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1.)
2.) Unit and Dimension ✓
3.) Gravitation ✓
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5.) motion in one dimension ✓
6.) Graph ✓
7.) Acc^n motion along straight line ✓
8.) motion under gravity ✓
Relative motion ✓

(9) Projectile motion ✓
- (i) In vertical plane
- (ii) In horizontal plane

10.) Newton's law of motion ✓
11.) Friction ✓
12.) Work-Power-energy. ✓

3/04/2018

जय माँ सरस्वती

Unit and Dimension

* Physical quantity →
The quantity by which we can describe the laws of physics are called physical quantities.

Base quantity	Derived quantity	Supplement quantity
Quantities which doesn't depends on each other are called base quantity	Quantities which are derived from the base quantities are called derived quantities	→ Plane angle → Solid angle

* Unit → standard of measurement
* Dimension: Dimension of a physical quantity are the power to which base quantity must be raise to represent that quantity

Q) Which can be assumed as personal base quantities?

Sol/n

length, area $= l^2$, mass	length current velocity $= \frac{l}{t}$] but it can't be defined by using length and current ✓ there is need to Time

नोट → जिन अलग-अलग quantity में आपस में कोई Relation नहीं है, वह personaly Independent होते उसे personal base quantity कहा जाता है।

read your previous not submit it
[dpk. g] Time

☆ Base quantity	S.I unit
① Length	metre (m)
② Mass	kilogram (kg)
③ Time	second (s)
④ Electric current	Ampere (Amp)
⑤ luminous Intensity of light	candla (cd)
⑥ Temperature	Kelvin (K)
⑦ Amount of substance	mol

☆ Dimension of some Important Physical Quantity

$[\text{Force}] = [mLT^{-2}]$

$[\text{work}] = [\text{Force}][\text{displacement}] = mL^2T^{-2}$

$[P] = [m][v] = [mLT^{-1}]$

$\text{acc}^n = \frac{v}{t} = [LT^{-2}]$

Torque $= R \times F = [F][\text{length}]$
$= [mL^2T^{-2}]$

Angular momentum $= [m][v][r]$
$= [mL^2T^{-2}]$

Angular Impulse = Torque × time

Angular velocity $(\omega) = \frac{[\text{angular disp}]}{[\text{time}]} = T^{-1}$

Angular displacement $= m^0L^0T^0$

Note → A dimension less quantity may have a unit [eg. Angle].

CAREER POINT

Target Course for NITs (JEE Main)-2014

DAILY PRACTICE PROBLEM SHEET

PHYSICS

Unit & Dimension

Q.1 The ratio of the SI unit to CGS unit of modulus of rigidity is–
(1) 10^2 (2) 10^{-2} (3) 10^{-1} (4) 10

Q.2 Parallactic second is equal to -
(1) 9.4605×10^{15} metre
(2) 3.084×10^{16} metre
(3) 1.496×10^{11} metre
(4) 3×10^{8} metre

Q.3 Given that $y = a\cos\left(\frac{t}{p} - qx\right)$, where t represents time in second and x represents distance in metre. Which of the following statements is true ?
(1) The unit of x is same as that of q.
(2) The unit of x is same as that of p.
(3) The unit of t is same as that of q
(4) The unit of t is same as that of p.

Q.4 With usual notation, amongst the following, the one which does not represent the dimensions of time is -
(1) $\frac{L}{R}$ (2) RC (3) $\sqrt{LC}$ (4) $\frac{1}{\sqrt{LC}}$

Q.5 If we choose velocity V, acceleration A and force F as the fundamental quantities, then the angular momentum in terms of V, A and F would be -
(1) $FA^{-1}V$ (2) FV^3A^{-2}
(3) FV^2A^{-1} (4) ML^2T^{-1}

Q.6 According to quantum mechanics light travels in the form of packets and energy associated with each packet is $E = h\nu$, where h is Planck's constant and ν is the frequency. The dimensional formula for Planck's constant is -
(1) $[M^1L^2T^{-1}]$ (2) $[M^2L^{-1}T^2]$
(3) $[ML^2T^{-1}]$ (4) $[ML^{-2}T^1]$

Q.1 अपरूपण गुणांक का SI मात्रक से CGS मात्रक का अनुपात क्या है–
(1) 10^2 (2) 10^{-2} (3) 10^{-1} (4) 10

Q.2 पैरालेक्टिक सेकण्ड तुल्य है -
(1) 9.4605×10^{15} metre
(2) 3.084×10^{16} metre
(3) 1.496×10^{11} metre
(4) 3×10^{8} metre

Q.3 दिया हुआ है $y = a\cos\left(\frac{t}{p} - qx\right)$, जहाँ t सेकण्ड में समय को दर्शाता है जबकि x मी में दूरी को दर्शाता है। निम्न में से कौनसा सही है ?
(1) x का मात्रक q के मात्रक के समान है
(2) x का मात्रक p के मात्रक के समान है
(3) t का मात्रक q के मात्रक के समान है
(4) t का मात्रक p के मात्रक के समान है

Q.4 सामान्य अर्थों सहित, निम्न में से कौनसा समय की विमा को नहीं दर्शाता–
(1) $\frac{L}{R}$ (2) RC (3) $\sqrt{LC}$ (4) $\frac{1}{\sqrt{LC}}$

Q.5 यदि हम वेग V त्वरण A व बल F को मूल राशियों के रूप में चुने तो V, A व F के पदों में कोणीय संवेग होगा -
(1) $FA^{-1}V$ (2) FV^3A^{-2}
(3) FV^2A^{-1} (4) ML^2T^{-1}

Q.6 क्वाण्टम यांत्रिकी के अनुसार प्रकाश ऊर्जा पैकेट के रूप में यात्रा करता है व प्रत्येक पैकेट के साथ सम्बन्धित ऊर्जा $E = h\nu$ है, जहाँ h प्लांक नियतांक है व ν आवृत्ति है। प्लांक नियतांक का विमीय सूत्र है–
(1) $[M^1L^2T^{-1}]$ (2) $[M^2L^{-1}T^2]$
(3) $[ML^2T^{-1}]$ (4) $[ML^{-2}T^1]$

Q.7 Given that v is the speed, r is the radius and g is the acceleration due to gravity. Which of the following is dimensionless ?
(1) v^2/rg (2) v^2r/g (3) v^2g/r (4) v^2rg

Q.8 If P represents radiation pressure, c represents speed of light and Q represents radiation energy striking a unit area per second then non-zero integers x, y and z such that $P^xQ^yC^z$ is dimensionless, are -
(1) $x = 1, y = 1, z = -1$ (2) $x = 1, y = -1, z = 1$
(3) $x = -1, y = 1, z = 1$ (4) $x = 1, y = 1, z = 1$

Q.9 A SI units of gas constant are -
(1) watt K^{-1} mol^{-1} (2) Newton K^{-1} mol^{-1}
(3) joule K^{-1} mol^{-1} (4) erg K^{-1} mol^{-1}

Q.10 If $x = at + bt^2$, where x is in metres and t is in hours, the units of b will be -
(1) metre (2) metre/hour
(3) metre/(hour)^2 (4) metre^2/hour

Q.7 दिया हुआ है v चाल है, r त्रिज्या है, g गुरुत्व है। निम्न में से कौनसा विमाहीन है ?
(1) v^2/rg (2) v^2r/g (3) v^2g/r (4) v^2rg

Q.8 यदि P विकिरण दाब को, c प्रकाश की चाल विकिरण ऊर्जा को दर्शाता है जो प्रति इकाई पर प्रति सैकण्ड टकराती है, तो अशून्य पूर्णांक इस प्रकार है कि $P^xQ^yC^z$ विमाहीन है, तो–
(1) $x = 1, y = 1, z = -1$ (2) $x = 1, y = -1, z =$
(3) $x = -1, y = 1, z = 1$ (4) $x = 1, y = 1, z =$

Q.9 गैस नियतांक का SI मात्रक है–
(1) watt K^{-1} mol^{-1} (2) Newton K^{-1} mol
(3) joule K^{-1} mol^{-1} (4) erg K^{-1} mol^{-1}

Q.10 यदि $x = at + bt^2$, जहाँ x मीटर में व t घण्टों में का मात्रक होगा -
(1) metre (2) metre/hour
(3) metre/(hour)^2 (4) metre^2/hour

HINTS & SOLUTION

1.[4] Modulus of rigidity, $\eta = \frac{T}{\theta}$

or $\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = [ML^{-1}T^{-2}]$

$\frac{\text{SI unit of } \eta}{\text{CGS unit of } \eta} = \frac{kgm^{-1}s^{-2}}{gm\,cm^{-1}s^{-2}}$

$= 10^3 \times 10^{-2} = 10$

2.[2] $1 \text{ parsec} = \frac{1.496\times10^{11} m}{(\pi/60\times60\times180) radian}$

$= 3.08 \times 10^{16}$ m.

3.[4] In $\cos\left(\frac{t}{p} - qx\right)$, the expression $\left(\frac{t}{p} - qx\right)$ is dimensionless. Hence t/p is also dimensionless. This is possible when units of t are same as that of p.

4.[4] $e = L\left[\frac{dl}{dt}\right]$ $\therefore [L] = \frac{[e][dt]}{[dt]} = \frac{[W][dt]}{[q][dl]} = \frac{[W]}{[dl]^2}$

$= \frac{[ML^2T^{-2}]}{[A]^2} = [ML^2T^{-2}A^{-2}]$

$[C] = \frac{[q]}{[V]} = \frac{[q]^2}{[W]} = \frac{[A^2T^2]}{[ML^2T^{-2}]} = [M^{-1}L^{-2}T^4A^2]$

$[\sqrt{LC}] = \{[ML^2T^{-2}A^{-2}] \times [M^{-1}T^{-4}A^2]\}^{1/2}$

$= [T^2]^{1/2} = [T]$ $\therefore \left[\frac{1}{\sqrt{LC}}\right] = [T^{-1}]$

5.[2] Angular momentum, mvr

$= \frac{\text{force}}{\text{acceleration}} \times V \times \frac{V^2}{A} = \frac{F}{A} \times V \times \frac{V^2}{A} = [FV^2A^{-1}]$

6.[3] $E = h\nu$

$h = \frac{E}{\nu} = \frac{mc^2}{\nu} = \frac{m(LT^{-1})^2}{T^{-1}} = ML^2T^{-1}$

7.[1] $\frac{v^2}{rg} = \frac{(LT^{-1})^2}{L.LT^{-2}} = \frac{L^2T^{-2}}{L^2T^{-2}} =$ dimensionless

8.[2] $[P] = \left[\frac{F}{A}\right] = [ML^{-1}T^{-2}]$, $[C] = [LT^{-1}]$

$[Q] = \frac{[E]}{[A][T]} = [MT^{-3}]$

Given that : $P^xQ^yC^z = M^0L^0T^0$

$M^{x+y}L^{-x+z}T^{-2x-z-3y} = M^0L^0T^0$

$\therefore x + y = 0; -2x - z - 3y = 0$

Solving we get : $x = 1, y = -1, z = 1$

9.[3] $[R] = [PV/T] = \frac{(Nm^{-2} \times m^3)}{(mol \times K)}$

$= N\,m\,K^{-1}\,mol^{-1} = J\,K^{-1}\,mol^{-1}$

10.[3] $x = at + bt^2$

Units of quantities x and bt^2 must be same.

$\therefore$ Unit of $b = \frac{\text{unit of } x}{\text{unit of } t^2} = \frac{\text{metre}}{(\text{hour})^2}$

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- while a unitless quantity can never have dimensions
  eg : refreeting index, dielecthe constant.

(i) E.m.f / voltage / potential difference (V)

$$[V] = \frac{[\omega]}{[q]} = \frac{m^1 L^2 T^{-2}}{[AT]}$$

$$V = [m^1 L^2 T^{-3} A^{-1}]$$

(ii) Electhe field intensity $(E) = \frac{[F]}{[q]} = \frac{[m^1 L^1 T^{-2}]}{[AT]}$

$$= [m L T^{-3} A^{-1}]$$

(iii) Permitivity (ε_0) =

$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$ → Physical relation

$[\varepsilon_0] = \frac{q_1 q_2}{F r^2}$ → Dimensional relation

$$[\varepsilon_0] = \frac{[A^2 T^2]}{[m L T^{-2} L^2]} \rightarrow [A^2 T^4 m^{-1} L^{-3}]$$

(IV) Relative permittivity (ε_r) / Di-electric constant

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

$$\varepsilon_r = [m^0 L^0 T^0]$$

(V) Resistance (R):

$$[R] = \frac{[V]}{[I]} = \frac{[mL^2 T^{-3} A^{-1}]}{[A]}$$

$$= [mL^2 T^{-3} A^{-2}]$$

(VI) Capacitance (C) =

$$q = \frac{C}{V}$$

$$C = q \cdot V$$

$$= [A \cdot T] \cdot [mL^2 T^{-3} A^{-1}]$$

$$= [T^2 m L^2]$$

(VII) Inductance (L) =

$$[L] = \frac{[\varepsilon m f]}{(di/dt)} = \frac{mL^2 T^{-3} A^{-1}}{(AT^{-1})} = mL^2 T^{-2} A^{-2}$$

(VII) Permiability (μ_0) =

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

speed of light

$$[\mu_0] = \frac{1}{[\varepsilon_0][c^2]}$$

$$= \frac{1}{M^{-1} L^{-3} T^2 \cdot L^2 T^{-2}}$$

or

$$B = \frac{\mu_0}{4\pi} \frac{i dl \sin\theta}{r^2}$$

$$[\mu_0] = \frac{[B][r^2]}{[i][dl]}$$

(VIII) magnetic field/ magnetic induction/ magnetic field intensity (B)

velocity

$$[F] = [q v B]$$

$$[B] = \frac{[F]}{[qv]} = \frac{[M L T^{-2}]}{[A T][L T^{-1}]}$$

(IX) Universal gravitational constant [G]

$$F = G \frac{m_1 m_2}{r^2}$$

$$[G] = \frac{F r^2}{m_1 m_2} = \frac{M L T^{-2} L^2}{M^2}$$

$$= M^{-1} L^3 T^{-2}$$

(X) Gas constant (R) =

$$PV = nRT$$

$$R = \frac{PV}{nT} = \frac{P \cdot V}{n \cdot T} = \frac{M L^2 T^{-2}}{mol \cdot K}$$

$$= M L^2 T^{-2} \cdot mol^{-1} K^{-1}$$

(xi) Boltzmann constant (K) :-

$PV = KT$

$K = \frac{PV}{T}$ → dimensions of energy के बराबर होता है।

$= [ML^2T^{-2}K^{-1}]$

(xii) Pressure / stress / elastic constant / energy density,

→ Young's modulus (Y)

→ Bulk modulus (B)

Stress = $[ML^{-1}T^{-2}]$

(xiii) [Strain] = $[M^0L^0T^0]$

Note :-

Young's modulus (Y) = $\frac{\text{stress}}{\text{strain}}$

Q] Dimensions of the following are :-

(a) $\frac{L}{R} = \frac{[ML^2T^{-2}A^{-2}]}{[ML^2T^{-3}A^{-2}]} = [T]$

(b) RC ✓ (k) $\sqrt{LC}$

(c) $\frac{L}{RCV}$

(d) $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

(e) $\frac{\omega_0 L}{R}$

(f) $\frac{1}{\omega_0 RC}$

(g) $\frac{1}{2}\varepsilon_0 E^2$

(h) $\frac{B^2}{2\mu_0}$

(i) $\frac{q^2}{2C}$ (j) $\frac{1}{2}Li^2$

energy density.

(II) $RC = [M^2 L^{-3} T^{-3} A^{-2}][T^{-2}][mL^2]$

$\Rightarrow M^2 L^4 T^{-6} A^{-2}$

(II) $[RC] = [R][C]$

$= [T]$

Capacitance

(III)

(c) $\left[\frac{L}{RCV}\right] = \frac{1}{RC} \times \frac{1}{V/L}$

Note $\Rightarrow$ $[\mathcal{E}] = \left[L\frac{di}{dt}\right]$

$\left[\frac{L}{RCV}\right] = \frac{1}{T} \times \frac{1}{AT^{-1}} \Rightarrow A^{-1}$

Potential difference

(d) $C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} =$

$\left[\frac{1}{\mu_0 \varepsilon_0}\right] = [LT^{-1}]$

(e) $\omega_0 \frac{L}{R} = T^{-1} \cdot T = 0$

(f) $= \frac{1}{\omega RC} \Rightarrow \frac{1}{T^{-1}T} = 0 = [m^0 L^0 T^0]$

(g) $\frac{1}{2}\epsilon_0 E^2 =$

$[\epsilon_0 E^2] = [A^2 T^4 m^1 L^{-3}]$

$\frac{1}{2}\epsilon_0 E^2 =$ (energy density or energy per unit volume. ↳ very Important)

$[\frac{1}{2}\epsilon_0 E^2] = [m^1 L^{-1} T^{-2}]$

(h) $\frac{B^2}{2\mu_0} = [m^1 L^{-1} T^{-2}]$

↳ (energy density of magnetic field

(i) $\frac{q^2}{2C}$

↳ Potential energy

$\left[\frac{q^2}{2C}\right] = [m^1 L^2 T^{-2}]$

(j) $\frac{1}{2} L i^2$

↳ Potential energy of Inductor

$\left[\frac{1}{2} L i^2\right] = [m^1 L^2 T^{-2}]$

(k) $[\sqrt{LC}] = [T]$

Note →

(I) sin(------) → Dimensionless
→ Dimensionless

(II) cos(---) → Dimens. less
→ D.L

(III) tan(---) → D.L
→ D.L

(IV) log(---) → D.L
→ D.L

(V) (---) → D.L
a → numerical value.

(VI) e^(---) → D.L
→ D.L

Q. $P = \frac{\alpha}{\beta} \frac{-dz}{dQ}$ (alpha ✗)

Dimension of ∇ di

P → Pressure
Z → distance
R → Boltzmann const.
Q → temp.

$$\nabla = \frac{\alpha}{P} e^{\left(\frac{-dz}{d\theta}\right)}$$

D.L (exponent) → D.L

$$\frac{dz}{d\theta} = \frac{L}{K} \Rightarrow LK^{-1}$$

$$\nabla = \frac{\alpha}{[mL^{-1}T^{-2}]}$$

$$\frac{\alpha}{P} = P \Rightarrow \left[\frac{\alpha}{P}\right] = [mL^{-1}T^{-2}] \quad \text{(i)}$$

$$\frac{dz}{d\theta} = m^0L^0T^0$$

$PV = KT = K\theta$ → it is given that it is temp. energy.

$$[\alpha] = [mL^{-1}T^{-2}]$$

$$[\nabla] = \frac{[\alpha]}{[mL^{-1}T^{-2}]}$$

#

Eg

quantity similar & its means their dimensions are equal.

$\left(P + \frac{a}{V^2}\right)(V-b) = RT$

P → Pressure
V → volume

Dimensions of a and b are

$\frac{PV}{n} = RT \Rightarrow [ML^{-1}T^{-2}]$

Concept :-

A physical quantity can be added or subtracted with similar quantity.

$\left[\frac{a}{V^2}\right] = [P]$

$a = [ML^{-1}T^{-2}][L^6]$

$[V] = [b] = L^3$

Uses of Dimensions :-

- (i) homogenity of Dimensions
- (ii) Conversion of unit
- To decide a relation b/w Physical quantities.

(a) Homogenity of dimensions.

Principle of homogenity → A/q to the principle each term on L.H.S and R.H.S of an equation has same dimension.

eg.] $v = at + \frac{bt}{l^2} + c$

find dimensions of a and b

t → time
l → length
v → speed

Sol^n

$LT^{-2} = a \cdot v$

$\frac{bt}{l^2} = LT^{-1}$; $c = LT^{-1}$

$b = L^3 T^{-2}$

concept

$\therefore [v] = [at] = \left[\frac{bt}{l^2}\right] = [c]$

∴ A/q homogenity of each term has same dimension.

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Eg.] $y = A\sin(kx - ct)$

y = displacement
x = position
t = time
} for find dimensional of A, k & c

Sol^n

~~Angle~~

~~$kx - ct \neq$~~

$[A] = [L]$ ✓

$\therefore$ $y = A\sin(kx - ct)$ → D.L; → D.L; → D.L

$[kx] = [ct] = [M^0 L^0 T^0]$

$k = L^{-1}$ | $c = T^{-1}$

* Important point :-

(i) A dimensionally correct equation may or may not be actually correct

(ii) A dimensionally incorrect equation must be actually Incorrect

(1) Conversion of units :-

5 m/s → magnitude
numerical value ← 5 m/s → units

* magnitude of a physical quantity does not depend upon system of unit

Note

$$n\,u \Rightarrow \text{Constant}$$

n → numerical value, u → units

i.e. $$n \propto \frac{1}{u}$$

larger the unit smaller will be the numerical value and vice-versa.

i.e. $$n_1 u_1 = n_2 u_2$$

eg.) 1 newton = ? dyne.

1 newton → S.I unit of force
dyne → C.G.S unit of force

Sol^n

$$[F] = [m^1 L^1 T^{-2}]$$

$$1 \text{ newton} = \frac{kg \cdot m}{s^2}$$

eg.

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ans. 1 dyne = $gm \cdot cm/s^2$

Note → यहाँ पर "n" मान कर question को solve करें

$1\ kg\ m/s^2 = n\ gm\ cm/s^2$

$\frac{1000\ (gm) \times 100\ (cm)}{s^2} = n\ \frac{gm\ cm}{s^2}$

$n = 10^5$

eg 2.) 1 Joule = ? erg

Sol/ Own

$\frac{kg\ m^2}{s^2} = n\ \frac{g\ cm^2}{s^2}$

$1000 \times (100)^2 = n$

$n = 10^7$

Teache

$[W] = [m^1 L^2 T^{-2}]$

$1\ Joule = \frac{kg\ m^2}{s^2}$

$1\ erg = \frac{g\ cm^2}{s^2}$

$1\ kg \cdot \frac{m^2}{s^2}$

eg 3.) If unit of mass (m) and velocity (v) is doubled then unit of linear momentum is

Sol/ $p = mv \Rightarrow [m^1 v^1] \Rightarrow kg\ m\ s^{-1}$

$P = 4mv$

$\Rightarrow 4mv$

unit of P_{final} is 4 times its initial.

c) In the above question number value of momentum will be

Ans $\therefore nu = \text{constant}$

$n \propto \frac{1}{4}$

So,

Numerical value becomes $\frac{1}{4}$

(c) To deduce a relation b/w physical quantities.

Time period of simple pendulum depends on its length and acc^n due to gravity find the relation time period, length and g.

$T \propto l^a g^b$

$T = k l^a g^b$

$[T] = [l]^a [g]^b$

$L^{1/2}$ $g^{-1/2}$

$\sqrt{L}$ $\frac{1}{\sqrt{g}}$

$[m^0 L^0 T^1] = [L]^a [LT^{-2}]^b$

$[m^0 L^0 T^1] = [m^0 L^{a+b} T^{-2b}]$

$a+b=0$ | $-2b = 1$, $b = -\frac{1}{2}$

$a = -b$

$a = \frac{1}{2}$

$\theta^0 \Rightarrow 1$

$T = \frac{1}{T} = T$

Now

$T = L^{1/2} g^{-1/2}$

$\Rightarrow \sqrt{L} \times \frac{1}{\sqrt{g}}$

$$T = K\sqrt{\frac{l}{g}}$$

K → experimentally determined.

Q.) Given that mass depends on force, length and time find physical relation b/w these quantities

Sol^n

$m \propto [F]^a [L]^b [T]^c$

$\Rightarrow [mLT^{-2}]^a [L]^b [T]^c$

$\Rightarrow m^a L^{a+b} T^{-2a+c}$

$[m] \Rightarrow [m^a L^{a+b} T^{-2a+c}]$

$[M^0 L^0 T^0] = [m^a L^{a+b} T^{-2a+c}]$

$a+b=0$ | $-2a+c=0$

$a=1$ $\quad a=$

$m \propto FL^{-1}T^2$

$\propto \frac{FT^2}{L}$ $\quad +2=+c$

$m = \frac{KFT^2}{L}$

Units of Some Important Physical Quantities

Physical Quantities	S.I units
① velocity	m/s
② Accⁿ.	m/s²
③ Force	newton
④ work / Energy	Joule
⑤ Torque	N-m
⑥ Impulse	N-s
⑦ linear momentum	kg m/s

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(8) Force — watt (W) = J/s
(9) ang. displacement — rad
(10) ang. velocity — rad/sec.
(11) Angular acc^n — radian/sec^2
(12) Angular Impulse — N-m-s or J-s
(13) m.o.I — kg.m^2
(14) Surface tension
- force / length → N/m
- Energy / Area → J/m^2

Note →
$$\frac{\Delta N}{m} = \frac{\Delta J}{m^2}$$

(15) Elastic coefficient — N/m^2
- Young modulus
- Bulk modulus
- Shear modulus

(16) Pressure — Pascal (Pa)
(17) Steafans constant — watt/m^{2}K^4
(18) Resistance — ohm (Ω)
(19) Capacitance — farad (F)
(20) Inductance — henry (H)
(21) Specific resistance (ρ) or Resistivity — ohm.m or Ωm

(22)	Electric field	V/m or N/C
(23)	Electric potential	volt (V)
(24)	magnetic field	tesla (T)
(25)	magnetic flux	weber (wb)
(26)	specific ~~heat~~ heat	J/kg K or J/kg °C
(27)	Latent heat	J/kg
(28)	Redberg constant (R)	L^{-1}

(1) Some practical units.

1 femto = 10^{-15} m

1 a° = 10^{-10} m

1 light year = $3\times10^{8}\times365\times86400$ s

= $3\times10^{8}\times365\times24\times60\times60$

⇒

1 parsec = 3.26 light year.

1 ev = 1.6×10^{-19} Joule

(Note → 1 coulomb volt → 1 Joule)

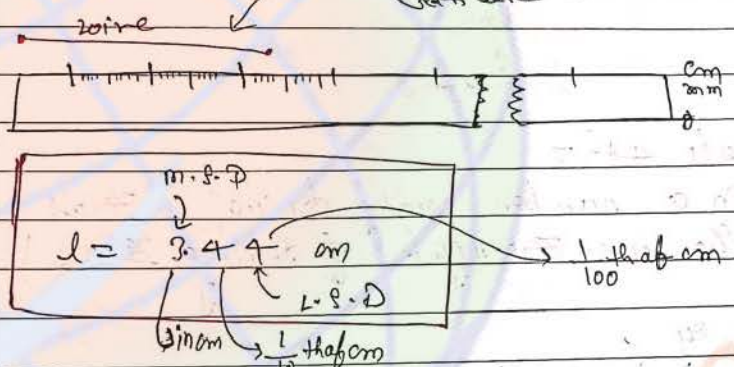
1 kilo watt hour → $(10^3 \text{ watt})(3600\ s)$

$= 3.6 \times 10^6$ J

1 Joule = 1 watt · s

Significant digit

Note: किसी भी wire का लंबाई हम least से least लेते हैं। मतलब हम सबसे छोटे - छोटे unit को लेकर करते हैं।



m.s.D → most significant digit

L.s.D → least " "

No of significant digit = 3

Rules for finding significant digit

Rule 1st → All non-zero digits are significant

eg 12378 , 3 D ⇒ 5

Rule 2nd → Zero's b/w non-zero digits are significant

eg 12045 , 5.D 25

→ Rule 3rd

In a number with decimal, zero left to the non-zero digit are insignificant

eg

0.0027 , S.D = 2

↳ Insignificant digits
↳ leading zero

→ Rule 4th

In a number with decimal, zero's right to the non-zero digit (& trailing zero's) are significant

eg

12.270 S.D = 5

↳ this zero is significant
↳ trailing zero is counted

eg

0.00330 , S.D = 3

12400.0 , S.D = 6

→ Rule 5th

In a number without Decimal; trailing zeros are not significant

eg. 12400 S.D = 3 (without decimal)

12400.0 S.D → 6 (with decimal)

→ Rule 6th →
If we change the unit of measurement number of significant digits do not changed

Significant digits in calculation

(1) In division or multiplication :-

Rule → final Answer में question में smallest significant digits जितना होगा उतना ही significant digit answer में consider करना है

eg - 1 a = 3.22
b = 2.2

ab = ?

ab = 7.084 → Insignificant
L.S.D
m.s.D

Rounding off of 7.084 is 7.1
or
ab = 7.1

3.22 × 2.2
6 4
6
7.084

Answer

eg (2) m = 3.513 kg
v = 5.00 m/s
p = ?

(A) 17.565 kg m/s
(B) 17.56 "
(C) 17.57 "
(D) 17.6

p = 17.6

Ans = (D)

35.13 × 5
17.565
m.s.D L.S.D Significant

Note→ In division or multiplication number of significant digits in the answer is equal to the (given) quantity which has least number of significant digits

Note

Counting का significant digit Infinite होता है

eg→ 7 सेब, 5 बच्चे etc

Note→ अगर "5" के case में significant digit लेना है तो दो case है

Rule 1st→ odd number के case में +1 करना है

eg→ find three S.D in following number

7.37592 → 7.38

+1 in case of odd.

Rule 2nd→ even number के case में "0" add करना होता है अर्थात कोई change नहीं।

eg→ find 5 S.D in following numbe

7.38572 → 7.38...

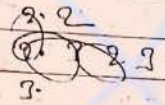
even के case में no change

③ In addition or substraction.

eg → a = 2.2
b = 2.323
c = 3.2

a+b+c = ?

Soln



* Step 1st
arrange the number one-below the other w.r.t decimal

2.2
2.323
3.2

* Step 2nd
Identify the column with doubtful digit

1st column (Rounding off) with doubtful digit → जो सबसे कम significant digit वाला है उसके respect में Rounding off निकाले।

2.2
2.323
3.2

2.2
2.3
3.2
5.5 → find answer

* Step's followed in this question.
(i) arrange the number one-below the other
(ii) Identify the 1st column with doubtful digit

गुणा → odd → add 1
even → 0 (not add)

(iii). Rounding off is done to the 1st column with doubtful digits

(iv) Addition is taken place

Q) Round off the following to three significant digits
Soln

(a) 12730 → 12700 → बिना decimal के दायें number को zero कर देते हैं
(b) 127.30 → 127 → decimal के दायें number को हटा देते हैं
(c) 4301 → 4300 = 4.30×10^3
(d) 43.273 → 43.3
(e) 127.80 → 12800

Note Round off upto two significant digits

(f) 107.8431 → 110

10 के power को significant में count किया जाता है

Note → Error में हम always maximum error निकालते हैं। क्योंकि यह अधिक-से अधिक कितना गलती होने की संभावना है। इसलिए इस बात का ध्यान रखें कि -ve को भी modulus में लेकर +ve कर देना है क्योंकि maximum value नहीं निकाला जा सकता है।

Absolute error, relative error and Percentage error.

Let observation taken of a physical quantity are,

$a_1, a_2, a_3, \ldots a_n$

$$a_{mean} = \frac{a_1 + a_2 + a_3 + \cdots + a_n}{n}$$

a_{mean} → True value or mean value

(1) Absolute error:→

$$\Delta a_1 = a_{mean} - a_1$$

$$\Delta a_2 = a_{mean} - a_2$$

$$\Delta a_n = a_{mean} - a_n$$

Note

mean absolute error (Δa) → maximum possible error

$$\Delta a_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \cdots + |\Delta a_n|}{n}$$

so,

$$a = a_{mean} \pm \Delta a_{mean}$$

eg :- $R = (5.5 \pm 0.5)\Omega$

$R_{mean} \Rightarrow 5.5\Omega$

$\Delta R_{mean} = 0.5\Omega$

or

$(5.5 - 0.5)\Omega \leq R \leq (5.5 + 0.5)\Omega$

(2) Relative error (fractional error)

$$\text{Relative error} = \pm \frac{\Delta a_{mean}}{a_{mean}}$$

(3) Percentage error $\Rightarrow$ $\pm \frac{\Delta a_{mean}}{a_{mean}} \times 100$

Error determination in mathematical operations:

(a) In multiplication

$$A = xy$$

$$\log A = \log x + \log y$$

$$\boxed{\frac{\Delta A}{A} = \frac{\Delta x}{x} + \frac{\Delta y}{y}}$$

↳ Relative error of A

and also

Percentage error in A

$$\frac{\Delta A}{A} \times 100 = \frac{\Delta x}{x} \times 100 + \frac{\Delta y}{y} \times 100$$

(b) In division:

$$A = \frac{x}{y}$$

$$\log A = \log x - \log y$$

$$\frac{\Delta A}{A} = \frac{\Delta x}{x} - \frac{\Delta y}{y}$$

max. possible relative error in A is

$$\boxed{\frac{\Delta A}{A} = \frac{\Delta x}{x} + \frac{\Delta y}{y}}$$

Notes :-

$$A = \frac{k x^a y^b}{z^c}$$

$$\log A = \log k + a \log x + b \log y - c \log z$$

$$\frac{\Delta a}{a} = 0 + a \cdot \frac{\Delta x}{x} + b \cdot \frac{\Delta y}{y} - c \cdot \frac{\Delta y}{y}$$

Sol

maximum possible relative error =

$$\frac{\Delta a}{a} = a \cdot \frac{\Delta x}{x} + b \frac{\Delta y}{y} + |c| \frac{\Delta y}{y}$$

Eg :- % error in the length of simple pendulum is 2% ; and error in the measurment of g is 3% ; find % error in the measurment of Time period.

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$T = 2\pi l^{1/2} g^{-1/2}$$

$$\frac{\Delta T}{T} \times 100 = \frac{1}{2} \cdot \frac{\Delta l}{l} \times 100 + \left|\frac{-1}{2}\right| \times \frac{\Delta g}{g} \times 100$$

Lvl 1 → 2 + 10 + 2,
Ex. (11) complete

Lvl 2 → 3, 6, 7, 8, 9 to 17, 21, 24, 25,
↳ 022
↳ correction
Ans → 1/4 times ΔT/T

$\frac{\Delta T}{T} \times 100 = \frac{1}{2} \times (\pm 2)\% + \frac{1}{2} \times (\pm 2)\%$

$= \pm 1\% \pm \frac{3}{2}\%$

$\Rightarrow \pm 2.5\%$

(1) Concept Q6.

$V = 10.0 \pm 0.1$

$I = 5.0 \pm 0.1$ A

% of R = ?

