA\sigma(\cancel{T^4 - T_0^4})$ — (1)

divide

$3\cancel{00} = P_{net_{emission}} = 1 \cdot \cancel{A\sigma}(\cancel{T^4 - T_0^4})$ — (2)

so,

$e = \frac{2}{3}$

(for black body $e = 1$)

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Wein's law :→

$$\boxed{\lambda_m \times T = b}$$

where

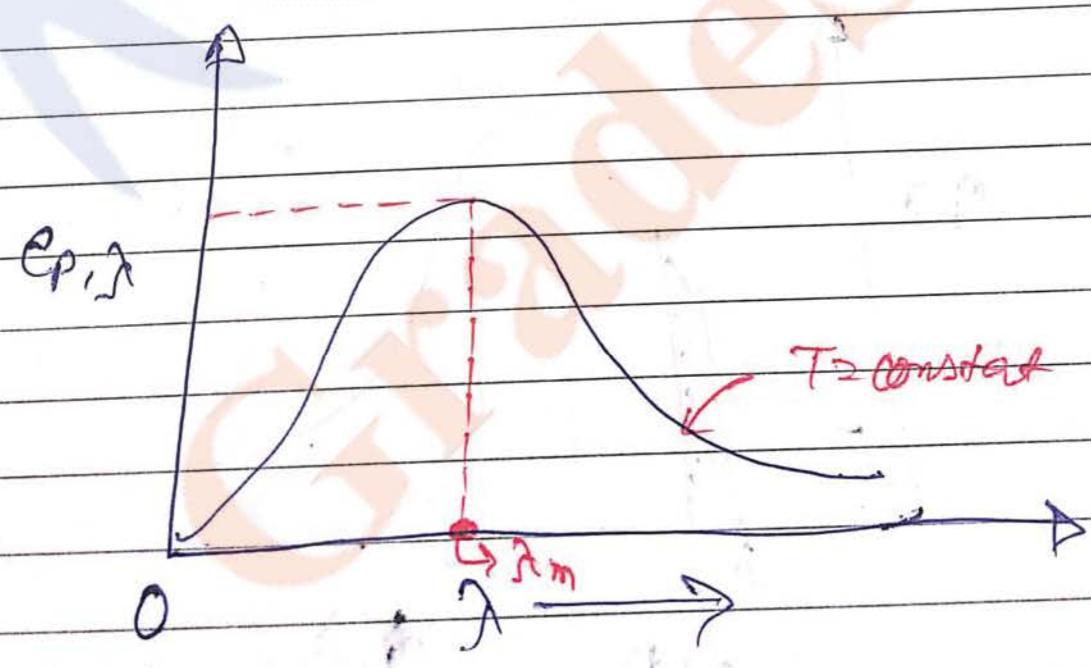
$b \rightarrow$ wein's constant

$b = 2.88 \times 10^{-3}$ m-K

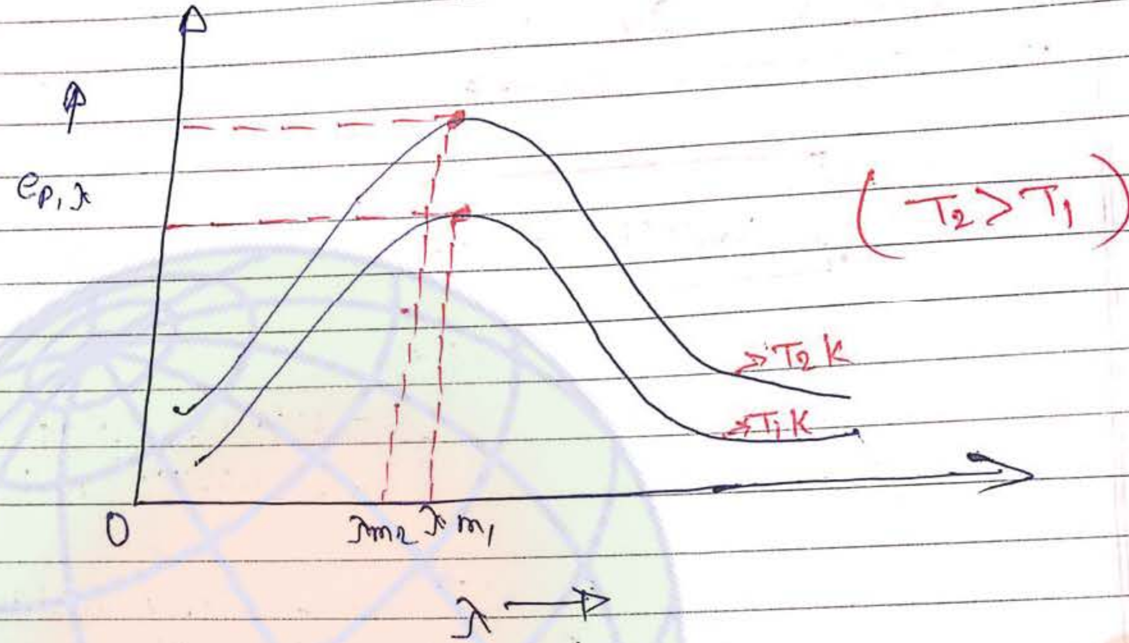
$T \Rightarrow$ Temp. of body in kelvin

$\lambda_m \Rightarrow$ It is the wavelength corresponding to maximum spectral emissive power / (spectral Intensity) / (spectral radiancy)

maxwell distribution law:→



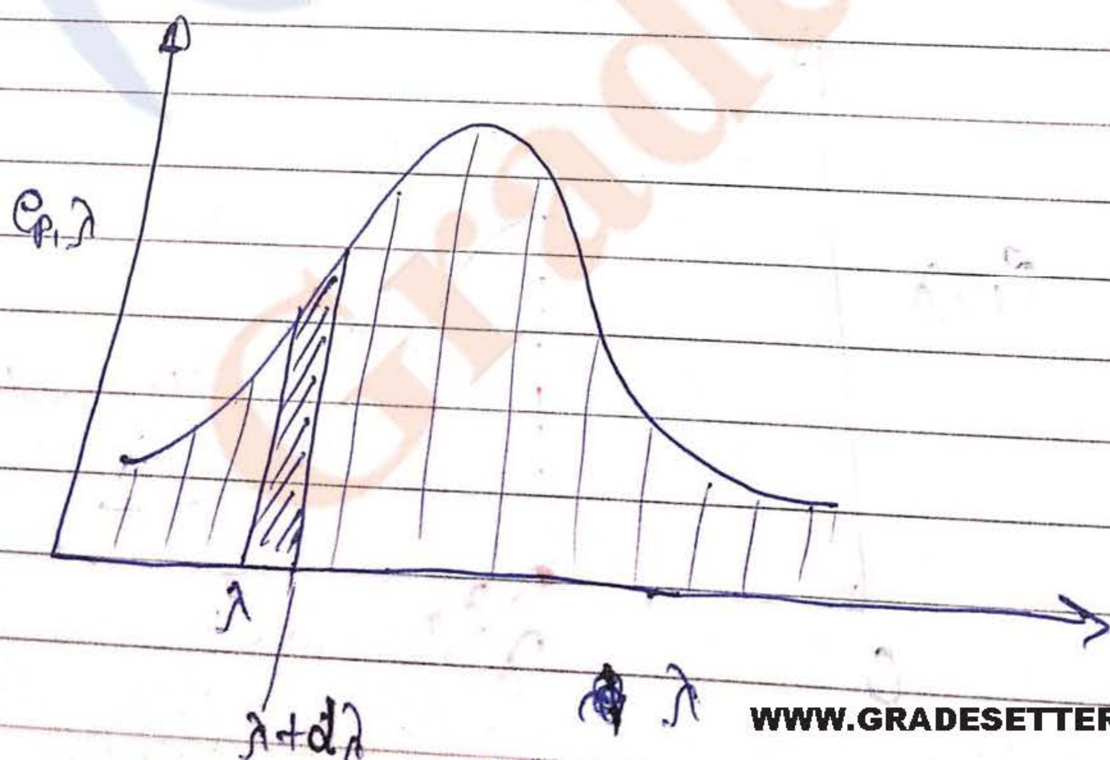
Wein's law



$$\boxed{\lambda_m \times T = b}$$

$$b = 2.88 \times 10^{-3}\ m \cdot K$$

$$\lambda_m \propto \frac{1}{T}$$



energy per unit time per unit area → intensity

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$e_P = e\sigma T^4$

Imp

$$\boxed{\begin{array}{l}\text{Area bounded by graph} = \int e_{P,\lambda}\, d\lambda \\ \Rightarrow \text{Total emissive power} = e_P \propto T^4\end{array}}$$

Plank's energy distribution law :⟹

Total Radiation energy is not uniformly distributed among its various wavelength

* Peak of graph :-

$$\boxed{\text{Peak of graph} \propto T^5}$$

$$\frac{e_P}{\lambda_m} \propto T^4$$

$$(e_{P,\lambda})_{max} \propto T^4 T$$

$$(e_{P,\lambda})_{max} \propto T^5$$

$\lambda_m \times T = b$

$\lambda_m \propto \frac{1}{T}$

nero make ग़लत

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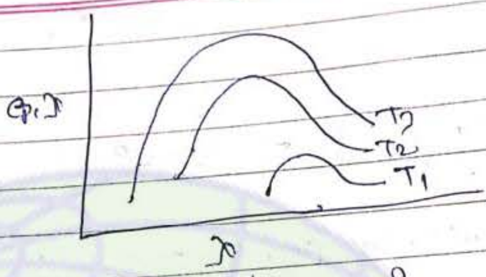


correct order :-

↳ $T_3 > T_2 > T_1$

115/58 Energy radiated by a ~~bad~~ body at a given temperature is E_1 in the range 499 nm to 500 nm, is E_2 in the range 999 nm to 1000 nm, and E_3 in the range 1499 nm to 1500 nm.

$b = 2.88 \times 10^{-3}$ mK

~~T = 2880 K~~

Temp. of body (T) = 2880 K

correct ?

(i) $E_2 > E_1$

(ii) $E_1 > E_2$

(iii) $E_2 > E_3$

(iv) $E_1 = 0$

(v) $E_3 = 0$

Ans $\lambda_m \times T = b$

$$\lambda m = \frac{b}{T} = \frac{2.88 \times 10^{-3}}{2880}$$

$$= 10^{-6} m$$

$1 m = 10^9 nm$

$$\lambda m = 10^{3} nm$$

$$= 1000 nm$$

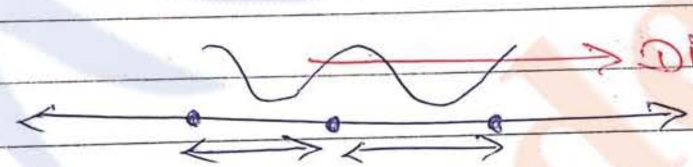
Waves

It is the disturbance produced in the medium which propagates from one place to another place without movement of material.
(waves transfer both energy and momentum.)

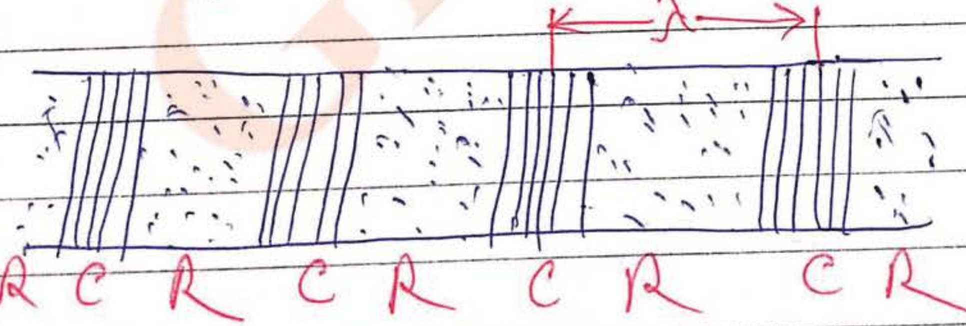
Waves

- Mechanical waves
  - Require medium for their propagation
  - eg → Sound waves
- Non-mechanical waves
  - Do not require any medium
  - eg → light waves.

1.) Longitudinal waves :- Particles of the medium oscillate along the direction of propagation of waves



* Longitudinal waves travels in the form of compression and rarefaction.



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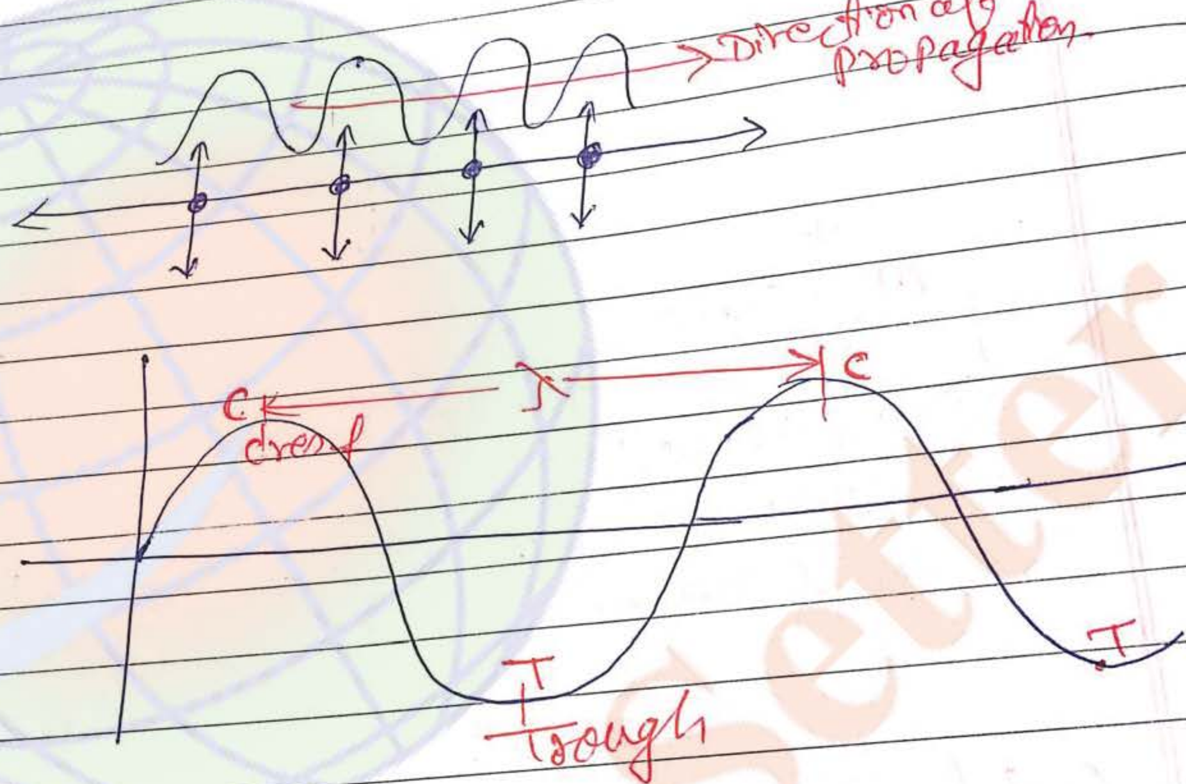
2.) Transversal waves → Particles of the medium oscillate "⊥" to the direction of propagation of waves.



On surface → "Ripple waves" neither transverse nor longitudinal

In water → Longitudinal waves.

velocity = frequency × wavelength

General equation of travelling wave :-

$$\boxed{y = f(x \pm vt)}$$

v = velocity of wave

$y = f(x, t)$

$$\boxed{y = f(ax \pm bt)}$$

* y must have finite value (Including zero). for all values of x and t.

$$\boxed{\text{velocity of wave}(v) = \frac{\text{Coeff. of "t"}}{\text{Coeff. of "x"}}}$$

* '+' ⇒ If the wave is travelling along −ve direction (X axis)

'−' ⇒ wave is travelling along "+ve" direction (+X axis)

Ex) $y = \ln(x - 3t)$

$y = f(x - vt)$

Put $x = \infty$, $t = \infty$

($y \neq$ Infinite)

All traveling waves is a wave but all waves in not travelling waves.

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(x) $y = e^{-(x-3t)^2}$

$t = \infty$, $x = \infty$

$y = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

(yes)

(x) $y = \frac{10}{2+(x-3t)^2}$

$v = 3\,m/s$ along $+x$-axis

$y = max = 5m$

x $y = A\sin(0.3x - 0.4t)$

General Equation of "wave" ⇒

$$\boxed{\frac{d^2y}{dt^2} = (v^2)\frac{d^2y}{dx^2}}$$

v ⇒ velocity of wave.

Equation of Plane Progressive/travelling harmonic wave :⇒

$$\boxed{y = A\sin(\omega t \pm kx \pm \phi_0)}$$

Note :⇒ Eq of Simple harmonic motion
$x = A\sin\underbrace{(\omega t + \phi_0)}_{Phase}$

"y" ⇒ y is the displacement of particles of the medium (w.r.t. mean position) oscillating simple harmonically (but it can represent electric field, magnetic field, pressure, density).

A ⇒ A is the amplitude of wave, It is maxⁿ value of 'y'.

ϕ_0 ⇒ It is Initial phase at $t = 0$

x ⇒ location or position of different particles of medium

HO (60, 61) D.P.P.-5

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$K \rightarrow$ It is wave number.

$$\boxed{k = \frac{2\pi}{\lambda}}$$

where
$\lambda \rightarrow$ wavelength of wave

$\omega \rightarrow$ Angular frequency.

~~For the given medium~~

$$\boxed{v = \frac{\omega}{K} = f\lambda}$$

$v \rightarrow$ velocity of wave

Imp

Frequency of wave only depends on source produced in the wave Independent of all other factor.

$$\boxed{y = A\sin(\omega t \pm kx \pm \phi_0)}$$

$\phi_0 = 0$

$y = A\sin(\omega t - kx)$

$y = A\sin(kx - \omega t)$

$\sin[(\omega t - kx) + \pi] = -\sin(\omega t - kx)$

Also

$$\boxed{y = A\sin(\omega t \pm kx \pm \phi_0)}$$

$\rightarrow \phi_0$

$$y = A\sin\left(kx - \omega t + \frac{\pi}{2}\right)$$

$$y = A\sin k\left(x - \frac{\omega}{k}t + \frac{\pi}{2k}\right)$$

$$y = f(x - vt)$$

so,

$$\boxed{y = A\cos(kx + \omega t)}$$

$$y = A\sin(\omega t - kx)$$

$\rightarrow$ constant (x-constant)

diff w.r.t "t",

(y-t graph) $\boxed{V_{particle} = \frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)}$ ← (1)

diff w.r.t "t"

$$\boxed{\begin{aligned} a_{particle} &= -\omega^2 A\sin(\omega t - kx) \\ &= -\omega^2 \cdot y \end{aligned}}$$

$y = 0$, $a_{particle} = 0$ (mean Position.)

(velocity of wave depends on the medium.)

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* $y = A\sin(\omega t - kx)$

diff w.r.t. "x" (t = constant)

$$\boxed{\text{slope of tangent line at any point of y-x graph} = \frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx)} \quad \text{—— (2)}$$

Now

eq (1) ÷ eq (2)

$$V_{Particle} = -\left(\frac{\omega}{k}\right)\cdot(\text{slope})$$

Imp

N.B.

$$\boxed{V_{Particle} = -v\,(\text{slope})}$$

(इस case में same होगा)

(more no. of questn is solved by this formula)

Notes ->

v = +ve ⇒ If wave is travelling along "+" x-axis

v = −ve ⇒ If wave is travelling along "−ve" x-axis

* $y = A\sin(\omega t - kx)$

$$y = A\sin\omega\left(t - \frac{k}{\omega}\cdot x\right)$$

$$\boxed{y = A\sin\omega\left(t - \frac{x}{v}\right)} \text{ or } \boxed{y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)}$$

$\frac{3\times0.01}{0.03}$ Page No. Date / /

Q.) $y = 4\sin 3(t - 0.01x)$ (find $k = ?$)

Now

$y = 4\sin(3t - 0.03x)$

$\rightarrow k = 0.03 = \frac{2\pi}{\lambda}$

$\lambda =$ ___

Q.1)

$y = A\sin(\omega t - kx)$

at $t = 0$

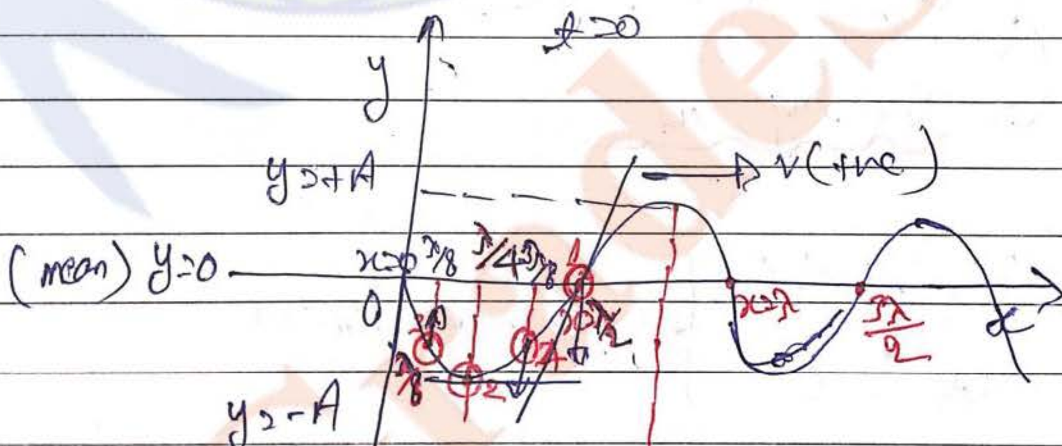
Find the nature of velocity and acc^n of different particles of medium located at the following position.

i) $x = \frac{\pi}{k}$ ii) $x = \frac{\pi}{2k}$ iii) $x = \frac{\pi}{4k}$ iv) $x = \frac{3\pi}{4k}$

Ans:> $t = 0$

$y = A\sin(-kx)$

$y = -A\sin kx$ $[\sin(-\theta) = -\sin\theta]$



i) $x = \frac{\pi}{k} = \frac{\pi\lambda}{2\pi} = \frac{\lambda}{2}$ (similarly other.)

iv) $x = \frac{3\pi}{4k} = \frac{3\lambda}{8}$

sol

(i) $x = \frac{\lambda \pi}{2\pi} = \frac{\lambda}{2}$; $y > 0$

$\because V_{parti} = -v(\text{slope})$

~~V_{pa} = -ive~~ , ~~a_{pa}~~

$(V_{pa} = -ve)$, $(a_{pa} = 0)$

ii) $V_{2\,min} = 0$

$\left(a_{pa\,max}\right) = |\omega^2 A|$

$a_{pa} = -\omega^2 y$

$y < 0$

$a_{pa} > 0$

iii) $(V_{pa})_3 = +ve$

$a_{pa} = +ve$, $y < 0$

→ speeding up

iv) $(V_{pa})_4 < 0$
$a_{pa} > 0$] → slowing down

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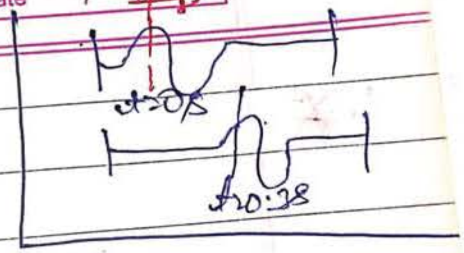
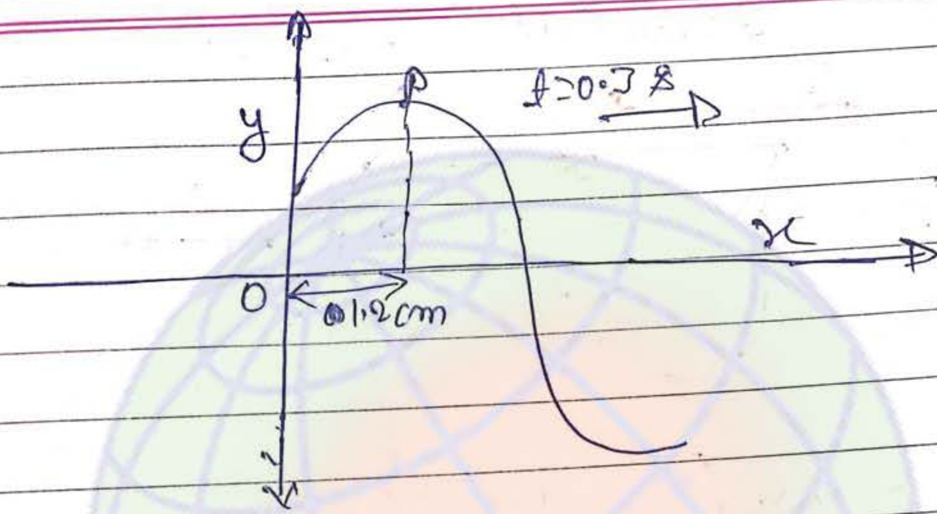


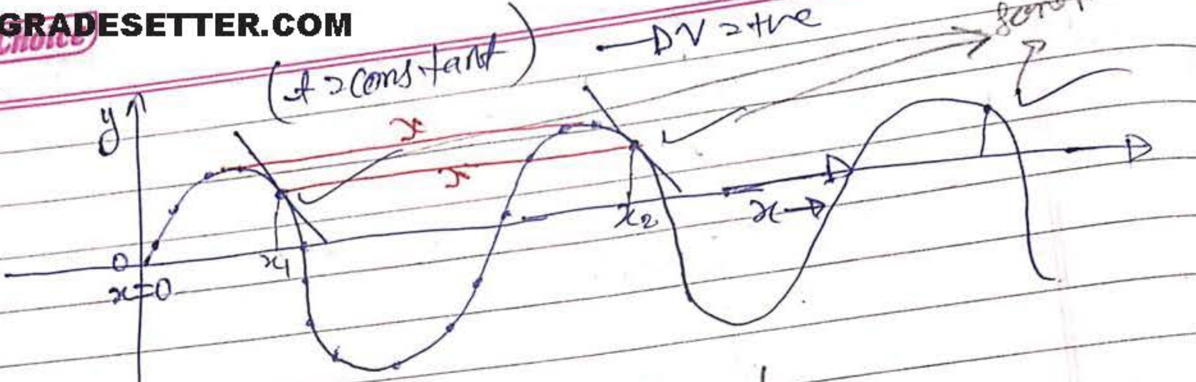


At time $t=0$ crest 'P' of the wave was at $x=0$

1) Find the speed of wave.

$$V = \frac{1.2\ \text{cm}}{0.3\ \text{s}}$$

$$= 4\ \text{cm/s}$$

1.) 

constant time (एक time पर सभी particle का graph)

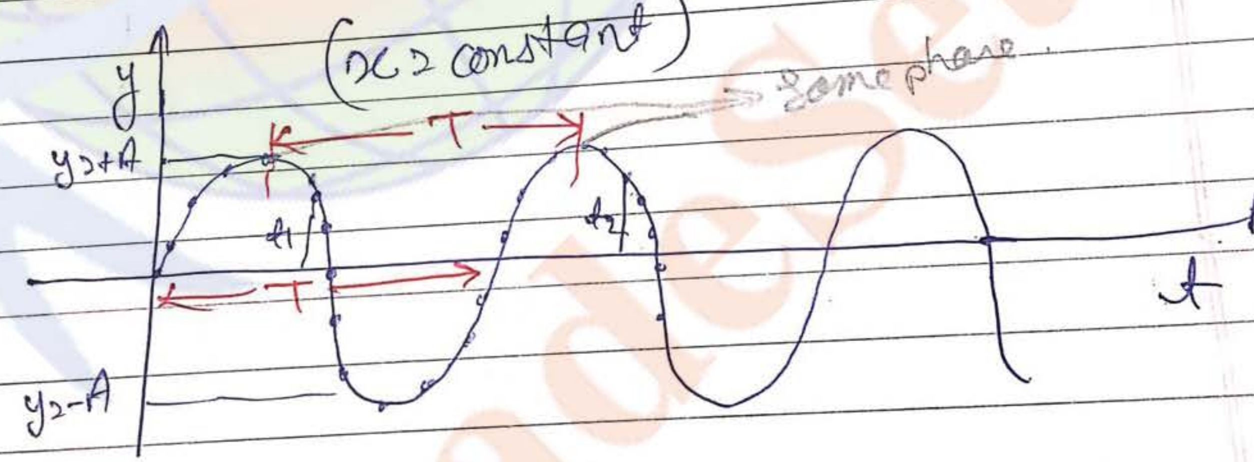
$$y_1 = A\sin(\omega t - kx_1) \rightarrow \phi_1$$

$$y_2 = A\sin(\omega t - kx_2) \rightarrow \phi_2$$

$$\boxed{\text{Phase diff} = \Delta\phi = k(\Delta x)}$$

$$\Delta x = x_2 \sim x_1$$

$$k = \frac{2\pi}{\lambda} \rightarrow \text{mean same phase.}$$

2.) 

(एक ही particle का अलग अलग time पर graph)

$$y_1 = A\sin(\omega t_1 - kx) \rightarrow \text{phase.}$$

$$y_2 = A_2\sin(\omega t_2 - kx)$$

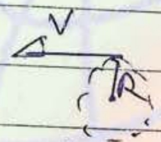
so,

$\boxed{\Delta\phi = \omega(\Delta t)}$ $\quad \Delta t = (t_2 - t_1)$

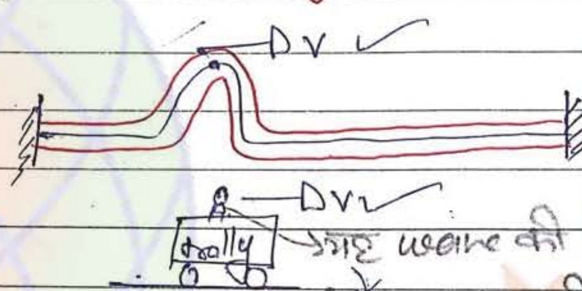
$\Delta\phi = \frac{2\pi}{T}(\Delta t)$ → same phase ।

3/1/2018

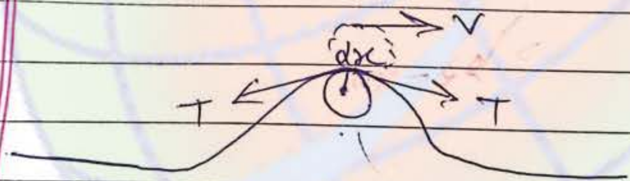
Speed of Transverse wave on Stretched String :⟹

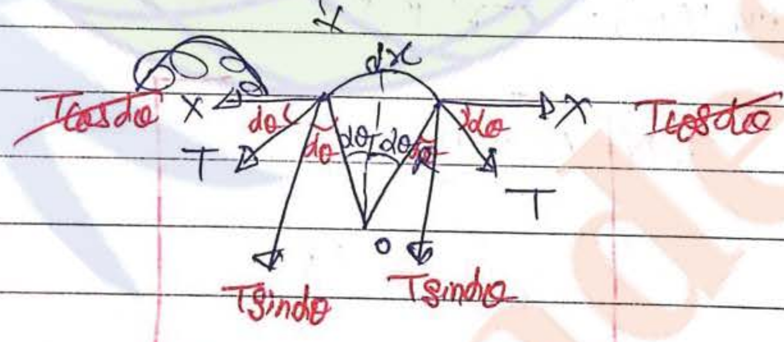


$2T\sin d\theta = \frac{(dm)v^2}{R}$



यह wave की देखरहा हूँ (observer की string पीछे की ओर खींच की तरह भागता दिखाई देगा





for very small

$\sin d\theta \approx d\theta$

$dx = R(2d\theta)$

$dm = \mu(dx)$

so,

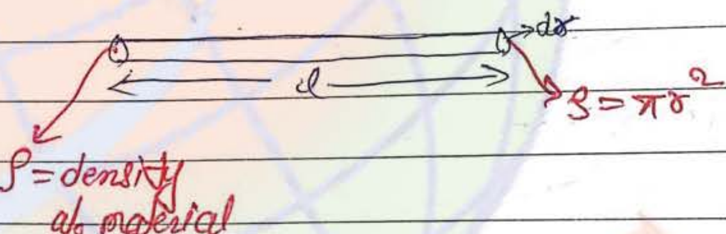
$$2T\sin d\theta = \frac{(dm)v^2}{R}$$

$$2T(d\theta) = \mu \frac{(dx)v^2}{R}$$

$$T(2(d\theta)) = \frac{\mu R\, 2(d\theta)v^2}{R}$$

Proof
$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

where
$F = T \Rightarrow$ Tension in the String
$\mu \Rightarrow$ mass per unit length or
(linear mass density).



$$\text{mass}(m) = \pi r^2 l \rho$$

$$\text{so}\quad \boxed{\mu = \frac{m}{l} = \pi r^2 \rho \qquad \mu = S\rho}$$

$$\boxed{v = \sqrt{\frac{T}{S\rho}} = \sqrt{\frac{T}{\pi r^2 \rho}}}$$

and also,

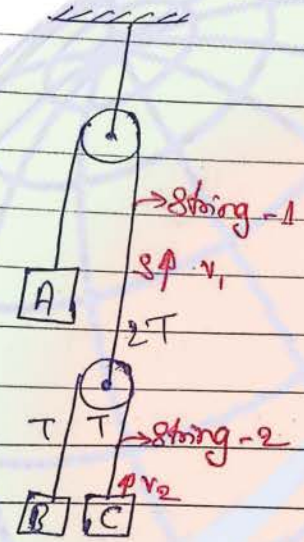
$$\boxed{v \propto \frac{1}{r}} \; ; \; (T, \rho) \rightarrow \text{Constant}$$

Tension in the string, density of material.

$$\boxed{v \propto \sqrt{T}} \; ; \; (r, \rho) \rightarrow \text{Constant}$$

Radius of material, density of material.

Ex:-



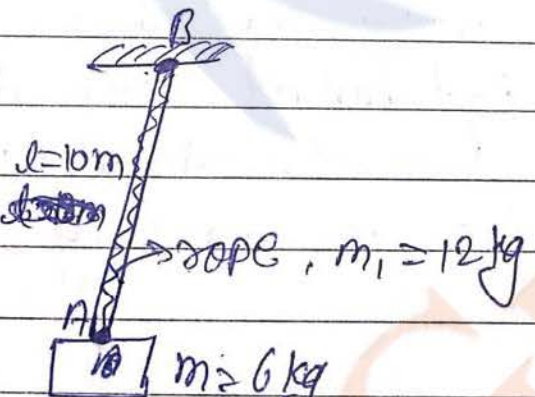
Block are at rest

$\frac{v_1}{v_2} = ?$

Ideal → Pulley and string.

Both string are made of same material and having same thickness.

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{2\cancel{T}}{\cancel{T}}} = \sqrt{2}$$



A Transverse wave pulse of wavelength 0.02m is produced at the end "A" and it travels along the rope AB.

