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Rotational mechanics

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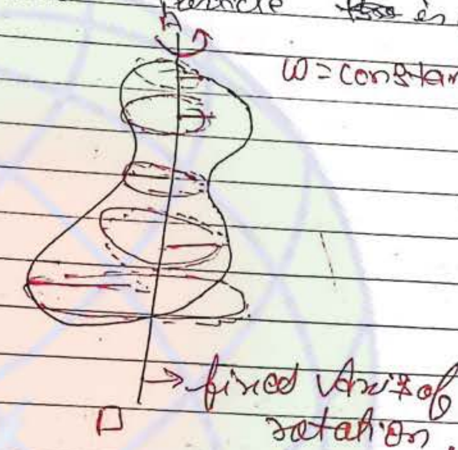
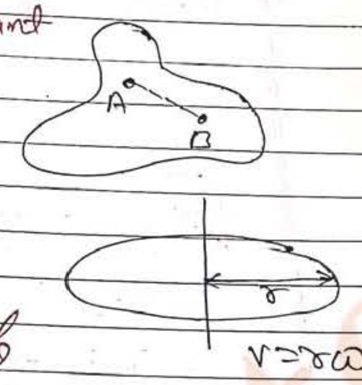
* Rigid body ⇒ The relative position of one particle with respect to other particle of a rigid body does not change during motion of rotation.
The relative velocity of one particle with respect to other particle along the line joining the two particle is always zero





* Moment of Inertia ⇒
It is the properties of body which always oppose the change in rotational motion or state of rigid body in rotation.

Note :⇒

i) M.O.I is not constant for a body. It depends on the axis of rotation.

ii) M.O.I depends on the mass of the body. The higher the mass, the higher the M.O.I.

iii) M.O.I depends on the distribution of the mass about an axis. The farther the mass is distributed from the axis, the higher will be M.O.I.

Now,

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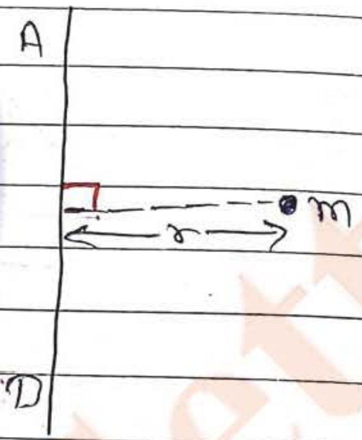
(M.O.I → moment of Inertia)

moment of Inertia does not change if the mass
(i) is shifted parallel to the axis of the rotation
(ii) Is rotated with constant radius about axis of rotation.

⇒ M.O.I of a Particle about given axis(I) ⇒ V.V.I



$$\boxed{I_{AB} = mr^2}$$

where
$r \Rightarrow \perp^r$ distance of mass(m) from the given axis.

Note :⇒ moment of Inertia can be treated as scalar quantity about the same given axis of rotation. VVI

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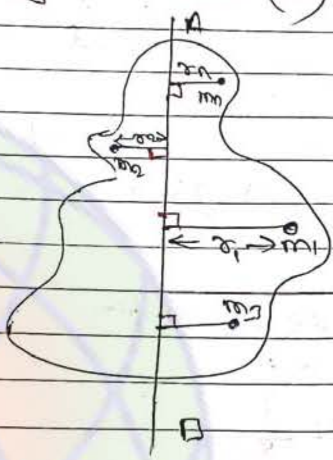
* Moment of Inertia (M.O.I) of system of Particle about the given axis (I) ⇒



$$\boxed{I_{AB} \Rightarrow m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \cdots\cdots m_n r_n^2 \\ \Rightarrow \sum_{i=1}^{i=N} m_i r_i^2}$$

$r_i \Rightarrow \perp^r$ distance of ith mass (m_i) from the given axis.

Note:→ moment of Inertia depends on the following factors:→

i) mass of body
ii) mass distribution of body or shape, size, density of body.
iii) on the position of axis of rotation.

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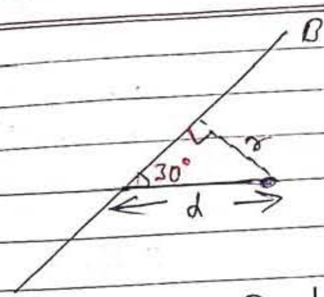
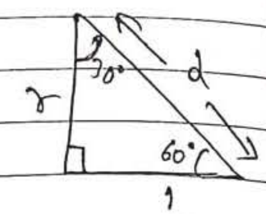
Eg!



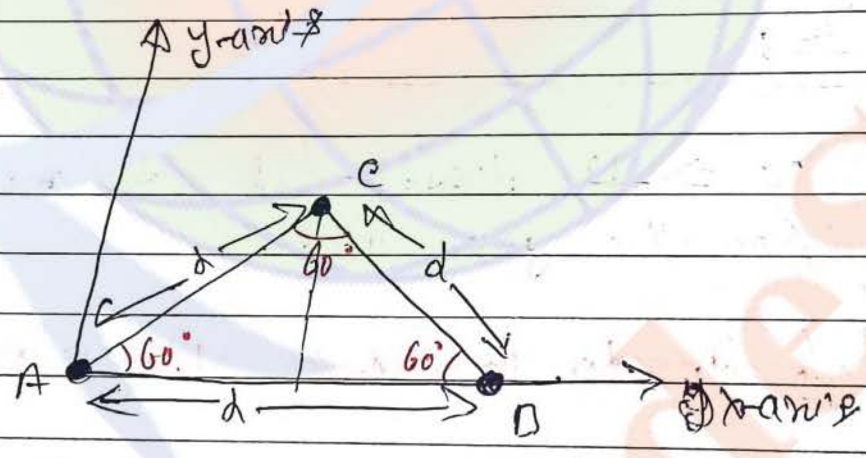
Find the m.o.I of mass 'm' about a given axis AB.



$r = d\sin 30°$
$= d\frac{1}{2}$
$\Rightarrow \frac{d}{2}$

So,
$I_{AB} = mr^2$
$= m\left(\frac{d}{2}\right)^2 \Rightarrow \frac{md^2}{4}$ Ans.

Q1)



Find the m.o.I of the given figure about:-

(i) About x-axis.
(ii) About y-axis in the plane of figure.
(iii) About z-axis

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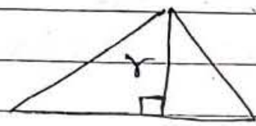
Ans: (i)



$r = d\cos 30 = d\cdot\frac{\sqrt{3}}{2}$ or $r = d\sin 60 = \frac{d\sqrt{3}}{2}$

Now

$\cancel{I = mr^2}$
$\cancel{= m\left(\frac{d\sqrt{3}}{2}\right)^2}$ $I = 0 + 0 + m\left(\frac{d\sqrt{3}}{2}\right)^2$

$= \frac{3md^2}{4}$

(ii) $I_y = 0 + md^2 + m\left(\frac{d}{2}\right)^2$

$= \frac{5md^2}{4}$

(iii) $I_z = 0 + md^2 + md^2$
$= 0 + 2md^2$
$= 2md^2$

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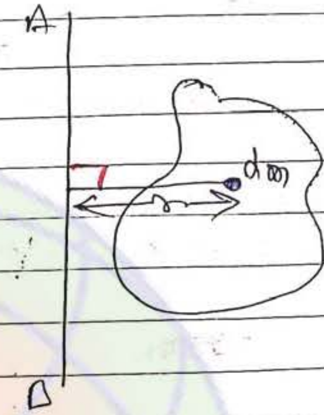
→ Moment of Inertia of continuous mass distribution about given axis ⇒



M.O.I of an element about given axis ⇒

⇒ dI

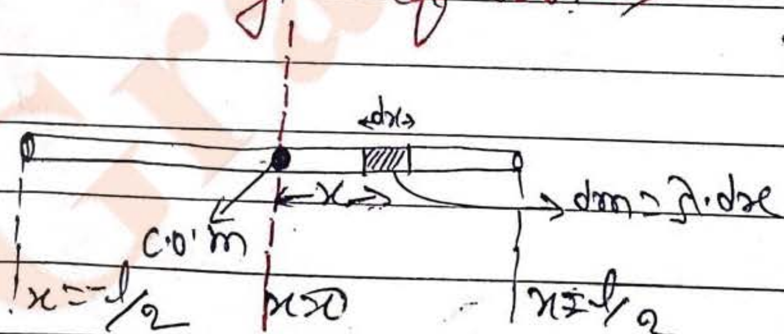
$$\boxed{I_{AB} = \int dI}$$

$$\boxed{I_{AB} = \int dI = \int dm \cdot r^2}$$

1.) Moment of Inertia of Uniform thin rod ⇒
(m, l)
mass, length

About an axis through its centre of mass and ⊥r to the length of rod ⇒

$\lambda = \frac{m}{l} = \text{const}$



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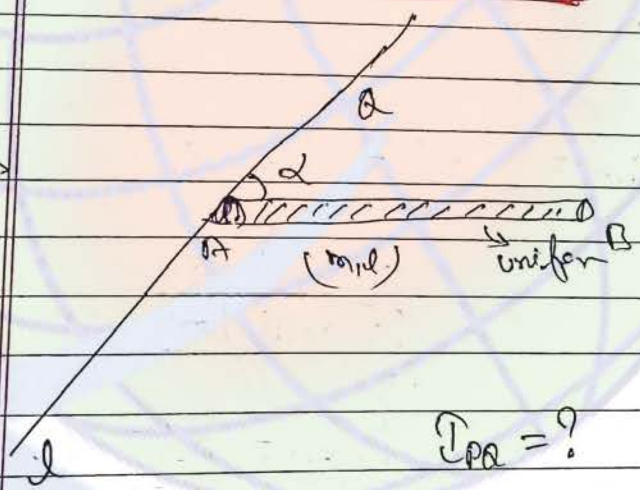
$I_{AB} = \int dI = \int (dm)\, x^2$

$$= \frac{m}{l}\int_{x=-l/2}^{x=l/2} x^2\, dx$$

$$\Rightarrow \frac{m}{l}\left[\frac{x^3}{3}\right]_{-l/2}^{l/2}$$

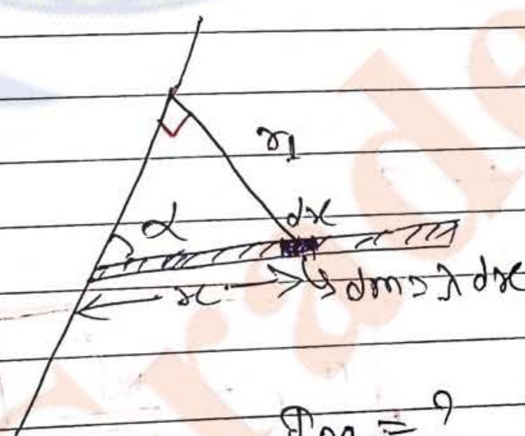
$$\boxed{I_{AB} = \frac{ml^2}{12}}$$

Ex:-



$I_{PQ} = ?$

Ans:-



$\lambda = \frac{m}{l}$

$I_{PQ} = ?$

$$I_{PQ} = \int dI = \int_{x=0}^{x=l} (dm)(x \sin\alpha)^2$$

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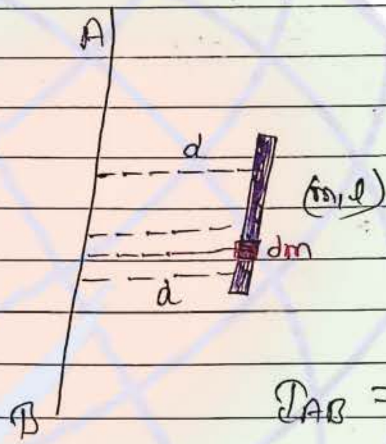
$= \lambda \sin^2\alpha \int x^2 dx$

महत्वपूर्ण

$$\boxed{I_{PQ} = \frac{ml^2}{3}\sin^2\alpha}$$ or $$\boxed{I_{PQ} = \frac{ml^2}{3}\sin^2\theta}$$

याद रखें

(X)



$I_{AB} = \int dI = \int (dm)\, d^2$

$= d^2 \int dm$

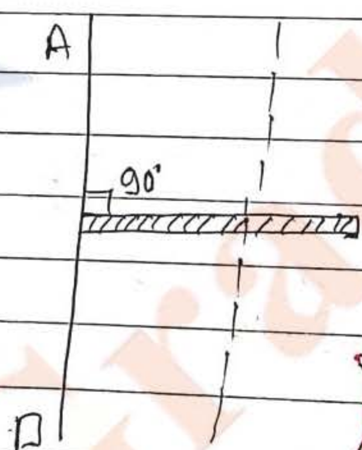
$\boxed{= md^2}$ or

or

$$\boxed{I_{AB} = ml^2}$$

याद रखें

(XI)



$$\boxed{I_{AB} = \frac{ml^2}{3}}$$

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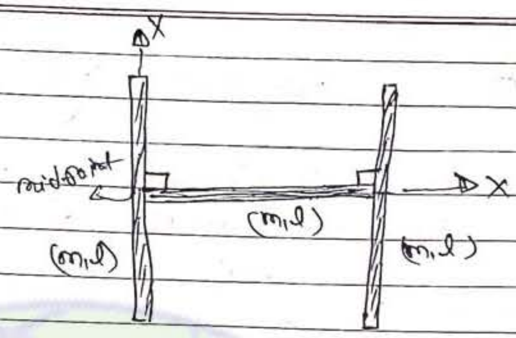
Ex:1)



Find m.o.I of given figure about
i) X-axis and
ii) Y-axis

Ans $I_X = 2\left(\frac{ml^2}{12}\right) + 0$

$\Rightarrow \frac{ml^2}{6}$

$I_Y = 0 + \frac{ml^2}{3} + ml^2$

$\Rightarrow \frac{4ml^2}{3}$

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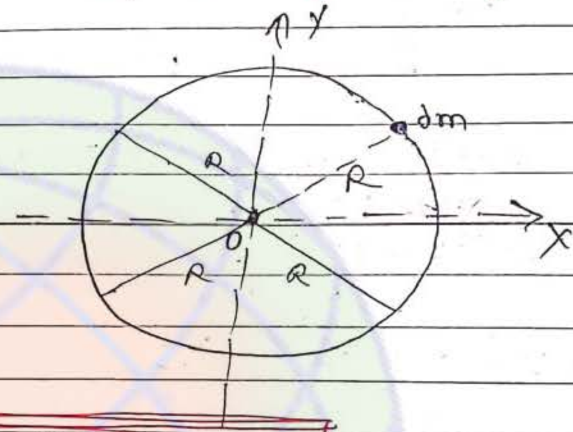
formula 2.)

M.O.I of Uniform Ring (m, R)

about an axis through its C.O.M and $\perp^r$ to the plane of ring :->



$$\boxed{I_{cm} = I_z = mR^2}$$

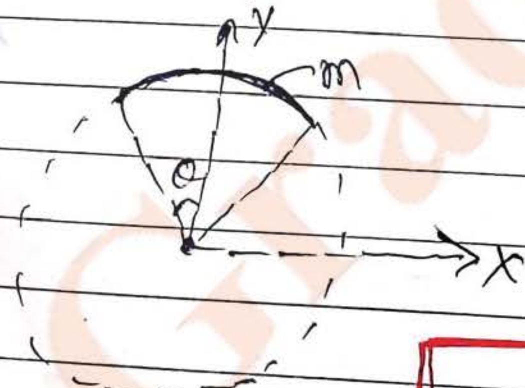
$$I = \int dI = \int (dm) R^2$$

$$= R^2 \int dm$$

$$\Rightarrow mR^2$$

Note :-> Subpoints :->

(a)



$$\boxed{I_z = mR^2}$$

-> mass of circular arc.

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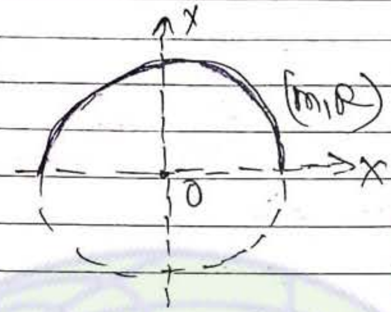
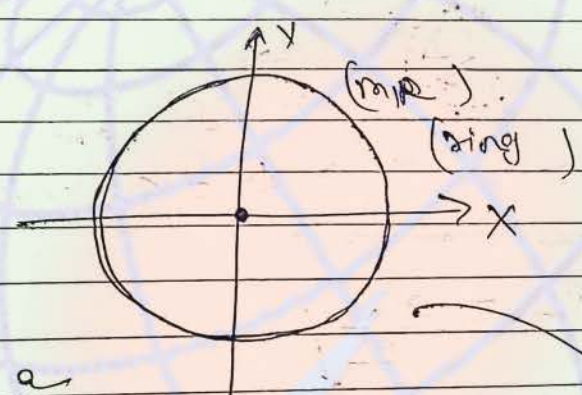
(b)



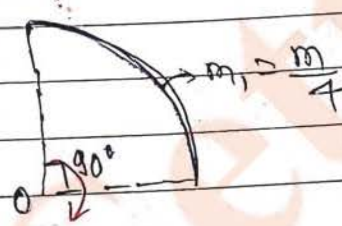
$$\boxed{I_z = mR^2}$$

Ex



$I = mR^2$

$I_z = I$



$$I'_z = m_1 R^2$$
$$= \frac{I}{4}$$

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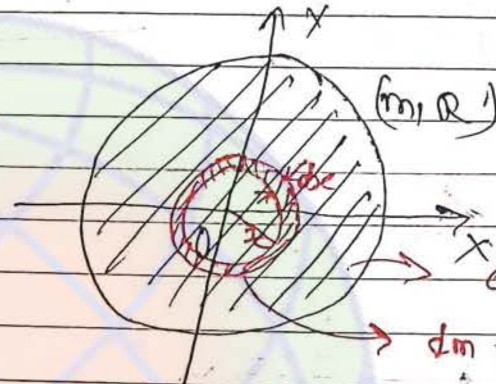
3.) M.O.I of Uniform circular Disc about an Axis through its C.O.M and perpendicular to its plan ⇒



$\sigma = \frac{m}{\pi R^2}$ = constant

$dm = \sigma 2\pi x dx$

$$\boxed{I_{cm} = I_z = \frac{mR^2}{2}}$$

Proof :→

$$I_z = \int dI = \int (dm) x^2$$

$$= \sigma 2\pi \int_{x=0}^{x=R} x^3 dx$$

$3x^2$

$\frac{3R^3}{3}$

$$\Rightarrow \sigma 2\pi$$

$$\Rightarrow \frac{mR^2}{2}$$

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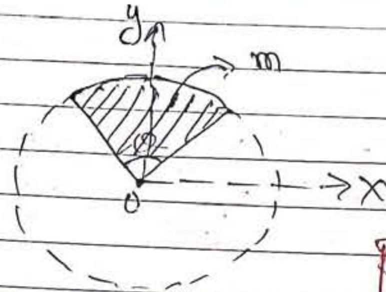
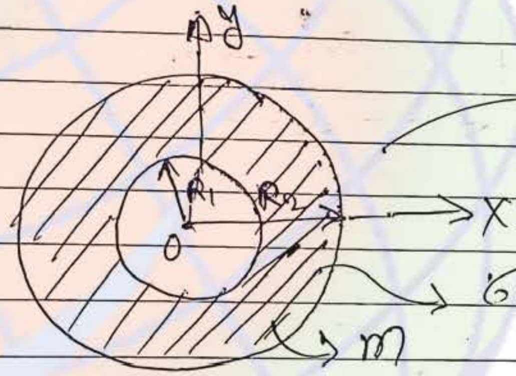
Note :-

(a)



$$\boxed{I_z = \frac{mR^2}{2}}$$

(b)



known as ("Annular disc")

$$\sigma = \frac{m}{\pi(R_2^2 - R_1^2)}$$

$I_0 = ?$

Find the M.O.I of hollow disc about an axis passing through point 'O' and ⊥ to the plane of disc.

$$I_0 = \sigma 2\pi \int_{x=R_1}^{x=R_2} x^3 \, dx$$

$$\boxed{I_0 = \frac{m}{2}(R_1^2 + R_2^2)}$$

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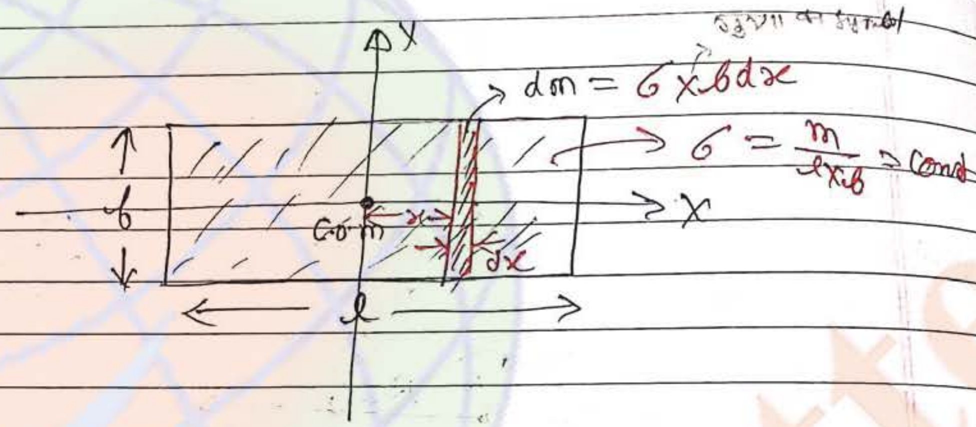
4) M.O.I of Uniform Rectangular Plate

about an Axis through its c.o.m and ~~perpendicular to the plane of Rectan~~ ~~Rectangular plate.~~

Parallel to one of two ~~axis~~ sides ⇒



$$\boxed{I_x = \frac{mb^2}{12}}$$

$$\boxed{I_y = \frac{ml^2}{12}}$$

$$\boxed{I_z = \frac{m}{12}(l^2+b^2)}$$

Now

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$$I_y = \int dI = \int_{x=-l/2}^{x=+l/2} (dm)\, x^2$$

$$= \frac{m\cancel{b}}{l\cancel{b}} \int_{-l/2}^{l/2} x^2\, dx$$

$$= \frac{ml^2}{12}$$

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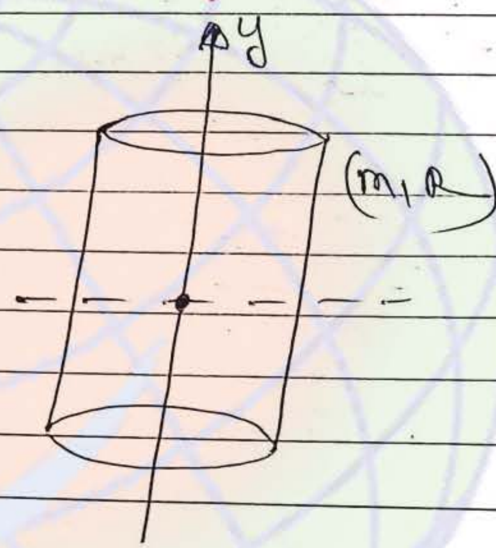
5) M.O.I of uniform hollow cylinder about an axis through c.o.m and parallel to length.



$$\boxed{I_y = mR^2}$$

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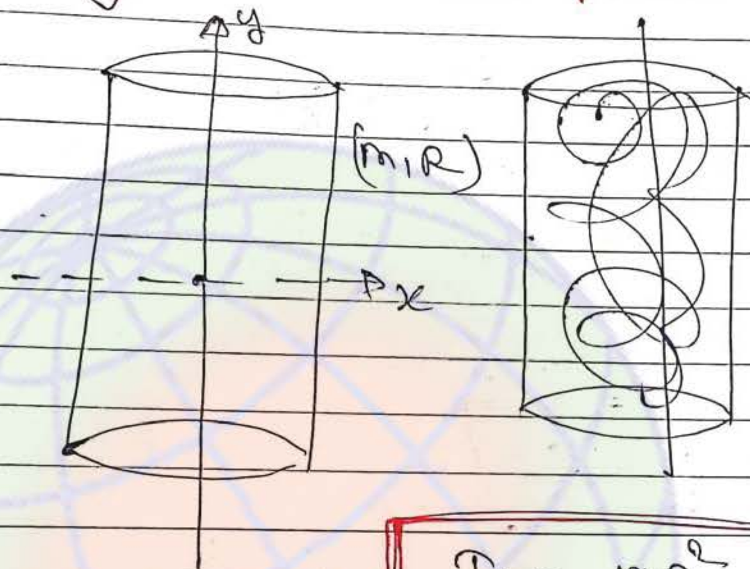
(c) M.O.I of uniform solid cylinder about an axis through c.o.m and parallel to length.



$$\boxed{I_y = \frac{MR^2}{2}}$$

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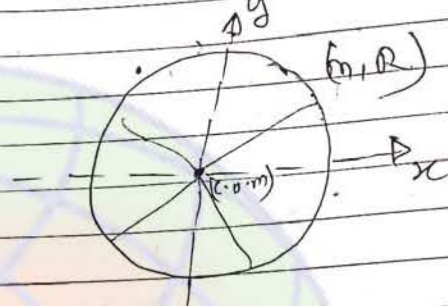
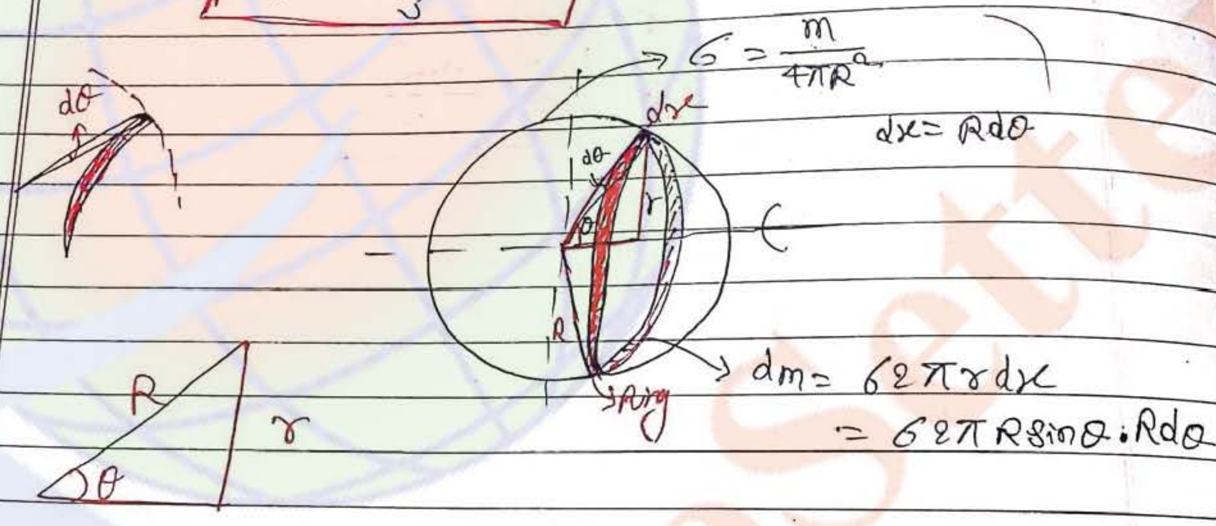
7) M.O.I of Uniform hollow Sphere or (thin) about an axis through its C.O.M

Q.>V

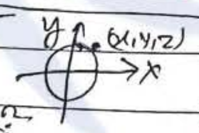


$$\boxed{I_{cm} = \frac{2}{3} mR^2}$$



so. $r = R\sin\theta$

other Proof



$I_y = m_i r_i^2$

$= \Sigma m_i (x^2+z^2)$

$I_x = I_y = I_z$

so,

$I_y = \Sigma m_i (y^2+z^2)$

$I_z = \Sigma m_i (x^2+y^2)$

so, $I_x + I_y + I_z = m\,2(x^2+y^2+z^2)$

$I_{cm} = \frac{2mR^2}{3}$

$I_{cm} = \int dI = \int_{\theta=\pi} (dm) r^2$

$= \sigma 2\pi R^4 \int_{\theta=0} \sin^3\theta\, d\theta$

$= \frac{2}{3} mR^2$

uses

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

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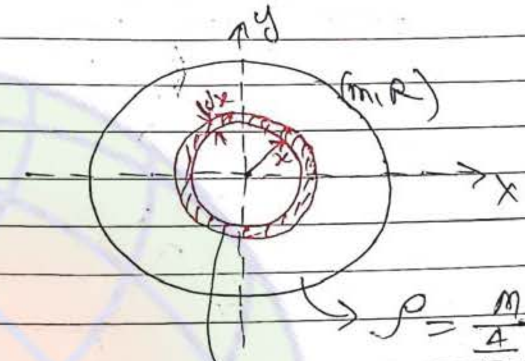
→ always in same direction

Q.) M.O.I of Uniform solid sphere about an axis through its C.O.M



$$\boxed{I_{cm} = \frac{2}{5} mR^2}$$

$\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$ = constant

$dm = \rho\, 4\pi x^2 dx$

$$I_{cm} = \int dI = \frac{2}{3}\int (dm)\, x^2$$

$$= \frac{2}{3} \times \rho\, 4\pi \int_{x=0}^{x=R} x^4 dx$$

$$= \frac{2}{3} mR^2$$

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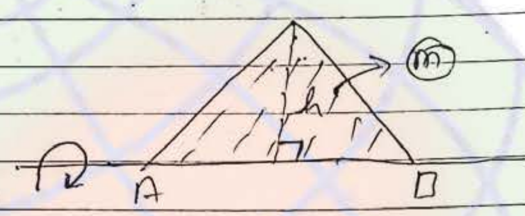
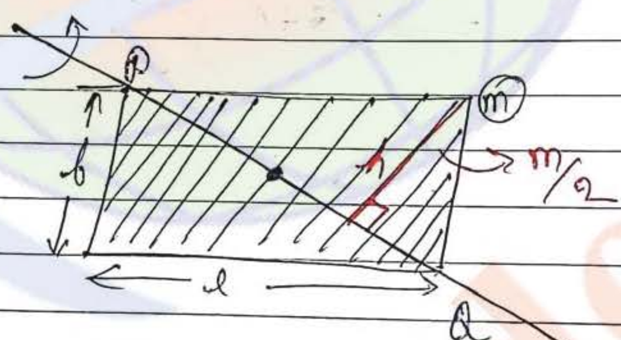
Note:> hollow का M.O.I ही ज्यादा होगा solid sphere से)

9.) M.O.I of uniform triangular plate about base (AB) :⇒



$$\boxed{I_{AB} = \frac{mh^2}{6}}$$

Ex:>



Find the M.O.I of a uniform triangular plate on axis through its diagonal or about the diagonal.

A- $I_{PQ} = \frac{2(m/2)h^2}{6}$

$= \frac{mh^2}{6}$

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Now,

$$l \times b = 2\left(\frac{1}{2} \times \sqrt{l^2+b^2} \times h\right)$$

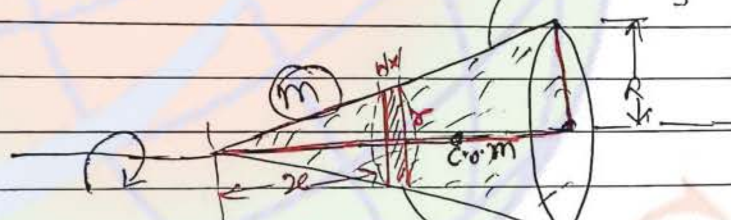
$$h = \frac{lb}{\sqrt{l^2+b^2}}$$

So,

$$\boxed{I_{PQ} = \frac{m}{6} \cdot \frac{l^2 b^2}{l^2+b^2}}$$

10.) M.O.I of uniform solid cone about an axis through its c.o.m and ⊥ to the circular base?

$$\rho = \frac{m}{\frac{1}{3}\pi R^2 h} = \text{constant}$$



$$dm = \rho \times \pi r^2 dx$$

$$\boxed{I_{cm} = \frac{3}{10} m R^2}$$

$$I = \int dI = \int_{x=0}^{x=h} \frac{(dm) r^2}{2}$$

$$\frac{r}{R} = \frac{x}{h}$$

$$r = \left(\frac{R}{h}\right) x$$

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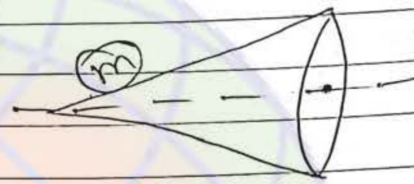
11) M.O.I of uniform hollow cone about an axis through its C.O.M and ⊥r to the circular base.

$$\boxed{I_{cm} = \frac{mR^2}{2}}$$



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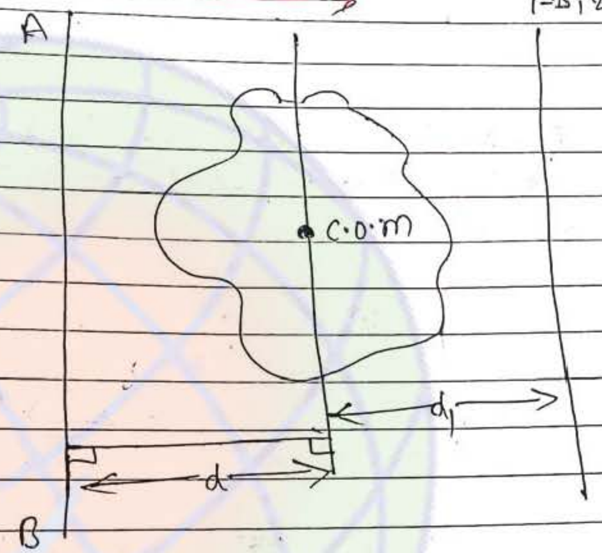
Theorem :-

1. Parallel Axis theorem :-

(This formula is also applicable in 1-D, 2D and 3-D)



$$\boxed{I_{AB} = I_{cm} + Md^2}$$

where, d ⇒ ⊥r distance b/w two Parallel axis

I_{cm} ⇒ M.O.I of a rigid body about an axis through its C.O.M

Note :- i) It is applicable to All type of a body 1-D, 2-D and 3-D

ii) one of the two Parallel Axises must passes through the C.O.M of the rigid body.

iii)

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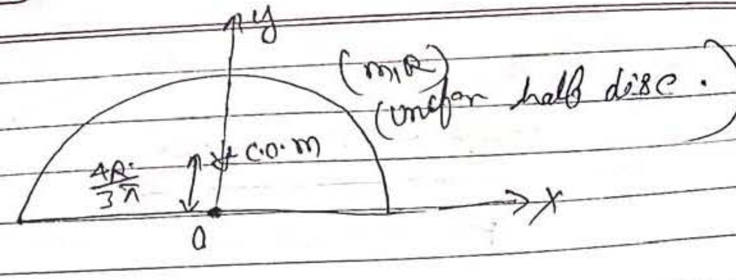
Ex:->



Find the m.o.I of half disc about an axis through its c.o.m and '⊥' to the plane of disc.

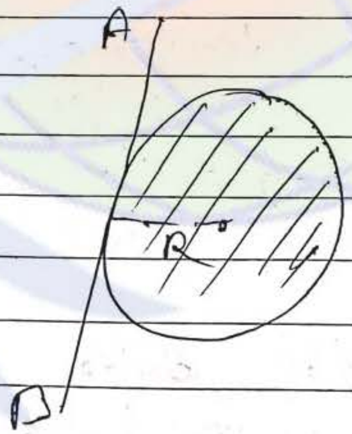
Ans->

$$I_0 = \frac{MR^2}{2}$$

$$I_0 = I_{cm} + m\left(\frac{4R}{3\pi}\right)^2$$

$$I_{cm} = \frac{MR^2}{2} - m\left(\frac{4R}{3\pi}\right)^2$$

Q->



solid sphere (m, R)

$$I_{AB} = I_{cm} + md^2$$

$$= \frac{2}{5}mR^2 + mR^2$$

$$= \frac{7mR^2}{5}$$

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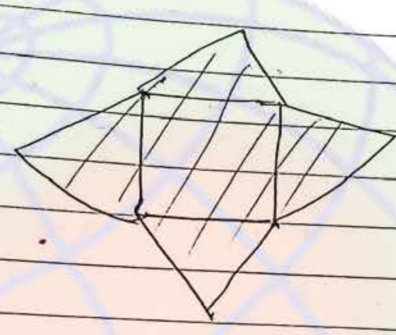
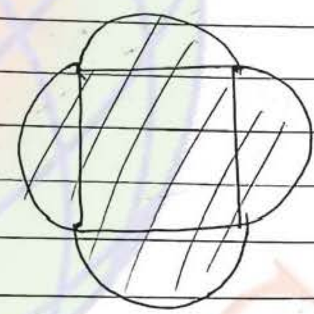
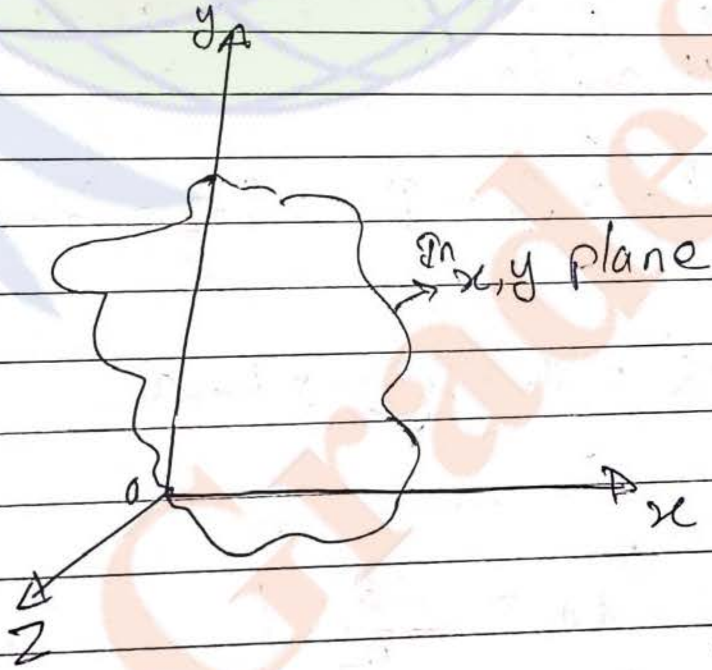
2.) Perpendicular Axis theorem :→

- It is only applicable for ~~2D~~ two dimensional (2-D or planar lamina body)

Ex :→







$$\boxed{I_z = I_x + I_y}$$

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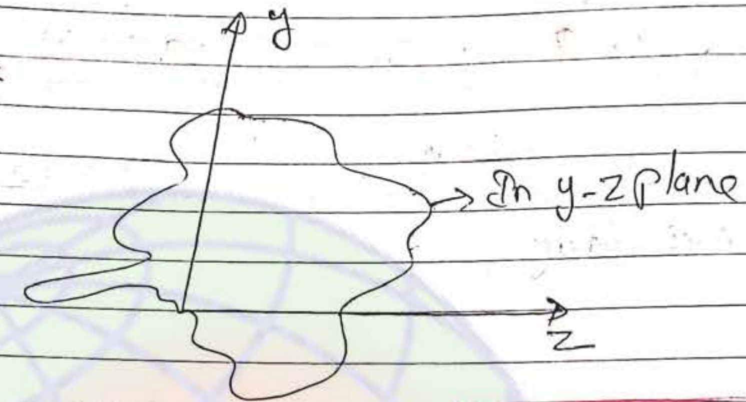
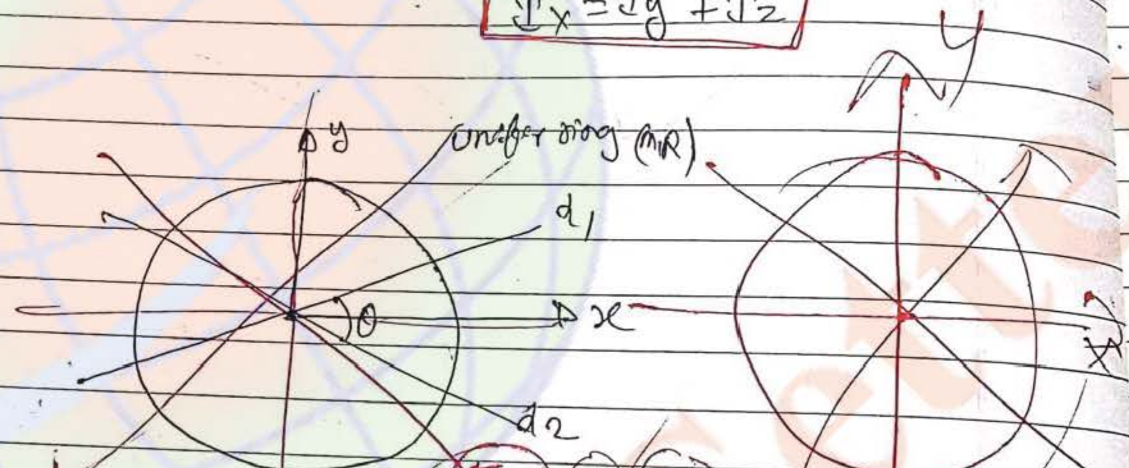


$$\boxed{I_x = I_y + I_z}$$



$$I_z = I_x + I_y = 2 I_d$$

Find the M.O.I of the ring about its diameter d_1 and d_2

$$I_x = I_y = I_{d_1} = I_{d_2} = I_{diameter}$$

$$\underbrace{MR^2}_{I_z} = I_x + I_y = 2\, I_{diameter}$$

$$\boxed{I_{dia} = \frac{MR^2}{2}}$$

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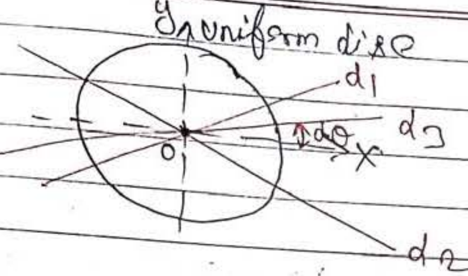
(Q:)



$I_{d_3} = ?$

Find the M.O.I of the disc about d_3

$I_x = I_y = I_{d_1} = I_{d_2} = I_{d_3} = I_{diam}$

$I_z = I_x + I_y$

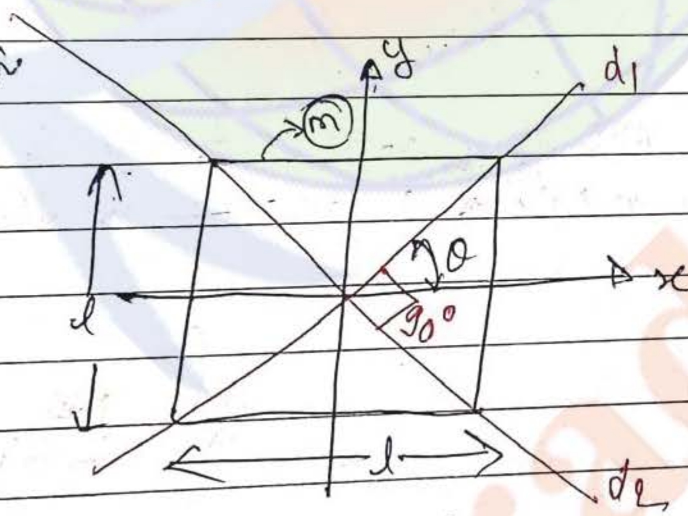
$\frac{mR^2}{2} = 2I_x = 2I_{diameter}$

$$\boxed{I_{dia} = \frac{mR^2}{4}}$$

$(I_z = I_x + I_{d_3})$

Here diameter is equal so we also write.
(But this is not any formula.)

H.Verma Exc



which of the following option are correct?

(i) $I_z = I_x + I_y$
(ii) $I_z = I_{d_1} + I_y$
(iii) $I_z = I_{d_1} + I_{d_2}$
(iv) $I_z = I_x + I_{d_1}$

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A.)

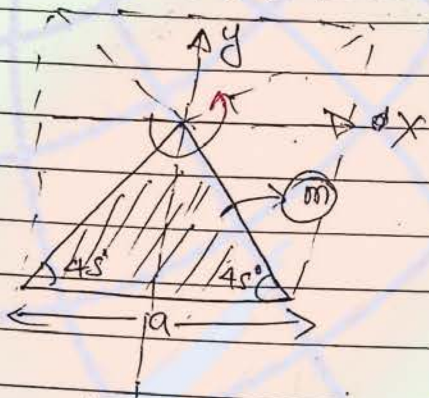
$$I_z = I_x + I_y$$

$$I_z = I_{d_1} + I_{d_2}$$

$$I_x + I_y = I_{d_1} + I_{d_2}$$
$$\cancel{2} I_x = \cancel{2} I_{d_1}$$
$$I_x = I_{d_1}$$

$$I_x = I_y$$
$$I_{d_1} = I_{d_2}$$

So, "All are correct"



Q. Find the M.O.I of this triangular plane about ~~through an~~ axis through 'O' and '⊥' to the plane.

A.)

$$(I_0)_z = \frac{ma^2}{6}$$

$$\frac{(4m)a^2}{6} \times \frac{1}{4}$$

$$= \frac{ma^2}{6}$$

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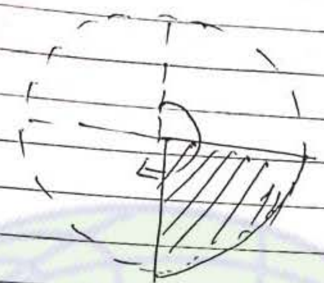
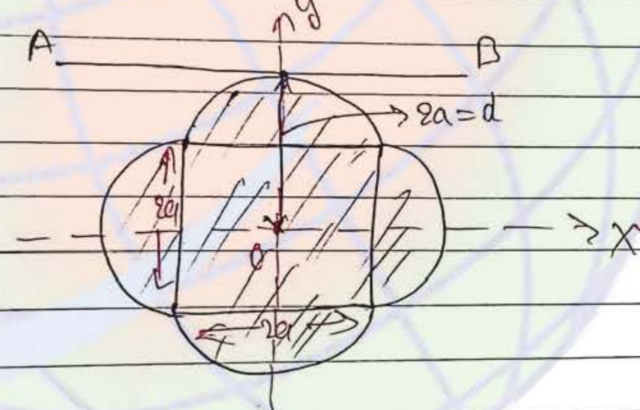
Ex)



$$I_0 = \frac{4mR^2}{2} \times \frac{1}{4}$$

$$= \frac{mR^2}{2}$$

Ex)



(Identical disk)

m is the mass of complete figure

M.O.I of given figure about ~~an axis~~ an axis passing through the centre 'O' and perpendicular to the plane of figure is

$$(I_0)_z = 1.6\, ma^2$$

1) Find the M.O.I of the given figure about an axis AB in the plane of figure.

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$$(I_z)_0 = I_x + I_y$$

$$1.6ma^2 = 2I_x$$

$$I_{cm} = I_x = 0.8ma^2$$

$$I_{AB} = 0.8ma^2 + m(2a)^2$$
$$= 4.8ma^2$$

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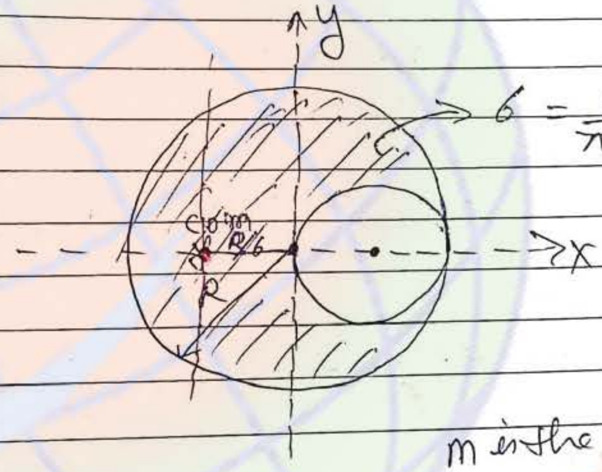
M.O.I of Remaining portion of body after cut :⇒

→ About the same given Axis ⇒

"same given axis"

$$\boxed{I_{remaining} = I_{complete} - I_{cut}}$$



$$\sigma = \frac{m}{\pi R^2 - \frac{\pi R^2}{4}} = \frac{4m}{3\pi R^2}$$

m is the mass of remaining part (after cut)

Find the M.O.I of the remaining portion of the disc about an axis passing throught 'O' and perpendicular to the plane of disc.

$I_O = I_{cm} + mR^2$

$\Rightarrow \frac{mR^2}{2} + \frac{m\left(\frac{R}{2}\right)^2}{2}$

$\Rightarrow \frac{mR^2}{2} + \frac{mR^2}{8}$

$\Rightarrow \frac{4mR^2 + mR^2}{8} = \frac{5mR^2}{8}$

Ans ⇒ mass of complete disc ⇒

$m_1 = \sigma \times \pi R^2$

$\Rightarrow \frac{4m}{3}$

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mass of cut out

$$m_2 = \frac{m}{3}$$

$$(I_{complete})_0 = \frac{m_1 R^2}{2} = \frac{4mR^2}{6} \quad \text{—(1)}$$

$$(I_{cut})_0 = \frac{m_2 (R/2)^2}{2} + m_2 (R/2)^2$$

$$= \left(\frac{mR^2}{24} + \frac{mR^2}{12}\right)$$

$$\Rightarrow \left(\frac{mR^2 + 2mR^2}{24}\right) \Rightarrow \frac{mR^2}{8} \quad \text{—(2)}$$

Now:

$$I_{sem} = \frac{4mR^2}{6} - \frac{mR^2}{8}$$

$$\Rightarrow \frac{13mR^2}{24}$$

Now:

$$I_0 = I_{cm} + m\left(\frac{R}{6}\right)^2$$

m.o.I about c.o.m along ⊥ to the plane (I body)

$$I_{cm} = I_0 - m\left(\frac{R}{6}\right)^2$$

$$\Rightarrow \frac{13mR^2}{24} - \frac{mR^2}{36}$$

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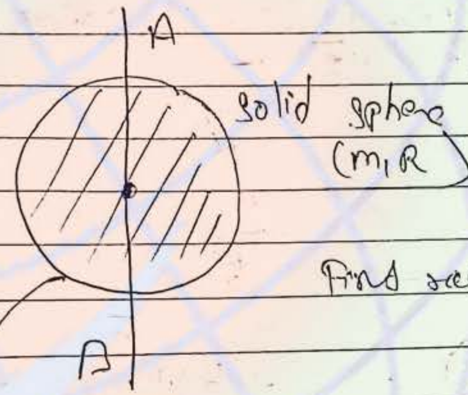
Radius of Gyration (k)

$$\boxed{I_{AB} = mk^2}$$

$$\boxed{k = \sqrt{\frac{I_{AB}}{m}}}$$

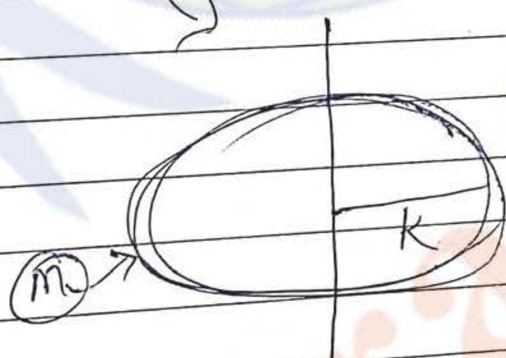
Q.