Sol^n $R = 15.0 \pm 0.2$ Ω

Explanation

$R = \frac{V}{I}$

$\log R = \log V - \log I$

$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 - \frac{\Delta I}{I} \times 100$

$= (10.0 \pm 0.1) \div (5.0 \pm 0.1)$

$= (15.0 \pm 0.2)$ Ω

$R = \frac{V}{I}$

$= \frac{10}{5} = 2$

NOW

% error =

$R = \frac{V}{I}$

$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$

$= \frac{0.1}{10} \times 100 + \frac{0.1}{5} \times 100$

$= 1 + 2$

$= \pm 3\%$

Ans

$R = 2.0 \pm 3\%$

② Concept use

In addition and subtraction

Note → ① $Z = xy$

$Z = \frac{x}{y}$

} % error of "Z" both case में same आएगा।

यानी $\frac{\Delta Z}{Z} \times 100 = (\text{const})$

लेकिन error in "Z" दोनों में same नहीं होगा।

② $A = x + y$

$\Delta x = \Delta x + \Delta y$

$\frac{\Delta x}{A} = \frac{\Delta x + \Delta y}{A}$

$$\frac{\Delta A}{A} = \frac{\Delta x + \Delta y}{x+y}$$

$$\boxed{\frac{\Delta A}{A} = \frac{\Delta x}{x+y} + \frac{\Delta y}{x+y}}$$

and

$$\boxed{\frac{\Delta A}{A} \times 100 = \frac{\Delta x}{x+y} \times 100 + \frac{\Delta y}{x+y} \times 100}$$

(ii) Subtraction

$$A = x - y$$

$$\Delta x = \Delta x - \Delta y$$

maximum possible error

$$\Delta x = \Delta x + \Delta y$$

and

$$\boxed{\frac{\Delta A}{A} = \frac{\Delta x + \Delta y}{x+y}}$$

$$\boxed{\frac{\Delta A}{A} \times 100 = \frac{\Delta x}{x+y} \times 100 + \frac{\Delta y}{x+y} \times 100}$$

eg 1 $\quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

relative error in R is

Sol/n

$$\frac{1}{R} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_0 = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta R_1 + R_2}{R_1 + R_2} \times 100$$

$$R = \frac{R_1 R_2}{(R_1 + R_2)}$$

$$R = R_1 R_2 (R_1 + R_2)^{-1}$$

$$\frac{\Delta R}{R} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + |-1| \frac{\Delta (R_1 + R_2)}{R_1 + R_2}$$

$$\boxed{\frac{\Delta R}{R} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}}$$

Eg 2)

$R_1 = (10.0 \pm 0.1)\Omega$

$R_2 = (5.0 \pm 0.2)\Omega$

find % error in the measurement of net resistance if

(i) R_1 and R_2 are in parallel combined

(ii) R_1 and R_2 are in series combine

Sol)

(i) R_1 ⊙ R_2

$$\frac{\Delta R}{R} \times 100 = \frac{0.1}{10} \times 100 + \frac{0.2}{5} \times 100 + \frac{0.3}{15} \times 100$$

$$\Rightarrow 1 + 4 + 2$$

$$\Rightarrow 7\%$$

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{5}$$

$$\Rightarrow \frac{1+2}{10}$$

$$\Rightarrow \frac{3}{10}$$

$$R = \frac{10}{3}$$

$$\Rightarrow 1 + 10 + 2$$

$$\Rightarrow 13$$

$$R_{net} = \left(\frac{10}{3} \pm 7.0\right)\Omega$$

H.W ∴ Complete question discuss

(17) $R = R_1 + R_2$

$\Rightarrow \frac{0.1}{10} \times 100 + \frac{0.2}{5} \times 100$

$\Rightarrow 1 + 4$

$= 5\%$

$\therefore R = 10 \pm 5$

~~Rect = (15.0 ± 5%) Ω~~

$\log R = \log (R_1 + R_2)$

$\frac{\Delta R}{R} \times 100 = \frac{\Delta (R_1 + R_2)}{R_1 + R_2}$

must be solve again

$\Rightarrow \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2} \times 100 = \frac{0.2}{15} \times 100 + \frac{20}{10}$

$= \pm 2\%$

$\therefore (R = 15 \pm 2\%)$

Gravitation

Newton law of gravitation :→

According to this law gravitational force of attraction b/w two particle is directly proportional to the product of their mass and inversly proportional to the square of the separation b/w them

$F \propto m_1 m_2$

$F \propto \frac{1}{r^2}$

$F \propto \frac{m_1 m_2}{r^2}$

$$F = G \frac{m_1 m_2}{r^2}$$

where

G is universal gravitational constant

$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

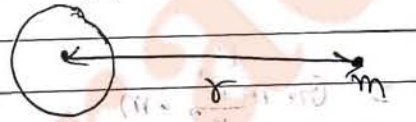
Note

(i)

(point mass)

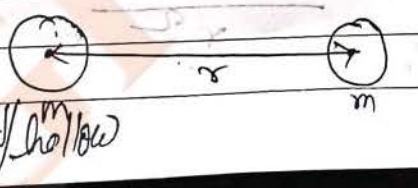
$F = G \frac{m_1 m_2}{r^2}$

(ii)



$F = \frac{G M m}{r^2}$

(iii)

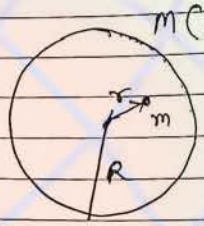


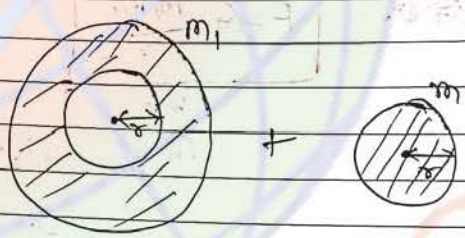
$F = \frac{G M m}{r^2}$

(iv) Spherical shell

$F = 0$

(v) M (solid)





$F = 0$ (concept of hollow)

$$F = \frac{G M m_2}{r^2}$$

$$m_2 = \frac{4}{3}\pi r^3 \cdot \rho$$

$$= \frac{4}{3}\pi r^3 \left(\frac{M}{\frac{4}{3}\pi R^3}\right)$$

$$= \frac{M r^3}{R^3}$$

$$F = \frac{G \cdot m \frac{M r^3}{R^3} \cdot m}{r^2}$$

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Target Course for NITs (JEE Main)-2014

DAILY PRACTICE PROBLEM SHEET

PHYSICS

Gravitation-1

Question based on Gravitational force

The distance of the centres of moon and earth is D. The mass of earth is 81 times the mass of the moon. At what distance from the centre or the earth, the gravitational force will be zero–

(1) $\frac{D}{2}$ (2) $\frac{2D}{3}$

(3) $\frac{4D}{3}$ (4) $\frac{9D}{10}$

Question based on Acceleration due to gravity

The mass of the moon is 7.34×10^{22} kg and the radius is 1.74×10^{6} m. The value of gravitation force will be-

(1) 1.45 N/kg (2) 1.55 N/kg

(3) 1.75 N/kg (4) 1.62 N/kg

Two planet have the same average density but their radii are R_1 and R_2. If acceleration due to gravity on these planets be g_1 and g_2. respectively, then

(1) $\frac{g_1}{g_2} = \frac{R_1}{R_2}$

(2) $\frac{g_1}{g_2} = \frac{R_2}{R_1}$

(3) $\frac{g_1}{g_2} = \frac{R_1^2}{R_2^2}$

(4) $\frac{g_1}{g_2} = \frac{R_1^3}{R_2^3}$

Question based on Gravitational force

Q.1 चन्द्रमा तथा पृथ्वी के केन्द्रों के बीच की दूरी D है। पृथ्वी का द्रव्यमान चन्द्रमा के द्रव्यमान का 81 गुना है पृथ्वी के केन्द्र से किस दूरी पर गुरूत्वाकर्षण बल शून्य होगा

(1) $\frac{D}{2}$ (2) $\frac{2D}{3}$

(3) $\frac{4D}{3}$ (4) $\frac{9D}{10}$

Question based on Acceleration due to gravity

Q.2 चंद्रमा का द्रव्यमान 7.34×10^{22} kg तथा त्रिज्या 1.74×10^{6} m है। इसकी सतह पर गुरूत्वीय त्वरण का मान होगा –

(1) 1.45 N/kg (2) 1.55 N/kg

(3) 1.75 N/kg (4) 1.62 N/kg

Q.3 दो ग्रहों के औसत घनत्व समान है परन्तु त्रिज्यायें R_1 तथा R_2 है। यदि इन ग्रहों पर गुरूत्वीय त्वरण क्रमशः g_1 तथा g_2 हो तो –

(1) $\frac{g_1}{g_2} = \frac{R_1}{R_2}$

(2) $\frac{g_1}{g_2} = \frac{R_2}{R_1}$

(3) $\frac{g_1}{g_2} = \frac{R_1^2}{R_2^2}$

(4) $\frac{g_1}{g_2} = \frac{R_1^3}{R_2^3}$

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The value of 'g' at a particular point is 9.8 m/s². Suppose the earth suddenly shrinks uniformly to half its present size without losing any mass. The value of 'g' at the same point (assuming that the distance of the point from the centre of earth does not shrink) will now be–

(1) 4.9 m/s² (2) 3.1 m/s²
(3) 9.8 m/s² (4) 19.6 m/s²

Question based on Gravity below the surface

Q.5 The depth d at which the value of acceleration due to gravity becomes $\frac{1}{n}$ times the value at the surface, is [R = radius of the earth]–

(1) $\frac{R}{n}$ (2) $R\left(\frac{n-1}{n}\right)$
(3) $\frac{R}{n^2}$ (4) $R\left(\frac{n}{n+1}\right)$

Question based on Gravity due to rotation

Q.6 What should be the velocity of earth due to rotation about its own axis so that the weight at equator become $\frac{3}{5}$ of initial value. Radius of earth on equator is 6400 km-

(1) 7.4×10^{-4} rad/sec
(2) 6.7×10^{-4} rad/sec
(3) 7.8×10^{-4} rad/sec
(4) 8.7×10^{-4} rad/sec

Q.7 R is the radius of the earth and ω is its angular velocity and g_p is the value of 'g' at the poles. The effective value of 'g' at the latitude $\lambda = 60°$ will be equal to -

(1) $g_p - \frac{1}{4}R\omega^2$
(2) $g_p - \frac{3}{4}R\omega^2$
(3) $g_p - R\omega^2$
(4) $g_p + \frac{1}{4}R\omega^2$

Q.4 पृथ्वी पर किसी विशेष बिन्दु पर 'g' का मान 9.8 m/s² है। माना कि अब पृथ्वी बिना द्रव्यमान क्षति के एक समान से प्रारम्भिक आकार के आधे आकार में सिकुड़ जाती है। इसी बिन्दु पर 'g' का मान होगा (माना कि इस बिन्दु पर पृथ्वी के केन्द्र के सापेक्ष नहीं सिकुड़ी है)

(1) 4.9 m/s² (2) 3.1 m/s²
(3) 9.8 m/s² (4) 19.6 m/s²

Question based on Gravity below the surface

Q.5 उस गहराई d का मान, जहाँ गुरुत्वीय त्वरण का मान सतह के मान का $\frac{1}{n}$ हो जाता है, होगा [R = पृथ्वी की त्रिज्या]–

(1) $\frac{R}{n}$ (2) $R\left(\frac{n-1}{n}\right)$
(3) $\frac{R}{n^2}$ (4) $R\left(\frac{n}{n+1}\right)$

Question based on Gravity due to rotation

Q.6 पृथ्वी का उसकी अक्ष के परितः कोणीय वेग क्या होना चाहिए ताकि भूमध्य रेखा पर किसी व्यक्ति का भार $\frac{3}{5}$ गुना हो जाये। भूमध्य रेखा पर पृथ्वी की त्रिज्या 6400 किलोमीटर है -

(1) 7.4×10^{-4} rad/sec
(2) 6.7×10^{-4} rad/sec
(3) 7.8×10^{-4} rad/sec
(4) 8.7×10^{-4} rad/sec

Q.7 पृथ्वी की त्रिज्या R, कोणीय वेग ω तथा ध्रुवों पर 'g' का मान g_p है। अक्षांश $\lambda = 60°$ पर 'g' का प्रभावी मान होगा -

(1) $g_p - \frac{1}{4}R\omega^2$
(2) $g_p - \frac{3}{4}R\omega^2$
(3) $g_p - R\omega^2$
(4) $g_p + \frac{1}{4}R\omega^2$

stion based on Energy

Energy required to move a bo
an orbit of radius 2R to 3R is
(1) GMm/12R² (2)
(3) $\frac{GMm}{8R}$ (4)

stion based on Escape veloc

The escape velocity from
11 km/second. The esca
planet having twice the r
mean density as the earth, i
(1) 22 km/sec (2
(3) 5.5 km/sec (4

stion based on Orbital ve

The distance between c
moon is 384000 km. If t
6×10^{24} kg and G = 6.6
orbital speed of the moo
(1) 1 km/sec
(3) 8 km/sec

Question based on Energy

Energy required to move a body of mass m from an orbit of radius 2R to 3R is

(1) $GMm/12R^2$ (2) $GMm/3R^2$

(3) $\frac{GMm}{8R}$ (4) $\frac{GMm}{6R}$

Question based on Escape velocity

The escape velocity from the earth is about 11 km/second. The escape velocity from a planet having twice the radius and the same mean density as the earth, is

(1) 22 km/sec (2) 11 km/sec

(3) 5.5 km/sec (4) 15.5 km/sec

Question based on Orbital velocity

The distance between centre of the earth and moon is 384000 km. If the mass of the earth is 6×10^{24} kg and $G = 6.66 \times 10^{-11}$ Nm^2/kg^2. The orbital speed of the moon is nearly.

(1) 1 km/sec (2) 4 km/sec

(3) 8 km/sec (4) 11.2 km/sec

Question based on Energy

Q.8 m द्रव्यमान के एक पिण्ड को 2R त्रिज्या की कक्षा से 3R त्रिज्या की कक्षा में भेजने के लिए आवश्यक ऊर्जा है (R = पृथ्वी की त्रिज्या)

(1) $GMm/12R^2$ (2) $GMm/3R^2$

(3) $\frac{GMm}{8R}$ (4) $\frac{GMm}{6R}$

Question based on Escape velocity

Q.9 पृथ्वी पर पलायन वेग का मान लगभग 11 किमी/सैकण्ड है। एक ऐसे ग्रह, जिसकी त्रिज्या पृथ्वी की त्रिज्या की दोगुनी है परन्तु माध्य घनत्व पृथ्वी के घनत्व के बराबर है, पर पलायन वेग का मान होगा

(1) 22 km/sec (2) 11 km/sec

(3) 5.5 km/sec (4) 15.5 km/sec

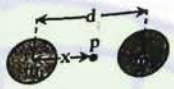
Question based on Orbital velocity

Q.10 पृथ्वी तथा चन्द्रमा के केन्द्रों के बीच की दूरी 384000 किलोमीटर है। यदि पृथ्वी का द्रव्यमान 6×10^{24} किलोग्राम तथा $G = 6.66 \times 10^{-11}$ न्यूटन मीटर^2/किलोग्राम^2 हो तो चन्द्रमा की कक्षीय चाल होगी लगभग -

(1) 1 km/sec (2) 4 km/sec

(3) 8 km/sec (4) 11.2 km/sec

1. [4]



शून्य तीव्रता वाले बिन्दु पर बल शून्य होगा

$$x=\frac{\sqrt{m_1}}{\sqrt{m_1}+\sqrt{m_2}}d=\frac{\sqrt{81M}}{\sqrt{81M}+\sqrt{M}}D=\frac{9}{10}D$$

2. [4] $g=\frac{GM}{R^2}=\frac{6.67\times10^{-11}\times7.34\times10^{22}}{(1.74\times10^6)^2}=1.62N/kg$

3. [1] $g=\frac{4}{3}\pi\rho GR$ यदि ρ = नियत हो तो $\frac{g_1}{g_2}=\frac{R_1}{R_2}$

4. [3] $g=\frac{GM}{r^2}$, चूंकि M तथा r नियत है

अतः $g = 9.8\ m/s^2$

5. [2]

$$g'=g\left(1-\frac{d}{R}\right)\Rightarrow\frac{g}{n}=g\left(1-\frac{d}{R}\right)\Rightarrow d=\left(\frac{n-1}{n}\right)R$$

6. [3] भूमध्य रेखा पर वस्तु का भार $=\frac{3}{5}$ प्रारंभिक भार

$\therefore g'=\frac{3}{5}g$ (क्योंकि द्रव्यमान नियत रहता है)

$$g'=g-\omega^2R\cos^2\lambda\Rightarrow\frac{3}{5}g=g-\omega^2R\cos^2(0°)$$

$$\Rightarrow\omega^2=\frac{2g}{5R}\Rightarrow\omega=\sqrt{\frac{2g}{5R}}=\sqrt{\frac{2\times10}{5\times6400\times10^3}}$$

$= 7.8\times10^{-4}\ \frac{rad}{sec}$

7. [1] $g=g_P-R\omega^2\cos^2\lambda=g_P-\omega^2R\cos^2 60°$

$=g_P-\frac{1}{4}R\omega^2$

8. [4] किसी वस्तु को r_1 से r_2 तक विस्थापित
ऊर्जा में परिवर्तन निम्न सूत्र से दि

$$\Delta U=GMm\left[\frac{1}{r_1}-\frac{1}{r_2}\right]=GMm\left(\frac{1}{2R}-\right.$$

9. [1] $v_e=\sqrt{\frac{2GM}{R}}=R\sqrt{\frac{8}{3}\pi G\rho}$ यदि ρ

$v_e\propto R$

10. [1] $v_0=\sqrt{\frac{GM_e}{r}}=\sqrt{\frac{6.66\times10^{-11}\times}{384000\times}}$

$=\sqrt{1.04\times10^6}$

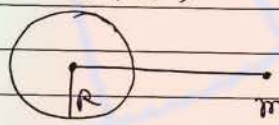
$= 1\ km/sec$

$\Delta U=GMm\left[\frac{1}{r_i}-\right.$

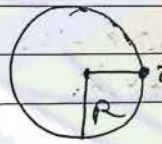
$$F = \frac{G M m r}{R^3}$$

M → mass of big body
m → mass of point body.

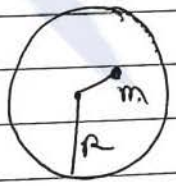
* Special cases



$$F = \frac{G M m}{r^2}, \quad r > R$$

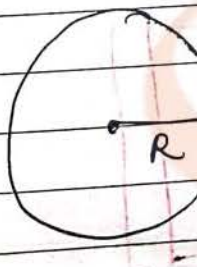


$$F = \frac{G M m}{R^2}, \quad r = R$$



$$F = \frac{G M m r}{R^3}, \quad r < R$$

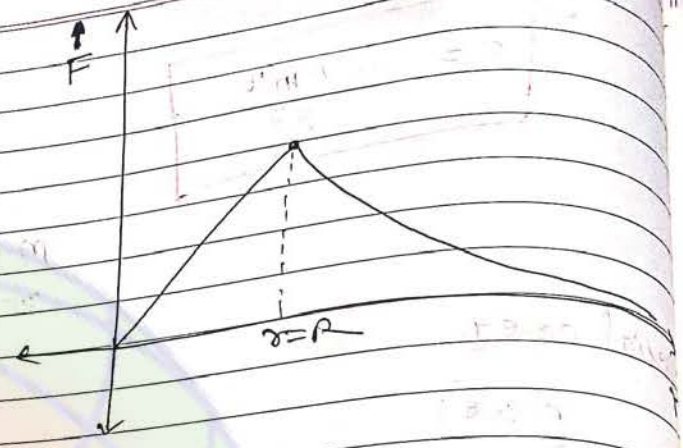
*



when "m" is on the surface of M then F is max.

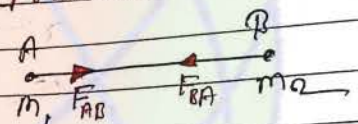
Note: "g" का value large height में "r" के large value के लिए

Qu Q.5



Vector Representation :-

Qu Q.6



Note :- Gravitation force is along the line joining the two particles

Q.7

F_{AB} → force on "A" due to "B"

F_{BA} → force on "B" due to "A"

$$F_{AB} = F_{BA} = \frac{G m_1 m_2}{r^2}$$

But in vector form

$$\vec{F}_{AB} = (-)\vec{F}_{BA}$$

Raj

classmate
Date

CAREER POINT

Target Course for NITs (JEE Main)-2014

DAILY PRACTICE PROBLEM SHEET

PHYSICS

Gravitation-2

on Gravitational force

articles of equal mass go round a circle of R under the action of their mutual tional attraction. The speed of each particle

$\frac{1}{2R}\sqrt{\frac{1}{Gm}}$

$\sqrt{\frac{Gm}{2R}}$

$\frac{1}{2}\sqrt{\frac{Gm}{R}}$

$\sqrt{\frac{4Gm}{R}}$

here of mass m and M are situated in air gravitational force between them is F. The round the masses is now filled with a liquid ific gravity 3. The gravitational force will

(2) $\frac{F}{3}$

(4) 3F

weighs 700 gm wt on the surface of the ow much will it weigh on the surface of a hose mass is 1/7 and radius is half that of

gm *wt* (2) 400 gm *wt*

m *wt* (4) 300 gm *wt*

Question based on Gravitational force

Q.1 परस्पर गुरूत्वीय आकर्षण के प्रभाव में दो समान द्रव्यमान कण त्रिज्या R के वृत्तीय पथ पर गति कर रहे हैं। प्रत्येक कण की चाल होगी -

(1) $v = \frac{1}{2R}\sqrt{\frac{1}{Gm}}$

(2) $v = \sqrt{\frac{Gm}{2R}}$

(3) $v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$

(4) $v = \sqrt{\frac{4Gm}{R}}$

Q.2 दो गोले जिनके द्रव्यमान क्रमशः m तथा M है, वायु में स्थित है तथा उनके मध्य गुत्वाकर्षण बल F है। यदि दोनों द्रव्यमानों के मध्य स्थान को आपेक्षिक घनत्व 3 वाले द्रव से भर दिया जाये तो गुरूत्वाकर्षण बल हो जायेगा -

(1) F (2) $\frac{F}{3}$

(3) $\frac{F}{9}$ (4) 3F

Q.3 एक वस्तु का पृथ्वी तल पर भार 700 ग्राम भार है। उस ग्रह के तल पर इसका भार क्या होगा जिसका द्रव्यमान पृथ्वी के द्रव्यमान का 1/7 गुना तथा त्रिज्या 1/2 गुनी है -

(1) 200 ग्राम भार (2) 400 ग्राम भार

(3) 50 ग्राम भार (4) 300 ग्राम भार

Q.4 A body has a weight 90 kg on the earth's surface, the mass of the moon is 1/9 that of the earth's mass and it radius is 1/2 that of the earth's radius. On the moon the weight of the body is–
(1) 45 kg (2) 202.5 kg
(3) 90 kg (4) 40 kg

Question based on Acceleration due to gravity

Q.5 The acceleration of a body due to the attraction of the earth (radius R) at a distance 2 R from the surface of the earth is (g = acceleration due to gravity at the surface of the earth) -
(1) $\frac{g}{9}$ (2) $\frac{g}{3}$
(3) $\frac{g}{4}$ (4) g

Question based on Energy

Q.6 A rocket is launched with velocity 10 km/s. If radius of earth is R, then maximum height attained by it will be-
(1) 2R (2) 3R
(3) 4R (4) 5R

Question based on Escape velocity

Q.7 If the radius of a planet is R and its density is ρ, the escape velocity from its surface will be.-
(1) $v_e \propto \rho R$
(2) $v_e \propto \sqrt{\rho}\, R$
(3) $v_e \propto \frac{\sqrt{\rho}}{R}$
(4) $v_e \propto \frac{1}{\sqrt{\rho} R}$

Q.8 The ratio of planets A and B is k_1 and ratio of acceleration due to gravity on then is k_2. The ratio of escape velocities from them will be -
(1) $k_1 k_2$ (2) $\sqrt{k_1 k_2}$
(3) $\sqrt{\frac{k_1}{k_2}}$ (4) $\sqrt{\frac{k_2}{k_1}}$

Q.4 एक वस्तु का पृथ्वी सतह पर भार 90 kg है। चन्द्रमा का द्रव्यमान पृथ्वी के द्रव्यमान का 1/9 तथा उसकी त्रिज्या की 1/2 है। तो चन्द्रमा पर वस्तु का भार होगा -
(1) 45 kg (2) 202.5 kg
(3) 90 kg (4) 40 kg

Question based on Acceleration due to gravity

Q.5 पृथ्वी तल से 2R दूरी पर पृथ्वी के गुरूत्वीय त्वरण के कारण किसी वस्तु में त्वरण [R = पृथ्वी की त्रिज्या] पृथ्वी तल पर गुरूत्वीय त्वरण का मान होगा -
(1) $\frac{g}{9}$ (2) $\frac{g}{3}$
(3) $\frac{g}{4}$ (4) g

Question based on Energy

Q.6 एक रॉकेट को 10 किमी/सैकण्ड से प्रक्षेपित किया जाता हैं यदि पृथ्वी की त्रिज्या R है तो रॉकेट द्वारा प्राप्त अधिकतम ऊँचाई होगी -
(1) 2R (2) 3R
(3) 4R (4) 5R

Question based on Escape velocity

Q.7 यदि किसी ग्रह की त्रिज्या R तथा घनत्व ρ है तो उसकी सतह से पलायन वेग का मान होगा -
(1) $v_e \propto \rho R$
(2) $v_e \propto \sqrt{\rho}\, R$
(3) $v_e \propto \frac{\sqrt{\rho}}{R}$
(4) $v_e \propto \frac{1}{\sqrt{\rho} R}$

Q.8 दो ग्रहों की त्रिज्याओं का अनुपात k_1 है तथा गुरूत्वीय त्वरणों का अनुपात k_2 है। तो उनके पलायन वेगों का अनुपात होगा -
(1) $k_1 k_2$ (2) $\sqrt{k_1 k_2}$
(3) $\sqrt{\frac{k_1}{k_2}}$ (4) $\sqrt{\frac{k_2}{k_1}}$

ion based on Keplar's law

A satellite A of mass m is at a d the centre of the earth. Anoth mass 2m is at a distance of 2 centre. Their time periods are
(1) 1 : 2 (2) 1
(3) 1 : 32 (4)

Two planet of radii in the from the material of densi Then the ratio of acceler g_1/g_2 at the surface of the
(1) 1 (
(3) 4/9 (

Question based on Keplar's law

Q.9 A satellite A of mass m is at a distance of r from the centre of the earth. Another satellite B of mass 2m is at a distance of 2r from the earth's centre. Their time periods are in the ratio of -

(1) 1 : 2 (2) 1 : 16

(3) 1 : 32 (4) 1 :2 $\sqrt{2}$

10 Two planet of radii in the ratio 2 : 3 are made from the material of density in the ratio 3 : 2 . Then the ratio of acceleration due to gravity g_1/g_2 at the surface of the two planets will be -

(1) 1 (2) 2.25

(3) 4/9 (4) 0.12

Question based on Keplar's law

Q.9 उपग्रह A जिसका द्रव्यमान m तथा पृथ्वी के केन्द्र से दूरी r है। उपग्रह B जिसका द्रव्यमान 2m तथा पृथ्वी के केन्द्र से दूरी 2r है। उनके परिक्रमा कालों में अनुपात होगा -

(1) 1 : 2 (2) 1 : 16

(3) 1 : 32 (4) 1 :2 $\sqrt{2}$

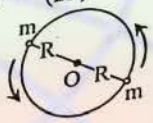
Q.10 दो ग्रहो की त्रिज्याओं का अनुपात 2 : 3 तथा घनत्वों का अनुपात 3 : 2 है। तब दोनों ग्रहों की सतह पर उनके गुरूत्वीय त्वरणों का अनुपात g_1/g_2 होगा -

(1) 1 (2) 2.25

(3) 4/9 (4) 0.12

1.[3] अभिकेन्द्रिय बल दो कणों के बीच गुरूत्वाकर्षण बल द्वारा प्रदान किया जायेगा

अर्थात् $\frac{mv^2}{R} = \frac{Gm \times m}{(2R)^2}$



$\Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$

2. [1] गुरूत्वाकर्षण बल माध्यम पर निर्भर नहीं करता है।

3. [2] हम जानते हैं कि $g = \frac{GM}{R^2}$

ग्रह पर $g_P = \frac{GM/7}{R^2/4} = \frac{4g}{7} = \frac{4}{7}g$

अतः ग्रह पर भार $= 700\frac{4}{7} = 400$ भार

4. [4] $\frac{g_m}{g_e} = \frac{M_m}{M_e} \times \left(\frac{R_e}{R_m}\right)^2 = \left(\frac{1}{9}\right)\left(\frac{1}{9}\right) = \left(\frac{1}{9}\right)$

$\Rightarrow g_m = \frac{4}{9} g_e$

$\therefore W_m = \frac{4}{9} \times W_e = \frac{4}{9} \times 90 = 40$ kg

5. [1] $g' = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{R+2R}\right)^2 = \frac{g}{9}$

6. [3]

7. [2] $v_e = R\sqrt{\frac{8}{3}G\pi\rho} \Rightarrow v_e \propto R\sqrt{\rho}$

8. [2] $v = \sqrt{2gR} \Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{g_A}{g_B} \times \frac{R_A}{R_B}} = \sqrt{k_1 \times k_2}$

$= \sqrt{k_1 k_2}$

9. [1] उपग्रह के द्रव्यमानका इसके आवर्त प्रभाव नहीं होता है।

$\frac{T_A}{T_B} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{r}{2r}\right)^{3/2} =$

10. [1] $\frac{g_1}{g_2} = \frac{\rho_1}{\rho_2} \times \frac{R_1}{R_2} = \frac{3}{2} \times$

By

$$\vec{F}_{AB} = \frac{G m_1 m_2}{r^2} (\text{unit vector along } \vec{AB})$$

Note $\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r}$

By

$$\vec{F}_{AB} = \frac{G m_1 m_2}{r^2} \left(\frac{\vec{AB}}{|\vec{AB}|}\right)$$

$$\vec{F}_{AB} = \frac{G m_1 m_2}{r^3} (\vec{AB}) \quad \because |AB| = r$$

$$\vec{F}_{BA} = \frac{G m_1 m_2}{r^3} (\vec{BA})$$

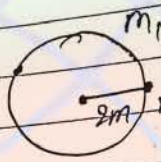
$$F_{AB} = (-) \frac{G m_1 m_2}{r^3} (\vec{BA})$$

Or

$$F_{BA} = (-) \frac{G m_1 m_2}{r^3} (\vec{AB})$$

Q) Force experience by a particle is "F" when it is kept on the surface of sphere of radius find
(i) fore experienced by it in 1m below the surface
(ii) Force experienced by it in 1m above the surface

Sol/q

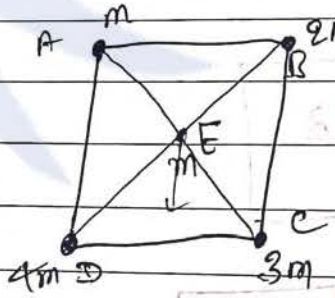


$F = \frac{G m_1 m_2}{4}$ ⇒ $G m_1 m_2 =$

(a) $F_1 = \frac{G m_1 m_2}{8^2} \Rightarrow \frac{4F}{8} = \frac{F}{2}$

(b) $F_2 = \frac{G m_1 m_2}{9} \Rightarrow \frac{4F}{9}$

Q)

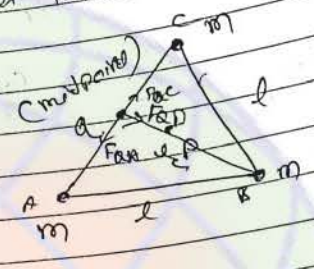


find fore experience by E due to A, B & C and D

Sol/n

$F = \frac{G 4m^2}{2^2} \rightarrow$

Q. Find gravitational force on the particle on another particle of mass 'm' by the given system if (i) it is kept at Q (ii) it is kept at P



(i) $\vec{F}_{net} = \vec{F}_{Q/A} + \vec{F}_{Q/B} + \vec{F}_{Q/C}$

$F_{Q/C} = F_{Q/A}$

and

$F_{Q/C} = (-)\, F_{Q/A}$

$\vec{F}_{Q/A} + \vec{F}_{Q/C} = 0$

so $|\vec{F}_{net}| = |\vec{F}_{Q/B}|$

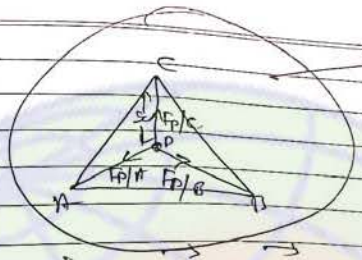
$F_Q = F_{Q/B}$

$= \frac{Gm^2}{r^2}$

where $r = \sqrt{l^2 - \left(\frac{l}{2}\right)^2}$

$r = \frac{\sqrt{3}l}{2}$

(11)

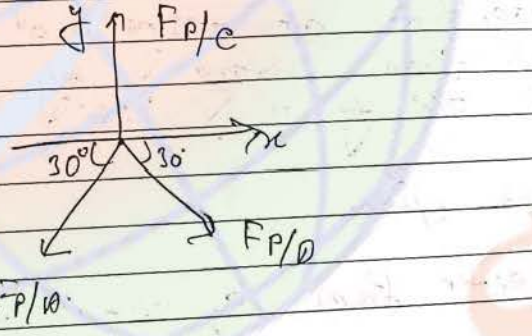


Note

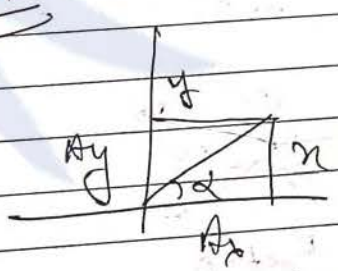
Plane के Case में x-y Co-ordinate system मान कर solve करें।

$$\vec{F}_{net} = \vec{F}_{P/A} + \vec{F}_{P/B} + \vec{F}_{P/C}$$

$$F_{P/A} = F_{P/B} = F_{P/C} = \frac{Gm^2}{r^2}$$



Note

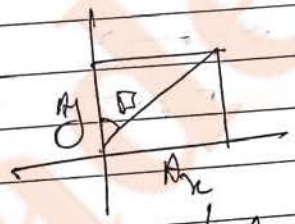


$$\cos\alpha = \frac{A_x}{A}$$

$$\sin\alpha = \frac{A_y}{A}$$

$\therefore A_x = A\cos\alpha$

$A_y = A\sin\alpha$



$A_x = A\sin\beta$

$A_y = A\cos\beta$

$\vec{F}_p = (-F\cos\theta\hat{i}$

Theorem 4b

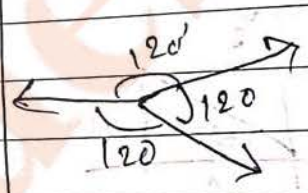
If "n" co-planar forces of equal magnitude are acting on a particle in such a way that any force makes an angle $\frac{360^\circ}{n}$ with its neighbouring force then,

vector sum of all the forces is zero

eg: if (n=2)

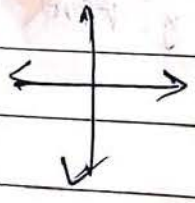
$F_{net} = 0$

n = 3



$F_{net} = 0$

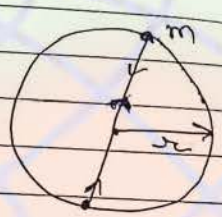
n = 4



$F_{net} = 0$

Image बनाओ

Q.) Two particles each of mass "m" are moving in a circle of radius "r" under mutual force of attraction with same speed. Speed of each particle is

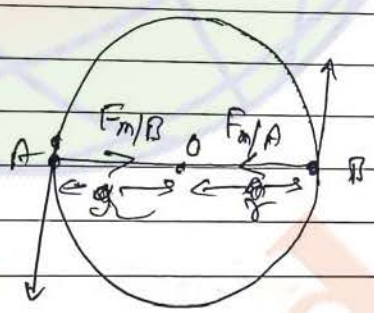


$$F_1 = \frac{G m_1 m_2}{4r^2}$$

$$F_2 = \frac{G m_1 m_2}{4r^2}$$

Now, $v = r\omega$

$$F = \frac{m v^2}{r}$$



Note

$F_{centripetal} = m\, a_{centripetal}$

and

$a_{centri} = \frac{v^2}{r} = \omega^2 r$

For A

$$F_{m/B} = \frac{mv^2}{r}$$

$$\frac{Gm^2}{(2r)^2} = \frac{mv^2}{r}$$

$$mv^2 = \frac{Gm^2}{4r^2} \times r$$

$$v = \sqrt{\frac{Gm}{4r}}$$

6/9/2013

Acceleration due to gravity (g) and factor affecting it

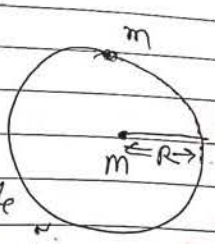
$$E_{or}g = \frac{F}{m}$$

where

F is gravitation force on the particle of "m" by the earth.

* on the surface of Earth

$$g_{do} = \frac{GM}{R^2}$$

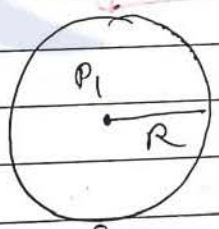


$$g_{do} = \frac{Gm}{R^2} = 9.8\ m/s^2$$

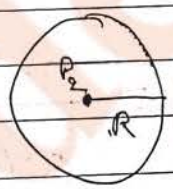
(m → mass of Earth or planet)

↳ This is only on the surface of earth.

Note → To find density (and concept of gradient)



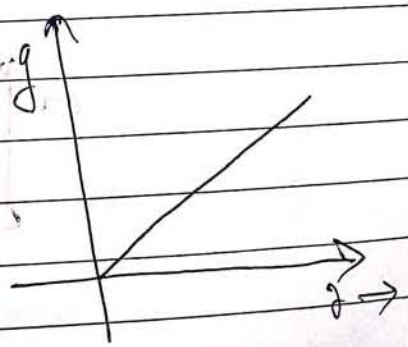
ρ_1



ρ_2

$$g_m = \frac{G \frac{4}{3}\pi R^3 \rho}{}$$

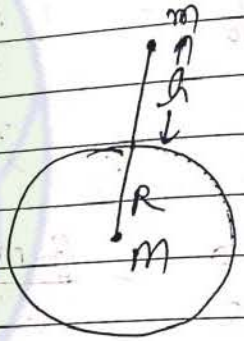
$$g_m \propto R\rho$$



$$\frac{(g_m)_{R_1}}{(g_m)_{R_2}} = \frac{R_1 \rho_1}{R_2 \rho_2}$$

★ Acc^n due to gravity at height "h"

$$g = \frac{F}{m}$$



$$g = \frac{\frac{GMm}{r^2}}{m}$$

$$\boxed{g = \frac{GM}{r^2} = \frac{GM}{(R+h)^2}}$$

$$g = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$\boxed{\text{or } g = g_0\left(1+\frac{h}{R}\right)^{-2}}$$

Effect of Depth

"h"

if $\frac{h}{R} \ll\ll\ll 1$

then

$$g = g_0\left(1 - \frac{2h}{R}\right)$$

Note

$$(1+x)^n = 1 + nx \quad \text{if } x \ll\ll 1$$

"x" का मान 1 से बहुत छोटा होना चाहिए।

$$\%\ \text{decrease in acc}^{\text{n}}\text{due to gravity} = \frac{g_0 - g}{g_0} \times 100$$

if $\frac{h}{R} \ll\ll 1$

then

$$\%\ \text{decrease} = \frac{g_0 - g_0\left(1 - \frac{2h}{R}\right)}{g_0} \times 100$$

$$\%\ \text{decrease} = \frac{2h}{R} \times 100$$

Q) A particle is taken to a height $h = \frac{R}{2}$ from the surface of earth. find % decrease in 'g'.

$g_0 \downarrow 4$

% decrease $= \frac{\frac{GM}{R^2} - \frac{GM}{(R/2)^2}}{\frac{GM}{R^2}} \times 100$

$\downarrow \frac{\frac{GM}{R^2} - \frac{4GM}{R^2}}{\frac{GM}{R^2}} \times 100$

$= \frac{-3\cancel{GM}}{\cancel{GM}} \times 100$

$\downarrow$ 300

$\downarrow$ 3%

$h = \frac{R}{2}$

$g = g_0\left(1 - \frac{h}{R}\right)^{n=2}$

$= g_0\left(1 + \frac{R/2}{R}\right)^{-2} = \frac{4}{9} g_0$

% decrease $= \frac{g_0 - g}{g_0} \times 100$

$= 5\%$

e) A particle is taken to some height from the surface of earth.
h = 1% of radius of earth, find % decrease g

R =

$g = g_0\left(1 + \frac{2 \times \frac{1}{100}R}{R}\right)$

$= g_0\left(1 + \frac{1}{50}\right)$

$= g_0\left(\frac{50}{51}\right)$

$\Rightarrow \frac{50}{51} g_0$

% decrease $= \frac{2 \times \frac{R}{100 \times 50} \times 100}{R}$

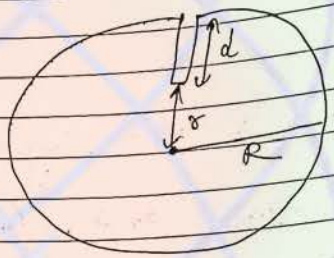
$= \frac{1}{50} \times 100$

$= 2\%$

use this formula upto 5%

☆ Acc^n due to gravity at a depth "d"

$$g = \frac{F}{m}$$



$$g = \frac{G M m r}{R^3 m}$$

$$\boxed{g = \frac{G M r}{R^3}}$$

r → distance from the centre of earth
d → depth from the surface of earth.

$$g = \left(\frac{GM}{R^2}\right)\left(\frac{r}{R}\right)$$

or

$$\boxed{g = g_0\left(1 - \frac{d}{R}\right)}$$

% decrease in g = $\frac{g_0 - g}{g_0} \times 100$

&

$$\% \text{ decrease in } g = \frac{d}{R} \times 100$$

Q) Acc" due to gravity at a height from the earth surface is same as at a distance "d" below the earth surface. where $\frac{d}{R} << << << 1$

which of the following is correct

(I) $h = d$

(II) $h = 2d$

(III) $h > d$

(IV) None of these

Sol"

$$g_h = g_d$$

$$g_0\left(1 - \frac{2h}{R}\right) = g_0\left(1 - \frac{d}{R}\right)$$

$$\boxed{h = d}$$

Q) At what depth below the earth surface acc" due to gravity is same as at a height $h = \frac{R}{2}$ from the earth surface.