1) Find the wavelength of the wave at Point 'B'

$$v = f \cdot \lambda$$

$$\sqrt{\frac{T}{\mu}} = f\lambda$$

$$\frac{\sqrt{T}}{\lambda} = \text{constant}$$

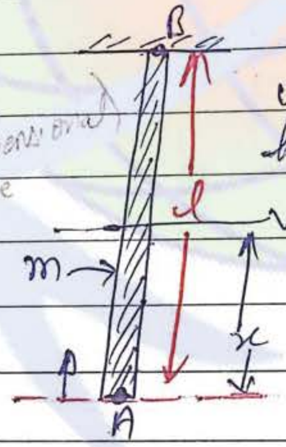
$$\frac{\sqrt{T_A}}{\lambda_A} = \frac{\sqrt{T_B}}{\lambda_B}$$

$$\lambda_B = \lambda_A \sqrt{\frac{T_B}{T_A}} = 0.02\sqrt{\frac{18\cancel{0}}{6\cancel{0}}}$$

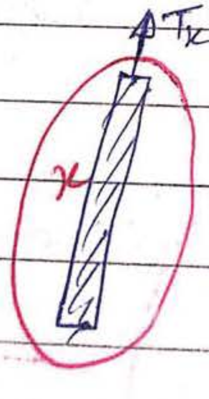
$$= (\sqrt{3})\,(0.02)\,m$$

$$= 1.7 \times 0.02\ m$$

Eg:



A transverse wave pulse is produced at end 'A' of the rope. Find the time taken by the wave pulse in reaching from 'A' to 'B'.



$$v = \sqrt{\frac{T_x}{\mu}} = \sqrt{\frac{\left(\frac{m}{l}\cdot x\right)g}{\frac{m}{l}}}$$

so,

$$v = \sqrt{gx}$$

$$\frac{dx}{dt} = (\sqrt{g})\sqrt{x}$$

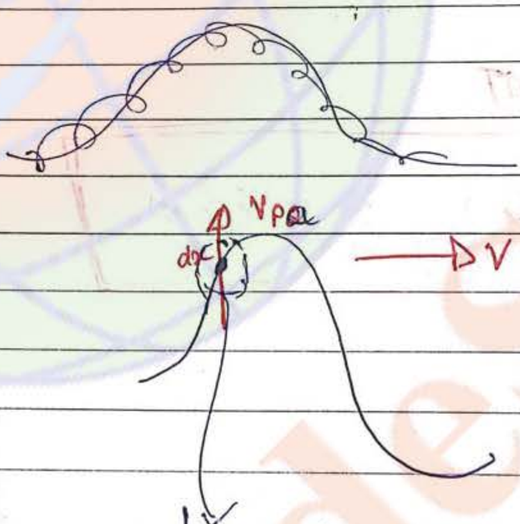
$$\int_{x=0}^{x=l} \frac{dx}{\sqrt{x}} = (\sqrt{g})\int_{t=0}^{t} dt$$

(sound में भी यही लागू होगा)

Energy in wave Motion

1.) K.E of an element (dx) Per unit volume → (K_V)

K.E Per unit volume



$$V_{p\alpha} = \frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$$

$dx \rightarrow S$

$$dV = S \cdot dx$$

$$(dm) = (dV)\rho$$

$$\left(\frac{dm}{dV}\right) = \rho$$

$$y = A\sin(\omega t - kx)$$

$$K_V = \frac{1}{2}\left(\frac{dm}{dV}\right)\cdot V_{p\alpha}^2$$

$$\boxed{K_V = \frac{1}{2}\rho V_{p\alpha}^2}$$

$$K_V = \frac{1}{2}\rho\omega^2 A^2\cos^2(\omega t - kx)$$

* For 1T / nxT ($n = 1, 2, 3, 4 \ldots$)

$$\boxed{(K_v)_{avg} = \frac{1}{4}\rho\omega^2A^2}$$

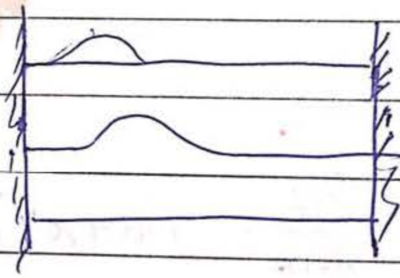
⇒ Potential energy (P.E.) of an element (dx) per unit volume (U_v)

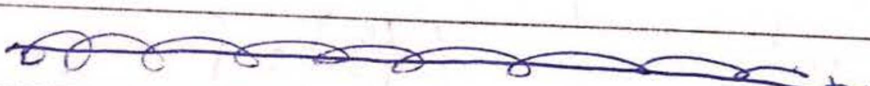
$$\boxed{U_v = \left(\frac{1}{2}\rho\omega^2A^2\right)\cos^2(\omega t - kx)}$$

For 1T / nT

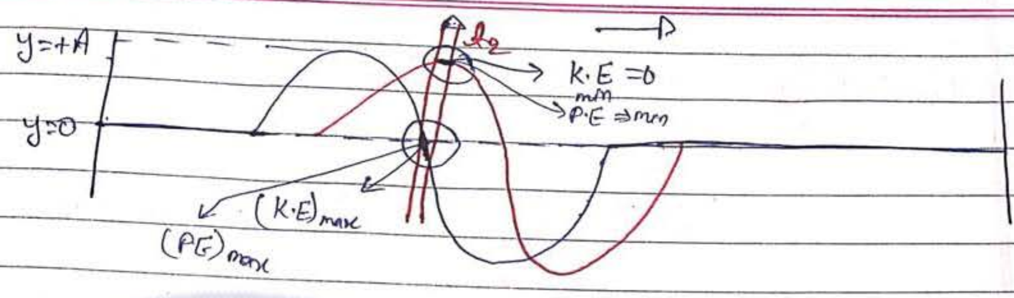
$$\boxed{(U_v)_{avg} = \frac{1}{4}\rho\omega^2A^2}$$

(rho)





Ex.)



⊛ Energy density (U_1)

(for element)

↳ (Total mechanical energy)avg per unit volume.

$$U_f = (K_v)_{avg} + (U_v)_{avg}$$

$$\boxed{U_1 = \frac{1}{2}\rho\omega^2A^2}$$

✱ Power (P) (P_{avg}) ⇒

$$\boxed{P_{avg} = U_{avg} \times S \cdot v}$$ → velocity

dx $S = \pi r^2$
vol $= S \cdot dx$

$$\boxed{P = \frac{1}{2}\rho\omega^2A^2 \cdot S \cdot v}$$

v → speed of wave

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⇒ Intensity of wave ⇒ (I)

$$I = \frac{P}{S}$$

$$\boxed{I = \frac{1}{2}\rho\omega^2 A^2 \cdot V}$$

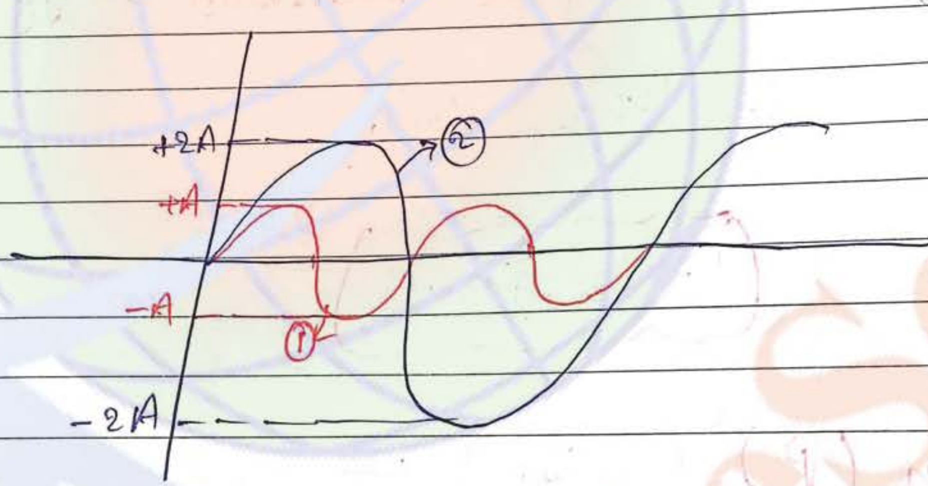
$\omega = 2\pi f$

$V = \sqrt{\frac{T}{\mu}}$

$V = \sqrt{\frac{F}{S\rho}}$

$F = V^2 S\rho$

For the medium ⇒



Find the ratio of Intensity of two medium?

$$\frac{I_1}{I_2} = ?$$

Ans –

$\omega_1 = 2\omega_1$

frequency

$A_2 = 2A_1$

Note ⇒

$\boxed{I \propto \frac{1}{r^2} \propto A^2}$ → (For Point source)

$\boxed{I \propto \frac{1}{r} \propto A^2}$ → (For line source)

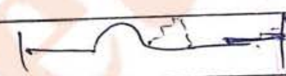
$$I \propto \omega^2 A^2$$

$$\frac{I_1}{I_2} = \left(\frac{\omega_1}{\omega_2}\right)^2 \left(\frac{A_1}{A_2}\right)^2$$

$$= \frac{1}{1} = 1:1$$

* Superposition of wave ⇒

when two or more than two waves reach simultinuously at a point the resultant displacement of that point is the vector sum of individual displacement of individual wave in the absence of other waves.



$$\vec{y} = \vec{y_1} + \vec{y_2} + \vec{y_3} + \cdots$$

component waves

$$y_1 = A\sin(\omega t - kx)$$

$$y_2 = A\sin(\omega t + kx)$$

$$y = y_1 + y_2$$

$$\boxed{y = 2A\cos kx \cdot \sin\omega t}$$

Equation of Standing.

7/1/2019

1st Choice

→ string की concept transverse standing wave Date में आते हैं

* Standing / Stationary wave ⟹ (Transverse standing wave)

when two waves of same frequency and same amplitude travelling in opposite direction superpose a new type of wave is form called standing wave.

component waves.
$$y_1 = A\sin(\omega t - kx)$$
$$y_2 = A\sin(\omega t + kx)$$

$$y = y_1 + y_2$$

$$\boxed{y = (2A\cos kx)\sin\omega t}$$

Equation of standing wave.

Ex. If

$$y_1 = A\sin(kx - \omega t)$$
$$y_2 = A\sin(kx + \omega t)$$

$$\boxed{y = (2A\sin kx)\cdot\cos\omega t}$$

दोनों eqn same है।

Trick to find wave eqn is standing or not :-

final eqn को trigonometry के formula से तोड़कर देखें की wave opp. direction में है या नहीं। यदि opp है तो standing

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1.) component wave / (displacement wave)

$$y = \boxed{2A\cos kx}\sin\omega t$$

$$\boxed{y = A_x \sin\omega t}$$

Amplitude. where $A_x = 2A\cos kx$

If $kx = (2n+1)\frac{\pi}{2}$; $n = 0,1,2,3 \ldots$

$$\frac{2\pi}{\lambda}\cdot x = (2n+1)\frac{\pi}{2}$$

$$\boxed{x = (2n+1)\frac{\lambda}{4}}$$

$n=0 \rightarrow x_1 = \frac{\lambda}{4} \rightarrow N$ (Nodes)

$\lambda/2$

$n=1 \rightarrow x_2 = \frac{3\lambda}{4} \rightarrow N$

$\lambda/2$

$n=2 \rightarrow x_3 = \frac{5\lambda}{4} \rightarrow N$

Nodes(N) ⇒

The particles of the medium located at the nodes will never vibrate.

They are at Permanent rest Position.

- The seperation b/w two consecutive nodes is "$\frac{\lambda}{2}$"

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$n = 0, 1, 2, \ldots$

2) If $kx = n\pi$

$(\cos(n\pi) = (-1)^n)$

$$\frac{2\cancel{\pi}}{\lambda} \cdot x = n\cancel{\pi}$$

$$\boxed{x = n\left(\frac{\lambda}{2}\right)}$$

(Highest position)

(max. Amplitude)

$$A_x = \pm 2A$$

$n = 0, \quad x = 0 \rightarrow (A)$

$n = 1, \quad x_2 = \frac{\lambda}{2} \rightarrow (A)$

$n = 2, \quad x_3 = 2\left(\frac{\lambda}{2}\right) \leftarrow (A)$

Antinodes

Antinodes :→ (A)

Particles of the medium located at the Antinodes vibrate with maximum Amplitude

- The seperation between two consecutive Antinodes is "$\lambda/2$".

e.:→ The seperation b/w consecutive nodes and Antinode is "$\frac{\lambda}{4}$"

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Point Related to Standing wave ⇒

1.) All the particles of the medium (except the nodale particles) reach their mean mean position simultaniously at same time Instant

2.) All the particles of the medium (except the nodle particles) vibrate with different Amplitudes (but some particles may have same Amplitude).

3.) All the particles except the nodle particles lying in the region b/w two consecutive nodes. vibrate in same phase.

4.) The phase difference b/w the particles of the medium is either 0° or 180° (π)

5.) Their is no energy transfer accross the nodes

It means the energy of one region (Region b/w two consecutive nodes) ~~energy of per~~ is confined within the ~~region~~ same region.

$$y = 2A \cos kx \cdot \sin \omega t$$

$$\boxed{V_{Particle} = \frac{\partial y}{\partial t} = (2A\cos kx)\ \omega \cos \omega t}$$

~~At~~ And also

1st Choice

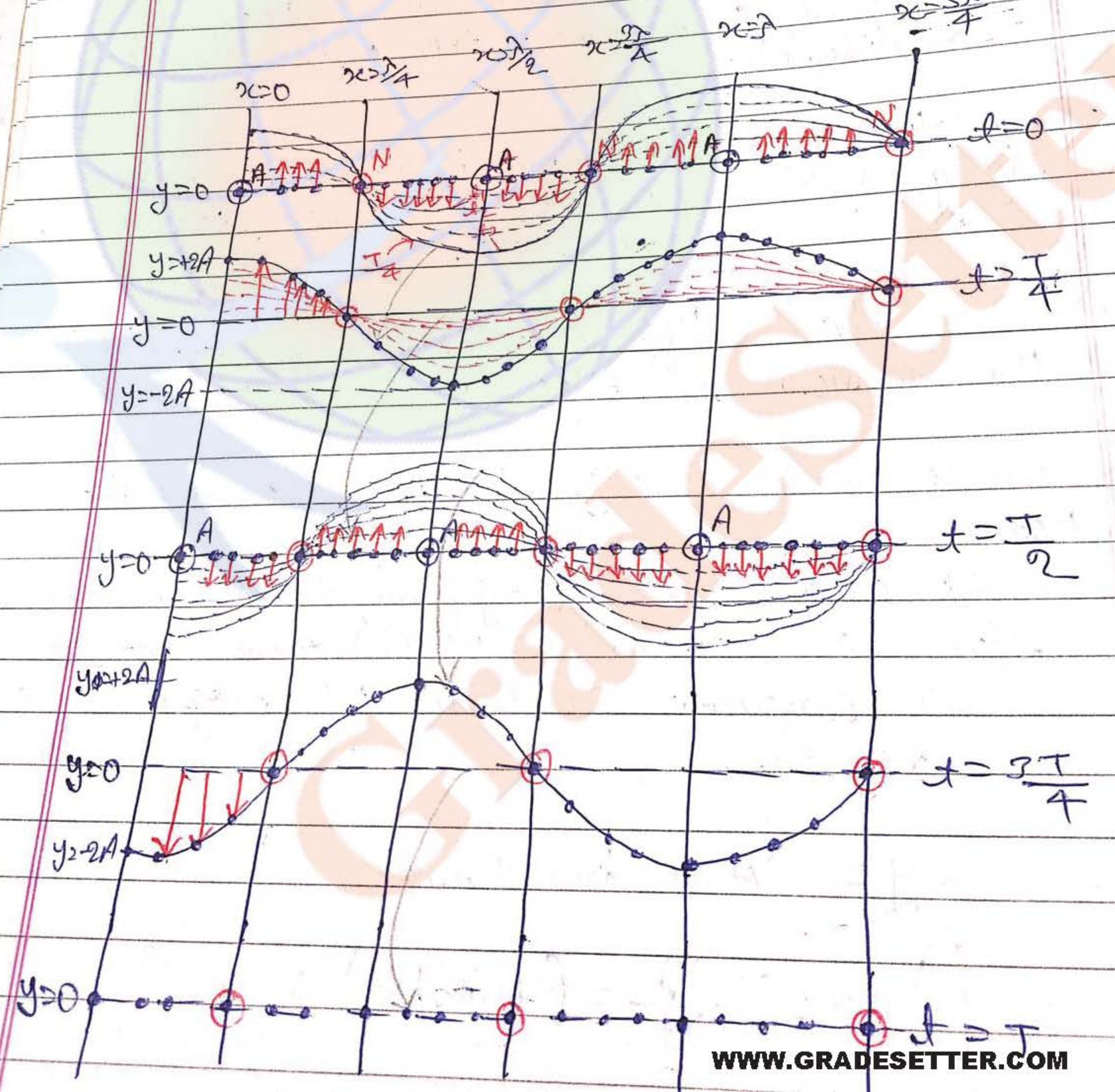
0 30 45 60 90 120 135 150 180

$$a_{particle} = -2A\cos kx\,\omega^2 \sin\omega t$$

$$a_{particle} = -\omega^2 y$$

$$y = 2A\cos kx \cdot \sin\omega t$$

$$y = A_x \sin\omega t$$



To make this graph we see two value:-
1st :-> we put time = ?
And then we put $x = ?$

$$y = 2A\cos kx \cdot \sin\omega t$$

$$y = A_x \sin\omega t$$

$$\frac{2\pi}{T} \cdot \frac{T}{4} = \frac{\pi}{2}$$

$$\omega t = \frac{2\pi}{T} \cdot \frac{T}{2} = \pi$$

$$y = 0$$

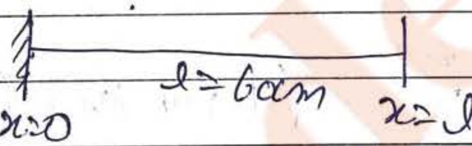
$$\omega t = \frac{2\pi}{T} \cdot \frac{3T}{4} = \frac{3\pi}{2}$$

$$y = -A_x$$

Q)

$$\overset{\text{In cm}}{y} = 4\sin\left(\frac{\pi x}{15}\right) \cdot \cos(96\pi t)$$

$x \to$ In cm.
$t \to$ In second



i) Find the max. displacement of the particle located at $x = 5cm$.
ii) Find the nodal position and the total no. of nodes in the string.
iii) Find the velocity of particle located at $x = 7.5cm$ and $t = 0.25s$
iv) Find the component waves.

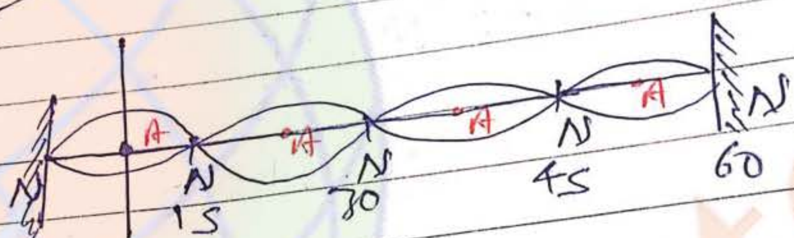
$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$ (3)

Ans i) $y = 4\sin\left(\frac{\pi x}{15}\right)$

$y_{max} = (2\sqrt{3}) \times 4$ cm.

ii) $\frac{2\pi}{\lambda} = k = \frac{\pi}{15}$

$\frac{\lambda}{2} = 15$ cm



80 Nodes ⇒ 5

Antinodes = 5 − 1
= 4

iii) differentiate
$V_{particle} = -(4\sin kx)(96\pi)\sin 96\pi t$

$t = 0.25$

$\sin 96\pi \times 0.25$
$= \sin 24\pi$

$V_{particle} = 0$

iv) $y_1 = 2\sin(kx - \omega t)$
$y_2 = 2\sin(kx + \omega t)$

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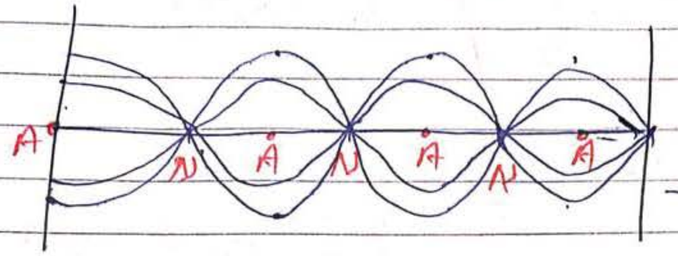
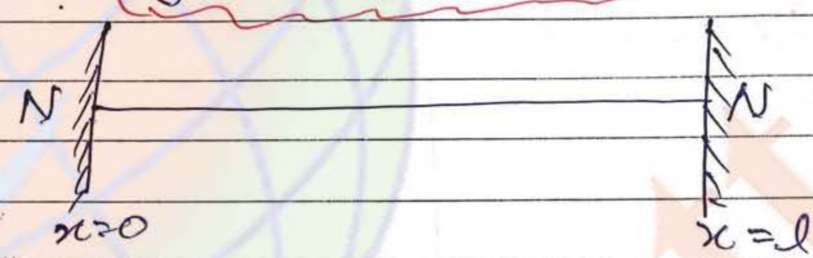


(Sonometer का आधार)

Vibration of String fixed at both ends:-

$$\boxed{y = 2A\sin kx \cdot \cos\omega t}$$



Boundary condition:
- $x=0$, Nodes , $y=0$
- $x=l$, Nodes , $y=0$

$$\sin kx = 0$$

$$kx = n\pi$$

$$\frac{2\cancel{\pi}}{\lambda}\cdot x = n\cancel{\pi}$$

$$\boxed{x = n\cdot\frac{\lambda}{2}}$$

And also

$l \rightarrow$ No. of nodes

$L = (l-1)\frac{\lambda}{2}$

(distance between)

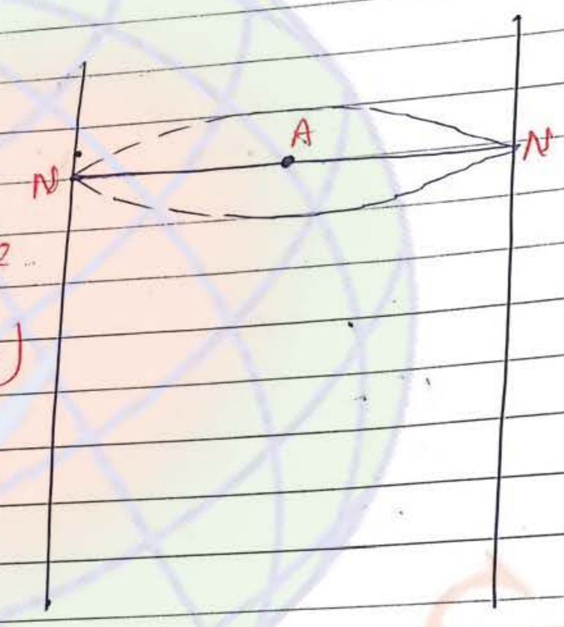
1st Choice

No. of Antinodes $(x=0)$

Imp $\boxed{l = n \cdot \frac{\lambda}{2}}$ $n = 1, 2, 3$ ——

$$\boxed{\lambda = \frac{2l}{n}}$$

where
$n \rightarrow$ No. of loops (= No. of antinodes / No. of segments)



$n = 1$
(Fundamental node of vibration or 1st harmonic)

* Frequency of vibration $(f) \Rightarrow$

$$f = \frac{v}{\lambda}$$

$$\boxed{f = n\left(\frac{v}{2l}\right)} \quad n = 1, 2, 3 \ldots$$

Natural frequency

where :-
$v \rightarrow$ speed of wave $= \sqrt{\frac{T}{\mu}}$

In case of string :-

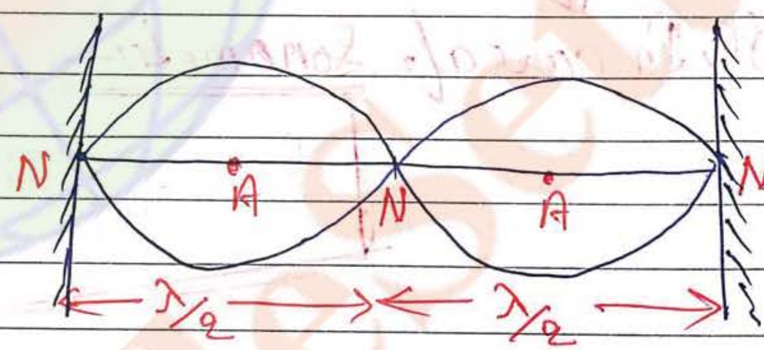
$$v = \sqrt{\frac{T}{\mu}}$$

$$\boxed{f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}}$$

⇒ $n = 2$

IInd Harmonic
or
1st overtone

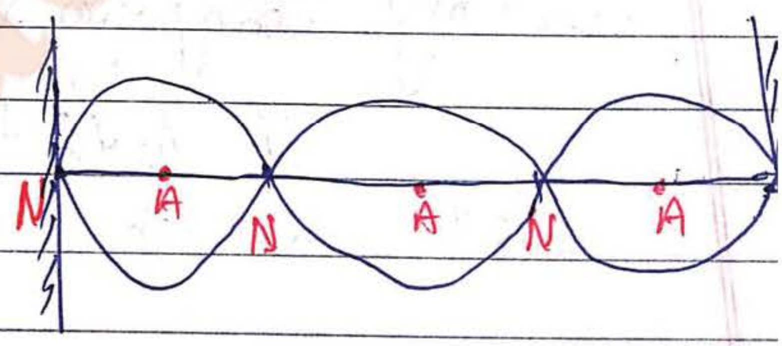
$$l = 2\left(\frac{\lambda}{2}\right)$$



⇒ $n = 3$

IIIrd Harmonic
or
IInd overtone

$$l = 3\left(\frac{\lambda}{2}\right)$$



$$\boxed{\text{Fundamental freq } (f_0) = \frac{v}{2l}}$$

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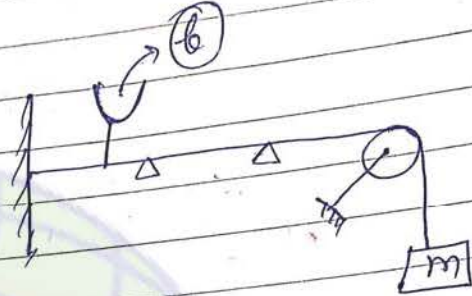
1st Choice

Sonometer ⇒



$$\boxed{f = n\left(\frac{v}{2l}\right)}$$ $n = 1, 2, 3, \ldots$

Natural frequency

where $v = \sqrt{\frac{T}{\mu}}$
velocity of transverse wave.

$$\boxed{l = n \cdot \frac{\lambda}{2}}$$

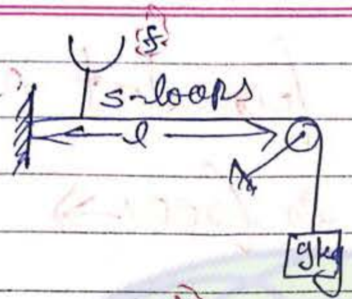
So, In case of Sonometer—

$$\boxed{f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}}$$

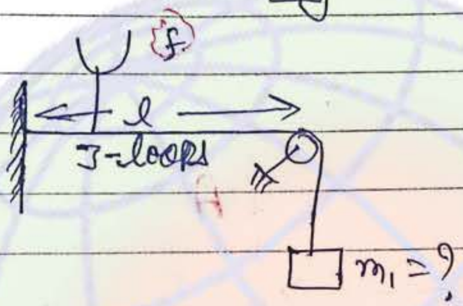
Frequency of tuning fork and frequency of vibrating string are same.

~~They both~~
If both are vibrated together both are in Unisson, mean same frequency

Ex.



Five no. of loops is obtaing



In both cases same tuning fork is used.

$$f = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$$

So,

$n_1\sqrt{T_1} = n_2\sqrt{T_2}$ ∵ Squaring both side

$25 \times mg = 9 \times m_2 g$

$25 \times \cancel{9} \times \cancel{g} = \cancel{9} \times m_2 \times \cancel{g}$

$m_2 = 25\,kg.$

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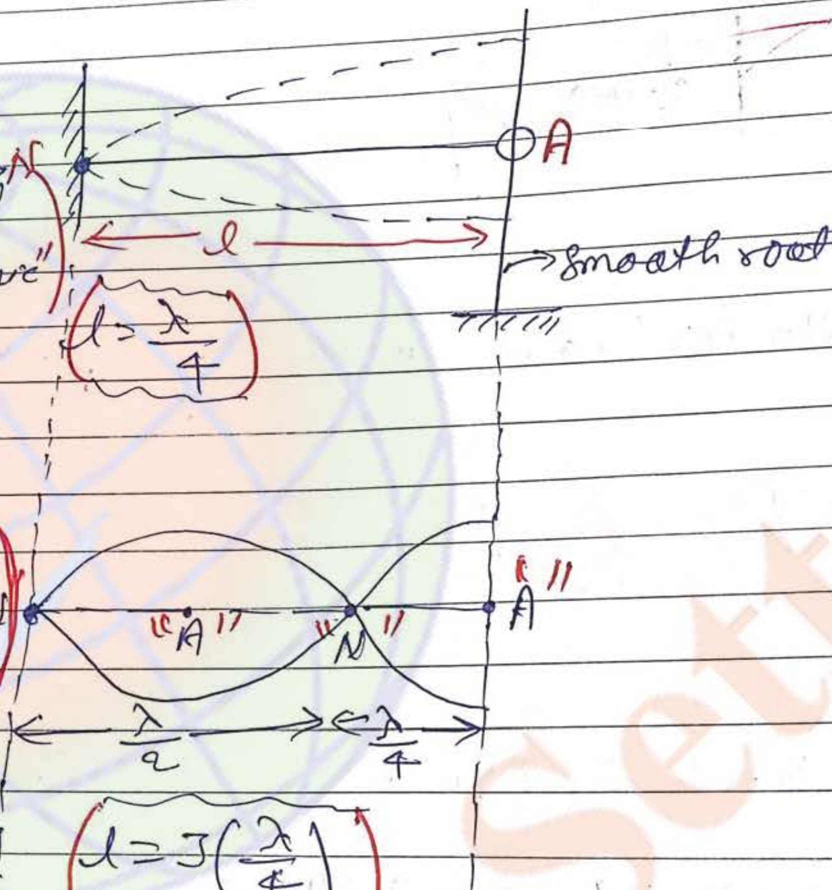
IIT ask this concept every year — 1st Choice

Vibration of String fixed at one end and other end is free :→



(fundamental node "1st Harmonic" (n=0) (f_0))

$$l = \frac{\lambda}{4}$$

Attention → (1st overtone or Third harmonic)

~~1st Harmonic~~ ~~1st har~~ (n=1)

$$l = 3\left(\frac{\lambda}{4}\right)$$

1st Choice

climbing means Node

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$$v = f\lambda$$

$$f = \frac{v}{\lambda} = \frac{v}{4l}(2n+1)$$

Imp

$$\boxed{f = (2n+1)\frac{v}{4l}}$$

where $n = 0, 1, 2, 3 \ldots$

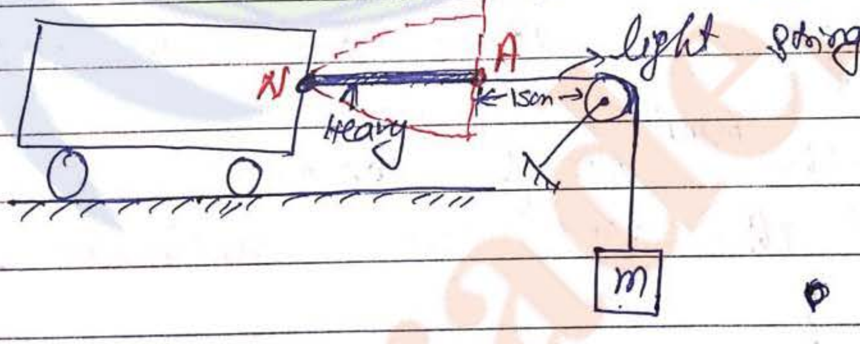
$n = 2, \quad f_2 = 5f_0$

$$\boxed{f_0 = \frac{v}{4l}}$$

"Fundamental frequency $(n=0)$"

$$f_0 = \frac{v}{4l} = 150$$

Q.

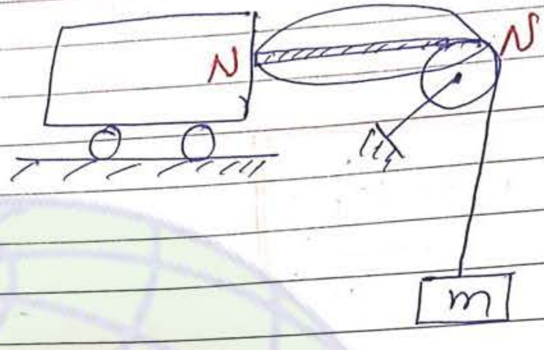


Heavy string vibrates with lowest frequency of 150 Hz in the given situation (as shown in figure)

1) Find the lowest frequency of vibration of heavy string If the trolley moves through a distance.

15 cm Rightward.

An.



$f_1 = \frac{v}{2l} = 2\left(\frac{v}{4l}\right)$

$= 300\ Hz$

Sol.

Note! ⇒ Plucking of wire is always "Antinode"

" Tune the word."

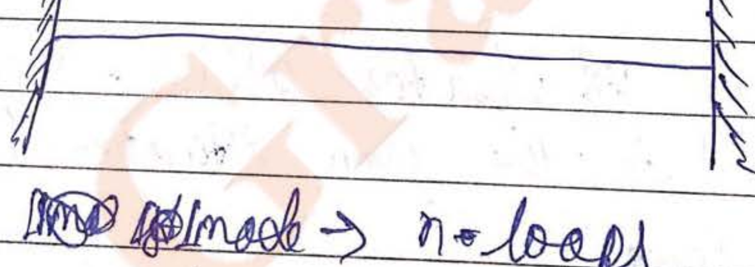
and climbing of wire is always "Node"

Q. The seperation b/w two conseculive nodes for the two consecutive modes of vibration of string are 18 cm and 16 cm respectively

i) Find the minimum no. of loops.

ii) And also find the minimum length of string

Ans.



1st mode → n loops

2nd mode → (n+1) loops

$$l = n\left(\frac{\lambda}{2}\right) = n \times 18 \quad \text{——} \quad (1)$$

$$l = (n+1) \times 16 \quad \text{——} \quad (2)$$

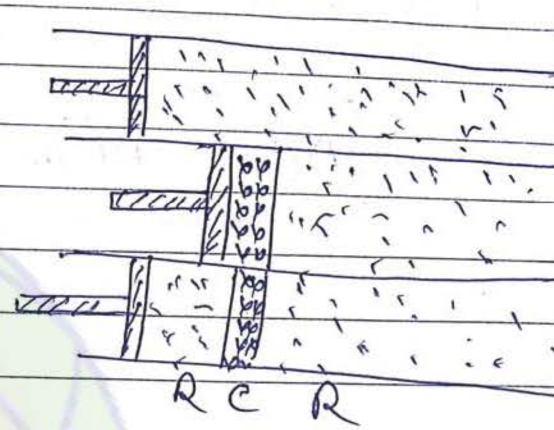
$$18n = (n+1)16$$

$$n = 8$$ Ar 1

so IInd Ans is 9

$$l = 18 \times n$$
$$= 18 \times 8$$
$$= 144 \text{ cm}$$

Sound wave



1) Displacement wave

$$\boxed{y = A\cos(kx-\omega t)}$$

2.) Pressure wave

$$\boxed{\Delta P = (\Delta P)_m \sin(kx-\omega t)}$$

where :–
$(\Delta P) \Rightarrow$ Excess pressure (variation in pressure from its Normal value)
$\pm(\Delta P)_m \Rightarrow$ Pressure Amplitude

3) Density wave :–

$$\boxed{\Delta \rho = (\Delta \rho)_m \sin(kx-\omega t)}$$

1) Pressure wave ⟹

$$\boxed{\Delta P = (\Delta P)_m \sin(kx - \omega t)}$$

where
(ΔP) = "Excess pressure" (variation in pressure from its Normal value)

$\pm(\Delta P)_m \Rightarrow$ "Pressure Amplitude" (max variation in pressure from its normal value)

Important

$$\boxed{(\Delta P)_m = BAK}$$

where
$B \Rightarrow$ Bulk modulus
$A \Rightarrow$ Displacement Amplitude
and $K = \frac{2\pi}{\lambda}$

II Task

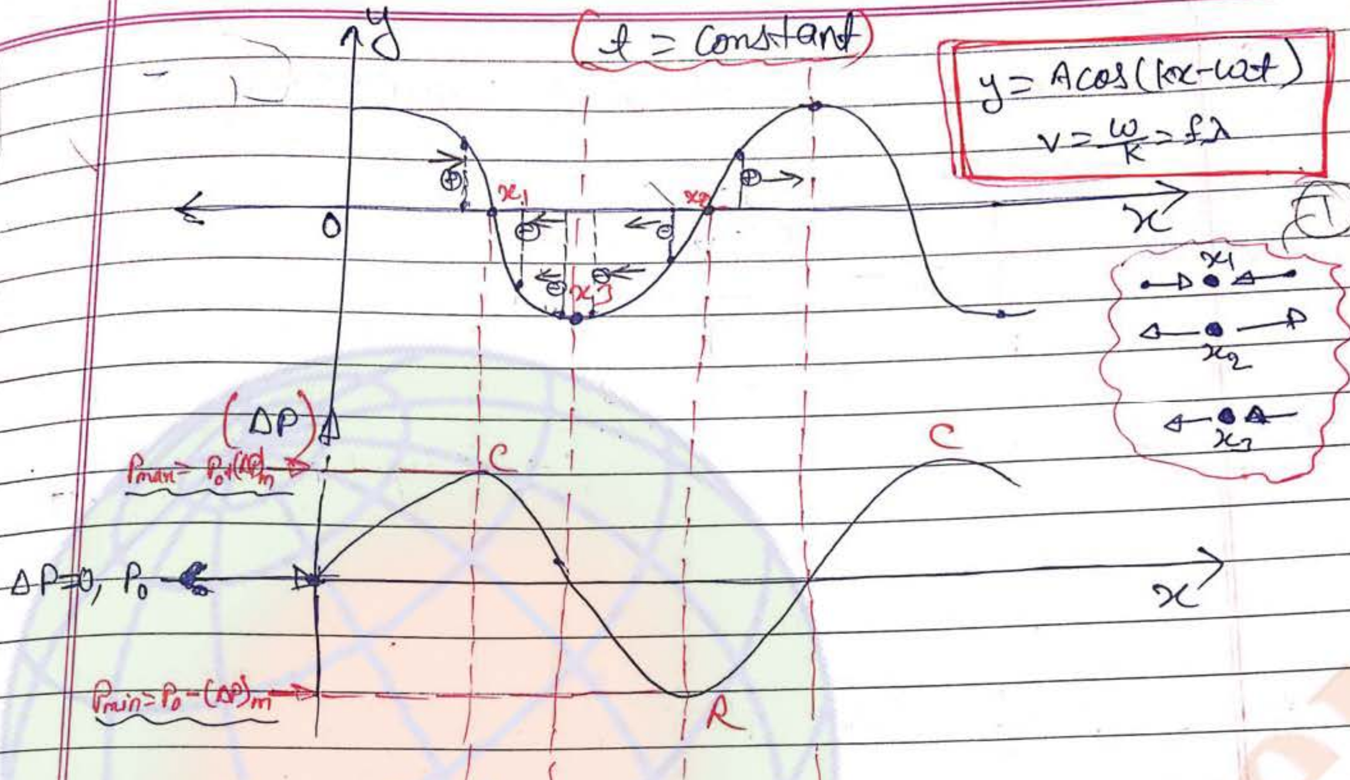
$$\boxed{\begin{aligned} P_{max} &= P_0 + (\Delta P)_m \\ P_{min} &= P_0 - (\Delta P)_m \end{aligned}}$$

P_0 → (Normal value of Pressure)

or

Imp

$$\boxed{\begin{aligned} P_{max} &= P_0 + BAK \\ P_{min} &= P_0 - BAK \end{aligned}}$$



⇒ Speed of longitidunal wave :→
(Sound wave)

In fluid :⇒ (liquid and gas)

Note :⇒ The speed of sound in medium depends on "The Elastic Property" as well as "Inertia Property"

$$\boxed{V = \sqrt{\frac{B}{\rho}} = \frac{\omega}{K} = f\lambda}$$

where
B = Bulk modlus
ρ ⇒ density of medium

Note :→
velocity of the ~~gas~~ sound in a gas is proportio to the square root of adiabatic elasticity.