Find radius of gyration.

$$I_{AB} = \frac{2}{5}mR^2 = mk^2$$

$$k = \left(\sqrt{\frac{2}{5}}\right)R$$



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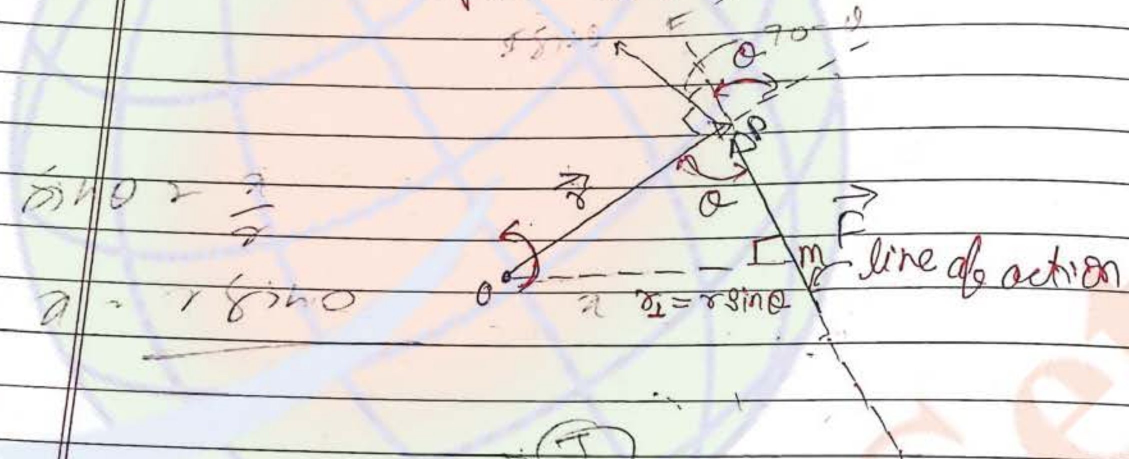
Torque ($\vec{\tau}$) ⇒ (moment of force)

It measures the capability of given force ~~vector~~ to change the rotational state.

or

It is the quantitative measurement of ~~given force~~ rotational effect of a given force.

Torque of a given force about some point ($\vec{F}$) ⇒



$$\boxed{\vec{\tau}_0 = \vec{r} \times \vec{F}}$$

where

$\vec{r} = \vec{OP}$

P → Point of Application of force

$|\vec{r}| = r$

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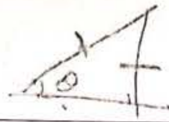

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magnitude

$$|\tau_0| = \tau_0 = r F \sin\theta$$

magnitude of torque

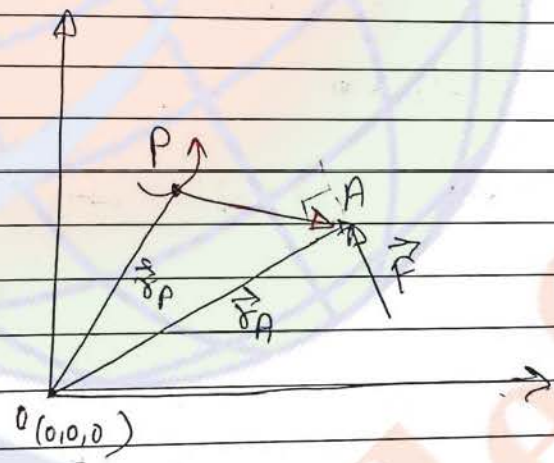
$$\boxed{\tau_0 = F(r\sin\theta)}$$

$$\boxed{\tau_0 = F r_\perp}$$

$r_\perp \rightarrow$ It is the perpendicular distance of point "O" from line of action.

(lever arm)

Ex →



~~what is the~~ $\vec{\tau}_P = ?$

Ans

$$\vec{\tau}_P = \vec{r} \times \vec{F}$$

$$= (\vec{r}_A - \vec{r}_P) \times \vec{F}$$

$\vec{r} = \vec{PA}$

$= \vec{OA} - \vec{OP}$

$\vec{r} = \vec{r}_A - \vec{r}_P$

जहाँ पर जाना

जहाँ से शुरू हुआ

(Final − Initial)

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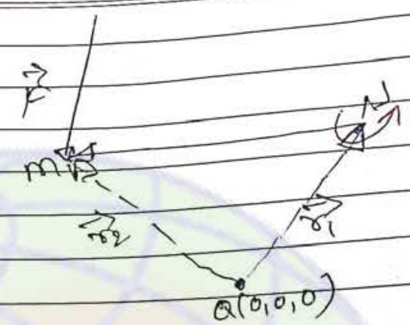
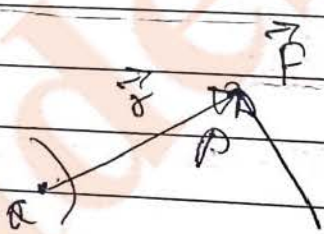
(e) →



$$\tau_N = \vec{r} \times \vec{F}$$
$$= (\vec{r}_2 - \vec{r}_1) \times \vec{F}$$

Q. A force $\vec{F} = (2\hat{i} - 3\hat{j} + \hat{k})$ N is acting at point P → (2m, 3m, −1m). Find the torque vector ($\vec{\tau}$) about point Q → (3m, −1m, 2m)

Ans → $\vec{\tau}_Q = \vec{r} \times \vec{F}$ | $\vec{r} = -1\hat{i} + 4\hat{j} - 3\hat{k}$



$$\vec{\tau}_Q = \begin{array}{ccc} + & - & + \end{array}$$
$$\vec{\tau}_Q = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & -3 \\ 2 & -3 & 1 \end{vmatrix} = \hat{i}(4-9) - \hat{j}(-1+6) + \hat{k}(3-8)$$
$$= (-5\hat{i} - 5\hat{j} - 5\hat{k})$$

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$$\vec{\tau}_{Q,x} = -5\hat{i}$$

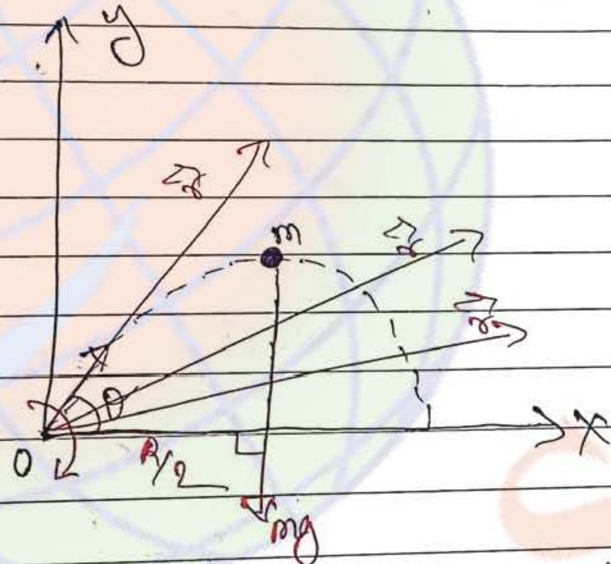
$$\vec{\tau}_{Q,y} = -5\hat{j}$$

$$\vec{\tau}_{Q,z} = -5\hat{k}$$

$$|\vec{\tau}_Q| = 5\sqrt{3}\ N\text{-}m$$

Ans

Q.)



Find the magnitude and direction of torque of gravitation force about point 'O'. when the projected ball reaches at its max. height.

Ans. $\vec{\tau}_0 = \cancel{mg}\ F\ r_\perp$

$$\Rightarrow mg\,\frac{R}{2}\,(-\hat{k})$$

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Torque of a force about given axis :⇒

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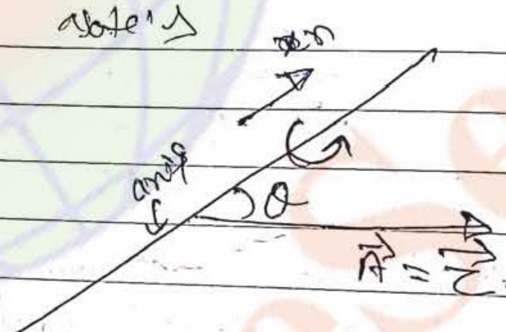
Step :⇒

1.) First find the torque of the given force about any point on the given axis ~~th~~ and then take the component of this torque along given axis.

$$\boxed{\vec{\tau}_{\underset{axis}{AB}} = (\vec{\tau}_0 \cdot \hat{n})\,\hat{n}}$$

note :⇒



Note :⇒

Torque of a given force $\vec{F}$ about the given axis is "Zero" :⇒

I) If applied force $(\vec{F})$ is Parallel/Antiparallel to the axis

$$\vec{F} \parallel AB\,(\text{axis})$$

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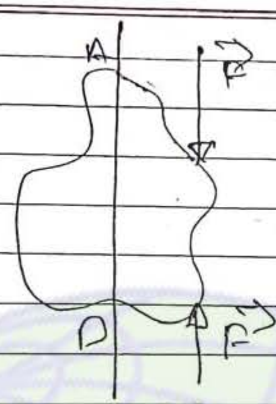
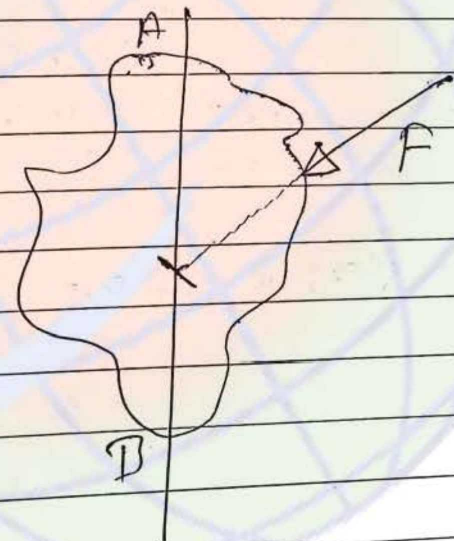


2.> If the applied force intersect the given axis.



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Q. Four forces ~~are~~ ($\vec{F_1}, \vec{F_2}, \vec{F_3}, \vec{F_4}$) are lying in x-y plane

$\vec{F_1} \Rightarrow P_1$ (2m, 3m)

$\vec{F_2} \Rightarrow P_2$ (3m, −2m)

$\vec{F_3} \Rightarrow P_3$ (−2m, −1m)

$\vec{F_4} \Rightarrow P_4$ (2m, 4m)

i) Find Net torque about Z-axis ?

And also.

ii) Find the torque about x and y-axis

Ans:

$$\vec{\tau}_{\substack{net \\ x\text{-axis}}} = \vec{0}$$

$$\vec{\tau}_{\substack{net \\ y\text{-axis}}} = \vec{0}$$

$$\vec{\tau}_{\substack{net \\ z\text{-axis}}} = \text{may or may not be zero.}$$

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Four forces ($\vec{F_1}, \vec{F_2}, \vec{F_3}, \vec{F_4}$) are lying in x-y plane

$\vec{F_1} \Rightarrow P_1\,(2m, 3m)$

$\vec{F_2} \Rightarrow P_2\,(3m, -2m)$

$\vec{F_3} \Rightarrow P_3\,(-2m, -1m)$

$\vec{F_4} \Rightarrow P_4\,(2m, 4m)$

i) Find Net torque about Z-axis ?

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Ans:

$$\vec{\tau}_{\substack{net \\ x\text{-axis}}} = \vec{0}$$

$$\vec{\tau}_{\substack{net \\ y\text{-axis}}} = \vec{0}$$

$$\vec{\tau}_{\substack{net \\ z\text{-axis}}} = \text{may or may not be zero.}$$

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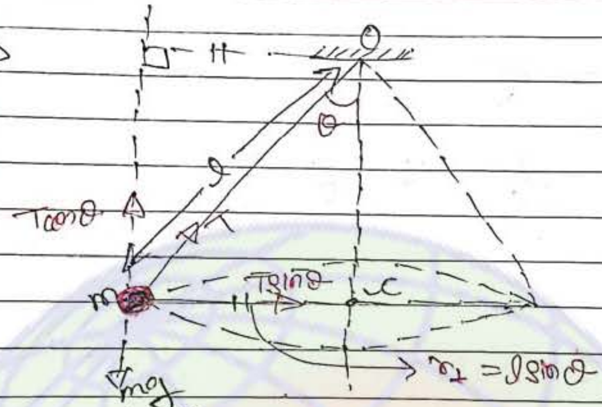
Ex :-



i) Find the magnitude of torque of gravitation force (mg) and tension in the string about point 'O' and 'C'.

ii) Find the magnitude of torque of mg about axis 'OC'

Ans :-

$$(\tau_{mg})_O = (\tau_{mg})_C = mg\, l \sin\theta$$

$$(\tau_T)_O = T \times \vec{0} = 0$$

$$(\tau_{T\sin\theta})_C = 0$$

$$(\tau_{T\cos\theta})_C = T\cos\theta \times l \sin\theta$$

$$(\tau_{mg})_{OC\,axis} = 0$$

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~~Rotation of Rigi~~

Equillibrium of Rigid body / system ⇒

→ (1) Translatory equillibrium.

$(\vec{F}_{net} = \vec{0})$

→ (2) Rotatory Equillibrium

$(\vec{\tau}_{net} = \vec{0})$

clockwise ← $\boxed{(\vec{\tau}_{cw})_{net} = (\tau_{A.c.w})_{net}}$ → Anticlock wise.

$(\vec{\tau}_{cw})_{net} = -(\vec{\tau}_{A\cdot cw})_{net}$

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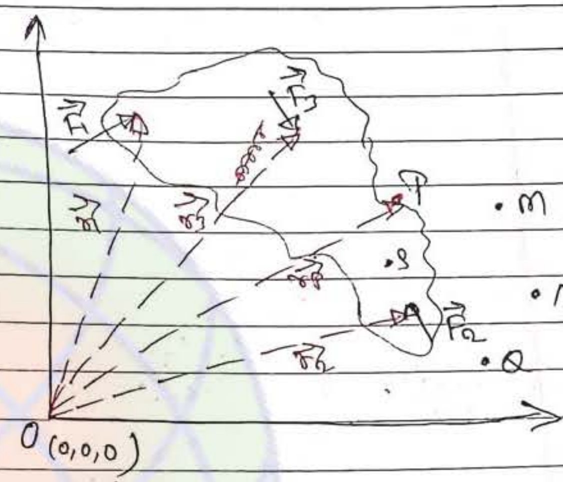
* Resultant torque of many forces about given point :⇒



$$\vec{\tau}_0 = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \dots \quad (1)$$

$$\vec{\tau}_P = (\vec{r}_1 - \vec{r}_P) \times \vec{F}_1 + (\vec{r}_2 - \vec{r}_P) \times \vec{F}_2 \dots$$

$$= (\vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \dots) - \vec{r}_P \times (\vec{F}_1 + \vec{F}_2 + \dots) \quad (2)$$

* I) If the rigid body is in translatory equillibrium than the resultant ~~to~~ torque of all the force ~~is zero~~ will be same about all the points in the space. (If $\vec{F}_{net} = \vec{0}$ ($\vec{F_1} + \vec{F_2} + \dots = \vec{0}$))

$$\boxed{\vec{\tau}_0 = \vec{\tau}_P = \text{Same}}$$

II) If the rigid body is in equillibrium (F_{net}) = 0 and ($\vec{\tau}_{net} = 0$)

~~(F_{net}) = 0 then (τ_{net}) = 0~~

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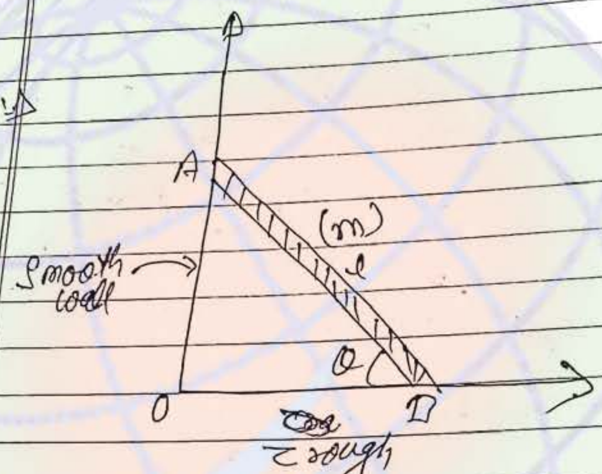
$$\boxed{\begin{aligned} &\text{If} \quad \vec{\tau}_0 = \vec{0} \\ &\vec{\tau}_{\text{any point}} = \vec{0} \end{aligned}}$$

Eg :-

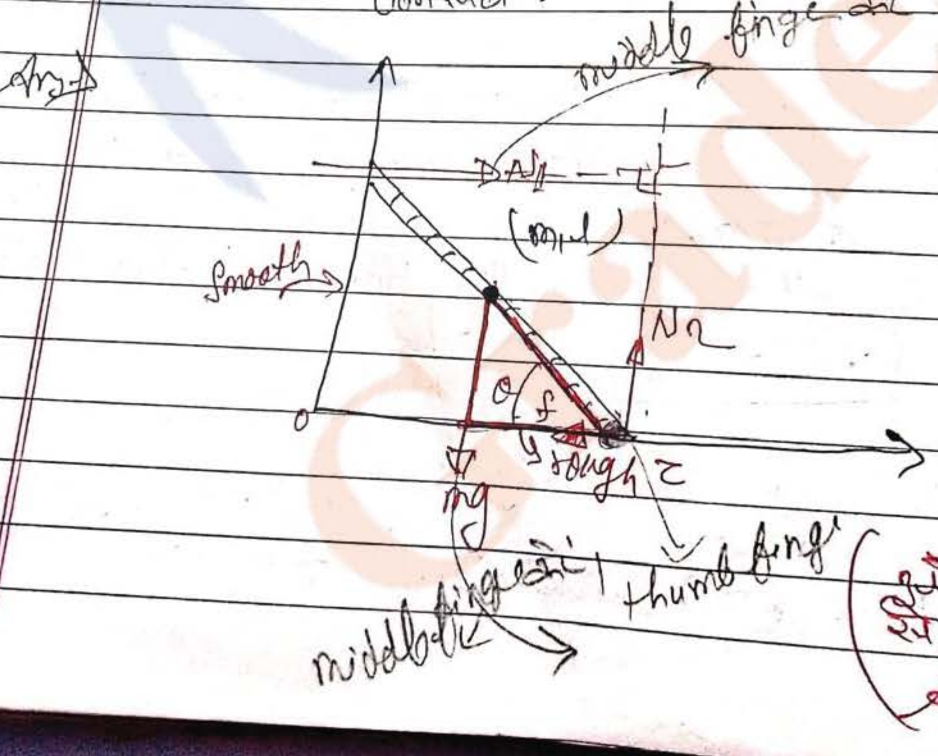


The rod is in equillibrium

i) Find the value of Normal contact force at the contact.

ii) And the value of friction force at the contact.

Ans :-



(अंगुलियों की सहायता से clockwise और Anticlockwise पता करें)

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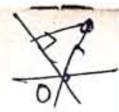


$F_{net} = 0$

$N_1 = f$ —(1)

$N_2 = mg$ —(2)

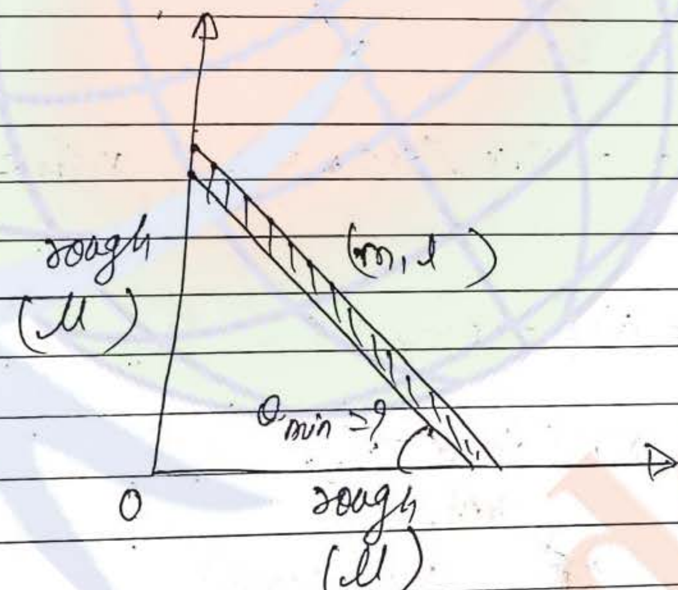
$(\tau_{net})_B = 0$

$\tau = F r_\perp$

$N_1 \cancel{l} \sin\theta = mg \frac{\cancel{l}}{2} \cos\theta$

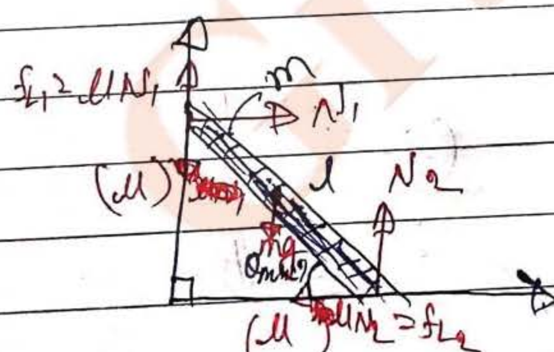
$N_1 = \frac{mg \cot\theta}{2} = f$

Ex:→



Find the minimum value of θ so that rod remains in equillibrium

Ans.



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$(F_{net}) = 0$

$N_1 = \mu N_2$ — (1)

$\mu N_1 + N_2 = mg$ — (2)

from eq (1) and (2)

$\mu^2 N_2 + N_2 = mg$

$$N_2 = \frac{mg}{(1+\mu^2)}$$

$$N_1 = \frac{\mu mg}{(1+\mu^2)}$$

$(\tau_{net})_B = 0$

$mg \times \frac{l}{2}\cos\theta = N_1 \times l\sin\theta + \mu N_1 \times l\cos\theta$

(ACW)

$$mg\frac{\cos\theta}{2} = N_1(\sin\theta + \mu\cos\theta)$$

$$\frac{mg}{2} = N_1(\tan\theta + \mu)$$

$$\frac{mg}{2} = \frac{\mu mg}{(1+\mu^2)}(\tan\theta + \mu)$$

$$\frac{\mu^2+1}{2\mu} - \mu = \tan\theta$$

$$\boxed{\tan\theta = \frac{(1-\mu^2)}{2\mu}}$$

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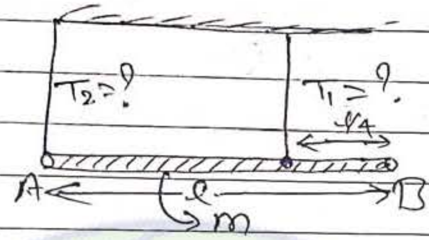
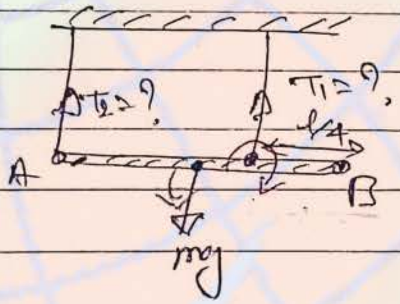
Ex!→



The Arrangement is in equilibrium. String is massless.

Find the tension in both the string.

Ans.



$$T_1 + T_2 = mg \quad \text{---(1)}$$

$$T_2 \frac{3l}{4} = mg \frac{l}{4}$$

$$T_2 = \frac{mg}{3}$$

$$T_1 = mg - \frac{mg}{3}$$

$$T_1 = \frac{2mg}{3}$$

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1st Choice

$m \to I$, $a \to \alpha$, $F \to \tau$

$p \to J$, $s \to \theta$, $v \to \omega$

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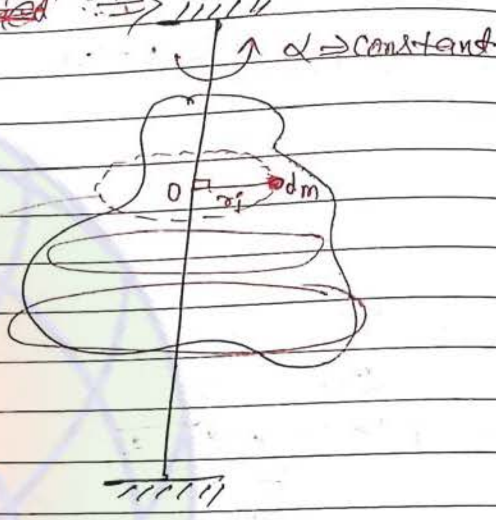
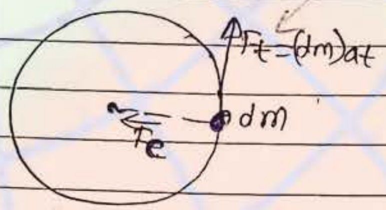
Rotation of rigid body about fixed axis





$F_t = (dm) r_i \alpha$

Torque of F_t on (dm) about 'O'.

$= F_t \cdot r_i$

$= (dm) r_i^2 \alpha$

$\tau_{net} = \alpha \int (dm) r_i^2$

$$\boxed{\tau_{net} = I \alpha}$$

$$\boxed{(\vec{\tau}_{ext})_{net} = I_{axis} \vec{\alpha}}$$ (Newton's law in Rotation)

Imp VVI

$$\boxed{(\tau_{axis})_{net} = I_{axis} \alpha}$$

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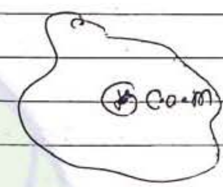
$$\alpha = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta}$$

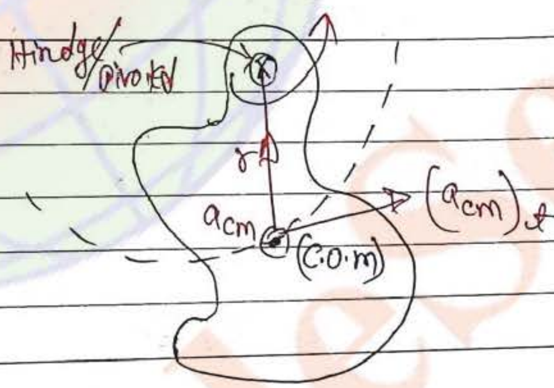
Notes:-

1) If the rigid body is hindge as its c.o.m then $a_{cm} = 0$
there is no acceleration

$$(a_{cm} = 0)$$



II) If the rigid body is not hindge as its c.o.m.
The c.o.m will always move along circular path



$$\boxed{(a_{cm})_t = r\alpha}$$

$$\boxed{(a_{cm})_c = \omega^2 r}$$

Rotation → $\boxed{\tau_H = I_H \alpha}$

Translation → $\boxed{F_{net} = M a_{cm}}$

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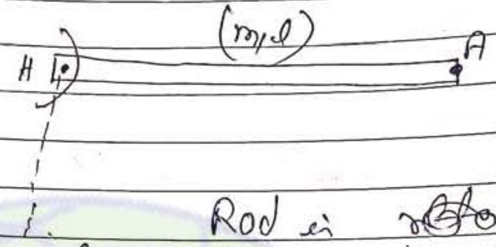
1st Choice

Ex:

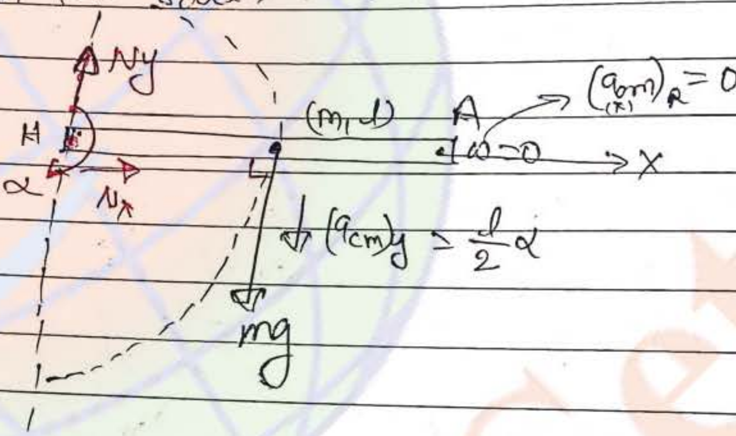


Rod is ~~…~~ released from rest from horizontal position.

i) Find the initial angular accⁿ of rod.

ii) Find the force exerted ~~…~~ by hindge on the rode.

Ans:



$$\tau_H = I_H \cdot \alpha$$

$$I_H = \frac{ml^2}{3}$$

$$mg\frac{l}{2} = \frac{ml^2}{3} \cdot \alpha$$

$$\alpha = \left(\frac{3g}{2l}\right)$$

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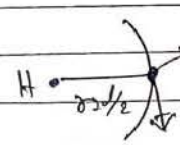
ii) $F = ma_{cm}$



$N_x = m\cancel{(a_{cm})_x} = 0.$

$mg - N_y = m(a_{cm})$

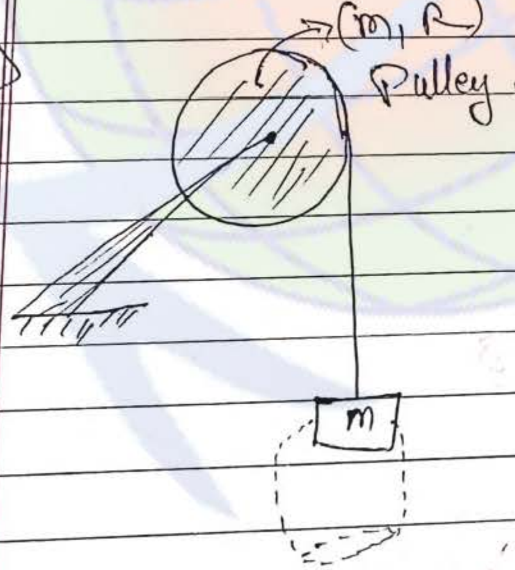
$mg - N_y = m \times \frac{l}{2} \times \alpha$

$mg - N_y = m \times \frac{l}{2} \times \frac{3g}{2l}$

$N_y = mg - \frac{3mg}{4}$

$N_y = \frac{mg}{4}$

Ex:



The string does not slip over the pulley. String is massless.

The system is released from rest.

i) Find the acc'n of the block.
ii) Find the Angular velocity of the pulley at any time 't' from the starting t=0.

Ans.

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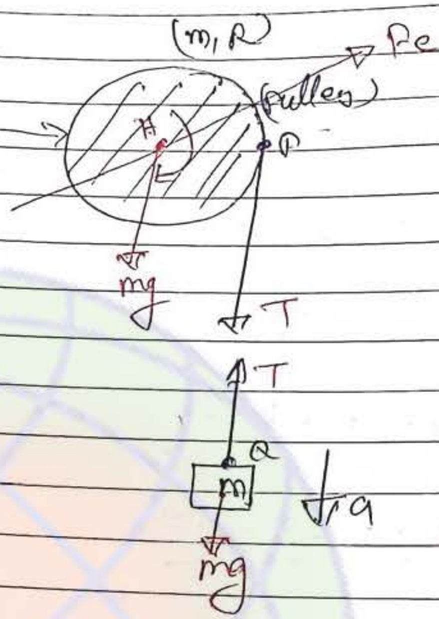
$F_{net} = ma_{cm} = 0$

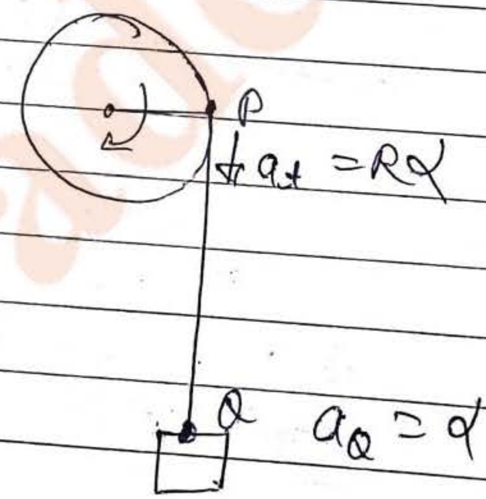


$$\tau_H = I_H \alpha$$

$$TR = \frac{mR^2}{2} \cdot \alpha$$

$$T = \frac{1}{2} m (R\alpha) \quad \text{—} (1)$$

$$mg - T = ma \quad \text{—} (2)$$



$$a = R\alpha$$

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From (i) →

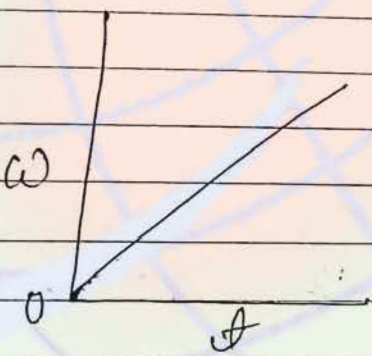
$$\begin{aligned} T &= \frac{ma}{2} \\ mg - T &= ma \end{aligned} \rightarrow a = \frac{2g}{3}$$

$$\alpha = \frac{a}{R} = \frac{2g}{3R} = \text{constant}$$

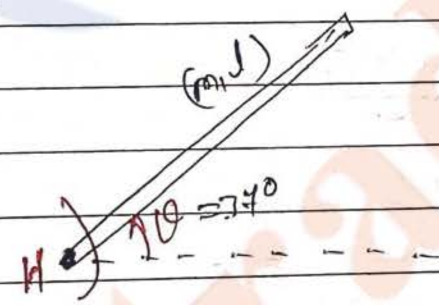
$$\omega = \omega_0 + \alpha t \quad (\omega_0 \to 0)$$

$$\omega = \alpha t$$

$$\omega = \left(\frac{2g}{3R}\right) t$$



Ex.→



Rod is released from rest from given position.

1) Find the Initial Angular acc^n of rod and the force exerted by hinge of the rod.

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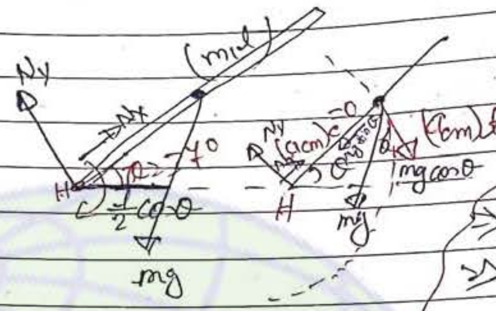
Ans.



$\tau_H = I_H \alpha$

$\Rightarrow mg\frac{\ell}{2}\cos\theta = \frac{m\ell^2}{3} \times \alpha$

$\Rightarrow \frac{g}{2}\cos\theta = \frac{\ell}{3}\alpha$

$\alpha = \left(\frac{3g\cos\theta}{2\ell}\right)$

$N_x - mg\sin\theta = m \times (a_{cm})_x$

$N_x = mg\sin\theta$

$N_x = \frac{3mg}{5}$

Now,

$mg\cos\theta - N_y = m(a_{cm})_t$

$N_y = mg\cos\theta - m(a_{cm})_t$

$= mg\cos\theta - \frac{m\ell}{2} \times \frac{3g\cos\theta}{2\ell}$

$N_y = \frac{mg\cos\theta}{4}$

$= \frac{mg}{5}$

$N = \sqrt{N_x^2 + N_y^2}$

$= \sqrt{\left(\frac{3mg}{5}\right)^2 + \left(\frac{mg}{5}\right)^2}$

$= \frac{mg}{5}\sqrt{10}$

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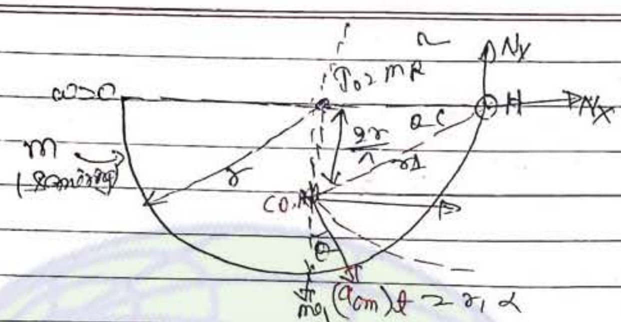
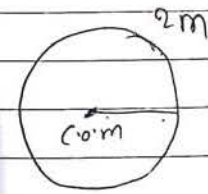
Ex! ⇒



1) Find the force exerted by the hinge on the semi ring initially

Ans $\tau_H = I_H \alpha$

Now,



$$\frac{\cancel{2}mR^2 + (2m)R^2}{\cancel{2}}$$
$$= 2mR^2$$

Now, -

$$I_{cm} = I_0 - m\left(\frac{2r}{\pi}\right)^2$$

$$I_H = I_{cm} + mr_1^2$$

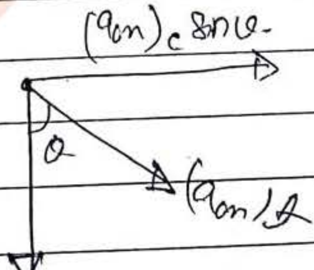
$$= I_0 - m\cancel{\left(\frac{2r}{\pi}\right)^2} + mr^2 + \cancel{m\left(\frac{2r}{\pi}\right)^2}$$

$$= mR^2 + mr^2 = 2mR^2$$

$\tau_H = I_H \alpha$

$\cancel{m}g r = 2\cancel{m}r^2 \alpha$

$\alpha = \frac{g}{2r}$



$mg - N_y = m(a_{cm})_t \cos\theta$

$N_y = mg - mr_1\alpha \cdot \cos\theta$

$= mg - mr_1\left(\frac{g}{2r}\right)\frac{r}{r_1}$

$N_y = \frac{mg}{2}$

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H.C.V ⇒ 5 to 27
10 to 45

$N_x = m(a_{cm})_x \sin\theta$

$= m \times \frac{80 g}{28} \cdot \frac{28}{...}$

80

$N = \sqrt{N_x^2 + N_y^2}$

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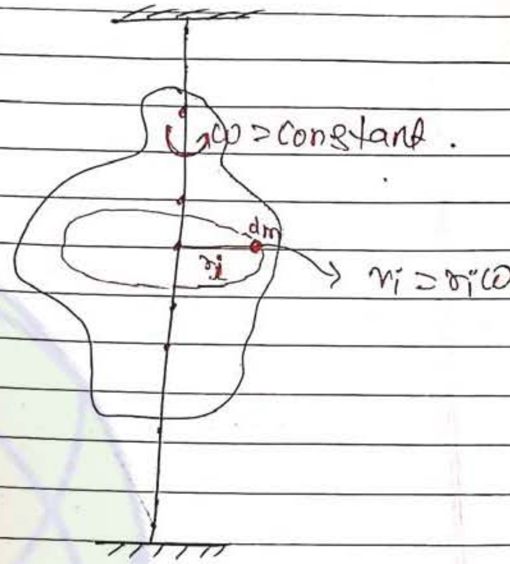
Rotational kinetic energy of ~~rot~~ a rotating body about fixed axis



K.E of an element (dm) ⇒

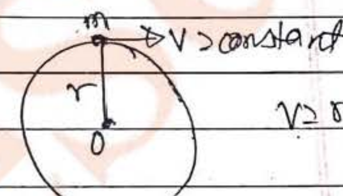
$dK = \frac{1}{2}(dm)v_i^2$

$(K.E)_R = \frac{1}{2}\int (dm) r_i^2 \omega^2$

$= \frac{1}{2}\omega^2 \int (dm) r_i^2$

$$\boxed{(K.E)_R = \frac{1}{2} I_{axis}\, \omega^2}$$

Note :⇒



$v = r\omega$

$K.E = \frac{1}{2}mv^2$

$(K.E)_R = \frac{1}{2} I\omega^2$

$= \frac{1}{2} m\cancel{r^2} \times \left(\frac{v}{\cancel{r}}\right)^2$

$= \frac{1}{2}mv^2$

Note :⇒ So,

K.E of a body in combined Rotational and translational

$$\boxed{K.E = \frac{1}{2}mv_{cm}^2 + \frac{1}{2} I_{cm}\omega^2}$$

attention

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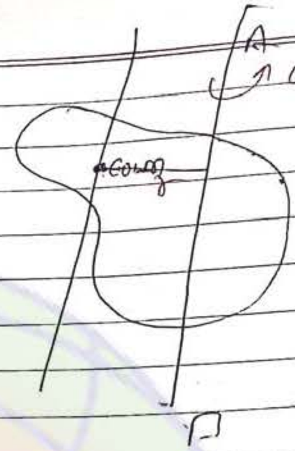


$V_{cm} \perp r\omega$

$$(K.E)_A = \frac{1}{2} I_{AB}\,\omega^2 \qquad \{\, I_{AB} = I_{cm} + mr^2$$

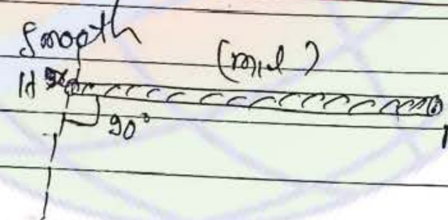
$$= \frac{1}{2}\left(I_{cm} + mr^2\right)\omega^2$$

$$= \frac{1}{2} I_{cm}\,\omega^2 + \frac{1}{2} m (r\omega)^2$$

$$= \frac{1}{2} I_{cm}\,\omega^2 + \frac{1}{2} m V_{cm}^2$$

$$\boxed{\text{So, } (K.E)_A = \frac{1}{2} I_{cm}\,\omega^2 + \frac{1}{2} m V_{cm}^2}$$

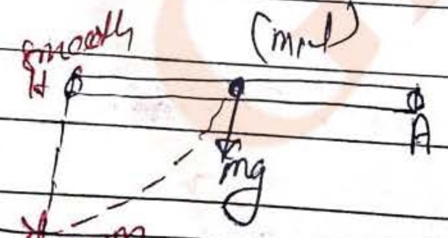
Ex:



Rod is released from rest from horizontal position. Find the angular velocity of the rod at the moment the rod becomes vertical.

ii) Find the velocity of point "A" of the rod at the moment the rod becomes vertical.

1)



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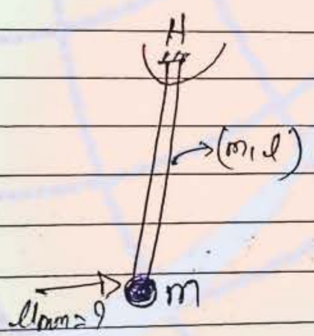
$$0 + mg\times\frac{l}{2} = \frac{1}{2} I_H \omega^2 - 0 + 0$$

$$\cancel{m}g\times\frac{l}{2} = \frac{1}{2}\left(\frac{\cancel{m}l^2}{3}\right)\omega^2$$

$$\omega = \sqrt{\frac{3g}{l}} \qquad \left\{ v = r\alpha \right.$$

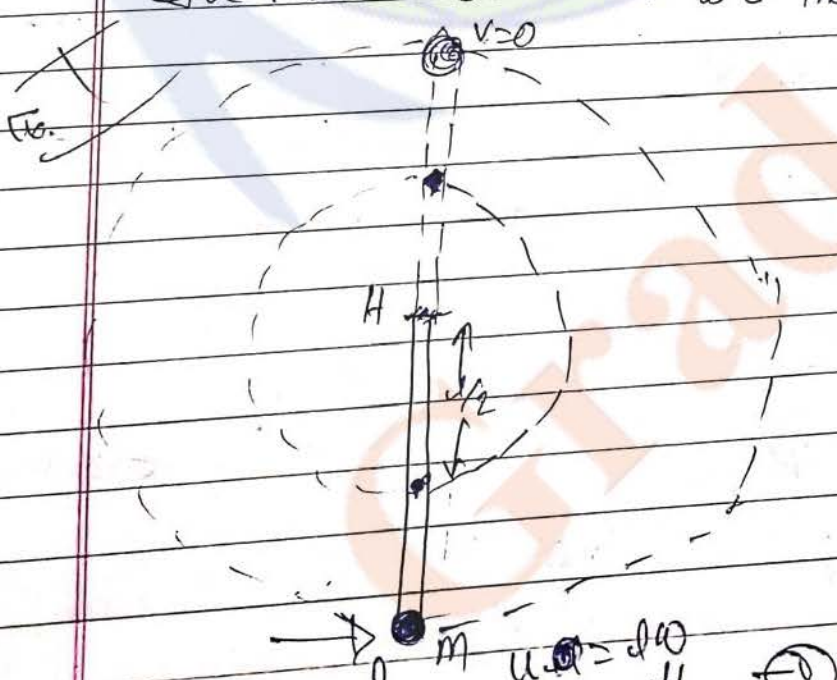
$$v_A = l\sqrt{\frac{3g}{l}} = \sqrt{3gl}$$

Ex.



Find the minimum horizontal velocity given to particle of mass 'm' at the lowest point show that the rod and particle can complete the vertical circle, w.e. Theo :-

Ex.



$$u = l\omega \quad (1)$$
$$\omega = \frac{u}{l}$$

$$\boxed{\because v = r\omega}$$

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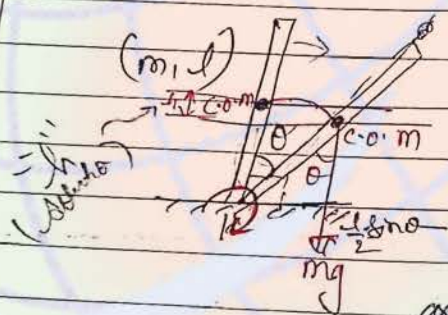
$$-mgl - mg(2l) = 0 - \frac{1}{2}(I_H)_{sys}\,\omega^2$$

$$3mgl = \frac{1}{2}\left(\frac{ml^2}{3} + ml^2\right)\cdot\left(\frac{u}{l}\right)^2 \quad \text{— decrease}$$

$$3mgl = \frac{4ml^2}{2\times 3}\cdot\frac{u^2}{l^2}$$

$$u = 3\sqrt{\frac{gl}{2}}$$

Q. 

Rod is slightly pushed find the angular acc^n and angular velocity of the rod at an angular position 'θ' with the vertical.

Ans.

$$\tau_H = I_H\,\alpha$$

$$mg\frac{l}{2}\sin\theta = \frac{ml^2}{3}\alpha$$

$$\alpha = \frac{g\sin\theta\cdot 3}{2l}$$

$$= \frac{3g\sin\theta}{2l}$$

Method - 1st :-

$$mgh = \frac{1}{2}I_H\,\omega^2$$

$$h = \frac{l}{2} - \frac{l}{2}\cos\theta$$

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m

$$mg\frac{l}{2}(1-\cos\theta) = \frac{1}{2}\frac{ml^2}{3}\cdot\omega^2$$

$$\omega^2 = (1-\cos\theta)\frac{3g}{l}$$

2b) method - 2 →

$$\alpha = \omega\cdot\frac{d\omega}{d\theta}$$

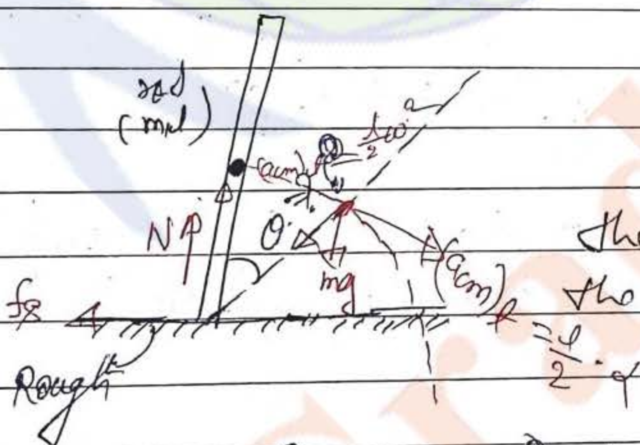
$$\int \omega\, d\omega = \int \alpha\, d\theta$$

$$\int_0^\omega \omega\, d\omega = \frac{3g}{2l}\int_{\theta=0}^{\theta} \sin\theta\, d\theta$$

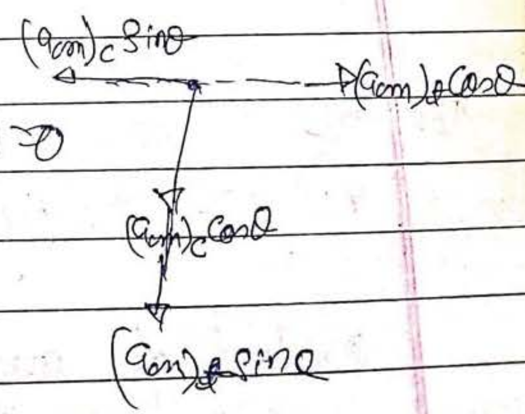
$$\frac{\omega^2}{2} = \frac{3g}{2l}(-\cos\theta)_0^\theta = \frac{3g}{l}\{(-\cos\theta)-(-\cos 0)\}$$

$$\omega^2 = \frac{3g}{l}(1-\cos\theta)$$

Q6.