Sol"

$g_0\left(1-\frac{d}{R}\right) = g_0\left(1-\frac{h}{R}\right)^{-2}$

$\left(1-\frac{d}{R}\right) = \left(1+\frac{R}{2R}\right)^{-2}$

$\left(1-\frac{d}{R}\right) = 1 - \frac{2}{2}$

$\frac{d}{R} = 1 + 1$

$d = 2R$

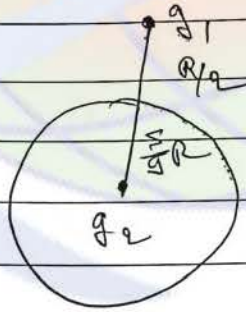
$1-\frac{d}{R} = \frac{g}{g}$

$\frac{d}{R} = \frac{g}{g} - 1$

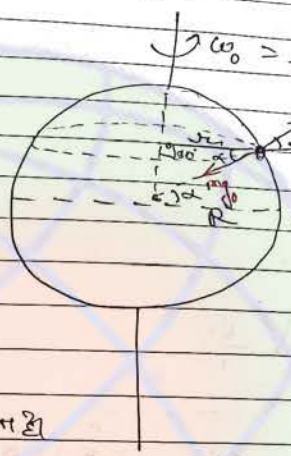
$= 1 - \frac{g}{g}$

$\frac{d}{R} = \frac{5}{g}$

$d = \frac{5R}{g}$



(c) Rotation of earth about its axis :-



$\omega_0 = \frac{2\pi}{T} = \frac{2\pi \text{ rad}}{24 \times 3600 \text{ s}}$

Imp
W → Actual weight = mg
N → app. weight
↳ free falling body का app. weight Zero होता है क्योंकि उसमें कोई Normal force नहीं लगता है

Acc due to gravity = $\frac{\text{weight}}{m} = \frac{W}{m}$

App. weight = $\frac{N}{m}$

$N + mR\omega_0^2 \cos\alpha = mg_0$

$N = mg_0 - mR\omega_0^2 \cos\alpha$

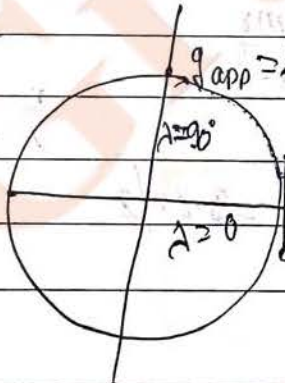
$= mg_0 - mR\omega_0^2 \cos^2\lambda$

= app. weight

But

$g_{app} = \frac{N}{m} = g_0 - R\omega_0^2 \cos^2\lambda$

$$\boxed{g_{app} = g_0 - R\omega_0^2 \cos^2\lambda}$$



g_{app} = Zero (0) or $\boxed{g_{app} = g_0}$ (maximum)

↑ g_{app} is Increase

$\boxed{g_{app} = g_0 - \omega_0^2 R}$ (minimum)

Q) What must be the time period of earth so that app. weight of the object at equator becomes zero

Sol'n $N = 0$ (at $\lambda = 0^\circ$)

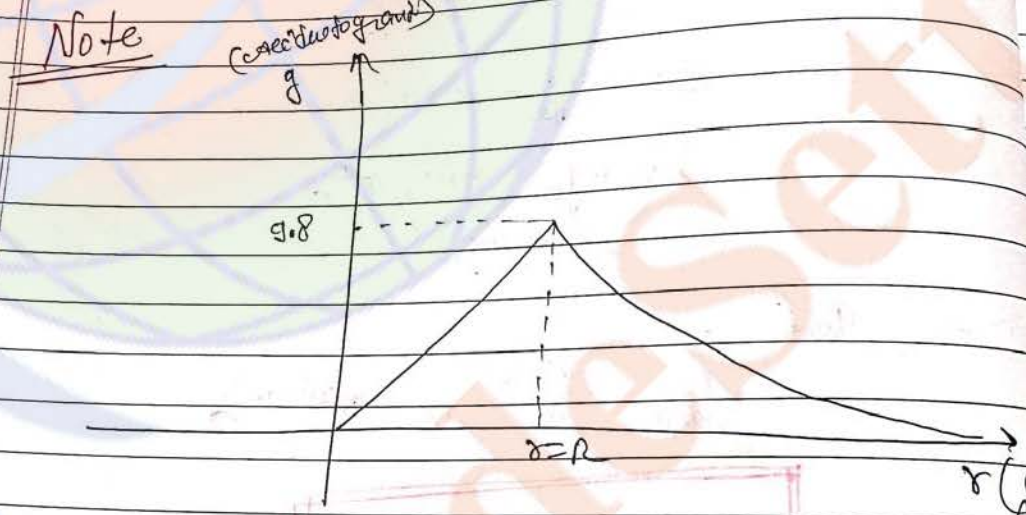
$$mg_0 - mR\omega^2 = 0$$

$$\omega = \sqrt{\frac{g_0}{R}}$$

and

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g_0}} \approx 1.41 \text{ hour}$$

Note



(i) If $r < R$, $g = \frac{GMr}{R^3}$

$g \propto r$

(ii) If $r = R$, $g = \frac{GM}{R^2} = 9.8 \text{ m/s}^2$

(iii) If $r > R$

$g = \frac{GM}{r^2}$

$g \propto \frac{1}{r^2}$

Note

- direction
- centre o
- Shape
- Inter
- It is
- In
- S
- from
- of th

Note

- Direction of Acc^n due to gravity is always towards the centre of earth.

Strength

Intensity of gravitational field (E)

It is defined as

$$E = \frac{F}{m}$$

(F: force exerted by any body)

$$g = \frac{F}{m}$$

(F: force exerted by earth)

where,

F = force exercised by the particle of mass "m" in gravitational field. (test mass)

- ~~S.I unit → N/kg~~
- S.I unit of E is $\frac{N}{kg}$
- S.I unit of g is $\frac{m}{s^2}$

Note :-
"g" is a special case of "E" (that is Intensity of earth's gravitational field.)

Intensity due to point mass (M)

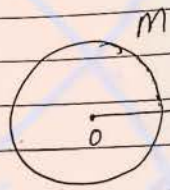
$$E = \frac{F}{m} \quad \frac{GMm/r^2}{m} = \frac{GM}{r^2}$$

$$\therefore E = \frac{GM}{r^2}$$

$$\vec{E} = \frac{GM}{r^2} \quad (\text{from } p \text{ to } O)$$

(c)

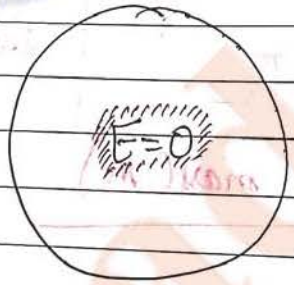
☆ Intensity due to this spherical shell



$$E = \frac{F}{m} = \frac{GMm/r^2}{m}$$

$$E = \frac{GM}{r^2}$$

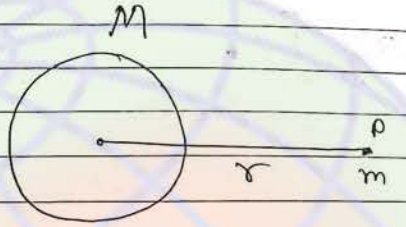
Note



(d)

(c) Intensity due to solid sphere

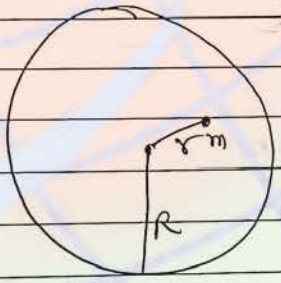
(i)



$$E = \frac{G m_1 m / r^2}{m}$$

$$\boxed{E = \frac{GM}{r^2}}$$

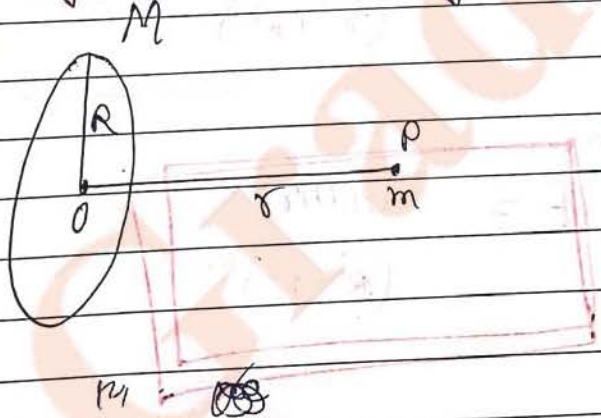
(ii)

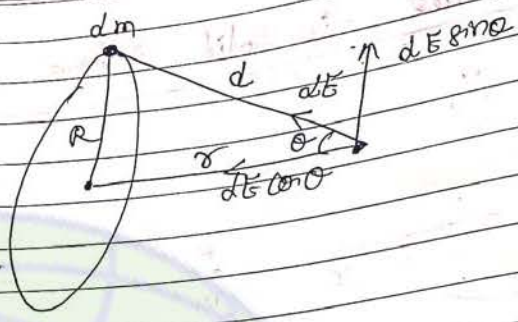


$$E = \frac{F}{m} = \frac{G m / r / R^3}{m}$$

$$\boxed{E = \frac{GMr}{R^3}}$$

(d) Intensity : due to Ring





$$E = \int dE \cos\theta$$

where

$$dE = \frac{G\,dm}{d^2}$$

$$E = \int \frac{G\,dm}{d^2} \cos\theta$$

$$E = \frac{G\cos\theta}{d^2} \int dm$$

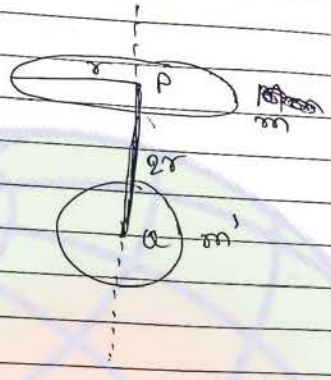
$$E = \frac{G\left(r/\sqrt{r^2+R^2}\right)\cdot m}{(r^2+R^2)} = \frac{GMr}{(r^2+R^2)^{3/2}}$$

Ans

$$\boxed{E = \frac{GMr}{(R^2+r^2)^{3/2}}}$$

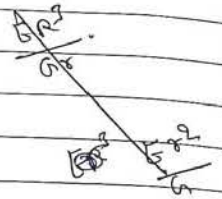
L-1 → 1 to 24
L-2 → 3, 5, 8, 9, 12, 14, 16, 17, 20, 21, 24, 25, 26, 28, 30,

Q.



$I = \frac{GMr}{R^3}$

$F = G$



Intensity due to Ring at the point Q is

$$I = \frac{GMr}{(R^2+r^2)^{3/2}}$$

$$= \frac{GM(2r}{((2r)^2+r^2)^{3/2}}$$

and

$F = I m'$ Ans

Gravitational Potential energy

Energy of a particle by virtue of its position in conservative force field is called potential energy.

Conservative force field:
If workdone by the field in a round trip of a particle is always zero than field is called conservative.
eg :- Gravitational field, Electric field etc.

Note: In conservative field workdone by the field on the particle does not depend upon path. it depends on initial and final position

Change in P.E = (−) $W_{\text{conservative field}}$

Change in P.E = (−) workdone by the conservative field

or

$$\Delta U = (-) W_{\text{conservative field}}$$

if $W_{\text{conservative force}} > 0$ then $\Delta U < 0$

$W_{\text{conservative force}} < 0$ then $\Delta U > 0$

$W_{\text{cons}} = 0$ then $\Delta U = 0$

$W = \Delta K$ → work energy theorem

change in gravitational potential energy (G.P.E) = (−) work done by the gravitational force

$\Delta U_g = (-) W_g$

Note

Proof:

$W = \int \vec{F} \cdot d\vec{r}$

$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$

$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$W = \int (F_x dx + F_y dy + F_z dz)$

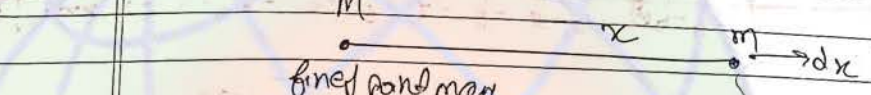
If

$F = f(x)$

$W = \int F_x dx$

$$U_f - U_i = (-)\int_i^f \vec{F}_g \cdot d\vec{x}$$

-) ve work done by gravitational force



$$U_\infty - U_x = (-)\int_x^\infty G m dx$$

$$U_\infty - U_x = (-)\int_x^\infty \frac{-G M m}{x^2} dx$$

$$U_\infty - U_x = GMm\left(\frac{-1}{x}\right)_x^\infty$$

$$U_\infty - U_x = \frac{GMm}{x}$$

or

$$U_x - U_\infty = \frac{-GMm}{x}$$

if $U_\infty = 0$ then

$$U_x = \frac{-GMm}{x}$$

classmate
Date
Page

★ Concept of Energy

$x \to \infty$

$V_\infty = 0$ # Pot

M — x

$U_x = -\frac{GMm}{x}$

इस Graph से पता चलता है कि Infinity (∞) पर Energy को zero (0) माना जाता है इसलिए Binding energy का value "-ve" में आता है। (जिसे Infinity पर Generally शून्य (0) ही मान लेते हैं)

← P.E decrease

P.E increase →

① M — x — $x=\infty$ — f

Note → एक साथ एवं आगे

$\Delta U = +\frac{GMm}{x}$

$U_g = (-)\frac{GMm}{x}$

② M — x — $x=\infty$ — f

$\Delta U = -\frac{GMm}{x}$

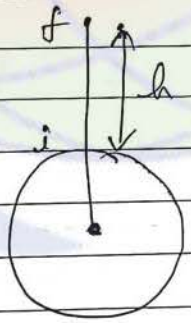
$U_g = \frac{GMm}{x}$

Potential energy due to earth

$$V = -\frac{GMm}{r}$$

$$V = -\frac{GMm}{R+h}$$

Special case:-
If particle is taken from earth surface to height "h" → "h" is from the earth surface



$$\Delta U = \left(\frac{-GMm}{R+h}\right) - \left(\frac{-GMm}{R}\right)$$

$$\Delta U = \frac{GMm}{R} - \frac{GMm}{R+h}$$

$$\Delta U = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$\Delta U = GMm\left[\frac{h}{(R)(R+h)}\right]$$

$$\Delta U = \frac{GMmh}{R^2\left(1+\frac{h}{R}\right)}$$

$$\Delta U = \frac{mgh}{\left(1+\frac{h}{R}\right)}$$

→ यह P.E में जाने का

If $\frac{h}{R} \ll 1$

$$\Delta U = mgh$$

Q) A particle of mass 'm' is taken from earth surface to a height $h = R$, where R is radius of earth. $\Delta U = ?$

Sol^n

$$\Delta U = \left(\frac{mgR}{1+\frac{R}{R}}\right)$$

$$= \frac{mgR}{2}$$

Q) In the above question w. done by gravity

$$= -\frac{mgR}{2}$$

$\therefore (\Delta U = -W_{gr}$

Q) In the above question w. done by ext. agent

$$\frac{mgR}{2}$$

$$W_{ext.agent} = W_{gravity\ agent}$$

Note

w done by stretching object from earth to height 'h' / w done by conservative

Note

(1) $W_{net} = \Delta K$
w done by all forces

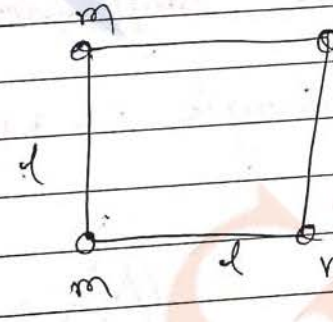
(2) $W_{ext} = \Delta U + \Delta K$

$W_{ext} = \Delta U + \Delta K$

$= mgh$

$= \frac{mgR}{2}$

a)

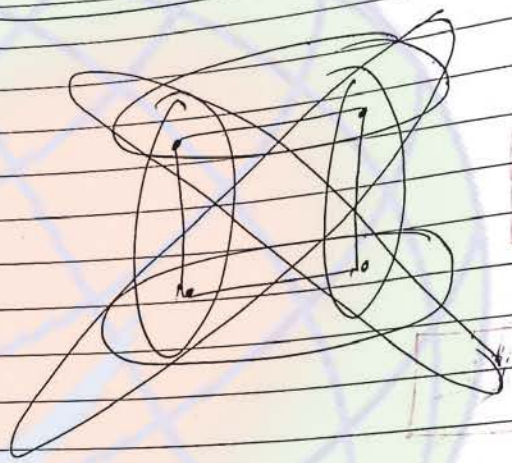


$\Delta U = ?$ if $U_\infty = 0$

$0 + mgh +$

Note

$m \quad r \quad m$, $U = \frac{-GMm}{r}$ if



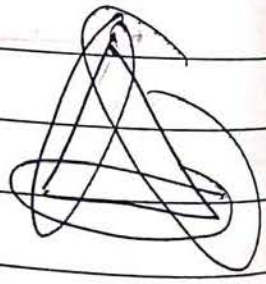
$$U_{system} = 4\left(\frac{-Gm^2}{l}\right) + 2\left(\frac{-Gm^2}{\sqrt{2}l}\right)$$

$$= \frac{-Gm^2}{l}(4+\sqrt{2})$$

a) three particles of same mass brought from ∞ vertices of an equilateral triangle. length a side is 'l'.

Sol:

$$U_{work} = 3\left(\frac{-Gm^2}{l}\right) J$$



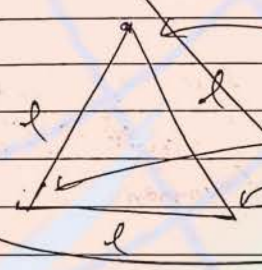
classmate
Date ____
Page ____

Note → $W_g = \frac{3Gm^2}{l}$

~~$W_{ext} = \frac{3Gm^2}{l}$~~

$\Rightarrow 0 + \left(\frac{-Gm^2}{l}\right) - \frac{2Gm^2}{l} = \frac{-3Gm^2}{l}$

~~$\frac{3Gm^2}{l}$~~



Q) A particle of mass "m" is released from a height $h = R$, R is Rad. of earth.
(measured from surf)

Find its speed when it reaches the earth surface

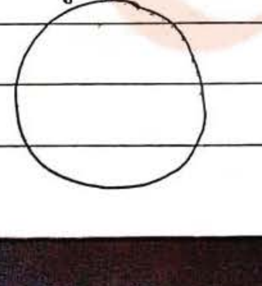
$\frac{1}{2} m v^2 = \frac{mgR}{2}$

$\left\{ \frac{mgh}{1 + \frac{h}{R}} = \frac{mgR}{1+1} \Rightarrow \frac{mgR}{2} \right.$

$$v = \sqrt{gR}$$

$v \not\propto m$

Teach



$W_{ext} = \Delta U + \Delta K$

$0 = \frac{-mgh}{\left(1+\frac{h}{R}\right)} + \left(\frac{1}{2} m v^2 - 0\right)$

Here,
$h = R$

$$v = \sqrt{gR}$$

$$= 8 \text{ km/s}$$

Q) A particle is released from very large distance as compared to Radius of earth (∞ means) find its speed when it reaches the earth surface

$$\Delta K.E = -\Delta P.E$$

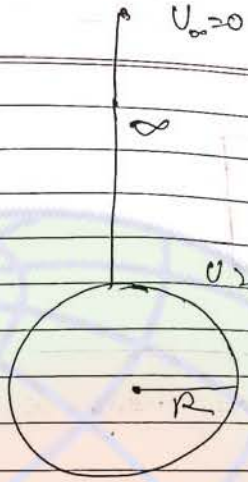
$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{R+x}$$

$$\frac{1}{2}mv^2 = mg - \frac{mgR}{1+\frac{x}{R}}$$

$$\frac{1}{2}mv^2 = \frac{mgRx}{R+x}$$

$$v = \sqrt{\frac{2GM}{R}}$$

$$v = \sqrt{\frac{2gRx}{R+x}}$$



$$\Delta W_{ext} = \Delta U + \Delta K$$

$$0 = \left(\frac{-GM_m}{R} - 0\right) + \left(\frac{1}{2}mv^2 - 0\right)$$

$$v_{\oplus} = \sqrt{\frac{2GM}{R}} = \sqrt{2g_0R}$$

$$= 11.2\ km/s$$

Note

$$W_{ext} = \Delta U + \Delta K$$

$$0 = \frac{-mg_0}{\frac{1}{R} + \frac{1}{\infty}} + \left(\frac{1}{2}mv^2 - 0\right)$$

$$v = \sqrt{2Rg_0}$$

Gravational Potential (V)

$$\Delta V = \frac{\Delta U}{m}$$

change in gravitational potential = change in potential energy per unit mass

Note

SI unit J/kg

$$\Delta U = (-) \int \vec{F} \cdot d\vec{s}$$

$$\therefore \Delta V = \frac{(-) \int \vec{F} \cdot d\vec{s}}{m}$$

$$\therefore \Delta V = -\int \vec{E} \cdot d\vec{s}$$

$$\Delta V = (-) \int \vec{E} \cdot d\vec{s}$$

potential difference = gravitational field intensity

$$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

and

$$d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\Delta V = (-)\int E_x dx + E_y dy + E_z dz$$

If $E = f(x)$ (In 1D)

$$\Delta V = (-)\int E_x dx$$

and

$$\frac{dV}{dx} = -E_x$$

or

$$E_x = \frac{-dV}{dx}$$

If $E = f(x, y, z)$ (in 3D)

$$\Delta V = (-)\int (E_x dx + E_y dy + E_z dz)$$

and

$$E_x = \frac{-\partial V}{\partial x} \; ; \; E_y = \frac{-\partial V}{\partial y} \; ; \; E_z = \frac{-\partial V}{\partial z}$$

(partial diff)

In very short

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

eg.) $V = K(x+y)$ (K constant)

find intensity at any point (x,y)

$$E_x = -\frac{\partial V}{\partial x} \Rightarrow -K$$

$$E_y = -\frac{\partial V}{\partial y} = -K$$

so

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j}$$

$$\vec{E} = -K\hat{i} - K\hat{j}$$

eg) In the above question find force experienced by a particle of mass "m" kept at the point $P(x,y)$.

$E = \frac{F}{m}$

$F = Em$

$= \sqrt{2}m|K|$

$= \sqrt{2}m|K|$

$= \sqrt{2}|K|m$

Q) if $\vec{E} = (3\hat{i} + 4\hat{j})\ \frac{N}{kg}$

find gravitational potential at a point P (3,4)m if V at origin = 0

Sol)

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$= -[3x + 4y]$

$= -[9 + 16]$

$= -25$

Recall

$\Delta V = -\left(\int 3dx + \int 4dy\right)$

$V_f - \Delta V_i = (-)\left(\int_0^3 3dx + \int_0^4 4dy\right)$

$V_f = -25\ \frac{J}{kg}$

E (Intensity)

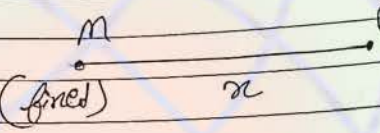
Integrate

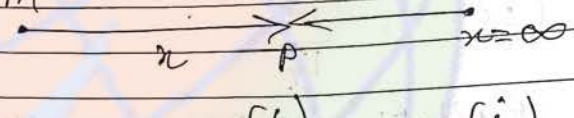
V (Pot. diff)

$E = -\frac{dV}{dr}$

$\Delta V = \frac{\Delta U}{m}$

☆ Gravitational Potential due to a point mass





(f) (i)

$$\Delta V = \frac{\Delta U}{m}$$

$$V_x - V_\infty = \frac{U_x - U_\infty}{m}$$

$$= \frac{-G m m/x}{m}$$

$$V_x - V_\infty = \frac{-GM}{x}$$

if $V_\infty = 0$ (assumption)

↓

Refrence point

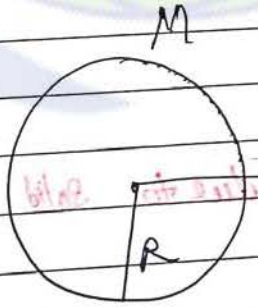
then formula is

$$V_r = \frac{-GM}{r}$$

Note

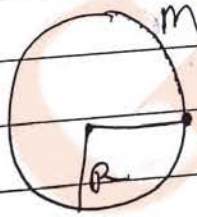
Refrence point : point at which potential is taken to be zero.

☆ Gravitational potential due to spherical shell :-



$$V_{outside} = \frac{-GM}{r}$$

taken $V_\infty = 0$



$$V_{surface} = \frac{-GM}{R}$$

E = 0

$V_{inside} = \frac{-GM}{R}$

Note

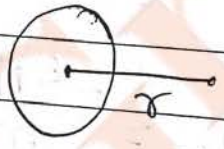
$V_{inside} = V_{surface}$

Graph :-

Gravitational potential due to solid sphere

① if $r \geq R$

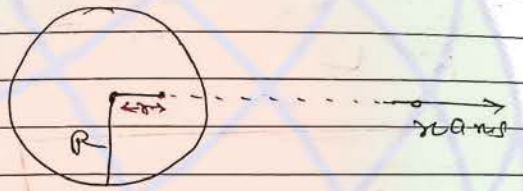
$V = \frac{-GM}{r}$ if $V_\infty = 0$



(i) If $r = R$

$$V_{surface} = \frac{-GM}{R}$$

(ii) if $r < R$



$$\Delta V = (-) \int E_x \, dx$$

$$V_r - V_R = (-) \int_R^r (-) \left(\frac{GMx}{R^3}\right) dx$$

$$V_r - V_R = \frac{GM}{R^3} \left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

$$V_r = \frac{GM}{R^3} \left[\frac{r^2}{2} - \frac{R^2}{2}\right] \cdot V \cdot R$$

$$= \frac{GM}{R^3} \left[\frac{r^2}{2} - \frac{R^2}{2}\right] \left(\frac{-GM}{R}\right)$$

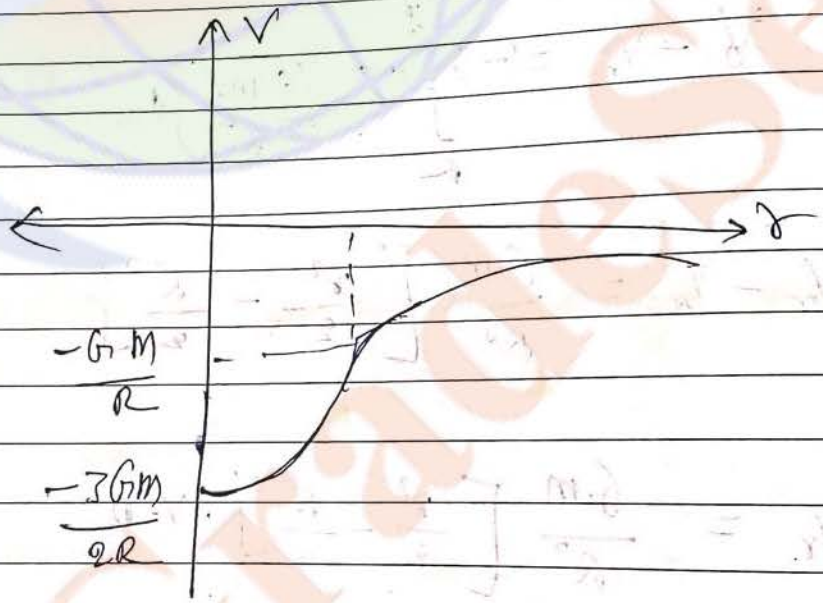
New L-1 → 25 to 35
L-2 → 4, 7, 22, 28, 31,
4A → 1, 4, 9, 11, 12, 13, 15, 16, 17, 18,

$$= \frac{GM}{R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right] + \left[\frac{-GM}{R}\right]$$

$$= \frac{GM}{R^3}\left[\frac{r^2}{2} - \frac{R^3}{2}\right] + \left[\frac{-GMR^2}{R^3}\right]$$

$$V_r = \frac{GM}{R^3}\left[\frac{r^2}{2} - \frac{3R^2}{2}\right]$$

$$\boxed{V_r = \frac{-GM}{2R^3}\left[3R^2 - r^2\right]}$$



$$\boxed{V_{centre} = \frac{-3GM}{2R}}$$

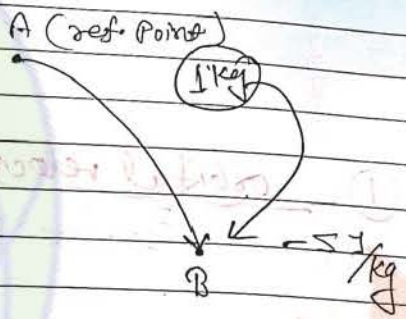
Note

Gravitation
change in
on the
the given

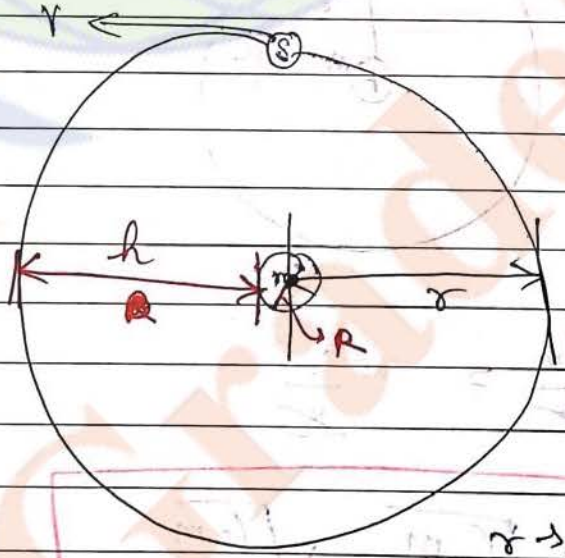
Sa

Note Gravitational potential at a point is equal to the change in gravitation potential energy per unit mass as the mass is brought from ref. point to the given point.

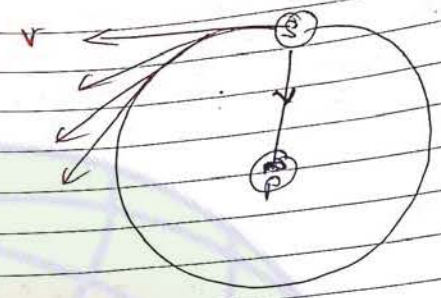
$V_{\text{ref. point}} = 0$



Satellite Motion



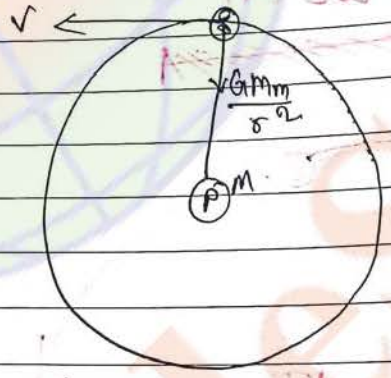
$r \rightarrow$ distance of satellite from centre of earth.



① orbital velocity →

$F_{centripetal} = m a_{centripetal}$

$\downarrow \frac{v^2}{r}$



$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+r}}$$

↳ orbital speed

Note Orbital speed of satellite does not depends on its own mass.

If Force is always ⊥ to the body then work done by this body is zero.

(2) Time period.

$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi r}{\sqrt{GM/r}} = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

$$\boxed{T^2 = \frac{4\pi^2 r^3}{GM}}$$

or

$$\boxed{T^2 \propto r^3}$$ if M = Const.

(3) Kinetic energy

$$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}m\left(\frac{GM}{r}\right)$$

or

$$\boxed{K.E = \frac{GMm}{2r}}$$

④ Potential energy

$$U = \frac{-GMm}{r}$$

⑤ Total mechanical energy

$$T = K.E + P.E$$

$$T = -\frac{GMm}{2r}$$

⑥ Binding energy

$$B.E = \frac{GMm}{2r}$$

Note

Standard Relation

$$T = -K.E = \frac{1}{2} P.E$$

Special case

① If $h \simeq 0$, that is satellite revolving near the earth surface.

then,

$$v = \sqrt{\frac{GM}{R}} = \sqrt{Rg}$$

$$= 7.8 \simeq 8 \text{ km/sec.}$$ (for earth surface)

(2) Time period $(T) = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi R}{\sqrt{Rg_0}} = 2\pi\sqrt{\frac{R}{g_0}}$

$\simeq 1.41$ hrs

(3) Kinetic energy $(K.E) = -\frac{GMm}{2R} = +\frac{mg_0R}{2}$

(4) Potential energy $(P.E) = -\frac{GMm}{R} = -mg_0R$

(5) Total energy $(T.E) = -\frac{mg_0R}{2}$

(6) Binding energy $= -$ (total energy)

$= \frac{mg_0R}{2}$

$\frac{1/2}{m}V\left(\frac{1}{g}\right) = \frac{T}{T}$

motion of Two planets

Some Important Case

$h \simeq 0$

(1) $v = \sqrt{\frac{Gm}{R}}$

$$\frac{v}{v'} = \sqrt{\frac{m}{m'}} \times \sqrt{\frac{R'}{R}}$$

(2) Time period (T) →

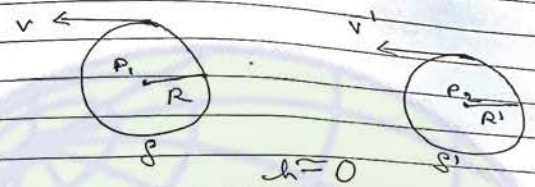
$$\frac{2\pi R}{\sqrt{\frac{Gm}{R}}} = \frac{2\pi R^{3/2}}{\sqrt{Gm}}$$

$$T \propto \frac{R^{3/2}}{\sqrt{m}}$$

$$\frac{T}{T'} = \left(\frac{R}{R'}\right)^{3/2} \sqrt{\frac{M_1}{m}}$$

(12)

(13)



(1) $$v = \sqrt{\frac{Gm}{R}} = \sqrt{\frac{G \cdot \frac{4}{3}\pi R^3 S}{R}}$$

$$\boxed{v \propto R\sqrt{\rho}}$$

$$\boxed{\frac{v}{v'} = \frac{R}{R'}\sqrt{\frac{\rho}{\rho'}}}$$

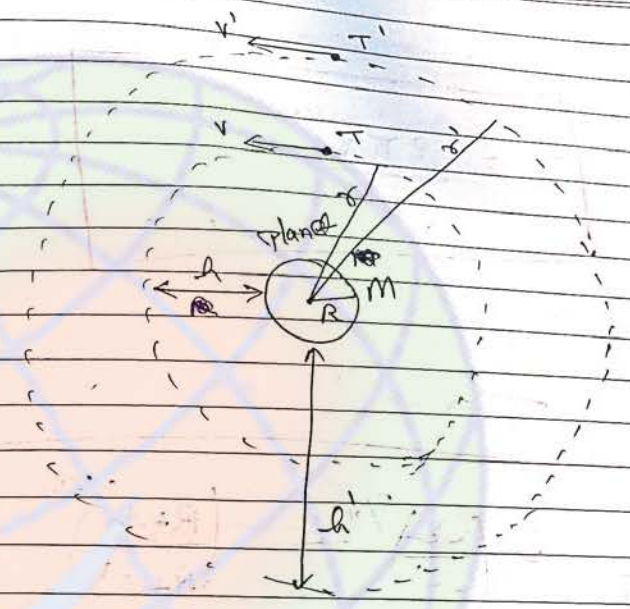
(2) $$T = \frac{2\pi R^{3/2}}{\sqrt{Gm}}$$

$$T = \frac{2\pi R^{3/2}}{\sqrt{G \cdot \frac{4}{3}\pi R^3 \rho}}$$

$$\boxed{T \propto \frac{1}{\sqrt{\rho}}}$$

$$\boxed{\frac{T}{T'} = \sqrt{\frac{\rho'}{\rho}}}$$

☆ Motion of two ~~planet~~ & satellite about same planet:-



① $V = \sqrt{\frac{GM}{r}}$ → "M" is same in both case of satellite

$$v \propto \frac{1}{\sqrt{r}}$$

$$\frac{V}{V'} = \sqrt{\frac{r'}{r}} = \sqrt{\frac{R+h'}{R+h}}$$

(2)

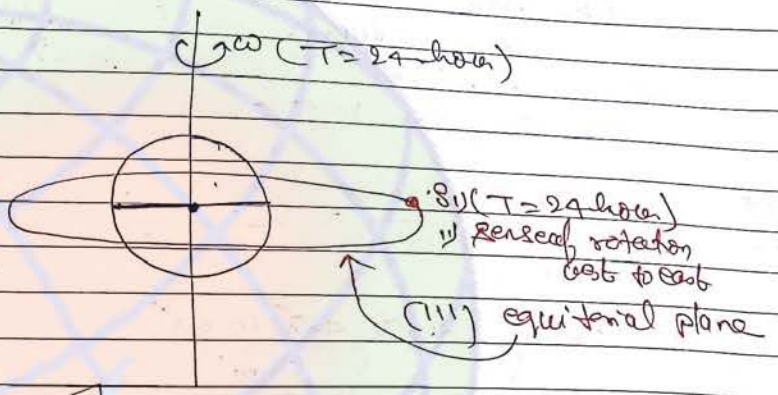
$$T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$ same

$$T \propto r^{3/2}$$

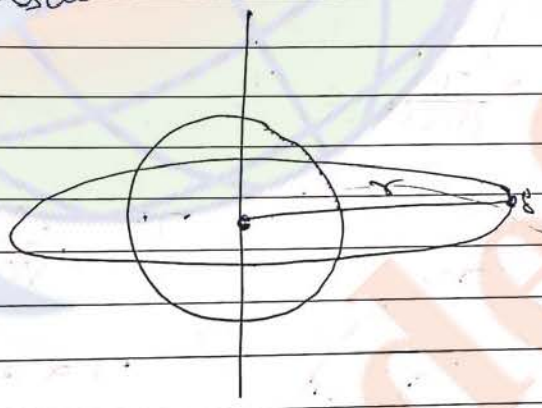
$$\frac{T}{T'} = \left(\frac{r}{r'}\right)^{3/2} = \left(\frac{R+h}{R+h'}\right)^{3/2}$$

Geo stationary satellite

A satellite which remain at rest with respect to earth is called Geo-stationary satellite.



Radius of earth 6,000 km



$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$r = \left(\frac{T^2 GM}{4\pi^2}\right)^{1/3}$$

$r \simeq 42,000$ km from centre of earth

or 36,000 km from surface of earth

Q.) A geo-stationary satellite is at a height 6R from the earth surface where R is radius of earth, find time period of a satellite which is at a height 2.5R from the earth surface.