* Speed of longitudinal wave (sound) in Solid ⇒

$$\boxed{v = \sqrt{\frac{Y}{\rho}}}$$

where $Y \Rightarrow$ Young's modulus of elasticity.

* Speed of sound wave in gas ⇒ (Ideal gas)

- Newton formula :⇒

$$\boxed{B = P}$$ (Isothermal process)

$$\boxed{v = \sqrt{\frac{P}{\rho}}}$$

correct करके laplace और formula दिया ⇒

- Laplace's formula :⇒

$$B = \gamma P$$

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$\frac{n_1 m_1}{n_2 m_2} = n_1$

$$\boxed{v = \sqrt{\frac{\gamma RT}{M}}}$$

$\gamma = \frac{C_P}{C_V}$

$T \Rightarrow$ Temp in K.

Ex) At the same temp. vel. of sound in $O_2 \rightarrow v$
Then find the velocity of sound in $H_2 \rightarrow v_1$?

Ans $\frac{v_1}{v} = \sqrt{\frac{M_{O_2}}{M_1}}$

$= \sqrt{\frac{32}{2}} = 4$

Longitudinal Standing wave ⇒

$y_1 = A \sin(\omega t - kx)$
$y_2 = A \sin(\omega t + kx)$

$y = y_1 + y_2$

$$\boxed{y = 2A \cos kx \cdot \sin \omega t}$$

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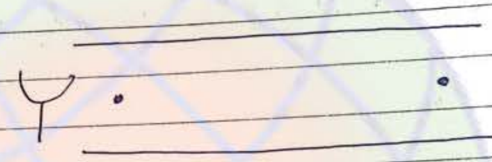
1st Choice

⊛ Application ⇒
(Vibration of air/gas column) ⇒

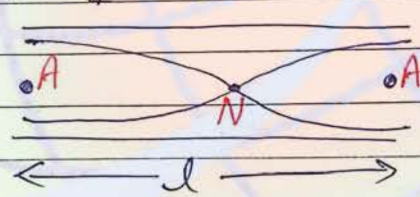
1.) Open organ pipe ⇒

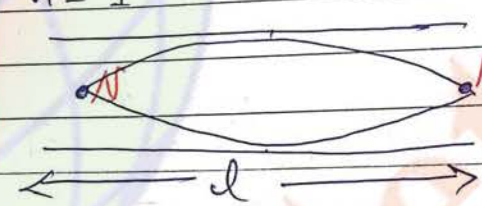


Attention IIT ask this concept

नोट :- जहाँ displacement का node होगा वहाँ pressure का Anti-node होगा (vice-versa)

Displacement node / Antinode	Pressure nodes / Antinodes
$n = 1$	$n = 1$





$l = \frac{\lambda}{2}$ $\qquad$ $l = \frac{\lambda}{2}$

$$\boxed{f = n\left(\frac{v}{2l}\right)} \quad n = 1, 2, 3, 4 \ldots\ldots$$

Natural frequency of vibration

where

$v \rightarrow$ velocity of sound in the given medium (air/gas)

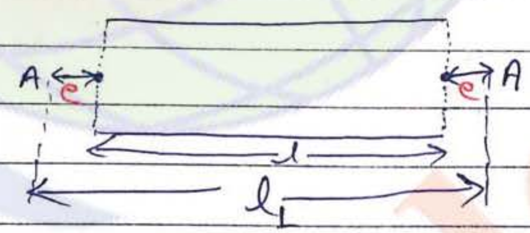
$l \rightarrow$ length of vibrating air/gas column

$n = 1$ (fundamental)

$$\boxed{f_0 = \frac{v}{2l}}$$

$n = 2$, $\boxed{f_2 = 2f_0}$

⇒ End Correction ⇒ (e)



(all overtones are harmonic)

$$\boxed{l_1 = (l + 2e)}$$

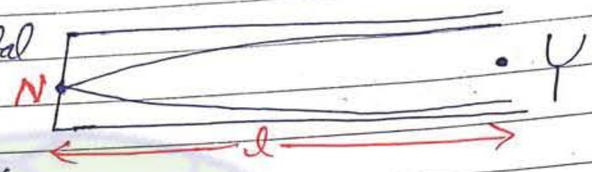
where

$r \rightarrow$ radius of tube

$$\boxed{e = 0.6r}$$ याद रखें

2.) closed Organ pipe ⇒

fundamental mode of vibration or 1st harmonic



$$\boxed{f = (2n+1)\frac{v}{4l}} \quad n = 0,1,2,3,4 \ldots$$

where $v \to$ velocity of sound

1st harmonic $\to f_0 \to \frac{v}{4l}$

3rd Harmonic $\to f_3 \to 3f_0$

Trick to learn

Harmonic → 1, 3, 5, 7
overtone → 0, 1, 2, 3

VVI

⇒ End Correction :→

$$\boxed{l_1 = l + e}$$

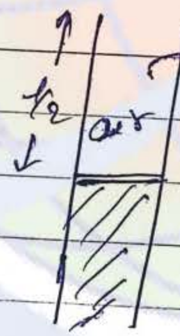
11) An open organ pipe of length "l" (air column of the pipe) vibrates in its fundamental mode. and its fundamental frequency is "f".
If the half length of this organ pipe is vertically dipped in the water the fundamental frequency of vibration of air column is f_1.
Find the value of f_1?

Surface of water behaves like rigid body.

Ans

$$f = \frac{v}{2l}$$

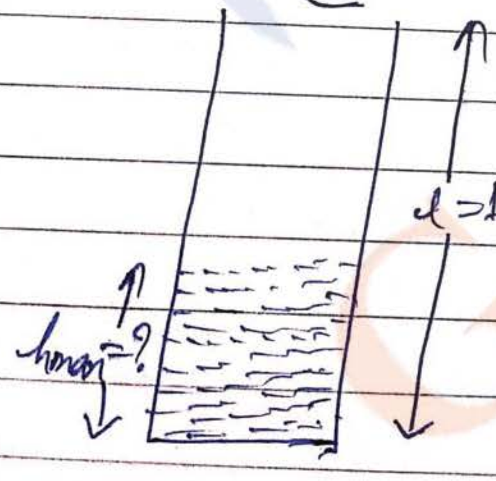
when



$$f_1 = \frac{v}{4(l/2)} = \frac{v}{2l} = f$$

So, same

a) $f = 660 Hz$



$V_{air} = 330\ m/s$

1) Find the max. level of wave in the organ pipe so that ~~tuning~~ frequency of tuning fork matches wi

the natural frequences of vibrating air ~~tuning form~~

for $f = (2n+1)\frac{v}{4l}$

$l_{air} = (2n+1)\frac{v}{4f}$

$= (2n+1)\frac{\cancel{330}}{4\times\cancel{660}_2} = \frac{1}{8}(\text{odd}) \times 100\,cm$

$= (12.5\,cm) \times \text{odd}$

$l_{1\,air} = 12.5\,cm$

$l_{2\,air} = 37.5\,cm$

$l_{3\,air} = 62.5\,cm$

$l_{4\,air} = 87.5\,cm$

$> 100\,cm$

so,

$h_{max} = 87.5\,cm$ Ans.

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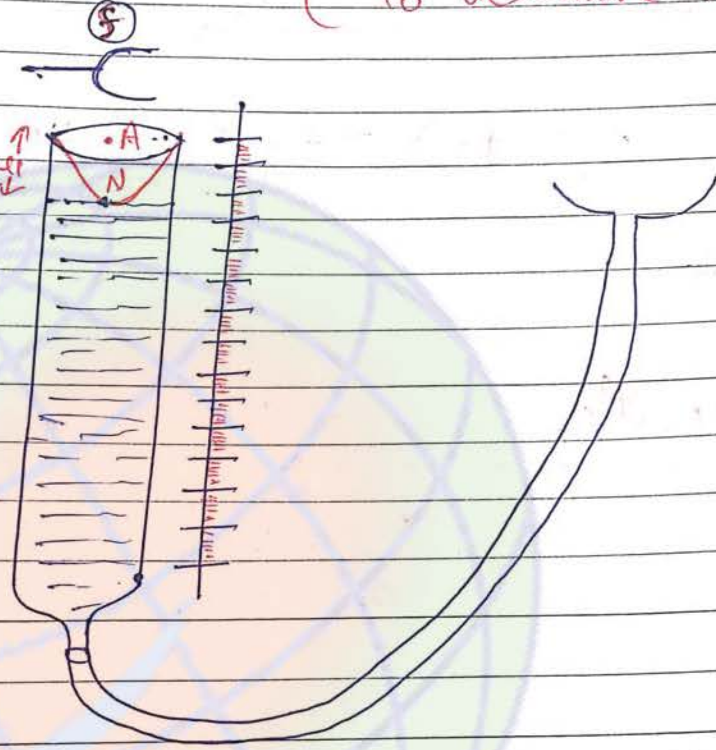
Resonance column method :→

(To determine the speed in air.)



$l_1 + e = \frac{\lambda}{4}$ — (1)

$l_2 + e = 3\left(\frac{\lambda}{4}\right)$ — (2)

eq (2) − eq (1)

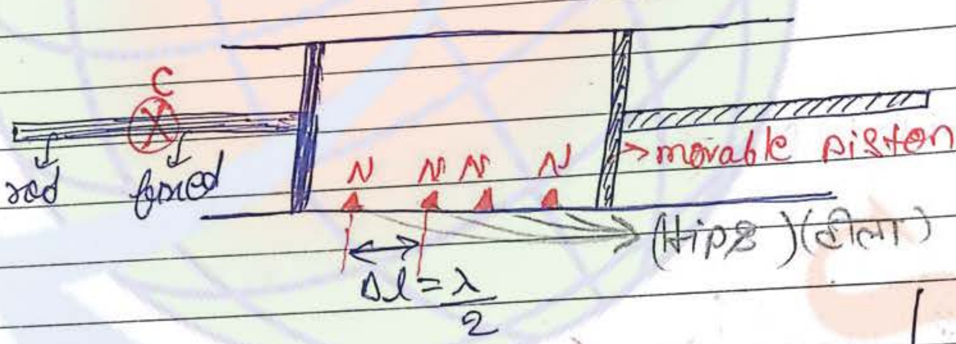
$$\boxed{\lambda = 2(l_2 - l_1)}$$

$$v = f\cdot\lambda = f\{2(l_2 - l_1)\}$$

100% IIT or other entrance test asks

Kundt's tube :⇒

(used to determine the speed of sound in air/gas)



$$v = f\cdot\lambda$$

$$\boxed{v = f\,2(\Delta l)}$$

$$(\Delta l)' = \frac{\lambda_{gas}}{2}$$

$$\lambda_{gas} = 2(\Delta l)'$$

If the compression/rarefaction pulse is reflected from the rigid body there is no phase change in the pulse.

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If the compression / rarefaction pulse is reflected from open boundry ($\Delta\phi = 0$)
There is phase change of $180°$.

Beats

$(x = 0)$

$y_1 = A \sin \omega_1 t$
$y_2 = A \sin \omega_2 t$

$y = y_1 + y_2$

$\boxed{\omega_1 \sim \omega_2 \leq 16\ Hz}$
→ for beat to be observed (Persistence of Hearing)

$$y = 2A \sin\left(\frac{\omega_1 + \omega_2}{2}\right)t \cdot \cos\left(\frac{\omega_1 - \omega_2}{2}\right)t$$

$$\boxed{y = 2A\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t \cdot \sin\left(\frac{\omega_1 + \omega_2}{2}\right)t}$$

$$\boxed{y = A_t \sin \omega_{avg} t}$$

Resultant Sound wave.

$I \propto A_t^2$

$$A_t = 2A\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t$$

$$\boxed{\omega_{avg} = \frac{\omega_1 + \omega_2}{2}}$$

Frequency of resultant wave

$$= \frac{2\pi(f_1 + f_2)}{2}$$

$$\boxed{\frac{\omega_1 - \omega_2}{2} = \omega_A}$$

→ frequency of vibration of Amplit

* maximum Intensity ⇒

$$\cos\left(\frac{\omega_1-\omega_2}{2}\right)t = \pm 1$$

so,

$$\boxed{\left(\frac{\omega_1-\omega_2}{2}\right)t = n\pi} \quad n = 0, 1, 2, \ldots$$

$n = 0$; $t_1 = 0$ ⟶ (maximum Intensity)

$n = 1$; $t_2 = \frac{2\pi}{\omega_1-\omega_2}$ ⟶ (maximum Intensity)

$n = 2$; $t_3 = \frac{4\pi}{\omega_1-\omega_2}$ ⟶ (maximum Intensity)

* Beat Period ⇒

$$\boxed{(\Delta t_b) = \frac{2\pi}{\omega_1-\omega_2} = \frac{\cancel{2\pi}}{\cancel{2\pi}(f_1-f_2)} = \frac{1}{(f_1-f_2)}}$$

Beat Period :-

Beat Period is defined as time Interval b/w two consecutive maxima and minima.

* Beat frequency ⇒

$\left(\frac{1}{\text{Beat Period}}\right)$ is defined as Beat frequency.

Imp

$$\boxed{f_b = f_1 - f_2 = \frac{1}{(\Delta t)_b}}$$

Beat frequency ⟶ (Number of beats Per second)

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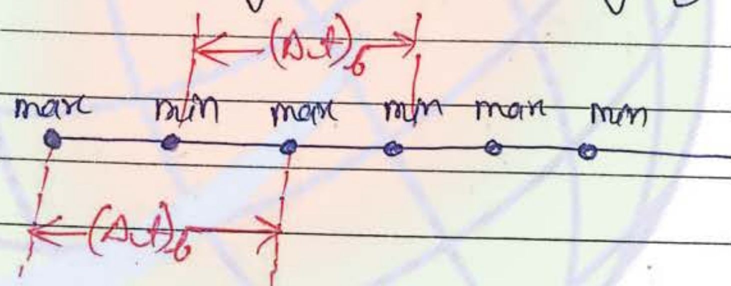
For minimum beat ⟹

$$\left(\frac{\omega_1-\omega_2}{2}\right)t = (2n+1)\frac{\pi}{2}$$

$$\boxed{t = \frac{(2n+1)\pi}{(\omega_1-\omega_2)}} \quad n = 0, 1, 2 \ldots$$

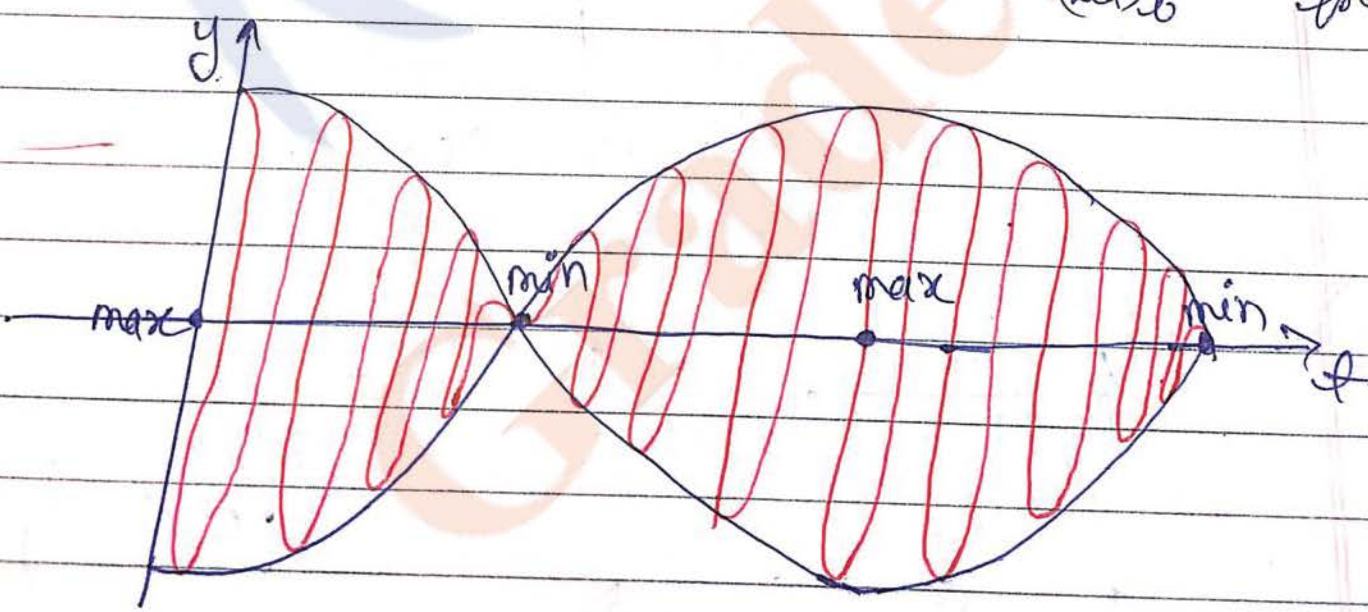
~~varing and~~

Periodic varing and vaining of Sound is Beat.



where
$(\Delta t)_b \rightarrow$ Beat Period

$\frac{1}{(\Delta t)_b} \rightarrow$ Beat frequency



Q. When two tuning forks A and B are vibrated together five (5) beats per second is observed. Frequency of fork "A" is 200 Hz.

1) Find the fork B is loaded with wax and ~~vibrated~~ again vibrated with fork A. The no. of beats observed remains same (i.e. equal to 5A). Find the original frequency of fork B.

Ans

$$f_A \sim f_B = 5 \quad (f_A = 200\,Hz)$$

$$f_B = 195\,Hz \text{ or } 205\,Hz$$

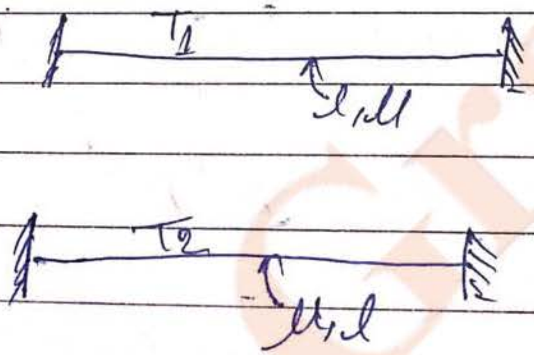
$200 - 195 = 5$ (B) | $205 - 200 = 5$ (B, A)

$\downarrow$

$195 \sim 200 = 5$

So, Ans 205 Hz.

Q2)



$(T_1 > T_2)$

$f_1 - f_2 = 5\,Hz$ → Remains constant.

Both beats are vibrated together. 5 beats are observed.

Ans D/o $T_1 \uparrow \rightarrow (X)$

D/o $T_1 \downarrow \rightarrow$ ✓

D/o $T_2 \uparrow \rightarrow$ ✓] ✓

D/o $T_2 \downarrow \rightarrow (X)$

Example of superposition of wave

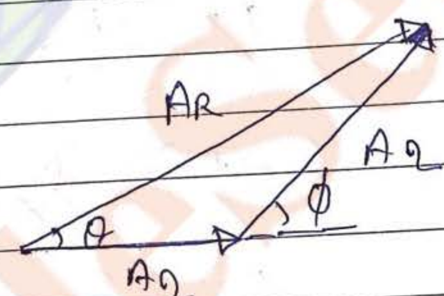
✱ Interference of "Sound waves"—

$$y_1 = A_1 \sin(\omega t - kx)$$

$$y_2 = A_2 \sin(\omega t - kx + \phi_0)$$

$$= y_1 + y_2$$

Note— According to principle of superposition when two or more sound waves pass through the "same point" then the "resultant disturbance" is the sum of disturbance produced by individual sound waves.
At some point intensity is maximum where as at some other point it is minimum making the condition of "constructive interference" and "destructive interference".



$$\boxed{A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}}$$

→ Amplitude of resultant sound wave.

$$A_R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

So,

$$\boxed{I_R = I_1 + I_2 + 2\left(\sqrt{I_1 I_2}\right)\cos\phi}$$

Resultant Intensity.

1) For maximum Intensity:- or Constructive Interference

$$\boxed{(I_R)_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2}$$

$$\cos\phi = +1$$

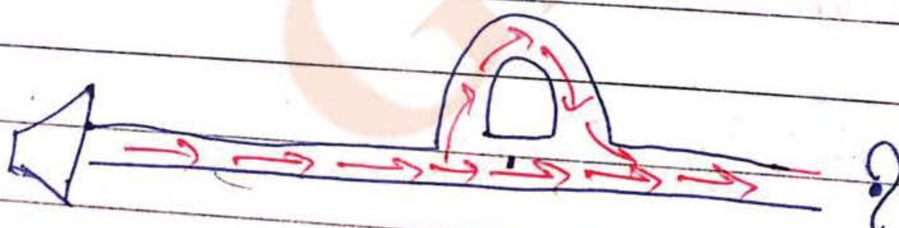
$$\boxed{\phi = 2n\pi} \quad n = 0, 1, 2, 3 \ldots$$

$$\phi = \frac{2\pi}{\lambda} \cdot \Delta x$$

$$\Delta x = \frac{\lambda}{\cancel{2\pi}} \cdot \cancel{2}n\cancel{\pi}$$

$$\boxed{\Delta x = n\lambda}$$

Path difference.



$$\Delta x = \pi r - 2r$$

Go: 1, 2, 4

1st Choice

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2.) For minimum Intensity :- or Destructive Interference

$$\boxed{\phi = (2n+1)\frac{\pi}{2}} \quad n = 0, 1, 2 \ldots$$

$$\boxed{\Delta x = (2n+1)\frac{\lambda}{2}}$$

$$\boxed{(I_R)_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

Note- For sustained Interference to be observed the phase difference (ϕ_0) must remain constant with respect to time.

This is observed in case of coherent ~~constant~~ source.

Sound level (S.L.) ⇒

$$\boxed{S.L = 10 \log_{10}\left(\frac{I}{I_0}\right)}$$

In dB (Decibell)

$$\boxed{I_0 = 10^{-12}\, W/m^2}$$

Refrence Intensity

$$S.L_1 = 10 \log_{10}\left(\frac{I_1}{I_0}\right) \quad \text{—} (1)$$

$$S.L_2 = 10 \log_{10}\left(\frac{I_2}{I_0}\right) \quad \text{—} (2)$$

$$\boxed{S.L_1 - S.L_2 = 10 \log_{10}\left(\frac{I_1}{I_2}\right)}$$

Note ⇒

$$I = 1\, W/m^2$$

$$S.L = 10 \log_{10}\left(\frac{1}{10^{-12}}\right)$$

$$= 120\ dB$$

1st Choice

Doppler's effect (In Sound)

whenever there is relative motion b/w sound, source and observer (detector). The frequency observed (apparent frequency) (means not actual) from the observer is different from the actual ~~fr~~ frequency emitted by sound source

(1) velocity of sound source (V_s) ~~and or obser~~ or ~~ob~~ velocity of observer (V_0) is always along the line joining sound source ~~to~~ and observer (line of sight)

(2) velocity of sound is always ~~is always~~ taken from sound source to observer.

$v \Rightarrow$ velocity of sound from source to observer.

$$\boxed{f_{AP} = f\,\frac{(\text{Rel. vel. of sound wave and observer})}{(\text{Rel. vel. of sound wave and sound source})}}$$

$$\boxed{f_{AP} = f\left(\frac{V \pm V_0}{V \mp V_s}\right)} ; \quad \left[(V_s, V_0) < V\right]$$

Note:- (V, V_0, V_s) $\hookrightarrow$ w.r.t. medium.

Note:- बस एक बात याद रखें
अगर object / source पास आएगा
तो frequency बढ़ेगा ||

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1st Choice

Note: → when wind is blowing then the velocity of sound is taken as follows :⇒

(a)

velocity of sound (V)

wind (V_w)

$$\boxed{V \rightarrow (V + V_w)}$$

Note: – यह problem Relative motion का नहीं है। इसका case water-boat problem के तरह ही होगा (condition)

Note! – यह concept जस्ट Relative के संभवत: क्या यह सुख होता है।

(b)

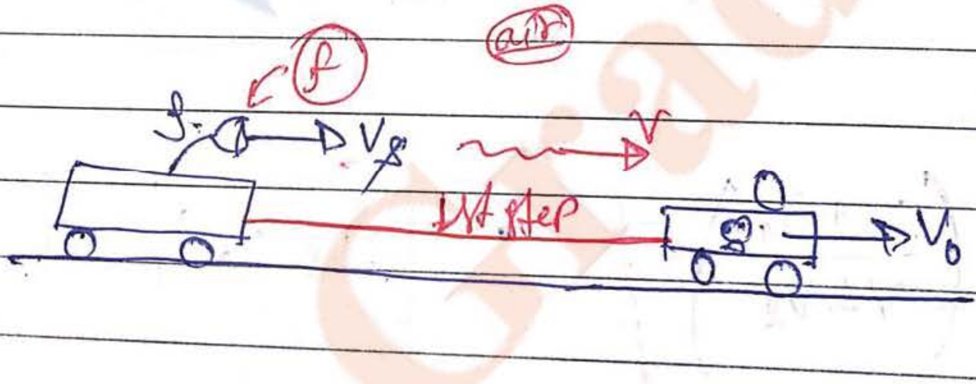
(V)

wind (V_w)

Note! → Relative velocity में यदि दो object same direction में जा रहे हो तो velocity (+) होगा तथा vice versa (लेकिन परिस्थिती को Relative समझें

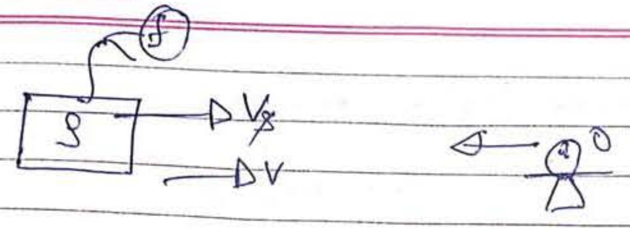
~~V → (V – …)~~

$$\boxed{V \rightarrow (V - V_w)}$$



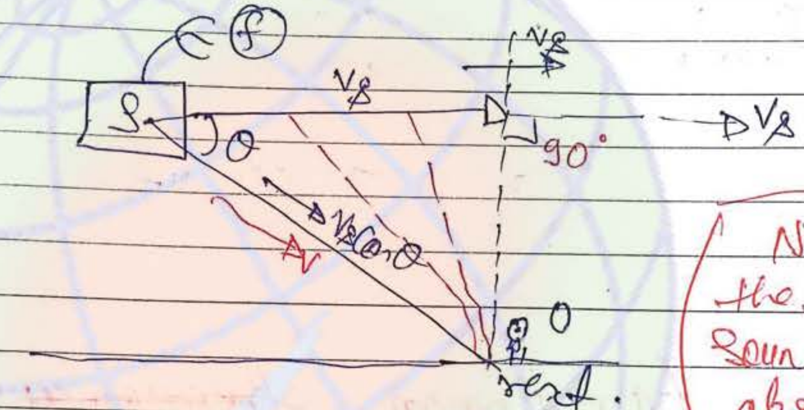
$$f_{Ap} = f\left(\frac{V - V_0}{V - V_s}\right)$$

(Q.)



$$f_{AP} = f\left(\frac{V+V_0}{V-V_s}\right)$$

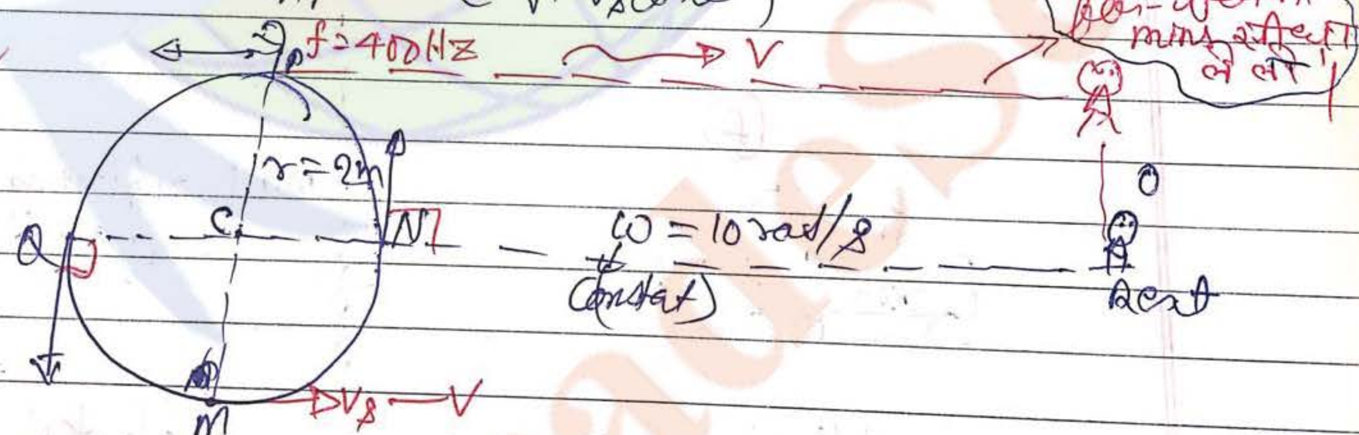
(Q.)



Note ⇒ Always the line joining the sound source and the observer

$$f_{AP} = f\left(\frac{V \pm 0}{V - V_s \cos\theta}\right)$$

IIT (Q.)



observer is standing ~~far from the obs~~ far-far away from the centre of the circle. And the max and minimum frequency observed by the observer.

vel. of sound = 340 m/s

1st Choice

velo. of source (V_s) $= r\omega$

$= 2\times 10$

$= 20\,m/s$

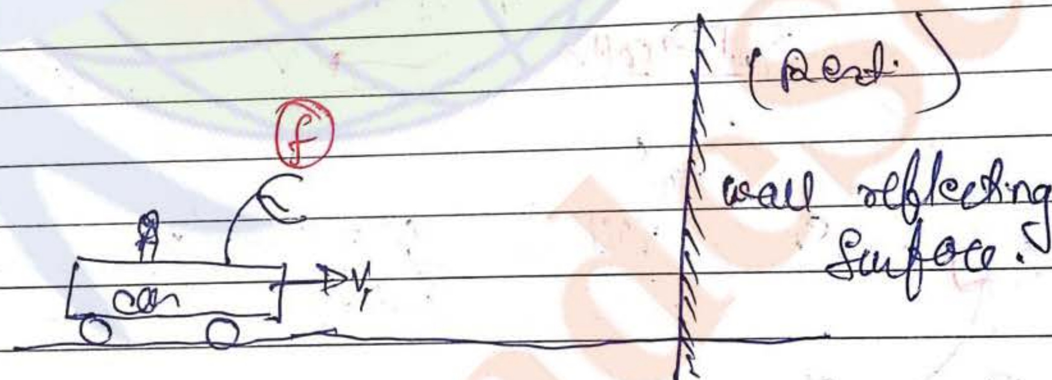
$$(f_p)_{min} = 400\left(\frac{340\pm 0}{340+20}\right) = \frac{400\times 340}{360}\ Hz$$

$$(f_{max})_m = 400\left(\frac{340}{340-20}\right)$$

$$= \frac{400\times 340}{320}\ Hz$$

✱ Problem based on ~~frequency~~ Reflection of sound wave:→

Ex.



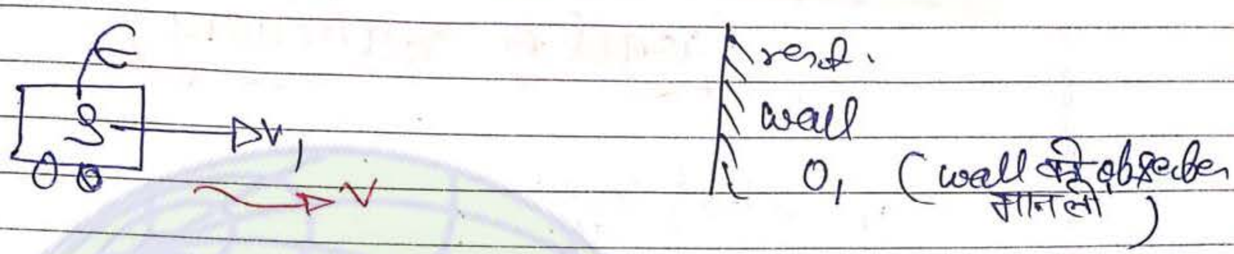
Find the frequency of reflected sound wave ~~obsd~~ observed by the driver of car.

'f' is the actual frequency emitted by the car.

Ans.

Approach →

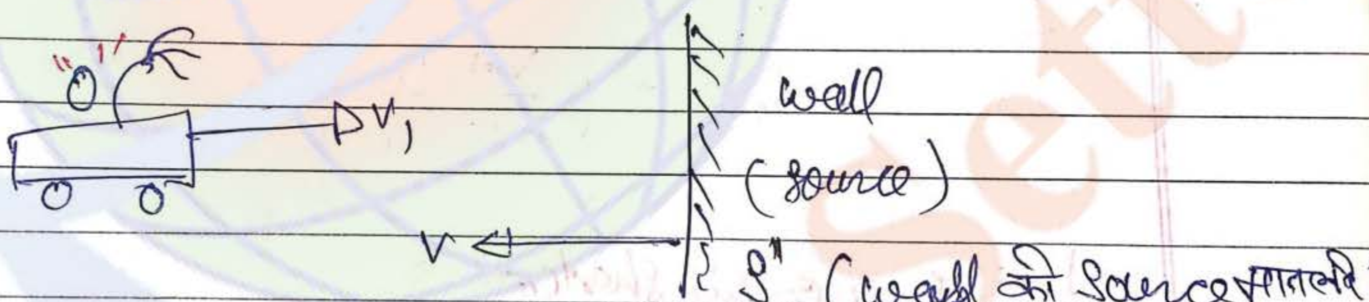
Ist Step: →



freq. observed by wall

$$f_{O_1} = f\left(\frac{V}{V-V_1}\right) \longrightarrow (1)$$

IInd Step: —



$$f_0 = f_{O_1}\left(\frac{V+V_1}{V \pm 0}\right)$$

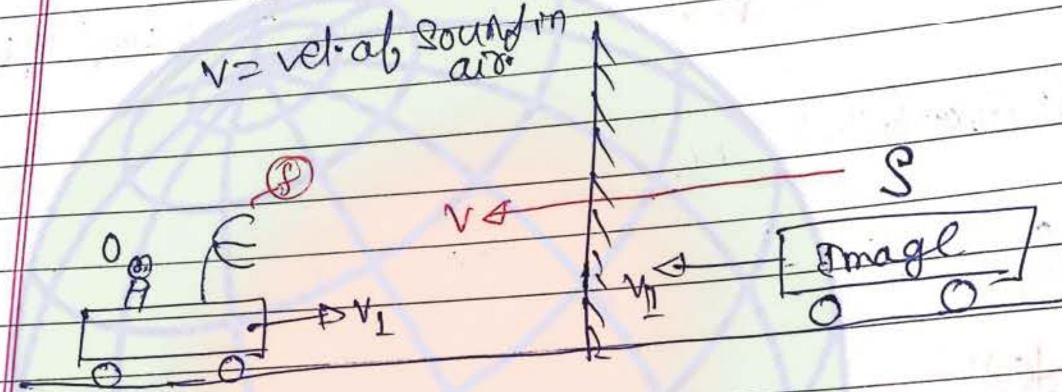
$$f_0 = f\,\frac{\cancel{V}}{(V-V_1)}\cdot\frac{V+V_1}{\cancel{V}}$$

Imp

$$\boxed{f_0 = f\,\frac{(V+V_1)}{(V-V_1)}}$$

Short trick

Image method :→
(wall is stationary)



$$f_0 = f\left(\frac{V+V_1}{V-V_1}\right)$$

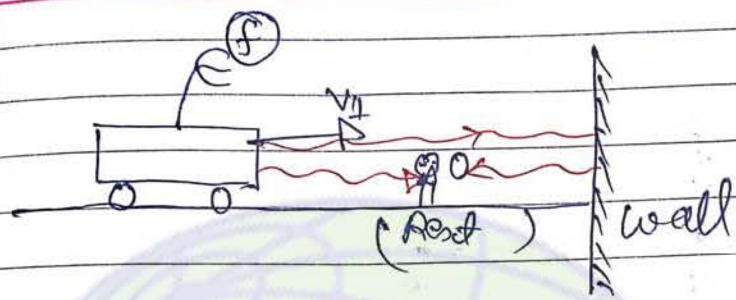
How to use this shortcut :→

शर्त ⇒ यह formula तभी Applicable है जब कोई एक Rest में हो।
इस method में जो Rest में है उसको mirror मान लो और उसके Image को source मान लो और जो वास्तविक observer है वह तो है ही। और इस प्रकार General Doppler's effect का formula लगा दो।
(तथा इस पर और कुछ Problem बना कर इसे आश्वस्त कर लो।)

If there is no relative motion frequency is same.