Find the value of 'θ' for which the friction force on the rod by the surface is zero.



$$f_s = m\{(a_{cm})_c\sin\theta - (a_{cm})_t\cos\theta\} = 0$$

$$\to \left(\cos\theta = \frac{2}{3}\right)$$

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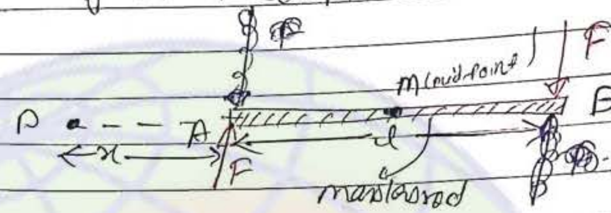
1st Choice

Force couple :⇒ when two force of equal magnitude act on different points and in opposite direction these force form a "couple"!



$$\left\{\begin{array}{c}\tau_{net} = ? \\ P, A, B, m\end{array}\right\}$$

$$(\tau_{net})_A = 0 + Fl \quad (c.\omega)$$

$$(\tau_{net})_B = 0 + Fl \quad (c.\omega)$$

$$(\tau_{net})_P = -Fx + F(l+x)$$
$$= Fl \quad (c.\omega)$$

$$(\tau_{net})_m = \frac{Fl}{2} + F\cdot\frac{l}{2} = Fl \quad (c.\omega)$$

Here we see that, in force couple "torque (τ)" about all the point is "same."

Note → 1) The couple cause the rotational motion in the body.
2) The effect of couple is known as its moment.
The moment of couple is equal to the product of magnitude of any force and perpendicular distance between the force.

$\boxed{\text{moment of couple} = Fr_\perp}$ → same as torque.

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Angular Momentum ($\vec{L}_0$)

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It is defined as moment of linear momentum about any point or about any axis.

(linear momentum :⇒ $\vec{P} = m\vec{v}$)

$V \Rightarrow \omega$
$a \Rightarrow \alpha$
$m \Rightarrow I$
$F \Rightarrow \tau$
$P \Rightarrow L$

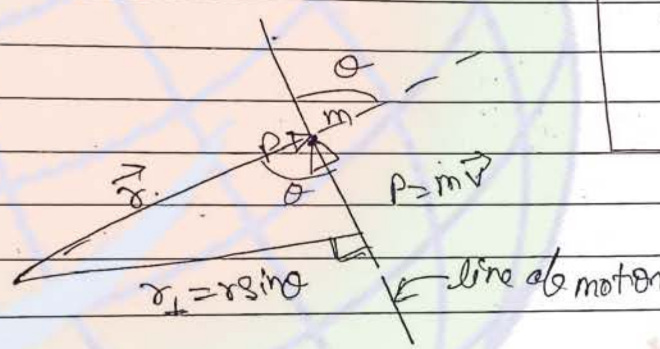
$\vec{P} = m\vec{v}$

$\vec{L}_{axis} = I_{axis}\vec{\omega}$

$\vec{F}_{net} = \frac{d\vec{P}}{dt}$

$\vec{\tau}_{net,ext} = \frac{dL}{dt}$

Angular momentum of a particle about some(any) point.



$$\boxed{\vec{L}_0 = \vec{r} \times \vec{P}}$$

$\vec{P} = m\vec{v}$

$$\boxed{|\vec{L}_0| = P \cdot r_\perp}$$

or $\boxed{L_0 = (mv\, r_\perp)}$ (where $r_\perp = r\sin\theta$)

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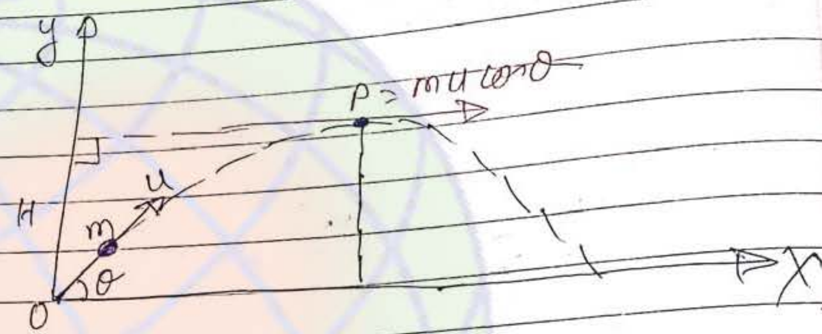
Q: A ball of 'm' is projected with initial velocity 'u' and angle 'θ' with horizontal. i) Find the angular momentum of the ball about the point of projection and the momentum when the ball reaches at its maximum height :-



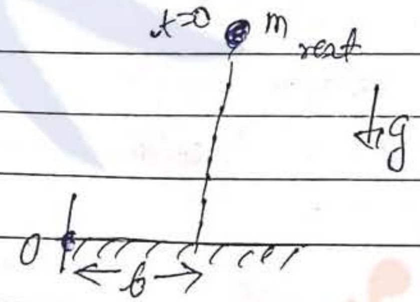
$(L_0 = P r_\perp)$

$L_0 = PH$

$L_0 = mu\cos\theta \cdot \frac{u^2\sin^2\theta}{2g}$

$\vec{L}_0 = L_0(-\hat{k})$

Ex:



Ans A ball is released from rest from certain height at time t=0, find the angular momentum of the ball at any 't' about point 'O'

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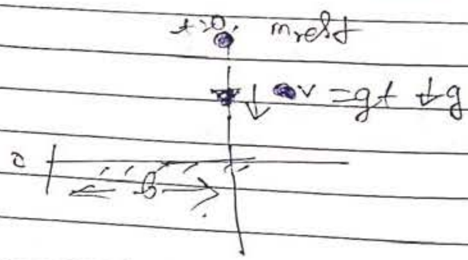
Ans:-



$L_O = m(gt)\,b$

~~Ex Angular momentum of a rigid body rotating about fixed axis.~~

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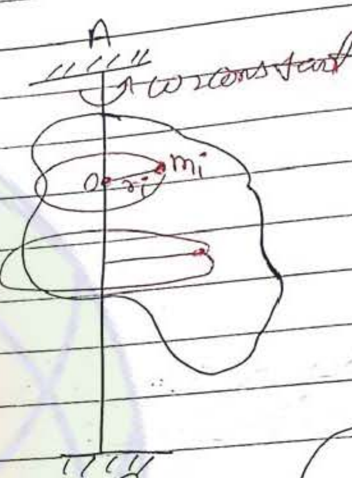
1st Choice

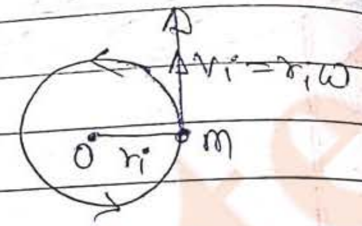
Angular momentum of a rigid body rotating about fixed Axis



$L_{axis} = \sum m_i r_i \omega$

$L_{axis} = \sum m_i v_i r_i$

$= \sum m_i r_i^2 \omega$



$$\boxed{L_{axis} = I_{axis}\,\omega}$$

$$\boxed{\vec{L}_{axis} = I_{axis}\,\vec{\omega}}$$

Angular momentum of particle about given Axis:→

1) First find the Angular momentum of the particle about any point on the given axis. then take the component of this Angular momentum along given axis.

$$\boxed{\vec{L}_{axis} = (\vec{L}_0 \cdot \hat{n})\,\hat{n}}$$

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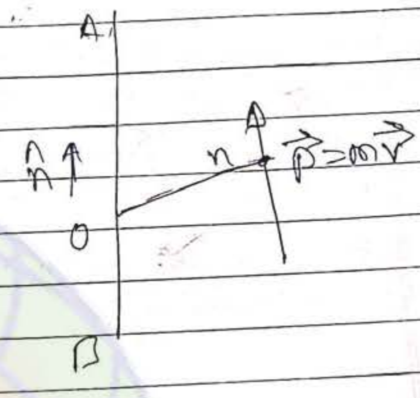
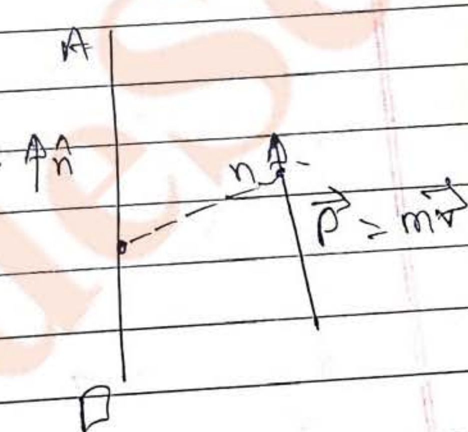


⇒ Angular momentum of a particle about given axis is "Zero" ⇒

→ (I) If $\vec{P}$ ∥ to axis

→ (II) If the line of motion of particle Intersects the Axis AB.



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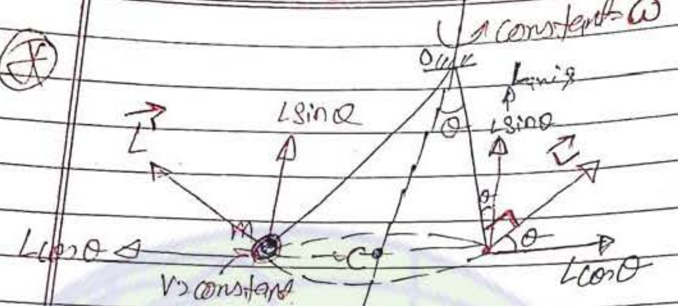
Angular momentum of a Conical section:-



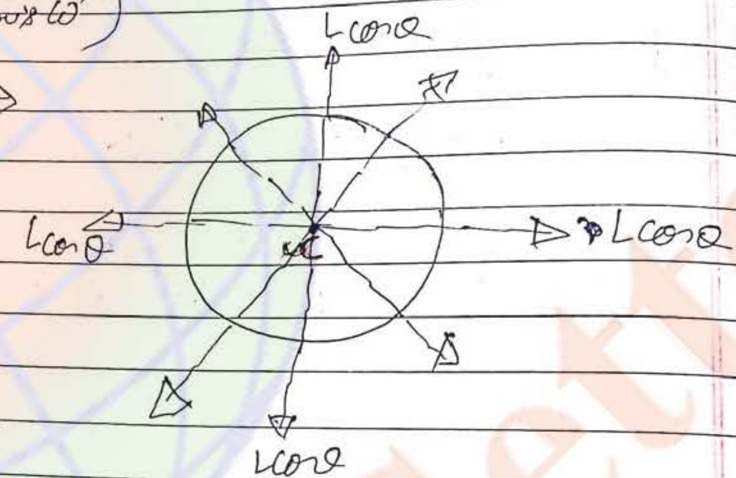
$\vec{L} = \vec{r} \times \vec{P}$

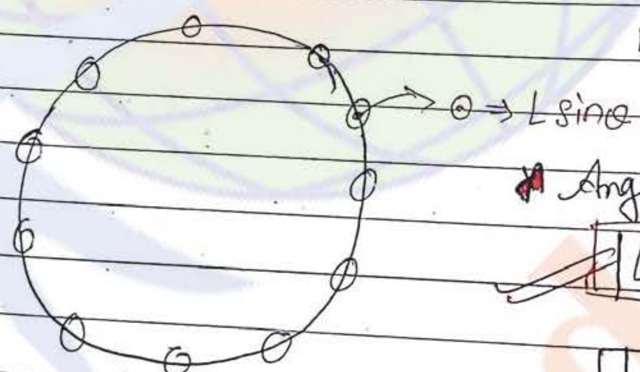
$$\vec{L} \cdot \vec{r} = 0$$

$$\vec{L} \cdot \vec{P} = 0$$

$$\left(\vec{L}_{axis} = I_{axis} \vec{\omega} \right)$$

$$\vec{L} \neq I\vec{\omega}$$



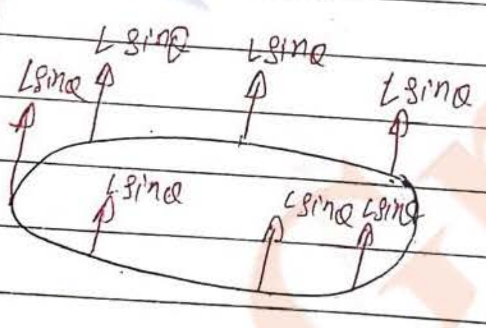


Angular momentum L_0 about point 'O' located at the hinge:→

$$\boxed{|L_0| = mvl}$$ where $v = r\omega$

so,

$$\boxed{|L_0| = ml r\omega}$$



The angular momentum about centre 'c' of the circle:-

$$L_c = r \times P$$
$$= r \times mr\omega$$
$$= mr^2\omega \hat{k}$$

where:-
$|P| = mv$
$= mr\omega$

Notes: for more see Resnick (Page-412)

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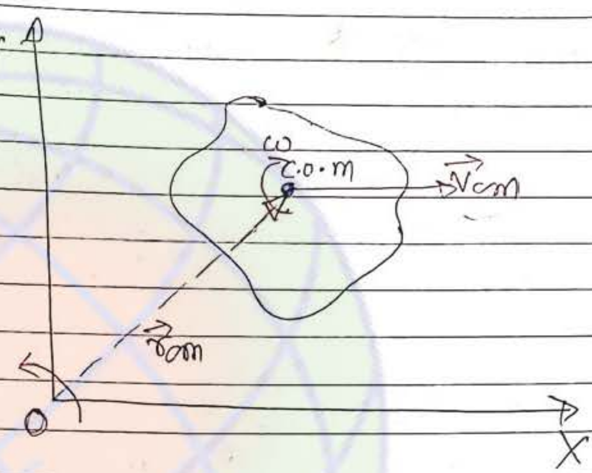
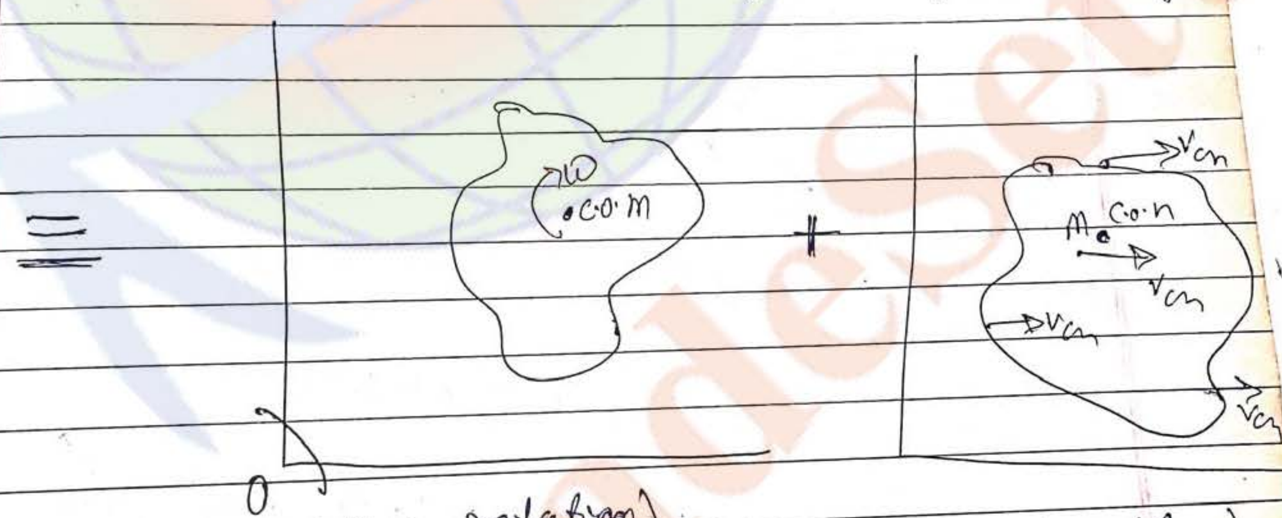
Angular momentum of a rigid body in Combined motion :⇒ (Translational + Rotational)



$\vec{r}_{cm}$ → Position vector of C.O.M of rigid body



(Pure rotation) (w.r.t. C.O.M as observed from C.O.M) (spin Angular momentum)

(Pure translation) (orbital momentum)

$$\boxed{\vec{L}_{cm} = \underbrace{I_{cm}\vec{\omega}}_{\text{Rotational Angular momentum about fixed axis}} + \underbrace{\vec{r}_{cm} \times M\vec{V}_{cm}}_{\text{Translational A.M of a body}}}$$

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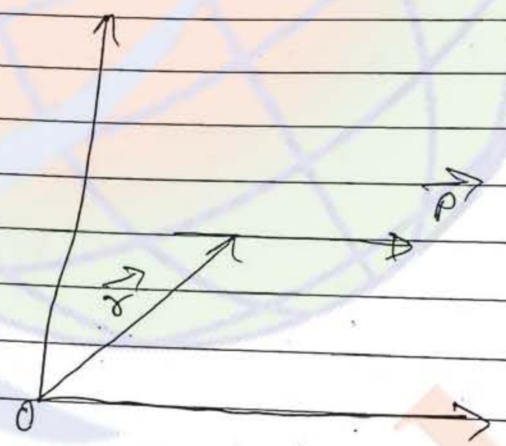
$$\boxed{\vec{L}_0 = \vec{L}_{cm} + \vec{r}_{cm} \times m\vec{V}_{cm}}$$

* $\vec{L}_{cm}$ ⇒ Angular momentum of rigid body with respect to C.O.M
* (It is "not" with respect to point O)

($\vec{L}_{cm} = I_{cm}\vec{\omega}$)

$\vec{r}_{cm} \times m\vec{V}_{cm}$ ⇒ Angular momentum of C.O.M w.r.t. point O'.



$$\vec{L}_0 = \vec{r} \times \vec{p}$$

$$\frac{dL}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

$$= \vec{r} \times \dot{\vec{p}} + \cancel{\vec{v} \times \vec{p}} \rightarrow 0 \quad \{ \vec{p} \parallel m\vec{v} \}$$

$$\Rightarrow \boxed{\vec{\tau}_{ext,net} = \frac{d\vec{L}_{sys}}{dt}}$$ (Newton IInd law in rotation)

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Principal of Conservation of Angular momentum :→

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If the net external torque acting on the System about any point, about any axis is zero. The Angular momentum of System will remain Conserved or Constant along that Point or that Axis.

If $\vec{\tau}_{net,ext} = \vec{0}$

$\vec{L}_{syst} =$ Constant

$\vec{L_i} = \vec{L_f} =$ Same.

Angular Impulse $(\vec{J})$

$$\left(\vec{\tau} = \frac{d\vec{L}}{dt}\right)$$

So

$$\boxed{\vec{\Delta L} = \vec{L_f} - \vec{L_i} = \int_{\vec{L_i}}^{\vec{L_f}} dL = \underbrace{\int_{t_1}^{t_2} \vec{\tau} dt}_{\text{Angular Impulse}}} \Rightarrow \text{Area of } \tau\text{-}t \text{ graph.}$$

$$\boxed{\vec{J} = \vec{\Delta L} = \vec{L_f} - \vec{L_i} = \int_{t_1}^{t_2} \vec{\tau} dt}$$

Angular-Impulse momentum theorem:

$$\boxed{\vec{J} = \vec{\tau}_{axis}(\Delta t)}$$

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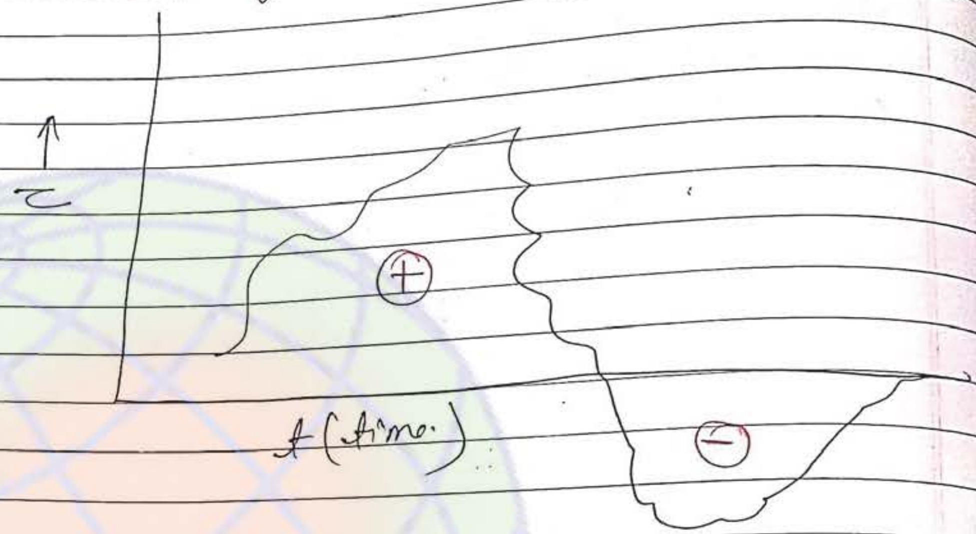
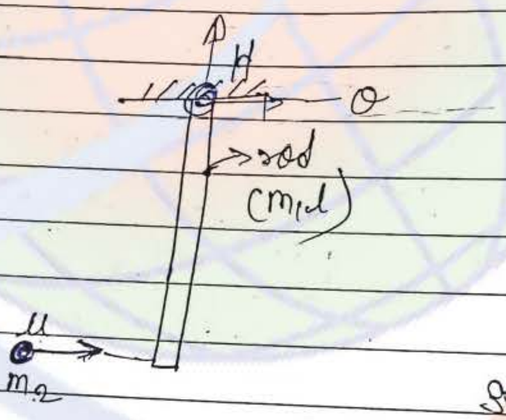
Note! → Graphical $\vec{\tau}$ - t !⇒



Eg:⇒



Before collision rod was at rest.
A particle of mass m_2 strikes the end of the rod with velocity u and stick to the rod after collision.

1) Find the Angular velocity of the system (rod + particle) Just after collision.

→ conservation of Angular momentum.

C.O.A.M of system (rod + Particle) about Hindge(H) ⇒

$$\vec{L_i} = \vec{L_f}$$

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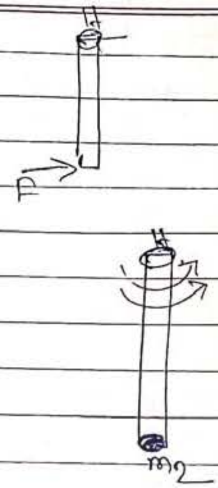
$J \cdot B \cdot C = J \cdot A \cdot C$

$0_{rod} + m_2 u l = (I_H)_{sys}\,\omega$

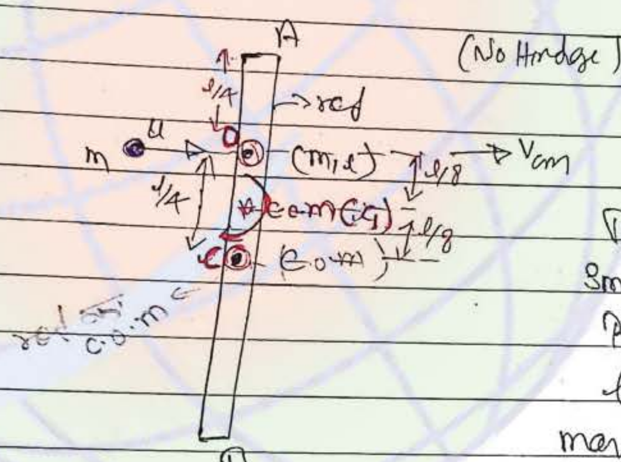
$m_2 u l = \left(\frac{m_1 l^2}{3} + m_2 l^2\right)\omega$

$\omega = \frac{m_2 u}{l\left(\frac{m_1}{3} + m_2\right)}$



Sol 4)

(No Hinge)



Particle and rod are on smooth horizontal surface. Rod was initially at rest before collision. Particle of mass 'm' strikes the rod with velocity 'u'. And stick to the rod after collision.

Ans:→ 1) Linear momentum of the system is conserved ...

i) ~~Linear~~

Now again :-> $mu + 0 = (2m) V_{cm}$

$v_{cm} = \frac{u}{2}$

(By linear momentum conservation.)

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 (C.A) also conv H·ω

Conservation of Angular momentum (C.O.A.M) about "C_1" (C.o.m of system)

$O_{rod} + mu\frac{l}{8} =$

$mv_{cm}(r_{cm})_\perp$

Easy method

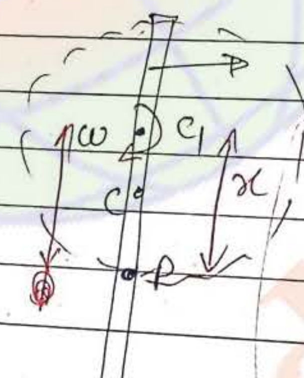
$O_{rod} + mu\frac{l}{8} = (I_{cm})_{sys}\,\omega + 0$

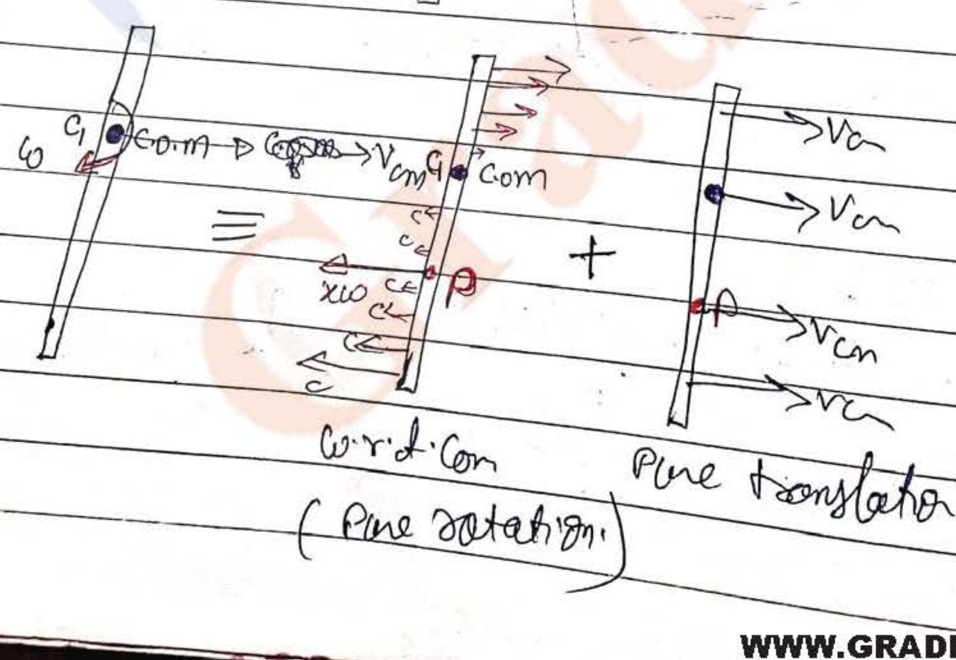
$$mu\frac{l}{8} = \left\{\underbrace{\left(\frac{ml^2}{12} + m\left(\frac{l}{8}\right)^2\right)}_{rod} + \underbrace{m\left(\frac{l}{8}\right)^2}_{particle}\right\}\omega$$

Direction is same direction

The complex method (2)

$\omega = \frac{12u}{11l}$





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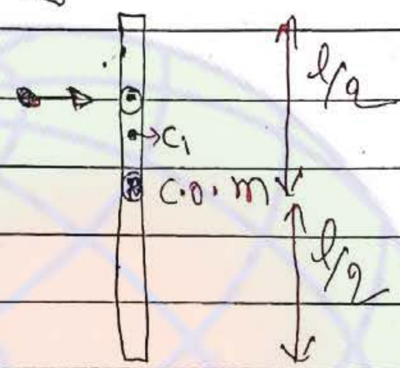
conservation of Angular momentum about c (C.O.M) of rod :—



C.O.M

$$mu + 0 = 2m V_{cm}$$ (C.O. linear momentum)

$$V_{cm} = \frac{u}{2}$$

Now :—

C.O.A.M about 'c' C.O.M of rod :—

$$0_{rod} + mu \times \frac{l}{4} = I_{cm}\omega + mV_{cm}(r_{cm})$$

$$\frac{mul}{4} = \left(\frac{ml^2}{12} + m\left(\frac{l}{8}\right)^2 + m\left(\frac{l}{8}\right)^2\right)\omega + (2m)\left(\frac{u}{2}\right)\left(\frac{l}{8}\right)$$

$$\frac{mul}{4} - \frac{mul}{8} = (\quad''\quad)\,\omega$$

conclusion ⟹ we get same result to conserve any Point.

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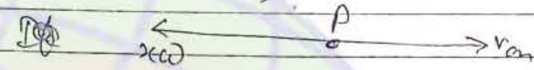
In the above question find the point 'p' whose velocity is zero after collision from $(C_1)' \rightarrow$



$v_{cm} = x\omega$

$x = \frac{v_{cm}}{\omega} = \cancel{\frac{u/2 \times 11l}{12u}} \Rightarrow \cancel{\frac{11l}{24}}$

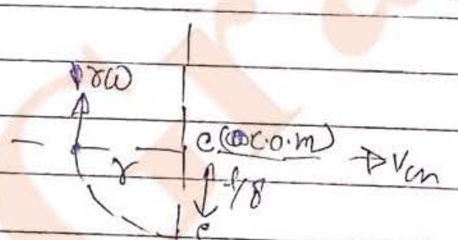
$= \frac{u/2 \times 11l}{12u} = \frac{11l}{24}$

In the above question find the time taken by the point 'c' to rotate through an angle '$\pi/2$'. At the same instance what is the speed of the particle

A. $\frac{\pi}{2} = \theta = \omega t + \cancel{\frac{1}{2}\alpha t^2} \rightarrow 0$

$t = \frac{\pi}{2\omega}$

$= \cancel{\frac{\pi \times 11l}{2 \times 12u}} \Rightarrow \frac{\pi \times 11l}{2 \times 12u}$



speed $= v_c = \sqrt{v_{cm}^2 + (r\omega)^2}$

$= \sqrt{\left(\frac{u}{2}\right)^2 + \left(\frac{l}{8} \times \frac{12u}{11l}\right)^2}$

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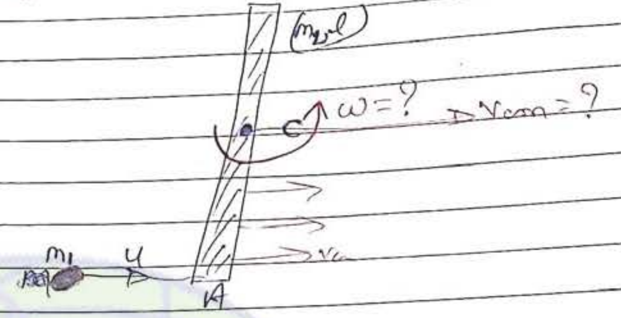
Eccentric collision (collision b/w particle and Rigid Body)

Ex



Rod and particle are on smooth horizontal surface. Rod is initially at rest before collision. Just after collision particles comes to rest. And the collision b/w the particle and rod is elastic.

1) Find the ratio of $\frac{m_1}{m_2} = ?$

ii). Find the Angular velocity of the rod just after collision.

Ans:- C.O.A.M about 'C'.

$$0_{rod} + m_1 u \times \frac{l}{2} = 0 + I_{cm}\omega + 0$$

$$m_1 u \times \frac{l}{2} = \frac{m_2 l^2}{12}\cdot\omega$$

$$\omega = \frac{6 m_1 u}{m_2 l}$$

$$l\omega = \frac{6 m_1}{m_2} \quad \text{---(1)}$$

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From C.O.L.M

$$m_1 u = m_1 \times 0 + m_2 V_{cm}$$

$$V_{cm} = \frac{m_1 \cdot u}{m_2} \quad \text{—— (2)}$$

$$e = 1 = \frac{V_A - 0}{u}$$

$$1 = \frac{\frac{l}{2}\omega + V_{cm}}{u}$$

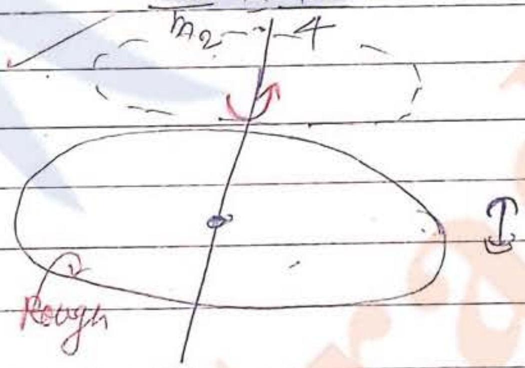
$$V_{cm} + \frac{l\omega}{2} = u \quad \text{—— (3)}$$

From eq (I), (II) and (III)

$$\frac{m_1 u}{m_2} + \frac{\overset{3}{\cancel{6}} m_1 u}{m_2 \, \cancel{2}} = u$$

$$\frac{m_1}{m_2} = \frac{1}{4}$$

Ex:-



Another Identical disk is co-Initially placed on the first disk. Find the common Angular velocity of the disk. (surfaces are rough.)

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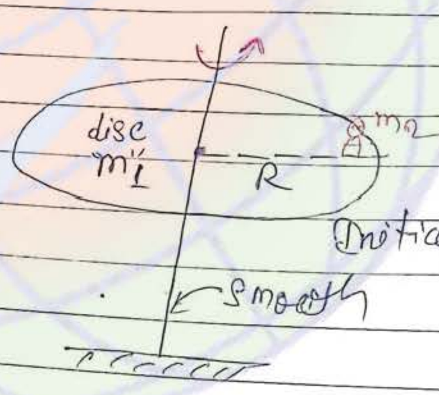
Ans.

$$I\omega + 0 = (2I)\omega_{com}$$

$$\omega_c = \frac{\omega}{2}$$

Find the time to achive common Acc^n.

Ex.



Initial Angula velocity $= \omega_0$.
Initially circula (disc and boy) system rotate with Angula velocity (ω_0) in Anticlockwise direction.

Now the boy starts running Along the circumference of the disc with velocity 'V' Relative to the disk (or circula platform) in

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Qs 1, 2, 3, 5, 6, 8, 9, 10, 13, 14, 16, 17, 18, 19, passage Qs 4, 1, 4, 5, 6, 7, 9, 10, 11, 12, 13, 16, 18, Page No. Date / /

the Same direction (Anti-clockwise).
Find the new Angular velocity of the disk w.r.t. to ground.

Ans.

$$I = \frac{m_1 R^2}{2}$$

Disc

$$(I + m_2 R^2)\omega_0 = I\omega + m_2 R^2\left(\omega + \frac{V}{R}\right)$$

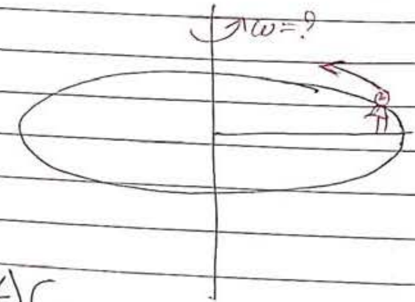
$$(I + m_2 R^2)\omega_0 = \omega(I + m_2 R^2) + m_2 R^2 \cdot \frac{V}{R}$$

$$\boxed{\omega = \omega_0 - \frac{m_2 R^{\cancel{2}} \cdot \frac{V}{\cancel{R}}}{(I + m_2 R^2)}}$$

$\vec{\omega}_{B,D} = \vec{\omega}_{B,g} - \vec{\omega}_{D,g}$

$\vec{\omega}_{B,g} = \vec{\omega}_{B,D} + \vec{\omega}_{D,g}$

$\vec{\omega}_{rel} = \frac{V_{rel}}{R}$



where,

$$I = \frac{m_1 R^2}{2}$$

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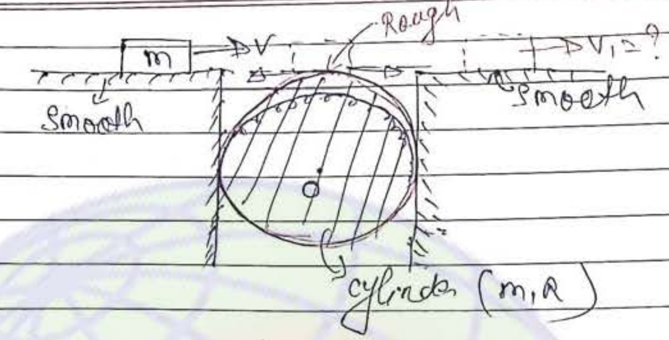
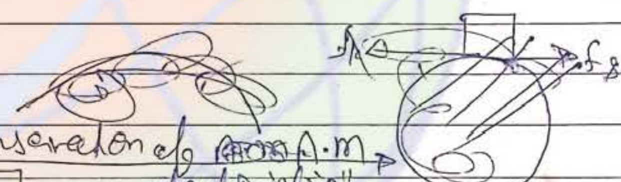
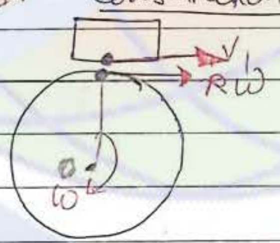


Finally the slipping b/w block and the cylinder is restricted and the block is seperated from the cylinder. Find the velocity of the block 'V_1' on the second surface.

Initially the cylinder was at rest.



method I: $\Rightarrow$ conservation of A.M about point "O".



slipping is when ceased than:-

$V_1 = R\omega$

$$mVR = \frac{m_1R^2}{2}\omega + mV_1R$$

$$mVR = \frac{m_1R}{2}\cdot V_1 + mV_1R$$

$$V_1 = \frac{mVR}{\left(\frac{m_1R}{2} + mR\right)}$$

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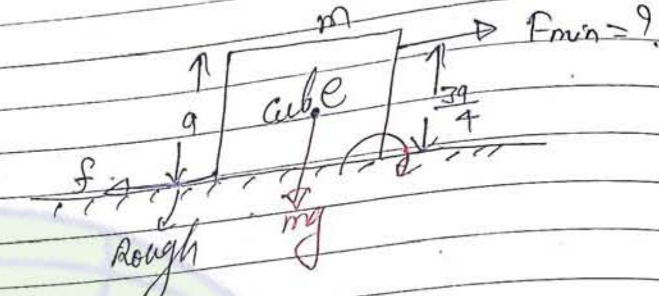
Q:

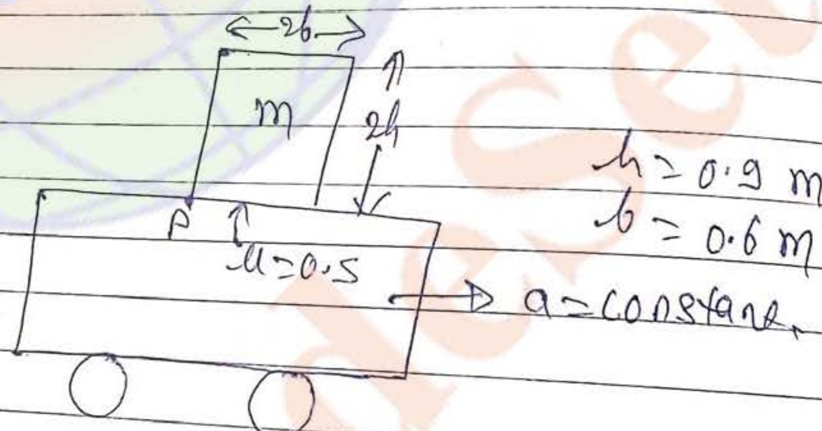


$$\tau_F > \tau_{mg}$$

$$F \times \frac{3a}{4} > mg\,\frac{a}{2}$$

$$\boxed{F > \frac{2mg}{3}}$$ → formula नहीं है।

Q:



$h = 0.9$ m

$b = 0.6$ m

i) Find the maximum ~~minimum~~ accⁿ of the truck so that the block does not slip with the surface of truck

ii) Find the minimum value of accⁿ of truck so that the block topples about point "P".

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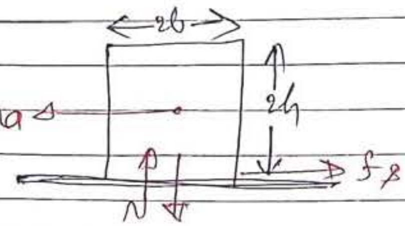
Ans



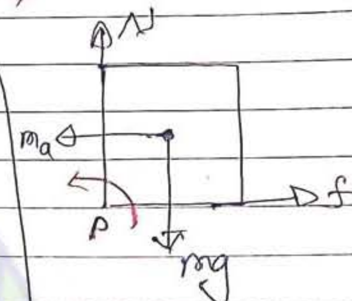
$ma > (f_s)_{max}$

$ma > f_L$

$ma > \mu mg$

$a > \mu g$

$a > 5\ m/s^2$



$\tau_{c.w} > \tau_{A.c.w}$

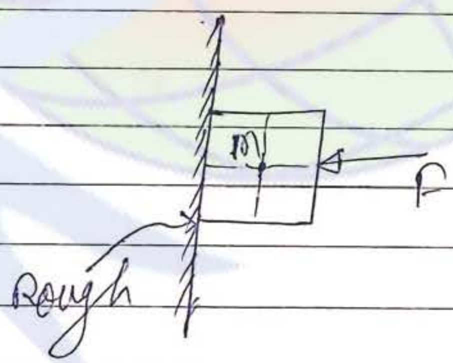
$ma \times h > mg\,b$

$a > \dfrac{gb}{h}$

$a > \dfrac{10 \times 0.6}{0.9}$

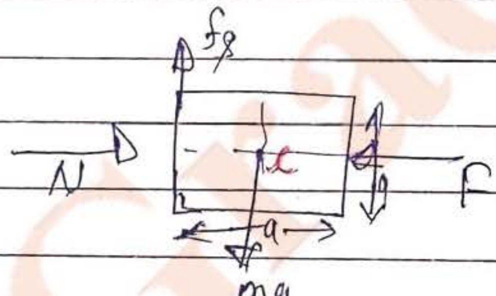
IIT (T)

Q! →



Block is in equillibria.

$(\vec{\tau}_{net}) = 0,\ F_{net} = 0$



$f_s = mg$

$\tau_{f_s} = mg \times \dfrac{a}{2}$

$\tau_{mg} = 0$

$\tau_F = 0$

$\tau_{f_s} \neq 0$

$\tau_f + \tau_N = 0$

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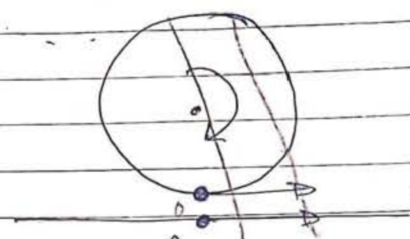
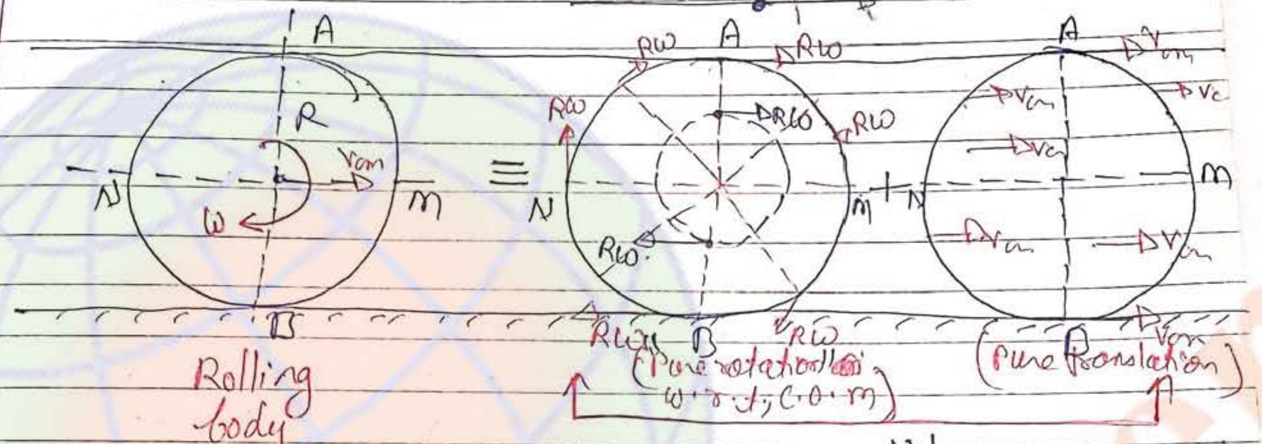
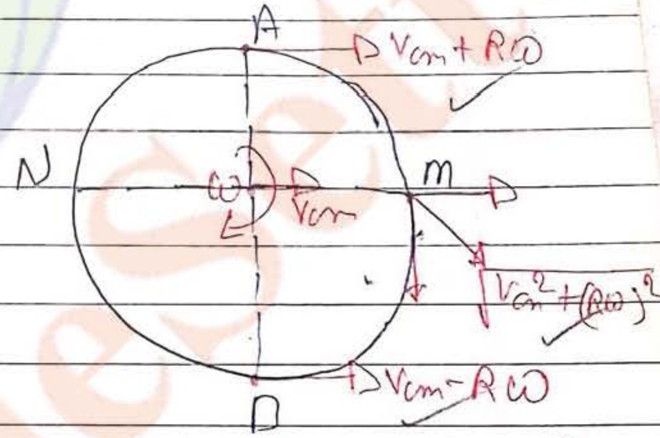
Rolling motion

↳ Rotation + Translation.





||| (superpose)



(Rolling body)

(Attention V.V.I)

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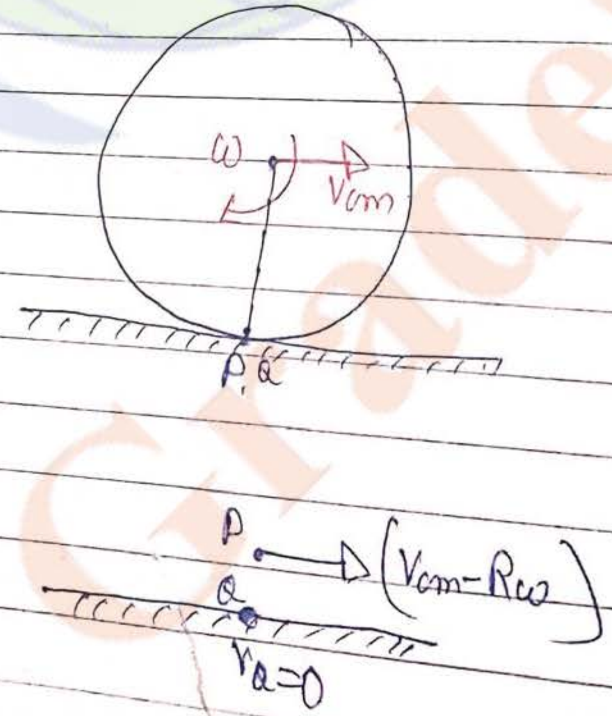
Condition of Pure rolling (Rolling without slipping/sliding)

1.) velocity of contact point of rolling body relative to the contact point of contact surface is zero.
There is no slipping at the contact point.