80/-

$T^2 = \frac{4\pi^2}{GM} r^3$

$T_1^2 = \frac{4\pi^2}{GM} (3.5R)^3$

$\left(\frac{T}{T_1}\right)^2 = \frac{T^2}{(3.5)^3}$

$T = 3.5 \times 3.5 \times 3.5$

$T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$

$T \propto r^{3/2}$

$\frac{T'}{T} = \left(\frac{r'}{r}\right)^{3/2}$

$r' = (24)\left(\frac{2.5R}{4R}\right)$

$\frac{24}{2\sqrt{2}} G - 6\sqrt{2} M$

earth

Kepler

(1) law of ellip

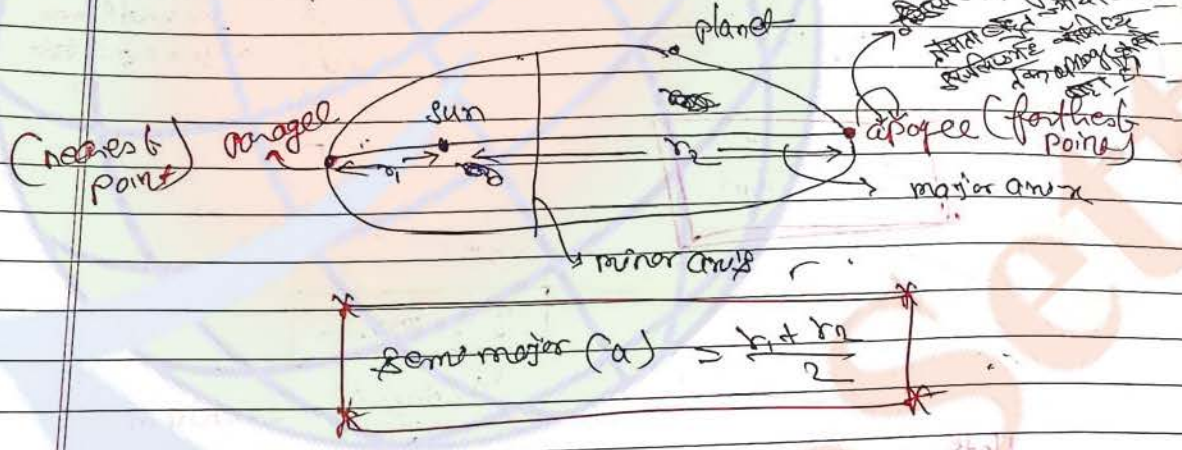
(2)

(nearest point)

Keepler's law

- law of orbit
- law of area
- law of time period.

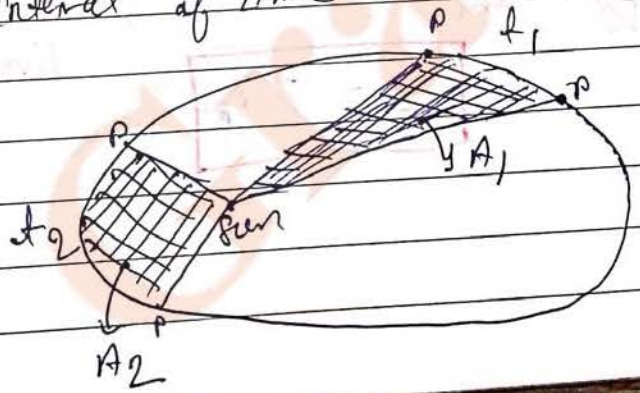
(1) law of orbit :→ each planet moves about the elliptical path about the sun on the focus



$$\text{semi major } (a) = \frac{r_1 + r_2}{2}$$

(2) law of Area :-

Acc to this law a line joining planet to the sun sweeps out equal area in equal Interval of time

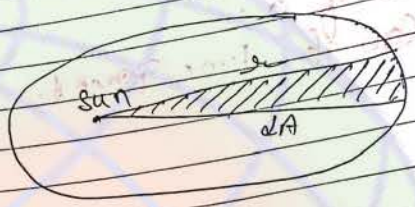


$A_1 = A_2$ if $t_1 = t_2$

and

$t_1 = t_2$ if $A_1 = A_2$

That is same area is same time.



$$dA = \frac{1}{2}(vdt)r$$

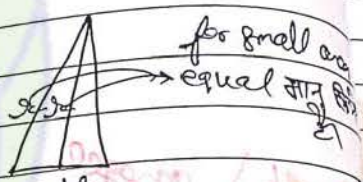
$$\frac{dA}{dt} = \frac{vr}{2}$$

$$\boxed{\frac{dA}{dt} = \frac{J}{2m}}$$

where

$$\boxed{J = mvr}$$

↳ Angular momentum



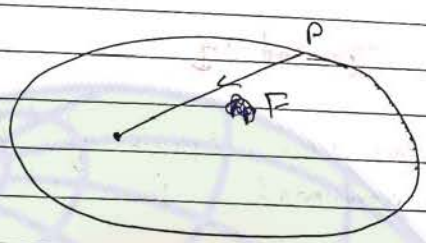
Note

(1) $\tau = \frac{dJ}{dt}$

and

$$\tau = F r_{\perp}$$

this is know as lever arm



Here

$\tau = 0$

$\therefore L = 0$

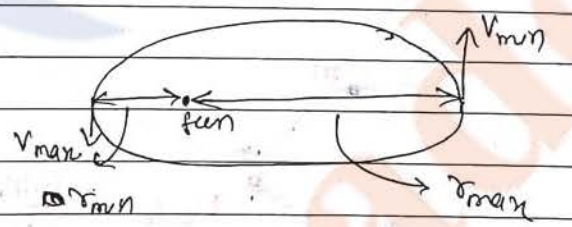
or $\frac{dJ}{dt} = 0$

J = constant

Note: This law (law of area) is also known as Angular momentum Conservation Principle.

(1) Law of Area is based on principle of conservation of Angular momentum during the motion angular momentum of the planet remains unchanged.

(2)



$m V_{max} \cdot r_{min} = m V_{min} \cdot r_{max}$

$V_{max} \cdot r_{min} = V_{min} \cdot r_{max}$

(c) Law of time period :-

According to this law square of time period is directly proportional to the third of semimajor axis.

$$T^2 \propto a^3 \text{, where } a = \left(\frac{r_1 + r_2}{2}\right)$$

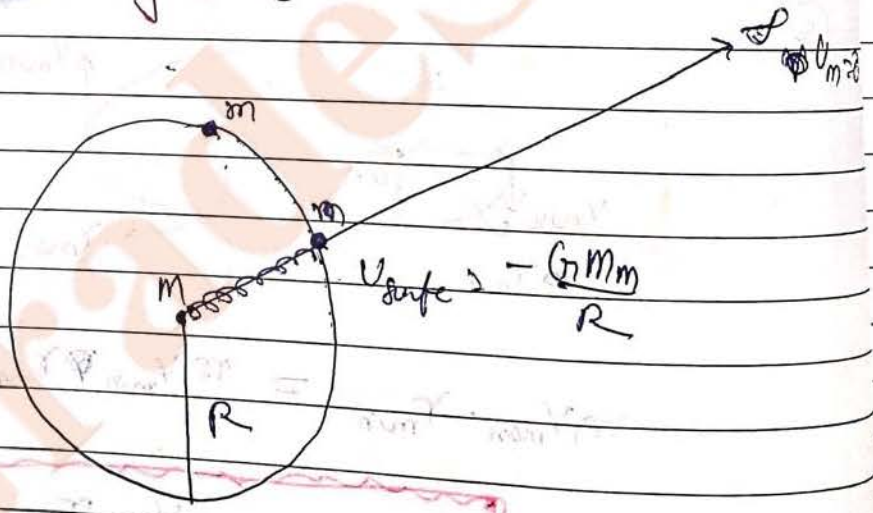
Note

In case of circular motion

$$T^2 \propto r^3$$

r → radius of orbit

☆ Escape velocity :- (v_e)



NOW

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$$T.E = K.E + P.E$$

$$= \frac{1}{2}mv^2 + \left(\frac{-GMm}{R}\right)$$

(a) if $T.E < 0$, object can not escape

(b) if $T.E \geq 0$, object escape from the earth gravitation field.

$$\frac{1}{2}mv^2 - \frac{GMm}{R} \geq 0$$

$$v \geq \sqrt{\frac{2GM}{R}}$$

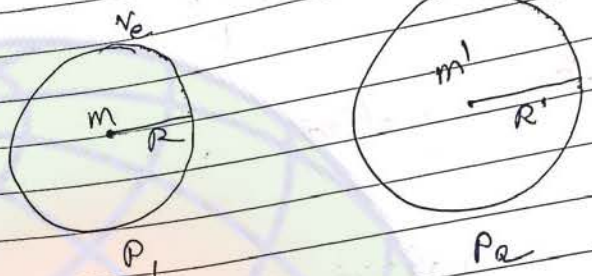
$$v_{escape} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} = 11.2 \text{ km/s}$$

(On the earth surface)

$$v_e = \sqrt{\frac{2G\frac{4}{3}\pi R^3 \rho}{R}}$$

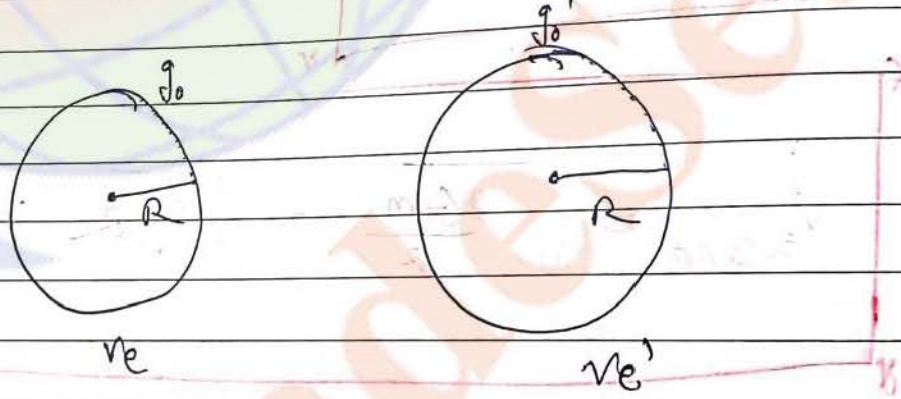
$$v_e \geq \sqrt{\frac{8G\pi R^2 \rho}{3}}$$

★C # Imp. Point:-



$$v_e \propto \sqrt{m/R}$$

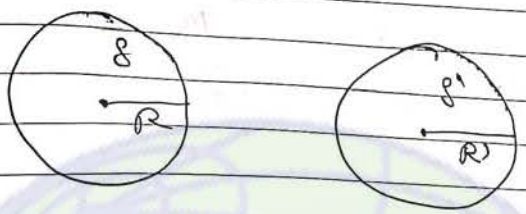
$$\frac{v_e}{v_e'} = \sqrt{\frac{m}{m'} \times \frac{R'}{R}}$$



$$v_e \propto \sqrt{g_0 R}$$

$$\frac{v_e}{v_e'} = \sqrt{\frac{g_0 R}{g_0' R'}}$$

(3)



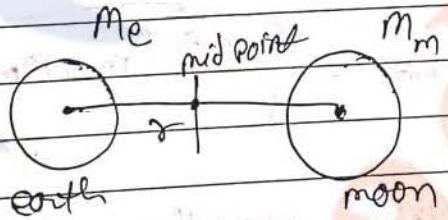
$$V_e \propto R\sqrt{\rho}$$

$$\boxed{\frac{V_e}{V_e'} = \frac{R}{R'}\sqrt{\frac{\rho}{\rho'}}}$$

(1) Escape velocity does not depend on the mass of body.

(2) Escape velocity does not depends upon angle of projection.

Q.

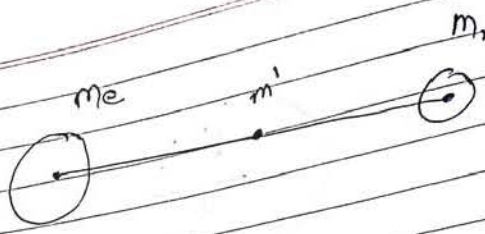


what minimum velocity must be given to to a particle so that it may escape from the mid-point into the space.

Sol

$$v \geq \sqrt{\frac{2GM}{R}}$$

$$\int \frac{1}{x^2}$$



$$U_i = \left(\frac{-G m_e m'}{r/2}\right) + \left(\frac{-G m_m m'}{r/2}\right)$$

$$\Rightarrow \frac{-2Gm'}{r}(m_e + m_m)$$

$$T.E = \frac{1}{2}mv^2 + U_i$$

$$T.E \geq 0$$

$$\frac{1}{2}mv^2 + \left(\frac{-2Gm}{r}\right)(m_e + m_m) \geq 0$$

$$v \geq \sqrt{\frac{4G}{r}(m_e + m_m)}$$

$$v_{escape} = \sqrt{\frac{4G}{r}(m_e + m_m)}$$

Q) A satellite revolves near the earth surface. By how % increase in its velocity so that it may escape into the space

$$v = \sqrt{\frac{GM}{R}}$$

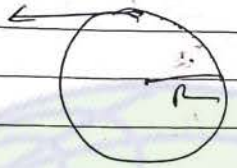
$$v' = \sqrt{\frac{2GM}{R}}$$

$$\frac{v}{v'} = \frac{1}{\sqrt{2}}$$

$$v' = \sqrt{2}\,v$$

$$= \sqrt{2} \times$$

Sol^n $v = \sqrt{g_0 R}$



% Decrease in velocity $= \frac{v_f - v_i}{v_i} \times 100$

$= \frac{\sqrt{2g_0 R} - \sqrt{g_0 R}}{\sqrt{g_0 R}} \times 100$

$\approx 41\%$

Q) In the above question find % Decrease in kinetic energy

Sol^n % Decr. $= \frac{K_f - K_i}{K_i} \times 100$

$K_f = \frac{1}{2} m (v_e)^2$

$= \frac{1}{2} \times (\sqrt{2g_0 R})^2$

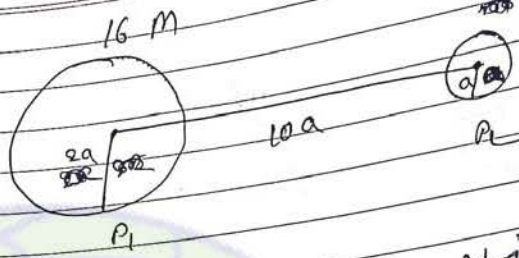
$= g_0 R$

$\% \downarrow = \frac{mg_0 R - mg_0 R/2}{mg_0 R/2} \times 100$

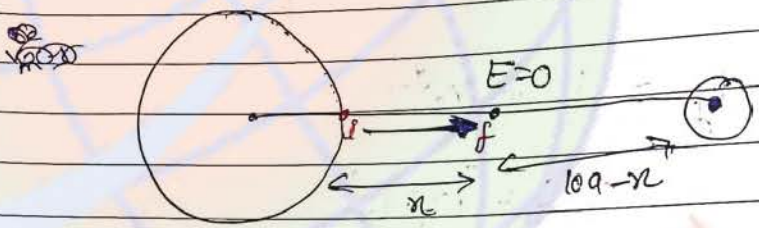
$= 100\%$

Note

$$\% \text{ decrease} = \frac{A_i - A_f}{A_i} \times 100$$



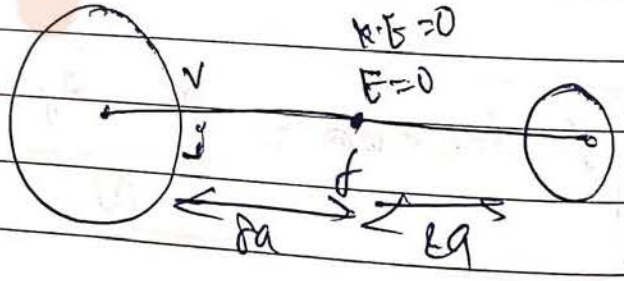
A body is projected straight from the surface of bigger plane towards the small Planet. What vel. must be given to it show that it reaches the surface of small plane.



$$\frac{G(16m)}{x^2} = \frac{Gm}{(10a-x)^2}$$

$$\frac{4}{x} = \frac{1}{10a-x}$$

$$\boxed{x = 8a}$$



Now

$$V_i = \frac{-G(16m)m'}{2a} + \frac{-G(m)(m')}{8a}$$

$$V_i = \frac{-65}{8}\frac{Gmm'}{a}$$

$$V_f = \frac{-G(16m)m'}{8a} + \frac{-G(m)(m')}{2a}$$

$$= \frac{-20}{8}\frac{Gmm'}{a}$$

$$\Delta U = V_f - V_i$$

$$= \frac{45}{8}\frac{Gmm'}{a}$$

but

$$\Delta U + \Delta K = W_{ext}$$

$$\frac{45}{8}\frac{Gmm'}{a} + \left(0 - \frac{1}{2}m'v^2\right) = 0$$

$$v = 2\sqrt{\frac{45}{4}\frac{Gm}{a}} = \frac{3}{2}\sqrt{\frac{5Gm}{a}}$$

Vector

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vector quantities → The quantities which have magnitude and direction and can be added or subtracted According to triangle Rule are called vector quantities.

eg → displacement, velocity, Acc^n, force, etc.

Note symbol of vector → $\vec{V}$, V (bold symbol — bold में लिखा जाता है)

$$\vec{A} = |\vec{A}|\hat{A}$$
$$= A\hat{A}$$

where

$|\vec{A}| = A$ = magnitude of $\vec{A}$

and

$\hat{A}$ = unit vector in the direction of $\vec{A}$

Note
$$|\hat{A}| = 1$$
↳ modulus of unit vector

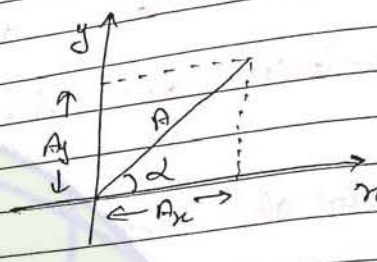
unit vector :-

in $+x$ direction $= \hat{i}$	and in $-x$ direction $= -\hat{i}$
" $+y$ " $= \hat{j}$	" $-y$ " " $= -\hat{j}$
" $+z$ " $= \hat{k}$	" $-z$ " " $= -\hat{k}$

Ans

☆ Resolution of a vectors :-

Concept



A_x → X-component of vector.

A_y → Y-component of vector

$\frac{A_x}{A}$ = Cos θ ⇒ $A_x = A\cos\theta$

$\frac{A_y}{A}$ = Sin θ ⇒ $A_y = A\sin\theta$

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

⇒

$\vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}$

Ans

$A^2 = A_x^2 + A_y^2$

$A = \sqrt{A_x^2 + A_y^2}$

CAREER POINT

Target Course for NITs (JEE Main)-2014

DAILY PRACTICE PROBLEM SHEET

PHYSICS

Vector

1 A vector $\vec{A}$ makes an angle of 20° and $\vec{B}$ makes an angle of 110° with the X-axis The magnitude of these vectors are 3 m and 4 m respectively. Find the resultant.

Q.1 एक सदिश $\vec{A}$, 20° का कोण तथा सदिश $\vec{B}$, 110° का कोण X-अक्ष के साथ बनाते हैं। इन सदिशों के परिमाण क्रमशः 3 m व 4 m है तो परिणामी ज्ञात कीजिए–

2 Let $\vec{A}$ and $\vec{B}$ be the two vectors of magnitude 10 unit each. If they are inclined to the x-axis at angles 30° and 60° respectively, find the resultant.

Q.2 मानाकि $\vec{A}$ तथा $\vec{B}$ प्रत्येक 10 इकाई परिणाम के दो सदिश है। यदि यह x-अक्ष से क्रमशः 30° व 60° कोण बनाते हैं, तो इनका परिणामी ज्ञात कीजिए –

3 Add vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ each having magnitude of 100 unit and inclined to the x-axis at angles 45°, 135° and 315° respectively.

Q.3 तीन सदिश $\vec{A}$, $\vec{B}$ व $\vec{C}$ प्रत्येक का परिणाम 100 इकाई है तथा यह x-अक्ष से क्रमशः 45°, 135° तथा 315° है कोण बनाते हैं, इनका योग कीजिए।

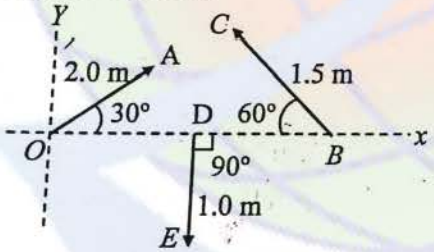
.4 Let $\vec{a} = 4\vec{i} + 3\vec{j}$ and $\vec{b} = 3\vec{i} + 4\vec{j}$ Find the magnitude of

(a) $\vec{a}$, (b) $\vec{b}$, (c) $\vec{a} + \vec{b}$ and (d) $\vec{a} - \vec{b}$.

Q.4 मानाकि $\vec{a} = 4\vec{i} + 3\vec{j}$ तथा $\vec{b} = 3\vec{i} + 4\vec{j}$ इनके परिमाण ज्ञात कीजिए–

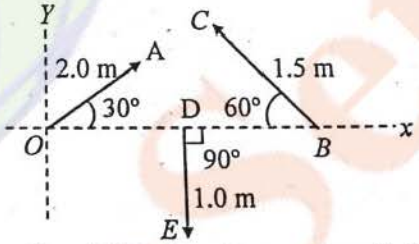
(a) $\vec{a}$, (b) $\vec{b}$, (c) $\vec{a} + \vec{b}$ तथा (d) $\vec{a} - \vec{b}$.

.5 Refer to figure. Find x and y components of $\vec{OA}$, $\vec{BC}$ and $\vec{DE}$.



Q.5 नीचे दर्शाये चित्र के सम्बन्ध ज्ञात कीजिए–

$\vec{OA}$, $\vec{BC}$ तथा $\vec{DE}$ के x व y घटक क्या होंगे –



.6 A carom board (4 ft × 4ft square) has the queen at the centre. The queen, hit by the striker moves to the front edge, rebounds and goes in the hole behind the striking line. Find the magnitude of displacement of the queen

(a) from the centre to the front edge,

(b) from the front edge to the hole and

(c) from the centre to the hole.

Q.6 एक कैरम बोर्ड (4 ft × 4ft square) इसके केन्द्र पर क्वीन रखता है, क्वीन को स्ट्राइकर द्वारा हिट किया गया है, यह गति करते हुए सामने वाले किनारे से टकराती है, व टकराकर स्ट्राइकिंग रेखा के पीछे वाले छिद्र में चली जाती है। क्वीन के विस्थापन का परिमाण ज्ञात कीजिए

(a) केन्द्र से सामने वाले किनारे तक

(b) सामने वाले किनारे से छिद्र तक

(c) केन्द्र से छिद्र तक

7 Two vectors have magnitudes 2m and 3m. The angle between them is 60°. Find

(a) the scalar product of the two vectors,

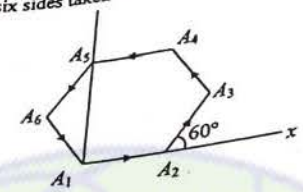
(b) the magnitude of their vector product.

Q.7 दो सदिशों के परिणाम 2m व 3m है। इनके मध्य कोण 60° है ज्ञात कीजिए

(a) दोनों सदिशों का अदिश गुणा

(b) इनके सदिश गुणा का परिणाम

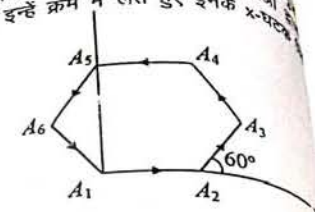
Q.8 Let $A_1A_2A_3A_4A_5A_6A_1$ be a regular hexagon. Write the x-components of the vectors represented by the six sides taken in order.



Q.9 Let $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{b} = 3\vec{i} + 4\vec{j} + 5\vec{k}$. Find the angle between them.

Q.10 If $\vec{A} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{B} = 4\vec{i} + 3\vec{j} + 2\vec{k}$, find $\vec{A} \times \vec{B}$.

Q.8 मानाकि $A_1A_2A_3A_4A_5A_6A_1$ एक समषट् सदिशों को षटभुज की छह भुजाओं है। इन्हें क्रम में लेते हुए इनके x-घटक



Q.9 मानाकि $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ तथा $\vec{b} = 3\vec{i} + 4\vec{j} + 5\vec{k}$.तो इनके मध्य का कोण

Q.10 यदि $\vec{A} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ तथा $\vec{B} = 4\vec{i} + 3\vec{j} + 2\vec{k}$ तो $\vec{A} \times \vec{B}$ ज्ञात

HINTS & SOLUTION

1. 5 m.

2. $20 \cos 15°$ unit

3. 100 unit at 45° with x-axis

4. (a) $|\vec{a}| = 5$ (b) $|\vec{b}| = 5$

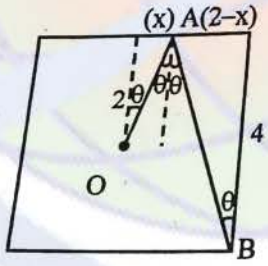
(c) $|\vec{a} + \vec{b}| = 7\sqrt{2}$ (d) $|\vec{a} - \vec{b}| = \sqrt{2}$

5. $\overrightarrow{OA} = (2\cos 30°\hat{i} + 2\sin 30°\hat{j})m$

$\overrightarrow{BC} = (-1.5\cos 60°\hat{i} + 1.5\sin 60°\hat{j})m$

$\overrightarrow{DE} = -\hat{j}$

6.



$\tan\theta = \frac{x}{2}$ and $\tan\theta = \frac{2-x}{4}$

$\therefore \quad \frac{x}{2} = \frac{2-x}{4}; \quad x = \frac{2}{3}$

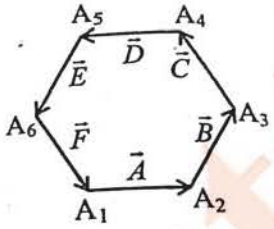
(a) $|\overrightarrow{OA}| = \sqrt{2^2 + (2/3)^2} = \sqrt{\frac{40}{3}} = \frac{2}{3}\sqrt{10}\,ft$

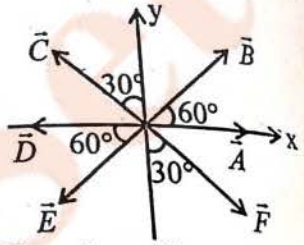
(b) $|\overrightarrow{AB}| = \sqrt{4^2 + (4/3)^2} = \frac{4\sqrt{10}}{3}$

(c) $|\overrightarrow{OB}| = 2\sqrt{2}\,ft$

7. (a) $3m^2$ (b) $3\sqrt{3}\,m^2$

8.





Let $|\vec{A}| = |\vec{B}| = |\vec{C}| = |\vec{D}| = |\vec{E}| = |\vec{F}| = \ell$

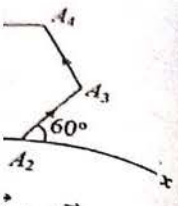
$\therefore A_x = \ell$ $B_x = \ell\cos 60°$

$C_x = -\ell \sin 30°$ $D_x = -\ell$

$E_x = -\ell \cos 60°$ $F_x = \ell \sin 30°$

9. $\theta = \cos^{-1}\left(\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right)$

10. $-6\hat{i} + 12\hat{j} - 6\hat{k}$



$+ 4\vec{k}$ तथा

इनके मध्य का कोण ज्ञात कीजि

$\vec{k}$ तथा

$\vec{A} \times \vec{B}$ ज्ञात कीजि

$= \frac{4\sqrt{10}}{3}$ ft

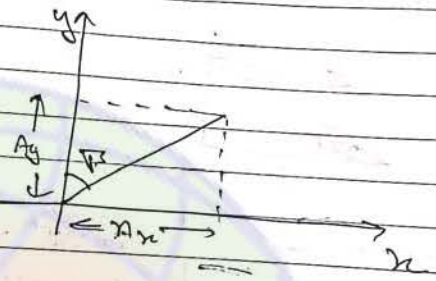
$|\vec{F}| =$

cos 60°

sin 30°

KOTA

Concepts

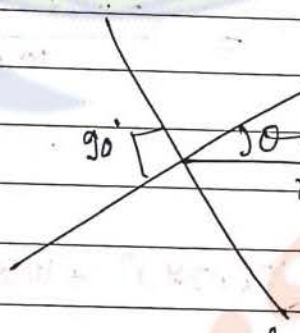


$\frac{A_x}{A} = \sin\theta$, $A_x = A\sin\theta$

$\frac{A_y}{A} = \cos\theta$, $A_y = A\cos\theta$

$\vec{A} = A\sin\theta\,\hat{i} + A\cos\theta\,\hat{j}$

Case 3rd



Projection of $\vec{A}$ along l_1 i.e. $= A\cos\theta$

" " " l_2 " $= A\sin\theta$

Case 4th

2D coordinate

(−, +)	(+, +)
(−, −)	(+, −)

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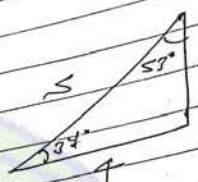
Page

Cos 53°



$\sin 37° = \frac{3}{5}$

$\cos 37° = \frac{4}{5}$

$\tan 37° = \frac{3}{4}$

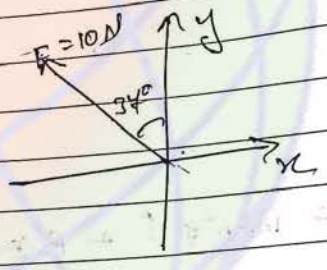
$\sin 53° = 4/5$

$\sin \cos 53° = 3/5$

$\tan 53° = 4/3$

A block
exte
a
m

Eg 1



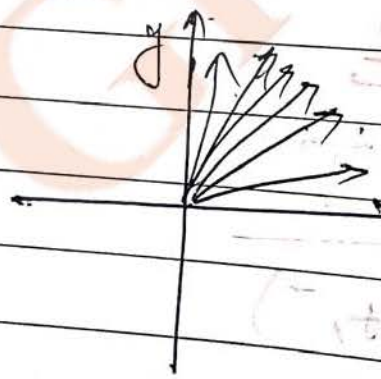
Unit vector in direction of $\vec{F}$ is

$$\hat{F} = \frac{\vec{F}}{|\vec{F}|} = \frac{(-6\hat{i} + 8\hat{j})\,N}{10\,N} = \frac{-3}{5}\hat{i} + \frac{4}{5}\hat{j}$$

take $\vec{F} = (-10\sin 37\hat{i} + 10\cos 37\hat{j})\,N$

$= (-6\hat{i} + 8\hat{j})$

Note

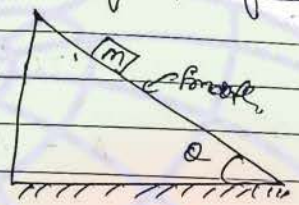


$|\hat{F}| = 1$ (Always)

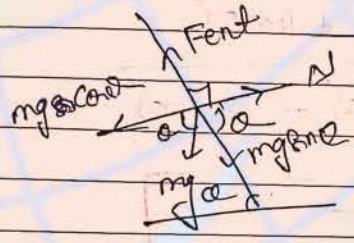
Ques

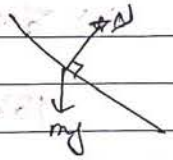
classmate
Date
Page

a) A block of mass "m" is kept on smooth inclined plane external force is applied along inclined to make a stationary ~~mag of~~ mag of this force.

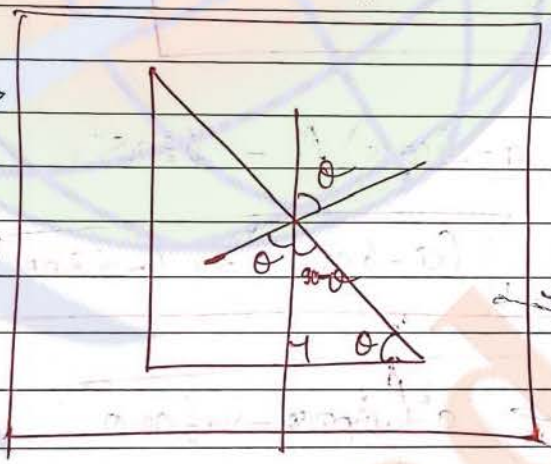


Sol^n





Ques



$F_{ext} = mg\sin\theta$

$N = mg\cos\theta$

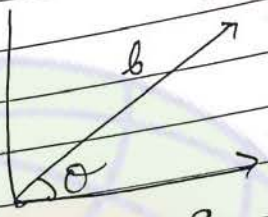
Addition and Substraction of vectors →

(i) $|\vec{a}+\vec{b}| = ?$

(ii) $|\vec{a}-\vec{b}| = ?$

Sol^n

(1) A



$a = a\hat{i}$

$b = b\cos\theta\hat{i} + b\sin\theta\hat{j}$

(2)

(i) $\vec{a}+\vec{b} = (a+b\cos\theta)\hat{i} + b\sin\theta\hat{j}$

$|\vec{a}+\vec{b}| = \sqrt{(a+b\cos\theta)^2 + (b\sin\theta)^2}$

$$\boxed{|\vec{a}+\vec{b}| = \sqrt{a^2+b^2+2ab\cos\theta}}$$

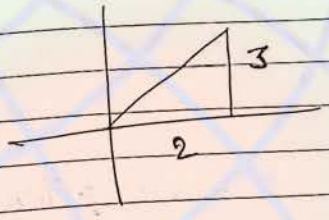
(ii) $\vec{a}-\vec{b} = (a-b\cos\theta)\hat{i} - b\sin\theta$

$|\vec{a}-\vec{b}| = \sqrt{(a-b\cos\theta)^2 + (-b\sin\theta)^2}$

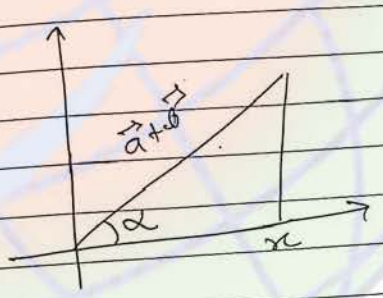
$$\boxed{|\vec{a}-\vec{b}| = \sqrt{a^2+b^2-2ab\cos\theta}}$$

Note

(1) $A = 2\hat{i} + 3\hat{j}$



(2)



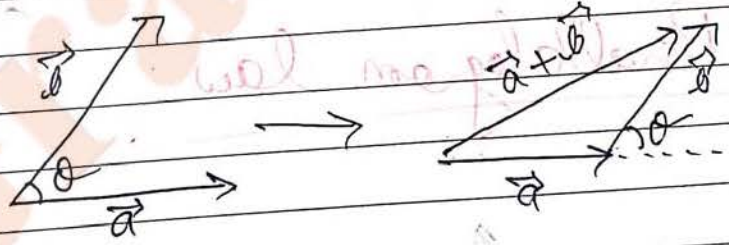
Ans

$$\tan\alpha = \frac{b\sin\theta}{a+b\cos\theta}$$

$$\alpha = \tan^{-1}\left(\frac{b\sin\theta}{a+b\cos\theta}\right)$$

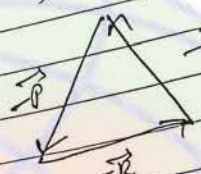
$\Rightarrow$ Angle b/w $\vec{a}+\vec{b}$ and $\vec{a}$

☆ Triangle Rule



(ii)

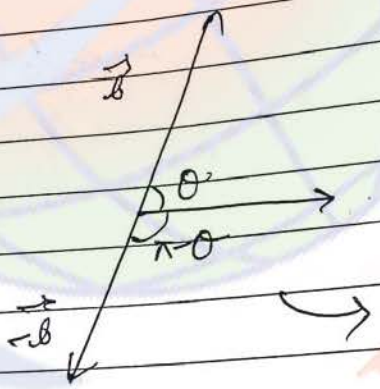
$\vec{P} + \vec{Q} + \vec{R} = 0$

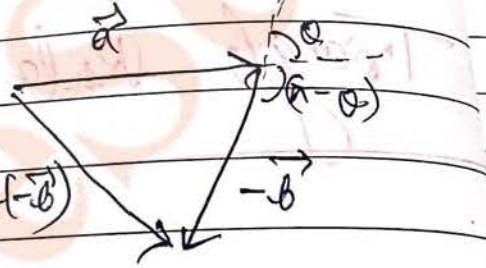


$-P + (-Q) + -R$

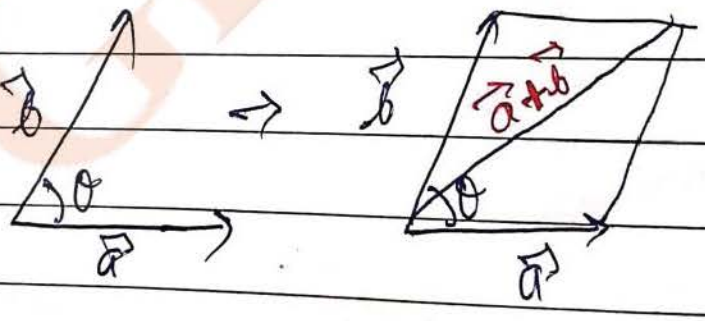
(iii) $\vec{a} - \vec{b}$

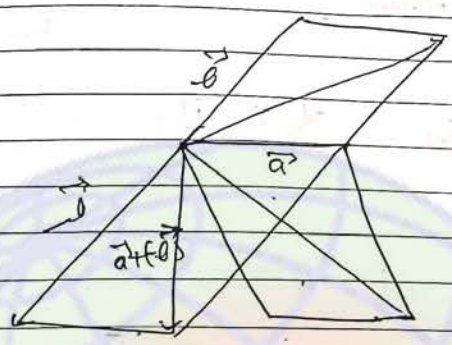
$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$



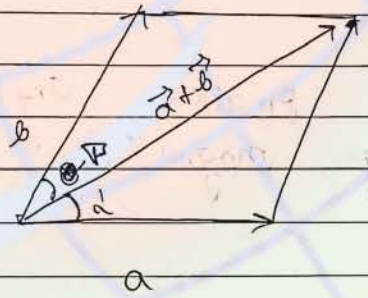


☆ Parallelogram law





Note



$$\tan\alpha = \frac{b\sin\theta}{a + b\cos\theta}$$

If $a = b = k$

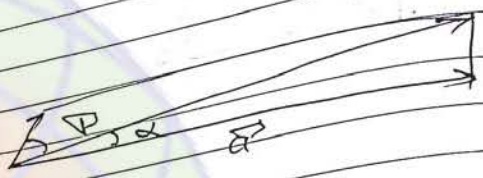
$$\tan\alpha = \frac{\sin\theta}{1+\cos\theta} = \frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\cos^2\frac{\theta}{2}}$$

$$\tan\alpha = \tan\frac{\theta}{2}$$

Ans $\alpha = \frac{\theta}{2}$

① if $a = b = k$, then $\alpha = \theta = \frac{\theta}{2}$

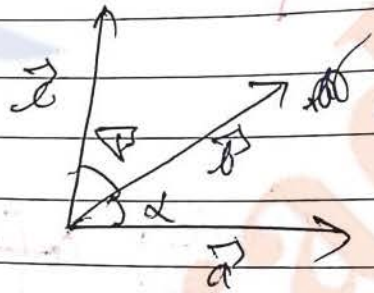
② if $a > b$, then $\alpha < \theta$



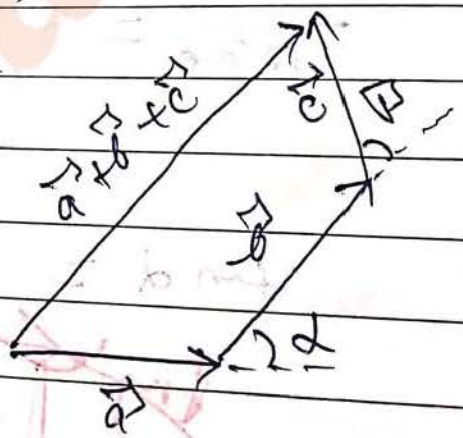
③ if $a < b$ then $\alpha > \theta$

④ Resultant makes smaller angle with vector of bigger magnitude.