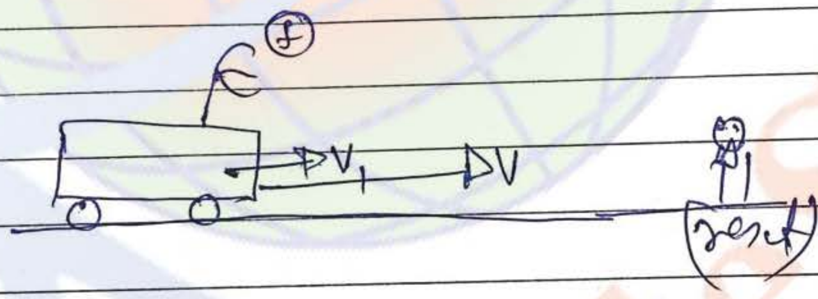
$v \rightarrow$ velocity of sound in air

(Q)



Find the beat frequency observed by the observer (In case of approach)
car approaches toward the static observer
observer observes two sound wave one is direct and other is reflected from the wall

Ans)



$$f_1 = f\left(\frac{v}{v-v_1}\right) \quad \text{--- (1)}$$

$$f_{wall} = f\left(\frac{v}{v-v_1}\right) \Rightarrow f_0 = f_1$$

$$\boxed{\Delta f = 0}$$

Ex 21, 2, 4
HW

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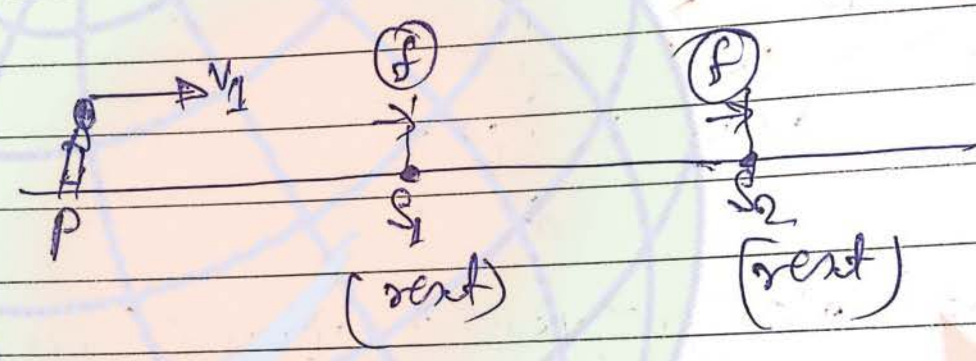

$$\boxed{\Delta f = 0}$$

 Fluid mechanics

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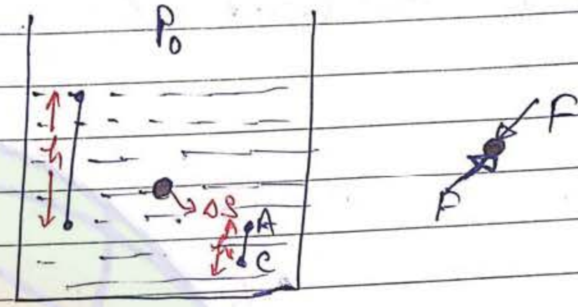
Pressure :⇒



$$\boxed{P = \lim_{\Delta S \to 0}\left(\frac{F}{\Delta S}\right)}$$

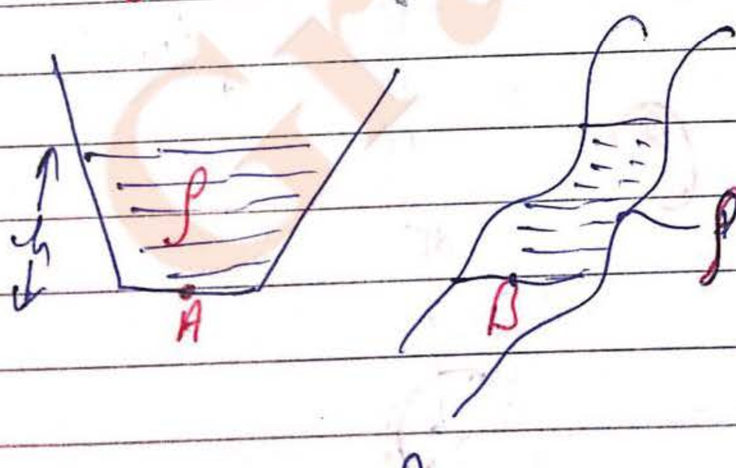
$$\boxed{P_B = P_0 + h\rho g}$$

Absolute Pressure

$$\boxed{P_C - P_A = h_1 \rho g}$$

$$\boxed{P_B - P_0 = h\rho g}$$

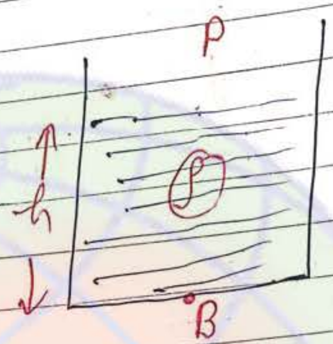
Gauge Pressure.



$P_A = P_B$

1st Choice

(Q) If the one fifth of the total height of water column is drilled out what is the new pressure at the bottom of vessel



$$P_B = 3P$$

Ans Initial :-

$$P_B = 3P = P + h\rho g$$

$$h\rho g = 2P$$

New

$$P_B'' = P + \left(\frac{4}{5}h\right)\rho g$$

$$= P + \frac{4}{5} \times 2P \Rightarrow \frac{13P}{5}$$

(Q)

Find the net force exerted by the water on the wall of the dam

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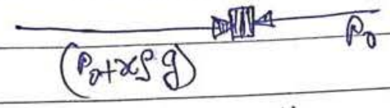
~~Rod~~ of length (l) and height (H)
Also find torque about "O".

Ans



$$F = \int dF = \int_{x=0}^{x=H} (x\rho g)\, l \cdot dx$$

$$F = \frac{\rho g l H^2}{2}$$

$$\tau_0 = \int_{x=0}^{x=H} (dF)\cdot(H-x)$$

$$= \int_0^H (x\rho g)\, l\, dx\, (H-x)$$

$$= \rho g l \int_0^H (xH - x^2)\, dx$$

$$= \rho g l \left[\frac{H^3}{2} - \frac{H^3}{3}\right]$$

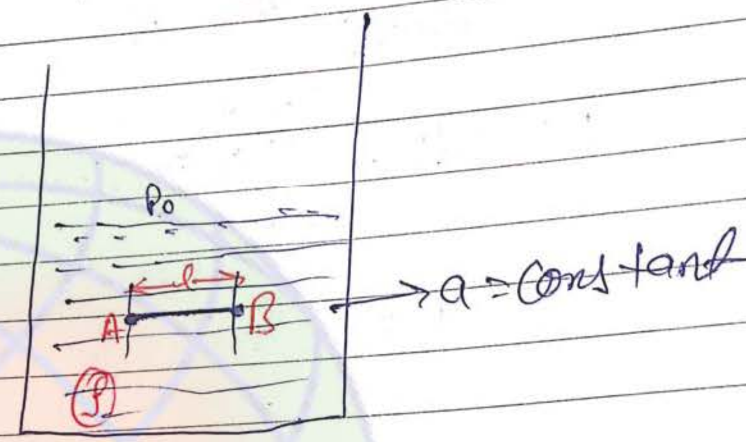
$$= \frac{\rho g l H^3}{6} \qquad \boxed{\tau_0 = \frac{\rho g l H^3}{6}}$$

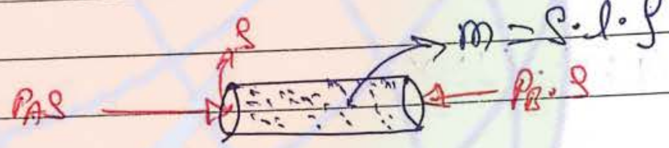
Note:- The height at which the resultant force would have to act to produce the same torque is $\boxed{x = \frac{H}{3}}$ where $x \Rightarrow$ point of application of force.

$$\boxed{F = P_{avg.} \times \text{Projected Area}}$$

$$= \frac{h\rho g}{2} \cdot l H$$

Pressure difference in accelerated frame ⇒



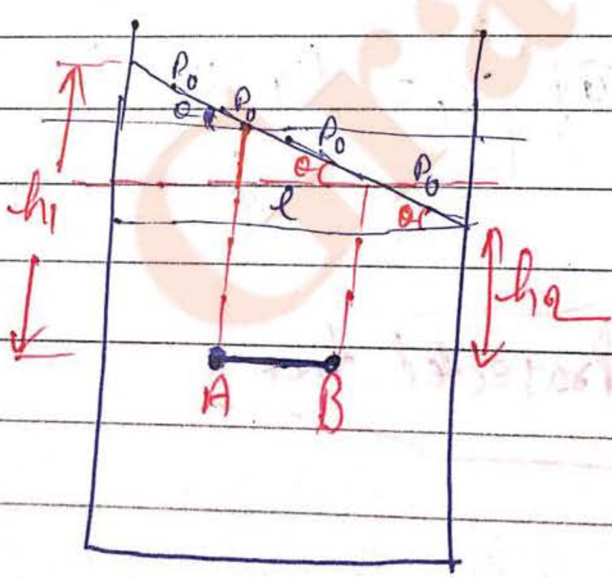


$$P_A S - P_B \cdot S = ma$$

$$(P_A - P_B)\cancel{S} = \cancel{S} \cdot l \cdot \rho \cdot a$$

$$\boxed{P_A - P_B = l \rho a}$$

a = 0
V = constant
= 0



$$P_A = P_0 + h_1 \rho g$$

$$P_B = P_0 + h_2 \rho g$$

$$\boxed{P_A - P_B} = (h_1 - h_2)\rho g$$

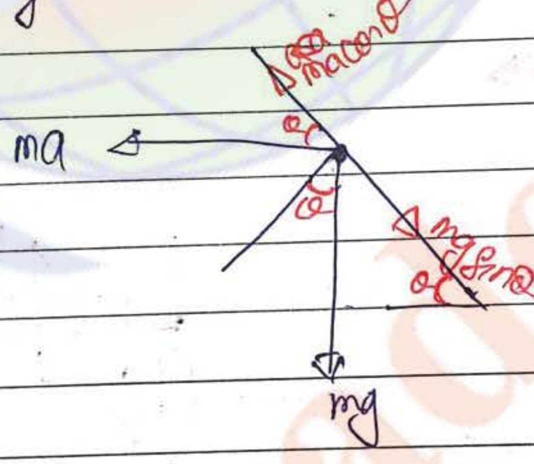
$$l \cancel{\rho} a = (h_1 - h_2)\cancel{\rho} g$$

$$\boxed{\frac{h_1 - h_2}{l} = \frac{a}{g} = \tan\theta}$$

Shortcut

Concept

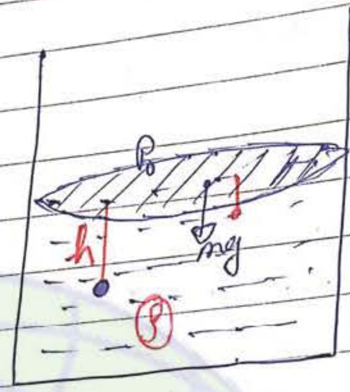
Net force along the surface of the liquid is always zero



$$\cancel{m} a\cos\theta = \cancel{m} g\sin\theta$$

$$\boxed{\frac{a}{g} = \tan\theta}$$

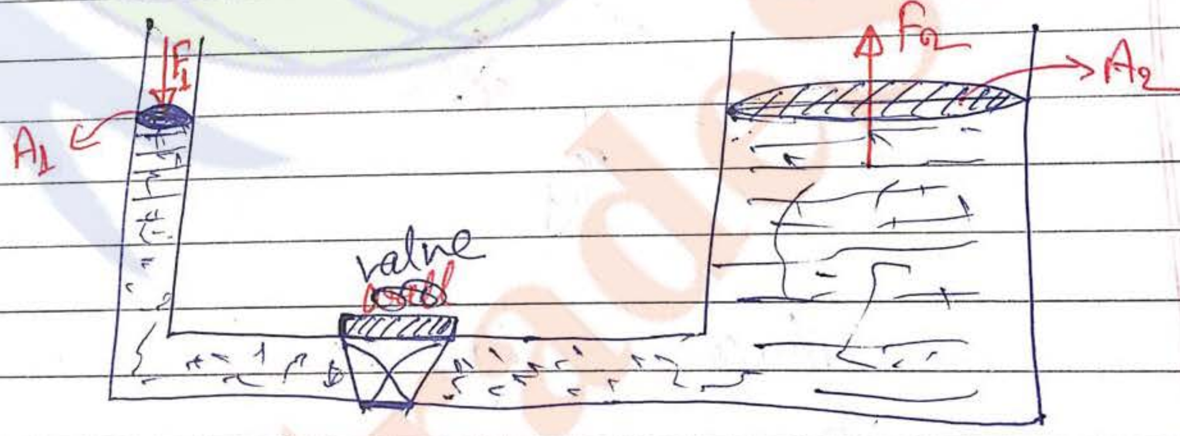
Pascal's law ⇒



If the Pressure at any point Inside the liquid is change this change is transmitted to all Point's Inside the liquid without being deminished.

Application :-

☆ Hydolic lift ⇒

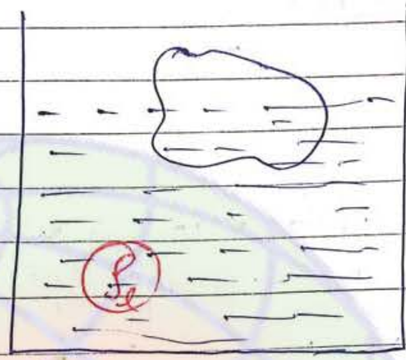


$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

so,

$$\boxed{F_2 = \left(\frac{A_2}{A_1}\right) F_1}$$

Boyant force (B)
(upthrust)



$$\boxed{\vec{g}_{eff} = |\vec{g} - \vec{a}|}$$

$$\boxed{B = V_{in}\, \rho_l\, g_{eff}}$$

where
$V_{in} \rightarrow$ vol. of body inside the liquid

$\rho_l =$ density of liquid.

Note:-

$\downarrow g = a$
freely falling ~~load~~ body

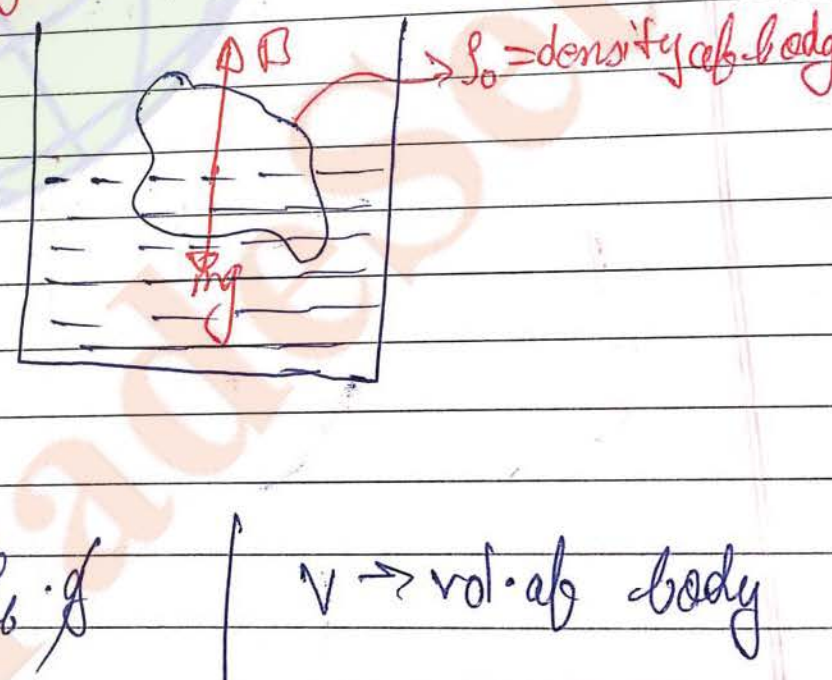
$g_{eff} = g - g$
$= 0$
$B = 0$

$$\boxed{B = mg}$$
or
$$V_{in}\rho_l g = M_{body}\, g$$

where $m \Rightarrow$ mass of whole body.

Boyant forces →
It is the resultant of ~~all~~ the contact ~~surface of surround~~ forces of liquid in contact with the emerged body in vertically upward direction.
It arises ~~due~~ due to Pressure difference.

Law of Buoyancy :→



$B = mg$

$V_{in}\, \rho_L \cdot g = V \cdot \rho_b \cdot g$ | $V \rightarrow$ vol. of body

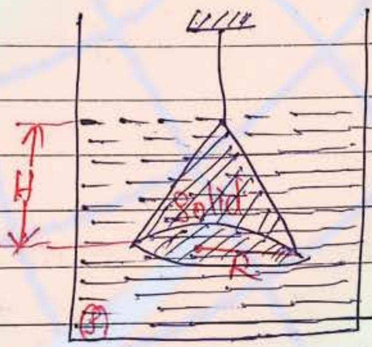
$$\boxed{f_{in} = \frac{V_{in}}{V} = \frac{\rho_{b \to body}}{\rho_{liquid}}}$$, $$\boxed{f_{out} = 1 - f_{in}}$$

Fraction of body Inside the ~~body~~ liquid

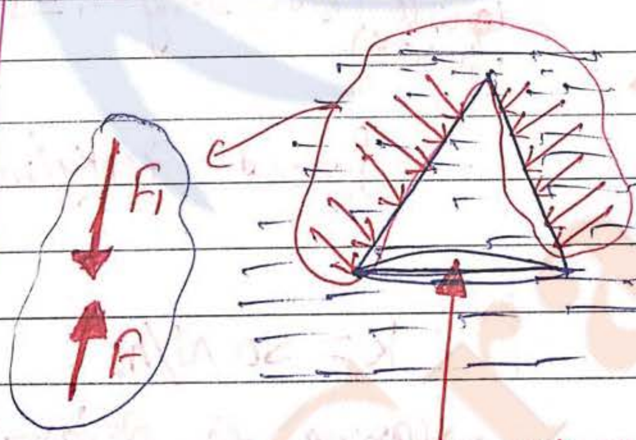
Relative density (R.D) or Specific gravity ⇒

$$\boxed{R.D = \frac{\rho_b}{\rho_{water}(4°C)}}$$

Q.)



Find the net force exerted by the liquid on the ~~state~~ slant side of the cone. (Neglect the Atmospheric Pressure.)



$$F = (\pi R^2) h\rho g$$

$$F - F_1 = B$$

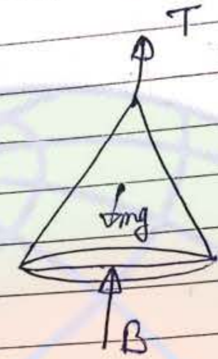
$$F_1 = F - B$$
$$= \pi R^2 \cdot H\rho g - \frac{1}{3}\pi R^2 H\rho g$$

Date / /

1st Choice

$F_1 = \frac{2}{3}\pi R^2 H \rho g$

Note: ⇒

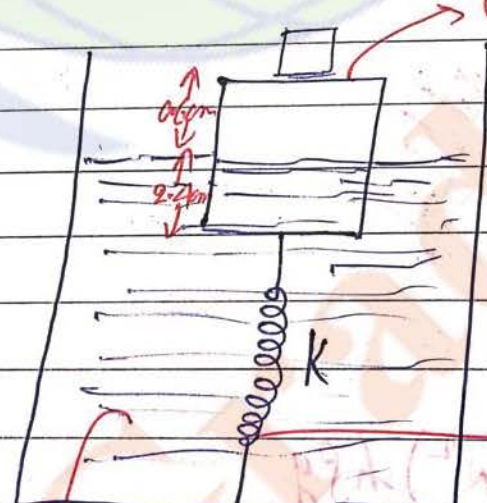


$m = \frac{1}{3}\pi R^2 \cdot H \cdot d_{cone}$ → density

$T + B = mg$

Note:→ Attention:—
Boyant is resultant force of All so if we use any other force then it is wrong. (Understand carefully.)

Q.)



→ cube (side length = 3cm) (wood)

$\rho_{wood} = 800 kg/m^3$.

$K = 50 N/m$

→ spring is Relaxed

$\rho_w = 10^3 kg/m^3$

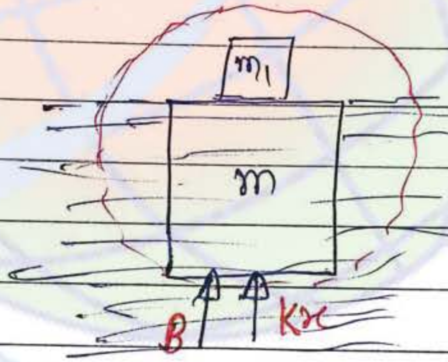
Find the maximum weight which

can be put on the upper surface of cubical block. So that the level of upper surface of cubical block and the level of liquid becomes same.

8.

$$R.D = \frac{0.8\cancel{00}}{1\cancel{000}}$$
$$= 0.8$$

So,



In equilibrium:

$$B + kx = mg + \cancel{m_1 g}\ w \xrightarrow{} m_1 g$$

$$(B + kx - mg) = w$$

$$0.27N - 0.3N - 0.21N = w$$

So,

$$w = 0.35\ N$$

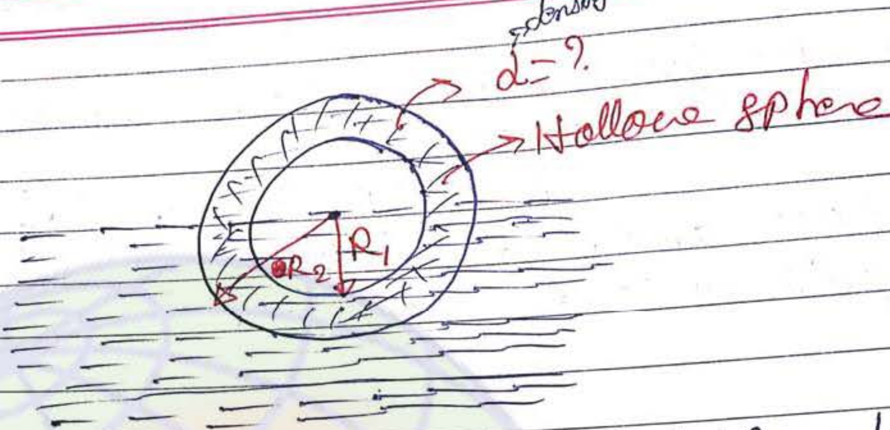
$$B = (27 \times 10^{-6} m^3) \times 1000 \times 10$$
$$= 0.27N$$

$$kx = 50 \times 0.6 \times 10^{-2}$$
$$= 0.3N$$

$$mg = (27 \times 10^{-6} m^3) \times 800 \times 10$$
$$= 0.21N$$

1st Choice

[Ex.]



Find the density of matter of the sphe. sphere is flotting where ~~Half~~ Half portion is Inside the liquid
Also find boyant fore.

Ans) $\frac{V_{in}}{V} = \frac{\rho_b}{\rho_l}$

$$\frac{\cancel{V}/2}{\cancel{V}} = \frac{d}{\rho}$$

$$d = \frac{\rho}{2}$$

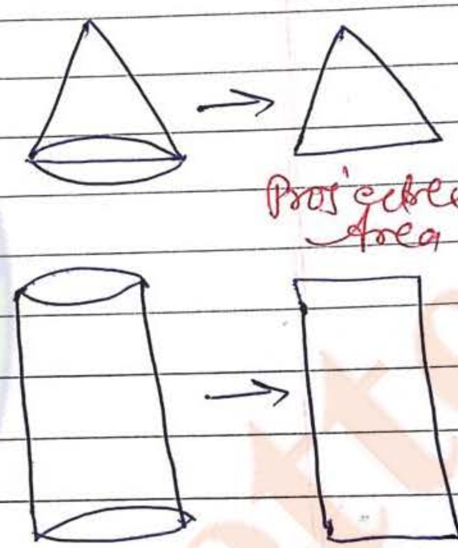
value of Boyant fore in term of

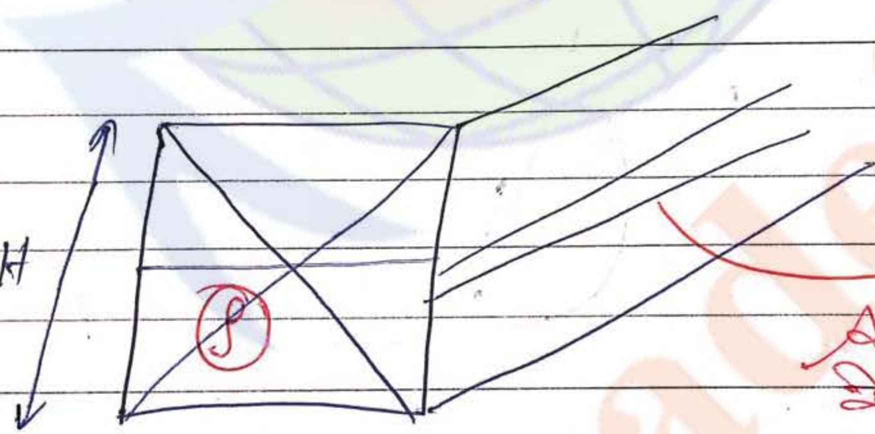
$$B = mg$$

$$\frac{2}{3}\pi(R_2)^3 \rho \cancel{g} = \frac{4}{3}\pi\left[(R_2)^3 - R_1^3\right] \times d \cancel{g}$$

Shortcut of (Dam):⇒

$$\boxed{F = P_{avg.} \times \text{Projected Area}}$$

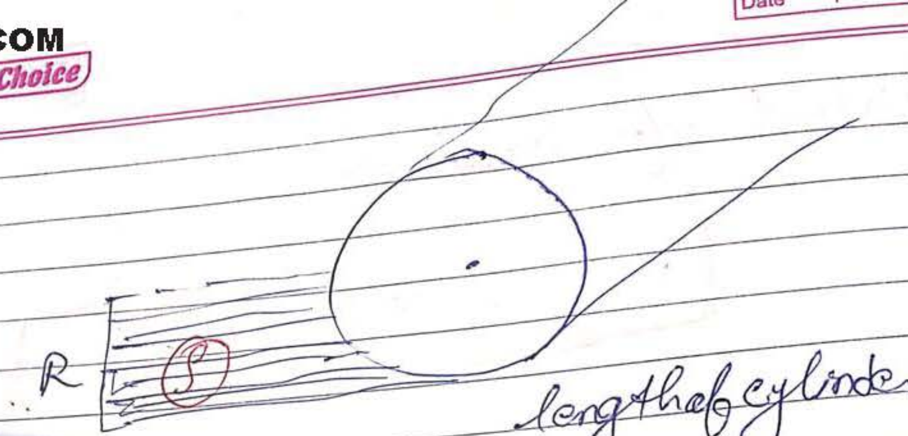




यह Projected Area एक Rectangle की तरह दिखाई देगा जिसका (Breadth) एक height (H) है और Length (l) है। इसलिए Projected Area अर्थात Rectangle का Area (H×l) होता है।

$$\boxed{F = \frac{H\rho g}{2} \times l \cdot H}$$

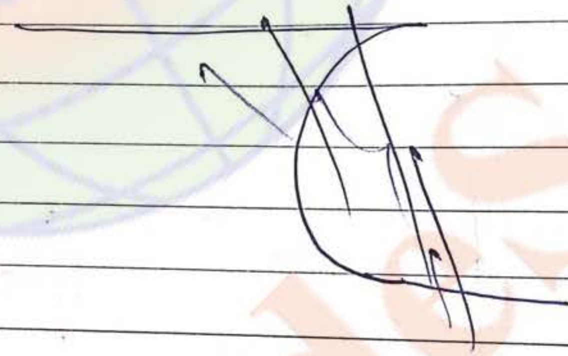
Ex.)

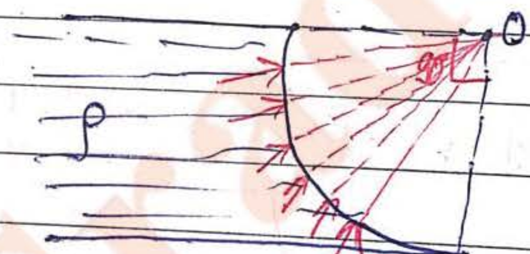


length of cylinder = l

$$F = \underbrace{\frac{R\rho g}{2}}_{\text{Half length का (Average } \mathbb{E})} \cdot R \cdot l = \left(\frac{\rho g R^2 l}{2}\right)$$

Ex.)





→ Circular Arc

Find the torque about "O".

τ_0 = Zero

1st Choice

Hydrodynamics
(flowing fluid)

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The branch of mechanics in which we study the flowing properties of fluid (liquid+gas) etc.

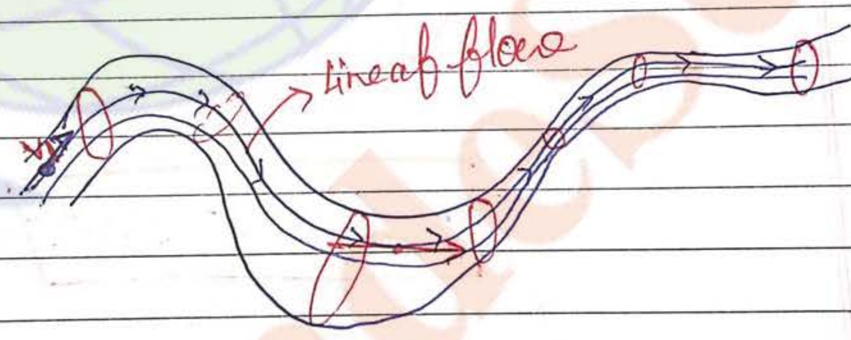
Ideal fluid ⇒

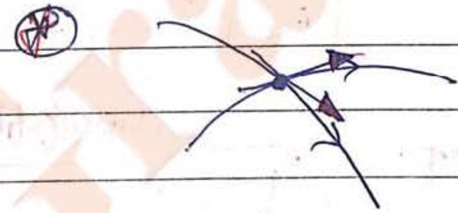
① Non viscous ⇒ There is no friction (or viscocity) b/w the two Adjacent layer's of fluid.

② Incompressible ⇒ Density of the fluid remain constant does not change during the flow.

③ Irrotational flow ⇒ Individual particle of fluid should not rotate.

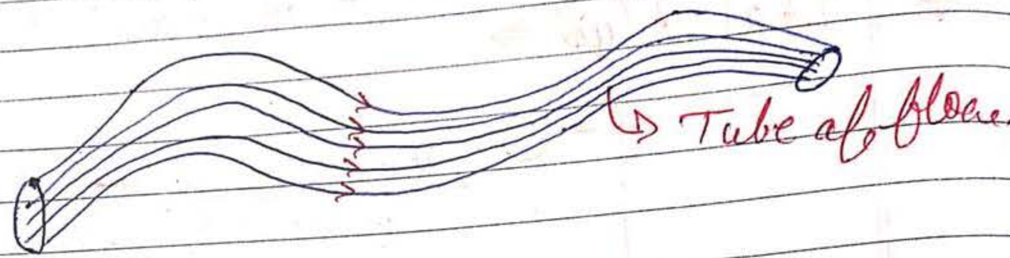
④ steady flow/streamline flow/laminar flow:—





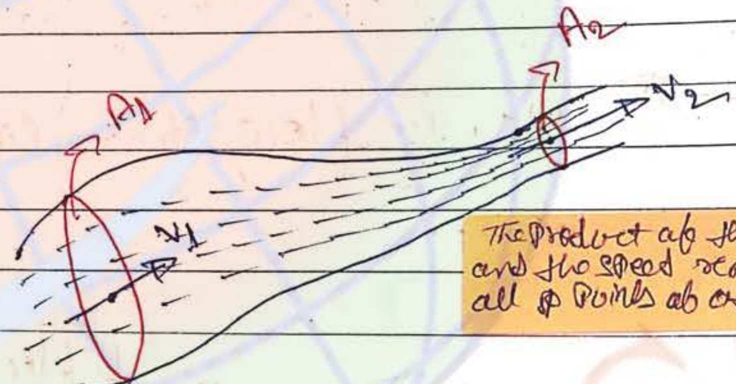
Two lines of flow can never Intersect each other

(because In one point not two direction velocity is possible)



Principle of Continuity ⇒

$\boxed{\text{It is Based on conservation of mass.}}$



The product of the area of cross-section and the speed remains the same at all points of tube of flow

$$\boxed{A \times V = \text{constant}} \rightarrow \boxed{V \propto \frac{1}{A}}$$

$$\boxed{A_1 V_1 = A_2 V_2 = \text{-----} = \text{constant}}$$

$$\boxed{\underbrace{\frac{dV}{dt}}_{\text{volume rate}} = A \times V = \text{constant}}$$

volume of fluid

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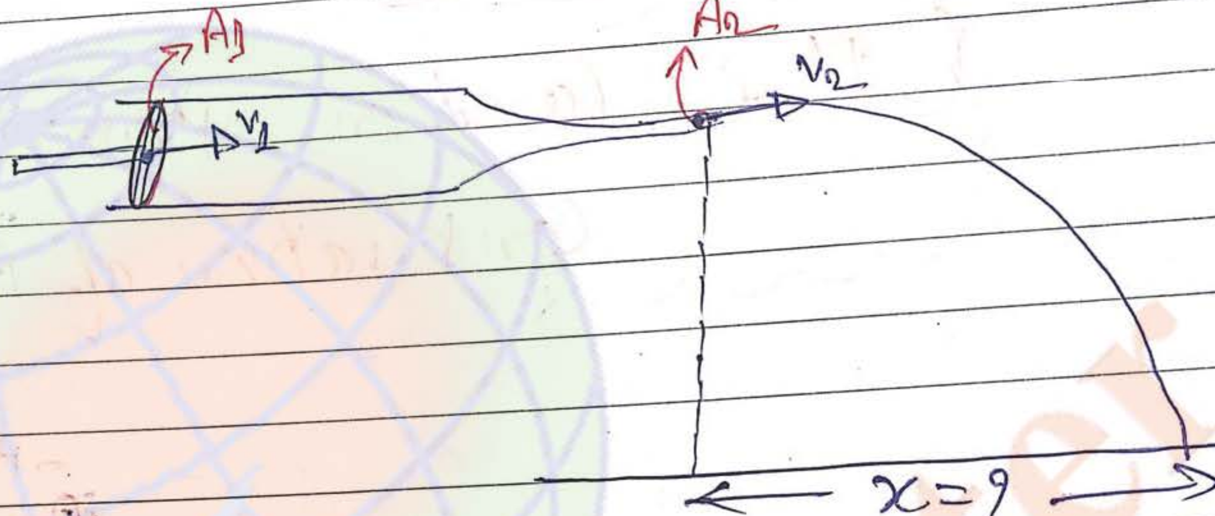
1st Choice

IIT 2004
[Ex.]