2.) Acc of contact point of rolling body along the contact surface and "relative" to contact surface is "zero".
V.V.I

1.) Pure Rolling on Stationary horizontal surface:



$V_{cm} - R\omega = 0$

$$\boxed{V_{cm} = R\omega}$$

or

$$\frac{dv}{dt} = R\frac{d\omega}{dt}$$

$$\boxed{a = \alpha R}$$

2.) Pure ro

Note

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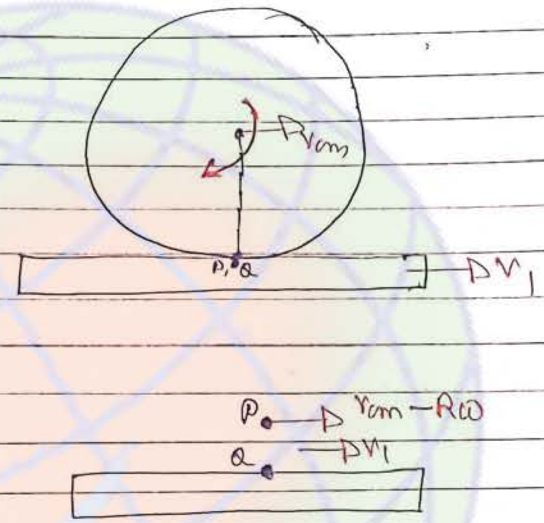
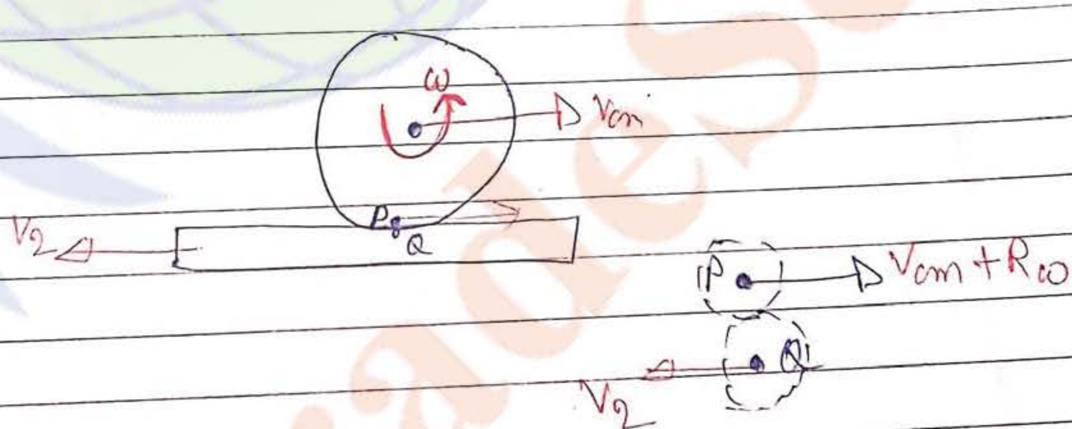
2) Pure rolling on moving horizontal surface ⇒



$V_P = V_Q$

$$\boxed{v_{cm} - R\omega = V_1}$$

Note :⇒



$$\boxed{V_{cm} + R\omega = -V_2}$$

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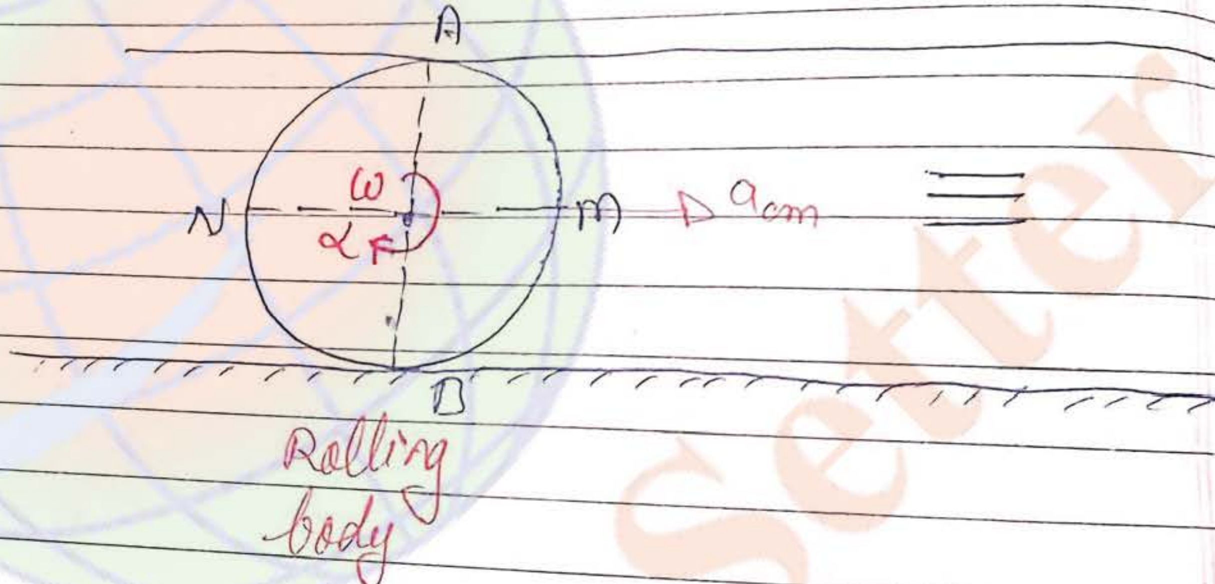
i) Uniform Pure Rolling :→

→ $\vec{V}_{cm} = \text{Constant}$

$\vec{a}_{cm} = \vec{0}$

ii) Non-Uniform / accelerated Pure rolling :→



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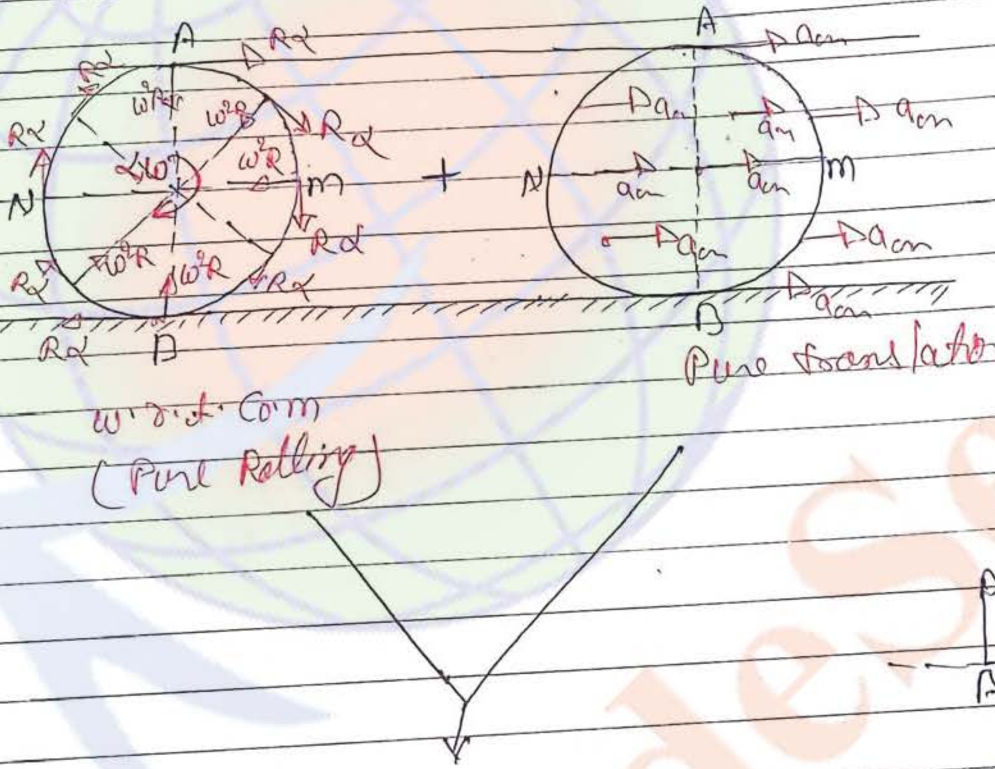
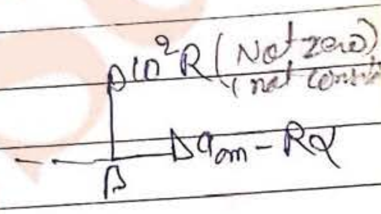
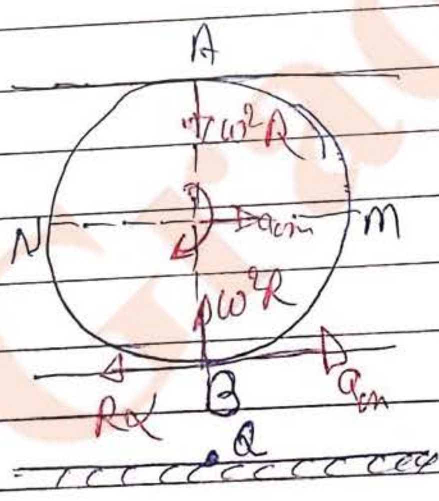




No slipping
(Pure rolling)



$a_{cm} - R\alpha = 0$

$\boxed{a_{cm} = R\alpha}$

$a_Q = 0$

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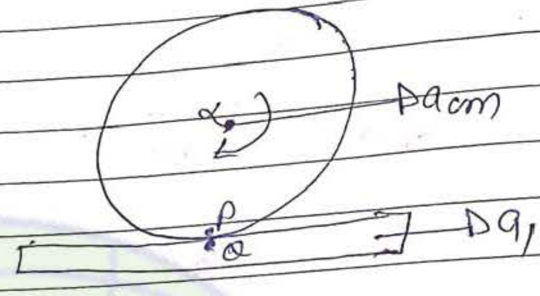
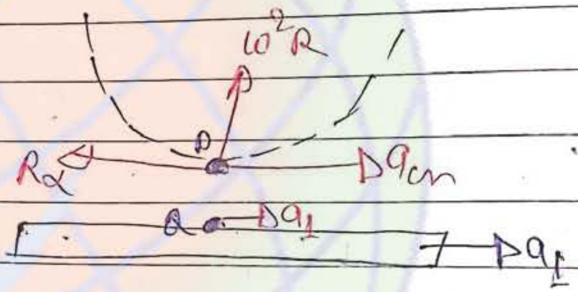
[Q]



Find the condition for no slipping



$$a_Q = a_1$$

$$a_{cm} - R\alpha = a_1$$

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7/12/2012

$K.E = \frac{1}{2}(mv^2 + I_{cm})\omega^2$

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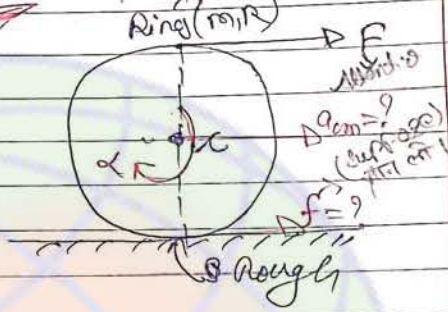
* Direction of friction ⇒

Imp. method.

1.) 1st method :⇒

Hollow cylinder (same Ans)

Ring (m, R)



Note :→ (Torque-force method)

* Step :→

1) $\boxed{\tau_{cm} = I_{cm}\alpha}$

2.) $\boxed{F_{net} = Ma_{cm}}$

3.) Condition of no slipping at all contacts or "constraint relation".

Find the value of friction force at the contact point of ring.

Find the acc^n of C.O.M of ring.

Step-1 :→

$$(F\cancel{R} - f\cancel{R}) = mR^{\cancel{2}}\alpha \quad (\text{At centre})$$

$$F - f = m(R\alpha) \quad \text{— (1)}$$

Step 2 :→

$$F + f = ma_{cm} \quad \text{— (2)}$$

Step 3 :→

$$a_{cm} = R\alpha \quad \text{— (3)} \quad (\text{Pure rolling}) \ (\text{At contact point})$$

From eq (1) and eq (II)

$$F - f = ma_{cm}$$
$$F + f = ma_{cm}$$
$$\cancel{2F} = \cancel{2}ma_{cm}$$
$$F = ma_{cm}$$

$$\cancel{F} - f = \cancel{M}a_{cm}$$
$$\cancel{F} + f = \cancel{M}a_{cm}$$

$$\boxed{f = 0, \quad a_{cm} = \frac{F}{m}}$$

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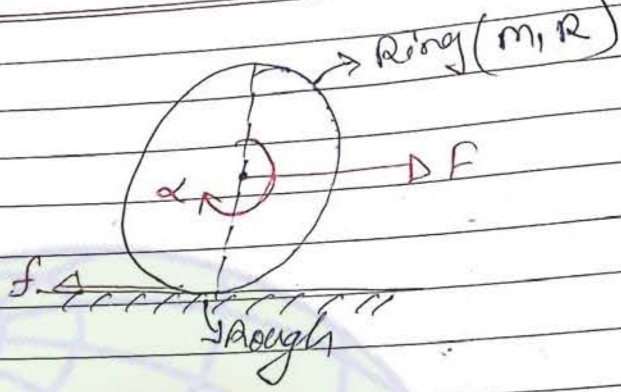
1st Choice

Sol:→



Step I :→

$$f \times R = mR^2\alpha \quad (\text{At centre})$$

$$f = m(R\alpha) \quad \text{—} (1)$$

$a_{cm} = R\alpha$

Step II:

$$F - f = m\, a_{cm} \quad \text{—} (2)$$

Now

$$-f = -m\, a_{cm} \quad \text{—} (1)$$

$$f = \frac{F}{2}$$

$$\boxed{a_{cm} = \frac{F}{2m}}$$

→ +ve Answer आया मतलब दिशा सही है।

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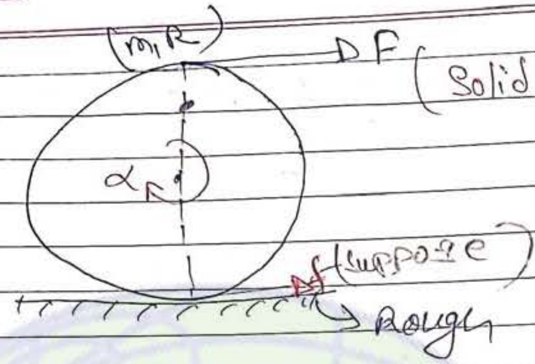
Sol:>



$\beta = \frac{2}{5}$

Find the acc^n of c.o.m and ~~force~~ direction value of friction.

Step I :>

$$F\cancel{R} - f \times \cancel{R} = \frac{2}{5} m R^2 \alpha$$

$$F - f = \frac{2}{5} m (\underbrace{R\alpha}_{a_{cm}}) \quad \text{—(1)}$$

Step III :>

$$a_{cm} = R\alpha$$

Step II :>

$$F + f = m a_{cm} \quad \text{—(2)}$$

Now,

$$F - f = \frac{2}{5} m a_{cm} \quad \text{—(1)}$$

$$2f = m a_{cm} - \frac{2}{5} m a_{cm}$$

$$2f = \frac{5 m a_{cm} - 2 m a_{cm}}{5}$$

$$2f = \frac{3 m a_{cm}}{5}$$

$$f = \frac{3 m a_{cm}}{10}$$

$$f = \frac{3(F+f)}{10} \quad \text{from eq (2)}$$

$$\boxed{f = \frac{3F}{7}}$$

Now :>

Add eq (1) and (2)

$$F + f = m a_{cm}$$

$$F - f = \frac{2}{5} m a_{cm}$$

$$2F = m a_{cm} + \frac{2}{5} m a_{cm}$$

$$2F = \frac{7 m a_{cm}}{5}$$

$$\boxed{a_{cm} = \frac{10F}{7m}}$$

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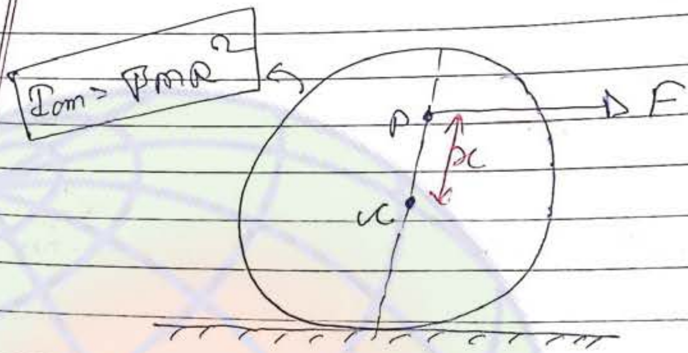
Easy method है third method की तुलना में |

2nd method to find direction:⇒ (short cut)



Step:→

I.) If $x = \beta R$, ($f = 0$)

II) If F(horizontal force) is applied above point "P" ⇒ friction is forward (Along F)

III) If F is applied below point "P". ⇒ friction is backward.

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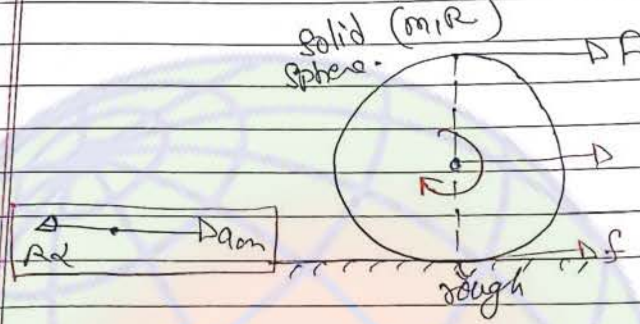
1st Choice

V.V.I → Note :→ Forward means in the direction of force applied and Backward means opposite to the direction of force Applied.

3rd method ⇒



V.V.I

Note :⇒ Assume the contact surface is "smooth"

$$a_{cm} = \frac{F}{m} \quad \text{—— (I)}$$

Now: –

$$F \times R = \frac{2}{5} m R^2 \alpha$$

$$R\alpha = \frac{5}{2} \frac{F}{m} \quad \text{—— (II)}$$

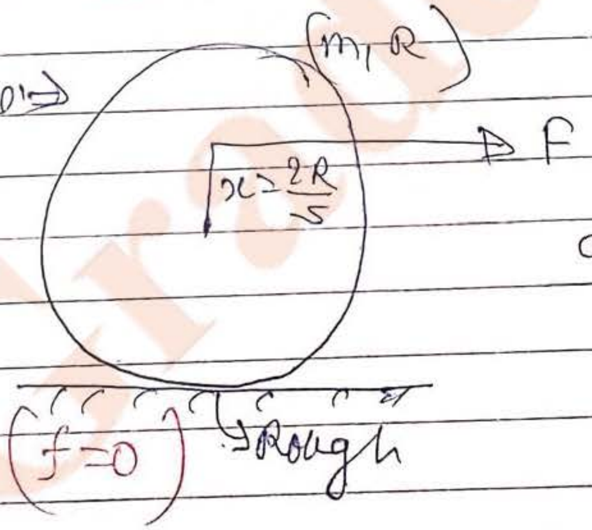
→ यह बता है इससे पुरा करने के लिए forward direction.

$$R\alpha = a_{cm}$$

$$\cancel{a_{cm} = \frac{5}{2} \frac{F}{m}}$$

Hence ⇒ Concept ⇒ From eq (I) and (II) we see that II is greater than Ist eq^n So, to balance this friction forces comes is in forward direction.

Ex ⇒ Solid sphere ⇒



$$a_{cm} = \frac{F}{m}$$

$$F \times \frac{2R}{5} = \frac{2}{5} m R^2 \alpha$$

$$R\alpha = \frac{F}{m} = a_{cm}$$

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(Pure rolling में $\vec{V}_{cm}$ const. है if $f=0$)

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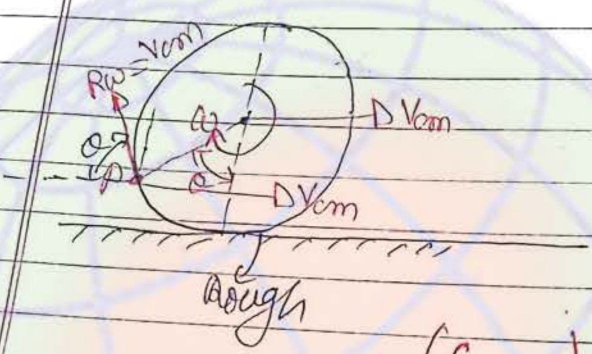
Uniform Pure rolling ⇒

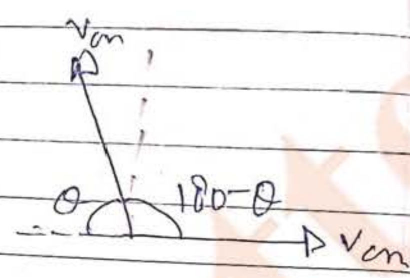
$$\left(\vec{V}_{cm} = \text{constant}\right)$$



$(f=0)$

Find $|\vec{V}_P| = V_P$

In pure rolling (At contact point)
$V_{cm} = R\omega$



$$V_P = \sqrt{V_{cm}^2 + V_{cm}^2 + 2V_{cm}^2\cos(180^\circ-\theta)}$$

$$\boxed{V_P = 2V_{cm}\sin\frac{\theta}{2}}$$ (T)

~~Find the vel~~

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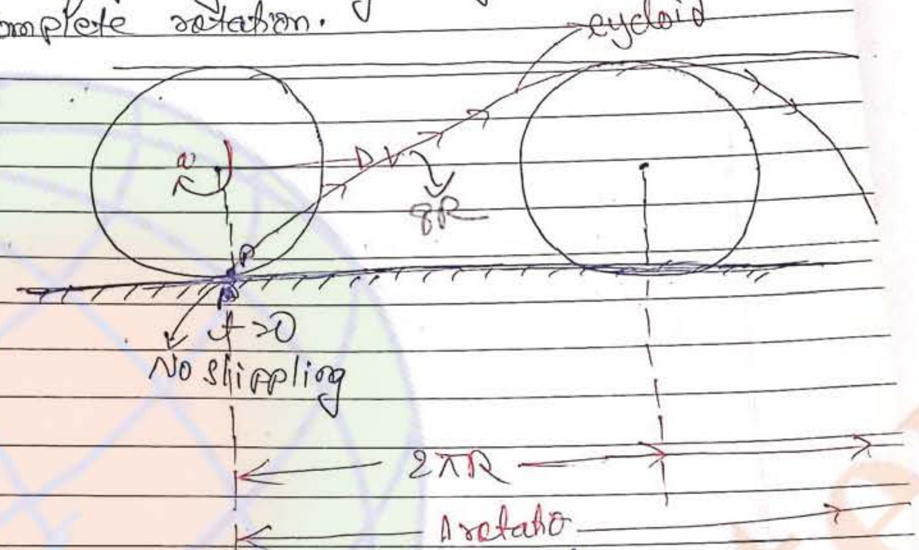
Q. Find the distance travelled by point 'P' on the circumference of purely rolling body. ~~ ~~ In one complete rotation.



Contact Point

Time for 1 Rotation

$$\boxed{T = \frac{2\pi R}{v_{cm}} = \frac{2\pi \cancel{R}}{\cancel{R}\omega} = \frac{2\pi}{\omega}}$$

In time t, $\theta = \omega t$

$$V_P = 2V_{cm} \sin\frac{\theta}{2}$$

$$\frac{ds}{dT} = 2V_{cm} \sin\frac{\omega t}{2}$$

$$s = \int ds = 2V_{cm} \int_{t=0}^{T=2\pi/\omega} \sin\left(\frac{\omega t}{2}\right) dt = 8R$$

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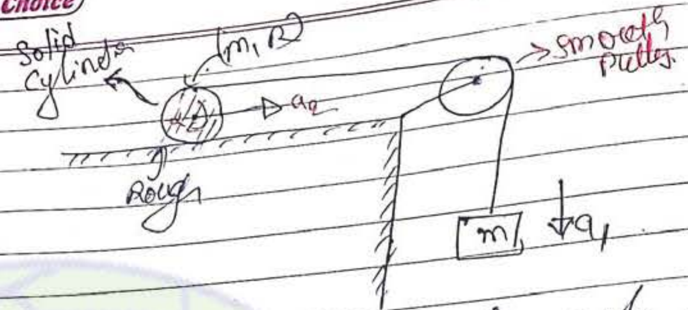
1st Choice

Q.)

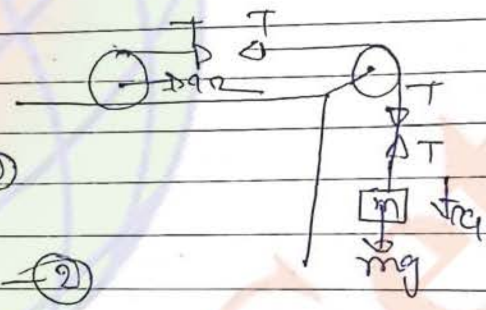


cylinder does not slip on ~~a~~ rough horizontal surface & string does not slip over the cylinder.

1) Find the "a_1" and "a_2".

~~No slipping condition of~~ cylinder.

For cylinder:—



1)

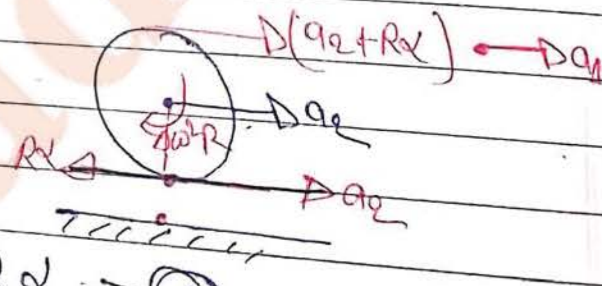
$$T + f = m_1 a_2 \quad \text{—— (1)}$$

$$T \times R - f \times R = \frac{m_1 R^2}{2}\alpha \quad \text{—— (2)}$$

For block:—

$$mg - T = m a_1 \quad \text{—— (3)}$$

No slipping condition:—



$$a_2 = R\alpha \quad \text{—— (4)}$$

so,

$$a_1 = a_2 + R\alpha \quad \text{—— (5)}$$

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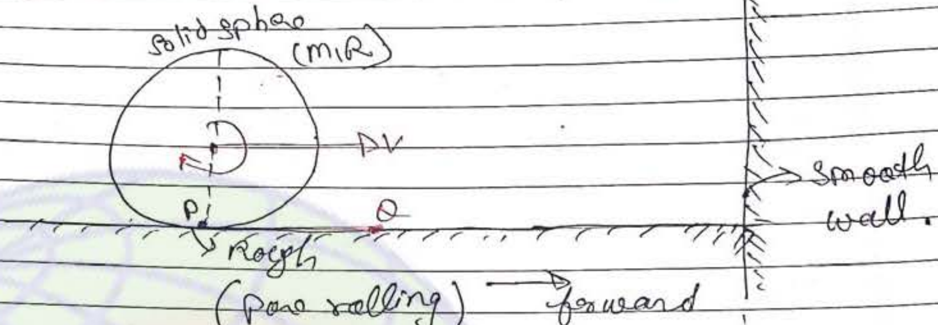
$N \to$ Force
$\tau \to \omega$

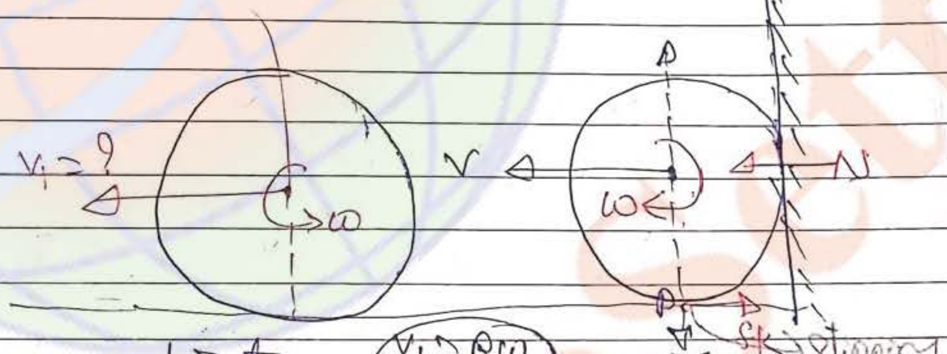
Q)



(Pure rolling) → forward

Collision b/w sphere and the wall is perfectly elastic.

i) Find the velocity of c.o.m of sphere at the time/moment ~~which~~ when the pure rolling starts in backward direction.



$t = t$ $\quad$ $V_1 = R\omega$

Pure rolling (in backward direction)

J.A.C

Conserving A.M, just after the sphere J.A.C and upto the start of pure rolling about point of contact 'P'.

$$\cancel{m}V\cancel{R} - \frac{2}{5}\cancel{m}R^{\cancel{2}} \times \frac{V}{\cancel{R}} = \frac{2}{5}\cancel{m}R^{\cancel{2}} \times \underbrace{\frac{V_1}{\cancel{R}}}_{\omega_1} + \cancel{m}V_1\cancel{R}$$

$$\boxed{V_1 = \frac{3V}{7}}$$

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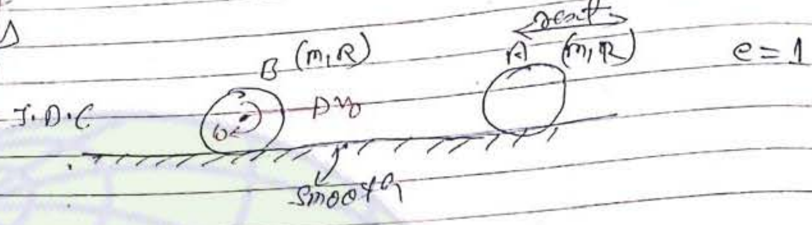
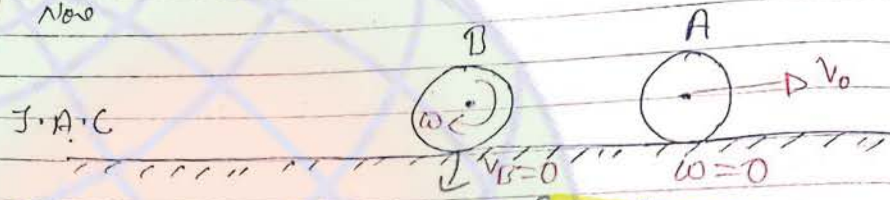
II-I (Q)

Collision Problem



Now

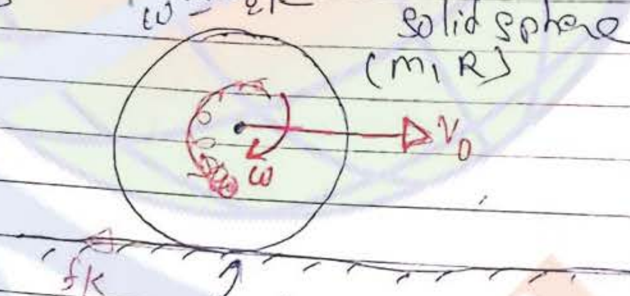


torque नहीं मिलने के कारण
उसी स्थान पर constant
से घूमता रहेगा।

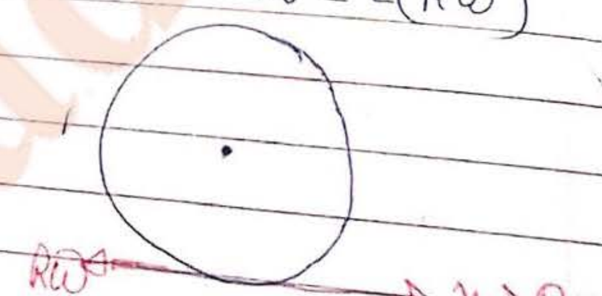
Angular velocity (ω) is only affected by torque (τ).

Eg!

$\omega = \frac{v_0}{2R}$ → (given)



$v_0 = 2(R\omega)$



$\boxed{v_{cm} > R\omega \rightarrow \text{condition for forward slipping}}$

$v_{cm} < R\omega \rightarrow$ condition for backward slipping

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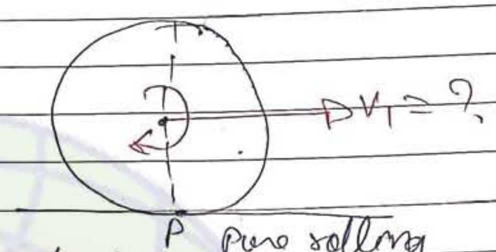
Find the velocity of c.o.m of sphere at the moment when pure rolling starts.



$v_1 = R\omega_1$

find.

$\omega_1 = \frac{v_1}{R}$

$$\frac{2}{5} \cancel{m}\cancel{R^2} \times \frac{v_0}{2\cancel{R}} + \cancel{m}v_0\cancel{R} = \frac{2}{5}\cancel{m}\cancel{R^2} \times \left(\frac{v_1}{\cancel{R}}\right) + \cancel{m}v_1\cancel{R}$$

$$\boxed{v_1 = \frac{6v_0}{7}}$$

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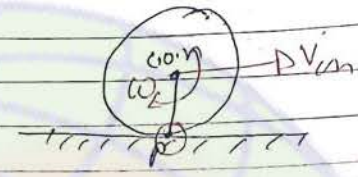
Kinetic Energy of rolling body without slipping :⇒



$$K_{roll} = \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

$$= \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I_{cm} \left(\frac{V_{cm}^2}{R^2}\right)$$

$$= \frac{1}{2} m R^2 \omega^2 + \frac{1}{2} I_{cm} \omega^2$$

$$K_{roll} = \frac{1}{2} (I_{cm} + m R^2) \omega^2$$

$$= \frac{1}{2} I_P \omega^2 \quad \Big| \quad I_P = I_{cm} + m R^2$$

so

$$\boxed{K_{roll} = \frac{1}{2} (I_{cm} + m R^2) \omega^2 = \frac{1}{2} I_P \omega^2}$$

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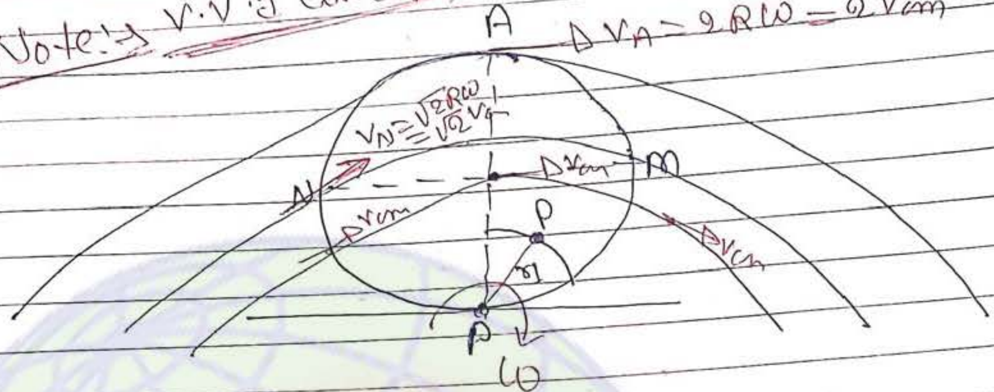
Note:→ V.V.I Concept



एक circle का एक ही
same speed होगा ।

$$\boxed{V_P = r_1\omega}$$

$$\boxed{\omega = \frac{V_P}{r_1} = \frac{V_A}{2R} = \frac{V_m}{\sqrt{2}R}}$$

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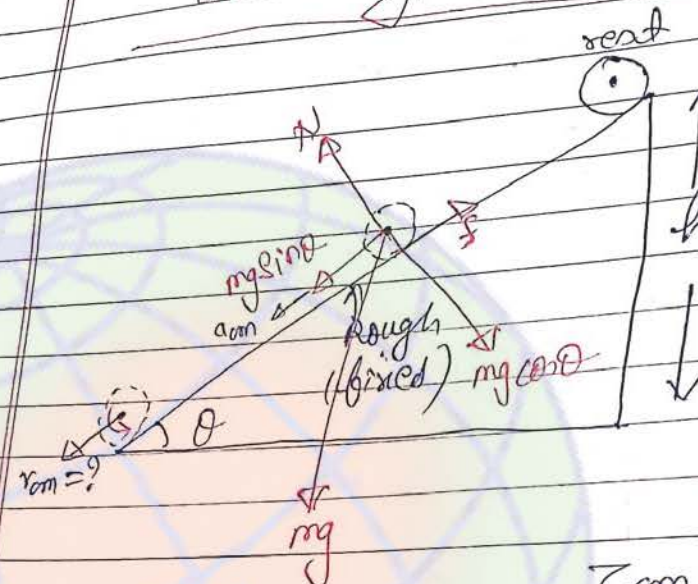
Pure rolling (rolling without slipping) on Rough Inclined Surface.



$$\boxed{I_{cm} = \beta m R^2}$$

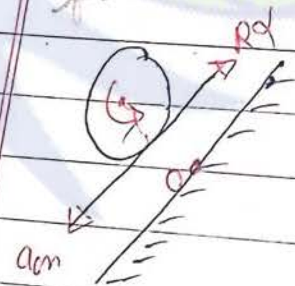
$$\tau_{cm} = I_{cm}\alpha$$

$$f \times \cancel{R} = \beta m R^{\cancel{2}}\alpha$$

$$f = \beta m R\alpha \quad \text{—(1)}$$

$$mg\sin\theta - f = m a_{cm} \quad \text{—(2)}$$

for no slipping:



$$a_{cm} = R\alpha \quad \text{—(3)}$$

$$\boxed{a_{cm} = \frac{g\sin\theta}{(1+\beta)}}$$

$$\boxed{f = \frac{\beta mg\sin\theta}{(1+\beta)}}$$

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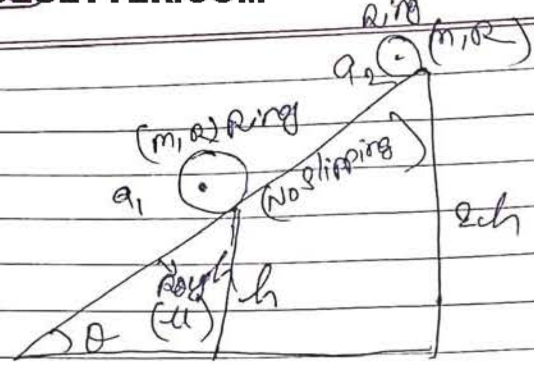
Ex:-

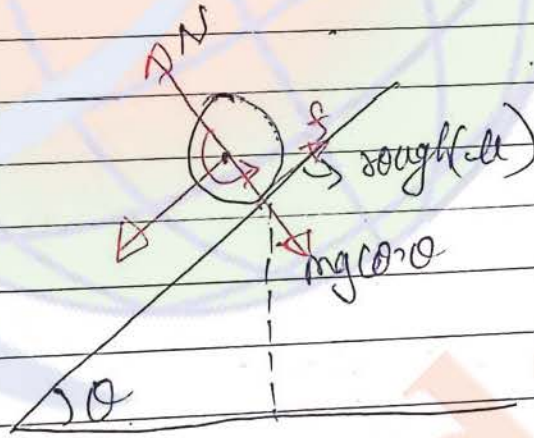


$$\boxed{a_{cm} = \frac{g \sin\theta}{(1+\beta)}}$$

So, In all the position acc^n is same

$(a_1 = a_2)$

acc^n is depends on Inclination (θ) and 'β'



$$f = \frac{\beta}{(1+\beta)} mg\sin\theta$$

$$f \le f_L$$

$$f \le \mu mg\cos\theta$$

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$$\frac{\beta}{1+\beta} mg\sin\theta \le \mu mg\cos\theta$$

$\Rightarrow$

For pure rolling on fixed rough Inclined Surface $\Rightarrow$ $\boxed{\mu \geq \frac{\beta}{(1+\beta)} \tan\theta}$

$$\boxed{\mu_{min} = \frac{\beta}{(1+\beta)} \tan\theta}$$

Note: $\Rightarrow$

$\boxed{\mu_{min} \quad (\theta = \text{constant})}$

1) Ring (M, R) $\rightarrow \frac{\tan\theta}{2}$

2.) disc (M, R) $\left(\beta = \frac{1}{2}\right)$ $\rightarrow \frac{\tan\theta}{3}$

3.) Solid sphere $\rightarrow \frac{2}{7}\tan\theta$

4.) hollow sphere $\Rightarrow \frac{2}{5}\tan\theta$.

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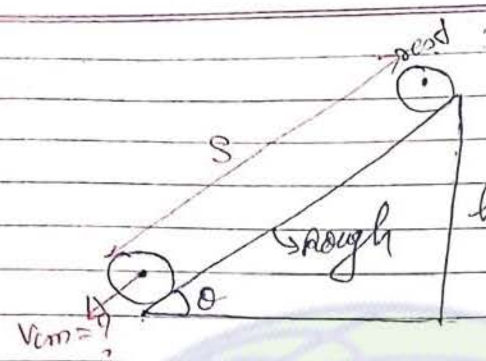

$$v_{cm}^2 = 0^2 + 2a_{cm}\times S$$

$$v_{cm} = \sqrt{2a_{cm}S}$$

$$= \sqrt{\frac{2g\cancel{\sin\theta}\,(h/\cancel{\sin\theta})}{(1+\beta)}}$$

$$\boxed{V_{cm} = \sqrt{\frac{2gh}{(1+\beta)}}}$$

$$S = \frac{1}{2}a_{cm}t^2$$

$$t = \sqrt{\frac{2S}{a_{cm}}}$$

$$= \sqrt{\frac{2h/\sin\theta\,(1+\beta)}{g\sin\theta}}$$

$$\boxed{t = \frac{1}{\sin\theta}\sqrt{\left(\frac{2h}{g}\right)(1+\beta)}}$$

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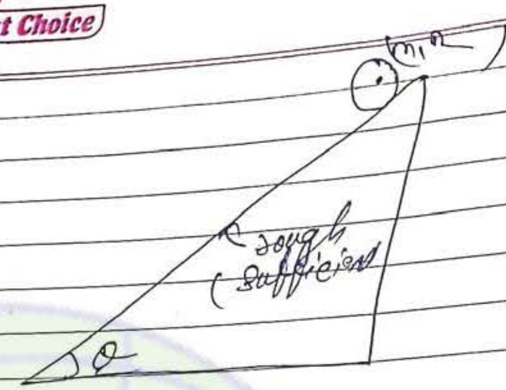
1st Choice

(Q.)

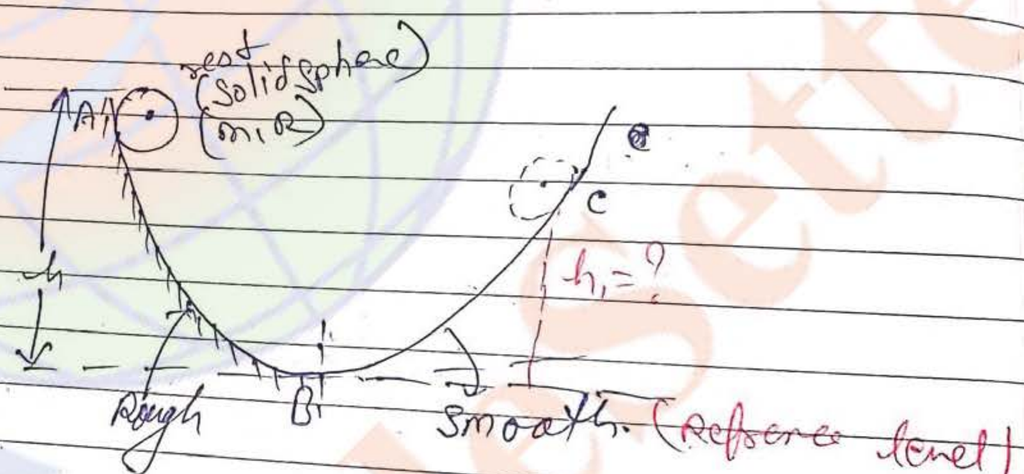


who reach first on the ground, a sphere, a ring and a disk of same (m, R)

Ans.:) Sphere :-

$$\because t = \frac{1}{\sin\theta}\sqrt{\left(\frac{2h}{g}\right)(1+\beta)}$$

IIT
(Q.)



i) Find the ratio translation and Rotation K.E. of sphere at point 'B'.

ii) Find the Rotational K.E. of sphere at point C.

iii) Find the height "h_1".

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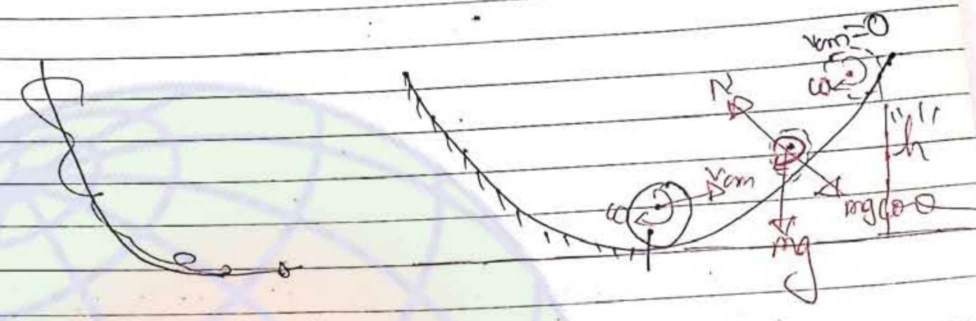
where sphere rolls without slipping in AB part.

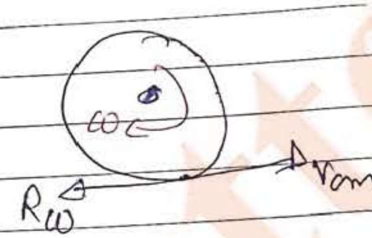
Ans.



In A and B

$$mgh + 0 = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} m v_{cm}^2$$

$$mgh = \frac{1}{2}\cdot\frac{2}{5} mR^2\left(\frac{v_{cm}^2}{R^2}\right) + \frac{1}{2} m v_{cm}^2$$



$$mgh = \frac{7}{10} m v_{cm}^2$$

$$K_T = \frac{1}{2} m v_{cm}^2 = \frac{5}{7}(mgh)$$

translational K.E

$$K_{Rotational} = \frac{1}{2}\cdot\frac{2}{5} mR^2\cdot\frac{v_{cm}^2}{R^2} = \frac{2}{7} mgh$$

Now

At "C"

$$\boxed{\frac{K_T}{K_R} = \frac{5}{2}}$$ → Always their Ratio is Same.

$$K_{rot} = \frac{2mgh}{7}$$

Now → B → C

$\frac{5mgh}{7} = mgh_1$, so, $\boxed{h_1 = \frac{5h}{7}}$

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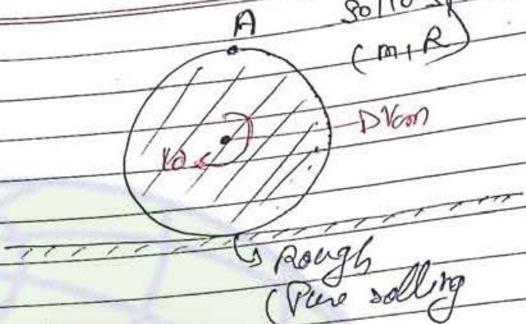
1st Choice

Ex:->



Find the radius of curvature of the path followed by top-most point of sphere.

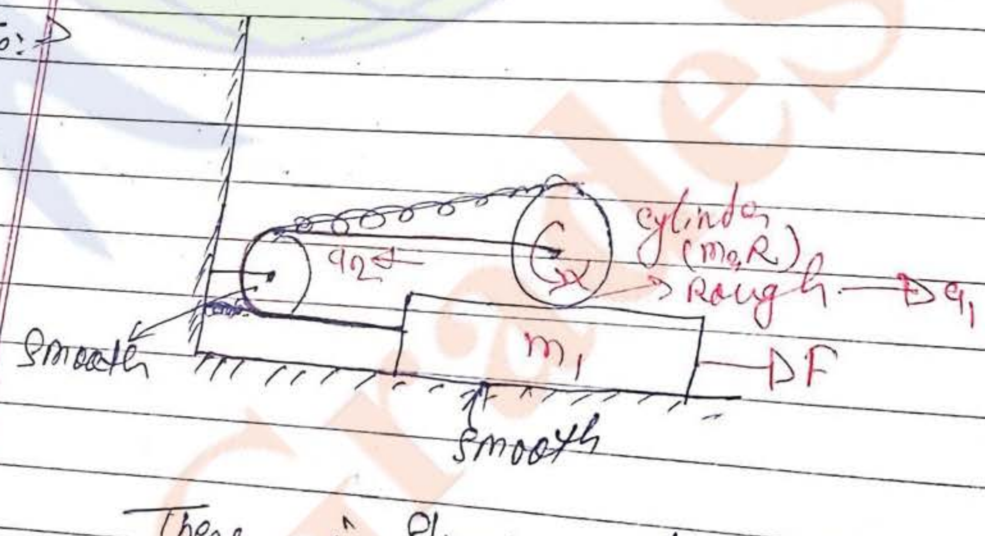
Ans

$$R_1 = \frac{v_A^2}{a_{cm}}$$

$$= \frac{(2v)^2}{(v^2/R)}$$

$$\boxed{R_1 = 4R}$$

Ex:->



There is slipping b/w cylinder and block

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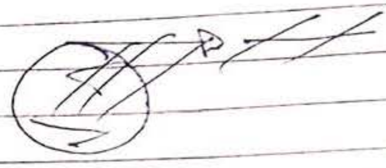
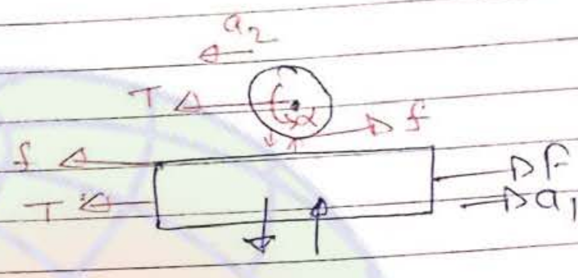
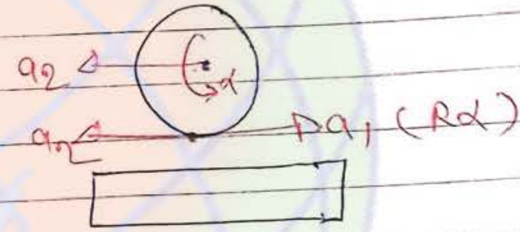
1st Choice

Ans







$\cancel{R\alpha - a_2 = a_2}$ $R\alpha - a_2 = a_1$

$a_1 + a_2 = R\alpha$ — (1)

$a_1 = a_2$ — (2)

$F - (f+T) = m_1 a_1$ — (3)

$T - f = m_2 a_2$ — (4)

$f \times R = \frac{m_2 R^2}{2}\alpha$ — (5)

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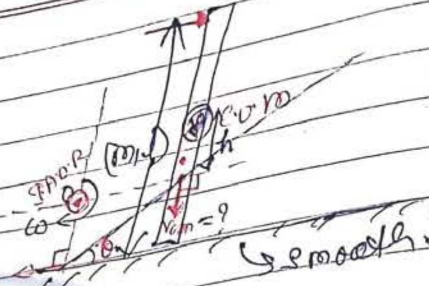
1st Choice

Q.



Rod is slightly disturbed and starts gliding on smooth surface.
Find the Angular velocity of rod when the rod makes an Angle 'θ' with the horizontal.

Ans

$$h = \frac{l}{2} - \frac{l}{2}\sin\theta$$

$$h = \frac{l}{2}(1-\sin\theta)$$

$$mgh = \frac{1}{2} I_0 \omega^2$$

$$I_0 = \frac{ml^2}{12} + m\left(\frac{l}{2}\cos\theta\right)^2$$

$$mg\frac{l}{2}(1-\sin\theta) = \frac{1}{2}\left[\frac{ml^2}{12} + \frac{ml^2}{4}\cos^2\theta\right]\omega^2$$

$$\omega =$$

$$v_{cm} = \frac{l}{2}\cos\theta\,\omega$$ Ans

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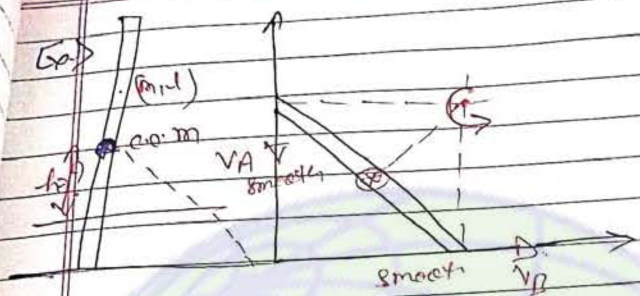
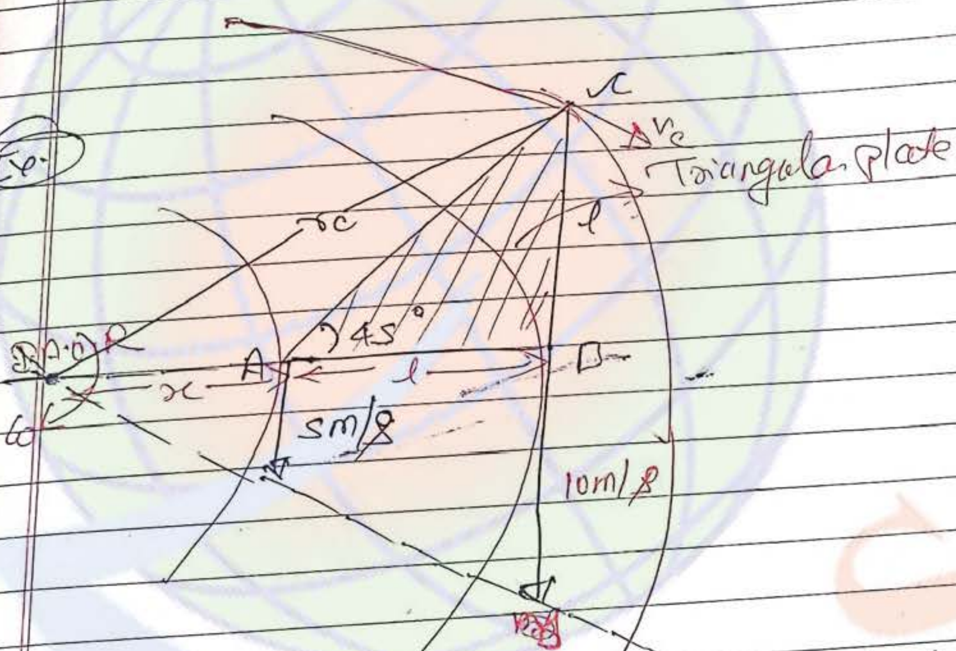
1st Choice

(Ex.)