Polygon law

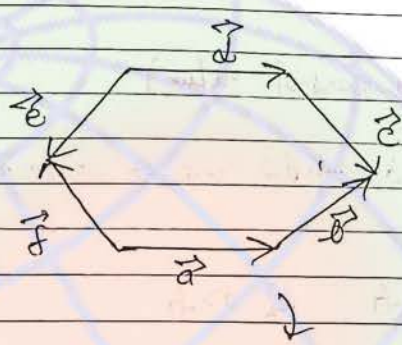


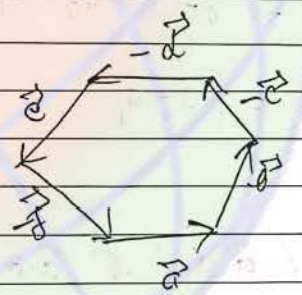
$\vec{a} + \vec{b} + \vec{c} = ?$



Note

(1)





$$\vec{a}+\vec{b}-\vec{c}-\vec{d}+\vec{e}-\vec{f}=0$$

$$\vec{a}+\vec{b}+\vec{e} = \vec{c}+\vec{d}+\vec{f}$$

☆ Some Special Case :-

(*) $|\vec{a}+\vec{b}| = \sqrt{\vec{a}+\vec{b}+2ab\cos\theta}$ (maximum possible value)

where

θ is angle b/w $\vec{a}$ and $\vec{b}$

Case (i), if $\theta = 0^\circ$ ($\vec{a}$ and $\vec{b}$ are on the same line)

$|\vec{a}+\vec{b}| = a+b = |\vec{a}|+|\vec{b}|$

(iii) Case 2nd : (minimum possible value)

if $\theta = 180$ ($\vec{a}$ and $\vec{b}$ are in opp. direction)

$|\vec{a}+\vec{b}| = a-b$ if $a>b$

$= b-a$ if $b>a$

$= |a-b|$ in general

(iii) if $\theta = \frac{\pi}{2}$ ($\vec{a}$ and $\vec{b}$ are mutually $\perp$r)

$|\vec{a}+\vec{b}| = \sqrt{a^2+b^2}$

Q) $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

angle b/w $\vec{a}$ and $\vec{b}$ is ——

$a^2+b^2+2ab\cos\theta = a^2+b^2+2ab\cos\theta$

$\cos\theta = -\cos\theta$

$4ab\cos\theta = 0$

$\theta = \frac{\pi}{2}$

[$\therefore \theta = \cos^{-1}(0) = \frac{\pi}{2}$]

Note

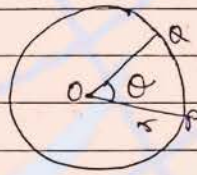
(i) $\vec{P} = \vec{Q}$ (ii) $P = Q$

Implies

उल्टा नहीं।

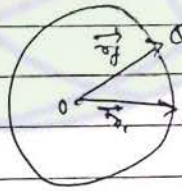
Here 1st Implies 2nd but 2nd does not imply 1st.

eg)



A particle moves in a circle of radius 'r'. find mag. of change in its position vector w.r.t. O as it moves from the point P to point Q.

so



$\vec{OP} - \vec{OQ} = \vec{PQ}$

Change in position vector $(\Delta\vec{r}) = \vec{r}_f - \vec{r}_i$

$$|\Delta\vec{r}| = |\vec{r}_f - \vec{r}_i|$$

$$|\Delta\vec{r}| = \sqrt{r_f^2 + r_i^2 - 2 r_f r_i \cos\theta}$$

where,

$$r_f = r_i = r$$

$$|\Delta\vec{r}| = \sqrt{r^2 + r^2 - 2r^2\cos\theta}$$

$= \sqrt{2}r \sqrt{(1-\cos\theta)}$

$= \sqrt{2}r \sqrt{2\sin^2(\theta/2)}$

$|\Delta\vec{r}| = 2r\sin(\theta/2)$

$\because 1-\cos\theta = 2\sin^2(\theta/2)$

Special Point :-

If θ is very small $(\sin\theta \simeq \theta)$

then,

$|\Delta\vec{r}| = 2r\sin\left(\frac{\theta}{2}\right)$

$= r\theta$ → in radian

Note

$\Delta r = r_f - r_i$

$= r - r = 0$

Note

$|\Delta\vec{r}|$ → change in magnitude

Δr → change in magnitude.

$1-\cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$

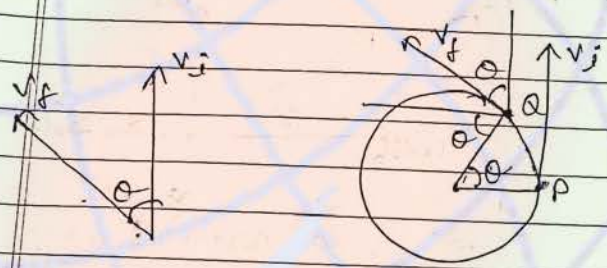
2014

Q) In the above question particle is moving with uniform speed "v" then magnitude of change in its velocity is

Sol)

change in velocity $(\Delta\vec{v}) = \vec{v_f} - \vec{v_i}$

$$|\Delta\vec{v}| = |\vec{v_f} - \vec{v_i}|$$



$v_f = v_i = v$

$$|\Delta\vec{v}| = \sqrt{v_f^2 + v_i^2 - 2v_f v_i \cos\theta}$$

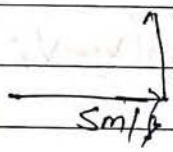
$v_f = v_i = v$

$$= 2v\sin\left(\frac{\theta}{2}\right)$$

2014

Q) A truck is moving towards east with speed of 5m/s after some time its velocity changes to 5m/s towards north find magnitude and direction of change in velocity.

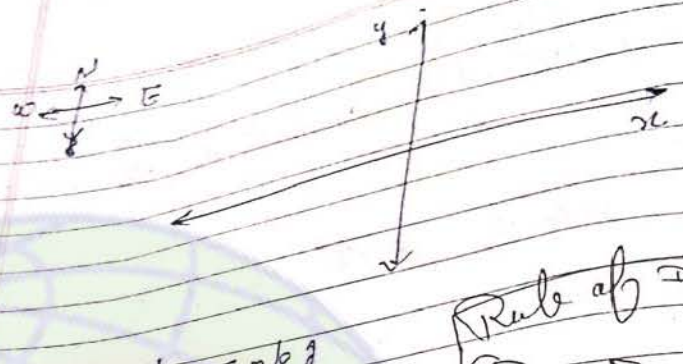
Sol)



$5\sqrt{2}$ m/s

$\frac{5}{\sqrt{\frac{10}{2}}}$

$\vec{\Delta v} = \vec{v_f} - \vec{v_i}$

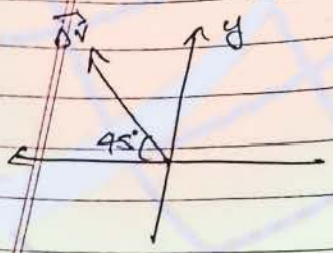


$\Delta\vec{v} = 5\,m/s(-\hat{j}) - 5\,m/s\,\hat{j}$

$\Delta\vec{v} = (-5\hat{j} + 5\hat{j})\,m/s$

$|\Delta\vec{v}| = \sqrt{(-5)^2 + (5)^2}\ m/s$

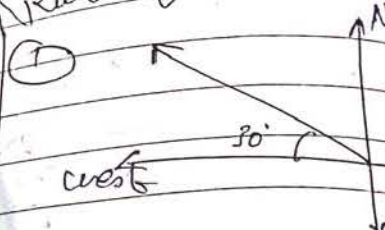
$= 5\sqrt{2}\ m/s$



Ans $\Delta\vec{v} = (5\sqrt{2}\ m/s)$ due N-W

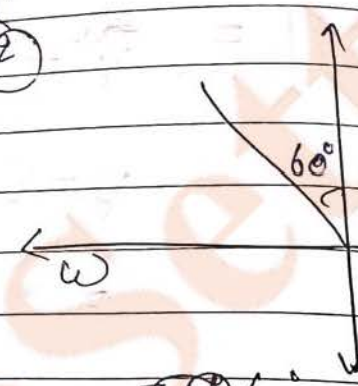
Rule of direction in vector

(1)



Ans → 30° north of west

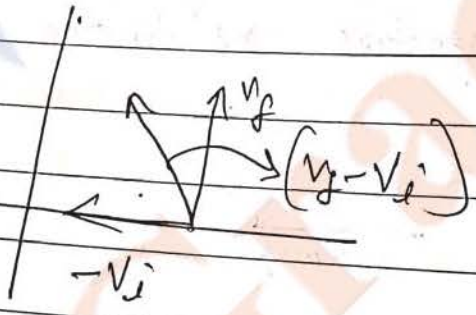
(2)



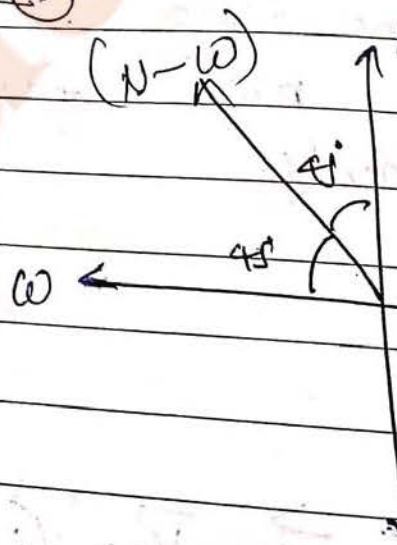
Ans 60° west of North

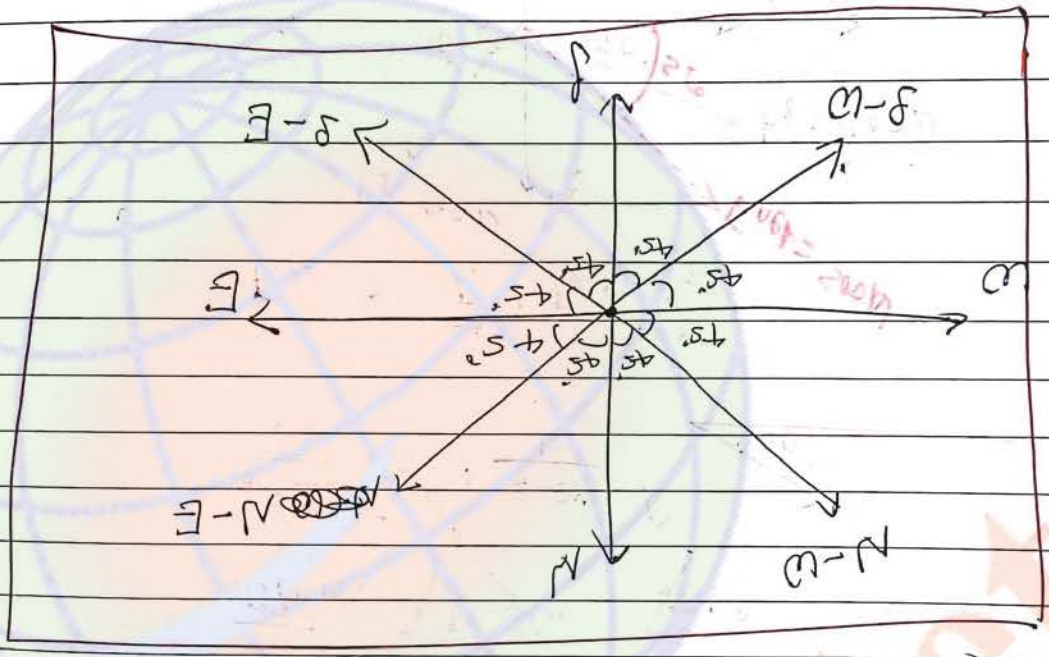
Note

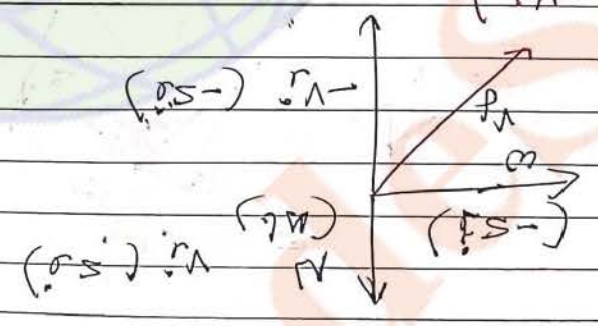
Ans

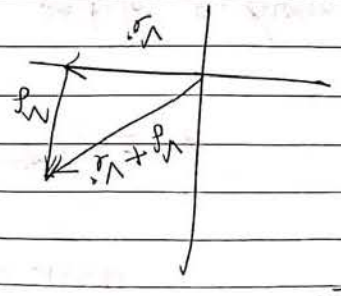


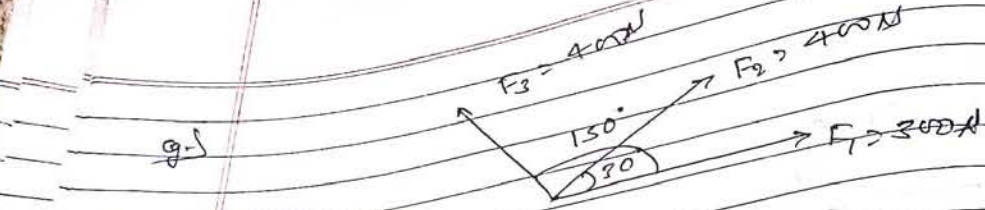
(3)



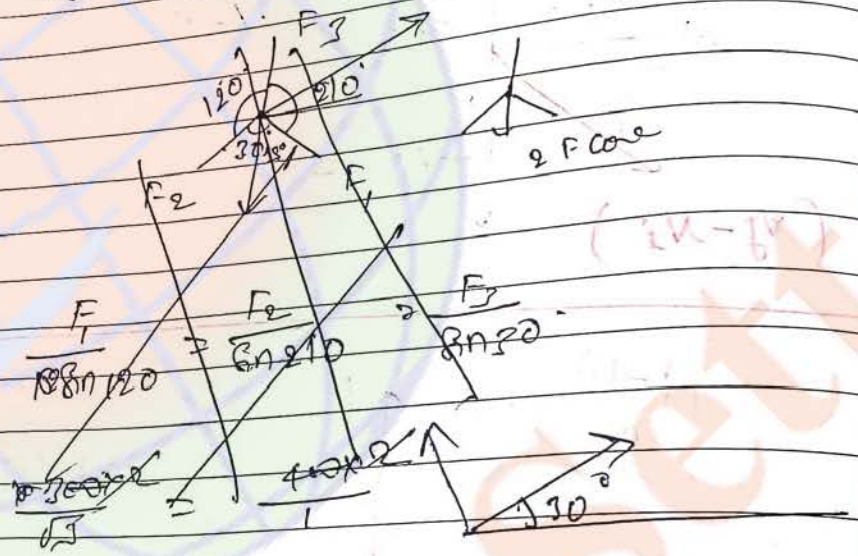




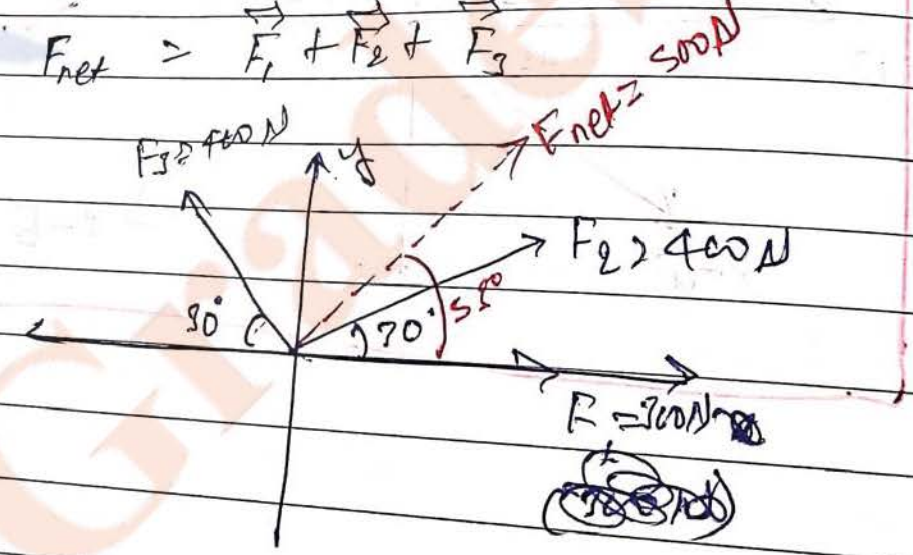




Sol:



$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$



$$F_1 = 300\,\hat{i}\ N$$

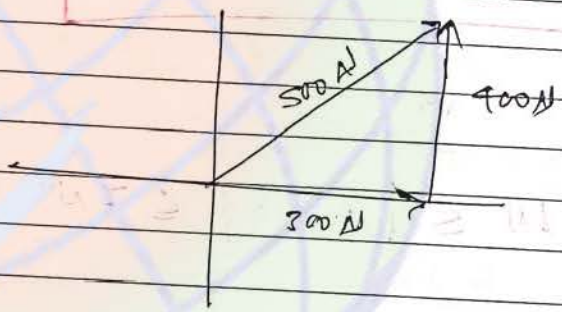
$\vec{F}_2 = 400\cos 30\,\hat{i} + 400\sin 30^\circ\,\hat{i}$

$\vec{F}_1 = (-400\cos 60\,\hat{i} + 400\sin 30^\circ\,\hat{j})\ N$

$\vec{F}_{net} = (300\,\hat{i} + 400\,\hat{j})\ N$

$|\vec{F}_{net}| = \sqrt{(300)^2 + (400)^2}\ N$

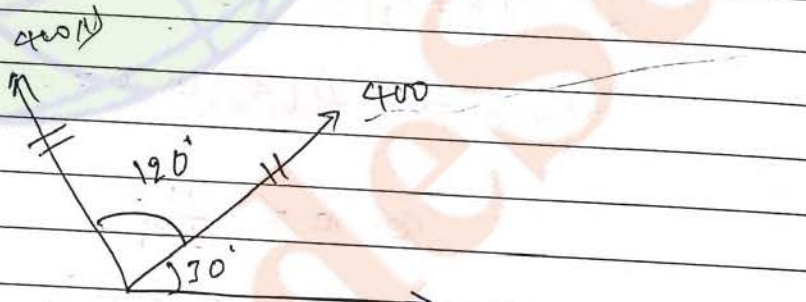
$= 500\ N$



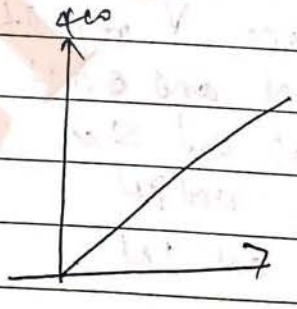
$\tan\theta = \frac{4}{3}$

$\theta = 53^\circ$

Note



$\sqrt{(400)^2 + (400)^2 + 2(400)(400)\cos 120^\circ}$



eg.) if $F_1 = 3N$ and $F_2 = 4N$ which of the following can not be the magnitude of resultant force

(i) 6N
(ii) 8N
(iii) 5N
(iv) 1N

Ans (iii)

Note

$$|A-B| \leq |\vec{A}+\vec{B}| \leq A+B$$

so,

$$1N \leq |\vec{F_1}+\vec{F_2}| \leq 7N$$

Note

if $|\vec{F_1}+\vec{F_2}| = 6N$

(Squaring both)

$$3^2 + 4^2 + 2(3)(4)\cos\theta = 36$$

$$\cos\theta = \frac{11}{24}$$

$$\theta = \cos^{-1}\left(\frac{11}{24}\right)$$

eg.) which of the following set of co-planar forces can not produce resultant of zero magnitude

(i) 3N, 4N and 6N
(ii) 3N, 4N, and 5N
(iii) 3N, 4N, and 8N
(iv) 3N, 4N, and 1N

the following resultant forces

Sol^n

$$|N| \leq |N+4N| \leq 8+N$$

Ans → (iii)

Note: If F_1, F_2 and F_3 are co-planar then they can produce resultant of zero magnitude

$$F_1 - F_2 \leq F_3 \leq F_1 + F_2$$

Q-> If $\vec{A} + \vec{B} = \vec{C}$ where, $A = 3$ unit, $B = 4$ unit, $C = 5$ unit

angle b/w $\vec{C}$ and $\vec{A}$ =

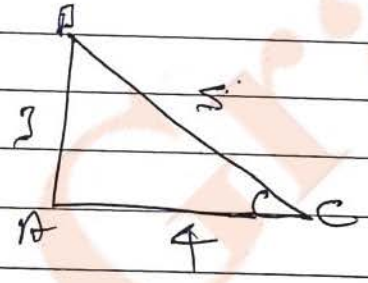
Sol^n

$$1 + 16 + 2 \times 3 \times 4 \times \cos\theta = 25$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1} 0$$

$$\theta = \pi/2$$



$$\vec{B} = |\vec{C} - \vec{A}|$$

$$B^2 = C^2 + A^2 - 2AC\cos\theta$$

where θ = angle b/w $\vec{C}$ and $\vec{A}$

Gravitation
L-3 → Revise
L-4 → Revise

L-1 → 1 to 8, 11 to 21
L-2 → 1 to 4, 6 to 14

$$\cos\theta = \frac{c^2 + b^2 - a^2}{2cb}$$

$$\cos\theta = \frac{18}{50} = \frac{3}{5}$$

$$\theta = 53^\circ$$

Note

$$1\ kgf = 10\ N$$

Products of vectors

(i) Scalar product or dot product
(ii) vector product or cross product.

(1) Scalar product :-

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$
$$= AB\cos\theta$$

where
θ is angle b/w $\vec{A}$ and $\vec{B}$

* If $\theta = 0$, $\vec{A} \cdot \vec{B} = AB$
* If $\theta = 180°$, $\vec{A} \cdot \vec{B} = -A \cdot B$
* If $\theta = \frac{\pi}{2}$, $\vec{A} \cdot \vec{B} = 0$ Imp

VVI

Scalar product of two vector quantities is = Scalar quantity

eg(1)

$$W = \vec{F} \cdot \vec{s}$$
$$W = \vec{F} \cdot \vec{V}$$

* $\vec{A} \cdot \vec{A} = |\vec{A}|\,|\vec{A}| = A^2$
* $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
* $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$
* $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

* if $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
and
$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y$$

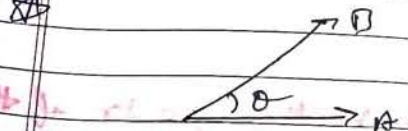
* $\vec{A}\cdot\vec{A} = |\vec{A}||\vec{A}|\cos 0$

* $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

* angle b/w vectors

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$$

$$\cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\cdot\sqrt{B_x^2 + B_y^2 + B_z^2}}$$



(i) Comp. of $\vec{A}$ along $\vec{B}$ = $A\cos\theta$

$= \frac{\vec{A}\cdot\vec{B}}{B}$

in vector form = $\left(\frac{\vec{A}\cdot\vec{B}}{B}\right)\cdot\hat{B}$

(ii) Comp. of $\vec{B}$ along $\vec{A}$ = $B\cos\theta$

$= \frac{\vec{A}\cdot\vec{B}}{A}$

in vector form = $\frac{\vec{A}\cdot\vec{B}}{A}(\hat{A})$

Eg:

If $\vec{A} = \hat{i}+\hat{j}+\hat{k}$

$\vec{B} = \hat{i}-\hat{j}+\hat{k}$

comp of $\vec{A}$ along $\vec{B}$ is

comp of $\vec{A}$ along $\vec{B}$ = $\frac{\vec{A}\cdot\vec{B}}{B}$

$= \frac{\vec{A}\vec{B}}{B}$

$= \frac{1-1+1}{\sqrt{1^2+(-1)^2+1^2}}$

$= \frac{1}{\sqrt{3}}$

In vect form:

$\frac{1}{\sqrt{3}}\vec{B}$

$= \frac{1}{\sqrt{3}}\left(\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}\right)$

$= \frac{1}{3}(\hat{i}-\hat{j}+\hat{k})$

Note

(1) If $\vec{A} \perp \vec{B}$

$\vec{A} \cdot \vec{B} = 0$

$A_x B_x + A_y B_y + A_z B_z = 0$

→ Note: Two vectors are ⊥ if dot product is zero. $\because \cos 90 = 0$

Q. if $|\vec{A}| = |\vec{B}|$

then

Angle b/w $(\vec{A}+\vec{B})$ and $(\vec{A}-\vec{B})$ is

$2\cos\theta$ & $-\cos\theta$

$2A^2 + 2A^2\cos\theta = 0$

$(1-\cos\theta) = 0$

$2\sin^2\frac{\theta}{2} = 0$

$\theta = 0$

$$\cos\theta = \frac{(\vec{A}+\vec{B})(\vec{A}-\vec{B})}{|\vec{A}+\vec{B}|\,|\vec{A}-\vec{B}|}$$

$$\cos\theta = \frac{A^2 - \vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{A} - B^2}{|\vec{A}+\vec{B}|\,|\vec{A}-\vec{B}|}$$

$$\cos\theta = \frac{A^2 - B^2}{|\vec{A}+\vec{B}|\,|\vec{A}-\vec{B}|}$$

If $|\vec{A}| = |\vec{B}|$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$ A.

eg) if $\vec{A} = \hat{i} + \hat{j} + \hat{k}$

$\vec{B} = -\hat{i} + \hat{j} + \hat{k}$ —

angle b/w $\vec{A}$ and $\vec{B}$ is

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{-1+1+1}{\sqrt{3}\sqrt{3}} = \frac{1}{3}$$

$\theta = \cos^{-1}\left(\frac{1}{3}\right)$ A

eg) if $\vec{P} = (2\sin\omega t\,\hat{i} + 2\cos\omega t\,\hat{j})\cos t$

ω = Constant

t = time

P = linear momentum

angle b/w $\vec{F}$ and $\vec{P}$ is

Sol^n

(i)

(ii)

80/7

$$\vec{F} = \frac{d\vec{p}}{dt} = 2\cos\omega t\cdot\frac{d\cos t}{dt} + \cdots \cos t$$

$$\vec{F} = 2\omega\cos\omega t\,\hat{i} - 2\omega\sin\omega t\,\hat{j}$$

$$\cos\theta = \frac{\vec{F}\cdot\vec{P}}{|\vec{F}|\,|\vec{P}|} = \frac{F_x P_x + F_y P_y}{|\vec{F}|\,|\vec{P}|} = \frac{0}{|\vec{F}|\,|\vec{P}|}$$

$\theta = \frac{\pi}{2}$ A

(B) Vector Product or Cross Product :-

Vector product of two vector quantity is a vector quantity

eg:- (i) $\vec{F} = q(\vec{v} \times \vec{B})$

(ii) $\vec{\tau} = \vec{r} \times \vec{F}$

(iii) $\vec{P} = \vec{m} \times \vec{v}$

$$\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}| \sin\theta\, \hat{n}$$

and

$$|\vec{A} \times \vec{B}| = |\vec{A}|\,|\vec{B}| \sin\theta$$

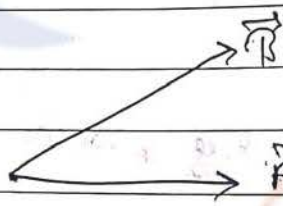
and

$\hat{n}$ ⇒ unit vector in the direction of $\vec{A}$

Note

(i) direction of $\hat{n}$ is determined by right thumb rule or screw rule

(ii)



$\vec{A} \times \vec{B}$ is directed ⊥r to the plane outward

and

$$\vec{B} \times \vec{A} = (-)(\vec{A} \times \vec{B})$$

and

$$|\vec{B} \times \vec{A}| = |\vec{A} \times \vec{B}|$$

(iii) $\vec{A}$ both

(iii) $\vec{A} \times \vec{B}$ is $\perp$ to the plane containing $\vec{A}$ and $\vec{B}$ both

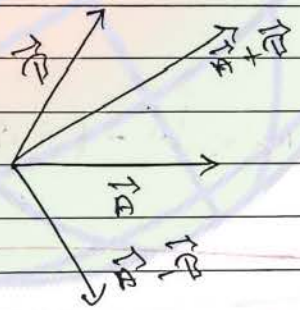
$$(\vec{A} \times \vec{B}) \cdot \vec{A} = 0$$

(the two are $\perp$)

$$(\vec{A} \times \vec{B}) \vec{B} = 0$$

$$(\vec{A} \times \vec{B})(\vec{A} + \vec{B}) = \vec{A} \times \vec{B} \cdot \vec{A} + \vec{A} \times \vec{B} \cdot \vec{B}$$
$$= 0 + 0$$
$$= 0$$

$$(\vec{A} \times \vec{B}) \cdot (\vec{A} - \vec{B}) = 0 - 0 = 0$$



* $|\vec{A} \times \vec{A}| = 0$

* $|\hat{j} \times \hat{j}| = |\hat{i} \times \hat{i}| = |\hat{k} \times \hat{k}| = 0$

* $\hat{i} \times \hat{j} = \hat{k}$
$\hat{j} \times \hat{i} = \hat{k}$
$\hat{i} \times \hat{k} = \hat{j}$
$\hat{k} \times \hat{j} = \hat{i}$

Ques 6 Ques 9

(1)

Compon

(1)

if $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{i} - (A_xB_z - A_zB_x)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

Ques 9

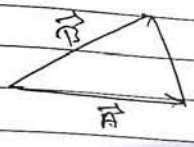
$$|\vec{A} \times \vec{B}| = \sqrt{(A_yb_z - A_zb_y)^2 + (A_xA_z - A_zb_x)^2 + (A_xb_y - A$$

and

Ques 6 →

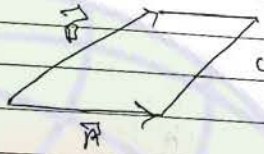
$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

(I)



$$\text{Area of triangle} = \frac{|\vec{A} \times \vec{B}|}{2}$$

(II)



$$\text{Area of llgm} = |\vec{A} \times \vec{B}|$$

Components of a 3D-vector

(I)

A = mag. of $\vec{A}$

α = angle b/w +ve x-axis and $\vec{A}$

∇ = " " +y " "

γ = " " +z " "

$$A_x = A\cos\alpha, \quad A_y = A\cos\nabla, \quad A_z = A\cos\gamma$$

$$\vec{A} = A(\cos\alpha\hat{i} + \cos\nabla\hat{j} + \cos\gamma\hat{k})$$

and,

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

$$\boxed{\hat{A} = \cos\alpha\hat{i} + \cos\nabla\hat{j} + \cos\gamma\hat{k}}$$

Note

(1) $\cos\alpha$, $\cos\nabla$ and $\cos\gamma$ are called direction cosines

(2) if $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

then,

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

and

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

$$\hat{A} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

$$\cos\alpha = \frac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

$$\cos\beta = \frac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

$$\cos\gamma = \frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Note

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

classmate
Date
Page

Q) If $\vec{A} = \hat{i} - \hat{j} + \hat{k}$

then,

Angle b/w $\vec{A}$ and +x-axis is

$$\cos\alpha = \frac{A_x}{|\vec{A}|} = \frac{1}{\sqrt{3}}$$

$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Q) If $\vec{A} = \hat{i} + \hat{j} + \hat{k}$

and $\vec{B} = \hat{i} - \hat{j} + \hat{k}$

find a vector $\perp$r to $\vec{A}$ and $\vec{B}$ both

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = 2\hat{j} - 2\hat{k}$$

$$|\vec{A} \times \vec{B}| = 2\sqrt{2}$$

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$\hat{n} = \frac{\hat{j} - \hat{k}}{\sqrt{2}}$$

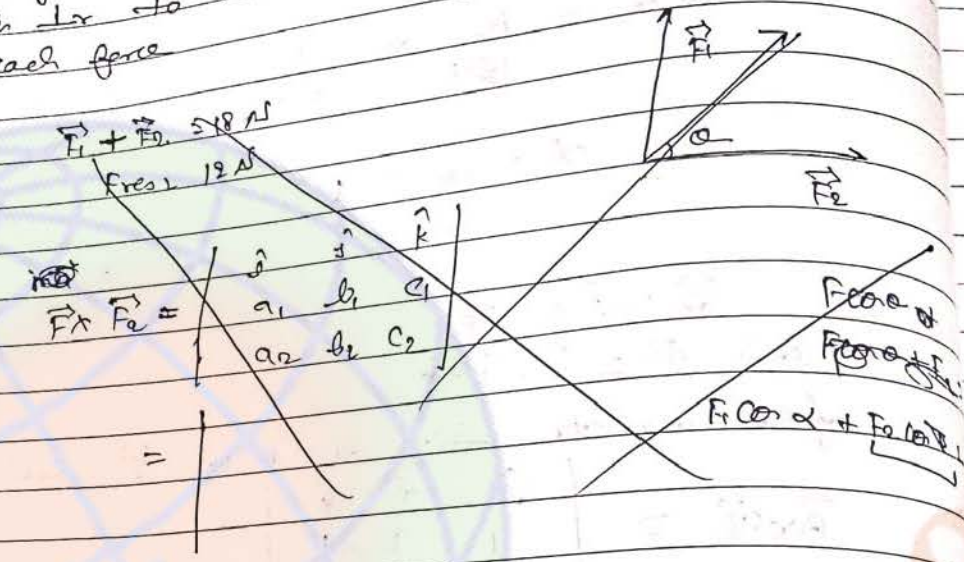
In general a vector $\perp$r to $\vec{A}$ and $\vec{B}$ both = $k\hat{n}$

↓

the same

Q. Sum of magnitude of two forces is 18 N and mag. of their resultant is 12 N. Smaller force is $\perp^{r}$ to the resultant force find mag. of each force

Sol/n



$$\vec{F_1} + \vec{F_2} = 18N \quad \text{—(i)}$$

and

$$|\vec{F_1} + \vec{F_2}| = 12N$$

$$F_1^2 + F_2^2 + 2F_1F_2\cos\theta = 144 \quad \text{—(ii)}$$

only.

$$\vec{F_1} \perp (\vec{F_1} + \vec{F_2})$$

or

$$\vec{F_1} \cdot (\vec{F_1} + \vec{F_2}) = 0$$

$$F_1^2 + F_1F_2\cos\theta = 0 \quad \text{—(iii)}$$

marks अधिक से दोनों eq. को हल करने की प्रक्रिया सर्वप्रथम दोनों में दिए इसके बाद eq" शुद्ध solve हो जाएगी

θ = angle b/w $\vec{F_1}$ and $\vec{F_2}$

From eq(2) and (3)

~~$F_1^2 + F_2^2 + F_1 F_2$~~

$$F_1^2 + F_2^2 + 2(-F_1^2) = 144$$

$$[F_1 F_2 \cos\theta = -F_1^2]$$

$$F_2^2 - F_1^2 = 144$$

$$(F_2 - F_1)(F_2 + F_1) = 144$$

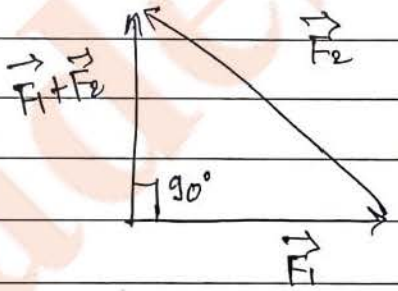
$$(F_2 - F_1)(18) = 144$$

$$F_2 - F_1 = 8 \quad \text{---(iv)}$$

Solving eq(i) and (iv)

$F_1 = 5\,N$

$F_2 = 13\,N$



Motion in One Dimensional

Distance and displacement →

1D → motion in straight line
2D → motion in plane
3D → motion in space

Distance :→

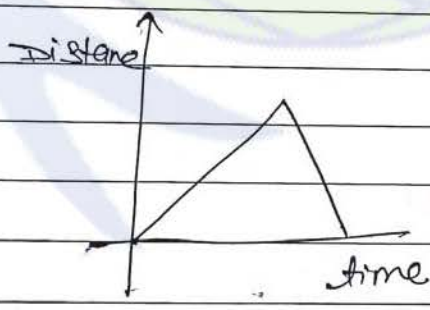
- Scalar quantity
- S.I unit mtr
- Dimension ⇒ $[m^0 L^1 T^0]$

* Actual length of path travells by particle is called Distance travelled by it.

(I) It can never be -ve;

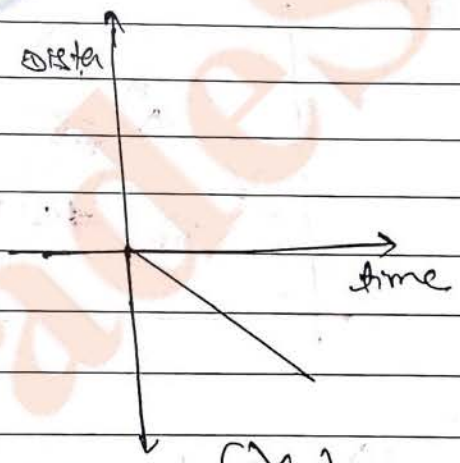
(II) with increase in time distance can never decrease

(IV) Distane - time graph



(X)

(Not possible)

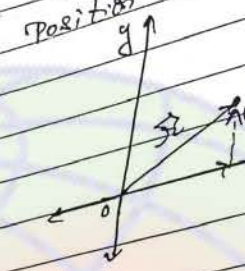


(X)

(Not possible.