$A_1 V_1 = A_2 V_2$

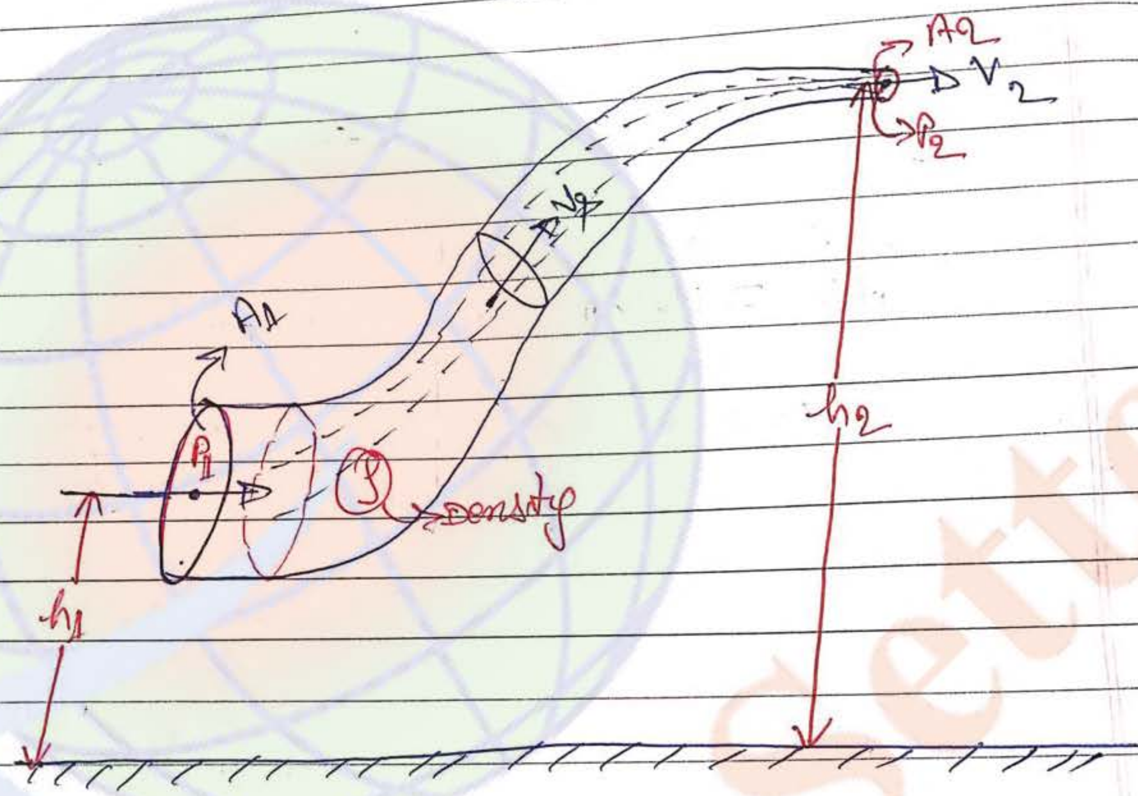
$V_2 =$

$x = V \times t$

$= V \sqrt{\frac{2h}{g}}$

Bernoulli's Theorem ⇒

It is based on Principle of Conservation of "Energy".



Per an Ideal flow of an Ideal fluid flowing through a tube of non-uniform crossectional area.

The sum of Pressure, The kinetic energy (K.E) per unit volume and Potential energy (P.E) per unit volume always remain constant at all points of the fluid Inside the tube.

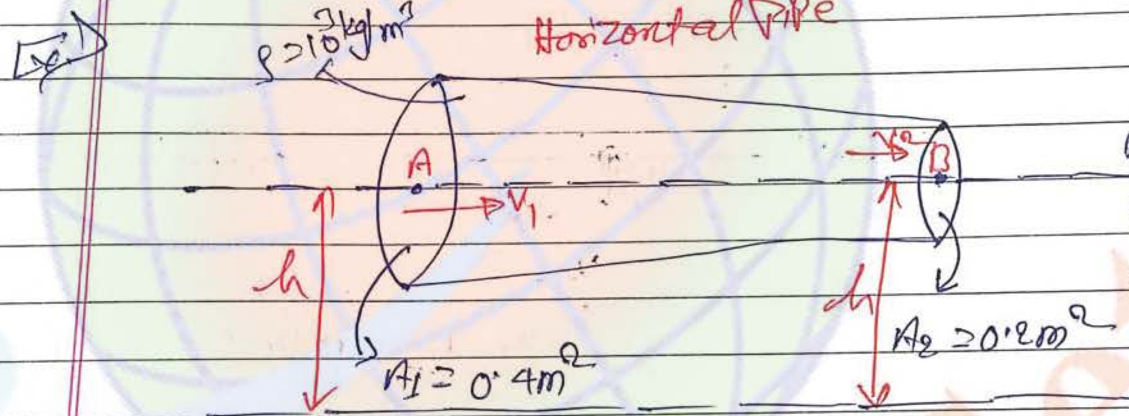
Ex 1 → 1 to 17, 22, 23
Ex 2 → 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18,
Ex 4 → 2, 3, Passage-1, 8, 9, 10, 11

Pressure ⇒ Pressure energy per unit volume

$$= \frac{Fl}{Al}$$

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}}$$

Ex)



$P_A - P_B = 10 N/m^2$
Find volume flow rate at point "A" = ?

$$\boxed{P + \frac{1}{2}\rho v^2 = \text{Constant}}$$

→ For Horizontal pipe.

(Pressure और velocity दोनों balance होते हैं)

$$P_A + \frac{1}{2}\rho v_A^2 + \cancel{\rho g h} = P_B + \frac{1}{2}\rho v_B^2 + \cancel{\rho g h}$$

$$P_A - P_B = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$10 = \frac{\cancel{1000}\ 50}{2}(4v_1^2 - v_1^2)$$

$A_1 v_1 = A_2 v_2$

$v_2 = 2v_1$

$3v_1^2 = \frac{1}{50}$

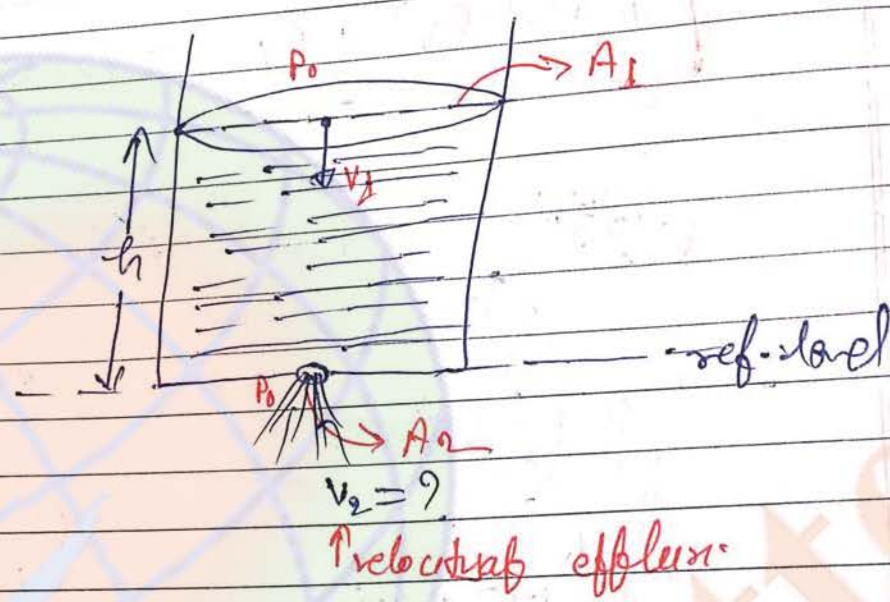
$v_1 = \ldots$ m/s

vol. rate $= \frac{0.4}{\sqrt{150}}$ m^3/s

IIT 20

IIT 2018

Application :— velocity of efflux (Torricell's theorem :⇒)



$A_1 v_1 = A_2 v_2$

$$\cancel{P_0} + \frac{1}{2}\rho v_1^2 + \rho g h = \cancel{P_0} + \frac{1}{2}\rho v_2^2$$

$$v_2^2 = v_1^2 + 2gh$$

$$v_2^2 = \frac{A_2^2}{A_1^2} v_2^2 + 2gh$$

$$\boxed{v_2 = \sqrt{\frac{2gh}{\left(1 - \frac{A_2^2}{A_1^2}\right)}}}$$

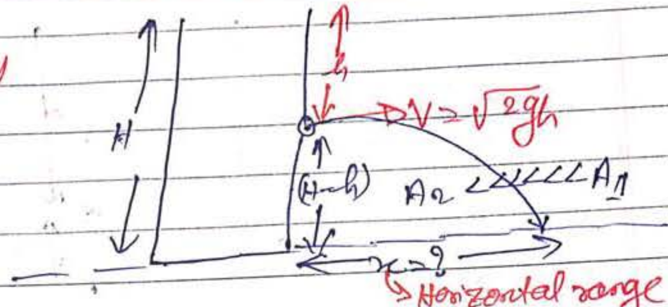
$$v_2 = \sqrt{\frac{2gh}{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

If $A_2 <<<< A_1$

$$v_2 = \sqrt{2gh}$$

IIT 2010

1.) Position of location of one hole and its Range:→



$$x = V \times t$$

$$= \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}}$$

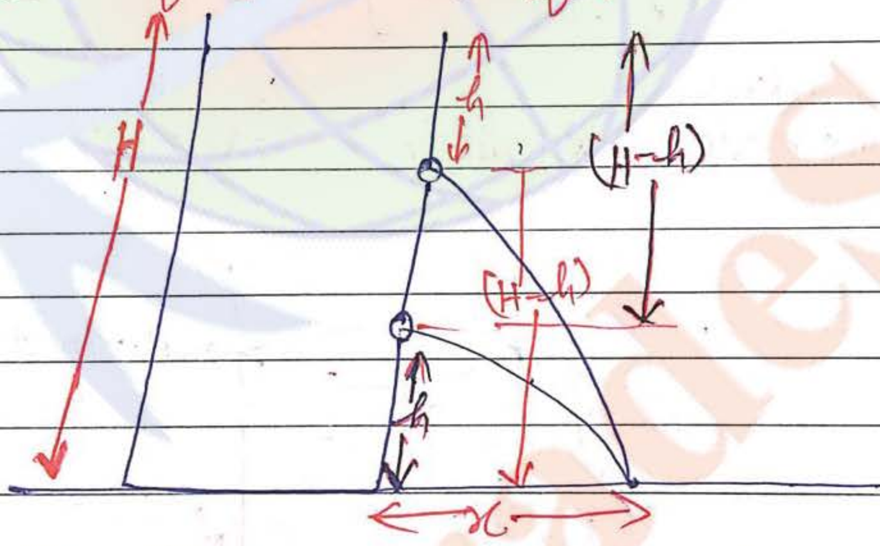
Imp:

$$\boxed{x = 2\sqrt{h(H-h)}}$$

Point के ऊपरवाला height

Point (hole) के नीचेवाला height

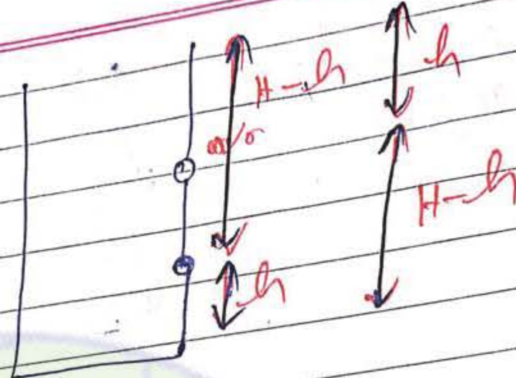
2.) location of two holes of the different height of same range:→



$h \to (H-h)$

$$x = 2\sqrt{h(H-h)}$$

मतलब एक बार ऊपर से small "h" पर hole कर दो और एक बार नीचे से small "h" का hole कर दो दोनों hole से same Range पर पानी गिरेगा यही formula से है।



III) Condition of max. horizontal range:-

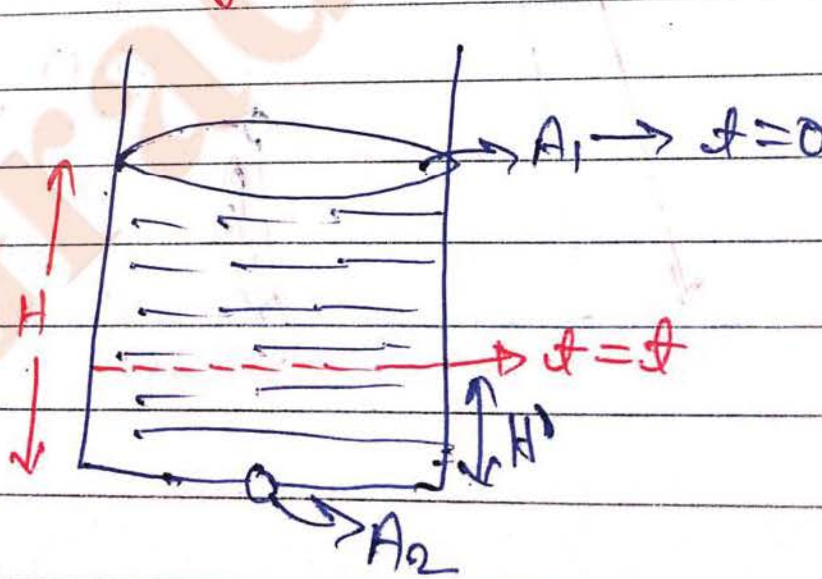
$$\frac{dx}{dh}=0 \Rightarrow \frac{d}{dh}(x^2)=0$$

$$\boxed{h=\frac{H}{2}}$$

$$\boxed{x_{max}=H}$$

very imp

IV) Time taken to empty the tank :-



- Time for $H \rightarrow H'$

$$\boxed{t = \frac{A_1}{A_2}\sqrt{\frac{2}{g}}\left(\sqrt{H} - \sqrt{H'}\right)}$$

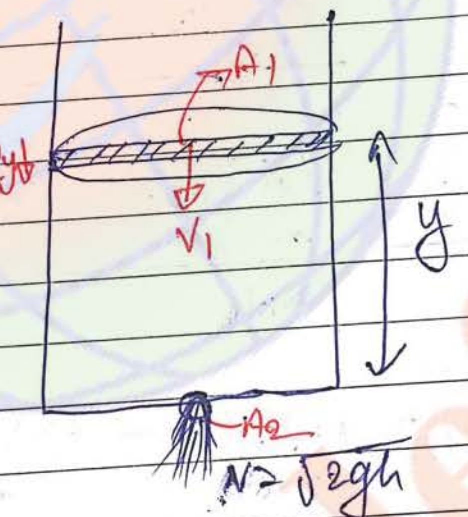
- Time to empty the tank:-

$$\boxed{t_{empty} = \frac{A_1}{A_2}\sqrt{\frac{2H}{g}}}$$

$H' = 0$

Derivation:-

→ Bodore



$$A_1 V_1 = \frac{dV}{dt} = -A_1\frac{dy}{dt} = A_2 V$$

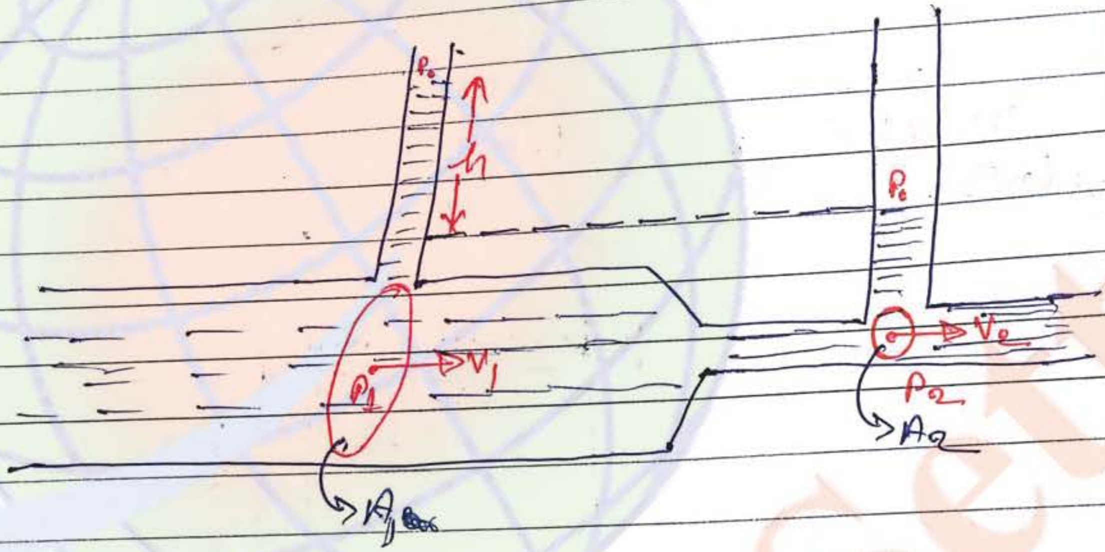
$$-A\frac{dy}{dt} = A_2\sqrt{2gy}$$

$$\int_{H}^{H'} \frac{-dy}{(\sqrt{2g})\sqrt{y}} = \frac{A_2}{A_1}\int_{t=0}^{t} dt$$

So,

$$\boxed{\int_{H}^{H'} \frac{-dy}{(\sqrt{2g})\sqrt{y}} = \frac{A_2}{A_1}\int_{t=0}^{t} dt}$$

2.) Venturimeter :-



$$A_1 V_1 = A_2 V_2 \quad \text{—— (1)}$$

$$\cancel{P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2}$$

$$\cancel{P_1 - P_2 \quad \frac{1}{2}}$$

so,

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$\boxed{P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)}$$

$$h\cancel{\rho} g = \frac{1}{2}\cancel{\rho}\left(V_2^2 - V_1^2\right)$$

$$\boxed{v_2^2 - v_1^2 = 2gh}$$

$$v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right) = 2gh$$

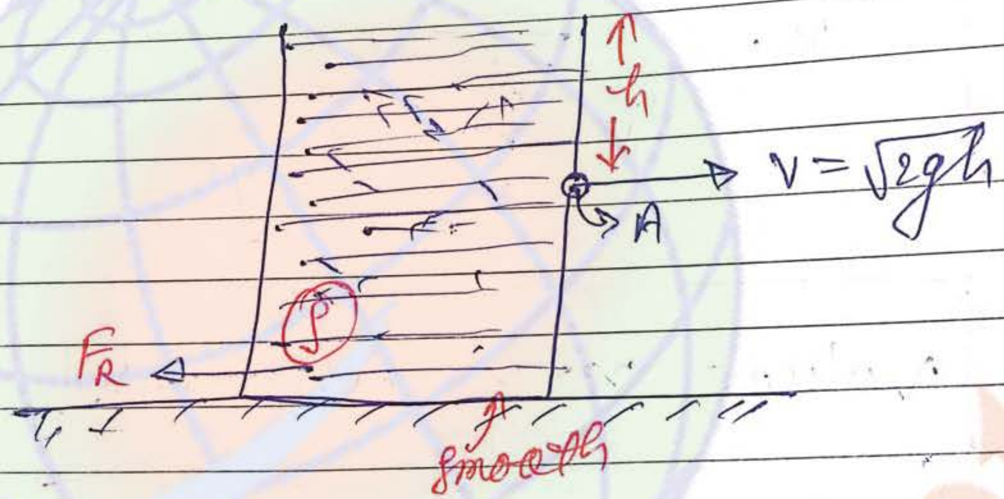
$$\boxed{v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1^2}{A_2^2} - 1\right)}}}$$

$$\boxed{\text{volume Rate} = A_1 v_1 = A_2 v_2}$$

Date / /

1st Choice

Reaction force or backward force ~~due to~~ of the vessel due to ejection of liquid from the vessel.



$$\boxed{F_R = \rho A v^2}$$

Reaction or backward force

Derivation :-

$$\frac{dv}{dt} = Av$$

$$F_R = \frac{\Delta P}{\Delta t} = \frac{m(v-0)}{\Delta t}$$

$$\frac{mass}{time} = \rho \frac{dv}{dt}$$

$$= \rho A v$$

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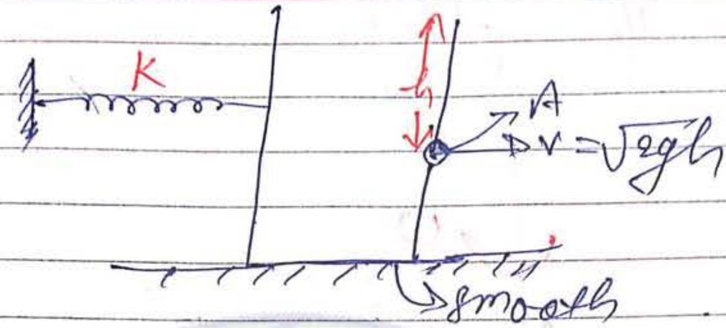
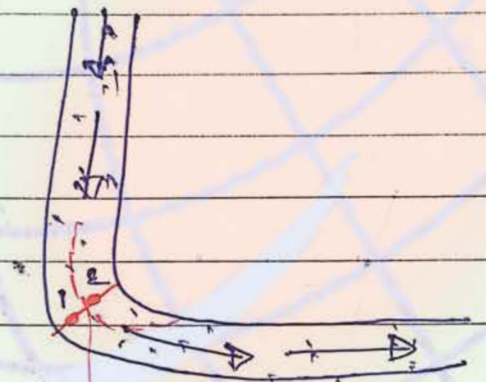
(6.)



$\rho A v^2 = k x_0$

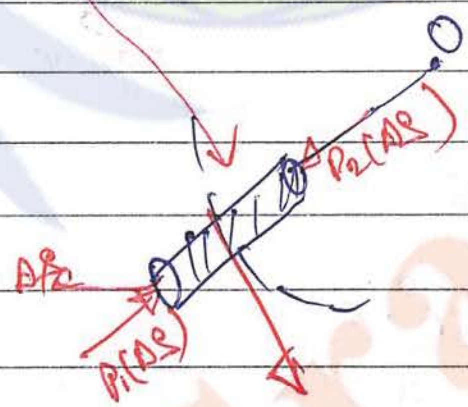
$x_0 = \checkmark$

(7.)



correct option:-

i) $P_1 > P_2$
ii) $P_2 > P_1$
iii) $P_1 = P_2$
iv) none.

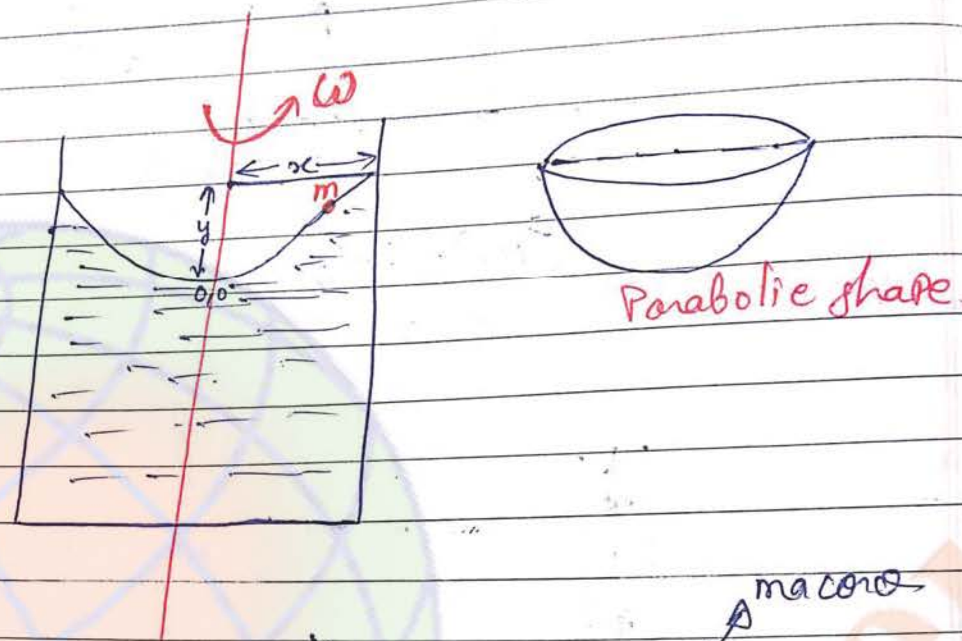


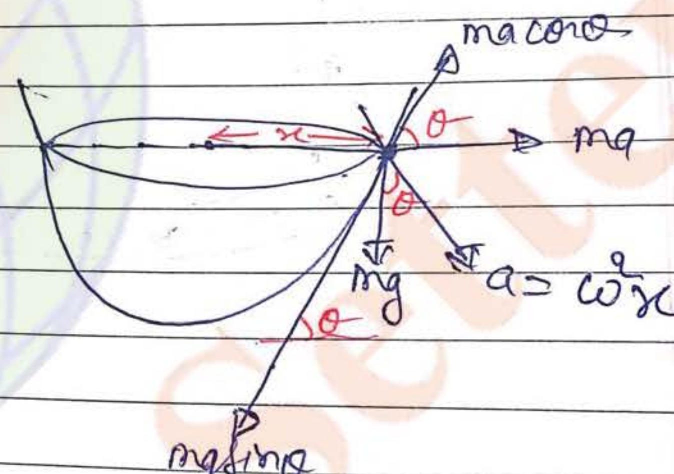
$P_1(\Delta S) - P_2(\Delta S) = (\Delta m) r \cdot \omega^2$

i) $P_1 > P_2$

Note:- यदि circula turn लेने पर उस पर centripetle force लगेगा।

Rotation of fluid:⇒





$$mg\sin\theta = ma\cos\theta$$

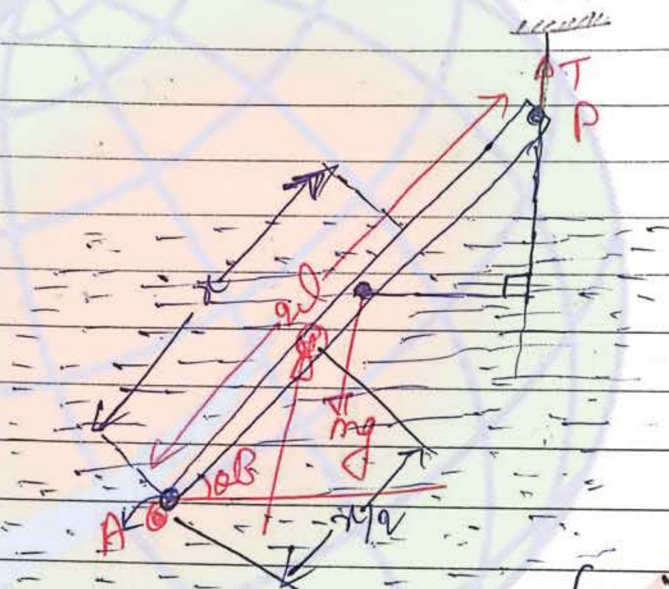
$$\tan\theta = \frac{a}{g} = \frac{\omega^2 x}{g}$$

$$\frac{dy}{dx} = \frac{\omega^2 x}{g}$$

$$\int dy = \frac{\omega^2}{g}\int x\,dx$$

$$\boxed{y = \frac{\omega^2 x^2}{2g}}$$

Q.) A slender homogeneous rod of length 2L floats partly immersed in water, being supported by a string fastened to one of its end, as shown. The specific gravity of the rod is 0.75. The length of rod that extends out of water is—



$$(\tau_P)_{net} = 0$$

$$mg\cancel{\cos\theta} = B\times\left(2L-\frac{x}{2}\right)\cancel{\cos\theta}$$

$$\cancel{A(2L)(0.75\,\rho_w)gL =}$$

$$\cancel{\Rightarrow A(2L)(0.75\rho)}$$

so,

$$\Rightarrow mg = B\left(2L-\frac{x}{2}\right)$$

$$\Rightarrow \cancel{A}\cdot(2L)(0.75\,\rho_w)\cancel{g}L = \cancel{A}\cdot x\,\cancel{\rho_w}\cancel{g}\left(2L-\frac{x}{2}\right)$$

$$1.5L = x\left(2L-\frac{x}{2}\right)$$

$\rho_\omega = 1 g/cm^3$

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1st Choice

$$1.5L^2 = 2L \cdot x - \frac{x^2}{2}$$

$$3L^2 = 4L \cdot x - x^2$$

$$x^2 - 4Lx + 3L^2$$

$$x^2 - 3lx - lx + 3L^2$$

$$x(x-3L) - l(x-3l)$$

$$x = 3L \,(\times) \qquad x = l$$

$$(a^2 \times 2) \times \rho_\omega \times \cancel{g} = 200 \times \cancel{g} \qquad \because \rho_\omega = 1 g/cm^3$$

$$a^2 = 100$$

$$a = 10 cm$$

Properties of matter

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Elasticity ⇒ It is the property of material (or matter) due to which a body regains its original shape and size after removal of deforming forces.

- On application of deforming forces :- Distance b/w atom's of material changes. because of change in separation b/w atom's, a restoring force is developed Inside the material.

Stress :-

It is defined as re-storing force per unit Area

$$\boxed{\text{Stress} = \frac{\text{Restoring force}(F)}{\text{Area}(A)}}$$

(S.I unit of stress is N/m^2)

Types of Stress :-

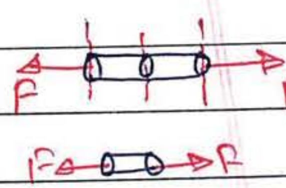
Stress

~~Normal Stress~~ S_n

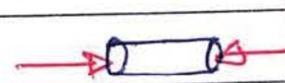
→ ① Normal stress (or Longitudinal stress)

$$\boxed{S_n = \frac{F_n}{A}}$$

→ Tensile

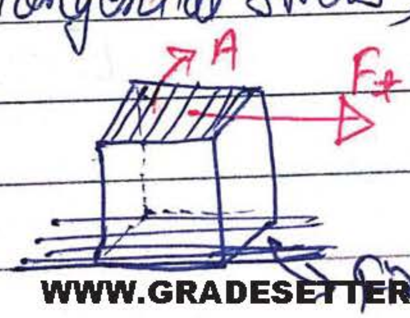


→ Compressive

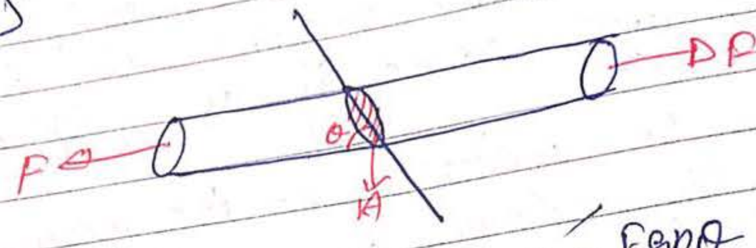


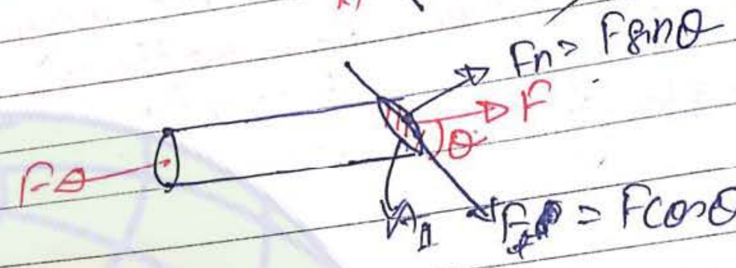
→ ② Shear stress (or Tangential stress)

$$\boxed{S_t = \frac{F_t}{A}}$$



Ex.)

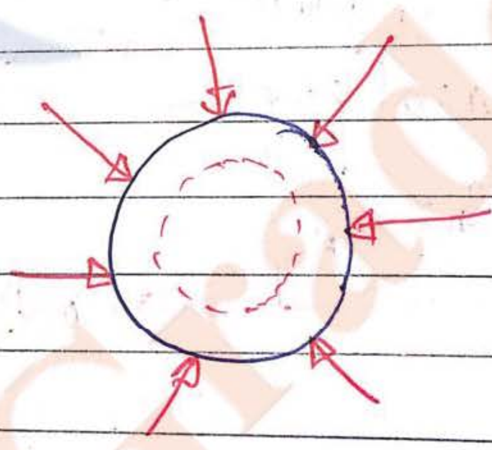




$$S_n = \frac{F\sin\theta}{A_1}$$

$$S_t = \frac{F\cos\theta}{A_1}$$

(3) volume stress –



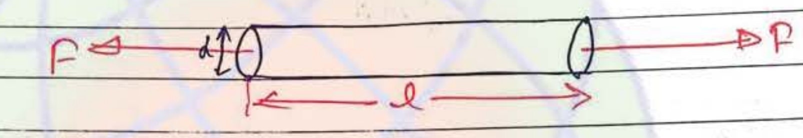
$$\boxed{\text{Forces Pressure} = \Delta P}$$

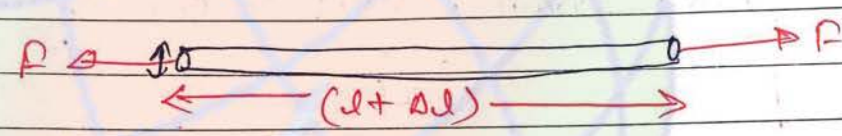
Strain ⇒

It is Relative change in "dimension" ~~and~~ or "Shape" of the body. It has no unit and no dimension.

Type of Strains:-

1) Longitudinal Strain:-





$$\boxed{\text{longitudinal Strain} = \frac{\Delta l}{l} = \frac{\text{change in length}}{\text{original length}}}$$

Poisson's ratio (σ)

(S)

change in dimension

Transverse strain

!- Here change in length and change in diameter both takes place. so there are two strains in the wire.

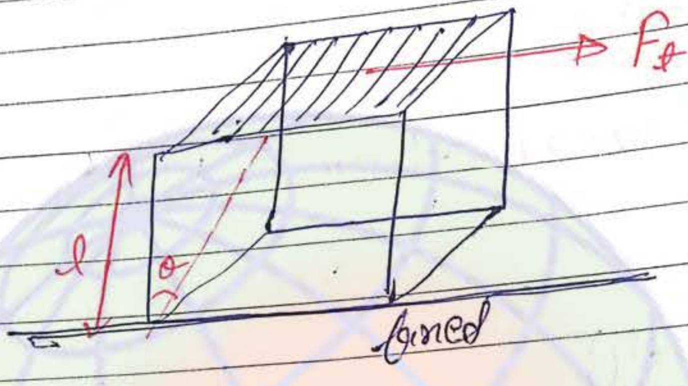
$$\text{ratio} = -\frac{(\Delta d)/d}{(\Delta l)/l}$$

or

$$\boxed{\text{Poisson ratio} = \frac{-(\Delta r)/r}{(\Delta l)/l} = \frac{-l}{r}\cdot\frac{\Delta r}{\Delta l}}$$

l = length of material, r = radius of material

2.) Shear Strain ⇒



$$\boxed{\text{Shear strain} = \tan\theta \simeq \theta = \frac{x}{l}}$$

when
$\theta \longrightarrow 0$
unit ratio
which is dimensionless

3.) volume strain

$$\boxed{\text{volume strain} = \frac{\Delta V}{V}}$$

Hooke's law :⇒

(For small deformation)

$\boxed{\text{Stress} \propto \text{Strain}}$ → (Not formula)

Stress = E Strain

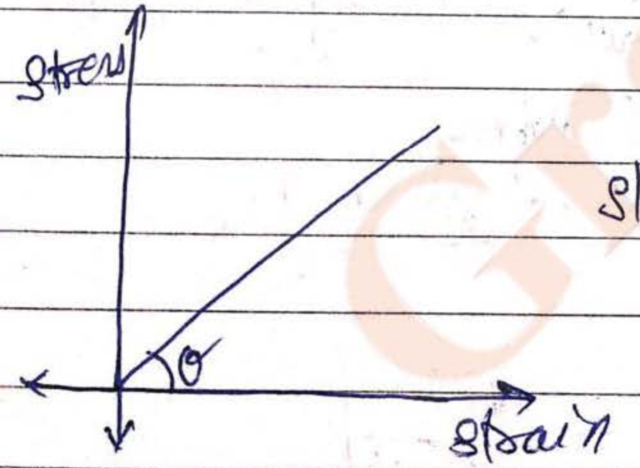
$$\boxed{E = \frac{\text{Stress}}{\text{Strain}}}$$

modulus of elasticity :⇒

→ It mainly depends on the nature of material and slightly depends on temperature.

On Increasing the temp. modulus of elasticity ~~Increase~~ ~~Increases~~ ~~decreases~~ Increases but their variation is small.