(Ex.)



Find the velocity of point C of this triangle.

Ans $\omega = \dfrac{V_A}{x} = \dfrac{V_B}{l+x} = \dfrac{V_C}{r_C}$

$\dfrac{5}{x} = \dfrac{10}{l+x}$ 2

$l+x = 2x$

$x = l$

Now! —

$r_C = \sqrt{4l^2 + l^2}$

$= l\sqrt{5}$

Again

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$$\frac{v_A}{x} = \frac{v_e}{r_e}$$

$$\frac{5}{x} = \frac{v_c}{x\sqrt{5}}$$

$$v_c = 5\sqrt{5} \text{ m/s}$$

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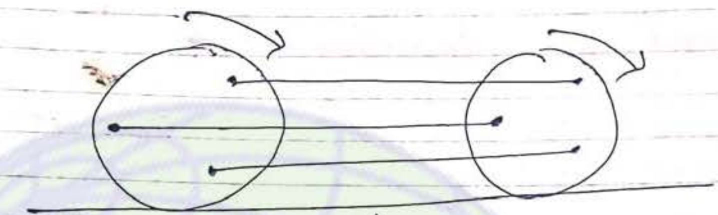
→ math: 8:05 to 9:35

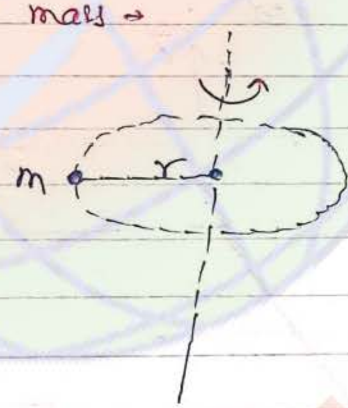
Start

Rotational motion
(Revision class)



Moment of Inertia ⇒

1) For point mass →



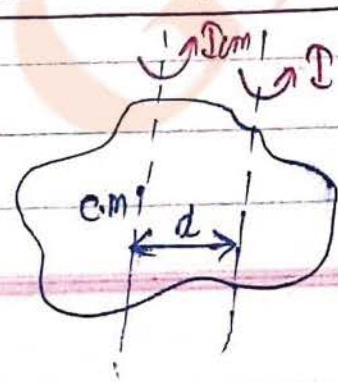
$$\boxed{I = mr^2}$$

i) M.O.I ↑ rolling capacity ↑ and vice versa.

ii) M.O.I is a Tensor quantity

Type of theorem

1) Parallel axis theorem →



$$\boxed{I = I_{cm} + Md^2}$$

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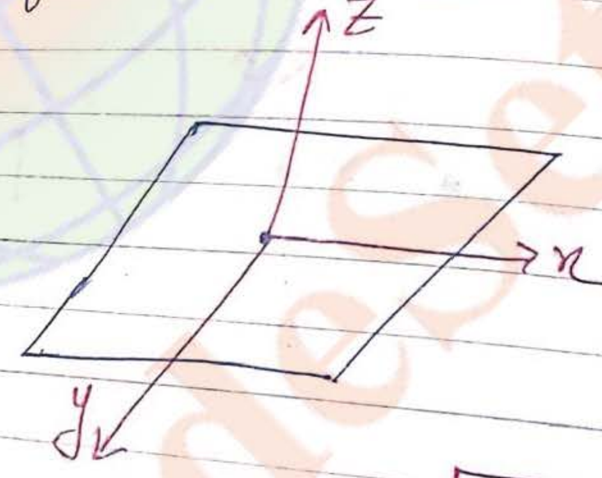
2.) Perpendicular Axis theorem →

i) This theorem is applicable only for planar bodies.

Disc (✓)
Ring (✓)
sphere (✗)

ii) This theorem says the M.O.I of the body about an axis ⊥r to plane of the body is simply the sum of M.O.I of the body about those two axis which are lying plane of the bodies.



$$\boxed{I_Z = I_x + I_y}$$

Concept →

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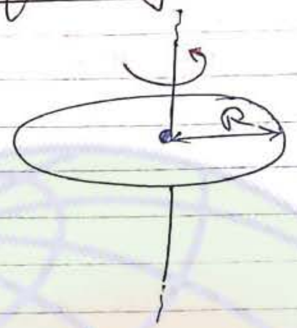
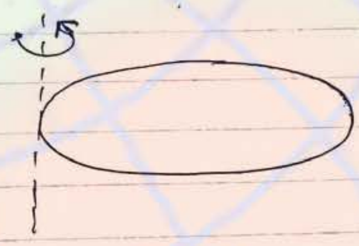
1) M.O.I of Ring

i)



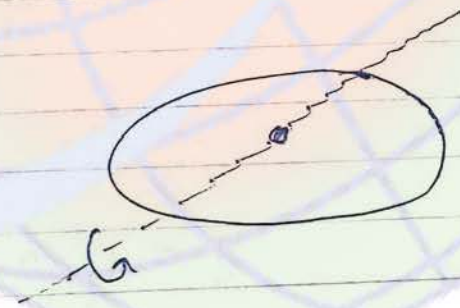
$$\boxed{I = MR^2}$$

ii)



$$\boxed{I = 2MR^2}$$

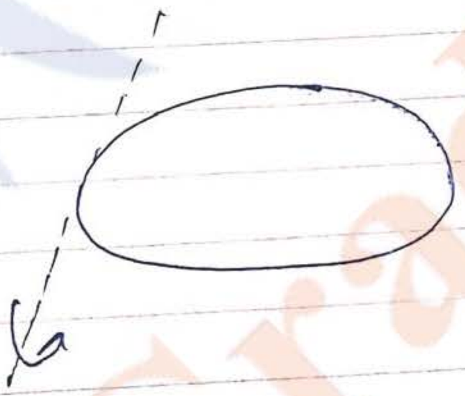
iii)



$$\boxed{I_z = MR^2 = I_x + I_x}$$

$$I_x = \frac{MR^2}{2}$$

iv)



$$I = \frac{MR^2}{2} + MR^2$$

$$= \frac{3}{2} MR^2$$

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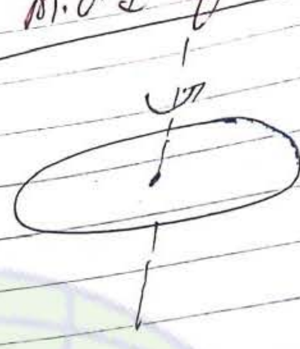
M.O.I of Disc

I)

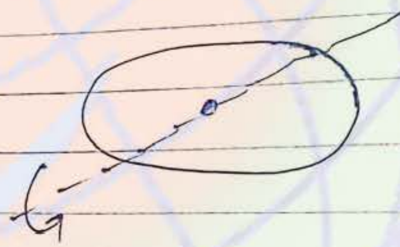


$I = \frac{MR^2}{2}$

II)

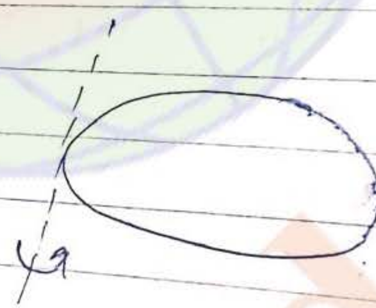
$I = \frac{3}{2}MR^2$, /4

III)



$I_x = \frac{MR^2}{4}$

IV)



$I = \frac{5}{4}MR^2$

Q.2

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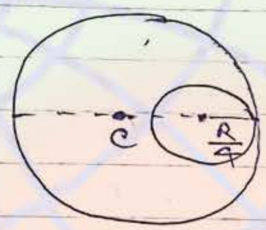
Q1.) From a disc of mass m and Radius R, a smaller disc of radius $\frac{R}{4}$ is Removed find out the M.O.I of Remaining disk, about an axis passing through Centre "C" and $\perp r$ to Plane of the disc.

Sol/n



$$\frac{m\left(\frac{R^2}{16}\right)}{2}$$

$$\Rightarrow \frac{MR^2}{32}$$

$$\frac{MR^2}{2} - \frac{MR^2}{32}$$

$$\Rightarrow \frac{16MR^2 - MR^2}{32} \Rightarrow \frac{15MR^2}{32}$$

M.O.I of original disc,

$$I_1 = \frac{MR^2}{2}$$

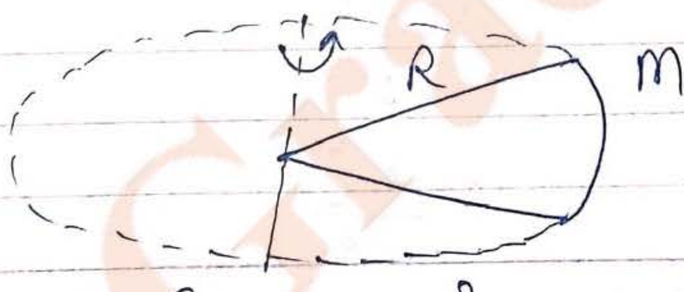
M.O.I of Removed disc, $I_2 = \frac{1}{2}\frac{M}{16}$

$$I_2 = \frac{1}{2}\cdot\frac{M}{16}\cdot\left(\frac{R}{4}\right)^2 + \frac{M}{16}\left(\frac{3R}{4}\right)^2$$

$$= \frac{MR^2}{512} + \frac{9MR^2}{256}$$

$$I_1 - I_2 = \frac{237MR^2}{512}$$

Q.2.)



i) ~~$\frac{MR^2}{2}$~~ ii) $\frac{MR^2}{4}$ iii) $\frac{MR^2}{8}$ iv) $\sqrt{2}MR^2$

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3. Rod

i)

$$I = \frac{ml^2}{12}$$

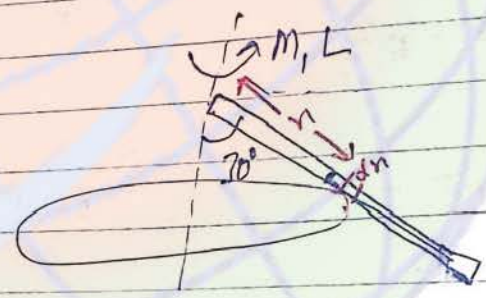
4)

(i)

ii)

$$I = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{3}$$

Q1.



Sol^n

$$dI = \left(\frac{M}{L}dx\right)\left(\frac{x}{2}\right)^2$$

$$= \frac{M}{4L}\int_0^L x^2 dx$$

$$= \frac{ML^2}{12}$$

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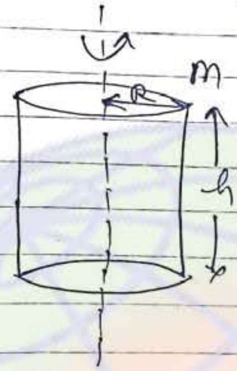
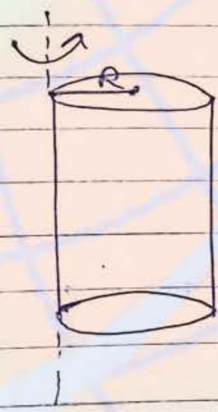
4) ~~Hollow~~ Hollow cylinder

(i)



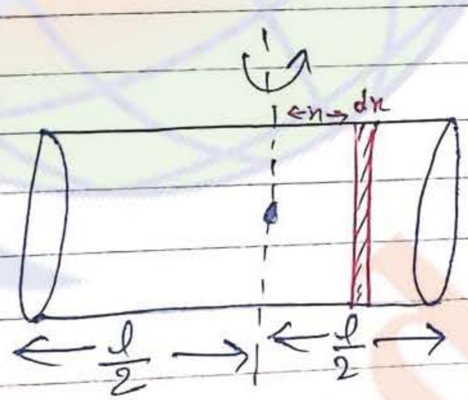
$$\boxed{I = MR^2}$$

ii)



$$\boxed{I = 2MR^2}$$

iii)



$$dM = \frac{M}{l}dx$$

$$= \frac{(dM)R^2}{2} + (dM)x^2$$

$$I = \frac{M}{l}\frac{R^2}{2}\int_{-l/2}^{l/2} dx + \frac{M}{l}\int_{-l/2}^{l/2} x^2 dx$$

so, $I = \frac{MR^2}{2} + \frac{Ml^2}{12}$

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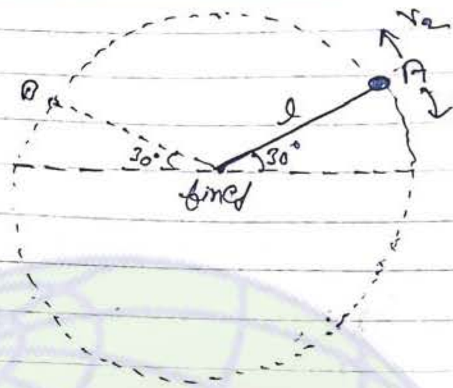
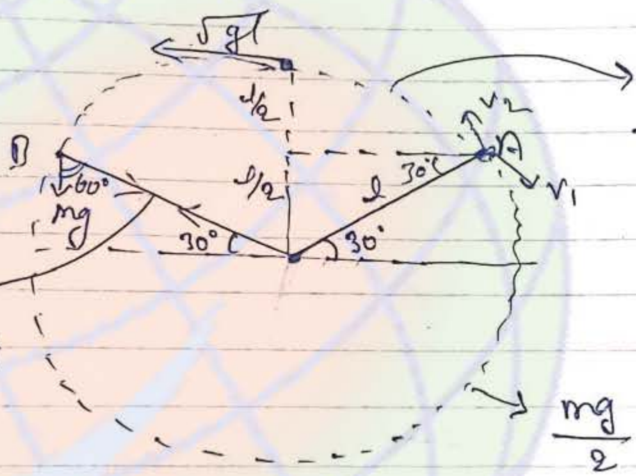
Q.)



In the fig. shown find out the ratio of min. value of v_1 and v_2 so that particle can just strike the point B.

Sol^n



$$\frac{1}{2}mv_2^2 = \frac{1}{2}m(gl) + \frac{mgl}{2}$$

$$v_2 = \sqrt{2gl}$$

Note →
(Here Tension को zero करेंगे)

$$\frac{mg}{2} = \frac{mv_1^2}{l} \Rightarrow v_1 = \sqrt{\frac{gl}{2}}$$

So, $$\frac{v_1}{v_2} = \sqrt{\frac{gl}{2} \times \frac{1}{2gl}} \Rightarrow \sqrt{\frac{1}{4}} = \frac{1}{2}$$

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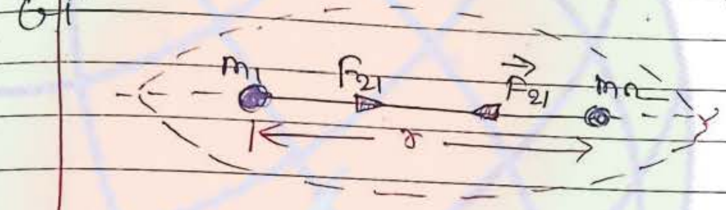
Gravitation

Newton's law of Gravitation :⇒

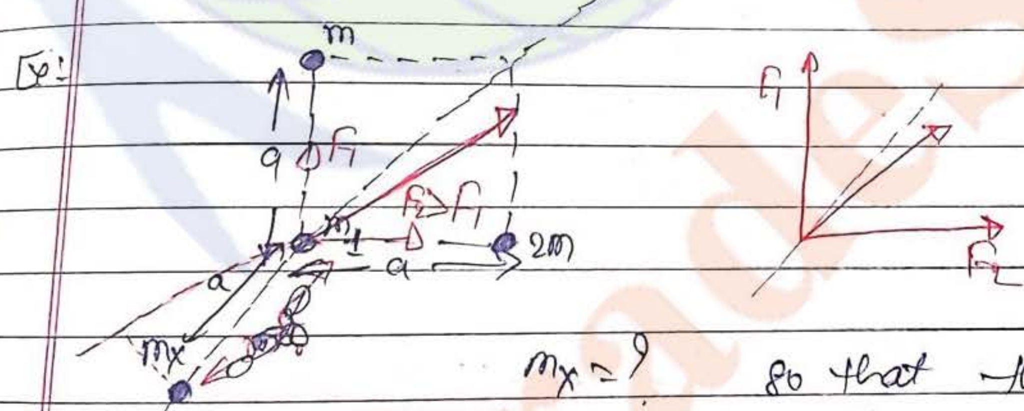
$|\vec{F}_{12}| = |\vec{F}_{21}| \rightarrow \boxed{F = G\dfrac{m_1 m_2}{r^2}}$ ⇒ This is only valid for point object.

$$G = 6.67 \times 10^{-11} \dfrac{Nm^2}{kg^2}$$



⇒ i) Conservative force.
⇒ ii) Action-reaction pair
⇒ iii) Independent of medium.

Q:



$m_x = ?$ so that the $\vec{F}_{net}$ on m_1 is zero.

Ans! ⇒ No value of m_x is possible.

Note! ⇒ Trick to learn Gravitational formula ⇒ i) Capital letter को "1" से denote करें तथा small letter को "2" से denote करें।

SNR! ⇒ यदि trick से याद नहीं रखोगे तो exam में याद रहेगा (g/G) [illegible] इससे तुम confuse कर जाओगे

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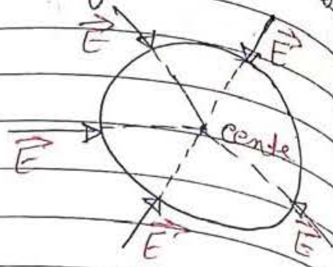
(तीव्रता)

Gravitational field Intensity ($\vec{E}$)

It is defined as the force experienced by unit mass placed at any point in the gravitational field.
~~It is defined as gravitation force~~



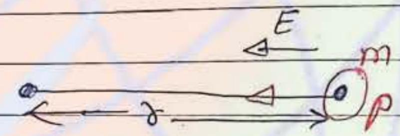
~~It is~~

$$\boxed{\vec{E} = \frac{\vec{F}}{m}}$$

$$\boxed{\vec{F} = m\vec{E}}$$

Force experienced on mass

Gravitational field Intensity due to "m".



$$F = \frac{GMm}{r^2}$$

$$\frac{F}{m} = E = \frac{GM}{r^2}$$

Notes:-

1) ~~Gravitation~~ field Intensity due to point

1) G.F.I always directed towards the centre of gravity of the body whose gravitational field is considered. Intensity of ~~gravity~~ gravitational field at a point is a vector quantity and is denoted by $\vec{E}$.

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1st Choice

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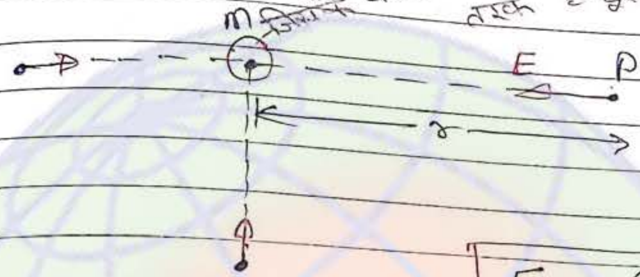
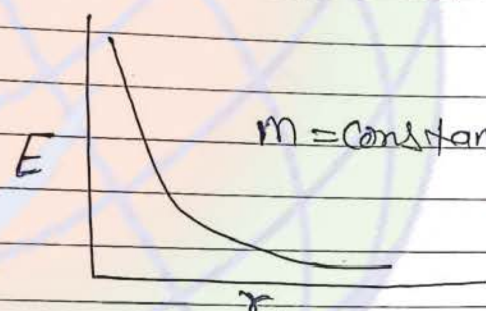
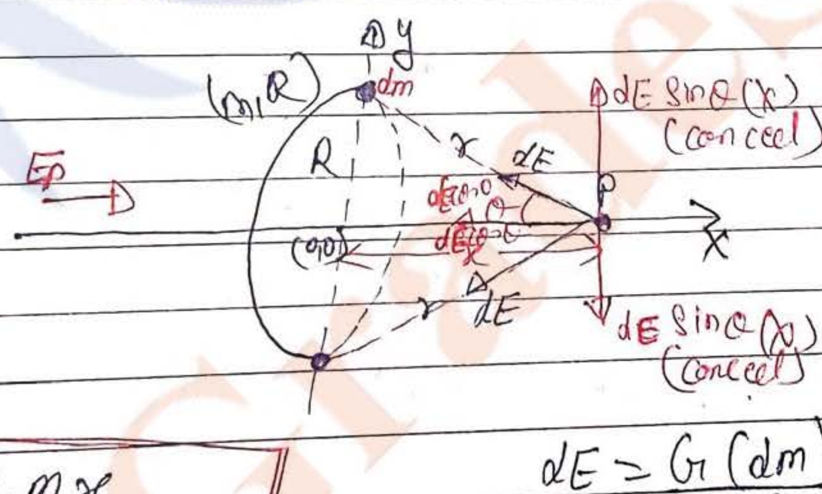
1.) Gravitational field Intensity (at point P) due to point mass (M) :→



$$\boxed{E = \frac{Gm}{r^2}}$$



2) G.F.I due to uniform circular ring (m, R) at any point on its Axis :→



$$dE = \frac{G(dm)}{r^2}$$

$$E_p = \int dE \cos\theta$$

$$\boxed{E_p = \frac{GMx}{(R^2 + x^2)^{3/2}}}$$

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$$= \int \frac{G (dm) \cos\theta}{r^2}$$

$$= \frac{G \cos\theta}{r^2} \int dm$$

$$= \frac{G m \cos\theta}{r^2}$$

Note:⇒

(*) E_P is maximum at $\boxed{x = \pm \frac{R}{2}}$

$$\frac{dE_P}{dx} = 0$$

$$\boxed{(E_P)_{max} = \frac{GM (R/\sqrt{2})}{\left[R^2 + (R/\sqrt{2})^2\right]^{3/2}}}$$

⇒ At the centre of ring

$x = 0$

$$(E_P)_{centre} = 0$$

⇒ If $x >>>>> R^2$

$$\boxed{E_P = \frac{GM}{x^2}}$$

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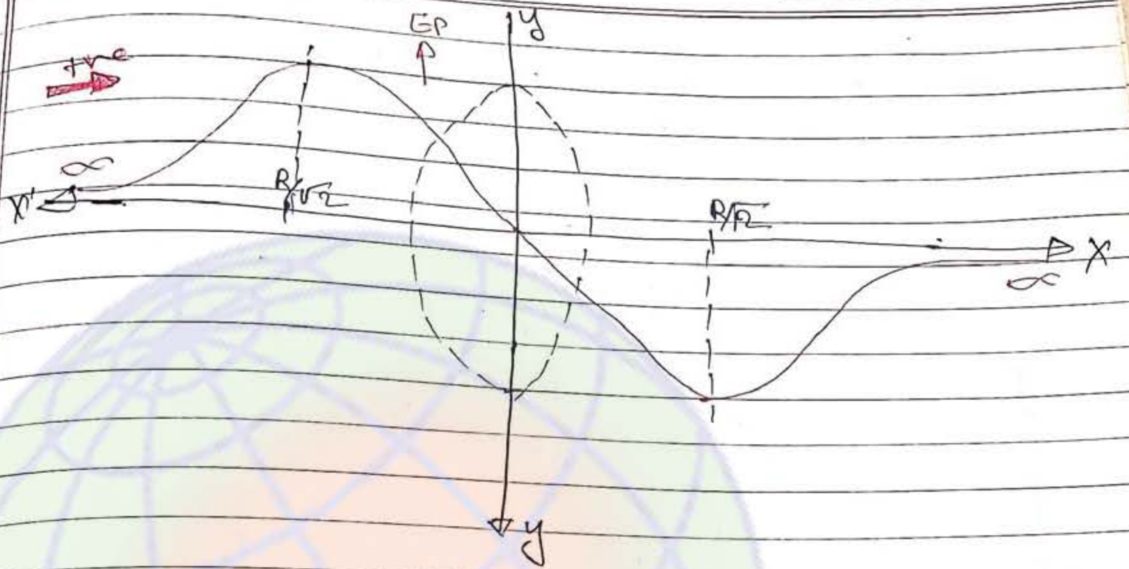
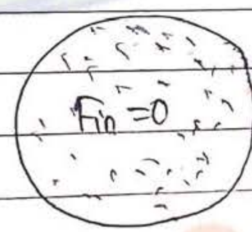


II Task

Shell theorem

1.) Gravitational field Intensity at any point Inside the uniform thin Spherical shell is always "zero."



~~uniform shell~~
Thin uniform Spherical shell

2.) For any external point $(r \geq R)$ the gravitational field Intensity due to ~~uniform~~ uniform hollow/solid sphere is

$$\boxed{E_{out} = \frac{GM}{r^2}} \qquad (r \geq R)$$

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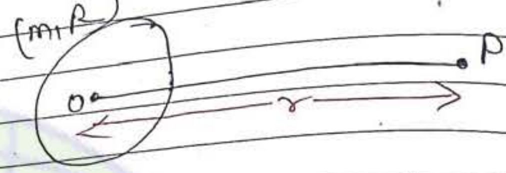
- Uniform hollow/solid sphere behaves like a point mass for external point.

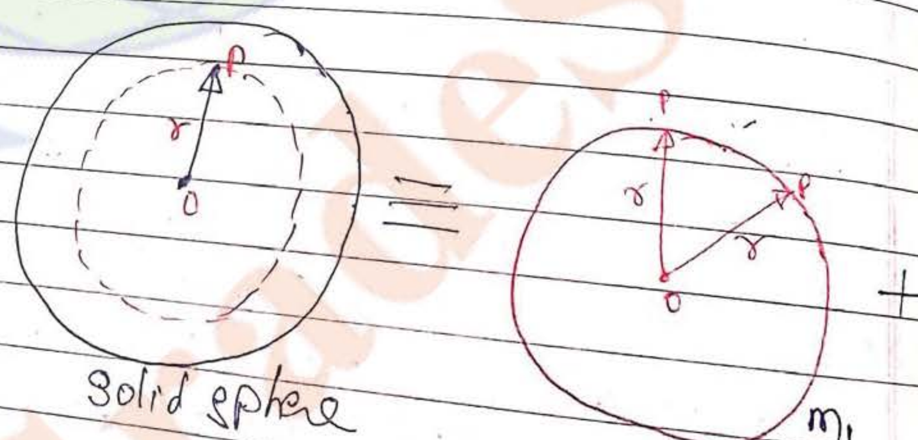


At the surface $(r=R)$

$$\boxed{E_{surface} = \frac{GM}{R^2}}$$

Gravitational field Intensity

3.) G. F. I due to Uniform Solid Sphere at Any Point Inside the Sphere:-



Solid sphere

$$E_{internal} = E_1 +$$

$$\Rightarrow \frac{GM_1}{r^2}$$

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$$m_1 = \frac{m}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3$$

$$= \frac{m r^3}{R^3}$$

$$\boxed{E_{in} = \left(\frac{GM}{R^3}\right) \cdot r}$$

At the surface, $r = R$

$$\boxed{E_{surface} = \frac{GM}{R^2}}$$

$$\rho = \frac{m}{\frac{4}{3}\pi R^3}$$

$$\rho = \frac{3m}{4\pi R^3}$$

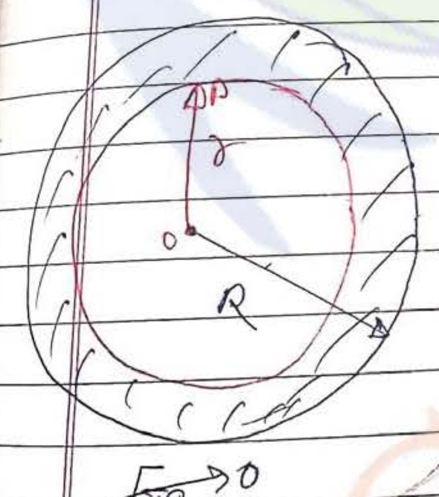
$$\frac{m}{R^3} = \frac{4\pi\rho}{3} \quad \text{--- (1)}$$

Now from eq (1)

$$\boxed{E_{in} = \left(\frac{4\pi\rho G}{3}\right) \cdot r}$$

(ρ = constant)

(Graph is in page = 11) Attend



$E_a \to 0$

Note :→ From shell theorem :-

If the point is situated inside the the hollow spherical shell of uniform density. The resultant Gr. F. I on the point mass is zero. Because the Gr. F. I on the point mass due to various small region of the spherical shell will be acting in various direction which cancel out each other

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IIT 2008 (3M)

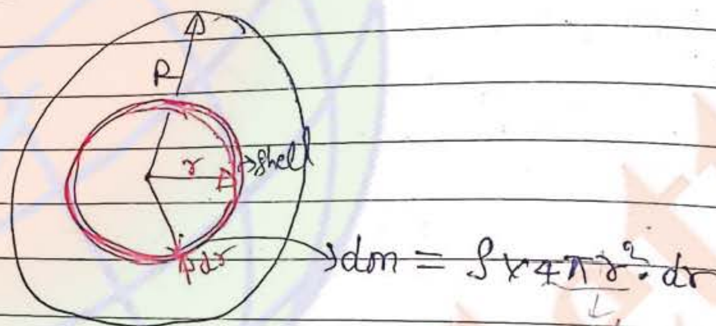
$\rho = \alpha r$

(α = constant)

Solid sphere

Find the G.F.I at external point "P". due to this sphere.

Ans $$E_P = \frac{G.M^{=?}}{(2R)^2}$$



$dm = \rho \times \underbrace{4\pi r^2}_{} \cdot dr$

Surface Area (का volume बनाने के लिए small 'dr')

$$M = \int dm$$

$$= \int_{r=0}^{r=R} \alpha r \cdot 4\pi r^2 dr$$

$$\Rightarrow 4\pi\alpha \int_0^R r^3 dr$$

$$M = \frac{4\pi\alpha}{\cancel{4}}(R^4) = \pi\alpha \cdot R^4$$

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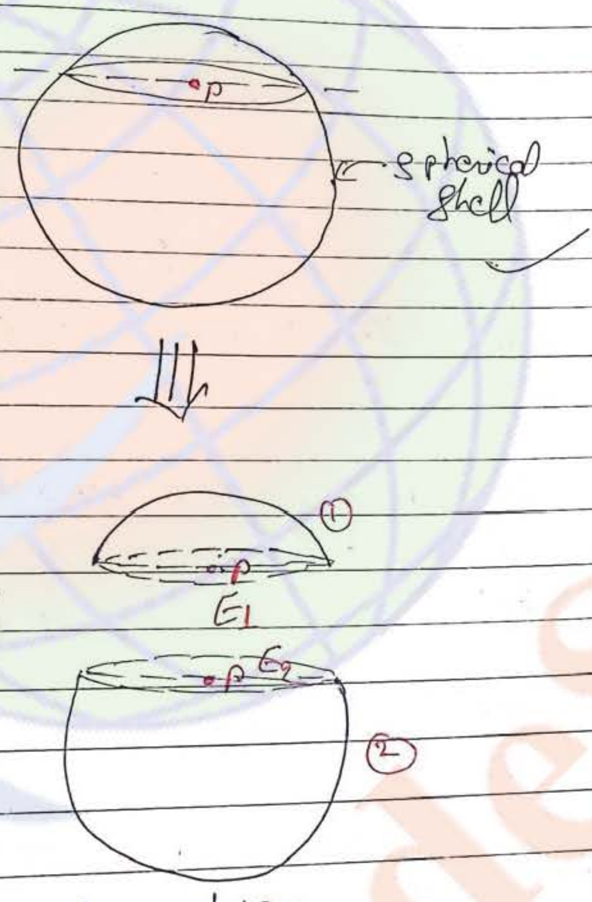
$$E = \frac{G(\pi \alpha R^4)}{(2R)^2}$$

$$= \frac{G \pi \alpha R^2}{4}$$

H.C.V(sub)
Q.1)



Correct option:-

i) $E_1 = E_2 \neq 0$

ii) $E_1 = E_2 = 0$

iii) $E_1 > E_2$

iv) $E_2 > E_1$

(Here two vects is cancell out)

Note:- Here we can not compare with $E = \frac{Gm}{R^2}$, ∵ Here 'm' and 'R' are balanced.

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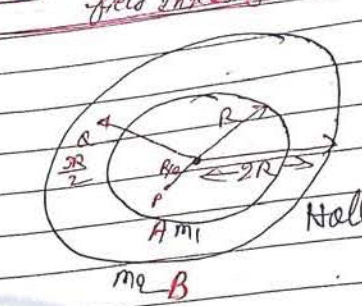
(E are g are same things, so their graph is same

$\boxed{E = g}$ Acc^n due to gravity. (Properties) Gravitational field Intensity



$$E_P = 0 + 0$$

$$E_Q = \cancel{E_B}^{\,0} + \frac{G M_1}{\left(\frac{3R}{2}\right)^2}$$

(Inside zero)

own write by concentrate teachers.

Relation b/w G.F.I (E) and Acc^n due to gravity. ~~and its variation~~ for own Understanding

So,
Intensity of gravitational field at a distance "r" from the centre of earth is:-

$$F = mg \quad \text{(I)}$$

Now,
$$F = mE \quad \text{(I)}$$

$$F = \frac{GMm}{r^2} \quad \text{(II)}$$

from eq. (I) and (II)

$$F = \underbrace{\frac{GM\cancel{m}}{r^2}}_{\text{"}E\text{"}} = \cancel{m}g$$

So, $\boxed{E = g}$

Gravitational field Intensity → Acc^n due to gravity.

Note:- E and "g" are same things. So their graph are Numerically value is also same. But they are different in their Properties.

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They are different in their properties.

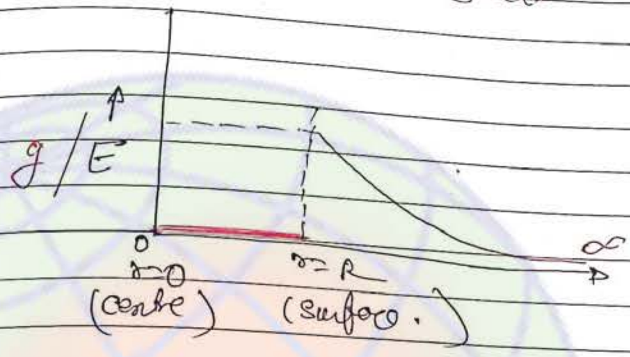
1st Choice

$F = mE$

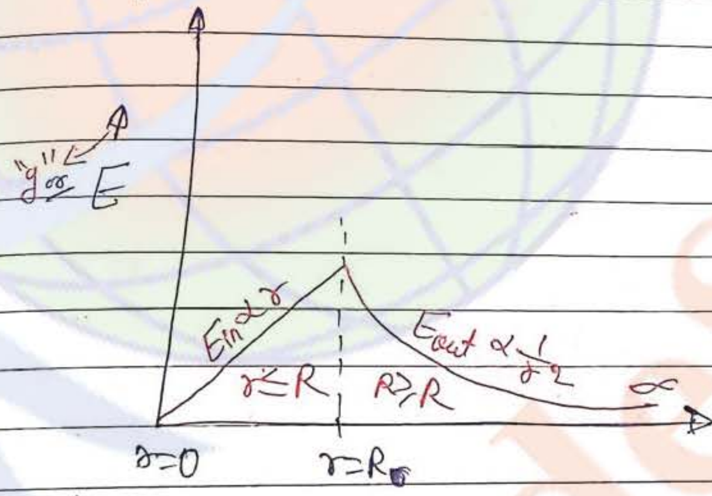
$F = \frac{GMm}{r^2} = mg$

For Uniform Spherical Shell

Graph of Page → 61 Spherical



For Uniform Solid Sphere :→



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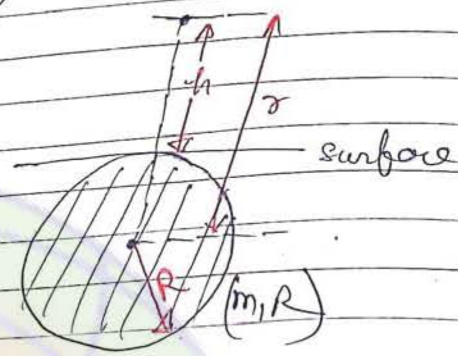
Acc^n due to gravity

1st Choice

Variation of acc^n. due to gravity:

1) With height! →



$$\boxed{g_h = E_h = \frac{GM}{(R+h)^2}} \quad \text{—— (1)}$$

At the surface →

$$\boxed{g_s = \frac{GM}{R^2}} \quad \text{—— (2)}$$

$$\boxed{g_h = \frac{g_s R_e^2}{(R_e+h)^2}}$$

or

$$\boxed{g_h = \frac{g_s}{\left(1+\frac{h}{R_e}\right)^2}} \rightarrow \text{safe in all place}$$

height

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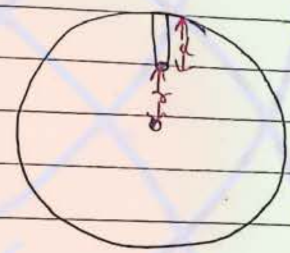
Note :→ If $h <<<< R$

$$\boxed{g_h = g_s\left(1 - \frac{2h}{R_e}\right)}$$ → इसे use करना safe नहीं है हर जगह (conditional)

(2) With depth :→



$$g_s = \frac{GM}{R^2}$$

$$g_{depth} = \frac{GM}{R^3} \cdot (R-d)$$

$$= g_s\left(\frac{R-d}{R}\right)$$

$$\boxed{g_{depth} = g_s\left(1 - \frac{d}{R_e}\right)}$$

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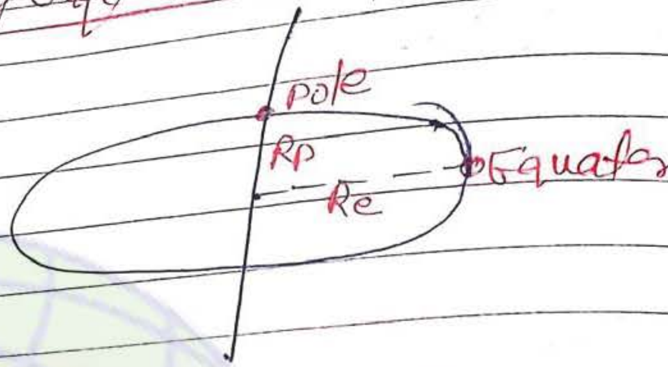
3.) With shape of earth



$$\left(g = \frac{GM}{R^2}\right)$$

$$g = \frac{G \cdot 4 \pi R^{\cancel{3}} \rho}{3 \cancel{R^2}}$$

$$\boxed{g = \left(\frac{4\pi G \rho}{3}\right) R}$$

$R_p < R_e$

$$\left(g_p > g_e\right)$$

$$m = \frac{4}{3} \pi R^3 \cdot \rho$$

$$\boxed{g = \frac{4}{3} \pi R \rho G}$$

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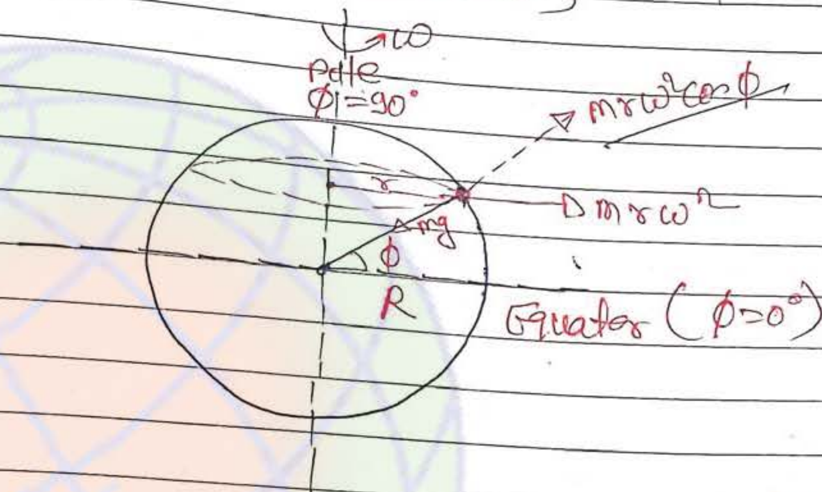
1st Choice

(4) Due to Rotation of earth:→
(with latitude)



$$\boxed{(g_p)_{effective} = g - \omega^2 R\cos^2\phi}$$

$$(mg_p = mg - mR\omega^2\cos^2\phi)$$

At pole ($\phi = 90°$)

$$g_p = g$$

At equator ($\phi = 0$)

$$g_{equa} = (g - \omega^2 R)$$

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Note:→
At equator:→
(Condition for of weightlessness.)

$$g_{eff} = 0$$

$$g - \omega^2 r = 0$$

$$\boxed{\omega = \sqrt{\frac{g}{R}}}$$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}} = 84.6\ min}$$

Concept:→ If earth rotate one round in 84.6 min then the person who stand on the pole of the earth feel weightlessness.

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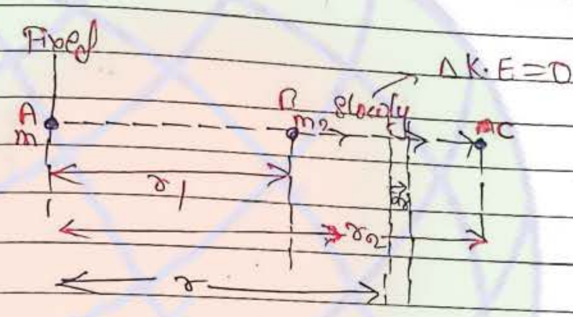
1st Choice

$\Rightarrow \cancel{\Delta K +} \Delta U = W_{ext}$

Change in Gravitational P.E $\Rightarrow$
(Potential energy)

$$\boxed{\Delta U = -W_c = -W_{gravity} = U_f - U_i}$$



$B \rightarrow C$

$$W_{gravity} = \int \vec{F} \cdot d\vec{r}$$

$$= -\int_{r=r_1}^{r=r_2} \frac{G m_1 m_2}{r^2}\, dr$$

$$\Rightarrow G m_1 m_2 \left[\frac{1}{r}\right]_{r_1}^{r_2} = G m_1 m_2 \left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$\cancel{\Delta U = -W_g}$

so,

$$\Delta U = \boxed{W_{gravity} = G m_1 m_2 \left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

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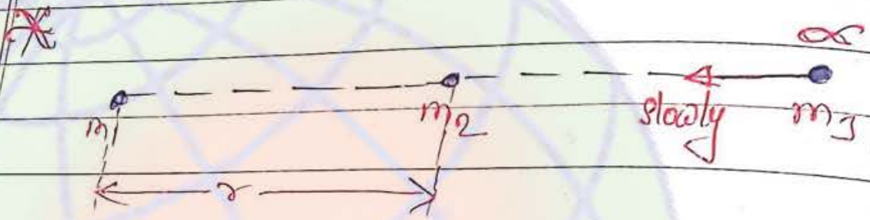
$$\Delta U = -W_g$$

$$\boxed{\Delta U = G m_1 m_2 \left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$



$$U_f - \cancel{U_i}^{\,0}_{\,\infty} = -\frac{G m_1 m_2}{r}$$

$$\boxed{U = -\frac{G m_1 m_2}{r}}$$

$$\boxed{\Delta K = 0}$$

$$\Delta U = -W_g = W_{ext}$$

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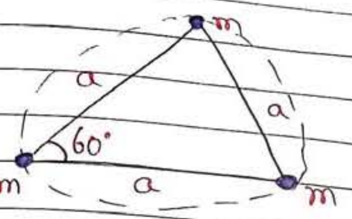
Q.15



$$U = -\frac{3Gm^2}{a}$$ ($3 \to {}^3C_2$)

$$\left(\text{No. of pair} = \frac{n(n-1)}{2} = {}^nC_2\right)$$

In the above ~~the~~ question.
1) Find the workdone by external agent in repeating the three mass for infinite distance.

Ans

$$\Delta U = U_f - U_i = W_{ext}$$

$$W_{ext} = 0 - \left(\frac{-3Gm^2}{a}\right)$$

$$= \frac{3Gm^2}{a}$$

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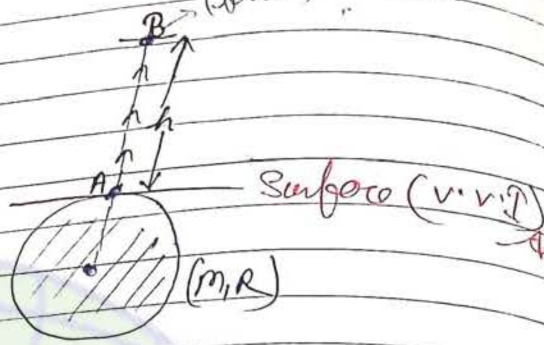
1st Choice

Change in Gravitational P.E (ΔU) : (Potential energy)



$$U_A = \frac{-GMm}{R}$$

$$U_B = \frac{-GMm}{(R+h)}$$

$$\Delta U = U_B - U_A$$

$$\Delta U = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\Delta = \frac{GMm}{R(R+h)} \qquad \left\{ g = \frac{GM}{R^2} \right.$$

$$\boxed{\Delta U = \frac{mgh}{\left(1+\frac{h}{R}\right)}}$$

If $h \lll\lll R$

$$\boxed{\Delta U = mgh}$$

1st Choice

Eg: A body ... from the ...

1) Find the ... from ...

Ans

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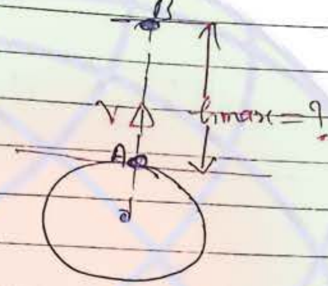
1st Choice

Q/ A body of mass (m) is projected vertically upward from the surface of earth at speed $v = \frac{1}{2}\sqrt{\frac{2GM}{R}}$

1) Find the max. height reached by the body from the surface of earth

Ans



$$\frac{1}{2}mv^2 = \Delta U$$

$$\cancel{\frac{1}{2} m \cdot \frac{1}{4} \cdot \frac{2GM}{G}}$$

$$\frac{1}{\cancel{2}}\cancel{m}\cdot\frac{1}{4}\cdot\frac{\cancel{2}\cancel{g}R^2}{\cancel{R}} = \frac{\cancel{m}\cancel{g}h}{\left(1+\frac{h}{R}\right)}$$

$g = \frac{GM}{R^2}$

$GM = gR^2$

$$R + h = 4h$$

$$h = \frac{R}{3}$$

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1st Choice

Page No.
Date

* Gravitational potential (V) at any field point →

Definition:

$$\boxed{V_P = \frac{W_{ext\ \infty \text{ to } 'P'}}{m}} \quad (\Delta K = 0)$$

Gravitational potential at any field point is defined as workdone by external agent in ~~caring~~ carrying unit mass (slowly without change in kinetic energy) from Infinity (∞) to that point (P)

$$\boxed{W_{ext} = U_f - U_i}$$

U_f (P), U_i (∞)

$$V_P = \frac{U_P}{m}$$

$$\boxed{U_P = m V_P}$$

($U = mV$)

carry A to B

$A \longrightarrow B$

$$V_B - V_A = \frac{W_{ext\ A\to B}}{m}$$

$$\boxed{W_{ext\ A\to B} = m(V_B - V_A)}$$

So

Imp

$$\boxed{W_{ext\ A\to B} = m(V_B - V_A) = U_B - U_A}$$

Imp

Very commonly asked in exam.

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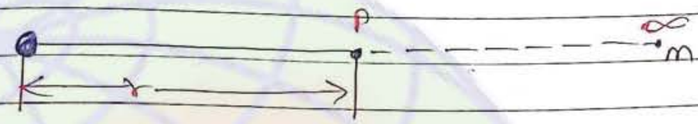
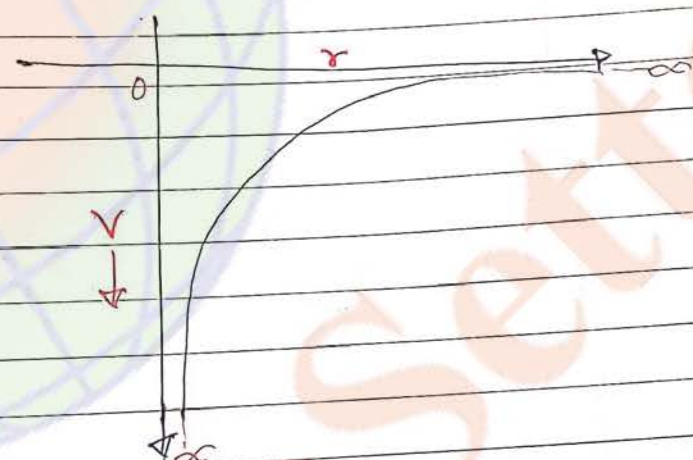
Gravitational potential at any field point due to point mass (m, r)

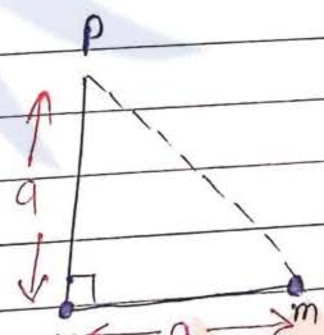


$$\boxed{V_P = \frac{-GM}{r}}$$

m = constant



Ex-1



(find Gravitational potential at point "P".)

$$V_P = \left(\frac{-GM}{a}\right) + \left(\frac{-GM}{(\sqrt{2})a}\right)$$

$$\Rightarrow \frac{-GM}{a}\left(1+\frac{1}{\sqrt{2}}\right)$$

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1st Choice

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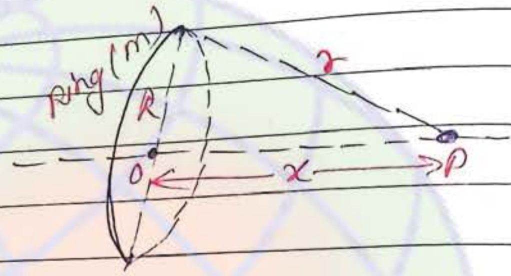
2) Gravitational potential at any axial point due to Uniform ring (m, R) :-



$$V_P = \int dV = -\int \frac{G\,(dm)}{r}$$

$$V_P = -\frac{G}{r}\int dm = \frac{-GM}{r}$$

$$\boxed{V_P = \frac{-GM}{(R^2+x^2)^{1/2}}}$$

$\left(x = 0 \text{ (at centre)}\right)$

$\left(V_{centre} = \frac{-GM}{R}\right)$

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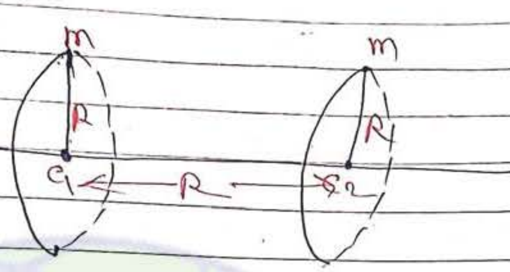
Q.1)



1) Find the gravitational potential at point C2.