(b) Position vector and displacement vector

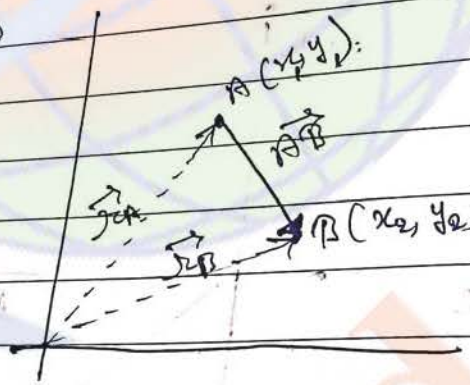
(i) Position vector:



Position vector of P w.r.t. $(\vec{OP}) = \vec{r} = x\hat{i} + y\hat{j}$

And $|\vec{r}| = \sqrt{x^2+y^2}$

(ii)



$\vec{AB}$ = Position vector of $\vec{B}$ w.r.t. A

but

$$\vec{r}_A + \vec{AB} = \vec{r}_B$$

CAREER POINT

Target Course for NITs (JEE Main)-2014

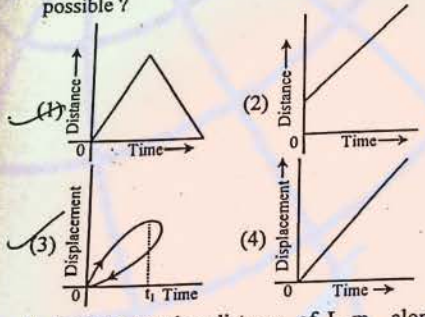
DAILY PRACTICE PROBLEM SHEET

PHYSICS

Motion in One Dimension-1

Sub topic : Distance, Displacement, Speed, Velocity

Q.1 Which of the following graph(s) is / are not possible ?



Q.2 A body covered a distance of L m along a curved path of a quarter circle. The ratio of distance to displacement is-

(1) $\pi/2\sqrt{2}$ (2) $2\sqrt{2}/\pi$

(3) $\pi/\sqrt{2}$ (4) $\sqrt{2}/\pi$

Q.3 An old man goes for morning walk on a semicircular track of radius 40 m ; if he starts from one end of the track and reaches to other end, the distance covered by the man and his displacement will respectively be-

(1) 126 m, 80 m (2) 80 m, 126 m

(3) 80 m, 252 m (4) 252 m, 80 m

Q.4 A particle moves along the x-axis in such a way that its x-coordinates varies with time as $x = 2 - 5t + 6t^2$. What will be its initial velocity ?

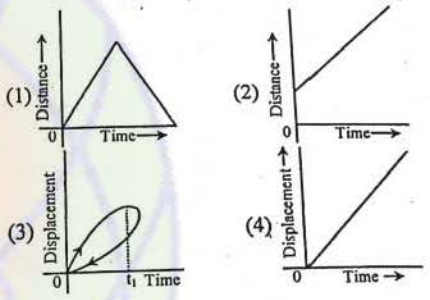
(1) 5 m/s (2) –5 m/s

(3) 2 m/s (4) –2 m/s

Q.5 The displacement of a particle moving in one-dimensional direction under a force at time t is given by $t = \sqrt{x} + 3$, where x is in m and t in sec. The displacement of the particle, when its velocity is zero, will be-

(1) 0 (2) 3m (3) –3m (4) 2m

Q.1 निम्न में से कौनसा/कौनसे ग्राफ सम्भव नहीं है ?



Q.2 एक वस्तु वृताकार मार्ग के चौथाई भाग पर L m दूरी तय करती है। दूरी व विस्थापन में अनुपात होगा-

(1) $\pi/2\sqrt{2}$ (2) $2\sqrt{2}/\pi$

(3) $\pi/\sqrt{2}$ (4) $\sqrt{2}/\pi$

Q.3 एक वृद्ध व्यक्ति 40 m त्रिज्या के अर्धवृत्ताकार मार्ग पर सुबह टहलने जाता है, यदि वह मार्ग के एक छोर से विस्थापन होगा-

(1) 126 m, 80 m (2) 80 m, 126 m

(3) 80 m, 252 m (4) 252 m, 80 m

Q.4 एक कण x-अक्ष के अनुदिश इस प्रकार गति करता है, कि इसका x-निर्देशांक समय t के साथ निम्न रूप में बदलता है $x = 2 - 5t + 6t^2$, कण का प्रारम्भिक वेग होगा ?

(1) 5 m/s (2) –5 m/s

(3) 2 m/s (4) –2 m/s

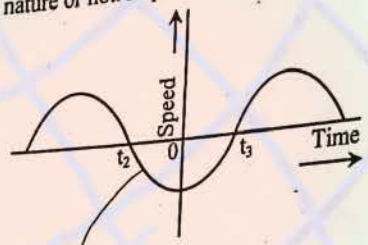
Q.5 एक कण, जो एक विमीय दिशा में किसी बल के अधीन गति कर रहा है तथा विस्थापन t समय में निम्न समीकरण से दिया जाता है, $t = \sqrt{x} + 3$ जहाँ x मीटर में तथा t सेकण्ड में है, जब कण का वेग शून्य हो, तो विस्थापन होगा–

(1) 0 (2) 3m (3) –3m (4) 2m

Q.6 Can a body have uniform velocity but non-uniform speed ?
(1) Yes
(2) No
(3) Depend on magnitude
(4) Unpredictable

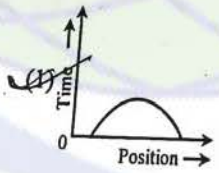
Q.7 Can a body have uniform speed but non-uniform velocity ?
(1) Yes
(2) No
(3) Unpredictable
(4) None of these

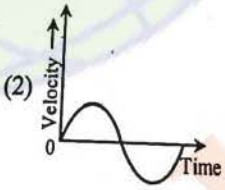
Q.8 State whether the following graph can be seen in nature or not. Explain.

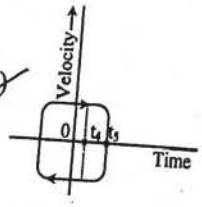


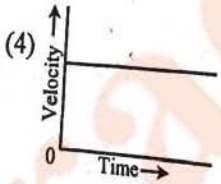
(1) Yes
(2) No
(3) Sometime
(4) At a particular instant

Q.9 Out of the following graph(s), which is / are not possible ?

(1)


(2)


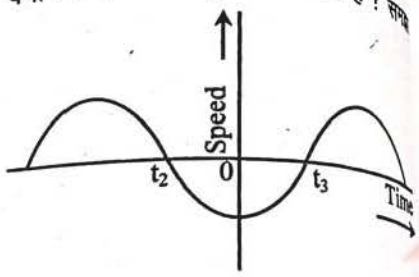
(3)


(4)


Q.6 क्या किसी वस्तु का वेग एकसमान असमान हो सकती है ?
(1) हाँ
(2) नहीं
(3) परिमाण पर निर्भर करता है
(4) बताया नहीं जा सकता

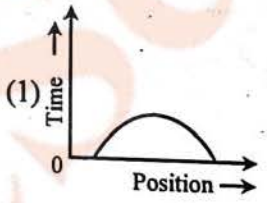
Q.7 क्या किसी वस्तु की चाल एकसमान वेग–असमान हो सकता है?
(1) हाँ
(2) नहीं
(3) बताया नहीं जा सकता
(4) इनमें से कोई नहीं

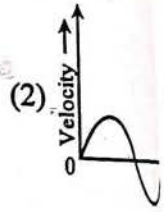
Q.8 क्या निम्न ग्राफ प्रकृति में सम्भव है ? स्पष्ट

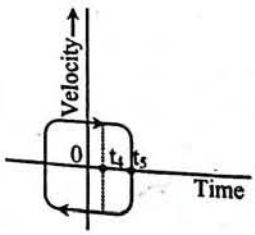


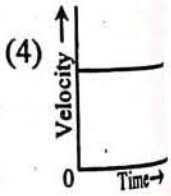
(1) हाँ
(2) नहीं
(3) कभी–कभी
(4) किसी एक निश्चित क्षण पर

Q.9 निम्न में से कौनसा ग्राफ सम्भव नहीं है ?

(1)


(2)


(3)


(4)


1.[1,3]

2.[1]

1.[1,3]

The (A) graph shows that with increase in time distance first increases and then decreases. However, distance can never decrease with time so this graph is not physically possible. The graph (C) shows that at certain instant of time (t_1) body is present at two positions. Also it shows that time first increases then decreases. These conditions are not possible physically.

Hence correct answer is (A) and (C).

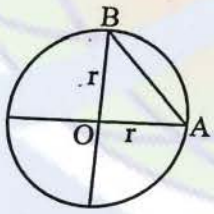
[ग्राफ (A) से प्रकट होता है कि समय के बढ़ने के साथ दूरी पहले बढ़ती है तथा फिर घटती है, जबकि दूरी समय बढ़ने के साथ कभी भी घट नहीं सकती अतः ग्राफ (A) प्रकृति में सम्भव नहीं है। ग्राफ (C) से प्रकट होता है कि समय के किसी मान (t_1) पर वस्तु की दो स्थितियाँ है, साथ ही यह भी पता चलता है कि समय पहले बढ़ता है फिर घटता है, ये दोनों स्थितियाँ प्रकृति में सम्भव नहीं है।

अतः सही उत्तर (A) तथा (C) हैं।]

2.[1] Length of quarter circle path = $L = 2\pi r/4$

$\therefore r = 2L/\pi$

Hence displacement $AB = \sqrt{r^2 + r^2} = \sqrt{2}\, r.$



From ΔOAB,

magnitude of displacement = $AB = \sqrt{2}\, r$

$\therefore \dfrac{\text{Distance}}{\text{Displacement}} = \dfrac{2\pi r/4}{\sqrt{2}.r} = \pi/2\sqrt{2}$

Hence correct answer is (A).

3.[1] Distance covered by man = Length of the path = $\pi R = \pi \times 40 = 126$ m

Displacement of the man = The least distance between initial and final points = Diameter of semicircular path = $2R = 2 \times 40 = 80$ m

The direction of displacement will be from initial point to final point.

Hence correct answer is (A).

4.[2] $v = \dfrac{dx}{dt} = -5 + 12\,t$, here $t = 0$

5.[1] $x = (t-3)^2 = t^2 + 9 - 6t$

$v = 2t - 6 = 0 \Rightarrow t = 3$ second

at $t = 3$ sec, $x = 0$

6.[2] No.

Velocity = (Speed) + (Direction)

Hence correct answer is (B).

7.[1] Yes, hence correct answer is (A).

e.g. Speed of a particle in circular path is constant but due to change in direction its velocity changes.

[हाँ, जैसे–वृतीय पथ पर गति करते कण की चाल नियत हो सकती है, लेकिन दिशा परिवर्तन के कारण वेग समान नहीं हो सकता। अतः सही उत्तर (A) है।]

8.[2] This graph shows that speed is negative for some interval of time (t_2 to t_3). Since speed can never be negative, so this graph is physically not possible.

Hence correct answer is (B).

[इस ग्राफ से प्रकट होता है कि चाल समयान्तराल (t_2 से t_3) पर ऋणात्मक है, चूंकि चाल ऋणात्मक नहीं हो सकती है, अत एवं ग्राफ सम्भव नहीं है। अतः सही उत्तर (B) हैं।]

9.[1,3] *The graph (A) shows that on increasing position (x), time first increases, then decreases, which is impossible. The graph (C) shows that at a given instant of time (t_4) particle has two velocities. Also it shows that at time (t_5) the acceleration is infinite (= slope of $\vec{v}$ /t curve). Since both these conditions cannot be achieved practically, then these graphs are not possible.*

Hence correct answer is (A) and (C).

[ग्राफ (A) से प्रकट होता है, कि स्थिति (x), बढने पर समय पहले बढ़ता है फिर घटने लगता है जो कि असम्भव है ग्राफ (C) से प्रकट होता है कि किसी समय (t_4) पर कण के दो वेग है। पुनः समय (t_5) पर त्वरण अनन्त है, (= /t वक्र का झुकाव)] चूँकि दोनों स्थितियाँ प्रकृति में नहीं देखी जा सकती है, इसलिये यह सम्भव नहीं है। अतः सही उत्तर (A, C) हैं

$$\vec{AB} = \vec{r}_B - \vec{r}_A$$

$$\vec{AB} = (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$$

$$\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

and,

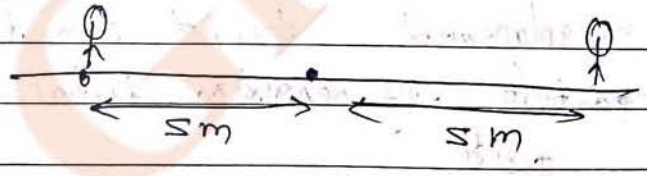
$$|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

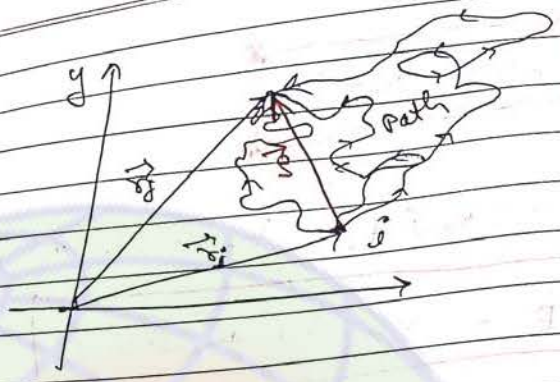
(iii) **Displacement :-**

* Vector Quantity.
  - SI Unit : "m"
  - Dimensions : $[m^0 L^1 T^0]$

* change in position is defined as **displacement**

$$\vec{s} = \vec{r}_f - \vec{r}_i$$





* shortest distance b/w final and initial position is equal to magnitude of displacement and it is directed from initial position to final position.

$$\vec{s} = \vec{r}_f - \vec{r}_i$$

$$\vec{s} = (x_f \hat{i} + y_f \hat{j}) - (x_i \hat{i} + y_i \hat{j})$$

$$\vec{s} = (x_f - x_i)\hat{i} + (y_f - y_i)\hat{j}$$

or

$$\boxed{\vec{s} = \Delta x \hat{i} + \Delta y \hat{j}}$$

$$\boxed{|\vec{s}| = \sqrt{(\Delta x)^2 + (\Delta y)^2}}$$

* Displacement does not depend on path the particle. It depends on initial and final positions.

CAREER POINT

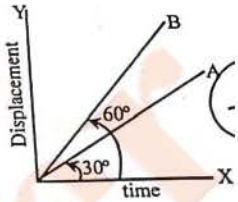
Target Course

DAILY PR

Mot

Sub topic : Grapl

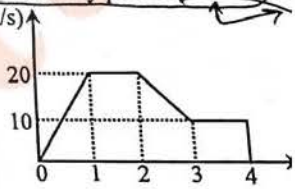
From the adjoining displacement-tim two particles A & B the ratio of velo will be-



(1) 1 : 2 (2) 1 : √

(3) √3 : 1 (4) 1 : 3

From the adjoining graph, traversed by particle in 4 sec is



(1) 60 m (2) 25 m (3)

A car travels first half dist places with a speed of 40 km distance with a speed of 60 speed of the car will be-

(1) 100 km/hr (2) 5

(3) 48 km/hr (4)

A table has its minute ha average velocity of the ti between 6.00 a.m. to 6.30 6.30 p.m. will respectively

(1) 4.4×10^{-3}, 1.8×10^{-4}

(2) 1.8×10^{-4}, 4.4×10^{-3}

(3) 8×10^{-3}, 4.4×10^{-3}

(4) 4.4×10^{-3}, 8×10^{-4}

CAREER POINT guruku

EER POINT, CP Tower, IPIA,

CAREER POINT

Target Course for NITs (JEE Main)-2014

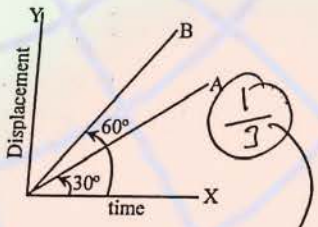
DAILY PRACTICE PROBLEM SHEET

PHYSICS

Motion in One Dimension-2

Sub topic : Graph, Average Speed, Average Velocity

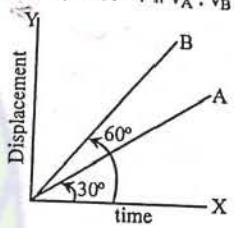
From the adjoining displacement-time graph for two particles A & B the ratio of velocities $v_A : v_B$ will be-



(1) 1 : 2 (2) 1 : $\sqrt{3}$

(3) $\sqrt{3}$: 1 (4) 1 : 3

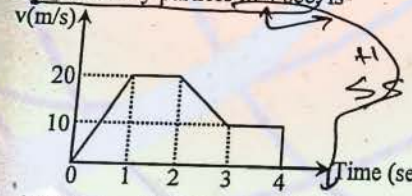
Q.1 संलग्न चित्र में दो कणों का विस्थापन व समय के बीच ग्राफ दिये गये हैं, उनके वेगों $v_A : v_B$ का अनुपात होगा-



(1) 1 : 2 (2) 1 : $\sqrt{3}$

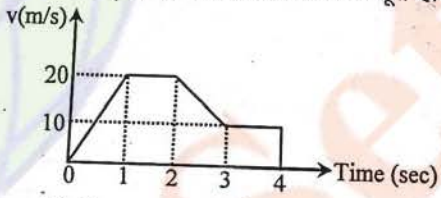
(3) $\sqrt{3}$: 1 (4) 1 : 3

From the adjoining graph, the distance traversed by particle in 4 sec is-



(1) 60 m (2) 25 m (3) 55 m (4) 30 m

Q.2 संलग्न चित्र से कण द्वारा 4 s में तय दूरी होगी-



(1) 60 m (2) 25 m (3) 55 m (4) 30 m

3. A car travels first half distance between two places with a speed of 40 km/h and the rest half distance with a speed of 60 km/h. The average speed of the car will be-

(1) 100 km/hr (2) 50 km/hr

(3) 48 km/hr (4) 200 km/hr

Q.3 एक कार दो स्थानों के बीच की पहली आधी दूरी 40 km/hr की चाल से तथा शेष आधी दूरी 60 km/hr की चाल से तय करती है, तो कार की औसत चाल होगी-

(1) 100 km/hr (2) 50 km/hr

(3) 48 km/hr (4) 200 km/hr

4. A table has its minute hand 4.0 cm long. The average velocity of the tip of the minute hand between 6.00 a.m. to 6.30 a.m. and 6.00 a.m. to 6.30 p.m. will respectively be- (in cm/s)

(1) 4.4×10^{-3}, 1.8×10^{-4}

(2) 1.8×10^{-4}, 4.4×10^{-3}

(3) 8×10^{-3}, 4.4×10^{-3}

(4) 4.4×10^{-3}, 8×10^{-4}

Q.4 एक टेबल घड़ी की मिनट की सुई 4.0 cm लम्बी है, मिनट की सुई के शीर्ष का, 6.00 a.m. से 6.30 a.m. तथा 6.00 a.m. से 6.30 p.m. के बीच, औसत वेग क्रमशः होगें (cm/s में)

(1) 4.4×10^{-3}, 1.8×10^{-4}

(2) 1.8×10^{-4}, 4.4×10^{-3}

(3) 8×10^{-3}, 4.4×10^{-3}

(4) 4.4×10^{-3}, 8×10^{-4}

Q.5 The average speed and average velocity during one complete cycle of radius R will respectively be- (T is the time to take one complete revolution)

(1) $\frac{\pi R}{T}$, 0 (2) $\frac{2\pi R}{T}$, $\frac{\pi R}{T}$

(3) $\frac{2\pi R}{T}$, 0 (4) 0, $\frac{2\pi R}{T}$

Q.6 A boy covers a distance AB of 2 km with speed of 2.5 km/h, while going from A to B and comes back from B to A with speed 0.5 km/hr, his average speed will be-

(1) 1.5 km/hr (2) 0.83 km/hr

(3) 1.2 km/hr (4) 3 km/hr

Q.7 Usually "average speed" means the ratio of total distance covered to the time elapsed. However some time the phrase " average speed" can mean the magnitude of the average velocity. Are the two same?

Q.8 If the displacement of a particle is proportional to the square of time, then-

(1) velocity is inversely proportional to t

(2) velocity is proportional to t

(3) velocity is proportional to $\sqrt{t}$

(4) acceleration is constant

Q.9 The position x of a particle varies with time (t) as $x = at^2 - bt^3$. The acceleration of the particle will be equal to zero at time –

(1) $\frac{2a}{3b}$ (2) $\frac{a}{b}$ (3) $\frac{a}{3b}$ (4) 0

Q.10 In the above example, the average acceleration of the particle in the interval t = 1 to t = 3 sec will be-

(1) 12 a – 2b (2) 2b – 12 a

(3) 2a – 12b (4) 12b – 2a

Q.11 It is possible to be accelerating if you are travelling at constant speed ? Is it possible to round a curve, with zero acceleration ? With constant acceleration ? With variable acceleration ?

(1) No, yes, no, no (2) Yes, no, yes, yes

(3) Yes, no, no, no (4) No, no, yes, yes

Q.5 R त्रिज्या के एक पूरे चक्कर में औसत वेग क्रमशः होगें (T एक पूरा चक्कर लगाने में लिया गया समय है)

(1) $\frac{\pi R}{T}$, 0 (2) $\frac{2\pi R}{T}$, $\frac{\pi R}{T}$

(3) $\frac{2\pi R}{T}$, 0 (4) 0, $\frac{2\pi R}{T}$

Q.6 एक बालक 2 km की दूरी AB, A से B तक 2.5 km/hr, की चाल से तय करता है तथा B से A तक 0.5 km/hr की चाल से लौटता है। उसकी औसत चाल होगी–

(1) 1.5 km/hr (2) 0.83 km/hr

(3) 1.2 km/hr (4) 3 km/hr

Q.7 प्रायः "औसत चाल" का अभिप्राय कुल लिया गया समय के अनुपात से होता है। कभी–कभी "औसत चाल" का अभिप्राय परिमाण से भी होता है। क्या दोनों समान हैं?

Q.8 यदि किसी कण का विस्थापन समय के अनुक्रमानुपाती हो, तो

(1) वेग समय t के व्युत्क्रमानुपाती है

(2) वेग, समय t के अनुक्रमानुपाती है

(3) वेग $\sqrt{t}$,के अनुक्रमानुपाती है

(4) त्वरण नियत है

Q.9 किसी कण की स्थिति x समय t के साथ इस प्रकार परिवर्तित होती है कि $x = at^2 - bt^3$, तो किस समय पर कण का त्वरण शून्य होगा –

(1) $\frac{2a}{3b}$ (2) $\frac{a}{b}$ (3) $\frac{a}{3b}$ (4)

Q.10 उपरोक्त उदा. में t = 1s से t = 3 s तक का औसत त्वरण होगा

(1) 12 a – 2b (2) 2b – 12 a

(3) 2a – 12b (4) 12b – 2a

Q.11 यदि तुम्हें नियत चाल से यात्रा करना है तो क्या त्वरित होना सम्भव है? क्या किसी वक्राकार पथ पर शून्य त्वरण से, नियत त्वरण से एवं परिवर्ती त्वरण से चक्कर लगाना सम्भव है?

(1) नहीं, हाँ, नहीं, नहीं (2) हाँ, नहीं, हाँ,

(3) हाँ, नहीं, नहीं, नहीं (4) नहीं, हाँ, हाँ,

HINTS & SOLUTION

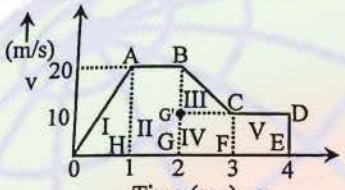
1.[4] The line having greater slope has greater velocity, hence the line making an angle 60° with time axis has greater velocity. Now,

$$\frac{v_A}{v_B} = \frac{\tan 30°}{\tan 60°} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

Hence correct answer is (D)

2.[3] The given graph can be drawn as shown in figure

Distance travelled = Area under v-t graph



= Area I + Area II + Area III + Area IV + Area V

= (1/2) (OH × AH) + HG × AH + 1/2 (G'C × BG') + (GF × GG') + (EF × CF)

= (1/2) (1 × 20) + (1 × 20) + 1/2 (1 × 10) + (1 × 10) + (1 × 10) = 10 + 20 + 5 + 10 + 10 = 55 m

Hence correct answer is (C).

3.[3] Let the total distance travelled be x.

Time taken to travel first half distance

$$t_1 = \frac{x/2}{40} = \frac{x}{80} \text{ hr}$$

Time taken to travel the rest half distance

$$t_2 = \frac{x/2}{60} = \frac{x}{120} \text{ hr}$$

$$\therefore \text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{x}{(x/80)+(x/120)} = 48 \text{ km/hr}$$

Hence correct answer is (C).

4.[4] At 6.00 a.m. the tip of the minute hand is at 12 mark and at 6.30 a.m. or 6.30 p.m. it is 180⁰ away. Thus the straight line distance between the initial and final positions of the tip is equal to the diameter of the clock.

Displacement = 2 R = 2 × 4 cm = 8 cm

Time taken from 6 a. m. to 6.30 a.m. is 30 minutes = 1800 s. The average velocity is

$$v_{av} = \frac{\text{Displacement}}{\text{time}} = \frac{8.0 \text{ cm}}{1800 \text{ s}}$$

$= 4.4 \times 10^{-3}$ cm/s

Again time taken from 6 am to 6.30 p.m. = 12 hrs + 30 minutes = 45000 s

$$\therefore v_{av} = \frac{\text{Displacement}}{\text{time}} = \frac{8}{45000}$$

$= 1.8 \times 10^{-4}$ cm/s

Hence correct answer is (A).

5.[3] Average speed $v_{av} = \frac{2\pi R}{T}$ and

average velocity $\vec{v}_{av} = 0/T = 0$

Hence correct answer is (C).

6.[2] As boy goes from A to B and then comes back from B to A hence his average speed

$$v_{av} = \frac{2v_1v_2}{v_1+v_2} = \frac{2\times 2.5\times 0.5}{2.5+0.5}$$

$$= \frac{2.5}{3} = 0.8 \text{ km/hr}$$

Hence correct answer is (B).

7. No, usually they have different meanings, as according to I-definition, $v_{av} = \frac{\text{distance}}{\text{time}}$, while according to II-definition $|\vec{v}_{av}| = \frac{|\text{displacement}|}{\text{time}}$.

Now as distance $\geq$ | displacement |,so $v_{av} \geq |\vec{v}_{av}|$

i.e. usually average speed is greater than the magnitude of average velocity

e.g. If a body returns to its starting point after some motion, then as distance travelled is finite while displacement is zero so $v_{av} > 0$ but $|\vec{v}_{av}| = 0$. However in case of motion along a straight-line without change in direction, as |displacement| = distance, the two definition will mean same $v_{av} = |\vec{v}_{av}|$.

[नहीं, प्रायः इन दोनों कथनों के भिन्न-भिन्न अर्थ होते है। I-परिभाषा के अनुसार $v_{av} = \frac{\text{दूरी}}{\text{समय}}$,

जबकि II-परिभाषा के अनुसार $|\vec{v}_{av}| = \frac{|\text{विस्थापन}|}{\text{समय}}$.

जैसे कि दूरी $\geq |\text{विस्थापन}|$ अतः $v_{av} \geq |\vec{v}_{av}|$

प्रायः औसत चाल, औसत वेग के परिमाण से अधिक होता है।

e.g. यदि कोई वस्तु गति के बाद अपने प्रारम्भिक बिन्दु पर आती है, तो तय दूरी सीमित होती है जबकि विस्थापन शून्य होता है, इसलिए

$v_{av} > 0$ लेकिन $|\vec{v}_{av}| = 0$

सीधी रेखा के अनुदिश गति में यदि दिशा परिवर्तित न हो, तो, | विस्थापन|= दूरी तब $v_{av} = |\vec{v}_{av}|$

8.[2,4] Given that $s \propto t^2 \Rightarrow s = kt^2$, where k is constant

$\therefore$ velocity $v = \frac{ds}{dt} = 2kt$, velocity varies with time

acceleration $a = \frac{dv}{dt} = 2k = \text{constant}$.

Hence acceleration of particle is constant

Hence correct answer are (B) & (D).

9.[3] Given that $x = at^2 - bt^3$

$\therefore$ Velocity $v = \frac{dx}{dt} = 2at - 3bt^2$ and

acceleration $a = \frac{d}{dt}\left(\frac{dx}{dt}\right)$

$\Rightarrow 0 = 2a - 6bt \Rightarrow t = \frac{2a}{6b} = \frac{a}{3b}$

Hence correct answer is (C).

10.[3] In the light of above example, we have

$\frac{dx}{dt} = 2at - 3bt^2$

Now velocity at $t = 1$ sec,

$v_1 = \left(\frac{dx}{dt}\right)_{t=1} = 2a - 3b$ and

that at $t = 3$ sec, $v_2 = \left(\frac{dx}{dt}\right)_{t=3} = 6a - 27b$

Thus average acceleration $a_{av} = \frac{v_2 - v_1}{3-1}$

$= \frac{6a - 27b - 2a + 3b}{3-1} = \frac{4a - 24b}{2}$

Hence correct answer is (C).

11.[2] If speed is constant, velocity may to change in direction and as rate of change of velocity so not be zero Actually in motion, speed = constant but For motion on a curve we at change the direction of motion require a force and hence, is not possible to round a curve acceleration. However, in acceleration may be constant case of projectile motion constant ($= \vec{g}$) while in case motion acceleration ≠ constant, change in direction or both direction and magnitude.

Hence correct answer is (B).

[यदि चाल नियत है, तो वेग दिशा कारण परिवर्तित हो सकता है तथा जैसे वेग परिवर्तन की दर होती है, इसलिए नियत है तो त्वरण आवश्यक रूप में सकता , जैसे– एकसमान वृत्तीय गति में, चाल = नियत लेकिन त्वरण ≠ 0. किसी गति के लिए हमे गति की दिशा बदलने इसलिए हमें एक बल की आवश्यकता पड़ती प्रकार त्वरण की भी। (i.e. किसी वक्राकार बिना त्वरण के चक्कर लगाना सम्भव नहीं। पर चक्कर लगाने में त्वरण नियत या की सकता है। परवलयकार गति में, त्वरण ($= \vec{g}$ रहता है, जबकि वृताकार गति में त्वरण ≠ नि अतः सही उत्तर (B) है।]

ration $a_{av} = \frac{v_2 - v_1}{t_2 - t_1}$

$= \frac{4a - 24b}{2} = 2a -$

is (C).

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- If there is no change in direction of motion during the given time interval, then

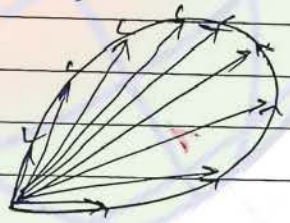
  |disp| = distance travelled

- If direction of motion changes

  |displacement| < distance travelled

- Displacement can never be greater than distance travelled.

- |displacement| may decrease with increase in time.



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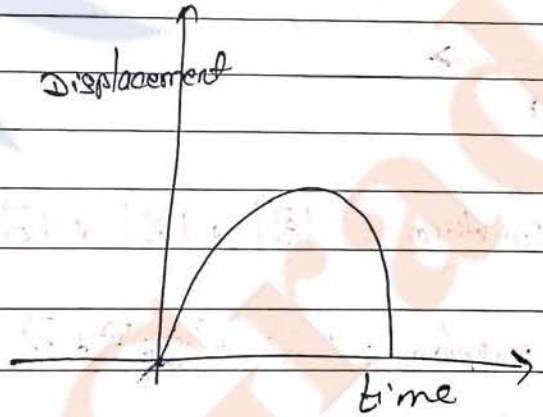

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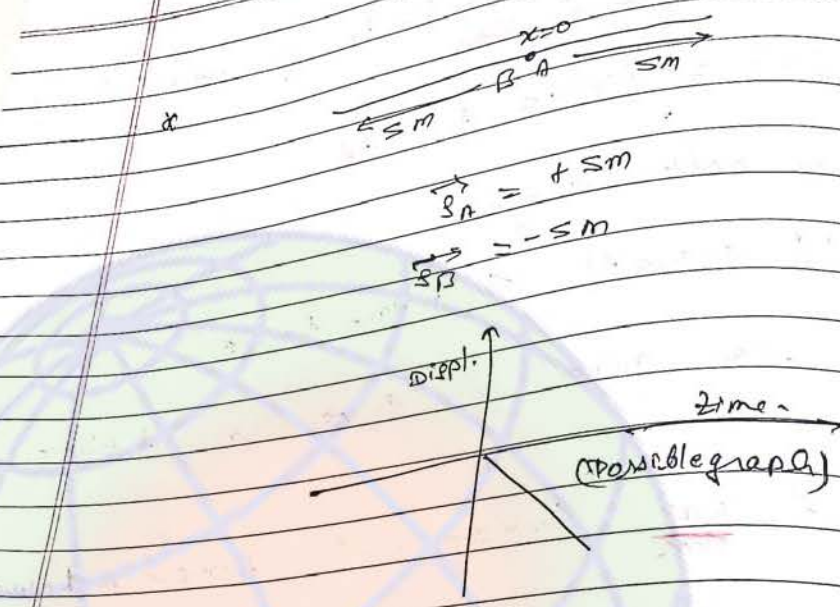

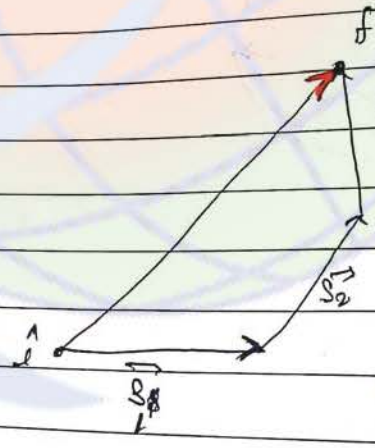

(i) Distance = $|\vec{S_1}| + |\vec{S_2}| + |\vec{S_3}|$

(ii) displacement $\vec{S} = \vec{S_1} + \vec{S_2} + \vec{S_3}$. (vohan

Note

$\vec{S} = \vec{r_f} - \vec{r_i}$ (vohan Positi

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DAILY

Sub topic :

A particle starts with an initial v
along the positive x-direction ar
uniformly at the rate 0.50 m/s²
reach the velocity 7.5 m/s will b
(1) 5 s (2) 2 s (3) 10

A particle starts with an initial
along the positive x-direction
uniformly at the rate 0.50 n
travelled by the particle in first t
(1) 4 m (2) 5n
(3) 1m (4) 6

A passenger is standing 'd' r
The bus begins to mo
acceleration a. To catch the
runs at a constant speed v t
minimum speed of the pass
catch the bus will be-
(1) 2ad (2)
(3) $\sqrt{2ad}$ (4)

A body moving with
describes 4 m in 3rd se
5th second. The distance
second is-
(1) 100 m (2) 80 m

A particle starts with a
along the positive x-di
uniformly at the rate
covered in reaching the
(1) 25 m (2) 50 m

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Target Course for NITs (JEE Main)-2014

DAILY PRACTICE PROBLEM SHEET

PHYSICS

Motion in One Dimension-3

Sub topic : Motion with uniform Acceleration

Q.1 A particle starts with an initial velocity 2.5 m/s along the positive x-direction and it accelerates uniformly at the rate 0.50 m/s^2. Time taken to reach the velocity 7.5 m/s will be-
(1) 5 s (2) 2 s (3) 10 s (4) 15 s

Q.1 एक कण 2.5 m/s के प्रारम्भिक वेग से धन x-अक्ष के अनुदिश गति करता है तथा यह समान दर 0.50 m/s^2. से त्वरित होता है। इसका वेग 7.5 m/s होने में लगा समय होगा-
(1) 5 s (2) 2 s (3) 10 s (4) 15 s

Q.2 A particle starts with an initial velocity 2.5 m/s along the positive x-direction and it accelerates uniformly at the rate 0.50 m/s^2. The distance travelled by the particle in first two seconds will be-
(1) 4 m (2) 5m
(3) 1m (4) 6 m

Q.2 एक कण 2.5 m/s के प्रारम्भिक वेग से धन x-अक्ष के अनुदिश गति करता है तथा यह समान दर 0.50 m/s^2 से त्वरित होता है, कण द्वारा पहले दो सेकण्डों में तय दूरी होगी-
(1) 4 m (2) 5m
(3) 1m (4) 6 m

Q.3 A passenger is standing 'd' m away from a bus. The bus begins to move with constant acceleration a. To catch the bus, the passenger runs at a constant speed v towards the bus. The minimum speed of the passenger so that he may catch the bus will be-
(1) 2ad (2) $\sqrt{ad}$
(3) $\sqrt{2ad}$ (4) ad

Q.3 एक यात्री किसी बस से 'd' m दूरी पर खड़ा है। बस नियत त्वरण a से चलना प्रारम्भ करती है, बस को पकड़ने के लिये यात्री को बस की ओर नियत चाल v से दौड़ता है, यात्री की न्यूनतम चाल क्या हो कि वह बस पकड़ ले-
(1) 2ad (2) $\sqrt{ad}$
(3) $\sqrt{2ad}$ (4) ad

Q.4 A body moving with uniform acceleration describes 4 m in 3rd second and 12 m in the 5th second. The distance described in next three second is-
(1) 100 m (2) 80 m (3) 60 m (4) 20 m

Q.4 एक वस्तु एकसमान त्वरण से चलना प्रारम्भ करती है, तथा 4m दूरी तीसरे सेकण्ड में तथा 12m दूरी पांचवें सेकण्ड में तय करती है तो अगले तीन सेकण्डों मे तय दूरी होगी-
(1) 100 m (2) 80 m (3) 60 m (4) 20 m

Q.5 A particle starts with an initial velocity 2.5 m/s along the positive x-direction and it accelerates uniformly at the rate 0.50 m/s^2. The distance covered in reaching the velocity 7.5 m/s will be-
(1) 25 m (2) 50 m (3) 75 m (4) 100 m

Q.5 एक कण 2.5 m/s के प्रारम्भिक वेग से धन x-अक्ष के अनुदिश गति करता है तथा यह समान दर 0.50 m/s^2 से त्वरित होता है, कण का वेग 7.5 m/s होने पर तय दूरी होगी-
(1) 25 m (2) 50 m (3) 75 m (4) 100 m

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Q.6 A particle starts moving from position of rest under a constant acceleration. If it travels a distance x in t sec. The distance it will travel in next t sec will be-
(1) 2x (2) 3x
(3) 4x (4) 5x

Q.7 A truck and a car are brought to a hault by application of same breaking force. Which one will come to stop in shorter distance if they are moving with same (a) velocity (b) K.E. (c) momentum
(1) Both car, truck (2) Truck, car, car
(3) Car, both, truck (4) Car, truck, truck

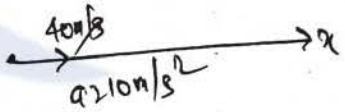
Passage # 1(Q. 8 to 11)
A particle is moving along x-axis. Its initial velocity is 40 m/s along positive x-axis and an acceleration of 10 m/s^2 along negative x-axis. Particle starts from x = 10 m.