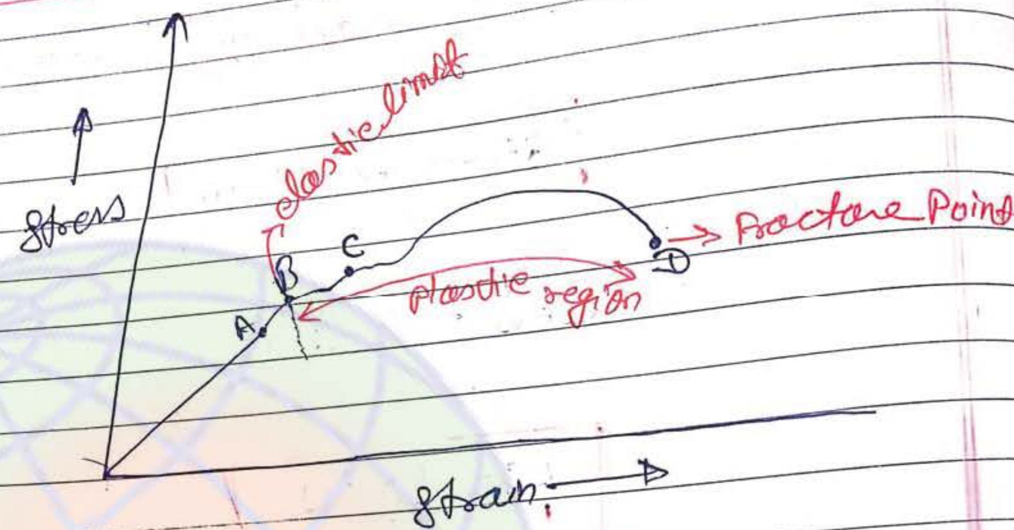
→ (T) Confuse.)



$$\text{slope} = \tan\theta = \frac{\text{Stress}}{\text{Strain}} = E$$

(Unit ⇒ N/m^2)

Metallic wire ⇒



O → A → Hooke's law
B → Elastic limit (yield point)

1. Youngs modulus of elasticity (Y) ⇒

(only for solid)

$$\boxed{Y = \frac{\text{Normal stress (longitudinal stress)}}{\text{longitudinal strain}}}$$

$$\boxed{Y = \frac{F/A}{\Delta l / l} = \frac{Fl}{A(\Delta l)}}$$

$$\boxed{Y = \frac{Fl}{A(\Delta l)}}$$ Imp

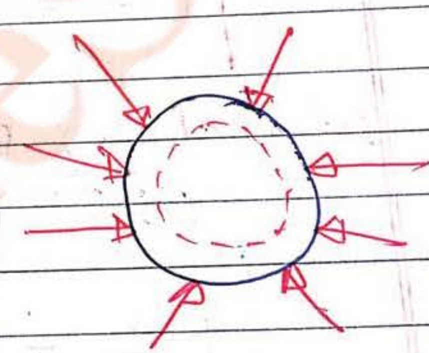
2.) Shear modulus or modulus of rigidity (η)
(only for solid)

$$\boxed{\eta = \frac{\text{shear stress}}{\text{shear strain}}}$$

$$\boxed{\eta = \frac{F_t/A}{\theta}}$$

3.) Bulk modulus → (B) or "K" (solid and fluid)

$$\boxed{B = \frac{\text{volume stress}}{\text{volume strain}}}$$



$$\boxed{B = -\frac{\Delta P}{\left(\Delta V/V\right)} = -V\cdot\frac{\Delta P}{\Delta V}}$$

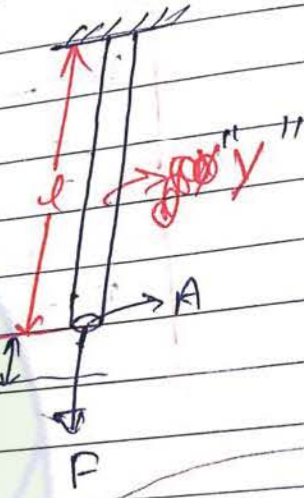
Imp

$$\boxed{\text{Compressibility } (C) = \frac{1}{B}}$$

Note :=> "-ve" sign is not included in calculation [on applying pressure volume decreases and, so, "-ve" sign makes "B" positive.

1st Choice

~~Potential~~ Potential energy stored in a ~~stretch~~ stretched wire (massless)



$$Y = \frac{Fl}{A(\Delta l)}$$

$$F = \underbrace{\left(\frac{YA}{l}\right)}_{K}(\Delta l)$$

so, $\boxed{K = \frac{YA}{l}}$

$$F = kx$$

Note – A rod behaves like a spring with equivalent spring constant,
$K = \frac{F}{x} = \frac{F}{(\Delta l)} = \frac{YA}{l}$ so, $\boxed{K = \frac{YA}{l}}$

★ Energy stored in stretched rod: ⟹

$$U = \frac{1}{2}kx^2$$

so,

$$\boxed{U = \frac{1}{2}\left(\frac{YA}{l}\right)(\Delta l)^2}$$

Potential energy (Energy stored in stretched rod)

Now

$$\boxed{U = \frac{1}{2}\left(\frac{YA}{l}\right)(\Delta l)^2} \times \frac{AR}{Al}$$

Then we get

Imp

VVNR

$$\boxed{U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}}$$

↳ Elastic Potential Energy

$$\text{volume} = A \cdot l$$

$$Y = \frac{\text{stress}}{\text{strain}}$$

$$\boxed{u = \frac{P.E}{\text{volume}} = \frac{1}{2} \times \text{stress} \times \text{strain}}$$

Energy density (u)

$$\therefore \boxed{\text{Energy density } (u) = \frac{\text{Energy}}{\text{volume}}}$$

Note:- A rod behaves like a spring with equivalent spring

Combination of rods:

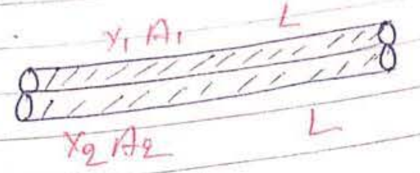
1) Series combination:

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} \qquad \because K = \frac{YA}{L}$$

$$\frac{(L_1 + L_2)}{Y_{eq} \cdot \cancel{A}} = \frac{L_1}{Y_1 \cancel{A}} + \frac{L_2}{Y_2 \cancel{A}}$$

$$\boxed{Y_{eq} = \left[\frac{L_1 + L_2 + \cdots\cdots}{\frac{L_1}{Y_1} + \frac{L_2}{Y_2} + \cdots\cdots} \right]}$$

2) Parallel combination :-

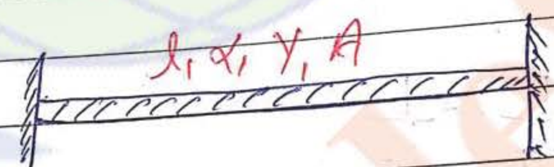


$$k_{eq} = k_1 + k_2$$

$$\frac{Y_{eq}(A_1 + A_2)}{L} = \frac{Y_1 A_1}{L} + \frac{Y_2 A_2}{L}$$

$$\boxed{Y_{eq} = \left(\frac{Y_1 A_1 + Y_2 A_2 + \cdots}{A_1 + A_2 + \cdots}\right)}$$

Thermal Stress



l = length of rod
A = Area of cross section of the rod
Y = Young's modulus
α = Thermal coefficient of linear expansion of the rod.

Now:-

1st Choice

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If temp. ↑ by Δt

$$\boxed{\Delta l = l\alpha\,\Delta t}$$ where, Δt = rise in temperature

$l_f = l(1 + \alpha\,\Delta t)$

$$\boxed{\text{Strain} = \frac{\Delta l}{l} = \frac{\alpha\,\Delta t}{1+\alpha\Delta t}}$$

∴ Thermal stress = Y × Strain

$$= \frac{Y\alpha\,\Delta t}{1+\alpha\,\Delta t}$$

$$\approx (Y\alpha\,\Delta t) \quad \text{when } \alpha \ll 1$$

∴ Force on wall/Rod

$F = \text{Stress} \times A$

$$\boxed{F = YA\alpha\,\Delta t}$$

force of the support by wall/Rod

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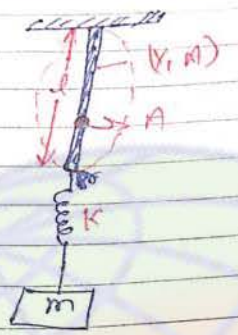
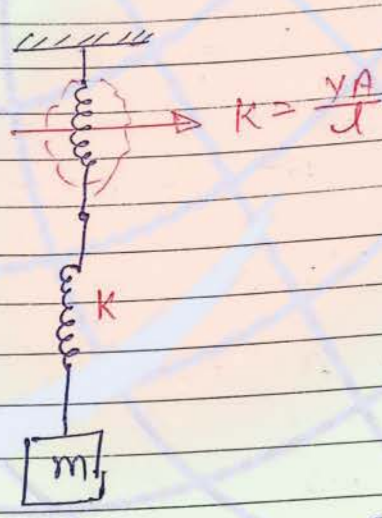
Q.



Block is slightly displaced in vertically downward direction and then released. Find the oscillation (S.H.M) of the block.

A.



नोट → Rode को spring मान लें और फिर Problem solve करें।

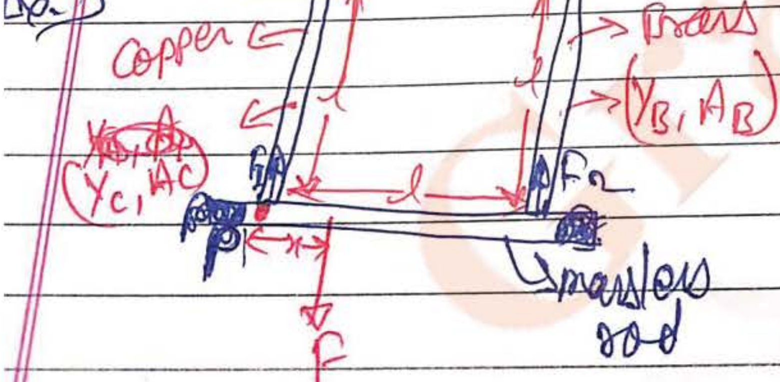
$$K_{eq} = \frac{K_1 K}{(K_1+K)} = \frac{\frac{YA}{l}\cdot K}{\frac{YA}{l}+K}$$

$$= \frac{YAK}{(AY+Kl)}$$

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}}$$

$$= 2\pi\sqrt{\frac{m(AY+Kl)}{YAK}}$$

Q.



Find the value of x for which the rod always remains in horizontal position in equilibrium.

As $Y = \frac{Fl}{A(\Delta l)}$

$$(\Delta l)_C = (\Delta l)_B$$

$$\frac{F_1 l}{A_C Y_C} = \frac{F_2 l}{A_B Y_B} \quad \text{—} (1)$$

$$F_1 + F_2 = F \quad \text{—} (2)$$

$$(\tau_{net})_P = 0$$

$$F \cdot x = F_2 l$$

$$x = \frac{F_2}{F} \cdot l \quad \text{—} (3)$$

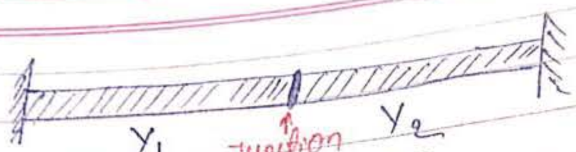
$$F_2\left(\frac{A_C Y_C}{A_B Y_B} + 1\right) = F$$

$$\frac{F_2}{F} = \frac{Y_B A_B}{(A_C Y_C + A_B Y_B)} \quad \text{—} (4)$$

from eq (3) and (4)

$$x = \frac{Y_B A_B \cdot l}{(A_C Y_C + A_B Y_B)} \text{ ur}$$

(b.)



Y_1 $\quad$ Y_2

α_1 $\quad$ α_2

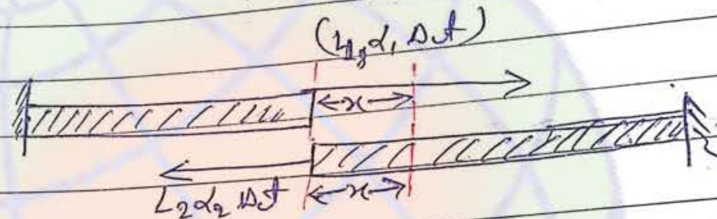
A_1 $\quad$ A_2

L_1 $\quad$ L_2

If temp is ↑ by Δt, find the shifting of junction

$\alpha_1 > \alpha_2$

Ans:-



$$\Delta L_1 = (L_1 \alpha_1 \Delta t - x)$$

$$\Delta L_2 = (L_2 \alpha_2 \Delta t + x)$$

Now;

$$F_1 = F_2$$

$$\left\{ \because Y = \frac{Fl}{A(\Delta l)} \right.$$ Youngs modulus

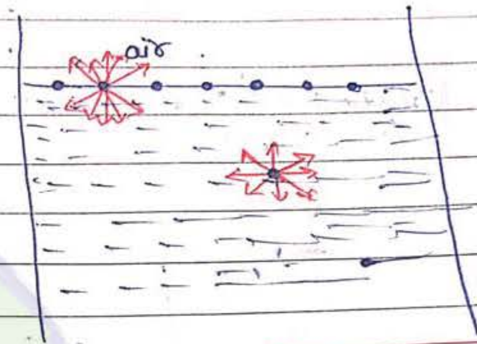
$$\frac{Y_1 A_1 (L_1 \alpha_1 \Delta t - x)}{L_1} = \frac{Y_2 A_2 (L_2 \alpha_2 \Delta t + x)}{L_2}$$

$$x = \frac{L_1 L_2 \Delta t (Y_1 A_1 \alpha_1 - Y_2 A_2 \alpha_2)}{(Y_1 A_1 L_2 + Y_2 A_2 L_1)}$$

$$\boxed{\text{density} = \rho_w \times \text{specific gravity}}$$

Surface Tension (T)

It is the property of liquid surface due to which a liquid try to occupy minimum surface area.

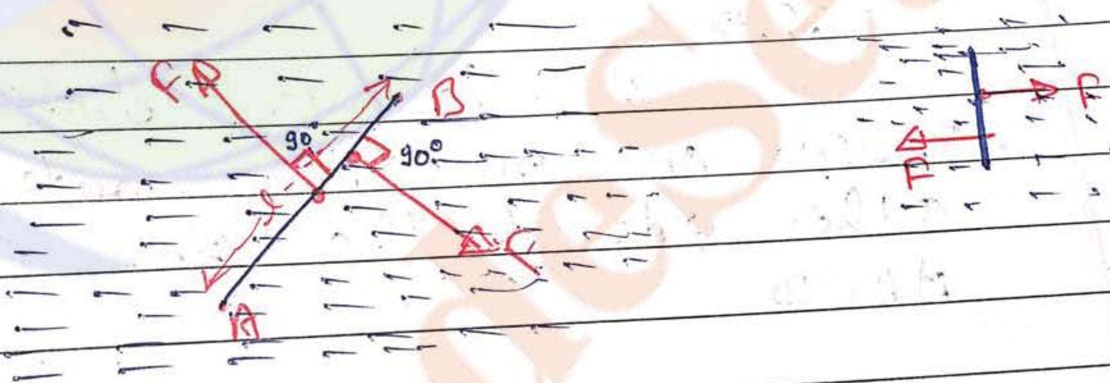


तना स्वर ज्ञान कर problem solve करें। use in life →

IMP

Definitions —

$$\boxed{T = \frac{F}{l}}$$



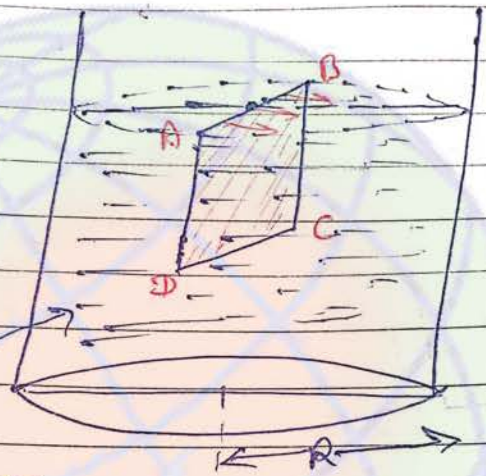
so,

$$\boxed{F = T \cdot l}$$

Always tangential to the surface and $\perp^{r}$ to the line (Imaginary)

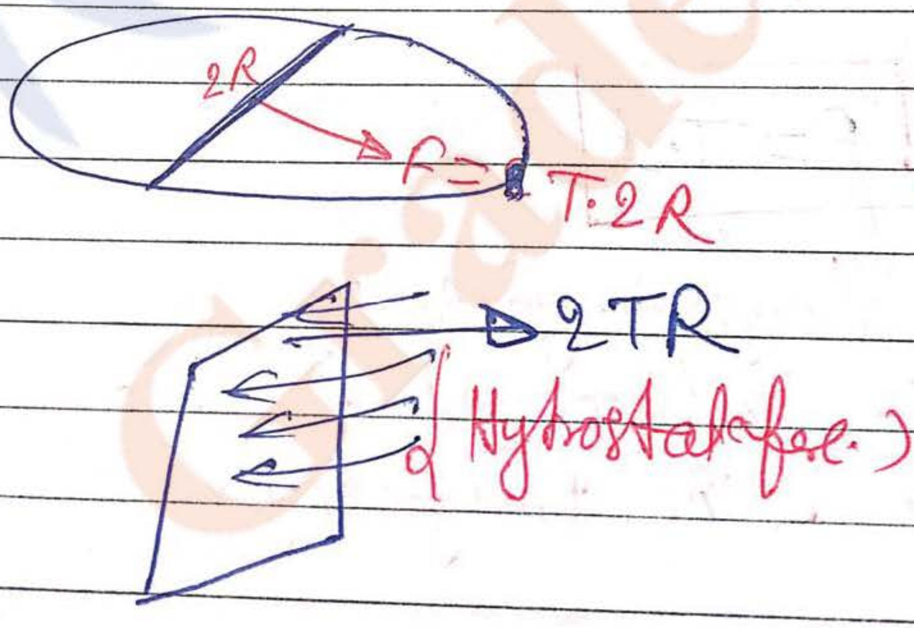
Surface Tension of liquid depends on the nature of liquid in ~~part~~ and its Temperature and it also depends on Temperature. And it does not depend on length (of Imaginary line) and dimension of the ~~length~~ (body.) Imagina line.

Ex.)



T = Surface Tension

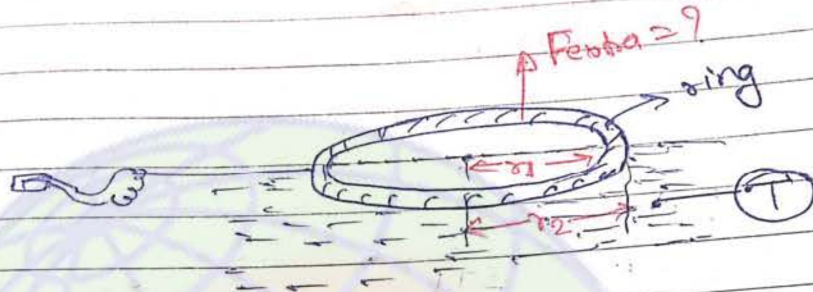
And the force due to surface tension on one side of water liquid due other side of the given cross-section ABCD

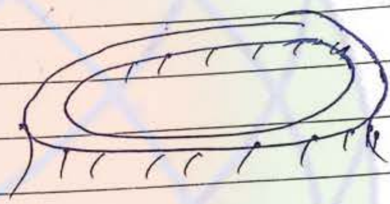


Cohesive (same nature) force · Adhesive force (diff) (air-wal)

Col 1 ⇒ 1 to 4
Col 2 ⇒ 5, 9, 4
Col 4 1, 2, 8, 10,

(6.)





$$F_{extra} = T(2\pi r_1 + 2\pi r_2)$$

Notes ⇒

At constant temperature surface tension is equal to the work done per unit increase in surface area of free liquid surface.

So

$$\boxed{T = \frac{W}{\Delta A}}$$

where – W ⇒ work
ΔA ⇒ Increase in Area

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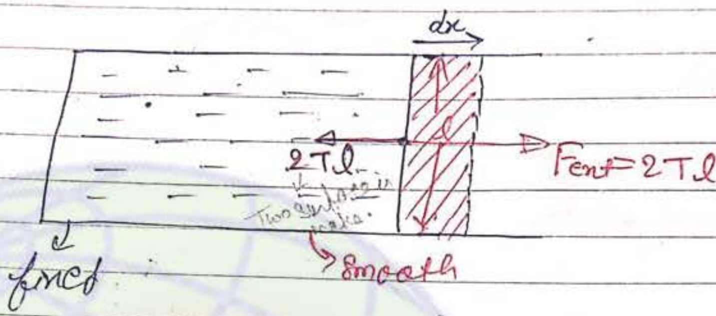
★ Surface Energy ⇒



$$W_{ext} = \Delta U = -W_c$$

$$W_{ext} = 2Tl\,dx$$

$$= T\,[2(l \times dx)]$$

$$\boxed{\Delta U = W_{ext} = T(\Delta A)}$$

→ change in surface area of liquid.

Imp.

$$\boxed{W = \Delta U = T\cdot(\Delta A)}$$

$$\cancel{W = \Delta U = T\cdot(\Delta A)}$$

Def. ST in term of Energy:-

$$\boxed{\begin{matrix}\Delta A > 0 \\ \Delta U > 0 \\ U_f > U_i\end{matrix}}$$

$$\boxed{T = \frac{W}{(\Delta A)}}$$ (under Isothermal condition)

Surface tension (T)

change in surface area of liquid.

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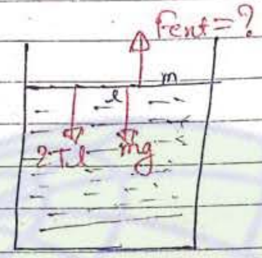
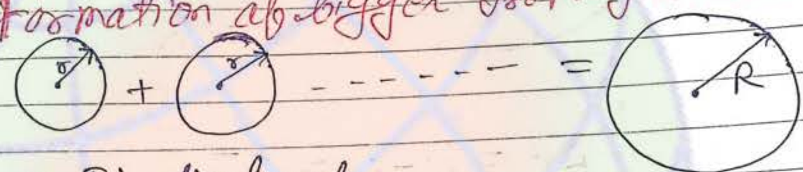
(36)

(Q.)



$$F_{ext} = (2Tl + mg)$$

Formation of bigger drop by a number of smaller drops :→



(Neglect gravitational force.)

n - Identical water drops.

$T \Rightarrow$ surface tension

$$W = \Delta U = ?$$

Note :- Increase in temperature of the bigger drop :-
$$\Delta t = \frac{3T}{s\rho}\left(\frac{R-r}{R \cdot r}\right)$$ where,
ρ = density of liquid, s = specific heat

$\ldots - 4\pi r^2)$

where, $R = n^{1/3} \cdot r$

छोटे drops से जुड़ कर बड़ा drop बनेगा तो "W" ve होगा

यहाँ पर -ve इसलिए है क्योंकि final का surface area initial drops से कम होगा

so,

~~W = 4πR²~~

$$\boxed{W = -4\pi R^2 T\left(\frac{R}{r} - 1\right)}$$

work done against surface tension.

$$n \cdot \left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

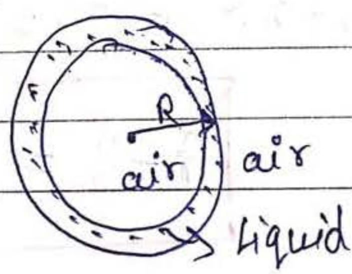
so,

No. of identical water droplets $\boxed{n = \left(\frac{R}{r}\right)^3}$ → or → $\boxed{n^{1/3} \cdot r = R}$ → Radius of bigger droplets.

※ For Soap bubbles (Two layers)

$$\boxed{W = T \cdot 2(\Delta A)}$$

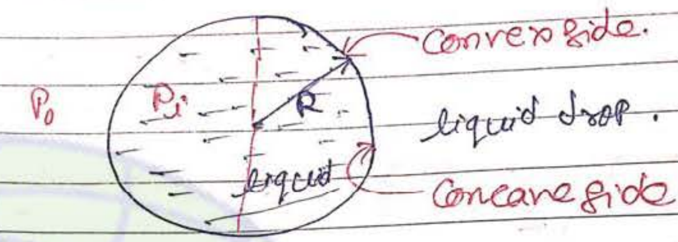
(यहाँ पर soap bubble का दो layer बनता है। एक अंदर में और एक बाहर में)



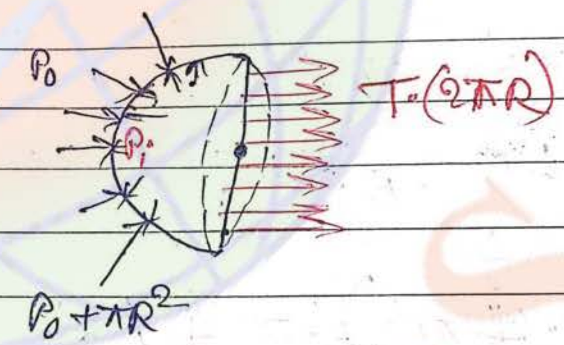
In this process energy is released.

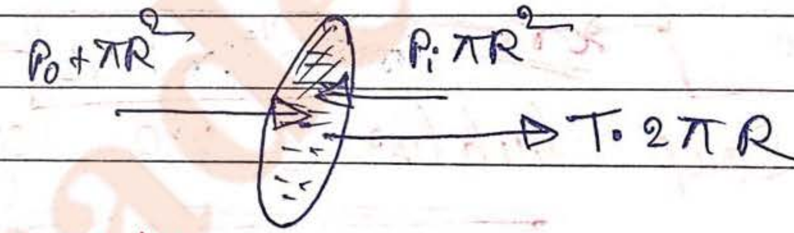
Excess Pressure Inside a liquid drop:

(neglect gravitational force.)



$$\boxed{\underbrace{\Delta P}_{\text{Excess pressure}} = P_i - P_0 = \frac{2T}{R}}$$





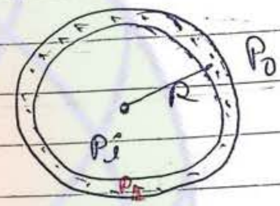
~~For Soap bubbles~~ so,

$$P_0 \cancel{\pi} R^{\cancel{2}} + T \cdot 2 \cancel{\pi} \cancel{R} = P_i \cancel{\pi} R^{\cancel{2}}$$

$$\boxed{P_i - P_0 = \frac{2T}{R}}$$

- For soap bubbles:⇒

$$\boxed{\Delta P = P_i - P_0 = \frac{4T}{R}}$$



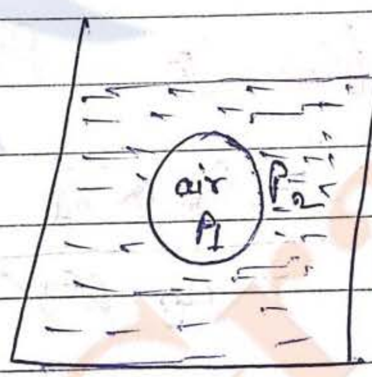
$$P_i - \cancel{P_1} = \frac{2T}{R}$$

$$\cancel{P_1} - P_0 = \frac{2T}{R}$$

$$P_1 - P_0 = \frac{4T}{R}$$

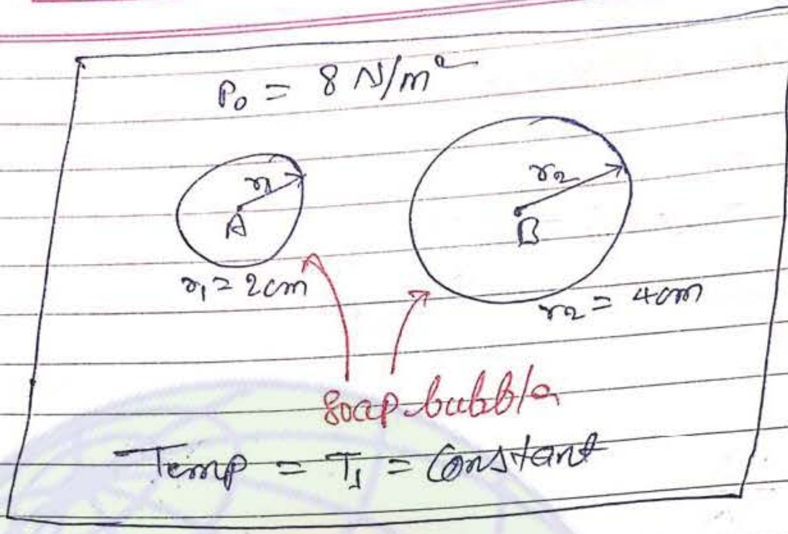
so, $$\boxed{P_1 - P_0 = \frac{4T}{R}}$$

Note:⇒ Concave side pressure is always greater than convex side pressure.



$$\boxed{P_1 > P_2}$$ — Always true.

IIT (Adv.)



$T = 0.04 \frac{N}{m}$

Find No. of moles $\frac{n_B}{n_A} = ?$

Ans

$$P_A = P_0 + \frac{4T}{r_1}$$

$$= 8 + \frac{4 \times 0.04}{2 \times 10^{-2}} = 16\ N/m^2$$

$$P_B = P_0 + \frac{4T}{r_2} = 12\ N/m^2$$

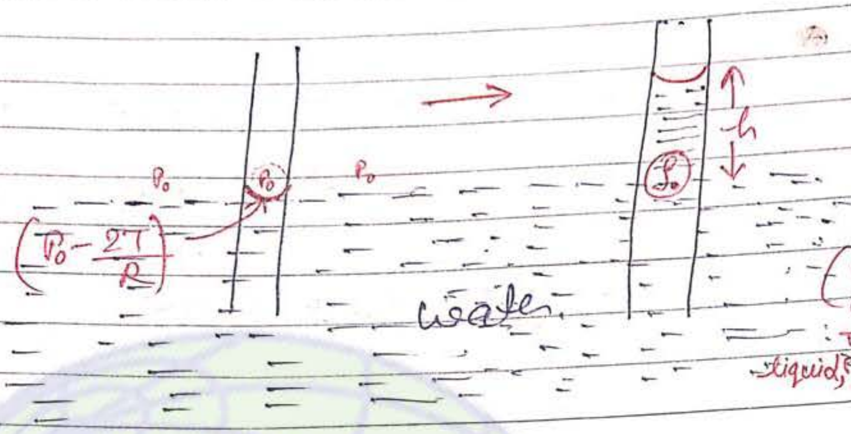
Here we use formula $PV = nRT \Rightarrow$

$$\frac{P_A \frac{4}{3}\cancel{\pi} r_1^3 = n_A \cancel{R} \cancel{T_1}}{P_B \frac{4}{3}\cancel{\pi} r_2^3 = n_B \cancel{R} \cancel{T_1}}$$

$$\frac{16}{12} \times \frac{(2)^3}{(4)^3} = \frac{n_A}{n_B}$$

So,

$$\frac{n_B}{n_A} = 6$$

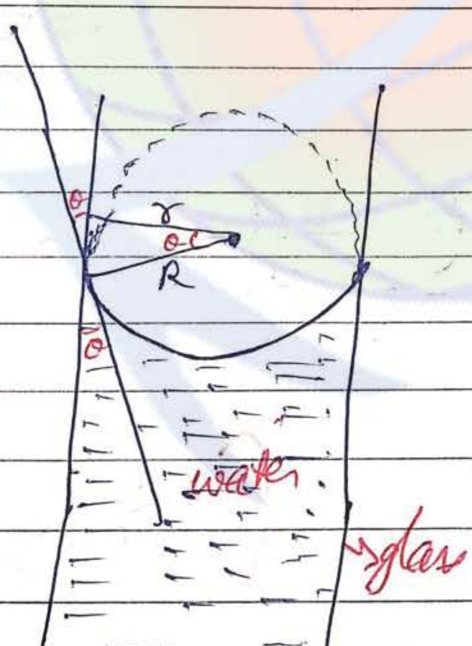


(Notes-> Concave वाले पर (water) liquid rise करता है)

$$\frac{2T}{R} = h\rho g$$

$$\boxed{h = \frac{2T}{R\rho g}}$$

where
$R \rightarrow$ Radius of meniscus.



$r \rightarrow$ radius of tube (मांगे तो $\cos\theta$ में)

$\theta \rightarrow$ Contact angle

$$\cos\theta = \frac{r}{R}$$

$$\boxed{h = \frac{2T\cos\theta}{r\rho g}}$$

where T = Surface tension

नोट याद रखे:- जब हमें directly या Inversely proportional पता करना हो तो जिससे जानना हो उसको एक तरफ तथा दूसरे को बाकी तरफ

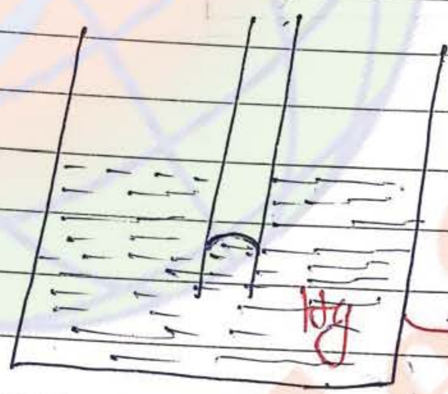
clean water – glass tube $\Rightarrow 0^\circ$
silver tube – clean water $\Rightarrow 90^\circ$

- $0^\circ \le \theta < 90^\circ \rightarrow h = +ve$ (rise)
  ↳ Concave meniscus.

  eg ↳ (glass – water)

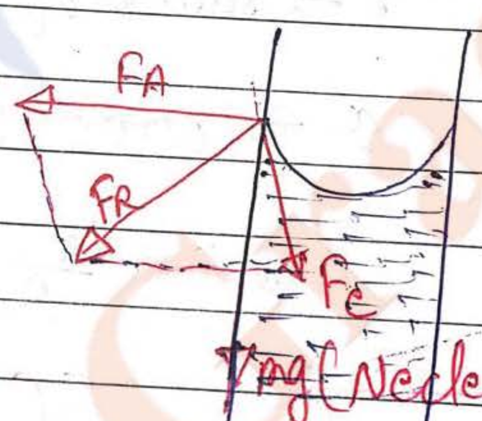
- $\theta > 90^\circ \rightarrow h = -ve$ (fall)
  ↳ (Convex meniscus)

  eg ↳ (glass – Hg)



(और इस convex बनने पर (Hg) liquid fall करता है)

Note :→

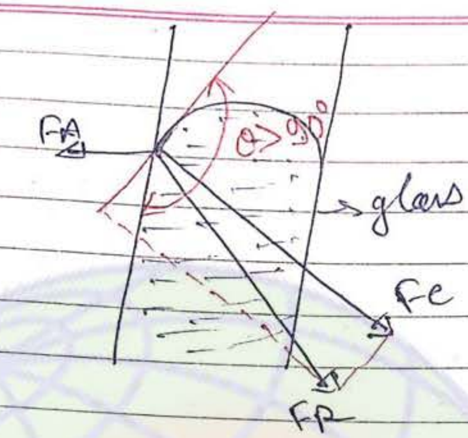


(Here $F_A > F_C$)

If $F_A > F_C \longrightarrow$ Concave meniscus

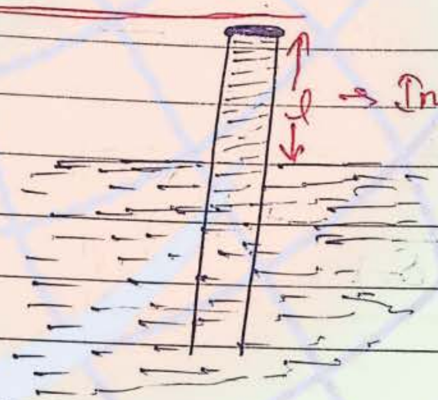
याद रखें

If $F_A < F_C \longrightarrow$ Convex meniscus



(Here $F_A < F_C$)

Zurin's law →



$$h = \frac{2T}{R\rho g}$$

$$\boxed{hR = \text{Constant}} \quad \because hR = \frac{2T}{\rho g}$$

$(T, \rho, g = \text{constant})$

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Note:-

1) For water and clean glass capillary, $\theta \approx 0°$

2.) The height "h" is measured from the bottom of the meniscus.

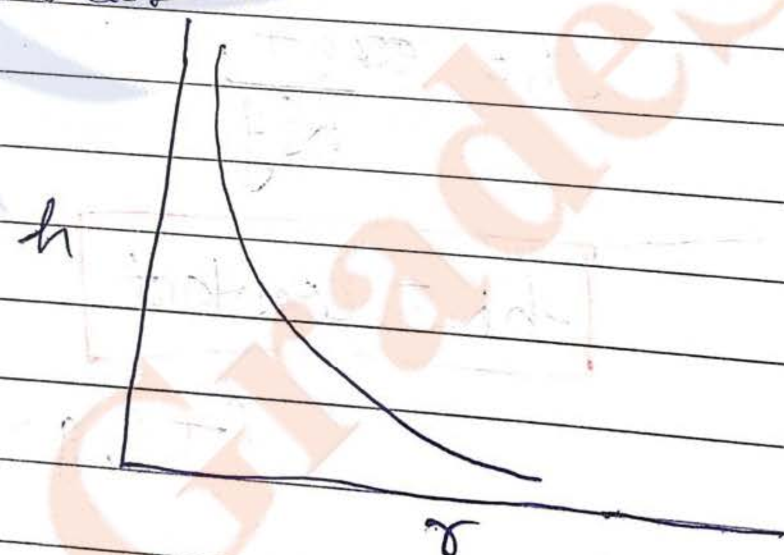
However there exist some liquid above this line also.