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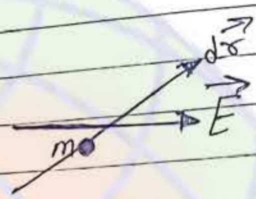
1st Choice

* Relation between Gravitational field Intensity ($\vec{E}$) and Gravitational Potential :->



$$\vec{F} = m\vec{E}$$

$$W_g = \int_{\vec{r}_A}^{\vec{r}_B} \vec{F} \cdot d\vec{r}$$

$$W_g = m \int_{\vec{r}_A}^{\vec{r}_B} \vec{E} \cdot d\vec{r}$$

$$W_{ext} = -m \int_{\vec{r}_A}^{\vec{r}_B} \vec{E} \cdot d\vec{r}$$

$$\cancel{W_{ext} = -m \int \vec{E} \cdot d\vec{r}}$$

$$\cancel{\frac{W_{ext}}{m} = V_B - V_A = -\int^{\vec{r}_B}}$$

$$\boxed{\frac{W_{ext}}{m} = V_B - V_A = -\int_{\vec{r}_A}^{\vec{r}_B} \vec{E} \cdot d\vec{r}}$$

$$W_{ext} = m(V_B - V_A)$$

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$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$\vec{dr} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\boxed{V_B - V_A = -\int_{(x_1, y_1, z_1)}^{(x_2, y_2, z_2)} \vec{E}\cdot\vec{dr}}$$

$$\boxed{V_B - V_A = -\left[\int_{x_1}^{x_2} E_x\,dx + \int_{y_1}^{y_2} E_y\,dy + \int_{z_1}^{z_2} E_z\,dz\right]}$$

(potential difference)

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$V = f(x, y, z)$ → function.

$$\boxed{E_x = -\frac{\partial V}{\partial x}}$$

$$\boxed{E_y = -\frac{\partial V}{\partial y}}$$

$$\boxed{E_x = -\frac{\partial V}{\partial z}}$$

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Ex:- Gravitational field Intensity due to some mass distribution is given by " $\frac{\alpha}{x^2}$ " along "the" x-axis" where α = constant

~~α = const~~

1) Find the ~~potent~~ Gravitational potential at position "x" ~~at P~~. If the potential Gravitational potential at Infinity is taken to be Zero.

Ans

$$V_B - \cancel{V_A}^{0} = -\int_{A \to \infty}^{B \to x} E_x\,dx$$

$$V_B = -\alpha \int_{\infty}^{x} \frac{1}{x^2}\,dx$$

$$= -\alpha \left[\frac{-1}{x}\right]_{\infty}^{x}$$

$$\Rightarrow \alpha \left[\frac{1}{x} - \frac{1}{\infty}\right] = \frac{\alpha}{x}$$

Q.)

$$\vec{E} = (2\hat{i} + 2\hat{j})\ \frac{N}{Kg}$$

$(m = 10kg)$

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1) Find the workdone by gravitational force during the displacement "O" to "P" for the given path.

$$\vec{F} = m\vec{E}$$

$$\vec{F} = (20\hat{i} + 20\hat{j})\,N$$

$O \longrightarrow A$

$W_1 = 20 \times 5 = 100J$

$A \rightarrow P$

$W_2 = 20 \times 4 = 80J$

So, Total workdone $= 180J$

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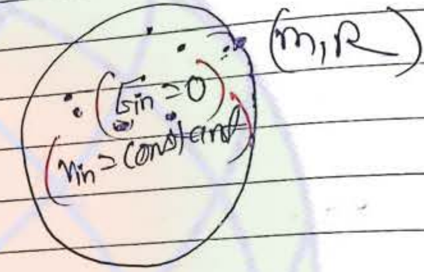
Square-centre=0
Sphere-centre=0

Q3) Gravitational potential due to thin Uniform Spherical Shell at any point Inside the shell.

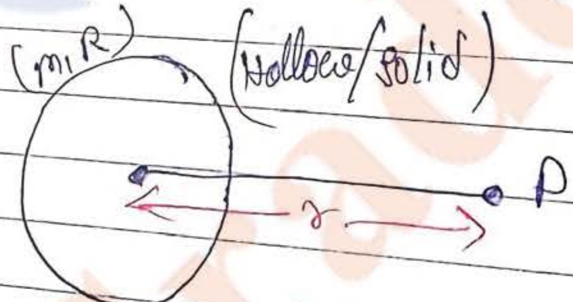
$(r \leq R)$



$$\boxed{V_{in} = V_{centre} = V_{surface} = -\frac{GM}{R}}$$

For external Points:-

$(r \geq R)$



$$\boxed{V_{out} = \frac{-GM}{r}}$$

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must SP / may SV

$dm = \frac{m}{(\ )}$ (S. A × thickness)

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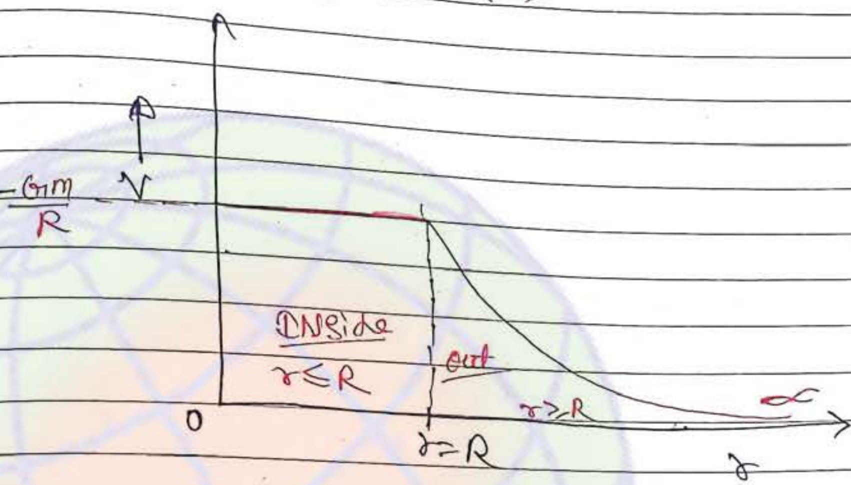
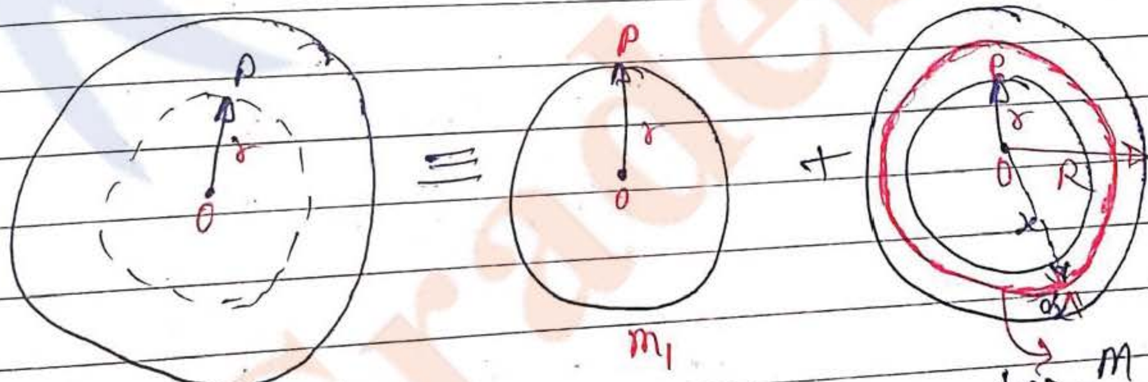
For Spherical Shell :→



Gravitational potential due to Uniform "Solid sphere" at Any Inside point ($r \le R$) :→



$dm = \frac{M}{\frac{4}{3}\pi R^3} \cdot (4\pi x^2 dx)$

$dm = \frac{3M}{R^3} x^2 dx$

$V_{in} = V_1 + V_2$

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$$V_1 = \frac{-GM_1}{r} \qquad \left\{ M_1 = \frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3 = \frac{Mr^3}{R^3} \right.$$

$$= \frac{-GMr^3}{R^3 \cdot r} = \frac{-GMr^2}{R^3}$$

$$V_2 = -\int_{x=r}^{x=R} \frac{G(dm)}{x}$$

$$= \frac{-3MG}{R^3}\int_r^R x\,dx$$

$$\Rightarrow \frac{-3GM}{2R^3}\left(R^2 - r^2\right)$$

$$\boxed{V_{in} = \frac{-GM}{2R}\left(3 - \frac{r^2}{R^2}\right)}$$

(i) At the centre $(r = 0)$

$$\boxed{V_{centre} = -\frac{3GM}{2R}}$$

(ii) At the surface $(r = R)$

$$\boxed{V_3 = -\frac{GM}{R}}$$

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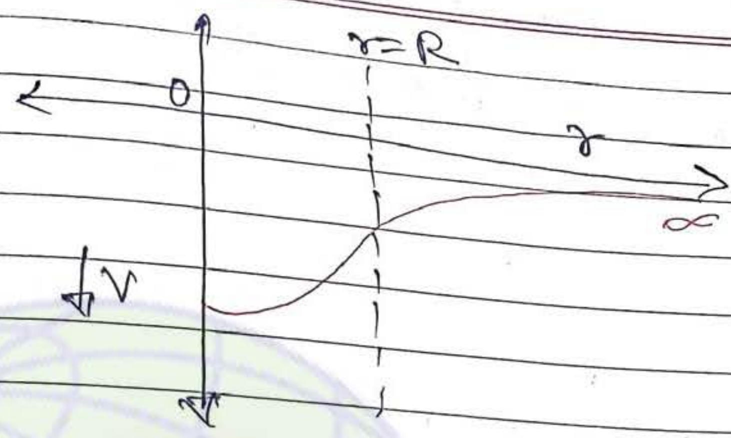

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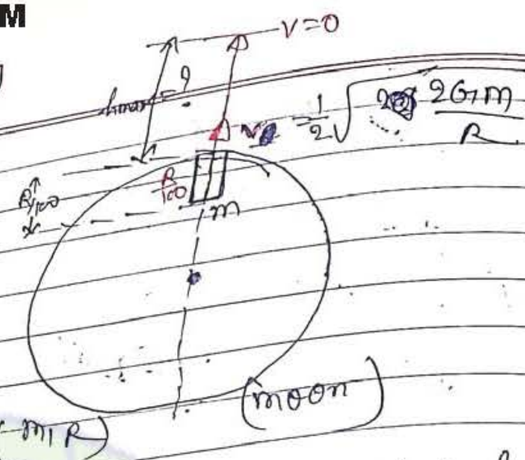
old IIT question

Q.



Find the max. height attained by the body.

$$\frac{1}{2}\sqrt{\frac{2GM}{R}}$$

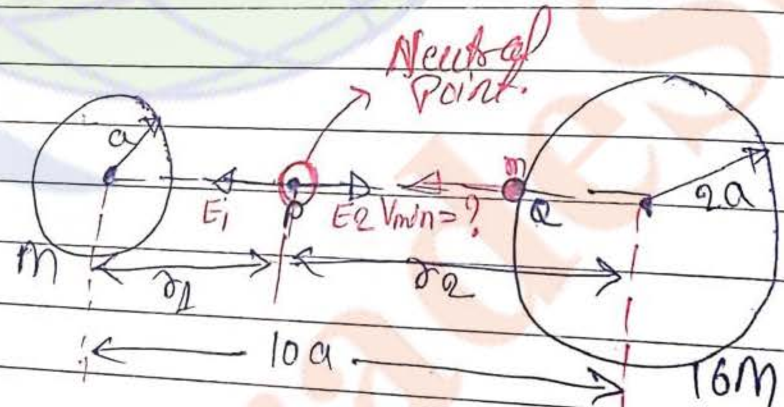
$$\frac{1}{2}\cancel{m}v^2 + \cancel{m}\left[\frac{-GM}{2R}\left(3 - \frac{\left(R - \frac{R}{100}\right)^2}{R}\right)\right] = \underset{K.E}{0} + \left(\frac{-GM\cancel{m}}{R+h}\right)$$

$h = \_\_$

(Ans 99 ...)

IIT question

Q.



Find the ~~min velocity~~ minimum velocity of projection from the surface of planet so that it can reach to the surface of large surface of larger planet.

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$\vec{F}_{net} = m\vec{E}_{net}$

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$E_1 = E_2$

$$\frac{\cancel{G}\cancel{M}}{r_1^2} = \frac{\cancel{G}(16\cancel{M})}{r_2^2}$$

$$\frac{r_1}{r_2} = \frac{1}{4} \qquad r_1 + r_2 = 10a$$

$r_1 = 2a$

$r_2 = 8a$

Now, $Q \to P$

$$\frac{1}{2} m v_{min}^2 + \cancel{m} V_Q = 0 + \cancel{m} V_P$$

$$\frac{v_{min}^2}{2} + \left[\frac{-G(16m)}{2a} + \left(\frac{-Gm}{8a}\right)\right] = 0 + \left[\left(\frac{-G(16m)}{8a}\right) + \left(\frac{-Gm}{2a}\right)\right]$$

$$\Rightarrow \frac{v_{min}^2}{2} + \left[\left(\frac{-4G(16m) + -Gm}{8a}\right)\right] = 0 + \left[\left(\frac{-G(16m) + -4Gm}{8a}\right)\right]$$

$$\Rightarrow v_{min}^2 + \left[\frac{-64Gm - Gm}{8a}\right] = \left[\left(\frac{-16Gm - 4Gm}{8a}\right)\right]$$

$$\Rightarrow v_{min}^2 + \left[\frac{-65Gm}{8a}\right] = \left[\frac{-20Gm}{8a}\right]$$

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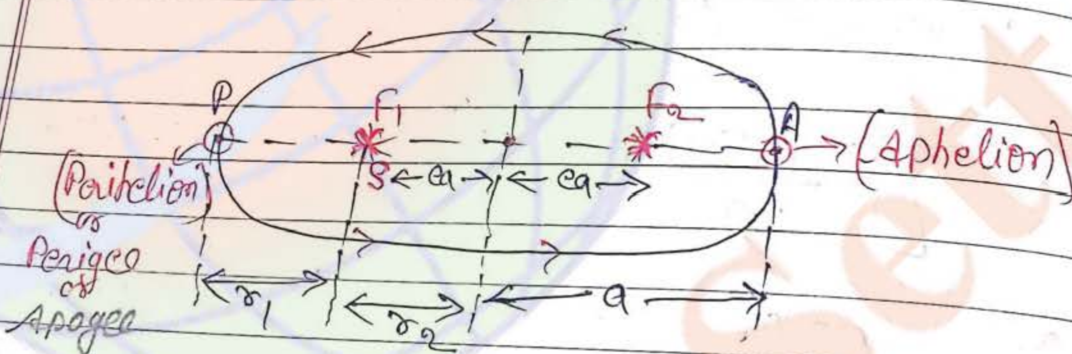
U may be -ve or +ve but absolute value of 'V' is always "+ve"

V Planetary Motion

* Kepler's law :-

I) 1st law :⇒ (or "law of orbit") All planets revolve around the ~~sun~~ Sun in an elliptical orbit in which ~~Sun is~~ Sun is located at one of the two focii.



$$\boxed{r_1 = a(1-e)}$$

$$\boxed{r_2 = a(1+e)}$$

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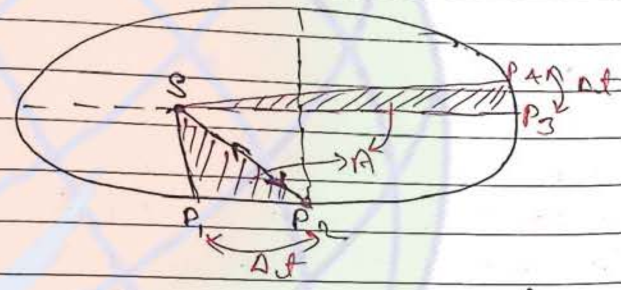
II) IInd law :⇒ (law of Area) :⇒

The line joining the sun and the planet sweeps out equal area in equal time interval.

(Areal speed of the planet remains constant)

$$\boxed{\frac{dA}{dt} = \frac{L}{2m}} \rightarrow \text{(constant)}$$

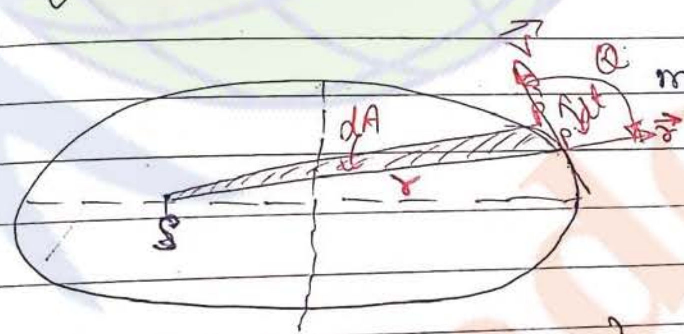


where :-

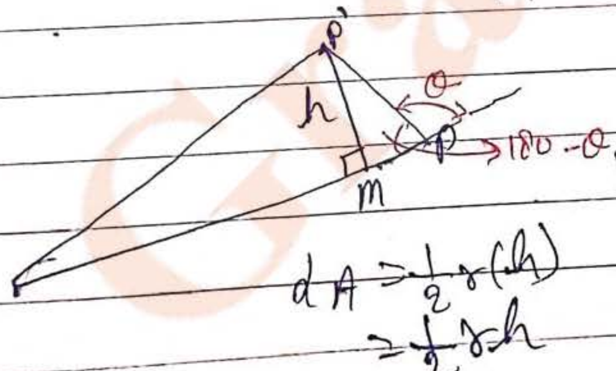
L ⇒ Angular Momentum of planet about sum.

m ⇒ mass of revolving planet.

Proof :⇒



$PP' = v\,dt$



$dA = \frac{1}{2} r(h)$

$= \frac{1}{2} rh$

Now

$\sin(180° - \theta) = \frac{h}{PP'}$

$h = (\sin\theta)\, v\,dt$

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1st Choice

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$$dA = \frac{1}{2} r v \sin\theta \, dt$$

$$|\vec{L}| = |\vec{r} \times m\vec{v}|$$

$$\boxed{L = mvr\sin\theta}$$

$$\frac{dA}{dt} = \frac{1}{2} \frac{(vr\sin\theta)(m)}{m}$$

$$\boxed{\frac{dA}{dt} = \frac{L}{2m}}$$

(III) IIIrd law :->

Square of time period of revolution of a planet revolving around the sun is directly proportional to the cubic power of semimajor axis.

$$\boxed{T^2 \propto a^3}$$

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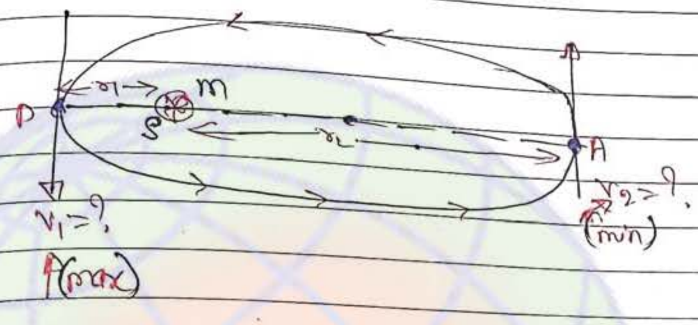
Note: ⟹



$$(L_P)_S = (L_A)_S$$

$$\cancel{m}v_1 r_1 = \cancel{m}v_2 r_2$$

$$v_1 \cancel{a}(1-e) = v_2 \cancel{a}(1+e)$$

$$v_1(1-e) = v_2(1+e) \quad \text{---(1)}$$

Now

$$E_P = E_A$$

$$\frac{1}{2}\cancel{m}v_1^2 + \left(\frac{-GM\cancel{m}}{r_1}\right) = \frac{1}{2}\cancel{m}v_2^2 + \left(\frac{-GM\cancel{m}}{r_2}\right) \quad \text{---(2)}$$

max velocity in Position "P"

$$\boxed{v_1 = \sqrt{\frac{GM(1+e)}{a(1-e)}}}$$

min velocity in position "A".

$$\boxed{v_2 = \sqrt{\frac{GM(1-e)}{a(1+e)}}}$$

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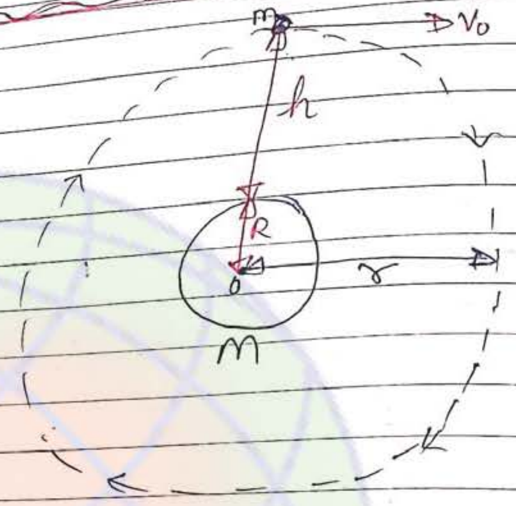
For Circular Orbit



$$\boxed{r = (R+h)}$$

$$\frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}v_0^2}{r}$$

$$\boxed{v_0 = \sqrt{\frac{GM}{r}}}$$

1) orbital speed is Independent of mass of orbiting body (जो Body घूम रहा है orbital speed उस पर depend नहीं करता है)

2.) $$\boxed{T = \frac{2\pi r}{v_0}} = \frac{2\pi r\sqrt{r}}{\sqrt{GM}}$$

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1st Choice

Total M.E. of body is "-ve" then this system is "bound system"

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$$T^2 = \frac{4\pi^2}{GM} \cdot r^3$$

$$\boxed{T^2 \propto r^3}$$

→ यह Relation circular मान कर ही proof किया जाता है।

Shortcut ⟶ If force exerted by two body is different then in this condition we find relation of "T and r" in this method:

Note:→ Generally दो body के बिच लगने वाला "force $\propto \frac{1}{r^2}$" लेकिन IIT Imagine करवाता है कि दो body है और उनके force और Radius में

Note:→ Relation कुछ और दूसरा है तब उस स्थान पर Time और Radius का Relation shortcut से पता करे

$$F \propto r^n$$

$$\boxed{T^2 \propto r^{(1-n)}}$$

IIT asks question (Imagination type)

check:→

$$F \propto \frac{1}{r^2}$$

$$F \propto r^{-2}$$

so

$$T^2 \propto r^{(1+2)}$$

$$T^2 \propto r^3$$

K.E (K)

$$K = \frac{1}{2} m v^2$$

$$\boxed{K = \frac{GMm}{2r}}$$

Gravitational Potential Energy (U)

$$\boxed{U = -\frac{GMm}{r}}$$

when satellight come from space to earth

and

when satellite go from earth to Atmosphere

VVI

$$\boxed{U = \frac{GMm}{r}}$$

Note: Generally P.E के formula में (-ve) sign होता है जिसका ही होता है -ve होता या +ve होता body के motion पर depend करता है।

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$g = \frac{GM}{R^2}$

1st Choice

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Total mechanical Energy (E)

$$E = K + U$$

$$\boxed{E = \frac{-GMm}{2r}}$$

$\int F\,d$

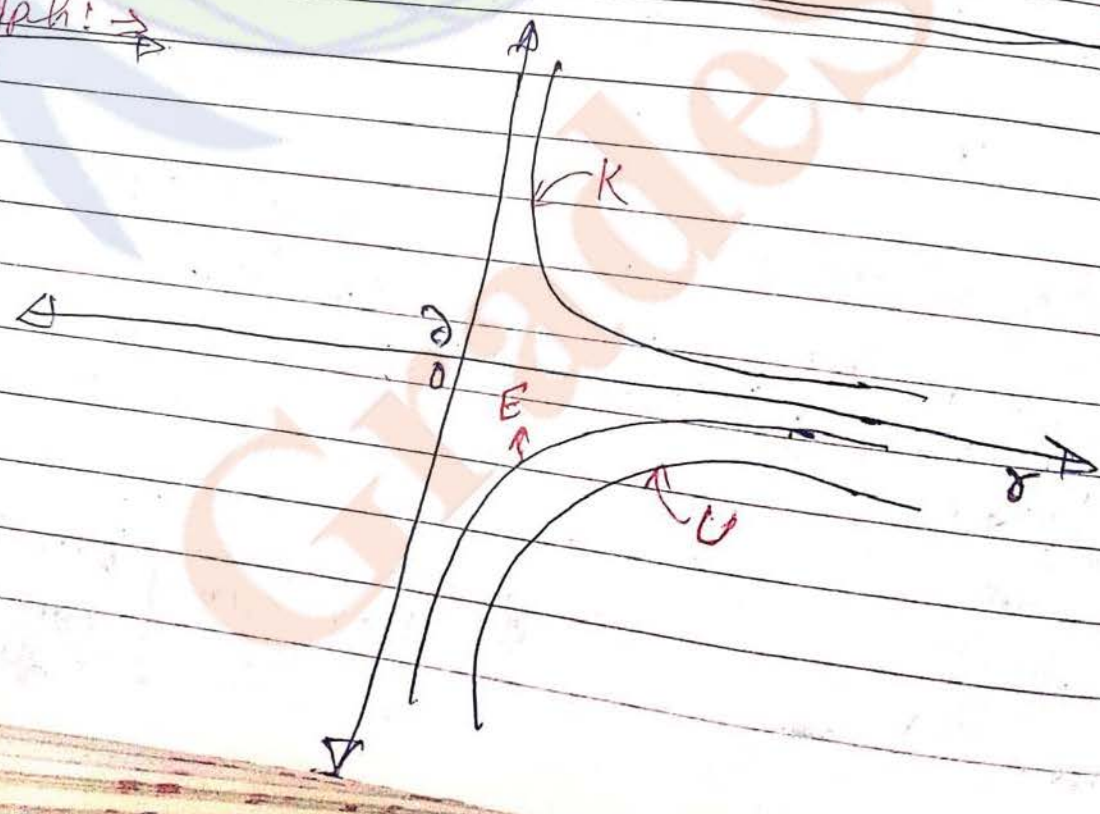
Note:→

$$\boxed{U = 2E}$$

$$\boxed{K = -E}$$

$\boxed{\text{Binding energy (B.E)} = -(\text{Total energy})}$

Graph:→



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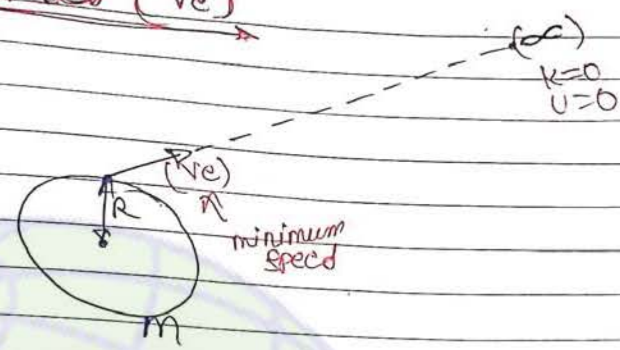
Escape Speed (V_e)



$$\frac{1}{2} m v_e^2 + \left(-\frac{GMm}{R}\right) = 0 + 0$$

$$\boxed{V_e = \sqrt{\frac{2GM}{R}}} = \sqrt{2}\, V_o$$

where V_o is orbital velocity.

only if h is very less.

$$\boxed{V_e = \sqrt{2gR}}$$

where

$$\boxed{g = \frac{GM}{R^2}}$$

Note :->
For earth surface

$$V_e = 11.2 \text{ km/s}$$

~~Scape~~ Escape speed is Independent of mass of ~~post~~ projected object and Angle of projection.

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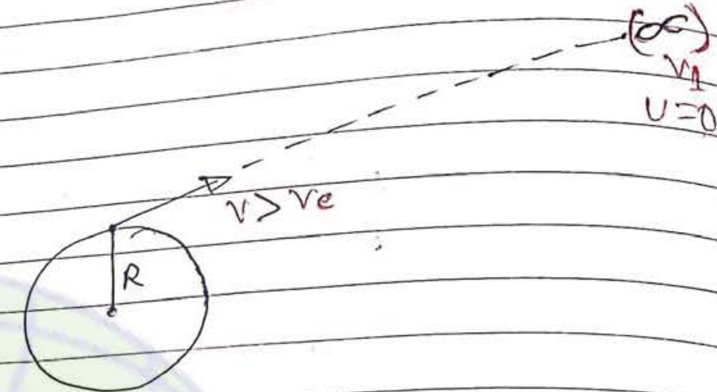
Note:→



$$\frac{1}{2}\cancel{m}v^2 + \left(\frac{-GM\cancel{m}}{R}\right) = \frac{1}{2}\cancel{m}v_1^2 + 0$$

$$\frac{v^2}{\cancel{2}} - \frac{1}{\cancel{2}}v_e^2 = \frac{1}{\cancel{2}}v_1^2$$

$$\boxed{v_1 = \sqrt{(v^2 - v_e^2)}}$$

$$v_e = \sqrt{\frac{2GM}{R}}$$

$$v_e^2 = \frac{2GM}{R}$$

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(55, 56, 57)

1st Choice Launching of Satellite

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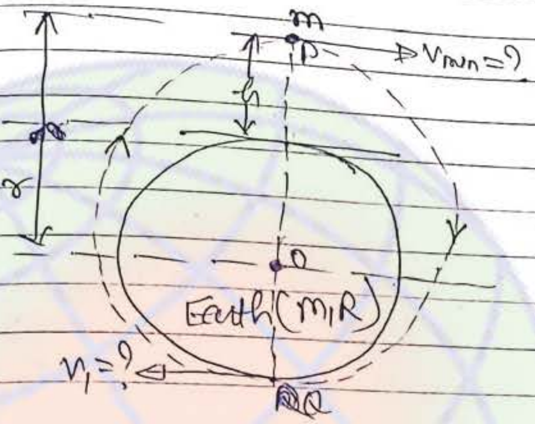
~~Launching of Satellite~~ →



Find the minimum velocity such that body just passes through point Q.

Ans:

$$(L)_P = (L)_Q$$

$$\Rightarrow \cancel{m} v_{min} r = \cancel{m} v_1 R$$

$$v_{min} r = v_1 R \quad \text{— (1)}$$

Now

$$E_P = E_Q$$

$$\frac{1}{2}\cancel{m} v_{min}^2 + \left(\frac{-GM\cancel{m}}{r}\right) = \frac{1}{2}\cancel{m} v_1^2 + \left(\frac{-GM\cancel{m}}{R}\right) \quad \text{— (2)}$$

$$\Rightarrow \frac{1}{2} v_{min}^2 + \left(-\frac{GM}{r}\right) = \frac{1}{2}\frac{v_{min} r}{R} + \left(\frac{-GM}{R}\right)$$

$$\Rightarrow \frac{1}{2} v_{min}^2 - \frac{1}{2}\frac{v_{min} r}{R} = \frac{-GM}{R} + \frac{GM}{r}$$

$$\Rightarrow \frac{v_{min}^2 R - 2 v_{min} r}{2R} = \frac{-GMr + GMR}{Rr}$$

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$$\boxed{v_{min} = \sqrt{\frac{2GMR_1}{r_2(r_2+R_1)}}}$$

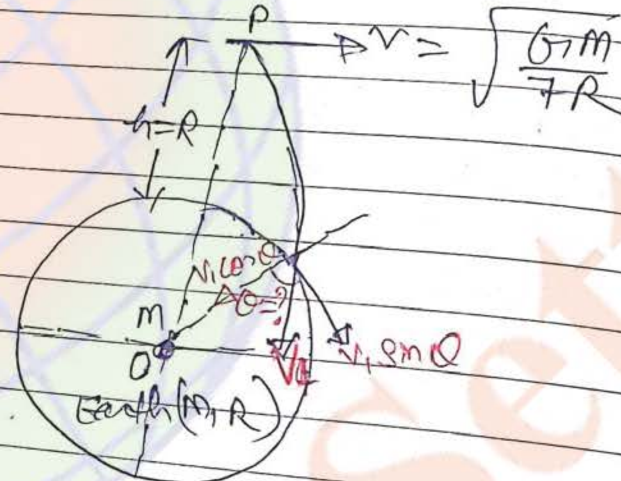
Now.

If $h = R$: →

$$v_{min} = \sqrt{\frac{2GMR}{2R \cdot 3R}}$$

$$\boxed{v_{min} = \sqrt{\frac{GM}{3R}}} \text{ or } \boxed{V_{min} = \sqrt{\frac{GM}{3R}}}$$

Eg 4)



Find the value of "θ" for which body collide with the earth surface.

Ans)

$$\cancel{m}v(2\cancel{R}) = \cancel{m}v_1 \sin\theta \times \cancel{R}$$

$$2v = v_1 \sin\theta$$

$$\sin\theta = \frac{2v}{v_1} \quad \text{—— (1)}$$

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1st Choice

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$$\frac{1}{2}\cancel{m}\frac{GM}{7R}+\left(\frac{-GM\cancel{m}}{2R}\right)=\frac{1}{2}\cancel{m}v_1^2+\left(\frac{-GM\cancel{m}}{R}\right)$$

$$\Rightarrow \frac{GM}{14R}-\frac{GM}{2R}+\frac{GM}{R}=\frac{1}{2}v_1^2$$

$$\Rightarrow \frac{GM-7GM+14GM}{14R}=\frac{1}{2}v_1^2$$

$$\Rightarrow \cancel{\frac{3GM}{2R}=\frac{1}{2}v_1^2} \Rightarrow \frac{8GM}{14R}=\frac{1}{2}v_1^2$$

$$\Rightarrow \cancel{v_1=\sqrt{\frac{3GM}{2R}}} \quad \text{So,} \quad v_1=\sqrt{\frac{8GM}{7R}}$$

$$=2\sqrt{\frac{2GM}{7R}}$$

$$v_1=2\sqrt{2}\,v$$

$$\boxed{\sin\theta=\frac{2v}{v_1}=\frac{1}{\sqrt{2}}}$$

$$\boxed{(\theta=45^\circ)}$$

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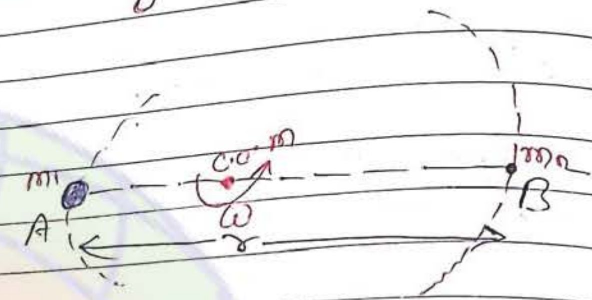
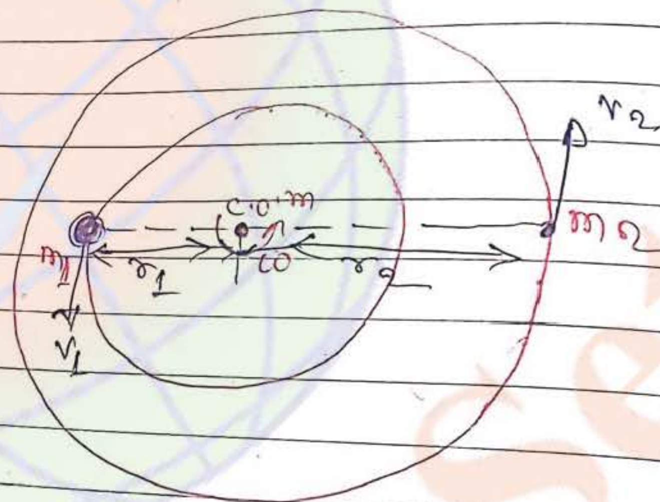
1st Choice

Binary Star System or (Double Star's System)





$\boxed{\omega = \text{Same}}$

Now
$(m_1 > m_2)$

$\boxed{T_A = T_B = \text{Same}}$

$\to$ time Period

$T = \frac{2\pi}{\omega}$

$\boxed{r_1 + r_2 = r}$

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$$\boxed{r_1 = r \cdot \frac{m_2}{(m_1+m_2)}}$$

$$\boxed{r_2 = r \cdot \frac{m_1}{(m_1+m_2)}}$$

$$m_1 r_1 \omega^2 = \frac{G m_1 m_2}{r^2} = m_2 r_2 \omega^2$$

$$\frac{G\, m_1\, m_2}{r^2} = \frac{m_1 r m_2\, \omega^2}{(m_1+m_2)}$$

$$\boxed{\omega = \sqrt{\frac{G(m_1+m_2)}{r^3}}}$$

$$\boxed{T = \frac{2\pi}{\omega}}$$

$$\boxed{T = 2\pi \sqrt{\frac{r^3}{G(m_1+m_2)}}}$$

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** Angular Momentum of "m_1" about C.O.M :⇒

$$\boxed{L = I_1 \omega}$$

*) Angular momentum of "m_2" about C.O.M:

$$\boxed{L_2 = I_2 \omega}$$

Now

$$\frac{L_1}{L_2} = \frac{I_1}{I_2} = \frac{m_1 r_1^2}{m_2 r_2^2} = \frac{m_2}{m_1}$$

$$\boxed{\frac{L_1}{L_2} = \frac{I_1}{I_2} = \frac{m_2}{m_1}}$$

so,
$$K = \frac{1}{2} I \omega^2$$

$$\boxed{\frac{K_1}{K_2} = \frac{I_1}{I_2} = \frac{m_2}{m_1} = \frac{L_1}{L_2}}$$

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1st Choice

Gravitation

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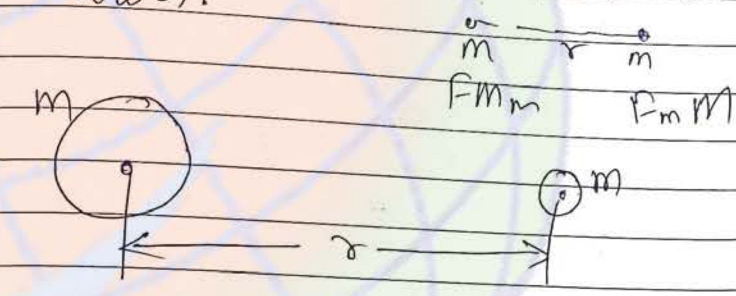
Universal Gravitation

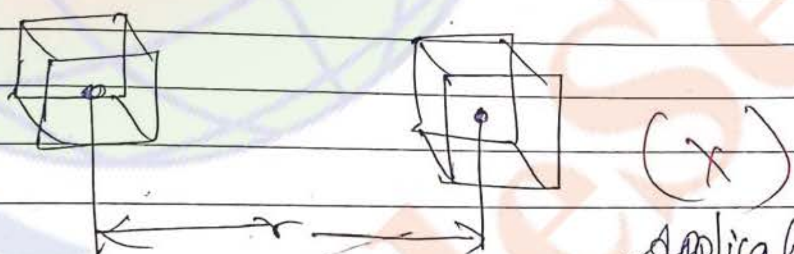
1) Newton law of universal Gravitation :->

$$\boxed{F_{m_1m_2} = \frac{G m_1 m_2}{r^2}}$$

i) Applicable to point masses or spherical bodies.

ii)





Applicable for far distance

Acc^n due to Gravity:—

$$F_{mM} = \frac{GMm}{r^2}$$

$$F_{mM} = m a_m$$

$$a_m = \frac{GM}{r^2}$$

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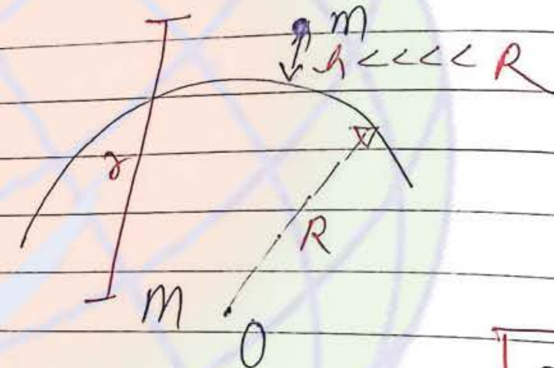
$$a_m = \frac{GM}{r^2}$$

$$\frac{a_M}{a_m} = \frac{M}{m} > 1$$

$$M > m$$

$$\boxed{a_M > a_m}$$



$$\boxed{r \simeq R}$$

$$a_m = \frac{GM}{R^2}$$

$$= 9.8\ m/s^2$$

$$G = 6.67 \times 10^{-11}\ N\,m^2/kg^2$$

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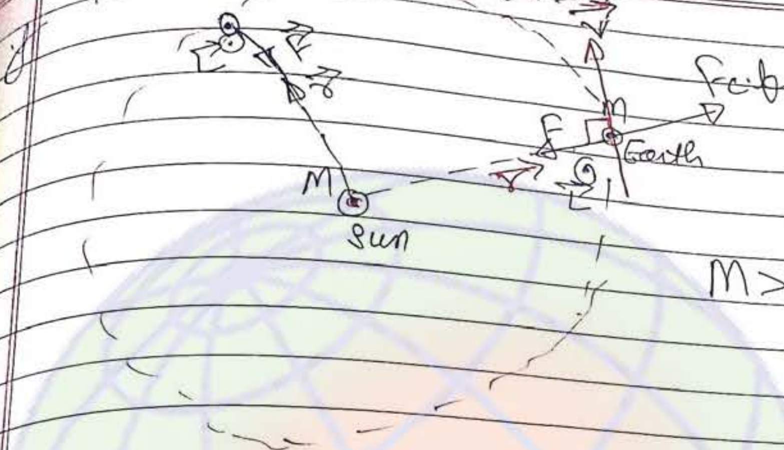
Planetary motion:→



// Torque of earth w.r.t. centre of sun:→

$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$= rF \sin 180°$$
$$= 0$$

$$\vec{\tau} = 0 \text{ (Always zero)}$$

so Angular moment is conserved.

Angular momentum of earth w.r.t. centre of ~~Sun~~ Sun is constant.

$$\vec{L} = \vec{r} \times m\vec{v}$$

$$|\vec{L}| = r m v \sin(90°)$$

$$|\vec{L}| = mvr$$

$$L = mvr$$

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$$\boxed{V = \frac{L}{mr}}$$ (Uniform circular motion)

Note:->

$$F = F_{c.f}$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$\boxed{v = \sqrt{\frac{GM}{r}}}$$ -> mass of sun

$$\boxed{V_{orbital} = \sqrt{\frac{GM}{r}}}$$

$$\boxed{V_{earth} \simeq 30\ km/sec}$$

$$v = \sqrt{\frac{GM}{(R+h)}}$$

$$h \to 0$$

$$V = \sqrt{\frac{GM}{R}}$$

$$= 8\ km/sec$$

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3.) Time Period:→

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

but Here not concell

$$\frac{1}{r} \times \frac{GMm}{r^2} = \frac{mv^2}{r \times r}$$

$$\frac{GM\cancel{m}}{r^3} = \frac{\cancel{m}v^2}{r^2}$$

$$\boxed{\omega^2 = \frac{GM}{r^3} = \left[\frac{2\pi}{T}\right]^2}$$

$$\boxed{T^2 = \frac{4\pi^2}{GM} r^3}$$

(mass of earth)

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Q1) Given ->

$\boxed{r_{jup} = 4\, r_{earth}}$

$M_{jup} = 100\, M_{earth}$

$T_{earth} = 1$ year

Find $T_{jupi} = ?$

Ar.)

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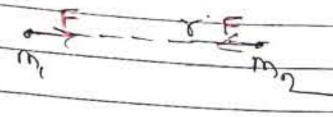
Gravitation

1) Newton law of gravitation: (Universal law of gravitation)

$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$



$G = 6.67 \times 10^{-11}\ N\,m^2\,kg^{-2}$

$m_1, m_2 \rightarrow$ Point mass.

i) Gravitation force is always attractive in nature
ii) It act along the line joining two point masses.
iii) It is Independent of medium in which masses are present
iv) It is a long range force and act even b/w two planet's

v)

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \quad \text{in electrostatic}$$

$$F = G \times \frac{m_1 m_2}{r^2} \quad \text{in Gravitation}$$

We have already have know electrostation.
and we know everything about electrostatics

and our gravitation force is almost similar in electrostatic force in terms of formula
Therefore in this chapter we will not derive any formula we will

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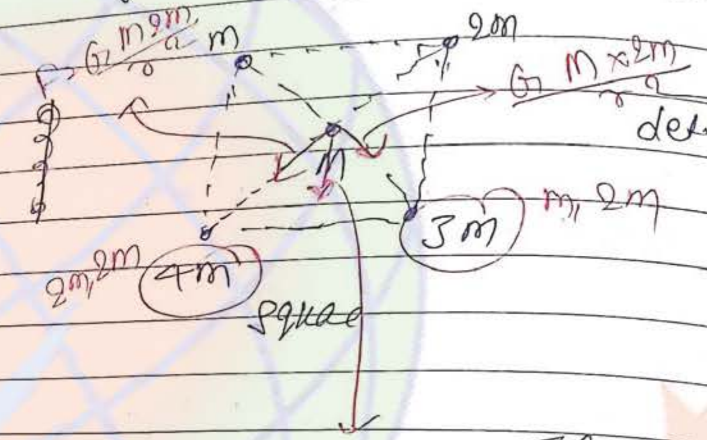
Simply replace by

i) $\frac{1}{4\pi\epsilon_0}$ by G

ii) charge by mass

iii) charge density by mass density

Q.1)



det F on M

Sol/n

$F_{net} = \frac{\sqrt{2}\, G M \times 2m}{r^2}$

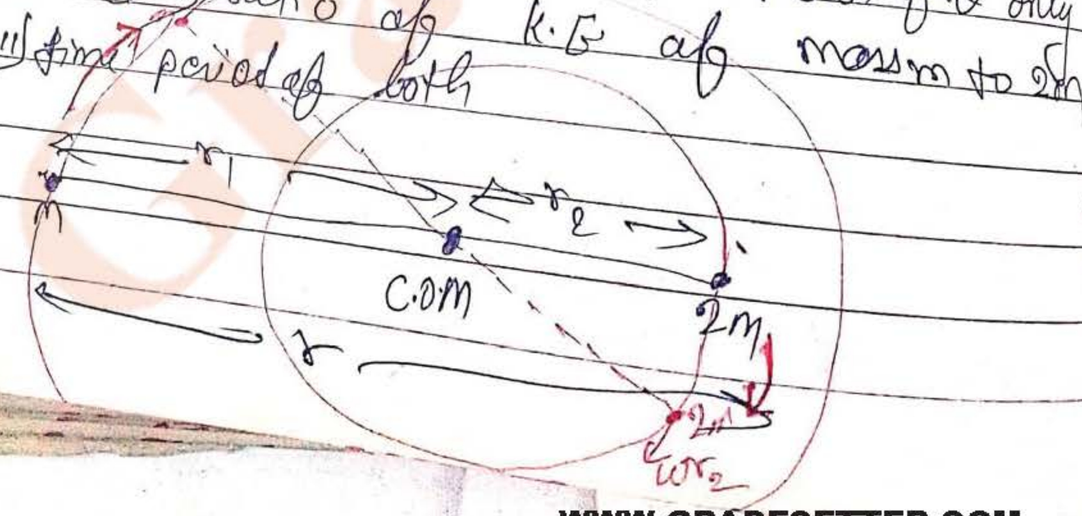
Q.2)

both mass are revolving about their common centre of mass with constant angular speed under the mutual gravitation force only

Det. i) ratio of K.E of mass m to 2m

ii) time period of both

Sol/n



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$mr_1 = mr_2$

$\frac{r_1}{r_2} = \frac{2}{1}$

$r_1 + r_2 = r$, $r_2 = \frac{r}{3}$

$F_{net} = 0$

$a_{cm} = 0$

Cm → चुपचाप रहेगा ।

$K.E_m = \frac{1}{2} m (\omega r_1)^2$

$K.E_{2m} \Rightarrow \frac{1}{2} \times 2m (\omega r_2)^2$

$\frac{K.E_m}{K.E_{2m}} = \frac{r_1^2}{2 r_2^2} \frac{2}{1}$

ii) $T = \frac{2\pi}{\omega}$

$F = 2m\omega^2 r_2$

$\frac{G \times m \times 2m}{r^2} = 2m\omega^2 r_2$

$\frac{G \times 2m^2}{r^2} = 2m\omega^2 \frac{r}{3}$

$\frac{G \times 2m^2}{r^2} = 2m\omega^2 r$

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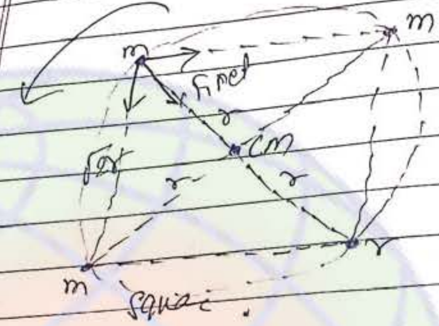
This two point mass system revolving about its common centre. is also known as Binary star.

eg 3)



Each of the 4 mass are revolving on circular path about C.O.M under the mutual gravitational force "only" C.M. Det. time period of revolution.