Q.8 Velocity of particle is zero at ..... second -
(1) 6 (2) 4
(3) 8 (4) 2

Q.9 Maximum x-co-ordinate of particle (in positive direction)
(1) 90 (2) 60
(3) 120 (4) 30

Q.10 Velocity of particle at origin is ..... m/s .
(1) $30\sqrt{2}$ (2) $20\sqrt{2}$
(3) $-20\sqrt{2}$ (4) $-30\sqrt{2}$

Q.11 Particle is at time ..... second at origin -
(1) $4+3\sqrt{2}$ (2) $4+\sqrt{2}$
(3) $2+3\sqrt{2}$ (4) $3+2\sqrt{2}$



Q.6 एक कण विराम से व नियत त्वरण से ... यदि कण दूरी x, t s में तय करता है तय दूरी होगी-
(1) 2x (2) 3x
(3) 4x (4) 5x

Q.7 एक ट्रक व कार ब्रेक लगाने से ... कम दूरी पर रूकेगा यदि वे समान ... गतिज ऊर्जा से (c) संवेग से, गति कर ...
(1) दोनों, कार, ट्रक (2) ट्रक, ...
(3) कार, दोनों, ट्रक (4) कार, ट्रक

गद्यांश # 1(Q. 8 से 11)
एक कण x-अक्ष के अनुदिश गति कर ... प्रारम्भिक वेग 40 m/s है, धनात्मक x ... तथा त्वरण 10 m/s^2 है ऋणात्मक x ... कण x = 10 m से गति करना प्रारम्भ ...

Q.8 कण का वेग ..... sec पर शून्य है ...
(1) 6 (2) 4
(3) 8 (4) 2

Q.9 कण का अधिकतम x निर्देशांक (धनात्मक ...
(1) 90 (2) 60
(3) 120 (4) 30

Q.10 मूलबिन्दु पर कण का वेग ..... m/s है.
(1) $30\sqrt{2}$ (2) $20\sqrt{2}$
(3) $-20\sqrt{2}$ (4) $-30\sqrt{2}$

Q.11 समय ..... पर कण पुन: मूल बिन्दु पर ...
(1) $4+3\sqrt{2}$ (2) $4+\sqrt{2}$
(3) $2+3\sqrt{2}$ (4) $3+2\sqrt{2}$.[2]

We have v = u + at or
$\Rightarrow t = 10$ s
Hence correct answer
We have, s = ut + (1/
= (2.5) (2
Since the particle d
also the distance tra
Hence correct answ

Let u is the ini
acceleration then
$S_n = u + (1/2)\, a(2$
$\therefore\ S_3 = u + (1/2)\, a(3$
$\Rightarrow 4 = u + \frac{5}{2}\, a$
similarly for 5th
$S_5 = u + (1/2)\, a$
$\Rightarrow 12 = u + (9$
From (i) & (ii
so, distance t
From "s = ut
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Similarly di
$= -6 \times 8 +$
So distance
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We have,
or $(7.5)^2 =$
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$\Rightarrow x' =$
$x' =$
This i
Also
t sec.

1.[3] We have $v = u + at$ or $7.5 = 2.5 + 0.50\ t$

$\Rightarrow t = 10$ s

Hence correct answer is (C)

2.[4] We have, $s = ut + (1/2)\ at^2$

$= (2.5)\ (2) + (1/2)\ (0.50)\ (2)^2 = 6$m

Since the particle does not return back, it is also the distance travelled.

Hence correct answer is (D)

3.[3]

4.[3] Let u is the initial velocity and a is the acceleration then

$S_n = u + (1/2)\ a(2n - 1)$

$\therefore\ S_3 = u + (1/2)\ a(3 \times 2 - 1)$

$\Rightarrow 4 = u + \frac{5}{2}\ a$ ...(i)

similarly for 5^{th} second

$S_5 = u + (1/2)\ a\ (2 \times 5 - 1)$

$\Rightarrow 12 = u + (9/2)\ a$ ...(ii)

From (i) & (ii) $u = -6$ m/s and $a = 4$ m/s^2,

so, distance travelled in 5 sec,

From "$s = ut + 1/2\ at^2$",

$s = -6 \times 5 + (1/2) \times 4 \times 5^2 = 20$ m

Similarly distance travelled in 8 sec

$= -6 \times 8 + (1/2)\ 4 \times 8^2 = 80$ m

So distance travelled in next 3 sec

$= 80 - 20 = 60$ m

Hence correct answer is (C)

5.[2] We have, $v^2 = u^2 + 2a\ x$

or $(7.5)^2 = (2.5)^2 + 2(0.50)\ x \Rightarrow x = 50$ m

Hence correct answer is (B)

6.[2] The velocity of particle after time t will be $v = u + at = 0 + at = at$

Now for next t sec, it will be the initial velocity,

From "$s = u't + (1/2)\ at^2$", we have

$\Rightarrow x' = (at)\ t + (1/2)\ at^2$ [Here $u' = at$]

$x' = 3/2\ at^2$ ...(1)

This is the distance travelled in next t sec

Also given that particle travels x distance in t sec. so again using "$s = ut + (1/2)\ at^2$"

We have, $x = \frac{1}{2}\ at^2$ ...(2)

From (1) & (2), we have, $x' = 3x$

Hence correct answer is (B)

7.[3] By breaking force the body is brought to rest so, $v = 0$ and $a = (-F/m)$ (as it is retardation)

If s is the distance travelled in stopping (called stopping distance), from $v^2 = u^2 + 2as$ we have, $0 = u^2 - 2(F/m)s$

$\Rightarrow s = \frac{mu^2}{2F}$,

But $KE = (1/2)\ mu^2$ and also

$KE = \frac{p^2}{2m}$ ($\because p = mu$)

So $s = \frac{mu^2}{2F} = \frac{KE}{F} = \frac{p^2}{2mF}$.

From this it is clear that,

(a) If u is same, $s \propto \frac{mu^2}{2F} \Rightarrow s \propto m$

Now as mass of car is lesser than that of truck, so car will stop in shorter distance.

(b) If K.E. is same, $s \propto \frac{KE}{F}$

So both will stop after travelling same distance.

(c) If p is same, $s \propto \frac{p^2}{2mF} \Rightarrow s \propto \frac{1}{m}$

Now as mass of truck is more than that of car so truck will stop in a shorter distance.

Hence correct answer is (C)

Note : As $s = \frac{mu^2}{2F}$, so for a given body if breaking force remains unchanged. $s \propto u^2$

[as m is constant]

i.e. if the speed of a moving body is made n times the stopping distance will become n^2 times.

8.[2] $\vec{v} = \vec{u} + \vec{a}\,t$

$0 = 40 - 10\,t$

$t = 4$ sec.

9.[1] $V^2 = 4^2 - 2|a|\,|\Delta x|$

$0 = 40^2 - 2(10)\,|\Delta x|$

$|\Delta x| = \dfrac{1600}{20} = 80$ m

i.e. $x_f - x_i = 80$

$x_f = 80 + 10 = 90$ m

10.[4] $\Delta x = \vec{u}\,t + \dfrac{1}{2}\,\vec{a}\,t^2$

$(x - 10) = 40t - 5t^2$

if $x = 0$

$5t^2 - 40\,t - 10 = 0$

$t = \dfrac{40 \pm \sqrt{1600 + 200}}{10}$

$t = 4 + 3\sqrt{2}$

i.e. at $t = 4 + 3\sqrt{2}$ particle its at origin and

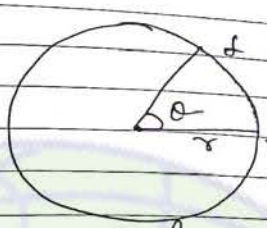
$v = u + at$

$v = 40 - 10(4 + 3\sqrt{2})$

$v = -30\sqrt{2}$ m/s

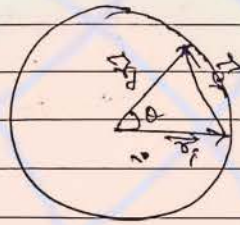
11.[1] at $t = 4 + 3\sqrt{2}$

e)



circular motion.

$|disp| = ?$



$|\vec{R}| = |\vec{r}_2 - \vec{r}_1|$

$|\vec{R}| = \sqrt{r^2 + r^2 - 2r^2\cos\theta}$

$= 2r\sin(\theta/2)$

and

Distance travelled $= r\theta$

↳ θ is in radian.

★ Special Case:-

(i) $\frac{1}{4}$ circle $(\theta = \pi/2)$

distance $= \frac{\pi r}{2}$ and $|\text{Displacement}| = \sqrt{2}r$

(ii) $\frac{1}{2}$ circle $(\theta = \pi)$

distance $= 2r$

$|\text{displ}| = 2r$

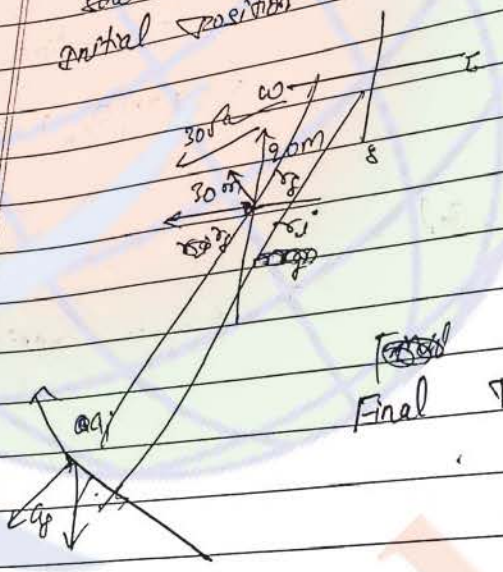
(Ex) Complete circle ($\theta = 2\pi$)

displacement = 0

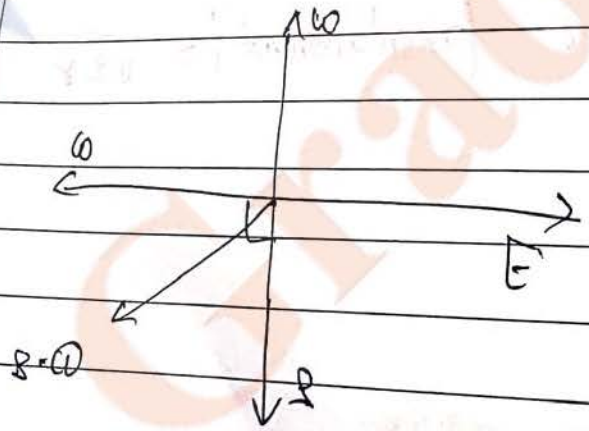
distance = $2\pi r$

displacement = 0

A person moves 30m towards east 20m towards north and finally 30$\sqrt{2}$ m south west. find its final position w.r.t initial position.



Final position w.r.t initial



$\vec{R} = 30\hat{i} + 20\hat{j} + (-30\hat{i})$

$\vec{R} = (-10\hat{i})$

CA

Target Co

DAI

Sub topic :

If a body travels half its
second of its fall from rest
of its fall, will respectively
($g = 9.8$ m/s^2)
(1) 0.59 s, 57 m
(3) 5.9 s, 5.7 m

A man standing on the e
straight up with initial
another stone straight
speed and from the sa
the speed the stones w
hit the ground at the b
(1) $\sqrt{2}$: 1
(3) 1 : 1

A pebble is thr
bridge with an
strikes the water
gravity is 9.8 m
and velocity w
water will resp
(1) 4.9 m, 1.4
(3) 49 m, 1.4

A rocket is
with a r
10 m/s^2. T
continues
reached.
will th
(Take g
(1) 36 k
(3) 36 k

CAR

EER PO

CAREER POINT

Target Course for NITs (JEE Main)-2014

DAILY PRACTICE PROBLEM SHEET

PHYSICS

Motion in One Dimension-4

Sub topic : Motion Under Gravity and Relative motion

Q.1 If a body travels half its total path in the last second of its fall from rest. The time and height of its fall, will respectively be- ($g = 9.8\ m/s^2$)

(1) 0.59 s, 57 m (2) 3.41 s, 57 m

(3) 5.9 s, 5.7 m (4) 5.9 s, 34.1 m

Q.2 A man standing on the edge of a cliff throws a stone straight up with initial speed u and then throws another stone straight down with the same initial speed and from the same position. Find the ratio of the speed the stones would have attained when they hit the ground at the base of the cliff.

(1) $\sqrt{2}$: 1 (2) 1 : $\sqrt{2}$

(3) 1 : 1 (4) 1 : 2

Q.3 A pebble is thrown vertically upwards from bridge with an initial velocity of 4.9 m/s. It strikes the water after 2s. If acceleration due to gravity is 9.8 m/s^2 . The height of the bridge and velocity with which the pebble strike the water will respectively be-

(1) 4.9 m, 1.47 m/s (2) 9.8 m, 14.7 m/s

(3) 49 m, 1.47 m/s (4) 1.47 m, 4.9 m/s

Q.4 A rocket is fired vertically up from the ground with a resultant vertical acceleration of 10 m/s^2. The fuel is finished in 1 minute and it continues to move up. (a) the maximum height reached. (b) After how much time from then will the maximum height be reached (Take $g = 10\ m/s^2$)

(1) 36 km, 1 min (2) 6 km, 1 min

(3) 36 km, 1 sec (4) 36 km, 1 sec

Q.1 यदि कोई वस्तु विराम से गिरना शुरू कर के कुल पथ का आधा भाग गति के अन्तिम सेकण्ड में तय करती हो तो गिरने से लगा समय व ऊँचाई क्रमशः क्या होगी– ($g = 9.8\ m/s^2$) -

(1) 0.59 s, 57 m (2) 3.41 s, 57 m

(3) 5.9 s, 5.7 m (4) 5.9 s, 34.1 m

Q.2 एक व्यक्ति किसी चोटी से u वेग से एक पत्थर को ऊर्ध्वाधर ऊपर की ओर प्रक्षेपित करता है तथा दूसरे पत्थर को समान चाल से समान ऊँचाई से ऊर्ध्वाधर नीचे की ओर प्रक्षेपित करता है। जमीन पर टकराते समय उनके वेगों में अनुपात क्या होगा

(1) $\sqrt{2}$: 1 (2) 1 : $\sqrt{2}$

(3) 1 : 1 (4) 1 : 2

Q.3 एक कण किसी पुल से 4.9 m/s के वेग से ऊपर की ओर फेंका जाता है, 2s बाद यह जल से टकराता है, यदि गुरूत्वीय त्वरण का मान 9.8 m/s^2 होतो पुल की ऊँचाई तथा कण जिस वेग से पानी से टकराता है क्रमशः होगें–

(1) 4.9 m, 1.47 m/s (2) 9.8 m, 14.7 m/s

(3) 49 m, 1.47 m/s (4) 1.47 m, 4.9 m/s

Q.4 एक रॉकेट जमीन से उपर की ओर परिणामी ऊर्ध्वाधर त्वरण 10 m/s^2 से उपर की ओर प्रक्षेपित किया जाता है। ईधन एक मिनट में खर्च हो जाता है, तथा यह लगाातार ऊपर की ओर बढ़ता रहता है, तो (a) पहुँच की महत्तम ऊँचाई क्या है

(b) इंधन खत्म होने के कितने समय बाद महत्तम ऊँचाई प्राप्त होगी ($g = 10\ m/s^2$)

(1) 36 km, 1 min (2) 6 km, 1 min

(3) 36 km, 1 sec (4) 36 km, 1 sec

Q.5 The acceleration of a particle is given as $a = 3x^2$. At $t = 0$, $v = 0$, $x = 0$. The velocity at $t = 2$ sec will be-
(1) 0.05 m/s (2) 0.5 m/s
(3) 5 m/s (4) 50 m/s

Q.6 The acceleration of a particle is given by $a = 3t$ and at $t = 0$, $v = 0$, $x = 0$. The velocity and displacement at $t = 2$ sec will be-
(1) 6 m/s, 4 m (2) 4 m/s, 6 m
(3) 3 m/s, 2 m (4) 2 m/s, 3 m

Q.7 Two cars are moving in the same direction with the same speed 30 km/hr. They are separated by a distance of 5 km, the speed of a car moving in the opposite direction if it meets these two cars at an interval of 4 minutes, will be –
(1) 40 km/hr (2) 45 km/hr
(3) 30 km/hr (4) 0 km/hr

Q.8 A man standing on a road holds his umbrella at 30° with the vertical (backward) to keep the rain away. He throws the umbrella and starts running at 10 km/h. He finds that rain drops are hitting his head vertically, the speed of raindrop with respect to the road will be-
(1) 10 km/h (2) 20 km/h
(3) 30 km/h (4) 40 km/h

Q.9 In the above example, the speed of raindrops w.r.t. the moving man, will be-
(1) $10/\sqrt{3}$ km/h (2) 5 km/h
(3) $10\sqrt{3}$ km/h (4) $5/\sqrt{3}$ km/h

Q.10 A ball is dropped from the roof of a tower of height h. The total distance covered by it in the last second of its motion is equal to the distance covered by it in first three seconds. The value of h in meters is : ($g = 10$ m/s^2)
(1) 125 (2) 200
(3) 100 (4) 80

Q.5 किसी कण का त्वरण $a = 3x^2$ है, पर $v = 0$, $x = 0$, तो $t = 2$s पर
(1) 0.05 m/s (2) 0.5 m/s
(3) 5 m/s (4) 50 m/s

Q.6 किसी कण का त्वरण $a = 3t$ है, $t = 0$, पर $v = 0$, $x = 0$, तो कण का वेग तथा विस्थापन $t = 2$ sec पर क्रमशः होंगे–
(1) 6 m/s, 4 m (2) 4 m/s, 6 m
(3) 3 m/s, 2 m (4) 2 m/s, 3 m

Q.7 दो कारे समान वेग 30 km/hr से चल कर रही है, उनके बीच की दूरी 5 km है, विपरीत दिशा में गति करती कार की यदि यह ट्रेनो को चार मिनट के मिलती है–
(1) 40 km/hr (2) 45 km/hr
(3) 30 km/hr (4) 0 km/hr

Q.8 एक व्यक्ति वर्षा से बचने के ऊर्ध्वाधर से 30° के कोण पर पीछे को फेंक कर 10 km/h की चाल से अब यदि वर्षा की बूंदे ऊर्ध्वाधर पड़ रही हों तो वर्षा की बूंदों सापेक्ष चाल होगी–
(1) 10 km/h (2) 20 km/h
(3) 30 km/h (4) 40 km/h

Q.9 उपरोक्त उदा. में वर्षा की बूंदो की चाल होगी -
(1) $10/\sqrt{3}$ km/h (2) 5 km/h
(3) $10\sqrt{3}$ km/h (4) $5/\sqrt{3}$ km/h

Q.10 एक गेंद को एक h ऊँचाई के टावर की गया है। इसकी गति के अन्तिम सेकण्ड में दूरी प्रथम तीन सैकण्डों में चली गई दूरी तो h का मान मी. मे है ($g = 10$ m/s^2)
(1) 125 (2) 200
(3) 100 (4) 80

If the body falls a he
equation of motion v
$h = 1/2\ gt^2$
[u = 0 as body starts
Now the distance f
$h' = 1/2\ g(t-1)^2$
So from eq. (1)
last second
$h - h' = (1/2)\ gt^2$
$h - h' = (1/2)\ g$ (
But according to
$(h - h') = h/2$
i.e. $(1/2)\ h = (1/$
or $(1/2)\ gt^2 =$
[as from eq. (1
or $t^2 - 4t + 2$
or $t = [4 \pm \sqrt{}$
or $t = 2 \pm \sqrt{}$
0.59 s is phy
total time t ta
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So $t = 3.4$
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So, th
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h =
(u
F
v
F

1.[2] If the body falls a height h in time t, from 2nd equation of motion we have

$h = 1/2\ gt^2$ ....(1)

[u = 0 as body starts from rest]

Now the distance fallen in (t – 1) s will be

$h' = 1/2\ g(t-1)^2$ ....(2)

So from eq. (1) & (2) distance fallen in the last second

$h - h' = (1/2)\ gt^2 - (1/2)\ g\ (t-1)^2$,

$h - h' = (1/2)\ g\ (2t-1)$

But according to given problem as

$(h - h') = h/2$

i.e. $(1/2)\ h = (1/2)g\ (2t-1)$

or $(1/2)\ gt^2 = g(2t-1)$

[as from eq. (1) $h = (1/2)\ gt^2$]

or $t^2 - 4t + 2 = 0$

or $t = [4 \pm \sqrt{(4^2 - 4\times2)}\ ]/2$

or $t = 2 \pm \sqrt{2}$ or $t = 0.59$ or 3.41 s

0.59 s is physically unacceptable as it gives the total time t taken by the body to reach ground is lesser than one sec while according to the given problem time of motion must be greater than 1 s.

So $t = 3.41$ s &

$h = (1/2) \times (9.8) \times (3.41)^2 = 57$ m

Hence correct answer is (B)

2.[3] As the stone thrown vertically up will come back to the point of projection with same speed, both the stones will move downward with same initial velocity, so both will hit the ground with velocity

$v^2 = u^2 + 2gh$ i.e., $v = \sqrt{(u^2 + 2gh)}$

So, the ratio of speeds attained when they hit the ground is 1 :1

Hence correct answer is (C)

3.[2] Taking the point of projection as origin and downward direction as positive. By 2nd equation of motion, i.e. $s = ut + (1/2)at^2$, We have,

$h = -4.9 \times 2 + (1/2)\ 9.8 \times 2^2 = 9.8$

(u is taken to be negative as it is upwards)

From 1st equation of motion i.e. $v = u + at$,

$v = -4.9 + 9.8 \times 2 = 14.7$ m/s

Hence correct answer is (B)

4.[1] (a) The distance travelled by the rocket during burning interval (1 minute = 60 s) in which resultant acceleration is vertically upwards is 10 m/s² will be

$h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2$

$= 18000$ m

And velocity acquired by it will be ....(1)

$v = 0 + 10 \times 60 = 600$ m/s

Now after 1 minute the rocket moves vertically up with initial velocity of 600 m/s and acceleration due to gravity oppose its motion. So, it will go to a height h_2 till its velocity becomes zero that ....(2)

$0 = (600)^2 - 2gh_2 \Rightarrow h_2 = 18000$ m

[as $g = 10$ m/s²] ....(3)

So from eq. (1) and (3) the maximum height reached by the rocket from the ground.

$H = h_1 + h_2 = 18 + 18 = 36$ km

(b) As after burning of fuel the initial velocity from Eq. (2) is 600 m/s and gravity opposes the motion of rocket, so from 1st equation of motion time taken by it to reach the maximum height (for which v = 0)

$0 = 600 - gt$, i.e. $t = 60$ s

after finishing of fuel, the rocket goes up for 60 sec i.e., 1 minute more.

Hence correct answer is (A)

5.[1] $a = 3x^2 \Rightarrow v\ \frac{dv}{dx} = 3x^2$

$\Rightarrow vdv = 3x^2\ dx$

$\Rightarrow \frac{v^2}{2} = 3\ \frac{x^2}{3} + c$

at $t = 0$, $v = 0$, $x = 0$

$\therefore\ c = 0$ Now, $\frac{v^2}{2} = x^3$

$v^2 = 2x^3 \Rightarrow v = \sqrt{2}\ x^{3/2}$ ... (1)

$\Rightarrow \frac{dx}{dt} = \sqrt{2}\ x^{3/2}$

Remember, when a is function of x.

use $a = \frac{vdv}{dx}$

when a is function of t, $a = \frac{dv}{dt}$,

$dx = \sqrt{2}\ x^{3/2}\ dt \Rightarrow \frac{dx}{x^{3/2}} = \sqrt{2}\ t + c'$,

at $t = 0, x = 0, v = 0$

$\therefore\ c' = 0$

Now $\frac{-2}{\sqrt{x}} = \sqrt{2}\,t \Rightarrow 4 = 2xt^2$ ...(2)

$\Rightarrow x = \frac{2}{t^2}$

From (1) and (2) $v = \sqrt{2}\left(\frac{2}{t^2}\right)^{3/2}$,

at $t = 2$ sec $\Rightarrow v = 1/2$ m/sec

Hence correct answer is (B)

6.[1] $a = 3t \Rightarrow \frac{dv}{dt} = 3t \Rightarrow \int dv = \int 3t\,dt$

$\Rightarrow v = \frac{3t^2}{2} + c$

Substituting the initial conditions,

At $t = 0$, $v = 0$ and $x = 0$

$\therefore\ c = 0$ Hence, $v = \frac{3t^2}{2}$,

Velocity at $t = 2$ sec is $\frac{3\times 2^2}{2} = 6$ m/s

Also, $\frac{dx}{dt} = \frac{3t^2}{3} \Rightarrow \int dx = \frac{3}{2}\int t^2 dt$

$\Rightarrow x = \frac{3}{2}\frac{t^3}{3} + c'$

at $t = 0, x = 0 \therefore c' = 0, \ \therefore x = \frac{t^3}{2}$,

Now displacement at $t = 2$ sec is $\frac{2^3}{2} = 4$ m

Hence correct answer is (A)

7.[2] Now if the velocity of car (say C) moving in opposite direction to A and B, is $\vec{v}_C$ relative to ground then the velocity of car C relative to A and B will be $\vec{v}_{rel.} = \vec{v}_C - \vec{v}$

But as $\vec{v}$ is opposite to v_C,

$v_{rel} = v_C - (-30)$

$= (v_C + 30)$ km/hr

So, the time taken by it to cross the cars A and B is

$$t = \frac{d}{v_{rel.}}$$

$$\Rightarrow \quad \frac{4}{60} = \frac{5}{v_C + 30}$$

$$\Rightarrow \quad v_C = 45 \text{ km/hr}$$

Hence correct answer is (B)

8.[2] When the man is at rest w.r.t. ground the rain comes to him at an angle of 30° with the vertical. This is the direction of relative velocity of rain drops with respect to the man.

Here $\vec{v}_{r,g}$ = velocity of rain with respect to ground

$\vec{v}_{m,g}$ = velocity of the man with respect to ground.

and $\vec{v}_{r,m}$ = velocity of the rain with respect to the man,

$\vec{v}_{r,m} = \vec{v}_{r,g} - \vec{v}_{m,g}$

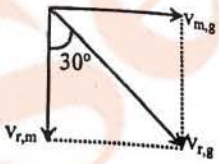
We have $\vec{v}_{r,g} = \vec{v}_{r,m} + \vec{v}_{m,g}$

Taking horizontal components eq. (1)

$v_{r,g} \sin 30° = v_{m,g} = 10$ km/h

or $v_{r,g} = \frac{10}{\sin 30°} = 20$ km/h

Hence correct answer is (B)



9.[3] Taking vertical components eq. (1)

$v_{r,g} \cos 30° = v_{r,m}$

or $v_{r,m} = 20\ \frac{\sqrt{3}}{2} = 10\sqrt{3}$ km/hr

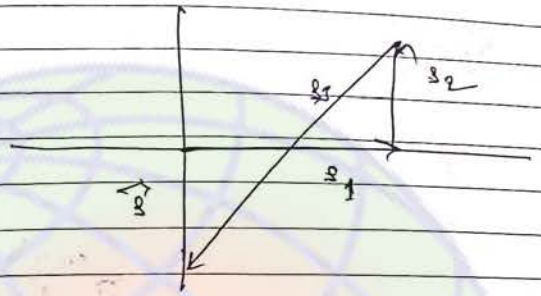
Hence correct answer is (C)

10.[1] $\frac{g}{2}(2n - 1) = \frac{1}{2}g\,(3)^2$

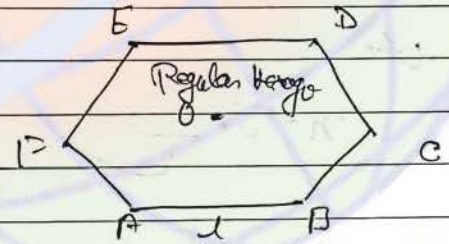
$n = 5$

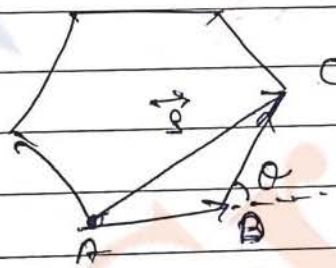
$\therefore\ h = \frac{1}{2}g\,(5)^2 = 125$ m.

Geometry method



Q) A person moves along the sides of the regular hexagon it moves from A to B then B to C the find mag. of displacement





$\vec{AC} = \vec{AB} + \vec{BC}$

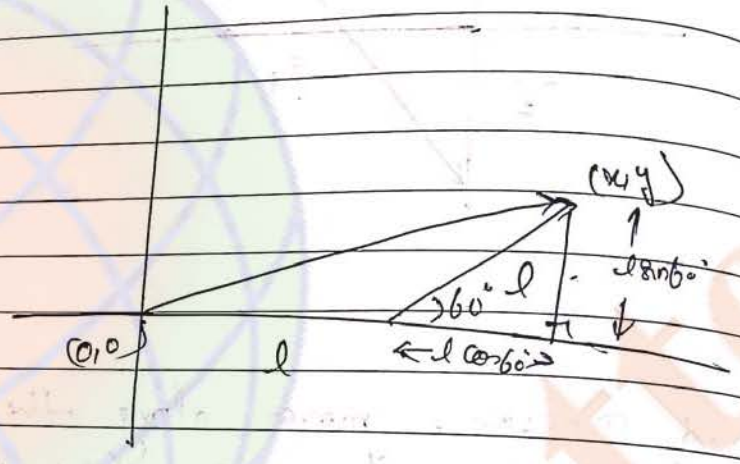
$|\vec{s}| = |\vec{s_1} + \vec{s_2}|$

$= \sqrt{l^2 + l^2 + 2l^2 \cos 60^\circ}$

$= \sqrt{3}l$

Alt

$\vec{s} = \vec{r}_f - \vec{r}_i$



$s = \sqrt{x^2 + y^2}$

where

$x = l + \frac{l}{2}$

$y = \frac{\sqrt{3}l}{2}$

$s = \sqrt{3}l$

Speed and velocity

① Speed :-
↳ scalar quantity
∴ Unit = m/s
[Speed] = $[m^0 L^1 T^{-1}]$

(i) Average speed (V_{avg})

$$V_{avg} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

* Cases :-

(i) $\frac{s}{t}$

$$V_{avg} = \frac{s}{t}$$

(ii) A — B — C
s_1 (t_1) s_2 t_2

$$V_{avg} = \frac{s_1 + s_2}{t_1 + t_2}$$ (from AtoC)

(iii) A — B — C
V_1 V_2

$$\frac{\frac{s_1}{t_1} + \frac{s_2}{t_2}}{\frac{s_1}{t_1} + \frac{s_2}{t_2}} \qquad \frac{s_1 t_2 + s_2 t_1}{t_1 t_2}$$

(Not able to find V_{avg} because Incomplete Info

Ra

Distance = $v_1 t_1$ Distance = $v_2 t_2$

(V) v_1 v_2 Ra

t_1

Ra

$v_{avg} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}$ (from Abdc

(V) A B C

s_1 s_2

v_1 v_2

$v_{avg} = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}} \rightarrow \frac{(s_1+s_2)\, v_1 v_2}{s_1 v_2 + s_2 v_1}$

Instantaneous speed (V)

$\Delta t \rightarrow 0$ (change in time)

At $t = 5s$ (time at Particals

V ⇒ Rate of change of distance w.r.t time

$$V = \frac{ds}{dt}$$

Velocity : v

reci

SI

i) Averag

Avg

Note

(1) Sp

(2)

(11)

Ra

B) Velocity :- $\vec{v}$

- vector quantity
- SI unit = m/s
- [velocity] = $[M^0 L T^{-1}]$

(i) Average velocity ($\vec{V}_{avg}$) :-

$$\text{Avg velocity } (\vec{V}_{avg}) = \frac{\text{Total displacement}}{\text{total time taken}}$$

Note

(I) If direction of motion does not change then

$$|\text{Avg velocity}| = \text{Avg speed}$$

or

$$|\vec{V}_{avg}| = V_{avg}$$

(2) If direction of motion changes

Ra

$$|\vec{V}_{avg}| < V_{avg}$$

→ Avg speed

(II) $|\vec{V}_{avg}|$ can not be greater than V_{avg}

(iv) $|\vec{V}_{avg}| = 0$

→ when particle does not move than

$V_{avg} = 0$

→ when initial and final position of particle is same than

$V_{avg} \neq 0$

So,

Case 1st is we say that if $|\vec{V}_{avg}| = 0$ then

V_{avg} may or may not be zero

Case 2nd is if $V_{avg} = 0$ then

$|\vec{V}_{avg}| = 0$

Instant

spec

(1)

Ra

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Instantaneous velocity

$\vec{V}$ = Rate of change of position w.r.t. time

$= \left(\frac{d\vec{r}}{dt}\right)$

OR

= Rate of change of displacement w.r.t. time,

$= \left(\frac{d\vec{s}}{dt}\right)$

Special case :-

(i) If motion is along x-axis :-

(a) $\vec{V} = \frac{dx}{dt}$

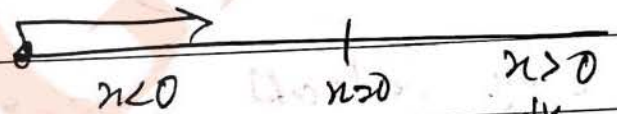
(b) $\vec{V}_{avg} = \frac{\Delta x}{\Delta t}$

Note

x → Represent location

$\frac{dx}{dt}$ Represent direction of motion

Note:



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(2) If motion takes place in x-y plane

(a) $\vec{V} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$

where $\frac{dx}{dt} = V_x$ = velocity along x-axis

and $\frac{dy}{dt} = V_y$ = velocity along y-axis

Note

(a) Instantaneous speed $= |\vec{V}|$

$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

(b) $\vec{V}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{dt}\hat{j}$

Q A particle moves along x-axis as

$x = (t^2)$ m.

x = Position (m)

t = time (s)

find

(i) Avg. velocity in 5 sec.

(ii) vel. at t = 5 sec.

(1) $V_{avg} = \frac{\Delta x}{\Delta t}$

$\vec{V} = \frac{dx}{dt}$

$\vec{V} \perp (dt)$

$\vec{V} = s$

A person ... 40 km/hour ... with ... Comp ...

(1) $\vec{V}_{avg} = \frac{\Delta x}{\Delta t}$

$= \frac{x_{t=5} - x_{t=0}}{\Delta t} = \frac{25 - 0}{5} = 5 \text{ m/s}$

$\vec{V} = \frac{dx}{dt}$

$\vec{V} = (2t)$ m/s

$\vec{V}_{t=5} = 10 \text{ m/s}$

(2) A person travells from kota to Indore with avg speed of 40 km/hour and returns back from the same path with avg. speed of 60 km/h. His avg. speed in complete journey

$V_1 = 40$ km/hour

$V_2 = 60$ km/h

$Avg = 40$

$\therefore h = 40$ km

$D = \frac{V}{t}$

$V_{avg} = \frac{2d}{\frac{d}{40} + \frac{d}{60}} = \frac{2(40)(60)}{60 + 40} = 48 \text{ km/h}$

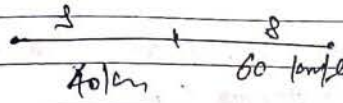
$V_{avg} = 0$

Q) A person travels 1st half of total dist. with Avg. speed of 40 km/h and next at time t.

In 1st half of time 't' its Avg. speed is and in next half its Avg. speed is 50 km/hour

Find Avg. speed for complete journey

A person travels south

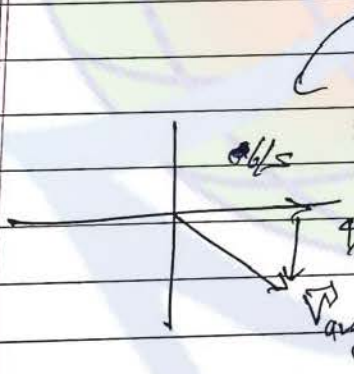


$$V_{avg} = \frac{2s}{\frac{1}{40} + \frac{1}{60}} = \frac{2 \times 40 \times 60}{100}$$

$$= 480$$

$$= 48 \text{ km/h}$$

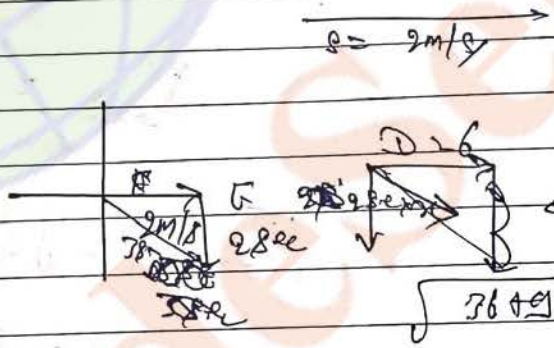
Eg.) A person moves with uniform speed of 2 m/s due east travels towards east for 3 sec then towards south for two second. $\vec{V}_{avg}$ = ?



$$\tan\theta = \frac{4}{6} = \frac{2}{3}$$

$$\theta = \tan^{-1}\left(\frac{2}{3}\right)$$

$|\vec{v}_1 + \vec{v}_2|$



$$= \frac{6\hat{i} - 4\hat{j}}{\sqrt{\left(\frac{6}{3}\right)^2 + \left(\frac{4}{2}\right)^2}}$$

$$= \frac{1}{\sqrt{52}} \text{ m/s}.$$

Q) A particle move in xy plane $x \perp y^2$ at
x → position (m)
t → time (s)

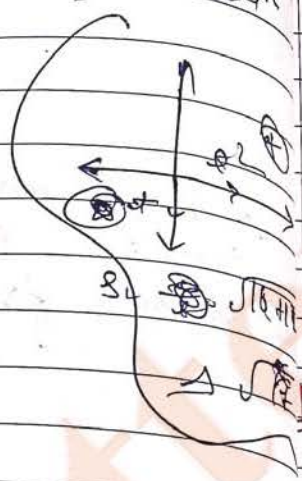
Speed of the particle at t = 2 sec?