If correction for this is applied then capillary formula will be :-

$$\boxed{T = \frac{r \rho g}{2\cos\theta}\left(h + \frac{r}{3}\right)}$$

where — $r \Rightarrow$ radius of capillary tube

The relation b/w h and r graphically represented as -



Viscosity

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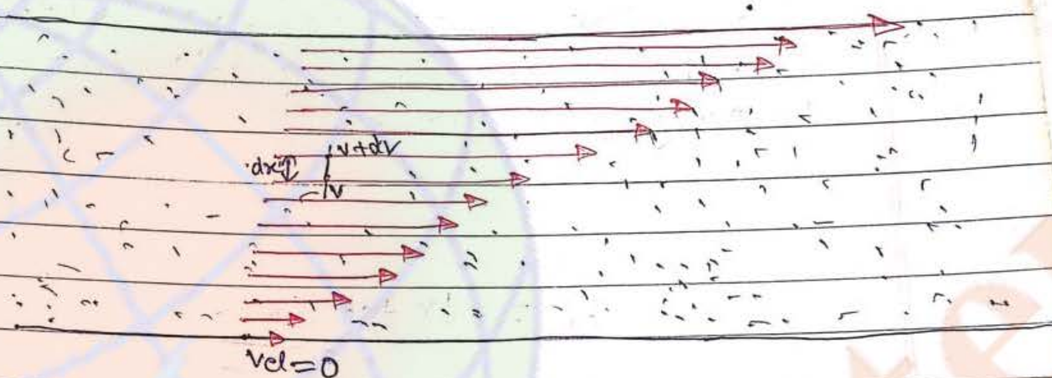
It is the property of liquid due to which a relative motion b/w adjacent layer's of liquid is opposed.

⇒ Newton law of viscous force ⇒



$$\boxed{F_v = -\eta A\left(\frac{dv}{dx}\right)}$$

viscous force.

$\frac{dv}{dx}$ = velocity gradient

A = surface area of liquid.

(ईटा) η = coefficient of viscosity
→ Depend's on nature of liquid.

Note → The viscosity of liquid decreases with rise in temperature but the viscosity of gases increases with rise in temperature.

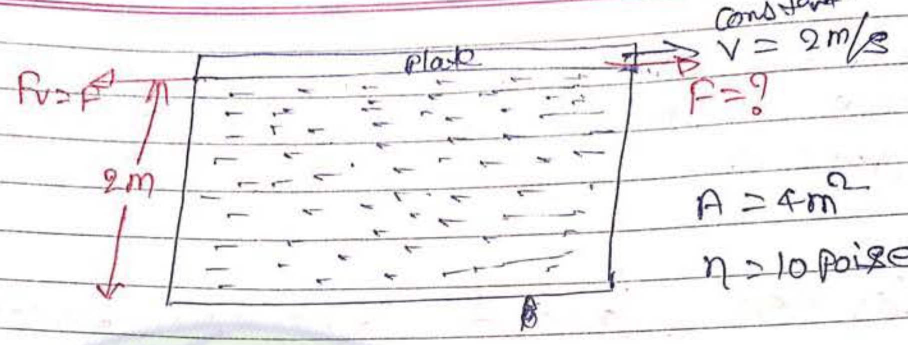
Unit of η → $\frac{N \cdot S}{m^2}$

Note → Another unit of coeff. of viscosity is —
$1\ Kg\, m^{-1} sec^{-1} = 10$ poise

$$\boxed{1\ \frac{N \cdot S}{m^2} = 10\ \text{Poise}}$$

Poise एक scientist का नाम है।

Eo.)



$A = 4m^2$

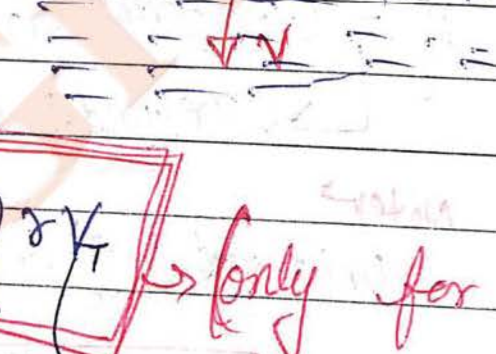
$\eta = 10$ poise

Find the horizontal force applied (exerted) on the plate. So that plate moves with constant velocity of 2m/s.

$$F_v = 1 \times 4 \times \frac{(2-0)}{2}$$

$$= 4N$$

* Stoke's law ⇒



$$\boxed{F_v = 6\pi\eta r V_T}$$ → (only for spherical object)

viscous force

→ Terminal velocity

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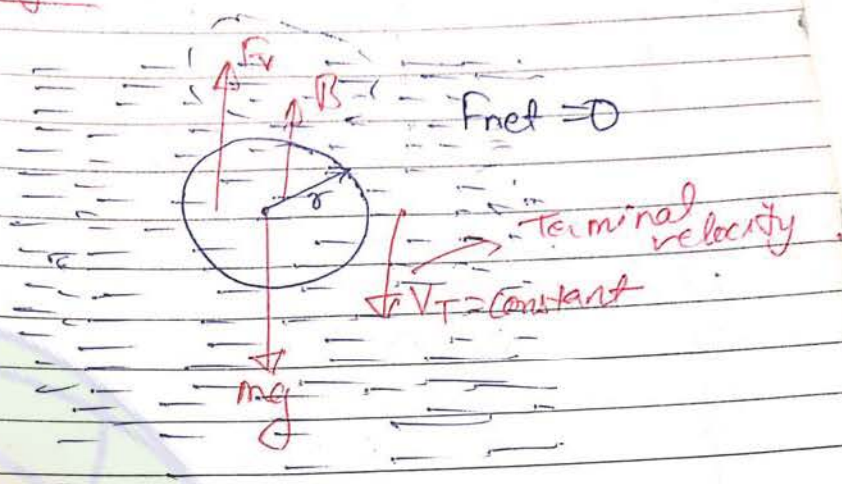
Imp

Thermal velocity



$$B + F_V = mg$$

$$\frac{4}{3}\cancel{\pi} r^3 \cdot \rho_l \cdot g + 6\pi\eta r V_T = \frac{4}{3}\cancel{\pi} r^3 \times \rho_b \times g$$

$$\boxed{V_T = \frac{2r^2 g}{9\eta}(\rho_b - \rho_l)}$$

Terminal velocity

$\rho_b \Rightarrow$ density of body

ρ_l = density of liquid

$r \Rightarrow$ radius of sphere

Imp

$$\boxed{V_T \propto r^2}$$

$(g, \eta, \rho_b, \rho_l) \Rightarrow$ constant

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(*) $$\boxed{P_{in} = \left| -F_V \times V_T \right|}$$

→ Rate of loss of energy.

IIT (Ex.) 4 prob 39

$$P_{ower} \propto r^{n=?}$$

$$P \propto F_V V_T$$

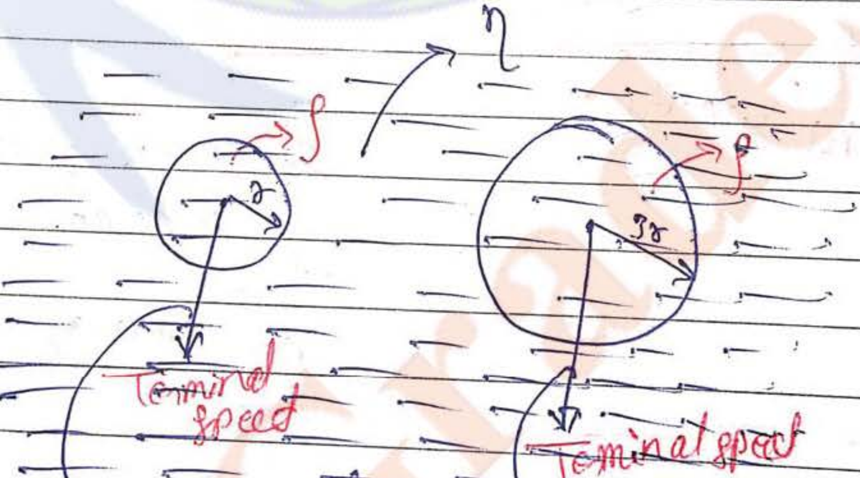
$$\propto r V_T^2 \quad \Big| \quad V_t \propto r^2$$

$$P \propto r^3$$

$8v_1$

$$\boxed{n=5} \text{ Ans.}$$

Ex.)



→ Terminal speed

$P \Rightarrow$ linear momentum

→ P_1 = linear momentum

$P = m V_T$ | $m \propto r^3$
$V_T \propto r^2$

$P \propto r^5$

$P_1 = (3)^5 \cdot P$
$= 243 \cdot P$

$\boxed{K.E(K) \propto r^7}$

Thermal expansion

when a substance is heated without change in its state it usually expand's because of greater Amplitude of vibration of atom's.

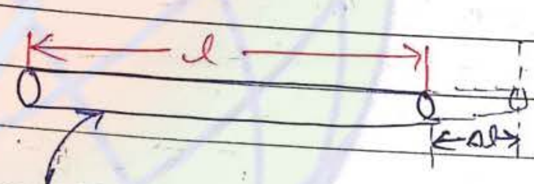
(*) Type of Thermal expansion :-

1) Linear expansion (1D) :⇒

2) Superficial / Area expansion (2D) ⇒

3) volume expansion (3D) ⇒

1) (*) Linear expansion ⇒



Temp rise = $\Delta\theta °C$

Δl = change in length (Increment)

$$\boxed{\Delta l = l \alpha \Delta\theta}$$

> Unit of "α" is per °C or per kelvin.
> +ve for solids except
> ..., wax, plastic etc.
> The value of "α" is -ve for ... because in plastic ... the temp Increases, length ...eases.

where :-

α ⇒ Coefficient of linear expansion

→ Depends on nature of material and (very slightly on Temperature)

→ Unit of "α" ⇒ $\frac{1}{C°}$ or $\frac{1}{K}$

$$l' = l + \Delta l$$

$$\boxed{l' = l\left[1 + \alpha\,\Delta\theta\right]}$$

Increased length

$$\left[\alpha(\Delta\theta) <<<< 1\right]$$

2.) Superficial expansion :⇒

$$\boxed{\Delta A = A \cdot \beta \cdot \Delta\theta}$$

where

$\beta \Rightarrow$ coeff. of superficial expansi

$$\boxed{A' = A\left[1 + \beta\,\Delta\theta\right]}$$

3.) volume expansion :⇒

$$\boxed{\Delta V = V \cdot \gamma \cdot \Delta\theta}$$

$$\boxed{V' = V\left[1 + \gamma\,\Delta\theta\right]}$$

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$$\boxed{\beta = 2\alpha}$$

$$\boxed{\gamma = 3\alpha}$$ (For Isotropic expansion) जब तीनों direction में expansion same हो।

Notes:

* $$\boxed{\gamma = (\alpha_x + \alpha_y + \alpha_z)}$$ (For Anisotropic expansion) (जब तीनों direction में expansion अलग-अलग हो।)

* $$\boxed{\begin{aligned} \beta_{xy} &= \alpha_x + \alpha_y \\ \beta_{yz} &= \alpha_y + \alpha_z \\ \beta_{zx} &= \alpha_z + \alpha_x \end{aligned}}$$

* $$\boxed{\frac{\Delta I}{I} = \frac{I' - I}{I} = 2\alpha \, \Delta\theta}$$

* $\Delta V = V \gamma_a \Delta\theta$

$$\boxed{l' = l\left[1 + (\gamma_m - 3\alpha_s)\Delta\theta\right]}$$

HT (10)

If the temp. of sphere is increased by $\Delta\theta$°C. Find the final angular velocity of the sphere at the final temp.

Find the fractional $\left(\frac{\text{change}}{\text{original}}\right)$ change in m.o.I of the sphere.

Ans :-

$$I\omega = I_f \omega_f \qquad R' = R[1+\alpha\Delta\theta]$$

$$\cancel{\tfrac{2}{5}}\cancel{m}\cancel{R^2}\omega = \cancel{\tfrac{2}{5}}\cancel{m}\cancel{R^2}(1+\alpha\Delta\theta)\cdot\omega_f$$

$$\omega_f = \frac{\omega}{1+2\alpha\Delta\theta} = \omega(1-2\alpha\Delta\theta)$$

↳ By use of binomial

$$I' = \frac{2}{5}mR^2(1+2\alpha\Delta\theta)$$

$$I' = I(1+2\alpha\Delta\theta)$$

Imp

$$\boxed{\frac{\Delta I}{I} = \frac{I'-I}{I} = 2\alpha\Delta\theta}$$

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IIT 2006

Q. Find the coeff. of volume expansion for an Ideal gas under Isobaric process.

Sol. $\Delta V = V \cdot \gamma \, \Delta \theta$

$dV = V \cdot \gamma \, dT$ for small variation.

Imp

$$\boxed{\gamma = \frac{1}{V}\left(\frac{dV}{dT}\right)}$$

$PV = nRT$

$P\frac{dV}{dT} = nR \times 1$

$\frac{dV}{dT} = \frac{nR}{P}$

$\gamma = \frac{nR}{PV}$

$\gamma = \frac{\cancel{nR}}{\cancel{nR}T}$

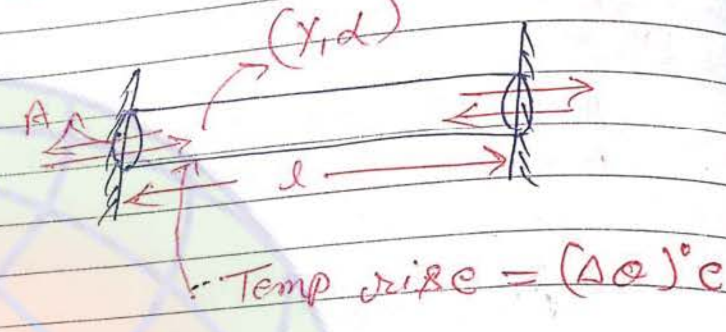
$$\boxed{\gamma = \frac{1}{T}}$$

Ans

* Application ⟹

IIT 2013

① Thermal stress ⟹



(Any position is the fixed position) ⟶ $\Delta l = l \alpha \Delta\theta$

$$\boxed{\text{Strain} = \frac{\Delta l}{l} = \alpha(\Delta\theta)}$$

Δl = restricted length

(Strain) Y = Stress

$$\boxed{\text{Thermal stress} = Y\alpha(\Delta\theta)}$$

change in temperature

$$\boxed{F = YA\alpha(\Delta\theta)}$$

नोट :⟹ In this stress If we hot ~~any~~ metal then metal expant, and that is the final position of metal.

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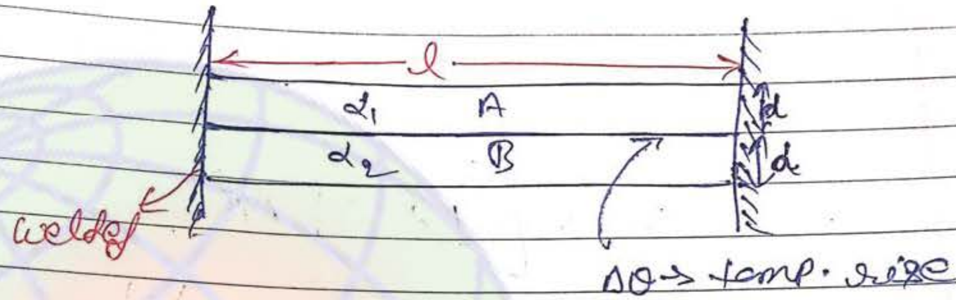
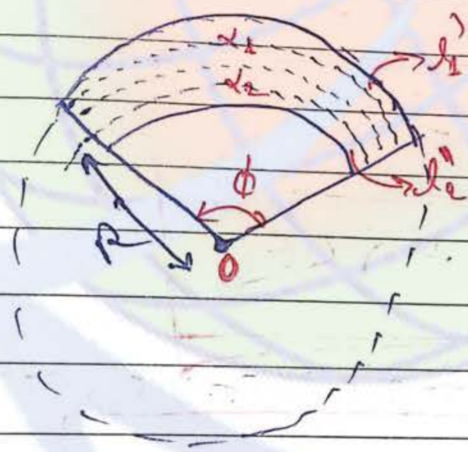
Bimetallic strip ⇒

$$(\alpha_1 > \alpha_2)$$

$$\Delta l = l\alpha\Delta\theta$$

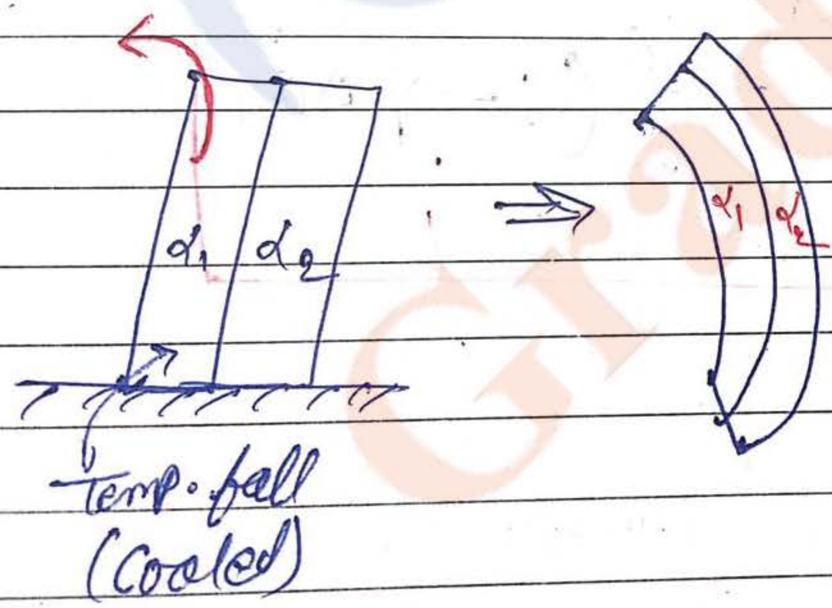


$$l_1' = l[1+\alpha_1\Delta\theta] =$$

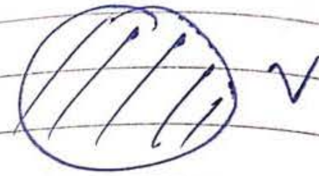


$$\boxed{l_1' = l[1+\alpha_1\Delta\theta] = \left(R+\frac{d}{2}\right)\phi}$$

$$\boxed{l_2' = l[1+\alpha_2\Delta\theta] = \left(R-\frac{d}{2}\right)\phi}$$



Effect of temperature on Solid/liquid



$$V' = V[1+\gamma\Delta\theta]$$

$$\text{mass} = m = V\rho = V'\rho'$$

$$\cancel{V}\rho = \cancel{V}[1+\gamma\Delta\theta]\rho'$$

V.V. I

$$\boxed{\rho' = \frac{\rho}{[1+\gamma\Delta\theta]}}$$

when "$\gamma\Delta\theta$" is very small then we neglect it ⇒

$$\boxed{\rho' = \rho(1-\gamma\Delta\theta)}$$

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Effect of temp. on Boyant force :→

$$B = V \rho_e g \quad \text{—} ①$$

$\Delta\theta \Rightarrow$ temp. rise

$$B' = V' \rho_e' g$$

$$B' = \frac{V[1 + \gamma_s \Delta\theta] \rho_e g}{[1 + \gamma_e \Delta\theta)}$$

$$\boxed{B' = \frac{B(1 + \gamma_s \Delta\theta)}{(1 + \gamma_e \Delta\theta)}}$$

$$B' = B(1 + \gamma_s \Delta\theta)(1 - \gamma_e \Delta\theta)$$

$$B' = B\left(1 - \gamma_e \Delta\theta + \gamma_s \Delta\theta - \cancel{\gamma_s \cdot \gamma_e (\Delta\theta)^2}\right)$$

Neglect

$$B' = B[1 + (\gamma_s - \gamma_e)\Delta\theta]$$

$$\frac{B'}{B} = 1 + (\gamma_s - \gamma_e)\Delta\theta$$

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$$\frac{B' - B}{B} = (\gamma_s - \gamma_l)\Delta\theta$$

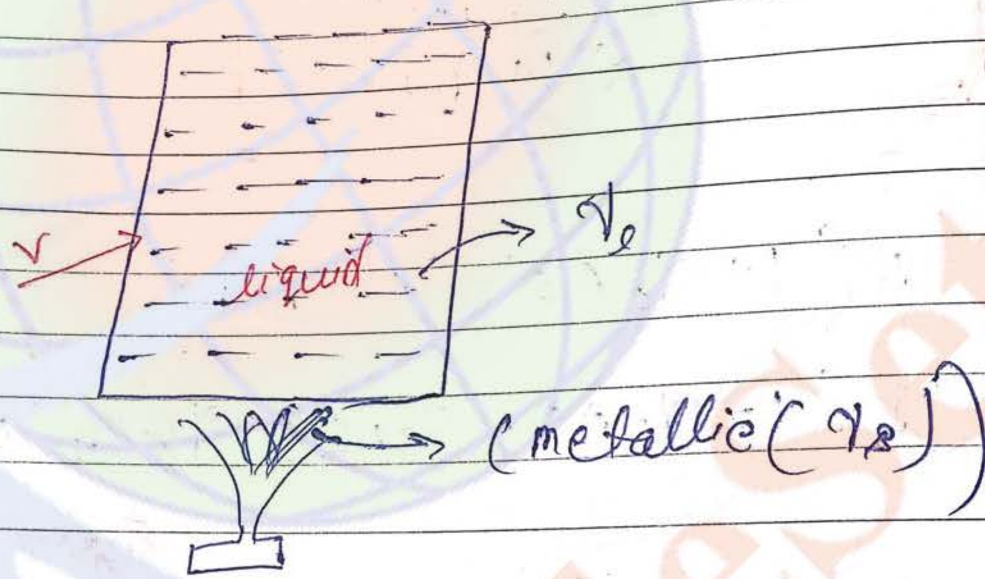
Imp

$$\boxed{\frac{\Delta B}{B} = (\gamma_s - \gamma_l)\,\Delta\theta}$$

Fractional change in ~~bouyant~~ buoyant force.

नोट

% change = fraction chang × 100



$$V_l' = V[1 + \gamma_l \Delta\theta]$$

$$V_s' = V[1 + \gamma_s \Delta\theta]$$

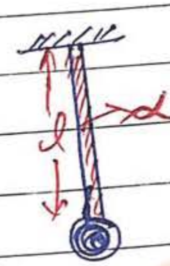
$$\Delta V = V_l' - V_s' = V(\gamma_l - \gamma_s)\Delta\theta$$

Imp

$$\boxed{\Delta V = V \cdot \gamma_{apparant} \, \Delta\theta}$$

$$\boxed{\gamma_{apparant} = (\gamma_l - \gamma_s)}$$

Simple Pendulum →



$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{—} (1)$$

$$T' = 2\pi\sqrt{\frac{l(1+\alpha\Delta\theta)}{g}}$$

$$T' = T\left(1 + \frac{\alpha\Delta\theta}{2}\right)$$

$$\frac{T' - T}{T} = \frac{1}{2}\alpha(\Delta\theta)$$

Imp

$$\boxed{\frac{\Delta T}{T} = \frac{1}{2}\alpha(\Delta\theta)}$$

Time lost / gained in given time "t"

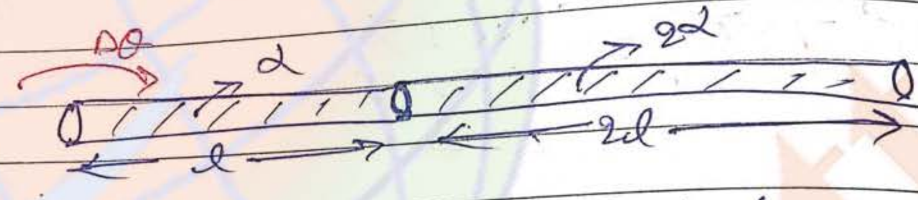
$$\boxed{\Delta t = \frac{1}{2}\alpha(\Delta\theta).t}$$

$T \rightarrow 1$ oscillation

$1 \rightarrow \frac{1}{T}$

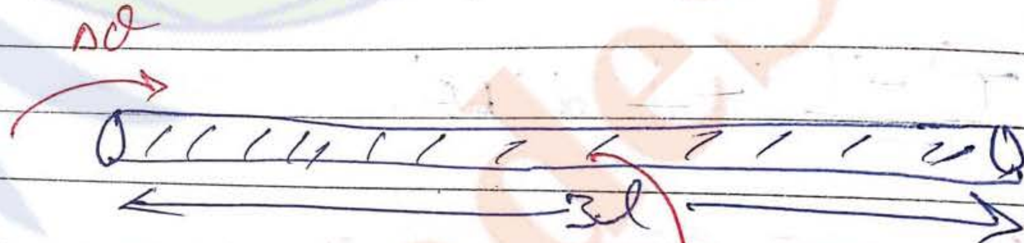
$t \rightarrow \frac{t}{T}$

Eg.



Find the eq. or Avg. coeff of linear expansion
(equivalent or Average)

Ans-



$\alpha_{eq} = ?$

$\Delta l_1 + \Delta l_2 = \Delta l$

$\cancel{l}\alpha\cancel{\Delta\theta} + (2\cancel{l})(2\alpha)\cancel{\Delta\theta} = (3\cancel{l})\alpha_{eq}\cancel{\Delta\theta}$

$$\alpha_{eq} = \frac{5\alpha}{3}$$ Ans

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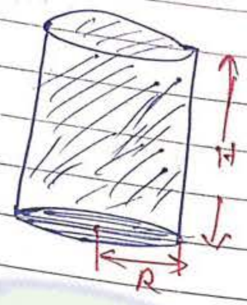
Q.)



$\alpha_1 \rightarrow$ Along radial direction

$\alpha_2 \rightarrow$ along height

i) Find the coeff. of superficial expansion for the curved surface.

ii) Find the coeff. of volume expansion for the cylinder.

Ans.)

i) $A = 2\pi RH$

$(\beta = \alpha_1 + \alpha_2)$

ii) $\gamma = \alpha_x + \alpha_y + \alpha_z$

$= 2\alpha_1 + \alpha_2$

V.V.I short for objective approach.

Other meth

i) $A' = 2\pi R' H'$

$R' = R[1 + \alpha_1 \Delta\theta]$

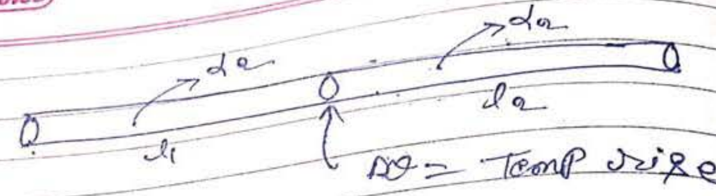
$H' = H[1 + \alpha_2 \Delta\theta]$

$A' = A[1 + \beta\Delta\theta]$

ii) $V = \pi R^2 H$

$V' = \pi R'^2 H'$ $\qquad V' = V[1 + \gamma\Delta\theta]$

Ex.



If the change in length in both rods is found to same. Then the value of

$$\frac{l_1}{l_1+l_2} = ?$$

Ans.

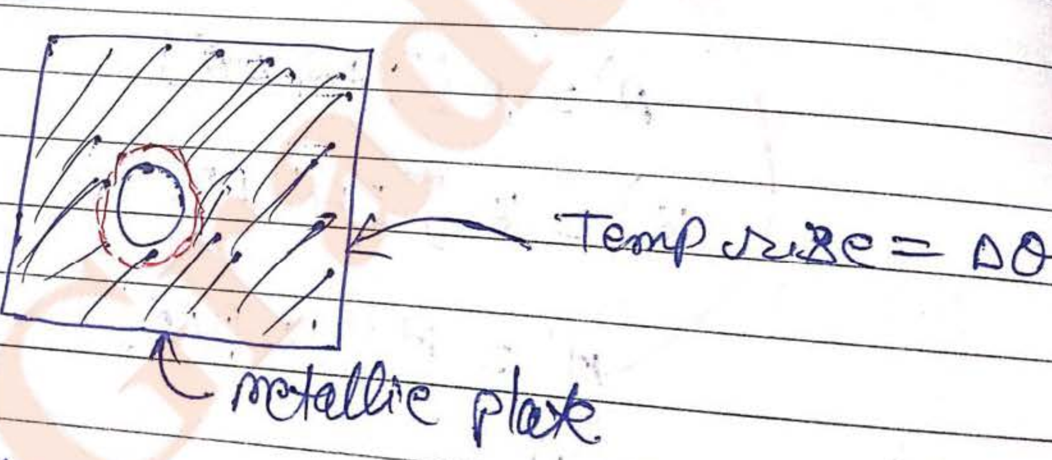
$$\Delta l_1 = \Delta l_2$$

$$l_1 \alpha_1 \Delta\theta = l_2 \alpha_2 \Delta\theta$$

$$1 + \frac{\alpha_1}{\alpha_2} = \frac{l_2}{l_1} + 1$$

$$\frac{l_1}{l_1+l_2} = \frac{\alpha_2}{\alpha_2+\alpha_2}$$ Ans

Ex. AIEEE 2013



Ans → If Temp. Increase then size of hole is Also Increase.

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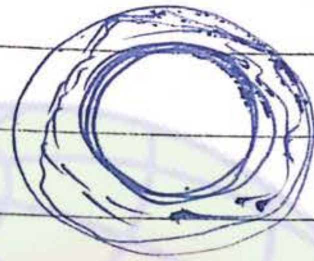
$2\pi r$



Revision classes

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$\vec{v} = \frac{d\vec{r}}{dt}$

$|\vec{v}| = \text{speed}$

$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$

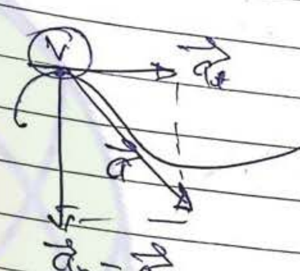
$|\vec{v}_{avg}| \rightarrow$ mag of avg. velocity

$v_{avg\ speed} \geq |\vec{v}_{avg}|$

v_{avg} speed $= \frac{\text{Total dist.}}{\text{Total time}}$

$a_t = \frac{d|\vec{v}|}{dt}$

$|\vec{a}| = \left|\frac{d\vec{v}}{dt}\right|$



$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$

Graphs →

Slope of x-t graph → vel

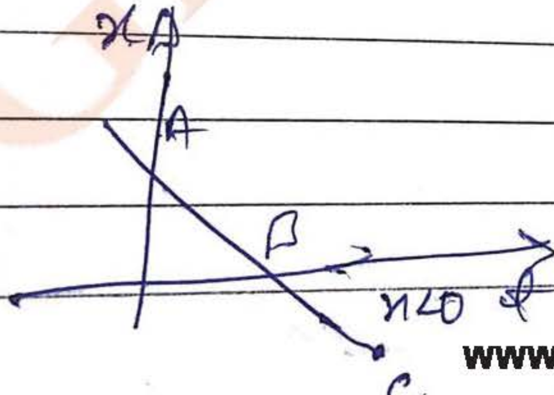
" " v-t " → acc^n

Area of a-t → change in vel.

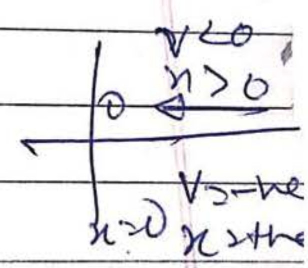
— v-t → Distance (without sign)

→ Displacement (with sign)

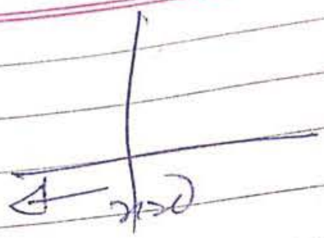
$a = \frac{dv}{dt} = v \cdot \frac{dv}{dx}$



A → B

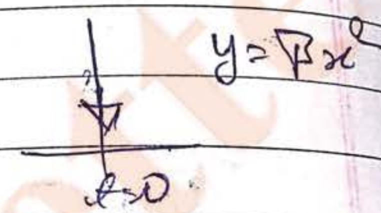


t $B \to C \Rightarrow$
$V = -ve$
$x < 0$



Ex ⇒ A Particle starts from origin at time t=0. The particle moves with const acc^n "α" which is directed along y-axis at time t=0. The particle moves in x-y plane ... The particle acc^n to given relation $y = \beta x^2$ whe β = constant

1) Find the x-component of vel. of particle at $t = 0$.



Ans $y = \beta x^2$

$\frac{dx}{dt} = V_x = ?$

diffe w.r.t. time

$$V_y = \frac{dy}{dx} = \beta\, 2x \left(\frac{dx}{dt}\right) = 2\beta x V_x \quad \text{(chain rule)}$$

(1) (2)

diffe

$$a_y = \frac{dV_y}{dt} = \cancel{2\beta x \frac{d^2x}{dt^2}}\; 2\beta\left[x\frac{V_x}{dt} + V_x^2\right]$$

0

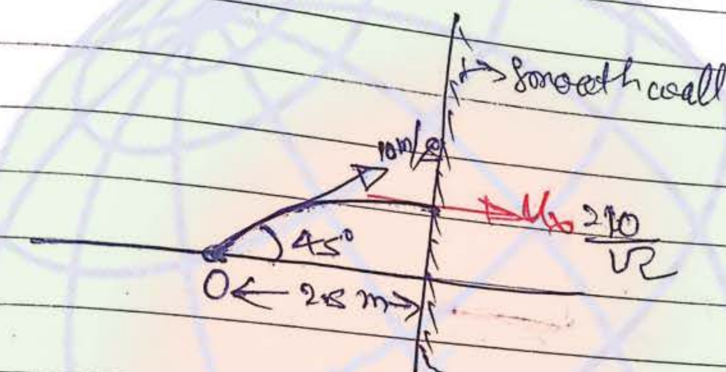
$$\alpha = 2\beta\left[V_x^2\right]$$

$$\boxed{V_x = \sqrt{\frac{\alpha}{2\beta}}}$$

$T = \frac{2u_y}{g}$

$H = \frac{u_y^2}{2g}$

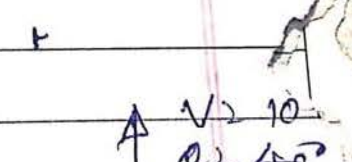
$R = \frac{2u_x u_y}{g}$



A particle is projected with initial velocity of 10 m/s at an angle of 45° with the horizontal. A smooth vertical wall is located at a distance of 2.5 m from the point of projection. Projected particle collides elastically with the wall. After collision the particle ~~lights on the~~ lands on the horizontal ground.

Find the position of the particle on the ground from the wall after collision.

Afte colli



$2 + 8 \sin^2$

$H = \frac{u^2}{2g}$

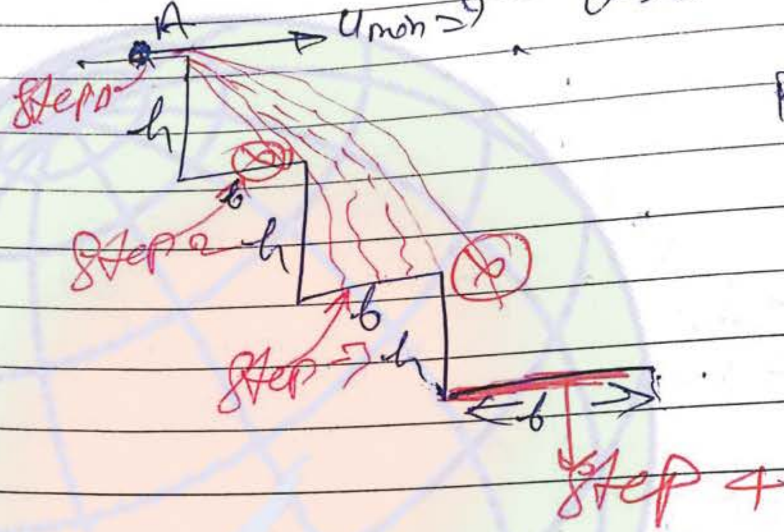
$\frac{\cancel{2}\times 10 \times 10}{10 \times 20}$

$\frac{100}{20}$ 5

25

6

(6)



Find the max. horizontal vel. of the ball so that the particle strikes on the third step.

$$y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}$$

$$y = \frac{g x^2}{2u^2}$$

$$u_{max} = x\sqrt{\frac{g}{2y}}$$

$$\boxed{u_{max} = b\sqrt{\frac{g}{2h}}}$$

Find the velocity of scooter w.r.t. moving belt, so that scooter appears stationary relative to ground observer.