Sol

$$F = \frac{Gm^2}{2r^2}$$

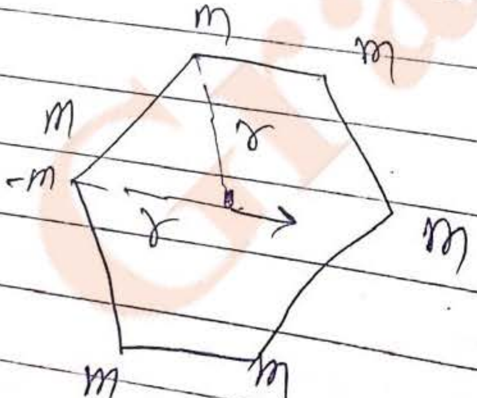
$$F_1 = \frac{Gm^2}{4r^2}$$

$$\sqrt{2}F + F_1 = m\omega^2 r$$

$$\omega = \text{—}$$

$$\boxed{T = \frac{2\pi}{\omega}}$$

eg 4)



$$\frac{GMm}{r^2}$$

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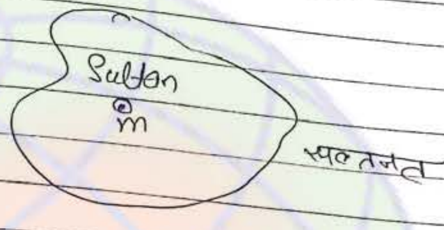
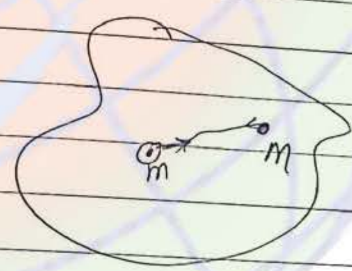
Gravitational field -

I) It is a region in which object or matter experience gravitation force



II) Every body has gravitational field i like Sultan have स्वतंत्र surrounding it. surround it

III)

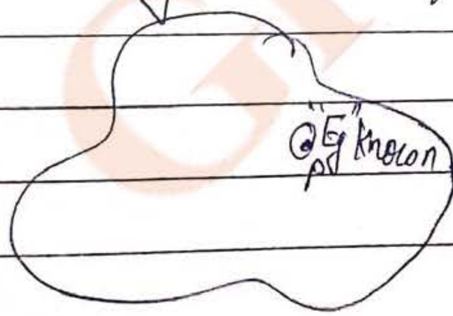


Gravitation field Intensity (Eg)

I) Gravitation field Intensity at any point is defined as gravitation force on unit mass if placed at that point.

II) Unit of Eg is N/kg

III)



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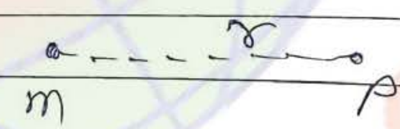
"F on mass "m" placed at P = mE_g

iv) Direction of force is always along E_g

v)

Direction of E_g due to any sultan object is always towards sultan.

vi) E_g due to point (sultan) mass ⟶



$$\boxed{E = \frac{Kq}{r^2}} \quad , \quad \boxed{E_g = \frac{GM}{r^2}}$$

Electrostatic.

$$\boxed{\cancel{\vec{E}_g = \frac{-GM\vec{r}}{r^3}}}$$

$$\boxed{\vec{E}_g = \frac{-Gm\vec{r}}{r^3}}$$

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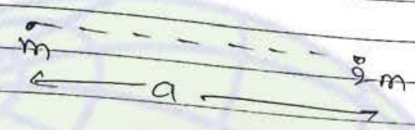
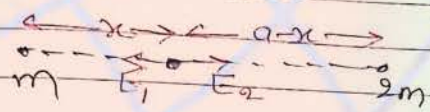
Note (In gravitation)

हर formula को जब vector form में convert करेंगे आगे "-ve" sign जरूर लगाएंगे।

Q)



Det. the point where Eg will be zero

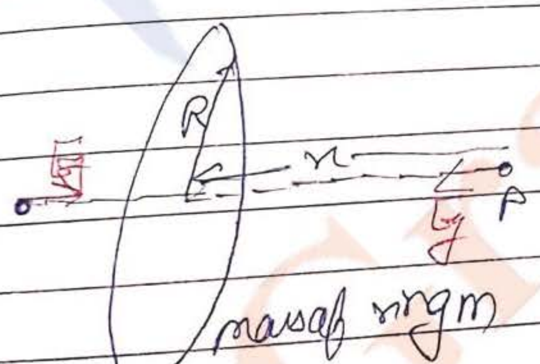


$$\frac{4m}{x^2} = \frac{4\times 2m}{(a-x)^2}$$

$$\cancel{x = \frac{4\times 2m}{(a-x)^2}}$$

$$x = \;—$$

Eg due to ring →



$$E = \frac{KQx}{(x^2+R^2)^{3/2}}\ , \quad \boxed{E_g = \frac{GMx}{(x^2+R^2)^{3/2}}}$$

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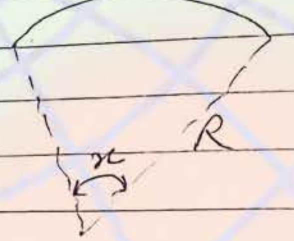
At the centre $E_g = 0$

at $x = \frac{R}{\sqrt{2}}$ E_g is max.

2. E_g due to arc



mass of Arc $= M$

E_g at $C = ?$

$$\boxed{E = \frac{\lambda \sin \alpha/2}{2\pi \varepsilon_0 R}} , \lambda = \frac{M}{R\alpha}$$

$$E_Q = \frac{\lambda \sin \alpha/2 \times 2}{2\pi \varepsilon_0 R \times 2}$$

$$\boxed{E_g = \frac{2G \sin \alpha/2 \times M}{R^2 \alpha}}$$

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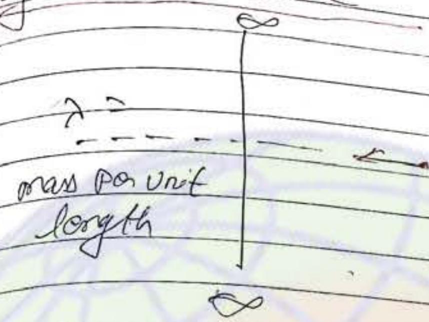
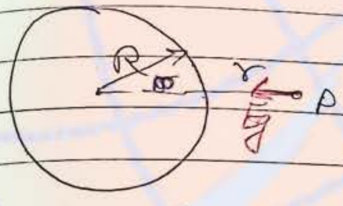
3) E_g due to wire



$$E_g = \frac{G \times 2\lambda}{r}$$

$$E = \frac{\lambda \times 2}{2\pi\varepsilon_0 r \times 2}$$

4) E_g due to hollow sphere



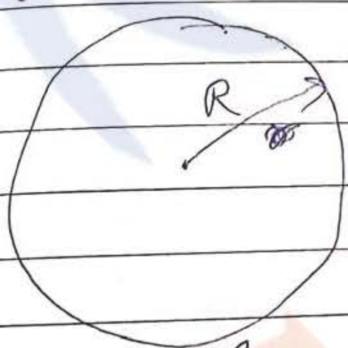
$r > R$, $E_g = \frac{GM}{r^2}$, $\frac{-GM\vec{r}}{r^3}$

$r = R$, $E_g = \frac{GM}{R^2}$

$r < R$, $E_g = 0$

mass of sphere M

5) E_g due to solid sphere



$r > R$, $E_g = \frac{GM}{r^2}$

$r = R$, $E_g = \frac{GM}{R^2}$

mass $= M$

$r < R$, $E_g = \frac{GMr}{R^3}$

$$E = \frac{\rho r}{3\varepsilon_0} = \frac{Qr}{\frac{4}{3}\pi R^3 \times 3\varepsilon_0}$$

$$\vec{E_g} = \frac{-GM\vec{r}}{R^3}$$

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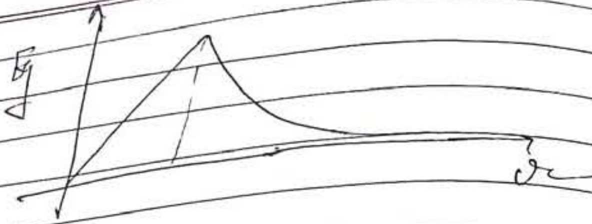
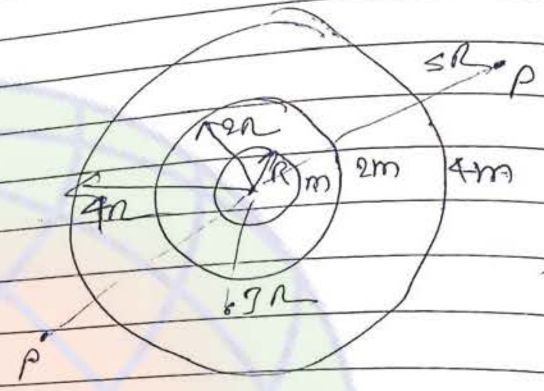


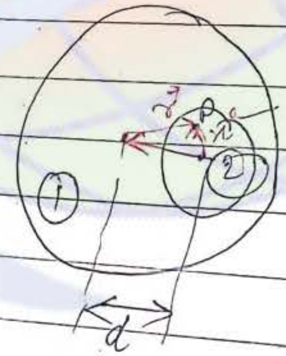
Q)



Det. Eg at
i) $r = 5R$
ii) $r = 3R$

i) $\frac{7Gm}{(5R)^2}$ ii) $\frac{3Gm}{(3R)^2}$

Q2) Det Eg at any point inside cavity → mass density ρ



① + ② → Complete गोला

$\vec{E_1} = \frac{4}{3}\pi G\rho \vec{x} \hookrightarrow -\frac{4\pi}{3}G\rho\vec{r}$

Sol^n $E_g = \frac{\rho r \times 4\pi}{3\varepsilon_0 \times 4\pi}$

$E_g = \frac{-4\pi G\rho \vec{r}}{3}$

$\vec{E_g} =$

$E_1 = \frac{4}{3}\pi G\rho \vec{d}$

$= \frac{4}{3}\pi G\rho(\vec{x} - \vec{r})$

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Simple Harmonic motion (S.H.M)

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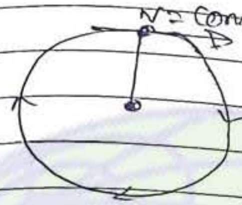
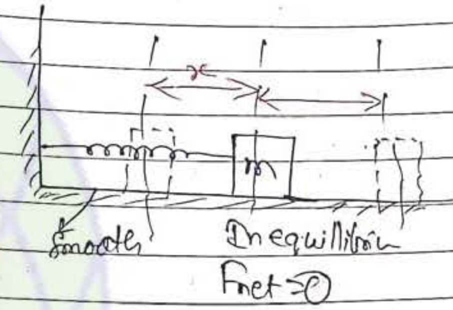
Periodic motion



Oscillatory / vibratory motion

↳ to and fro motion
or
back and forth



Note:→

$F = -kx$

linear Restoring force

Note:→ There must be Restoring force or Restoring torque on the "oscillatory system", (of linear motion)

Note:→ oscillatory must be Periodic. but Periodic may or may not be oscillatory

Note:→

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Note :→ which is Restoring force or not :→

i) $F = -2(x-2)^2$ — (✗) Not Restoring f...

ii) $F = -2(x-2)^3$ (✓)

iii) $F = -4x$ (✓)

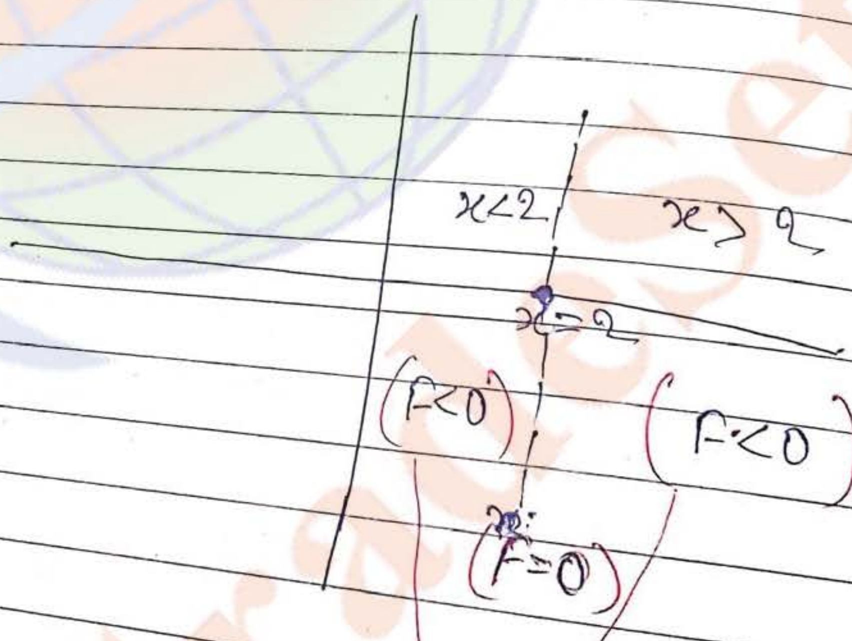
iv) $F = -3x^2$ (✗) ~~...~~ Not Restoring

v) $F = -4(x-3)^4$ (✗) 〃

vi) $F = -4x-8$ (✓)

Ans :→

i) $x = -2$, $F = 0$



→ Here force एकतरफा है। ~~...~~ ~~...~~ इसलिए मे है फिर back अपनी position पर नहीं आ पाएगा।

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iii) $F = -2(x-2)^3$

$x = 2, \quad F = 0$

$x<2$	$x>2$
$F>0$	$F<0$

$x=2$, $F=0$

→ (force ऋणात्मक नहीं है इसलिए प्रतिकरण back अपने position पर आ पायेगा।)

Note :- यदि 'x' पर Power even होगा तो force is not Restoring.

इसलिए

For Restoring force Power exponent of 'x' is odd.

iv) $F = -3x^2$

$F = 0$ in $x = 0$

$x<0$	$x>0$
$F<0$	$F<0$

$x=0$, $F=0$

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Condition of S.H.M ⇒

1) There must be a mean position / stable equilibrium position / centre of oscillation.

where
(F_{net} / τ_{net} = 0)
↓ linear S.H.M ↓ Angular S.H.M

2) There must be linear restoring force / ~~linear~~ Restoring torque acting on the oscillating particle or body.

$$\boxed{F = -Kx}$$

$$\boxed{\tau = -K \cdot \theta}$$

where
x → is the displacement of oscillating particle from mean position.

K → "+ve" Constant
→ Spring constant or force constant.

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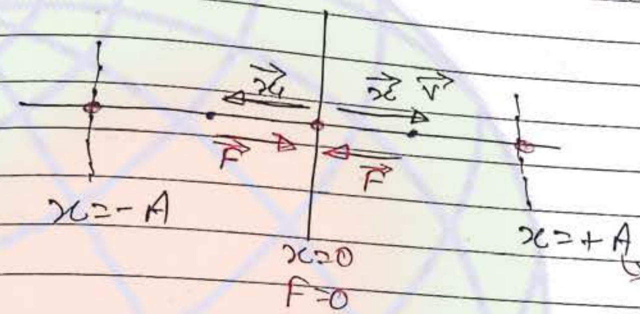
- Restoring force is always opposite of direction of displacement.
- Restoring force is always ~~oriented to~~ directed towards the mean position.



→ Amplitude of oscillator (max. displacement)

$$\vec{V} \times \vec{x} = \vec{0}$$
$$\vec{F} \times \vec{V} = \vec{0}$$

$a = \frac{-k}{m}$

$$\boxed{F = -kx}$$

$$a = -\left(\frac{k}{m}\right)x$$

$$\boxed{a = -\left(\frac{k}{m}\right)x}$$

$$\boxed{\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right) \cdot x = 0}$$

→ Differential equation of S.H.M.

Imp

$$\boxed{a = -\omega^2 \cdot x}$$

where

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

→ Angular frequency of oscillation.

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$$\boxed{T = \frac{2\pi}{\omega}}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

$$\boxed{T = \frac{1}{f}}$$

time period

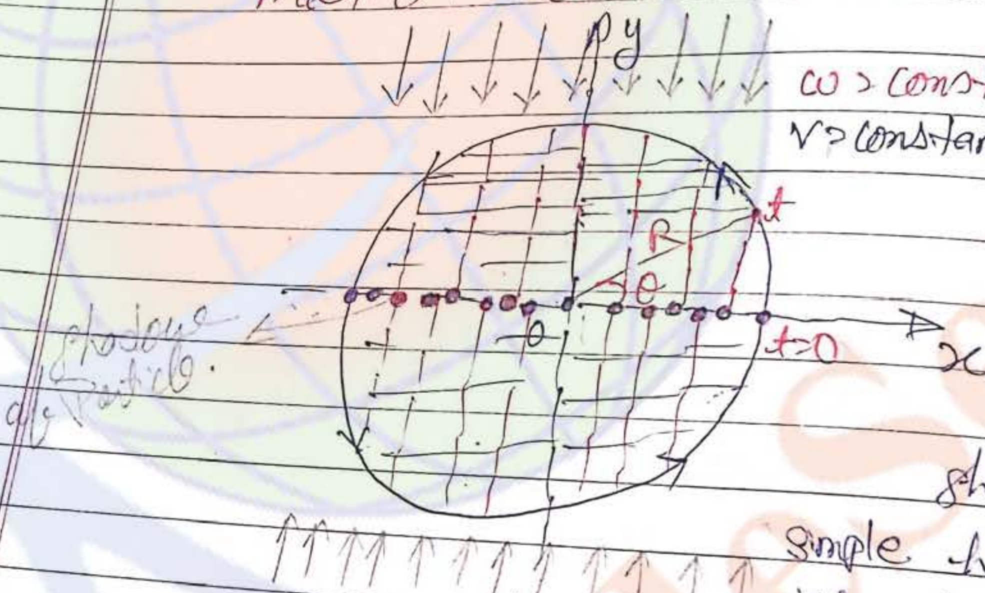
frequency

Relation between Uniform Circular motion and S.H.M :⇒

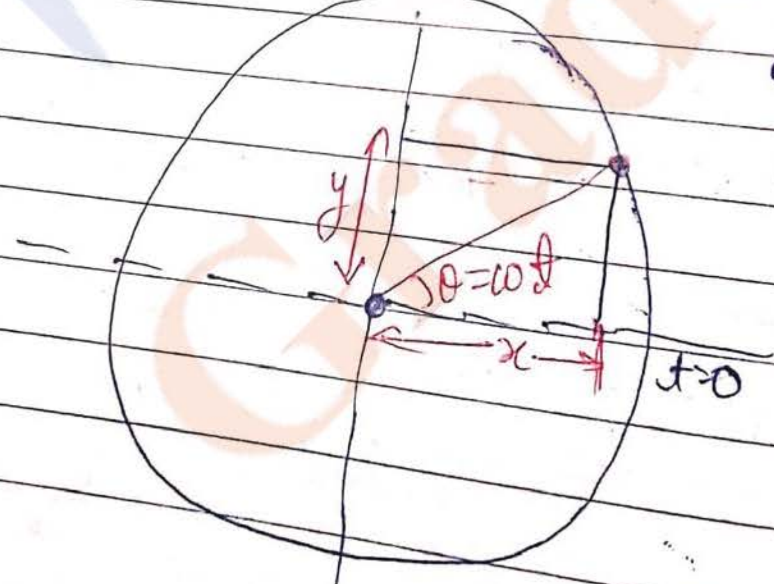
ω = constant.

v = constant.

$\theta = \omega t$



shadow is performing simple harmonic motion with Angular frequency (ω)



- For "shadow" "ω" is "Angular frequency" but
- for "particle" "ω" is constant angular speed

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$y = R\sin\omega t$

$v_y = \frac{dy}{dt} = (\omega R)\cos\omega t$

$a_y = -\omega^2 \cdot y$

$x = R\cos\omega t$

$a_x = -\omega^2 \cdot x$

Now :-

$$a_x = -\omega^2 \cdot x$$

$$v \cdot \frac{dv}{dx} = -\omega^2 \cdot x$$

$$\int_v^0 v \cdot dv = -\omega^2 \int_x^A x \cdot dx$$

$$\boxed{v = \pm\,\omega\sqrt{A^2 - x^2}}$$

$$\boxed{v = \omega\sqrt{A^2 - x^2}}$$

where $\omega \Rightarrow$ Angular frequency.

Now :-

$$\frac{dx}{dt} = \omega\sqrt{(A^2 - x^2)}$$

$$\int \frac{dx}{\sqrt{(A^2 - x^2)}} = \omega \int dt$$

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It is max. displacement of oscillating particle from the mean position on either side of mean position

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$$\sin^{-1}\frac{x}{A} = \omega t + \phi_0$$ w.r.t $t=0,\ x=0$

$$\frac{x}{A} = \sin(\omega t + \phi_0)$$

Imp

$$\boxed{x = A\sin(\omega t + \phi_0)}$$

Equation of S.H.M

where → $x \Rightarrow$ displacement at time "t" from mean position

~~where~~

$A \Rightarrow$ Amplitude of oscillation

or

It is max. displacement of oscillating particle from the mean position or either side of mean position.

velocity

$$\boxed{v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi_0)}$$

$$\boxed{a = \frac{dv}{dt} = -A\omega^2\sin(\omega t + \phi_0) = -\omega^2 x}$$

ϕ_0 → Initial phase or Phase constant.

→ ~~It gives the status of oscillating particle (position~~

It gives the status (position and direction of velocity) of oscillating particle at $(t = 0)$.

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$s \to v \to acc^n$

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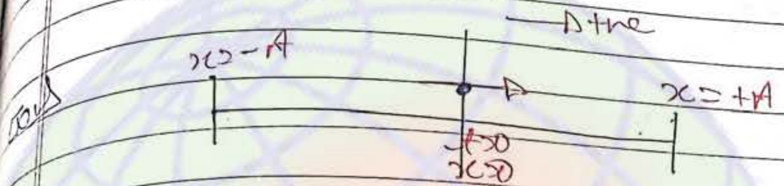
$(\omega t + \phi_0) \Rightarrow$ At any time "t" it gives the status of particle. (Phase)



, find $\phi_0 = ?$

at $t=0$
$x=0$

$0 = A\sin(\omega \times 0 + \phi_0)$

$\sin\phi_0 = 0$

$\phi_0 = 0, \pi, 2\pi \ldots$

$\cancel{x = A\sin(\omega t + \phi_0)}$

$v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi_0)$

$\frac{dv}{dt} = a = -A\omega^2\sin(\omega t + \phi_0)$

$\frac{dv}{dt} = a = -\omega^2 x$

Note:—

$x = A\sin(\omega t + \phi_0)$

$v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi_0)$

$v = A\omega\cos(\omega \times 0 + \pi)$

$(v = -ive)$

so,

$\boxed{\phi_0 = 0}$

$x = A\sin\omega t$

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Ex A particle oscillates simple harmonically along x-axis whose eqⁿ of S.H.M is given by

$$x = A\sin(\omega t + \phi_0)$$

at $t=0$, the particle passes through $x = +\frac{A}{2}$ and moving ~~fas~~ towards "-ve" x-axis.

Find the value of ϕ_0.

Ans: At $t=0$

$$\frac{A}{2} = A\sin\phi_0$$

$$\sin\phi_0 = \frac{1}{2}$$

$$\phi_0 = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$V = A\omega\cos(\omega t + \phi_0)_{t=0}$$

$$= A\omega\cos\left(\frac{5\pi}{6}\right) < 0$$

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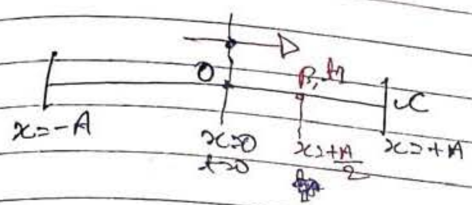

(T) → Time period of oscillation.

$(t_{OB})_{min} = t_1$

$(t_{BC})_{min} = t_2$

$(t_{OC})_{min} = t_3$

1) ~~tp~~

Correct Relation :-

$t_1 < t_2 < t_3$

$t_1 = ?$

$t_2 = ?$

$t_3 = ?$

Q.) Now, In the above question find t_1, t_2, and t_3?

$x = A\sin\omega t$

$\frac{\cancel{A}}{2} = \cancel{A}\sin\omega t_1$

$t_1 = \frac{\pi}{6\omega} = \frac{\cancel{\pi} T}{6 \times 2\cancel{\pi}} = \frac{T}{12}$, $t_2 = \frac{T}{6}$

$\cancel{A} = \cancel{A}\sin\omega t$

$t_3 = \frac{\pi}{2\omega} = \frac{\cancel{\pi} T}{2 \times 2\cancel{\pi}} = \frac{T}{4}$

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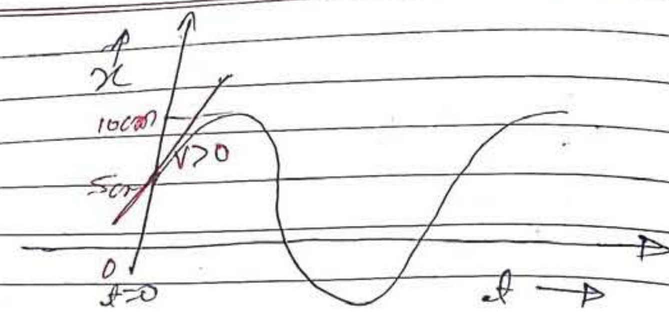
Q.18



$\omega = ?$

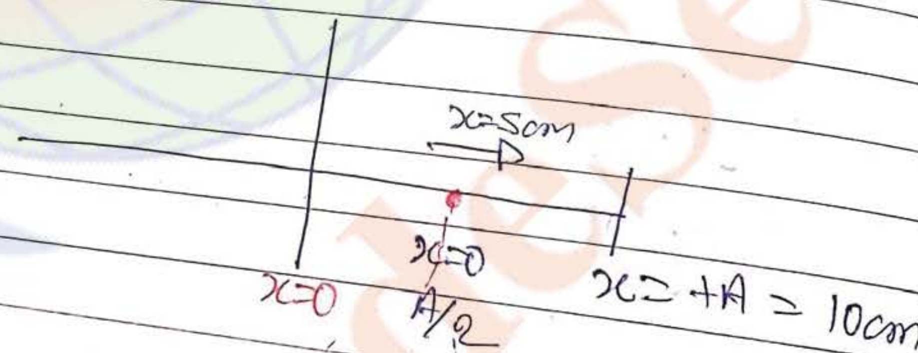
$$x = a\sin(\omega t + \phi_0)$$

$$\phi_0 = ?$$

Now,

$t=0, \quad x = 5cm$

$\hookrightarrow v > 0$



So,

$$\phi_0 = \pi/6$$

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1st Choice

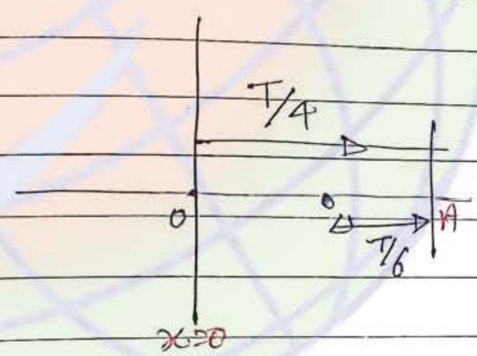
Q) A particle is oscillating simple harmonically and it passes through mean position at $t=0$ towards the x-axis.

1) Find the average speed of the particle during the time interval of $3/8$th oscillation.
where $T \rightarrow$ time period of oscillation
and $A \rightarrow$ Amplitude of oscillation.

Ans→

1 oscillation $\rightarrow 4A$

$\frac{3}{8}$ oscillation $= \cancel{4}A \times \frac{3}{\cancel{8}_{2}} = \frac{3A}{2}$



$$\Delta t = \frac{T}{4} + \frac{T}{6} = \frac{5T}{12}$$

$$V_{avg} = \frac{3A}{\cancel{2}} \times \frac{\cancel{12}^{6}}{5T} = \frac{18A}{5T}$$

$$|\vec{v}_{avg}| = \frac{A}{\cancel{2}} \times \frac{\cancel{12}^{6}}{5T} = \frac{6A}{5T}$$

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Equation of motion of S.H.M.

Note: ⇒ $x = A\sin(\omega t + \phi_0)$

$$v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi_0)$$

$$a = \frac{dv}{dt} = -\omega^2 x$$

$$F = -kx \quad \Big| \quad \omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega}$$

$$\left(\phi_0 = 0 \xrightarrow{} \text{at } t=0,\ x=0,\ v=+ve\right)$$

$$\left(\frac{x}{A}\right)^2 = \sin(\omega t + \phi_0)^2$$

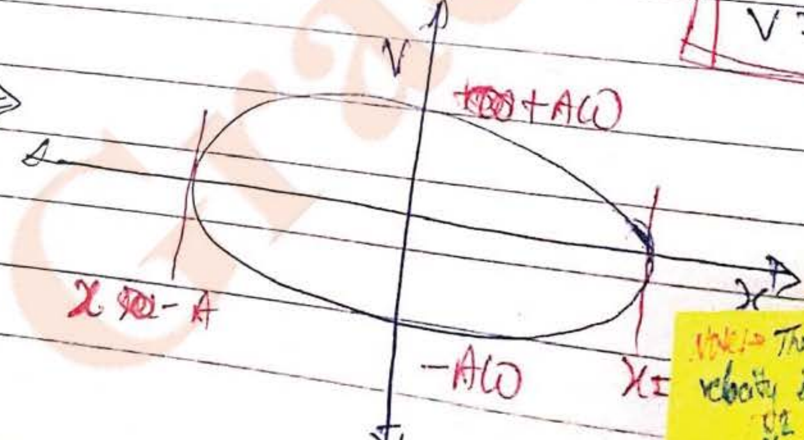
$$+ \quad \left(\frac{v}{A\omega}\right)^2 = \cos(\omega t + \phi_0)^2$$

$$\boxed{\frac{x^2}{A^2} + \frac{v^2}{(A\omega)^2} = 1} \Rightarrow v = \pm\omega\sqrt{(A^2 - x^2)}$$

Equation of ellipse.

$$\boxed{v = \omega\sqrt{A^2 - x^2}}$$

Graph of velocity/speed(v) vs displacement(x) ⇒



Note:- The graph b/w displacement and velocity is ellipse as

$$\frac{x^2}{A^2} + \frac{v^2}{(A\omega)^2} = 1$$

If $\omega = 1$, this curve would be a circle!

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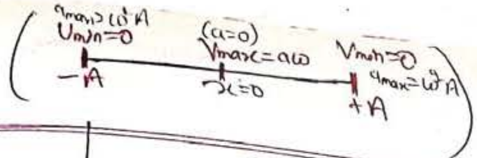
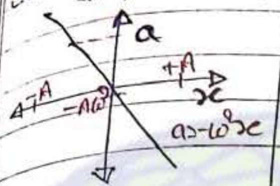
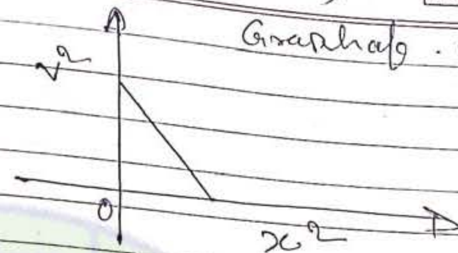


Note: Graph b/w Acc(a) vs Displacement(x)



Graph b/w speed(v^2) vs Displacement(x^2)



1) At the mean position :—

$$x=0$$

$$\boxed{V_{max} = A\omega}$$

$$\boxed{(a=0)}$$

2) At the extreme position :—

$$(x = \pm A)$$

$$\boxed{V=0}$$

$$\boxed{|a_{max}| = \omega^2 A}$$

$$\cancel{\phi_0} = 0$$

K.E (K)

Kinetic energy

$$K = \frac{1}{2} m v^2$$

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$U_{max} = \frac{1}{2}m\omega^2A^2$ at $x=-A$; $K_{max} = \frac{1}{2}m\omega^2A^2$ at $x=0$; $U_{max} = \frac{1}{2}m\omega^2A^2$ at $x=+A$

$$\boxed{K = \frac{1}{2}m\omega^2(A^2 - x^2)}$$

$K_{max} = \frac{1}{2}m\omega^2A^2$ (at $x=0$ mean)

Potential Energy (U)

$$F = -kx$$

$$\boxed{U = \frac{1}{2}kx^2}$$

$$\boxed{U = \frac{1}{2}m\omega^2x^2}$$

At the mean Position ($x=0$)

P.E $= 0$ (Assume) (मान लिया है)

$$\boxed{U_{max} = \frac{1}{2}m\omega^2A^2}$$

→ at $x = \pm a$

$K \Rightarrow K$

$$\boxed{E = K_{max} = U_{max} = \frac{1}{2}m\omega^2A^2}$$

$\because$ $\boxed{U_{min} = 0, (x=0)}$

→ $K+U$

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(P.E is min at mean position and max. at extreme position.)

If $\phi_0 = 0$

⇒ K.E (K)

$$K = \frac{1}{2} m v^2$$

$$\boxed{K = \frac{1}{2} m \omega^2 A^2 (\cos^2 \omega t)}$$

$$K = \left(\frac{1}{4} m \omega^2 A^2\right)(1 + \cos 2\omega t) \quad \text{--- (1)}$$

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$$

$$\boxed{f = \frac{1}{T}}$$

linear frequency

⇒ P.E

$$\boxed{U = \frac{1}{2} m \omega^2 A^2 (\sin^2 \omega t)}$$

$$U = \frac{1}{4} m \omega^2 A^2 (1 - \cos 2\omega t) \quad \text{--- (2)}$$

$$\boxed{E = K + U = \frac{1}{2} m \omega^2 A^2 = \text{constant}}$$

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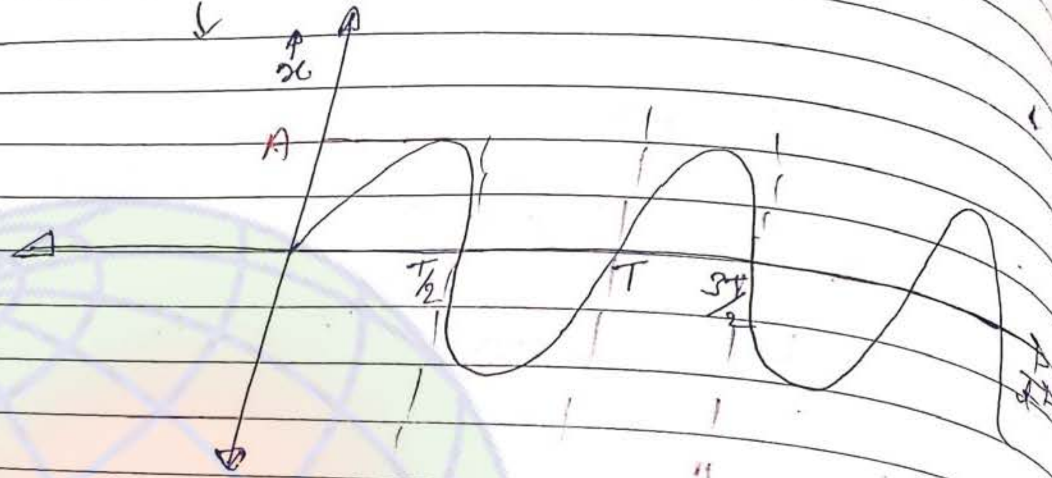
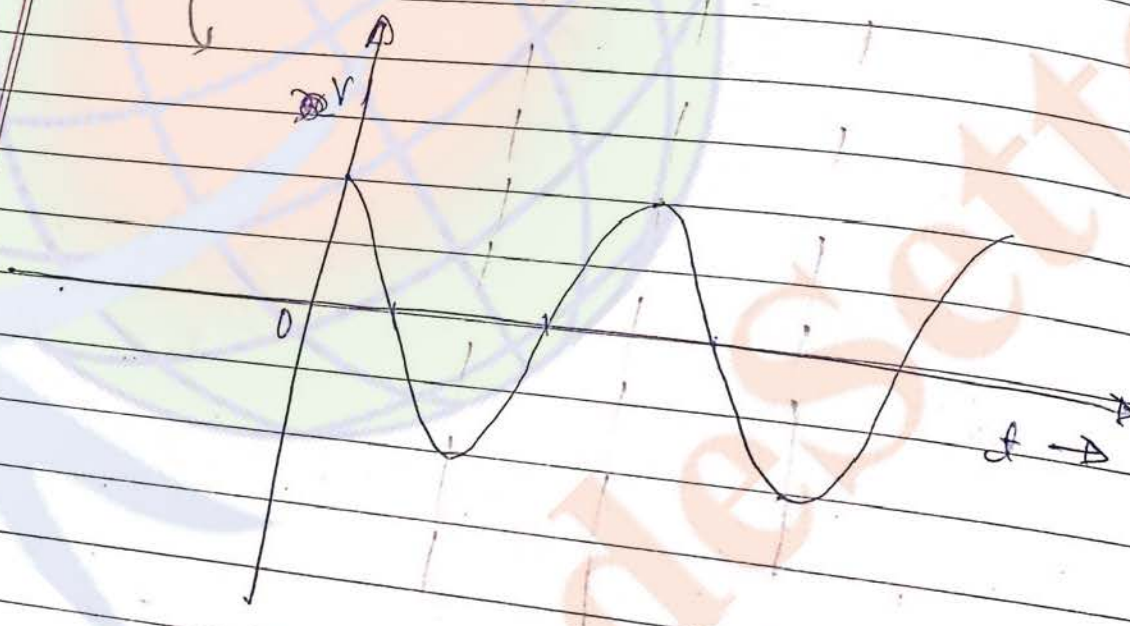
Graphs

$x = A \sin \omega t$



$v = A\omega \cos \omega t$



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1st Choice

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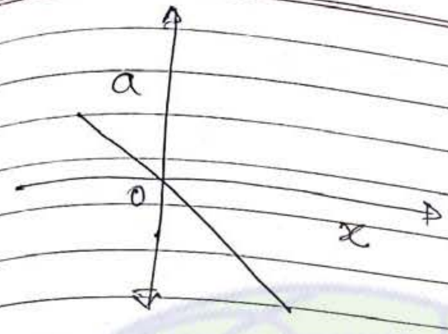

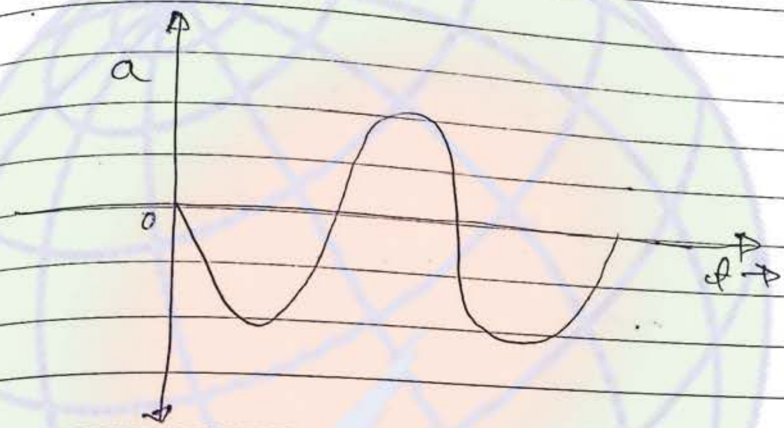

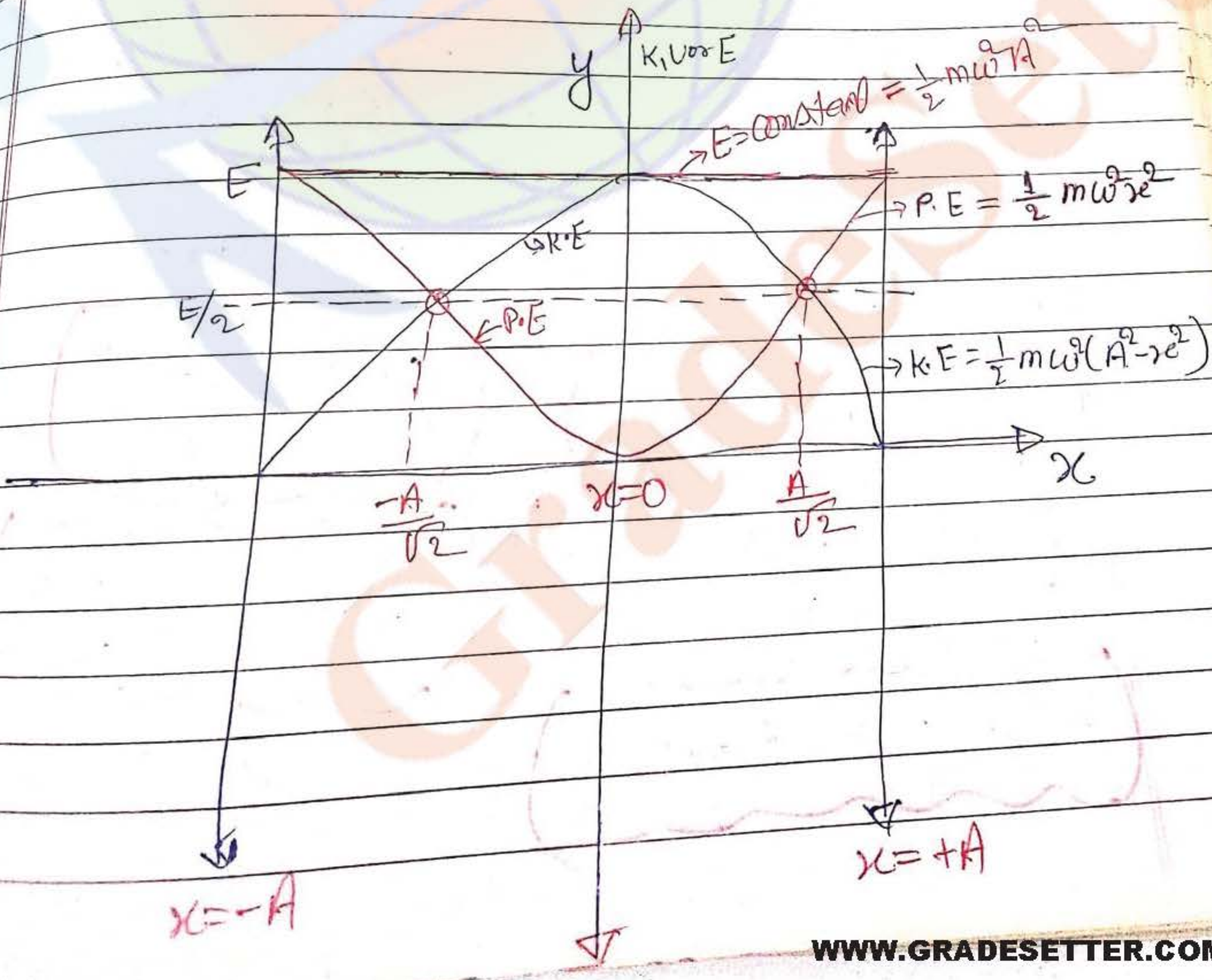

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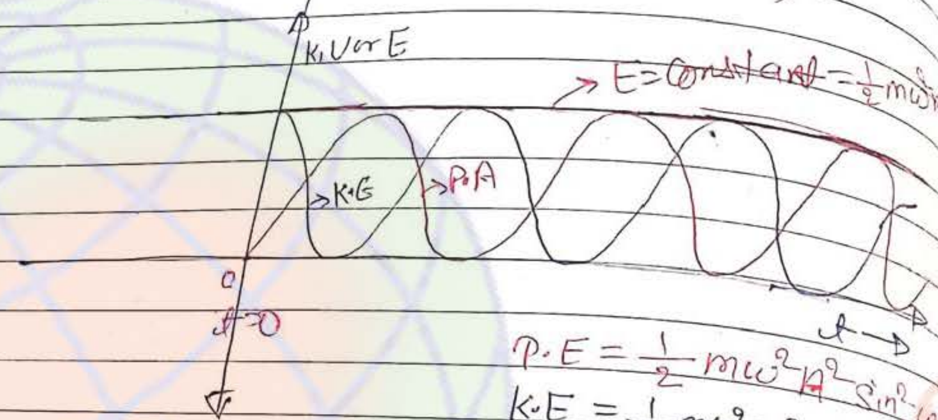
$$\left(K = \frac{1}{2} m\omega^2 (A^2 - x^2) \right.$$

$$= \frac{1}{2} m\omega^2 A^2 - \frac{1}{2} m\omega^2 x^2$$

$$\left. K = \alpha A^2 - \alpha x^2 \right)$$



$$P.E = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t$$

$$K.E = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$$

Q.) i) Find the ~~avgs~~ $(K.E)_{avg}$ and $(P.E)_{avg}$ for 1 time period (T). $\Rightarrow (x=0, \; t=T)$

ii) $K_{avg} \rightarrow$ Position average
$U_{avg} \rightarrow$ Position average $\Rightarrow$ for $x=0$ to $x=A$

Ans) For 1T or nT
$n = 1, 2, 3 \ldots$

$$\boxed{(\sin \omega t)_{avg} = (\cos(\omega t))_{avg} = 0}$$

~~$\Rightarrow \cos(\omega t)_{avg}$~~

$$\boxed{(\sin^2 \omega t)_{avg} = (\cos^2 \omega t)_{avg} = \frac{1}{2}}$$

$$\boxed{f(t)_{\text{time avg}} = \frac{\int_{t_1}^{t_2} f(t)\,dt}{\int_{t_1}^{t_2} dt}}$$

$$\boxed{f(x)_{\text{position avg.}} = \frac{\int f(x)\,dx}{\int dx}}$$

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sol

(i) $K = \frac{1}{2} m\omega^2 A^2 (\cos^2 \omega t)$

$\left(K_{avg} = \frac{1}{4} m\omega^2 A^2\right)$

$U = \frac{1}{2} m\omega^2 A^2 (\sin^2 \omega t)$

$\left(U_{avg} = \frac{1}{4} m\omega^2 A^2\right)$

$$K_{avg} = \frac{\frac{1}{2} m\omega^2 A^2 \int_0^{2\pi/\omega = T} \cos^2 \omega t \, dt}{\int_0^{T = \frac{2\pi}{\omega}} dt}$$

(ii)

$$K_{avg\ position} = \frac{\frac{1}{2} m\omega^2 \int_{x=0}^{x=A} (A^2 - x^2)\, dx}{\int_0^A dx}$$

sol

$$\boxed{K_{avg\ position} = \frac{1}{3} m\omega^2 A^2}$$

$$U_{position\ avg} = \frac{\frac{1}{2} m\omega^2 \int_0^A x^2\, dx}{\int_0^A dx} = \frac{1}{6} m\omega^2 A^2$$

sol

$$\boxed{U_{position\ avg} = \frac{1}{6} m\omega^2 A^2}$$

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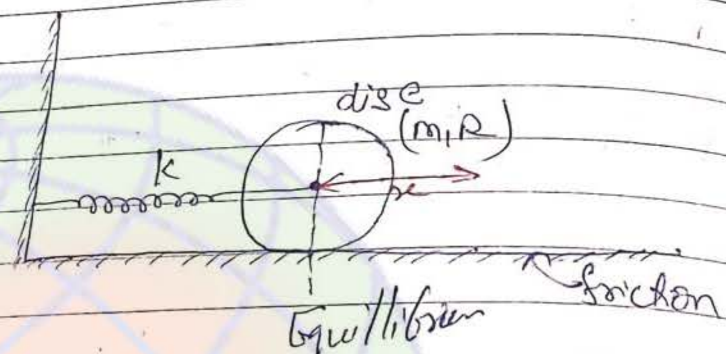
Force method to determine time Period or frequency of S.H.M ⇒

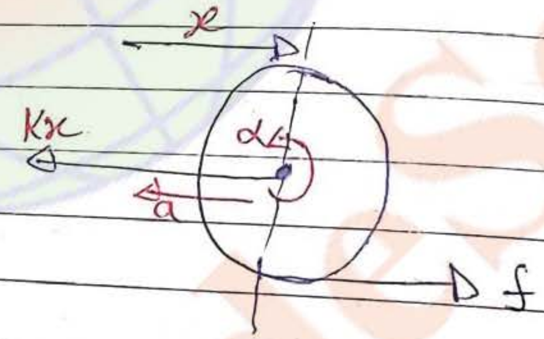
Ex:⇒



Disc rools over the surface without slipping.

1) Find the time period of oscillation.

Ans ⇒



$$Kx - f = ma \quad \text{—} \; (1) \qquad (\text{force})$$

$$f \times R = \frac{mR^2}{2} \cdot \alpha$$

$$(a = R\alpha)$$

$$f = \frac{ma}{2} \quad \text{—} \; (2) \qquad (\text{torque } \tau = I\alpha)$$

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$kx - f = 2f$ (from eq (1) and (2))

$f = \frac{kx}{3}$

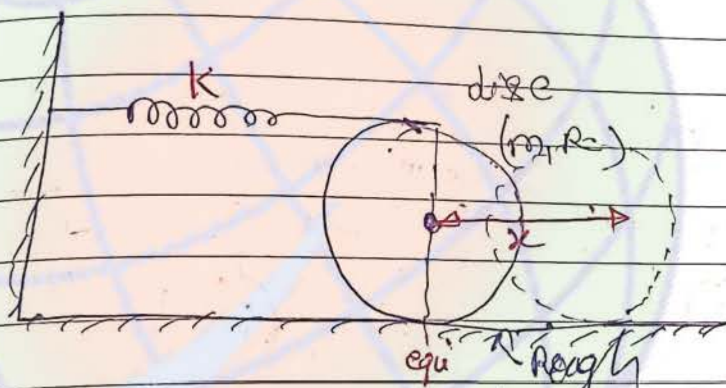
$F_{net} = f - kx$ (SHM)

$= \frac{kx}{3} - kx$

$= -\underbrace{\frac{2k}{3}}_{K_{eff} = \frac{2k}{3}} x$

$F = -\frac{2k}{3}x$

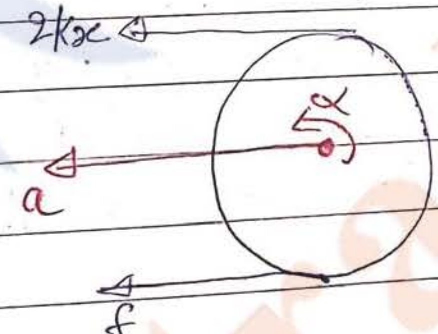
$T = 2\pi\sqrt{\frac{3m}{2k}}$



(Disc rolls without slipping.)