Find the vel. of scooter relative to belt so that vel. of scooter is 5 m/s rel. to ground observer. (along right ward)

S.)

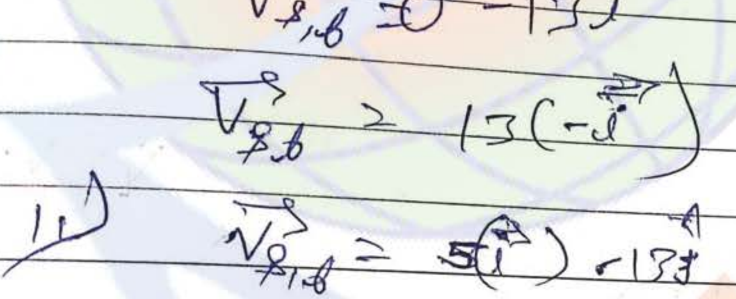
$$\vec{V}_{S,b} = \vec{V}_{S,g} - \vec{V}_{b,g}$$

1) $\vec{V}_{S,b} = 0 - 13\hat{i}$

$\vec{V}_{S,b} = 13(-\hat{i})$

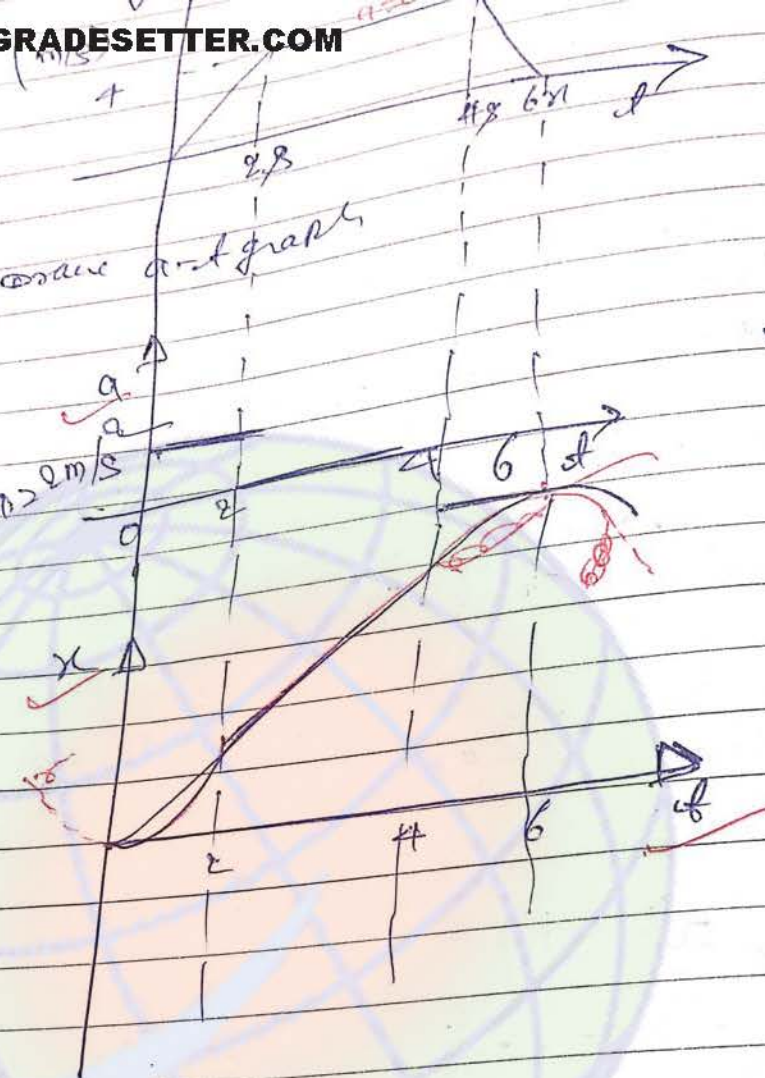
11) $\vec{V}_{S,b} = 5(\hat{i}) - 13\hat{i}$

$= 8(-\hat{i})$



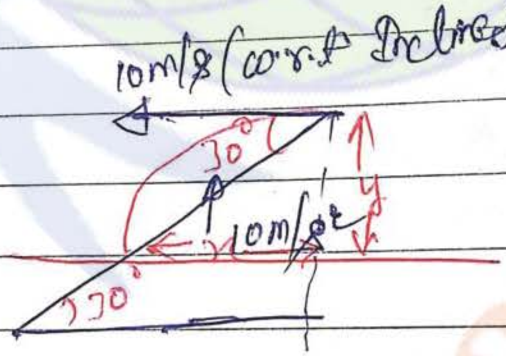
$\vec{V}_{R,M} = ?$

Find the angle of umbrella with the vertical so that man can keep the rain away



a-t graph

$\vec{V} = V \cdot \frac{dx}{dt}$



A ball is projected horizontally with vel. 10m/s w.r.t Inclined surface. The Inclided surface is moving with constant acc^n. of 10 m/s^2 in

 ... direction.

Find the time after which the ball lands on Inclined surface.

A) $S_{rel} = U_{rel}t + \frac{1}{2} a_{rel} t^2$ $\quad v = 10\,m/s$

$x = 10 \times t + \frac{1}{2} \times 0 \times t^2$

$x = 10t$ —— (1)

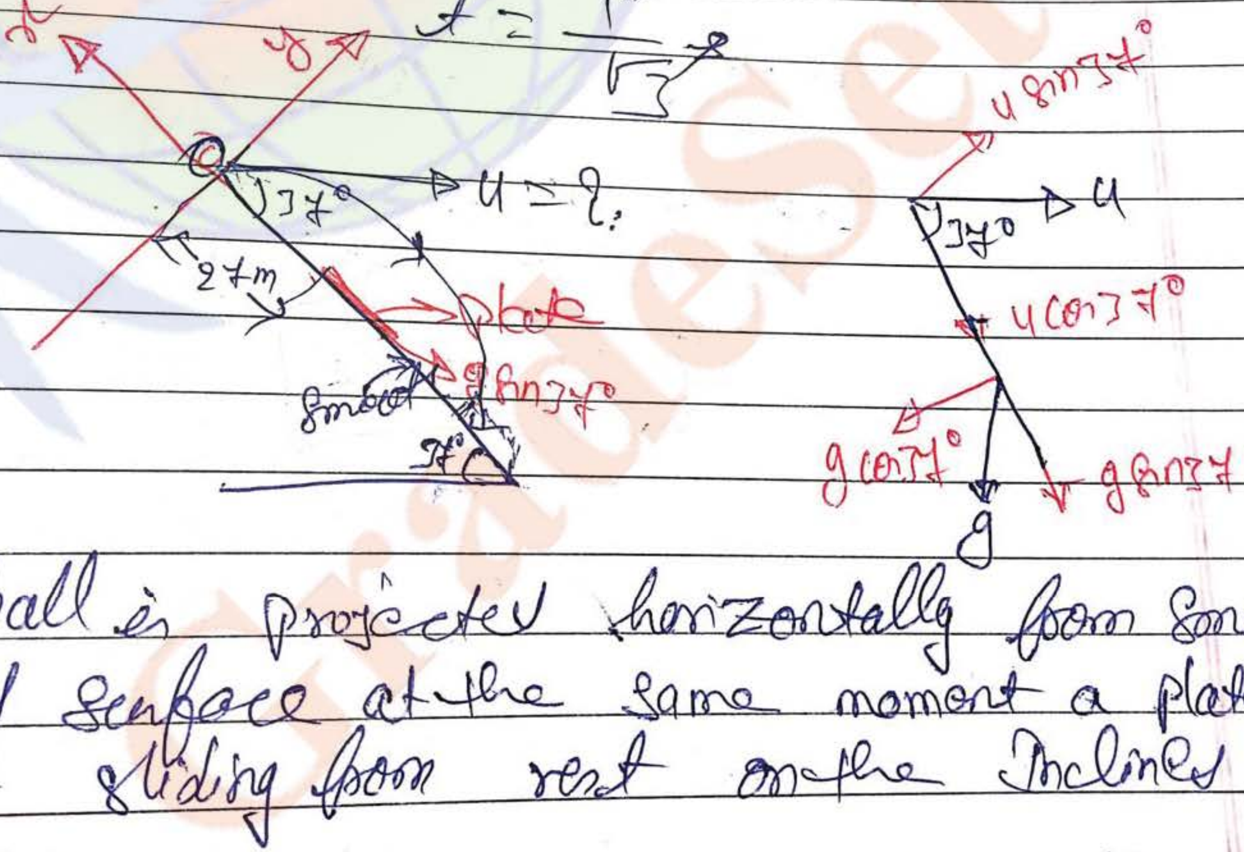
$y = \frac{1}{2}(\cancel{20}^{10}) t^2$

$= 10t^2$

$y = xt$

$t = \frac{y}{x} = \tan 30 = \frac{1}{\sqrt{3}}$

$t = \frac{1}{\sqrt{3}}\,s$



A ball is projected horizontally from some Inclined surface at the same moment a plate start sliding from rest on the Inclined surface.

Find the vel. of projection of the ball so that it lands on the plate

Along (+x) axis

~~$S_{rel_x} = u_{rel_x}$~~

$$S_{rel_x} = u_{rel_x} t + \frac{1}{2} a_{rel_x} t^2$$

$$27 = (u \cos 37^\circ) t + \frac{1}{2} \times 0 \times t^2 \quad \text{— (1)}$$

$$27 = \frac{4u}{5} \cdot t$$

Along (+y) axis

$$S_{rel_y} = u_{rel_y} t + \frac{1}{2} a_{rel_y} t^2$$

$$0 = (u \sin 37^\circ) t - \frac{1}{2} (g \cos 37^\circ) t^2$$

$$\frac{3u}{5} \cancel{t} = \frac{1}{\cancel{2}} \times 10 \times \frac{4}{\cancel{5}} \times t^{\cancel{2}}$$

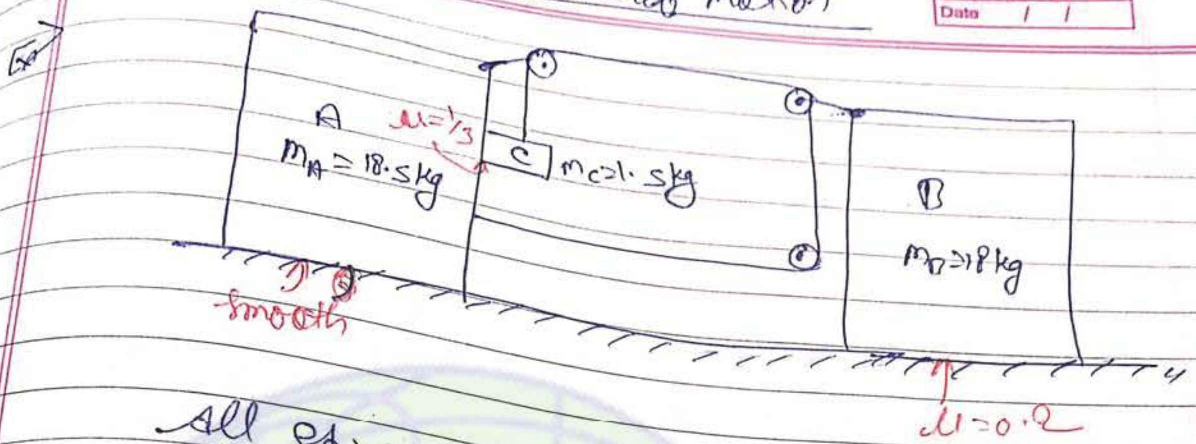
$$t = \frac{3u}{20} \quad \text{— (2)}$$

Using eq (1) and eq (2)

$$\frac{\cancel{27}}{9} = \frac{\cancel{4}u}{5} \times \frac{\cancel{3}u}{\cancel{20}_{5}}$$

$$\boxed{u = 15 \text{ m/s}}$$

Newton's laws of motion

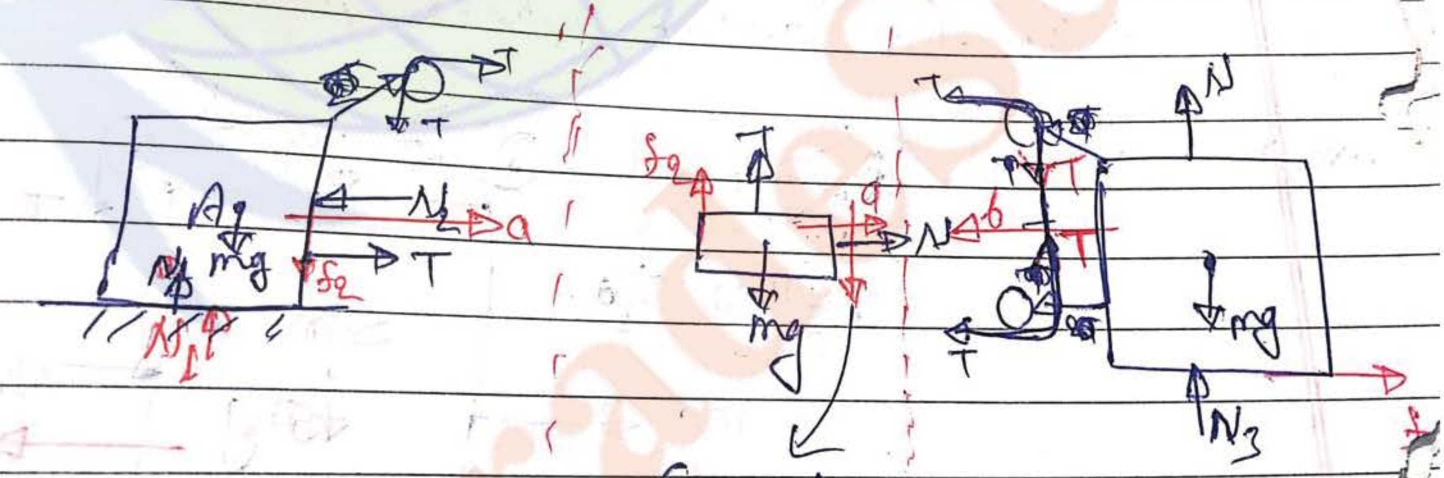


All string and pully are Ideal. String are inextensible.

i) The system is released from rest at time $t=0$. Find the acc^n of block A, B and C w.r.t ground

ii) find the acc^n of block C w.r.t block A.

iii) Find the total energy dissipated (loss) during the time $t=0$ to $t=0.2\,s$



$(2a+2b)$ w.r.t "A"

For "A"

$$N_1 - M_A g - \overset{\mu_2 N_2}{f_2} - T = 0 \quad \text{—— (1)}$$

$$2T - N_2 = M_A a \quad \text{—— (2)}$$

1st Choice

$N_3 = m_B g$ —— (3).

$2T - f_3 = m_B b$ —— (4)

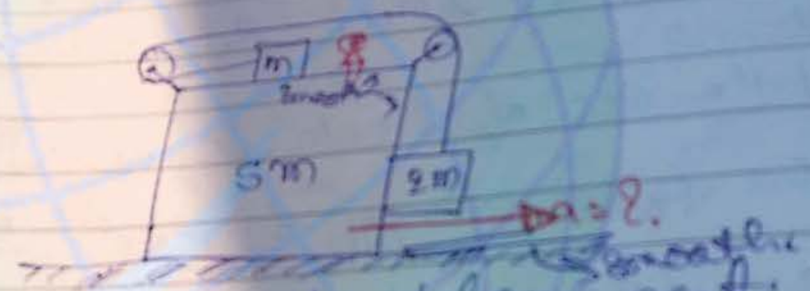
$\mu_1 N_3$

pseL

$N_2 = m_C a$ —— (5)

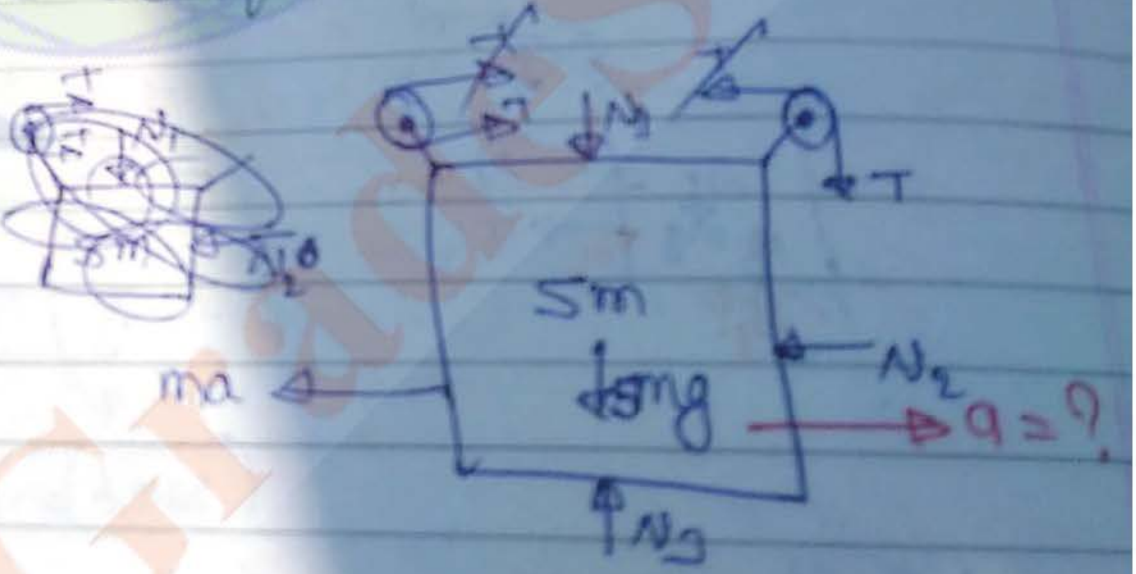
$m_C g - T - f_1 = m_C(a\ldots)$ —— (6)

[xi]



system is released from rest.
Find the acc^n of "5m" block just after release.
i) Find the acc^n of block "m" w.r.t. block "5m"

Ans



~~$5mg + N_1 + T = N_3$ —— (7)~~

~~$2T = T + N_2 + ma$~~

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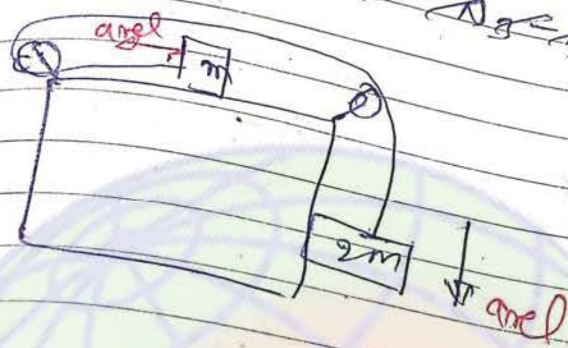
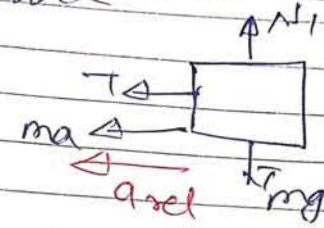
$T - N_2 = 5ma$ — (1)

$\cancel{a = \frac{T - N_2}{m}}$

$\cancel{a = N_2 - N_1}$

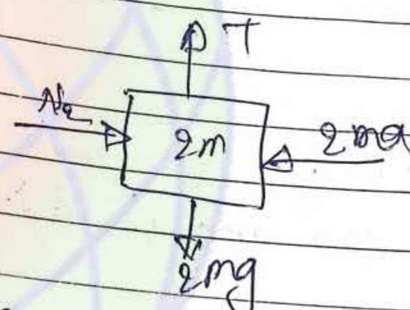
Now w.r.t. 5m





$T + ma = m a_{rel}$ — (2)

Now



$2mg - T = (2m) a_{rel}$ — (3)

$N_2 = 2ma$ — (4)

From eq (1) and (4)

$T - 2ma = 5ma$

$T = 7ma$ — (5)

$2mg - 7ma = 2(T + ma)$

$2mg - 7ma = 2T + 2ma$

$\Rightarrow 2mg - 9ma - 2T = 0$

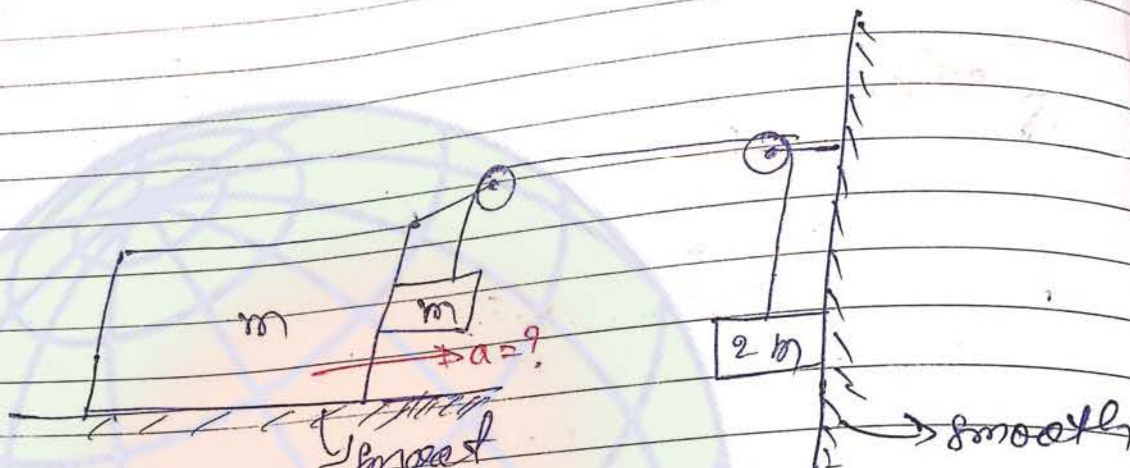
$\Rightarrow 2mg - 9ma - 14ma = 0$

1st Choice

$\cancel{2mg - 16ma}$

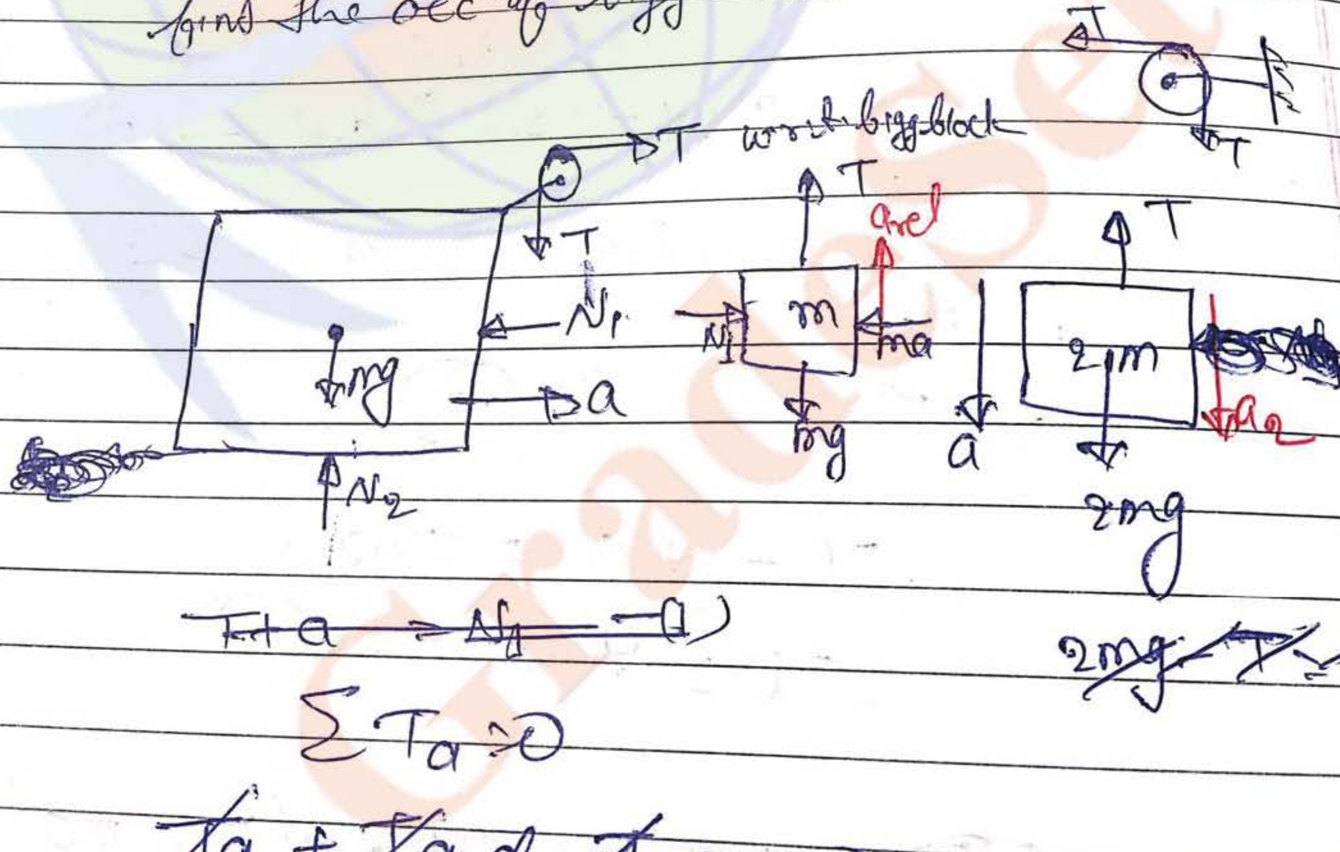
$a = \frac{2g}{23}$

(4)



String and pulley are Ideal.
find the acc^n of bigger block.

Ans:



$\cancel{T + a = N_1}$ —(1)

$\cancel{2mg - T = ma}$

$\Sigma T a = 0$

$\cancel{T}a + \cancel{T}a_{rel} - \cancel{T} \cdot a_2 = 0$

$a_2 = a + a_{rel}$ —(1)

1st Choice

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$T - N_1 = ma$ — (2)

$T = 2ma$

$T - mg = ma_{rel}$ — (3)

$N_1 = ma$ — (4)

$2mg - T = (2m)(a + a_{rel})$ — (8)

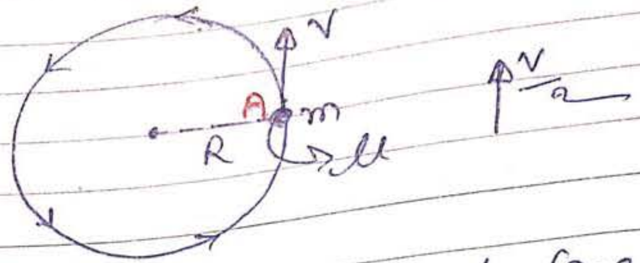
$\cancel{2mg - T = 2m}$

$\cancel{2}m g - \cancel{2}ma = (\cancel{2}m)a + \cancel{2}(2ma - mg)$

$$\boxed{a = \frac{g}{2}}$$

$a_{rel} = 0$

Ex.



Find the work done by the friction force when the particle "m" is projected with velocity v. It's velocity became $\frac{v}{2}$ when it again 1st time pass through "A".

Find the work done by the friction on particle "A". Find the value of μ.

Ans

$\Rightarrow$ K.E $= \frac{1}{2}mv^2$

$$W_{fk} = \frac{1}{2}m\left(\frac{v}{2}\right)^2 - \frac{1}{2}m$$

$$= \frac{1}{2} \times \frac{mv^2}{4} - \frac{1}{2}mv^2$$

$$\Rightarrow \frac{-3mv^2}{8}$$

Now

$$-\mu \cancel{m} g \times 2\pi R = \frac{-3\cancel{m}v^2}{8}$$

$$\boxed{\mu = \frac{3v^2}{16\pi R g}}$$

Spring is compressed by x_0 from natural length and then released from rest.

i) Find the displacement of block "A" and B when the compression is reduced to $\frac{x_0}{2}$.

ii) Find the ratio of kinetic energy of block "A" and "B" when the compression and spring is reduced by $\frac{x_0}{2}$.

iii) Find the workdone by the spring force during the time interval when the spring required its natural length.

iv) Find the ... from initial position x_0

$$m_1 \vec{\Delta x_1} + m_2 \vec{\Delta x_2} = \vec{0}$$

$$x_A + x_B = x_0 \quad \text{—(i)}$$

$$(m)\, x_A = 2m\, x_B \quad \text{—(ii)}$$

$$3x_B = x_0$$

$$x_B = \frac{x_0}{3} \qquad \bigg| \qquad x_A = \frac{2x_0}{3}$$

$$0 = P_A - P_B$$

$$(P_A = P_B)$$

1st Choice

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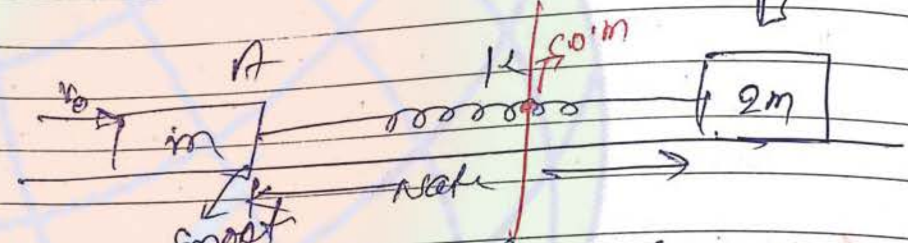
$$K = \frac{P^2}{2m}$$

$$\frac{K_A}{K_B} = \frac{m_B}{m_A} = \frac{2m}{m}$$

$W_{spring} = \frac{1}{2}mv_f^2 +$

Ex- block



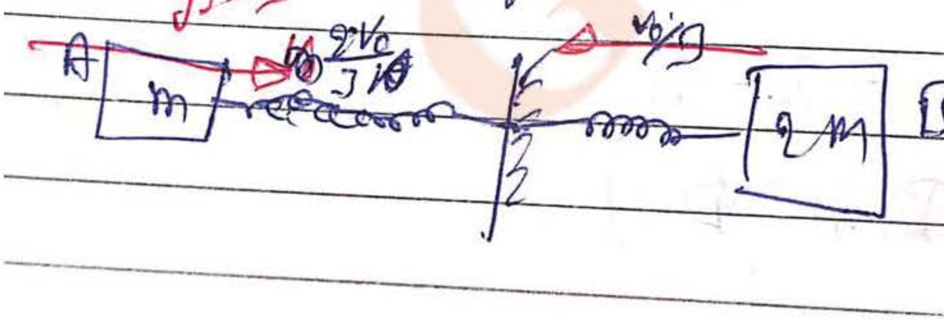
Block A is given initial velocity v_0. Initially the spring is relaxed.

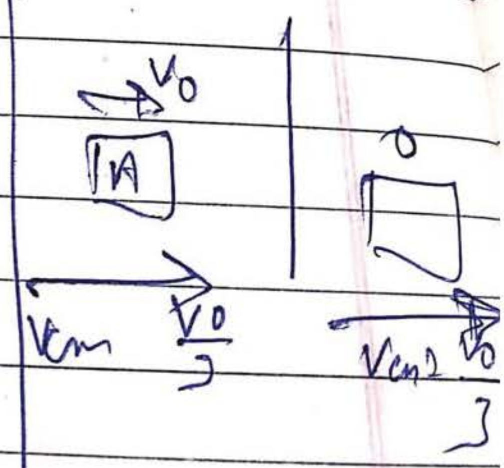
Find the max. speed of the block when the spring acquires its natural length again after first time.

$$V_{cm} = \frac{m v_0 + 2m \times 0}{3m}$$

$$= \frac{v_0}{3}$$

Initially, In C.O.M. frame —





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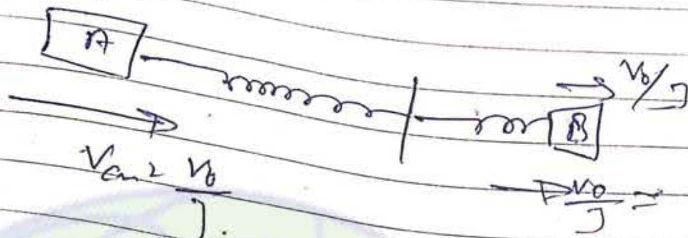
again at natural
$\frac{2v_0}{3}$



$V_{cm} = \frac{v_0}{3}$

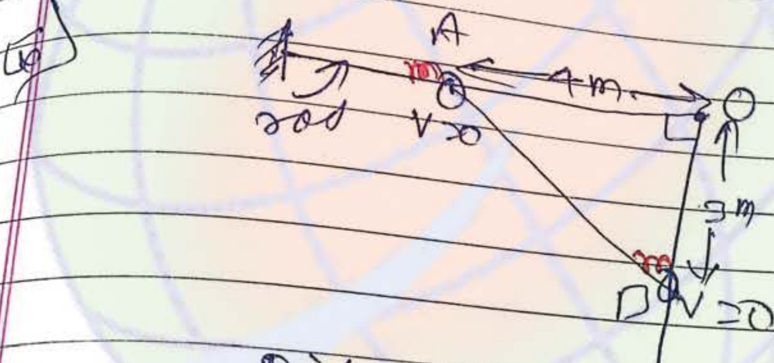
$\frac{v_0}{3}$

$\vec{V}_{A,g} = \vec{V}_{A,cm} + \vec{V}_{cm,g}$

$V_{A,g} = \frac{v_0}{3}$

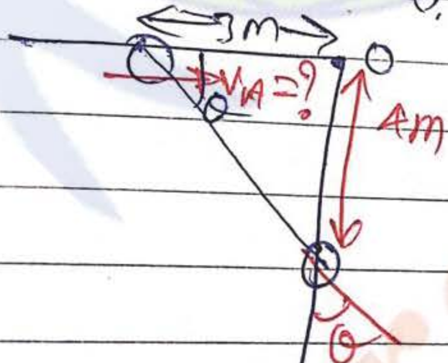
$\frac{v_0}{3} \; v_{cm}$

$V_{B,g} = \frac{2v_0}{3}$



All surfaces are smooth

Find the velocity of ring "A" and "B" when ring A is at the distance of 3m from point O.



$mg \times \ldots = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2$

$S_{rel} = u_{rel} t + \frac{1}{2} a_{rel} t^2$

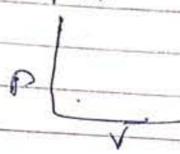
$\Rightarrow 24 = 0 + \frac{1}{2} x$

$\Rightarrow S = ut + \frac{1}{2} a t^2$

$\Rightarrow 24 = 0 + \frac{1}{2} a x$

$PV = nRT$

IAD



$\frac{\sqrt{RT}}{m}$

$\frac{\sqrt{8RT}}{\pi m}$

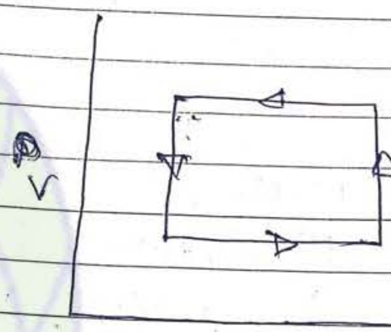
$\frac{T}{m}$

$PV = nRT$

$P = \frac{1}{V}$

P T D const.

$PV = nRT$



$\frac{400-300}{350-300}$ KBR^2

$\Rightarrow \frac{100}{50} = 2$ $t = \frac{1}{\alpha} \ln\left(\frac{50}{50}\right)$ $\frac{KA\Delta T}{l}$

$\Rightarrow \frac{K\pi r \Delta T}{l}$

$3\pi R (R_2 - R_1)$

$K\pi R (R_2^2 - R_1^2)$

$\pi r^2 \Rightarrow \pi R^2$

$\pi 4R^2$

$7R^2$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_1}$

$\frac{160.8}{T_{12}} \frac{2}{4_2}$ $R_{eq} = \frac{R}{2} + \frac{R}{2}$ $\Rightarrow \frac{2}{R} +$

$\Rightarrow \frac{1}{6}$ $\Rightarrow \frac{2R}{2}$ $R + R = 2R$

$v = u + at^2$ $\frac{4R^2}{R^2}$ $4:1$

$s = ut + \frac{1}{2}at^2$

$s =$

1st Choice

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6/11/2011 → Board Patte)

Physics :-> work Power energ, centre of mass, collision, momentum, circular motion, heat, calorimetr.

Chemistry :-> Gaseous State, chemical equillibrium chemical energetic (thermodynamics)

math :-> Logarithm, Progression, Quadratic, Point, Straight line, Parabola,

I) Physics :- Rotation, SHM, wave
II) math -> P. C. M, Trigo equations, solution of triangl
III) chemistry -> chemical, Ionic equillibrium, Redox,

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09852511582

① The story of English → XII → B.T.B.C (Bihar)

② English Grammar → XII → B.T.B.C (Bihar)