Find the time period of oscillation

Ans)



$2kx + f = ma$ —— (1) (force)

$2kx \times R - fR = \frac{mR^2}{2} \cdot \alpha$ (torque) ($\tau = I\alpha$)

$2kx - f = \frac{ma}{2}$ —— (2)

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from eq (I) and (II)

$$f = \frac{2kx}{3}$$

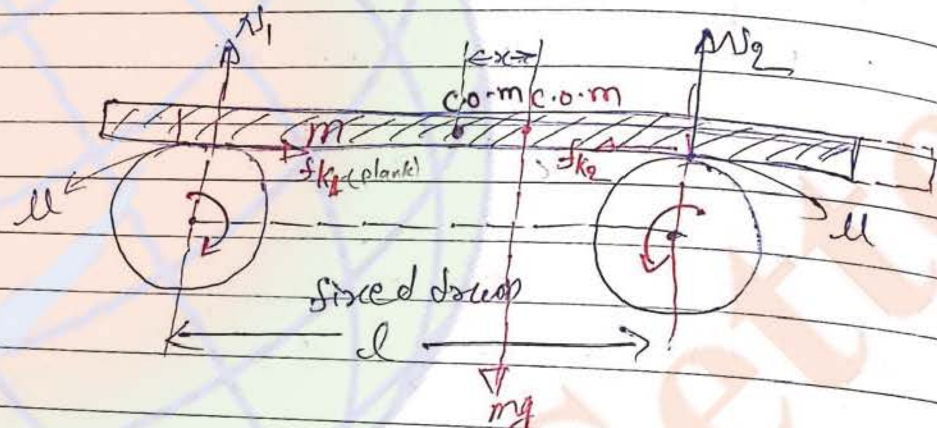
$$F_{net} = -\left(2kx + \frac{2kx}{3}\right)$$

$$= -\frac{8k}{3}x \rightarrow k_{eff} = \frac{8k}{3}$$

$$T = 2\pi\sqrt{\frac{3m}{8k}}$$

Irodov + HCV

Ex



Plank is in equillibrium position in the given condition.

If the plank is slightly disturb in horizontal direction it oscillate simple harmonically find the time period of oscillation

Ans :-

$$N_1 + N_2 = mg \quad \text{— (1)}$$

$$mg\left(\frac{l}{2} + x\right) = N_2 \cdot l$$

$$N_2 = \frac{mg}{l}\left(\frac{l}{2} + x\right)$$

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$N_1 = mg - N_2$

$\Rightarrow mg - \frac{mg}{l}\left(\frac{l}{2} + x\right)$

$\Rightarrow mg - \frac{mg}{2} - \frac{mgx}{l}$

$\Rightarrow \frac{2mgl - mgl - 2mgx}{2l}$

$\Rightarrow \frac{mg}{l}\left(\frac{l}{2} - x\right)$

$f_{k_2} > f_{k_1}$

$F_{net} = f_{k_1} - f_{k_2}$

$= \mu (N_1 - N_2)$

$= -\mu \frac{2mgx}{l}$

$T = 2\pi \sqrt{\frac{ml}{2\mu mg}}$

$T = 2\pi \sqrt{\frac{l}{2\mu g}}$

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Q: Speed of a particle oscillating simple harmonically along x-axis changes with position x according to given relation; $v = \sqrt{\alpha - \beta x^2}$ where α and β are some "+ve" constants.

i) Find the Angular frequency "ω"

Ans:

$$v = \sqrt{(\alpha - \beta x^2)}$$

$$v^2 = \alpha - \beta x^2$$

diff. w.r.t to x

$$\cancel{2} v \underbrace{\frac{dv}{dx}}_{(a)} = 0 - \cancel{2} \beta x$$

$$a = -\beta x$$

$$\omega = \sqrt{\beta}$$

$$a = -\omega^2 x$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\beta}}$$

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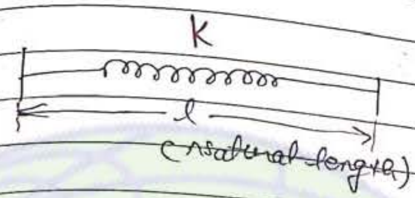
1st Choice

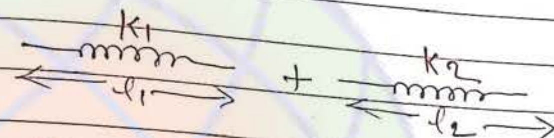
Problem based on Spring-mass system.

(i)



$$\boxed{k \propto \frac{1}{l}}$$

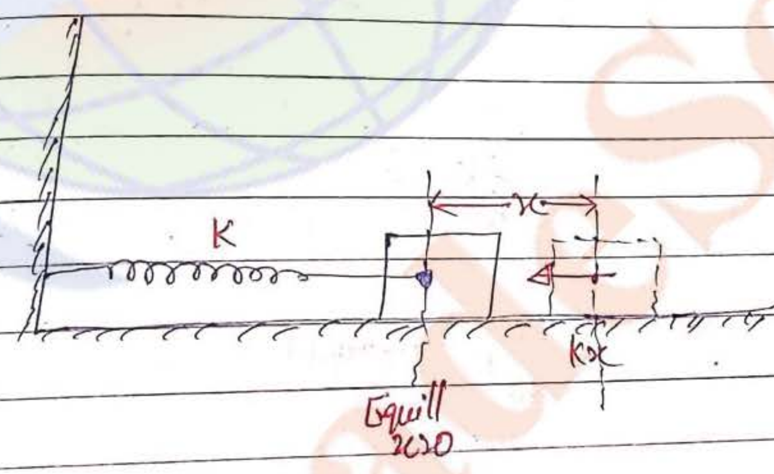
$\Downarrow$



$$\boxed{k_1 = k\left(1+\frac{l_2}{l_1}\right)} \quad \text{or} \quad \boxed{l_1 = l \cdot \frac{m_2}{m_1+m_2}}$$

$$\boxed{k_2 = k\left(1+\frac{l_1}{l_2}\right)} \quad \text{or} \quad \boxed{l_2 = l \cdot \frac{m_1}{m_1+m_2}}$$

(ii)



$$F = -kx$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}} \qquad \omega = \sqrt{\frac{k}{m}}$$

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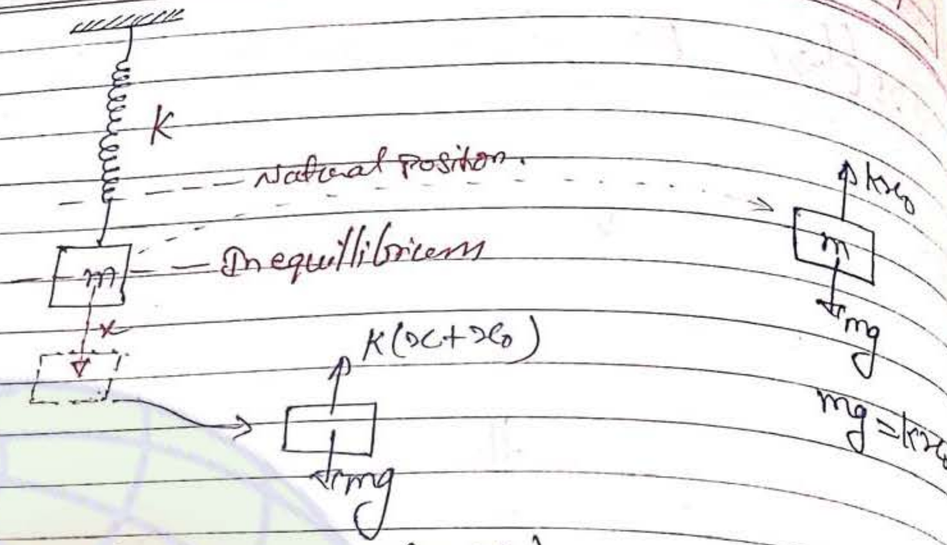
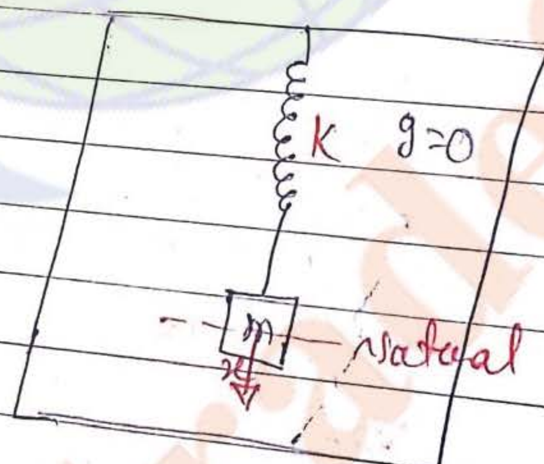


$$F_{net} = mg - K(x+x_0)$$
$$= -Kx$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

Note ⟹ when body is kept in Gravity free position.



$$\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$$

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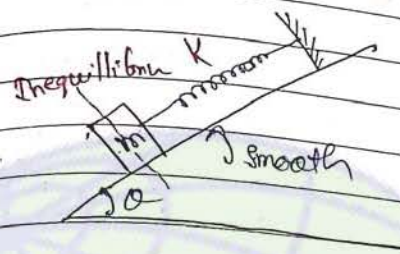
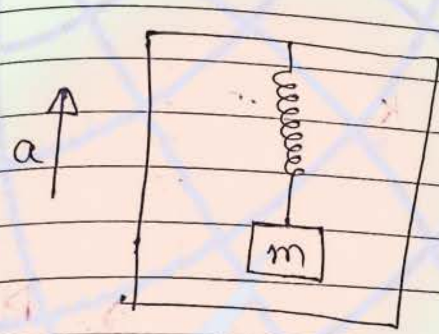
Spring Pendulum :-



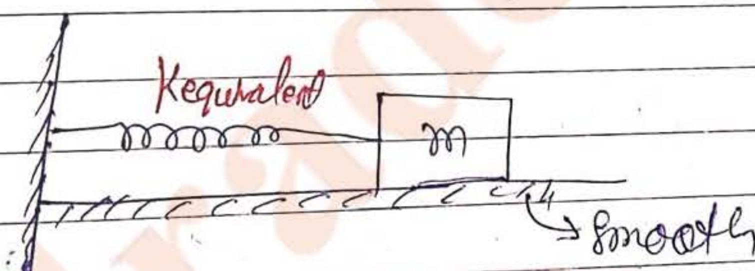
$$T = 2\pi\sqrt{\frac{m}{k}}$$



$$T = 2\pi\sqrt{\frac{m}{k}}$$

So, Here in this ~~period~~ position, Time period is not change in any case.

Note :⇒



$$T = 2\pi\sqrt{\frac{m}{K}}$$

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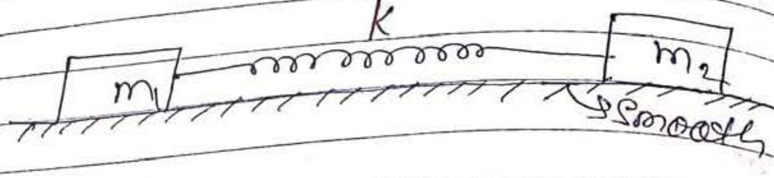
1st Choice

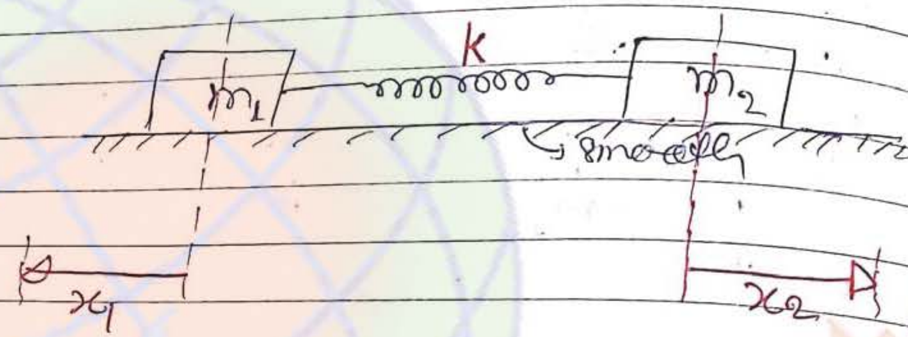
H.C.V

Eg:-



Find the Time period of oscillator (where spring is slightly displaced in opposite direction / compressed and then released and oscillates simple harmonically) values

Ans:-



$$x = x_1 + x_2 \quad \text{---(1)}$$

$$\frac{d^2x}{dt^2} = \frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2}$$ (Double diff. of Distance)

Double differentiation of distance gives acc^n $\frac{d^2x}{dt^2}$

$$m_1 \frac{d^2x_1}{dt^2} = -kx$$

$\because a = \frac{d^2x_1}{dt^2}$

$$m_2 \frac{d^2x_2}{dt^2} = -kx \quad \text{---(2)}$$

$$\{eq(1)\} \times m_2 + \{eq(2)\} \times m_1$$

$$m_1 m_2 \left(\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2}\right) = -kx(m_1 + m_2)$$

$$m_1 m_2 \frac{d^2x}{dt^2} = -k \cdot x\,(m_1 + m_2)$$

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$$a = \frac{d^2x}{dt^2} = -\left\{\frac{k(m_1+m_2)}{m_1 m_2}\right\} \cdot x$$

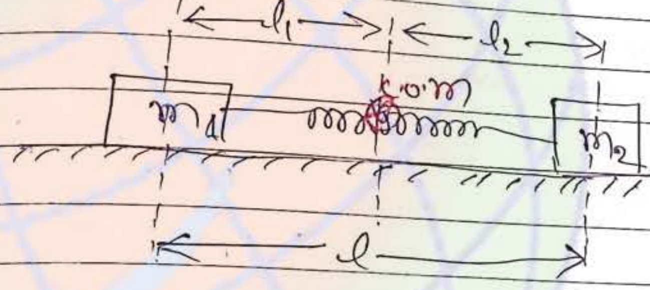
$$\omega = \sqrt{\frac{k}{\mu}} \qquad \mu = \frac{m_1 m_2}{(m_1+m_2)}$$

Reduced mass.

$$T = \frac{2\pi}{\omega}$$

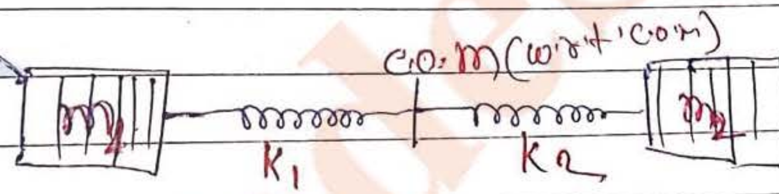
short method :->

Alt 1 ->



$$T = 2\pi\sqrt{\frac{m_2}{k_2}} \qquad l_1 = l \cdot \frac{m_2}{(m_1+m_2)} \qquad l_2 = l \cdot \frac{m_1}{(m_1+m_2)}$$

$$T = 2\pi\sqrt{\frac{m_1}{k_1}}$$



$$T = 2\pi\sqrt{\frac{m_1 m_2}{k(m_1+m_2)}}$$

$$k_1 = k\left(1 + \frac{l_2}{l_1}\right)$$

$$\omega = \frac{2\pi}{T} \qquad = k\left(1 + \frac{m_1}{m_2}\right) = k\left(\frac{m_1+m_2}{m_2}\right)$$

$$= \sqrt{\frac{k}{\mu}}$$

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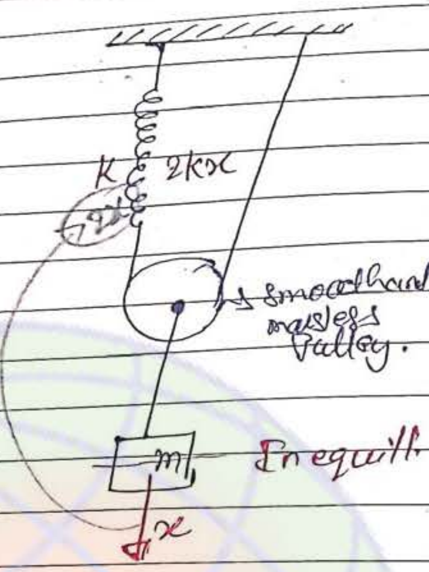
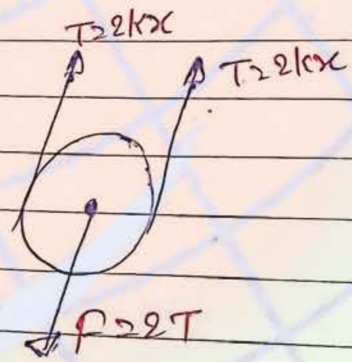
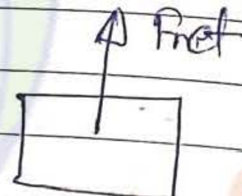


Block is slightly displaced vertically in downward direction.

1) Find the time period of oscillation of block.

FBD





$$F_{net} = -2T$$
$$= -4k\,x$$

$$T = 2\pi\sqrt{\frac{m}{4k}}$$

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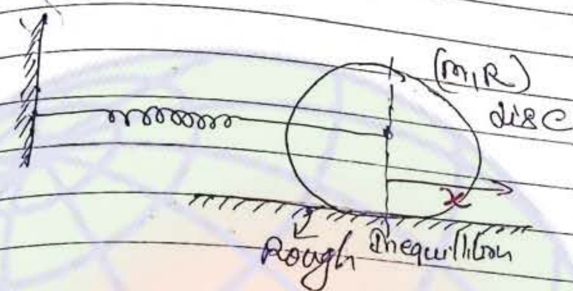
Energy Method to calculate the time period of oscillation or frequency of oscillation.



Disk rools without slipping over the surface
// Find the time period of oscillation.

Working Steps :-

Step "1" :⇒ Find the total mechanical energy (E) of the oscillating system in displaced position "x" (at mean position.)

Step 2 :- $\boxed{\frac{dE}{dt} = 0}$; $\boxed{v = \frac{dx}{dt}}$

$\boxed{a = \frac{dv}{dt}}$

Step 3 :-

$\boxed{a = -\omega^2 x}$

So, Solution :-

Step 1 :-

$$E = \underbrace{\frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\left(\frac{v^2}{R^2}\right)}_{\text{(Disk)}} + \underbrace{\frac{1}{2}kx^2}_{\text{Spring}}$$

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$$E = \frac{3}{4}mv^2 + \frac{1}{2}kx^2$$

Step 2 :—

$$\frac{dE}{dt} = \left(\frac{3m}{4_2}\right)\cdot 2v\cdot\frac{dv}{dt} + \frac{1}{2}k\cdot 2x\left(\frac{dx}{dt}\right) = 0$$

$$\left(\frac{3m}{2}\right)\cancel{v}\cdot(a) = -kx\cancel{v}$$

$$a = -\frac{2k}{3m}\cdot x$$

ω^2

$$\omega = \sqrt{\frac{2k}{m}}$$

$$T = 2\pi\sqrt{\frac{3m}{2k}}$$

Ex ⇒

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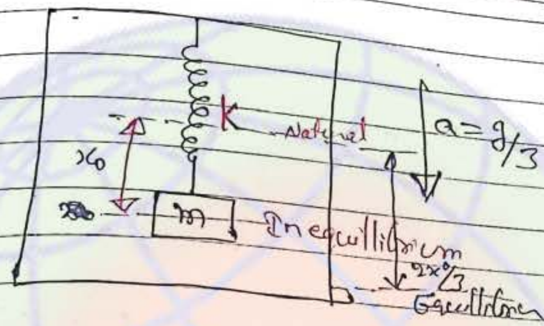
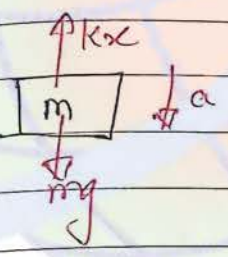
Ex 1 → 12, 13, 14
Ex 2 → 3, 5, 7, 8, 11, 12, 13, 14, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 33, 34, 35
Ex 3 → 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 - 21

Problem based on Amplitude



Initial equillibrium extension in the spring is $x_0 = \frac{mg}{K}$

Find the Amplitude of oscillation of the Block

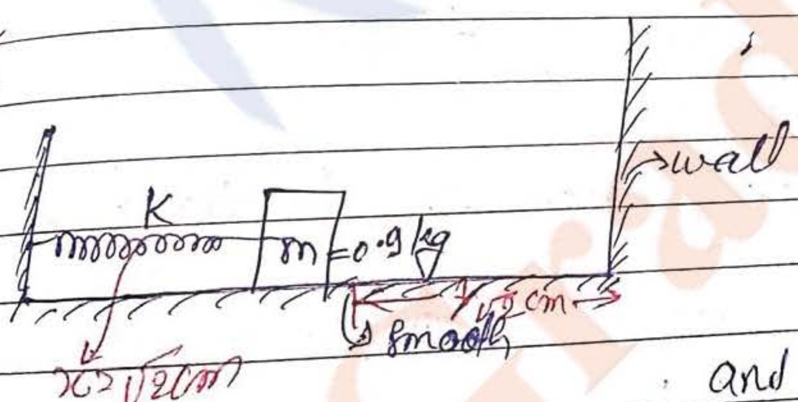


$mg - kx = ma$

$kx = mg - \frac{mg}{3}$

$x = \frac{2mg}{3K}$

$x = \frac{2x_0}{3}$ so, Amplitude $x_0 - \frac{2x_0}{3} = \frac{x_0}{3}$



$K = 100\ N/m$

Collision b/w the block and the wall is elastic.

1) Find the time period of oscillation of the block?

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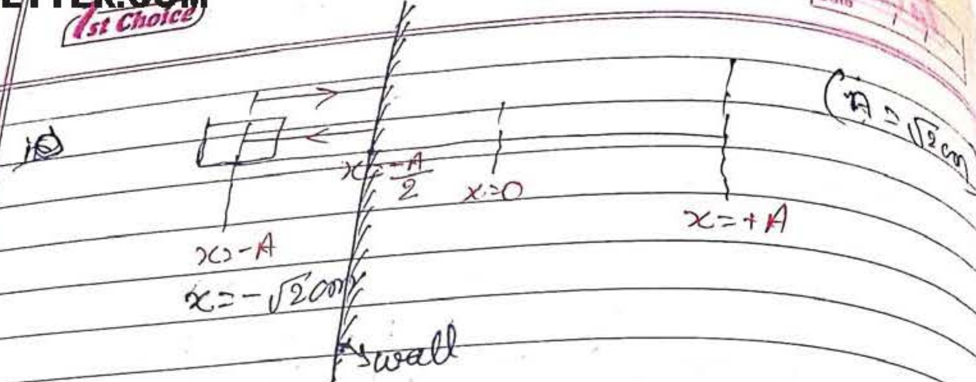


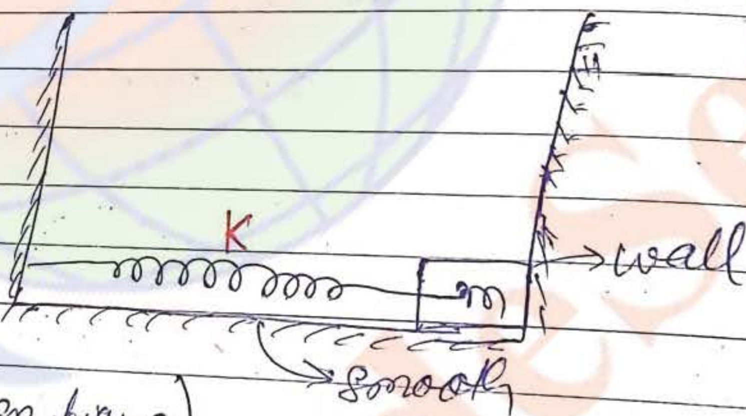
$t_0 = \frac{2T}{6} = \frac{T}{3}$

where $T = 2\pi\sqrt{\frac{m}{K}}$

so,

$T = \frac{2\pi}{3}\sqrt{\frac{m}{K}}$ Ans

Q5 -



Ans (In the given figure)
Initially the compression in the spring is x_0. Spring is further compressed by x_0 and then released,
1) Find the time period of oscillation of the block.
collision b/w the block and the wall is elastic.

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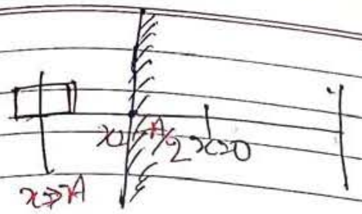
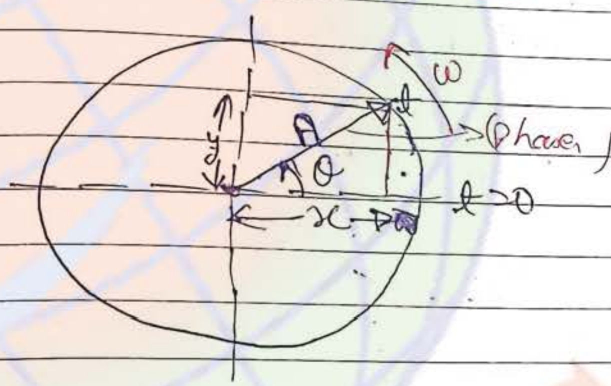
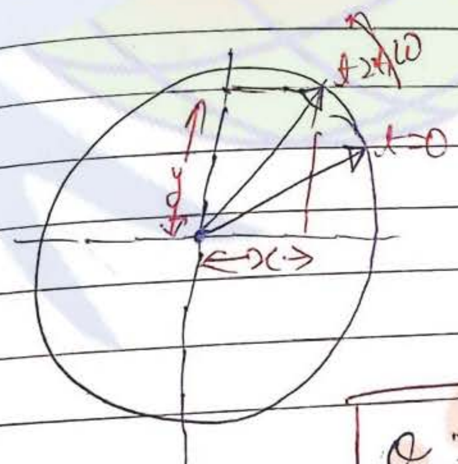


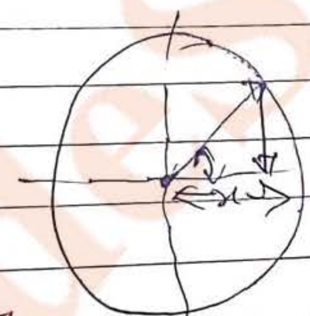
$$t_6 = \frac{2T}{6} = \frac{T}{3} = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$$

→ It Indicate phase (Status of particle)

Phasor method :→







$$\boxed{\theta = \omega t}$$

$$\boxed{\begin{aligned} x &= A\cos(\theta + \phi_0) \\ x &= A\cos(\omega t + \phi_0) \\ y &= A\sin(\omega t + \phi_0) \end{aligned}}$$

(std)

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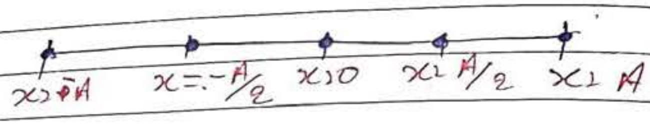
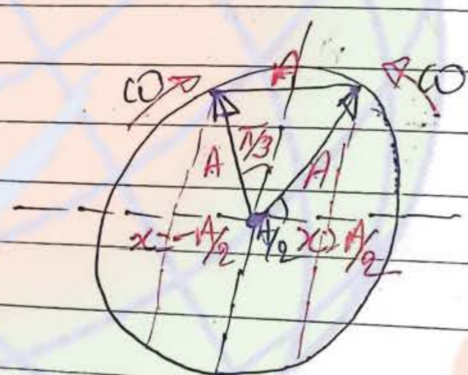
Q:>



Two particles are oscillating simple harmonic with same time period and about same mean position and with same Amplitude. Amplitude of oscillation is 20cm, max. distance b/w two particles is 20cm.

1) Find the time after which ~~they~~ they will meet.

Ans:-



$$\frac{\pi}{3} = \omega t + \omega t$$

$$\frac{\pi}{3} = 2\omega t$$

$$\omega t = \frac{\pi}{6}$$

$$t = \frac{\pi}{6\omega}$$

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Angular S.H.M

$$\boxed{\tau_{net} = -k\cdot\theta = I\alpha}$$

$m \to I_{m.o.I.}$

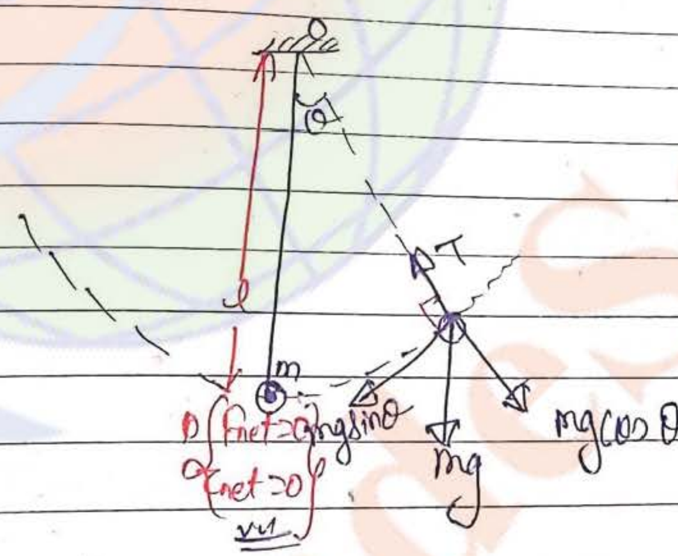
$$\boxed{\alpha = -\omega^2\theta}$$

$\therefore a = -\omega^2 x$

$$\boxed{\omega = \sqrt{\frac{k}{I}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{I}{k}}}$$

→ Simple Pendulum :⇒



$$\tau_0 = -(mg\sin\theta)\,l$$

$$\tau_0 = -(mgl)\sin\theta$$

→ (Not S.H.M)

For very small Angle "θ"

$$\sin\theta \simeq \theta$$

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$$\tau_0 = -(mgl)\theta$$

$$\left(\omega = \sqrt{\frac{mgl}{I_0}}\right)$$

$$\left(T = 2\pi\sqrt{\frac{l}{g}}\right)$$

$$I_0 = ml^2$$

$$\omega = \sqrt{\frac{\cancel{m}gl}{\cancel{m}l^2}}$$

$$\left(\omega = \sqrt{\frac{g}{l}}\right)$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g_{eff}}}}$$

Imp

$$\boxed{|g_{eff}| = |\vec{g} - \vec{a}|}$$

where :-

$\vec{a} \Rightarrow$ Acc^n of point of suspension "O".

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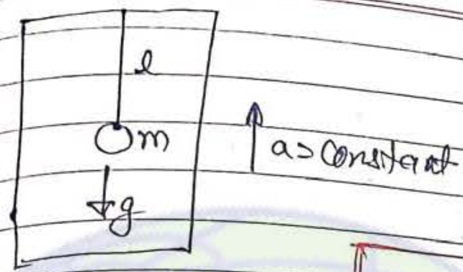
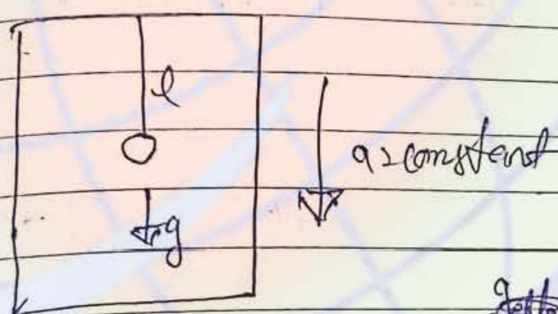


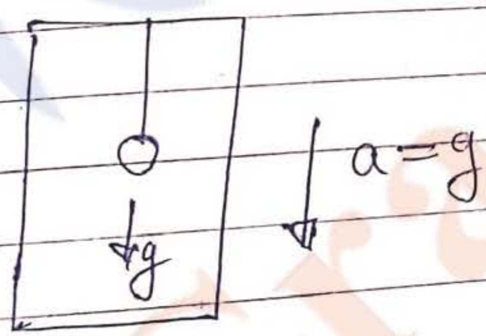
$$\boxed{g_{eff} = (g+a)}$$

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$



$$\cancel{g_{eff} = (g)}$$

$$\boxed{g_{eff} = (g-a)}$$



$g_{eff} = ($ Infinite $)$

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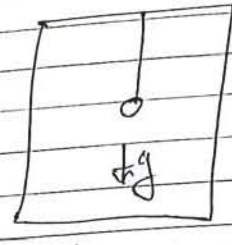
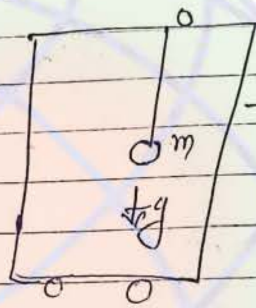
Ex-4.)



$a \leq g$

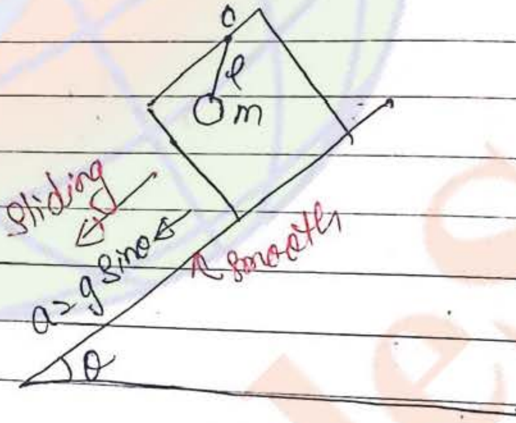
$$T = 2\pi\sqrt{\frac{l}{g-a}}$$

Ex-5.)

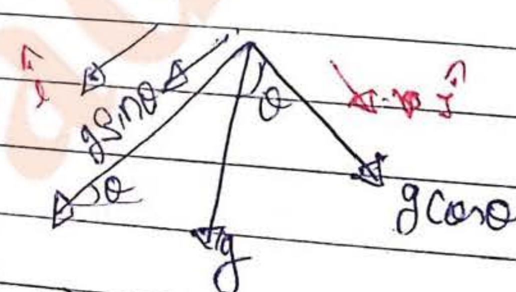


$$\boxed{g_{eff} = \sqrt{(g^2 + a^2)}}$$

ex-6.)



$$\boxed{T = 2\pi\sqrt{\frac{l}{g\cos\theta}}}$$



$$\vec{g} = (g\sin\theta)\hat{i} + (g\cos\theta)\hat{j}$$

$$|\vec{g}_{eff}| = |\vec{g} - \vec{a}| = g\cos\theta$$

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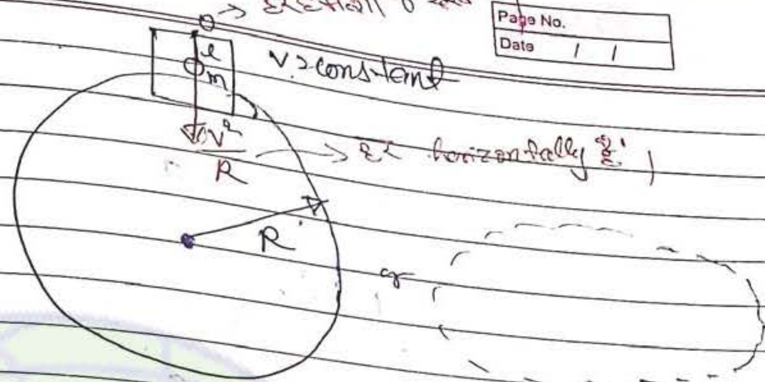
A body is moving in circular path with constant velocity.



$$\boxed{g_{eff} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}}$$

~~Physical~~

Second Pendulum

↳ It's time period is 2 seconds.

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}} \qquad l <<<<< R$$

R → Radius of earth.

Time period of simple pendulum whose length (l) is comparable to radius of earth (R)

$$\boxed{T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}}}$$ → (original formula)

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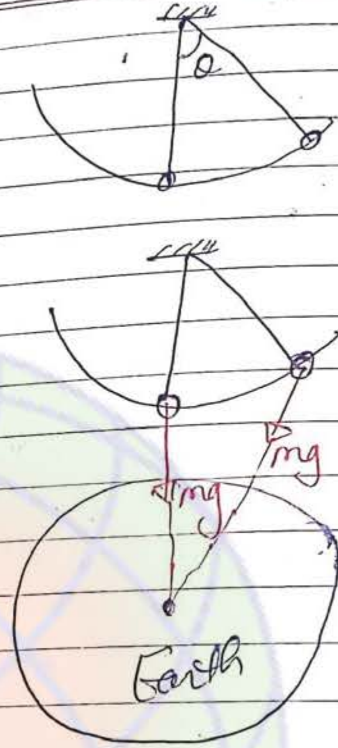

i) If $l >>>> R$

$$\frac{1}{l} <<<< \frac{1}{R}$$

$$\boxed{T = 2\pi\sqrt{\frac{R}{g}}}$$

$T = 84.6$ min

ii) If, $l = R$

$$\boxed{T = 2\pi\sqrt{\frac{R}{2g}}}$$

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iii) If $l <<<<< R$

$$\frac{1}{l} >>>>> \frac{1}{R}$$

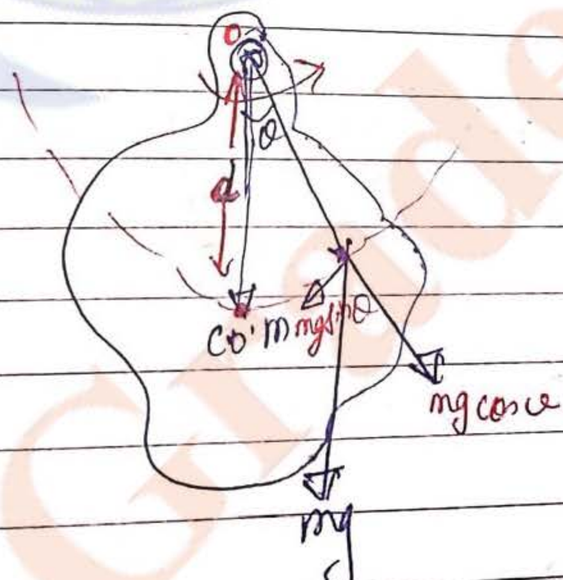
$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Compound / Physical Pendulum

$$\boxed{\omega = \sqrt{\frac{mgd}{I_0}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{I_0}{mgd}}}$$

where →
"d" → is the distance b/w C.O.M and the point of suspension.



Here :-
I_0 = It the M.O.I of oscillating body about an axis through point of suspension and "⊥" to the plane of oscillation.

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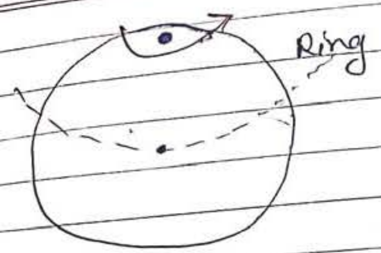
Ex:



$$T = 2\pi\sqrt{\frac{2mR^2}{mg \cdot R}}$$

$$T = 2\pi\sqrt{\frac{2R}{g}}$$

Torsional Pendulum:

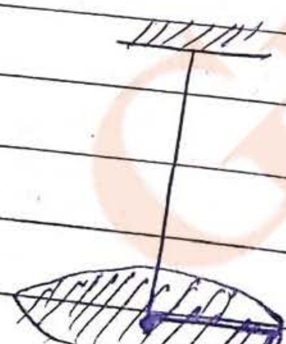
$$\boxed{\tau_{res} = -K\theta}$$

$K \rightarrow$ Torsional constant

$$\boxed{I\alpha = -K\cdot\theta}$$

$$\boxed{\alpha = \frac{-K}{I}\cdot\theta}$$

$$\boxed{\omega = \sqrt{\frac{K}{I}}}$$



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Shortcut

For Same phase:-

$$\boxed{n T_s = (n-1) T_l}$$

where:-

T_s = Time Period of "shorter" Pendulum

T_l = Time Period of "longer" Pendulum

For Simple Pendulum ⟹

$$\boxed{T = 2\pi \sqrt{\frac{l}{g}}}$$

For Spring Pendulum ⟹

$$\boxed{T = 2\pi \sqrt{\frac{m}{K}}}$$

* No. of oscillation $(n) = \frac{t}{T}$

$t \Rightarrow$ total time of observation

$T \Rightarrow$ Time Period.

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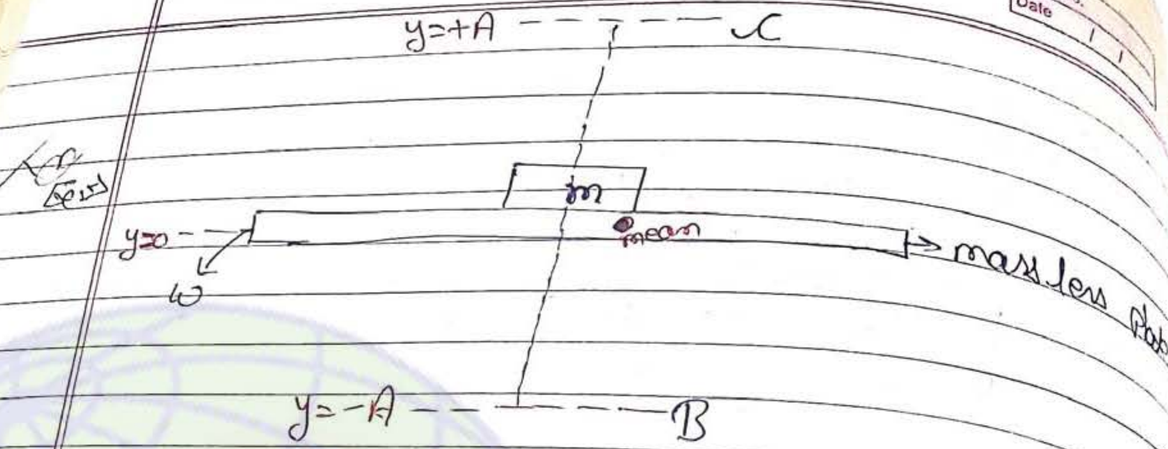
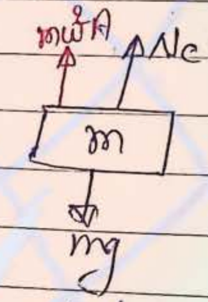


At → C



w.r.t. plate

$$m\omega^2 A \geq mg$$

$$A \geq \frac{g}{\omega^2}$$

w.r.t. ground

$$(mg - N_C) = m\omega^2 A$$

$$N_C = mg - m\omega^2 A$$

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Combination / Superposition of S.H.Ms

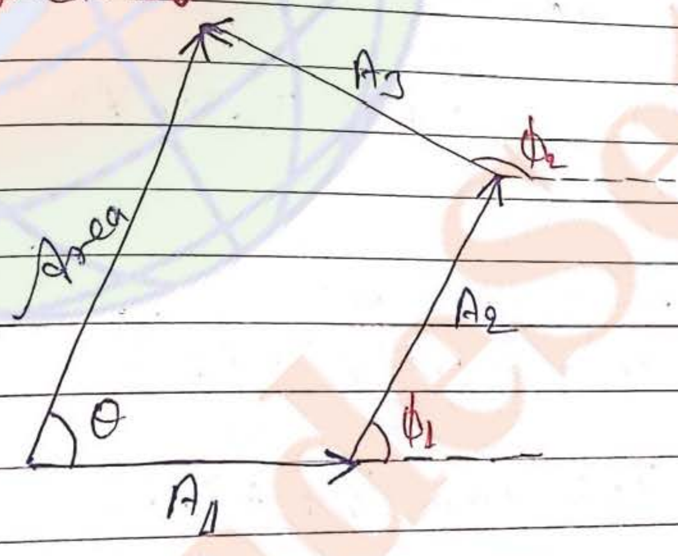
Case 1st :⇒ SHMs in same direction are combined (of same frequency of oscillation.)

$$\boxed{\begin{aligned} x_1 &= A_1 \sin \omega t \\ x_2 &= A_2 \sin(\omega t + \phi_1) \\ x_3 &= A_3 \sin(\omega t + \phi_2) \end{aligned}}$$

So

$$\boxed{\vec{x} = \vec{x_1} + \vec{x_2} + \vec{x_3}}$$

Vector method :⇒



$$\boxed{x = A_{res} \sin(\omega t + \theta)}$$

Now:—

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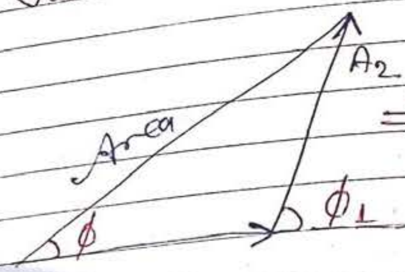
For two SHMs :-



$$\Rightarrow \boxed{A_{res} = \sqrt{(A_1^2 + A_2^2 + 2A_1A_2 \ldots}}$$

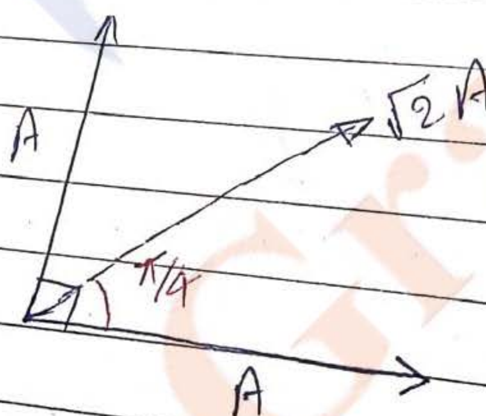
$$\boxed{\tan \phi = \frac{A_2 \sin \phi_1}{A_1 + A_2 \cos \phi_1}}$$

Ex1.) $x_1 = 3 \sin \omega t$
$x_2 = 4 \cos \omega t = 4 \sin\left(\omega t + \frac{\pi}{2}\right)$

$$A_{res} = \sqrt{3^2 + 4^2} = 5 \text{ units}$$

Ex2.) $x_1 = A \sin \omega t$
$x_2 = A \sin\left(\omega t + \pi/2\right)$

$x_3 = A \sin\left(\omega t + \pi/4\right)$



$$A_{res} = A + (\sqrt{2})A$$
$$= A(1 + \sqrt{2})$$

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No. 2 $x = A(1+\sqrt{2})\sin(\omega t + \pi/4)$

$x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t \cdot \cos\omega t$

$A_1 A_2 \cos\phi$

If $A = -B$ and $C = 2B$
then ~~thes~~ find the amplitude of resultant S.H.M

$x = B(\cos^2\omega t - \sin^2\omega t) + 2B\sin\omega t\cos\omega t$

$x = B\cos\omega t + B\sin 2\omega t$

$A_{res} = \sqrt{B^2 + B^2} = \sqrt{2}\, B$

Case : 2 ⟹

Two SHMs of same frequency and mutually "⊥" are combined

$x = A_1 \sin\omega t$

and $y = A_2 \sin(\omega t + \phi_0)$

$y = A_2[\sin\omega t\cos\phi_0 + \cos\omega t\sin\phi_0]$

$y = A_2\left[\frac{x}{A_1}\cos\phi_0 + \sqrt{\left(1 - \frac{x^2}{A_1^2}\right)}\sin\phi_0\right]$

so

$$\boxed{\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy\cos\phi_0}{A_1 A_2} = \sin^2\phi_0}$$

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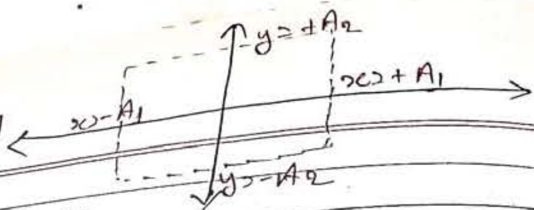


(i) If $\phi_0 = 0$

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-\left(\frac{x}{A_1}\right)\left(\frac{y}{A_2}\right)=0$$

$$\left(\frac{x}{A_1}-\frac{y}{A_2}\right)^2=0$$

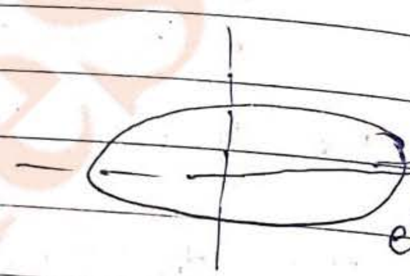
$$y = \frac{A_2}{A_1}x$$

ii) If $\phi_0 = \pi = 180$

$$\boxed{\left(\frac{x}{A_1}+\frac{y}{A_2}\right)^2=0}$$

iii) If $\phi_0 = \frac{\pi}{2}$

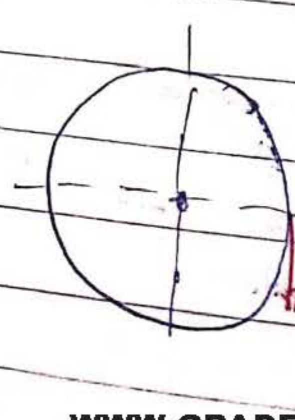
$$\boxed{\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}=1}$$



ellipse

If $\boxed{A_1 = A_2 = A}$

$$\left(\begin{array}{l} x = A\sin\omega t \\ y = A\cos\omega t \end{array}\right)$$



circle

$+ \Rightarrow -ve$

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$x = A \sin \omega t$

$y = A \cos 2\omega t$

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