Soln Teacher

$v_x = \frac{dx}{dt} = 2t =$

and

$v_y = 1$ m/s



$\vec{v} = (2t\hat{i} + \hat{j})$ m/s

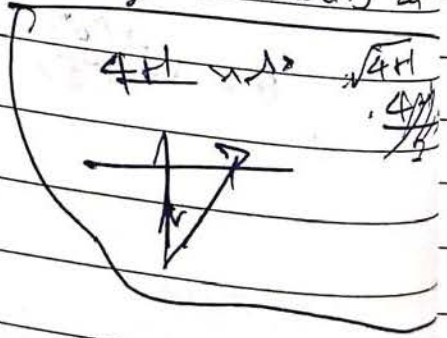
$\vec{v}_{t=2} = (4\hat{i} + \hat{j})$ m/s

$|\vec{v}_{t=2}| = \sqrt{4^2+1^2}$ m/s

$= \sqrt{17}$ m/s

Q) In the above question average velocity in 2 sec is

Teacher



$\frac{\Delta x}{\Delta t} = \frac{x_{t=2} - x_{t=0}}{2}$

$= \frac{4-0}{2} = 2 \text{ m/s}$

$\frac{\Delta y}{\Delta t} = \frac{2-0}{2}$

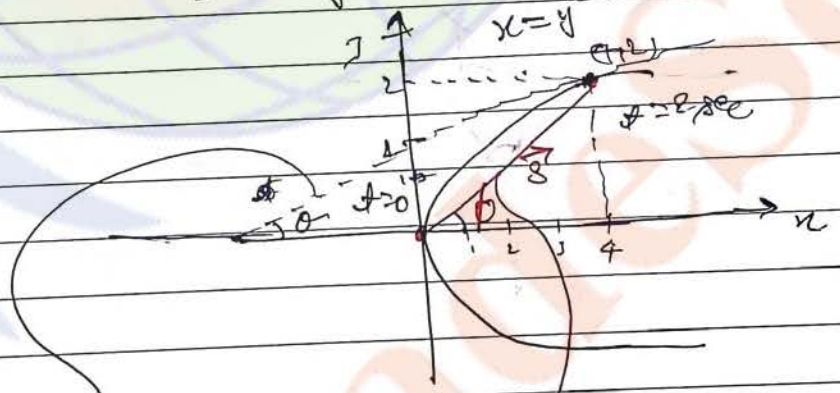
$\vec{V}_{avg} = (2\hat{i} + \hat{j})$ m/s

$|\vec{V}_{avg}| = \sqrt{2^2+1^2}$ m/s $= \sqrt{5}$ m/s

Note

$x = t^2$ and $y = t$

$\therefore x = y^2$



$\tan\theta = \frac{1}{4}$

$\theta = \tan^{-1}\left(\frac{1}{4}\right)$

$\tan\phi = \frac{1}{2}$

$\phi = \tan^{-1}\left(\frac{1}{2}\right)$

slope of chord $= \frac{\Delta y}{\Delta x}$

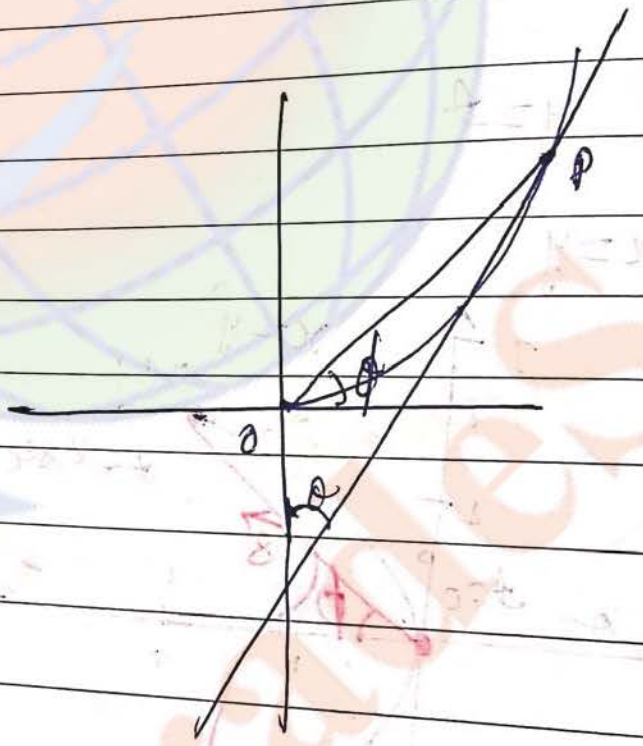
slope of tangent $= \frac{dy}{dx}$

(i) $y = \sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\left(\frac{dy}{dx}\right)_{x=4} = \frac{1}{4}$

(ii) $\frac{\Delta y}{\Delta x} = \frac{2-0}{4-0} = \frac{1}{2}$



Acceleration

→ vector quantity
→ SI unit → m/s²
[Acc] = [M⁰ L¹ T⁻²]

Average Acc^n ($\vec{a}_{avg}$)

$$\vec{a}_{avg} = \frac{\text{change in vel } (\Delta \vec{v})}{\text{time interval } (\Delta t)} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Instantaneous Acc^n

$\vec{a}$ = rate of change of vel. w.r.t. time

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= \frac{d^2\vec{r}}{dt^2} \text{ or } \frac{d^2 s}{dt^2}$$

$\because$ $v = \frac{dx}{dt}$ or $\frac{ds}{dt}$

SP Core : 7

(1) If particle moves along x-axis

$\vec{a}_{avg} = \frac{\Delta v_x}{\Delta t}$ where $v_x = \frac{dx}{dt}$

and

Acceleration

→ vector quantity
→ SI unit → m/s²
[Acc] = [M⁰L¹T⁻²]

* Average Acc^n ($\vec{a}_{avg}$)

$$\vec{a}_{avg} = \frac{\text{change in vel } (\Delta\vec{v})}{\text{time interval } (\Delta t)} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

* Instantaneous Acc^n

$\vec{a}$ = rate of change of vel. w.r.t. time

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= \frac{d^2\vec{r}}{dt^2} \text{ or } \frac{d^2s}{dt^2}$$

$\because$ $v = \frac{dx}{dt}$ or $\frac{ds}{dt}$

SP Core : 7

① If particle moves along x-axis

$\vec{a}_{avg} = \frac{\Delta v_x}{\Delta t}$ where $v_x = \frac{dx}{dt}$

and

Q2

$$\vec{a} = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

ii) If particle moves in x-y plane

$$\vec{a}_{xy} = \frac{\Delta v_x \hat{i}}{\Delta t} + \frac{\Delta v_y \hat{j}}{\Delta t}$$

and,

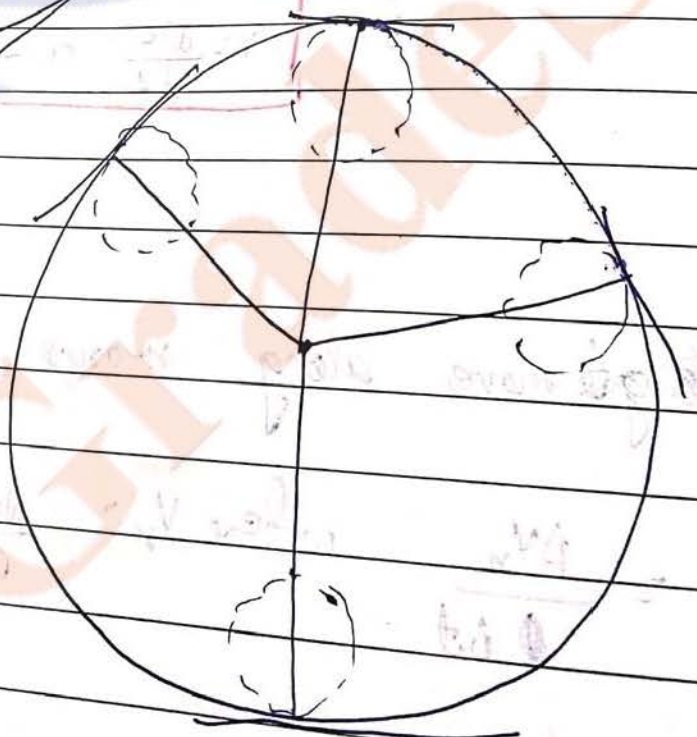
$$\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$= \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

$$= a_x\hat{i} + a_y\hat{j}$$

Notes

Q3 →



a_t → tangential acc^n

a_c → centripetal acc^n

Note:
- Centripetal acc^n is directed towards the instantaneous centre of curvature.

(i) a_t ⇒ rate of change of speed w.r.t. time

$$a_t = \frac{d|\vec{v}|}{dt}$$

(ii)
$$a_c = \frac{v^2}{r}$$

v = inst. speed

r = inst. radius of curvature.

Note ⇒ velocity

Note:
- $\vec{v}$ = velocity
- $|\vec{v}|$ = mag. of velocity or speed

(iii) $a = \frac{dv}{dt}$

$\vec{a} \rightarrow \vec{a_c} + \vec{a_t}$

and $a = \sqrt{a_c^2 + a_t^2}$

$$\left|\frac{d\vec{v}}{dt}\right| = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{d|\vec{v}|}{dt}\right)^2}$$

special case:

(1) If a particle moves along straight line

$a_c = 0$

i.e $a = a_t$

eg For a particle moving in a xy plane

$x = 2\sin 2t$

$y = 2\cos 2t$

when

x, y are position (m)

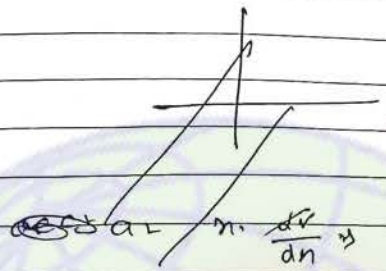
t is time (sec)

find (i) acc^n

(ii) tangential acc^n

(iii) Centripetal acc^{n}

Sol^{n}



$a_c = v \cdot \frac{dv}{dn}$

$\frac{x}{y} = 2\tan 2\theta$

$\tan 2\theta = \frac{x}{y}$

$\frac{d}{dx}\tan^{-1}$

$a = \frac{dv}{dt}$

$= \frac{dv}{dx}$

(i) $\frac{dx}{dt} = 4\cos 2\theta$ and

$\frac{dy}{dt} = -4\sin 2\theta$

Now

$\frac{d^2x}{dt^2} = -8\sin 2\theta$ and

$\frac{d^2y}{dt^2} = -4 \times 2\cos 2\theta = -8\cos 2\theta$

$\vec{a} = (-8\sin 2\theta\, \hat{j} - 8\cos 2\theta\, \hat{i})$

or

$|\vec{a}| = 8\ m/s^2$

(ii) $a_t = \frac{d}{dt}|\vec{v}|$

$\vec{v} = 4\cos 2\theta\, \hat{j} - 4\sin 2\theta\, \hat{i}$

$|\vec{v}| = 4\ m/s$

$\frac{d|\vec{v}|}{dt} = 0$ so $a_t = 0$

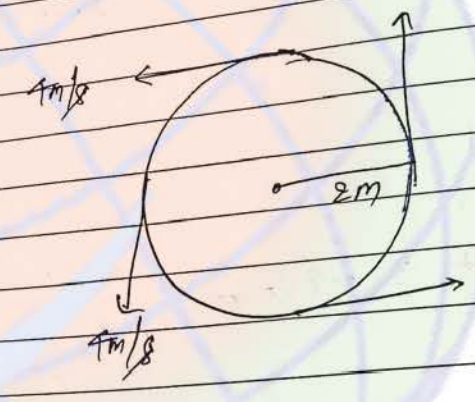
iii) $a_c = \sqrt{a^2 - a_t^2}$

$\therefore a_t = 0$

$a_c = a = 8\ m/s^2$

(iv) $x^2 + y^2 = 8^2$

4m/s $\therefore (|\vec{v}| = 4m/s)$



$a_c = \frac{v^2}{r} = \frac{4^2}{2} = 8\ m/s^2$

Note

(i) Uniform circular motion:-

circular motion with uniform speed

(ii) Non-uniform circular motion

circular motion with non-uniform speed

uniform motion

No

motion.

uniformly

No

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(iii) Uniform motion or Uniform velocity

No change in speed and no change in direction of motion.

Ra

Acc^n in this case

Acc^n in this case = 0

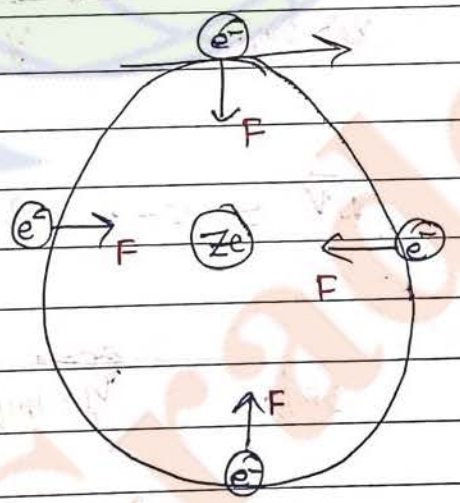
(iv) Uniformly Accelerated motion

No change in mag. of acc^n and No change in direction of acc^n

Note

$$\vec{a} = \frac{\vec{F}}{m}$$

Note eq^n of motion is valid only in case of uniformly accelerated motion



$$F = \frac{K(Ze)(e)}{r^2}$$

$\vec{a} = \frac{d\vec{v}}{dt}$

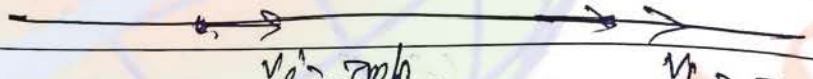
Note $d\vec{v}$ → small change in velocity

$\vec{v}$ → velocity

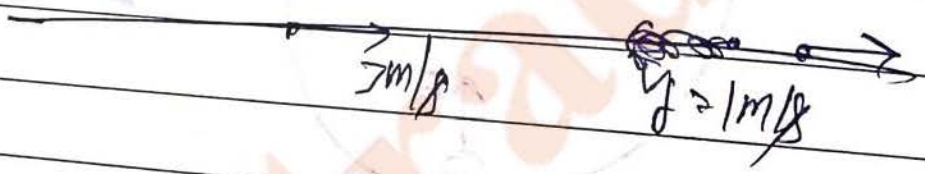
$\Delta\vec{v}$ → change in velocity in considerable interval

$d\vec{v}$ → " " " Instant

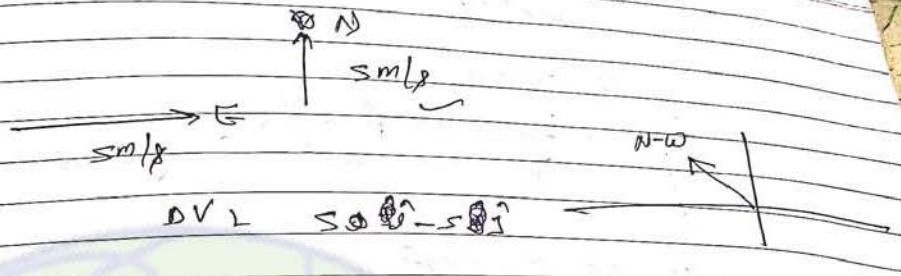
Note



$\Delta\vec{v} = 5 - 3 = 2m/s$



$\Delta v = -1m$

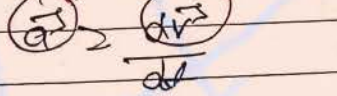


$\Delta \vec{V} = 5(\hat{i}) - 5(\hat{j})$

$\Rightarrow |\Delta \vec{V}| = 5\sqrt{2}$ m/s

Note

दोनों का direction same होना जरूरी

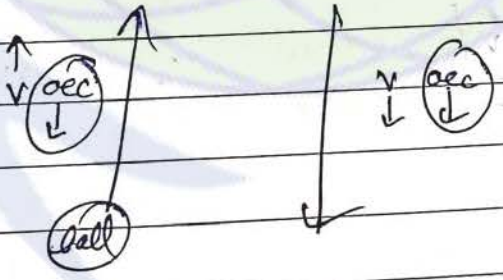


$\vec{a} = \frac{d\vec{v}}{dt}$

dir of $d\vec{v}$ ⊗ dirn of $\vec{v}$

↓ may or may not same

(1)



Acc" + ve change

V ↓ change

(Acc" always in downward direction)

⑦ Zero acc^n :-

if $\vec{acc} = 0$

(i) uniform motion
(ii) No motion (body is at rest)

so we say that if body is either in rest or in uniform motion then acc^n is Zero

Q) A particle moves in a circle with uniform speed which of the following parameter is Zero
(a) acc^n
(b) centripetal acc^n
(c) tangential acc^n
(d) None of these.

sol^n (c) tangential acc^n

$$a_t = \frac{d|\vec{v}|}{dt} \quad \because |\vec{v}| = v$$

$$= 0$$

Q) A particle moves in a circle with non-uniform speed, which of the following parameter is Zero
(a) acc^n
(b) centripetal acc
(c) tangential acc^n
(d) None of these

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(D) none of the

Q) For a particle moving along x-axis $v = \alpha\sqrt{x}$

α is constant $(\alpha > 0)$

$x \rightarrow$ position $(x \geq 0)$

$v \rightarrow$ velocity

Initially particle is at $x = 0$

$(t = 0)$

Concept:

Note

$t = 0$ means where we start observation.
or
those time when

concept that in all cases not take at all zero only term by without understanding

Kota — Bundi — Jaipur

$x = 0$ $x = 45$ km $x = 245$ km

Thus → $v = 0$ → $v = 40$ km/s

A

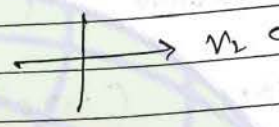
In this case at $t = 0$ $v = 0$

B

in this case at $t = 0$ $v = 40$ km/s $x = 45$

find
(i) vel. at any time 't'
(ii) dis. covered travelled in time t
(iii) acc.

Q/a



$v = \frac{dx}{dt}$

$\propto \frac{1}{\sqrt{x}}$

$\propto \frac{\alpha}{2\sqrt{x}}$

(i) $\frac{dx}{dt} = \alpha\sqrt{x}$

$$\int_{x=0}^{x} \frac{dx}{\sqrt{x}} = \int_{t=0}^{t} \alpha\, dt$$

$t = 0$

$2\sqrt{x} = \alpha t$

$x = \frac{\alpha^2 t^2}{4}$

dist. in time $= \frac{\alpha^2 t^2}{4}$

(iii) $\frac{dx}{dt} = \frac{\alpha^2 t}{2}$

$v = \frac{\alpha^2 t}{2}$

vector quantity = Direction req. each time

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(iii) $a = \frac{dv}{dt}$

$a = \frac{d^2}{2}$

Note

$$a = \frac{dv}{dt} = v \cdot \frac{dv}{dx}$$

$$a_x = \frac{dv_x}{dt}$$

$$a_x = \frac{dx}{dt} \cdot \left(\frac{dv_x}{dx}\right)$$

$$a_x = v_x \cdot \frac{dv_x}{dx}$$

In the given question

$$v = \alpha\sqrt{x}$$

$$\frac{dv}{dx} = \frac{\alpha}{2\sqrt{x}}$$

$$a = \frac{v\,dv}{dx} = \alpha\sqrt{x} \cdot \frac{\alpha}{2\sqrt{x}}$$

$$= \frac{\alpha^2}{2}$$

ex. For a particle moving along x-axis

$t = \alpha x^2 + \nabla x$

$\alpha, \nabla \rightarrow$ constants $\begin{cases} \alpha > 0 \\ \nabla > 0 \end{cases}$

$x \rightarrow$ Position

$t \rightarrow$ time

Retardation is ——

Note:

$a = \frac{dv}{dt} = v \cdot \frac{dv}{dx}$

Sol/n

$\frac{dt}{dx} = 2\alpha x + \nabla$

$\frac{dx}{dt} = \frac{1}{(2\alpha x + \nabla)}$

$v = \frac{1}{(2\alpha x + \nabla)}$

$\frac{dv}{dx} = \frac{-1}{(2\alpha x + \nabla)^2} \cdot 2\alpha$

$a = v \cdot \frac{dv}{dx}$

$a = \frac{-2\alpha}{(2\alpha x + \nabla)^3}$

$$a = -2\alpha(2\alpha x + \square)^{-3}$$

$$a = -2\alpha v^3$$

Retardation $= 2\alpha v^3$

Note: If acc^n is opp. to the vel. then its magnitude is called retardation.

Alt

$$\frac{dx}{dt} = \frac{1}{(2\alpha x + \square)}$$

$$\frac{d^2x}{dt^2} = \frac{-1}{(2\alpha x + \square)^2} \cdot 2\alpha \cdot \frac{dx}{dt}$$

$$= \frac{-2\alpha}{(2\alpha x + \square)^2}\left(\frac{1}{2\alpha + \square}\right)$$

Eg.) For a particle moving along x-axis —

$$f = kt$$

where

$f \rightarrow$ acc^n, $k \rightarrow$ constant, $t \rightarrow$ time

initially it at rest and $x = x_0$

Find :-

(i) vel. after time 't'

(ii) Position of the particle after time t —

(iii) Displacement of the particle in time 't'.

Sol^n (i) $\frac{dv}{dt} = kt$

$$\int_{v=0}^{v} dv = \int_{0}^{t} kt\,dt$$

$$v = \frac{kt^2}{2}$$

(ii) $\frac{dx}{dt} = \frac{kt^2}{2}$

$$\int_{x_0}^{x} dx = \int_{0}^{t} \frac{kt^2}{2}\,dt$$

$$x - x_0 = \frac{kt^3}{6}$$

$$x = x_0 + \frac{kt^3}{6}$$

(iii) Disp in time t $= x - x_0 = \frac{kt^3}{6}$

8/4/2015

Graphs:-

Ra Ra

$x = f(t)$

Integrate ∫ dt

$v = f(\theta)$

$\therefore$ ↓ d/dt

$a = f(\theta)$

① straight line:-

$y = mx + c$

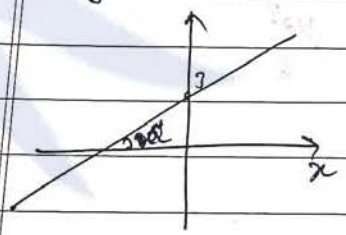
where, m = slope $= \tan\theta$

c = intercept on y-axis.

Ra

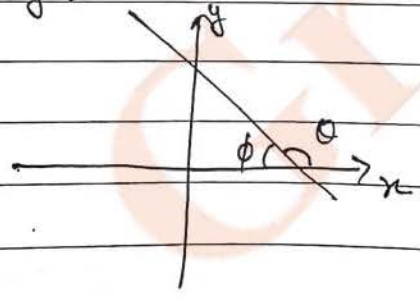
$m > 0$	$\Rightarrow$	$0 < \theta < 90$
$m < 0$	$\Rightarrow$	$90 < \theta < 180$
$m = 0$	$\Rightarrow$	$\theta = 0$
$m = \infty$	$\Rightarrow$	$\theta = 90$

(I) $y = 2x + 3$



$\tan\theta = 2$

(II) $y = -2x + 3$



$\tan\theta = -2$
$\tan\theta = 2$ } Ra

line का eq" → $y = mx + c$ or $ax + by + c = 0$ form में

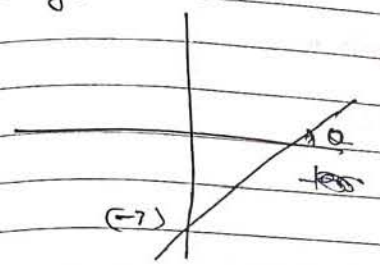
(III) $y = 2x - 1$

(IV) $y = -2$

(V) y

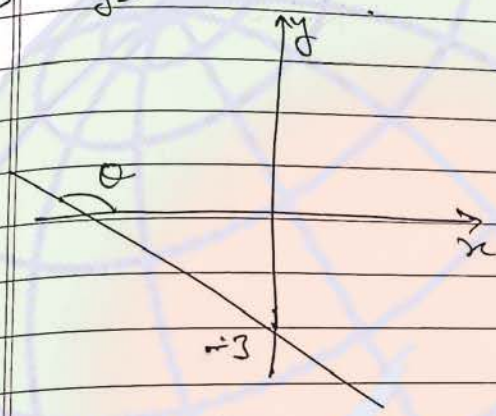
(VI)

(iii) $y = 2x - 3$



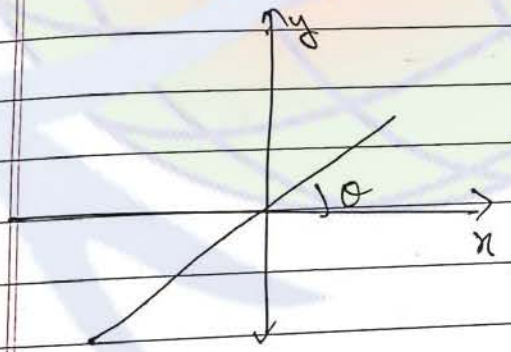
$\tan\theta = -2$

(iv) $y = -2x - 3$



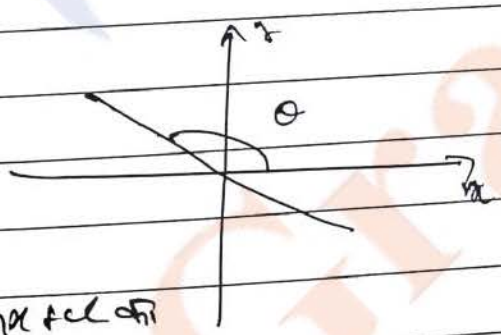
$\tan\theta = -2$

(v) $y = 2x$



$\tan\theta = 2$

(vi)



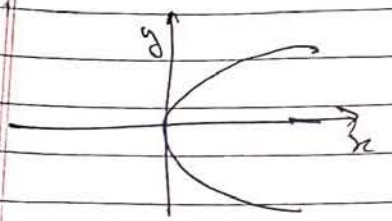
$\tan\theta = -2$

→ दी $y = mx + c$ का ग्राफ खींचकर slope "m" ज्ञात करो

Parabola :-

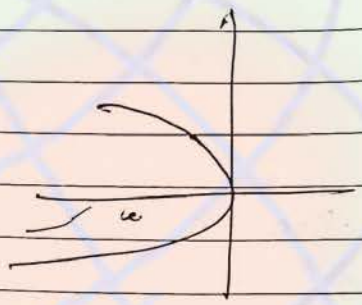
(i) $y^2 = kx$

$k > 0$



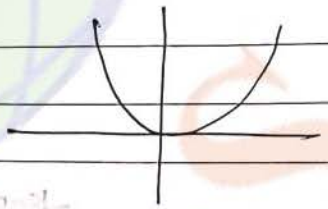
(ii) $y^2 = kx$

$k < 0$



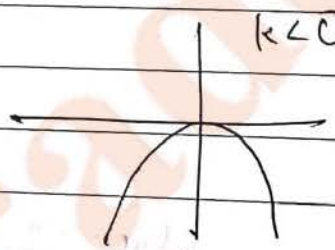
(iii) $x^2 = ky$

$k > 0$

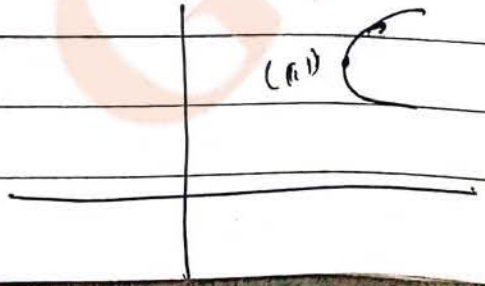


(iv) $x^2 = ky$

$k < 0$

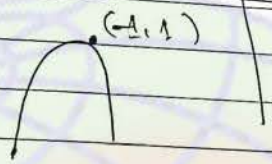


(vi) $(y-1)^2 = 2(x-1)$



Ans

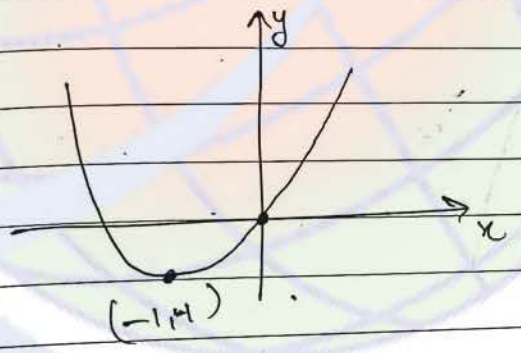
(7) $(x+1)^2 = -2(y+1)$



(8) $y = 2x + x^2$

$y + 1 = x^2 + 2x + 1$

$(y+1) = (x+1)^2$ Ans

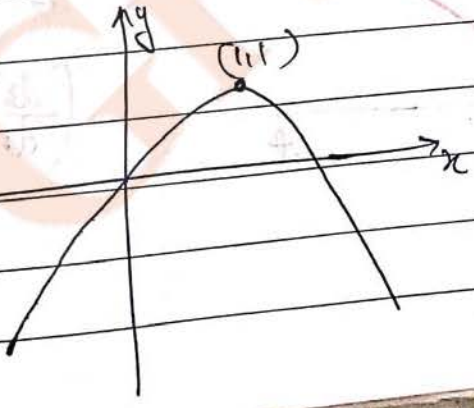


(9) $y = 2x - x^2$

$-y = x^2 - 2x$

$-y + 1 = x^2 - 2x + 1$

$-(y-1) = (x-1)^2$

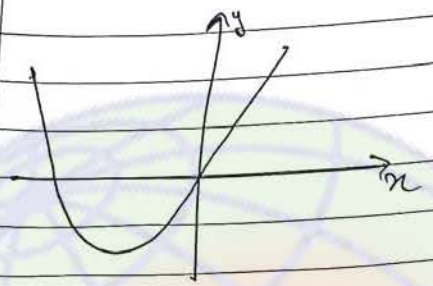


Note

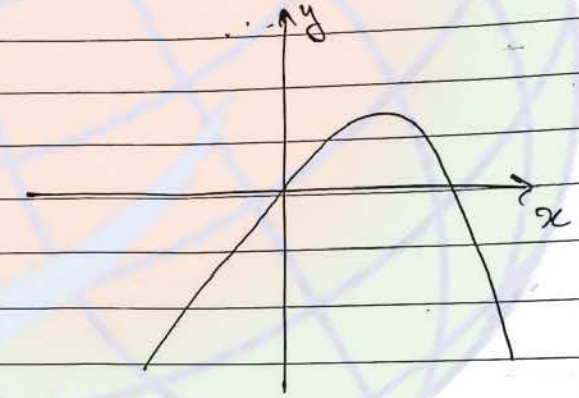
Imp

(i) $y = ax^2 + bx^2$

$a > 0,\ b > 0$



(ii) $y = ax - bx^2$, $a > 0$, $b > 0$



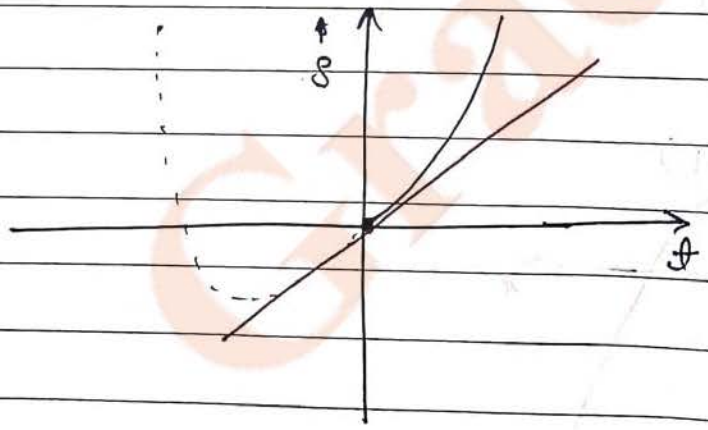
Important

(ii)

(iii) $s = ut + \frac{1}{2}at^2$

$s \rightarrow$ displacement

$t \rightarrow$ time

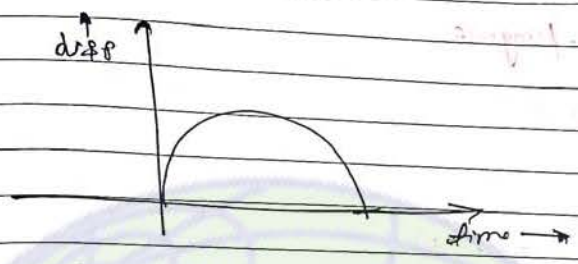


Note

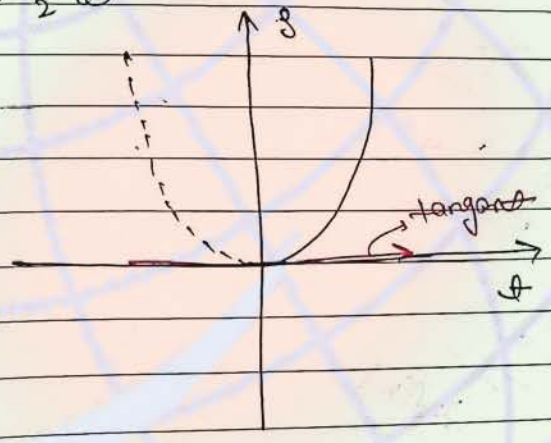
→ General eq" of this is $y = ax + bx^2$

$\frac{ds}{dt} > 0$ or $\left(\frac{ds}{dt}\right)_{t=0} > 0$

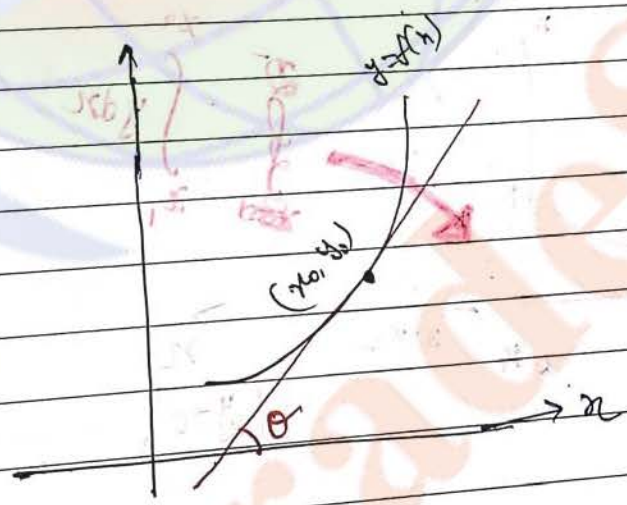
(ii) $s = ut - \frac{1}{2}at^2$ → General eq^n of this



(iii) $s = \frac{1}{2}at^2$



$\left(\frac{ds}{dt}\right)_{t=0} = 0$

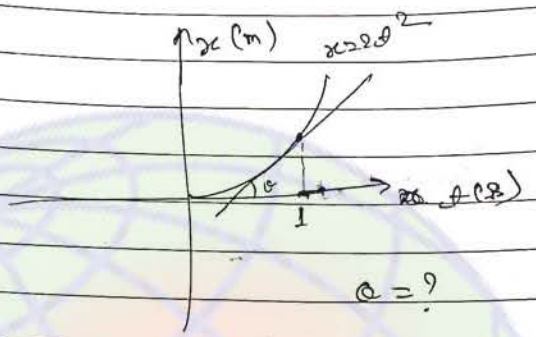


$\left(\frac{dy}{dx}\right)_{x=x_0} = \tan\theta$

Note

$\frac{dy}{dx}$ = slope of tangent

Eg:)



$\frac{dx}{d\theta} = 4\theta$

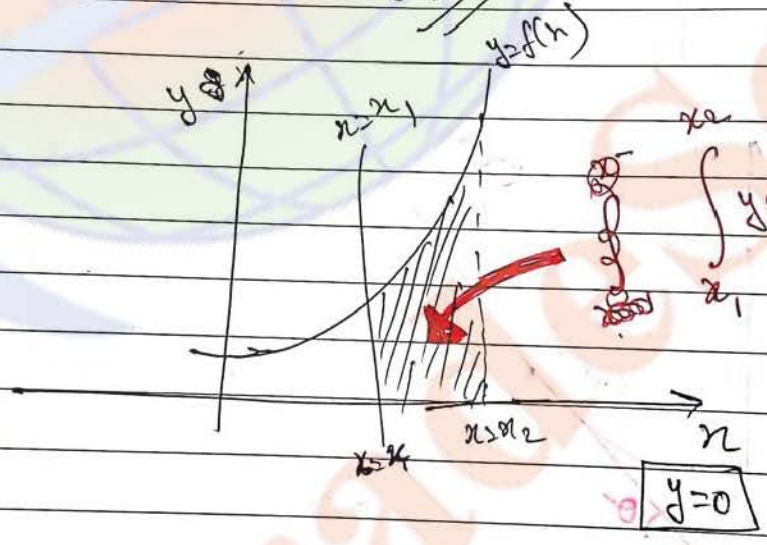
$\theta = ?$

$\frac{dx}{d\theta} = 4\theta$ //

$\left(\frac{dx}{d\theta}\right)_{\theta=1} = 4$

$\tan\theta = 4$

$\theta = \tan^{-1}(4)$ //

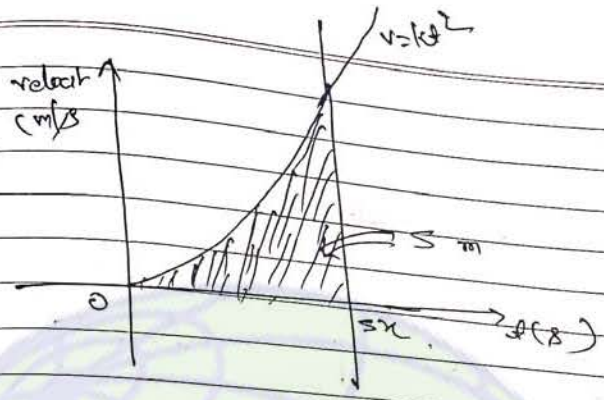


$\int_{x_1}^{x_2} y\,dx$ = Area enclosed by the curve $x=x_1$, $x=x_2$, $y=0$ and $y>0$

or

Area under the curve

eg.)



k=?

Sol:

$$\int_0^5 v\,dt = S$$

$v = f'(t)$

$$\int_0^5 kt^2\,dt = S$$

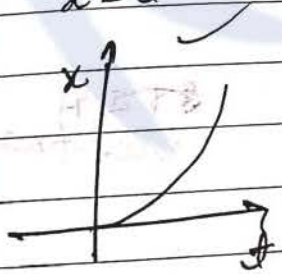
$$k\left[\frac{t^3}{3}\right]_0^5 = S$$

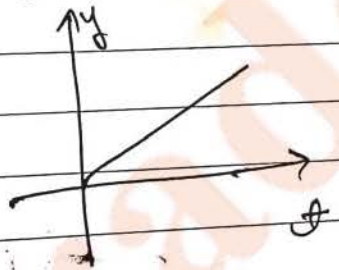
$$k\left[\frac{5^3}{3} - 0\right] = S$$

$$k = \frac{3}{25}$$

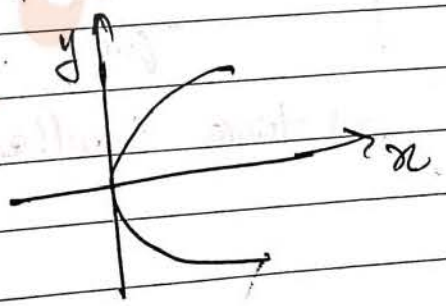
Note:

$x = t^2$; $y = t$



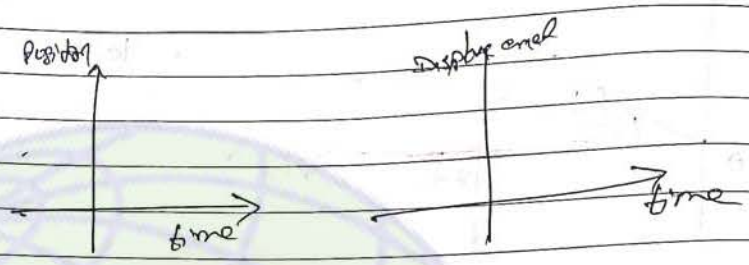


Now $x = y^2$



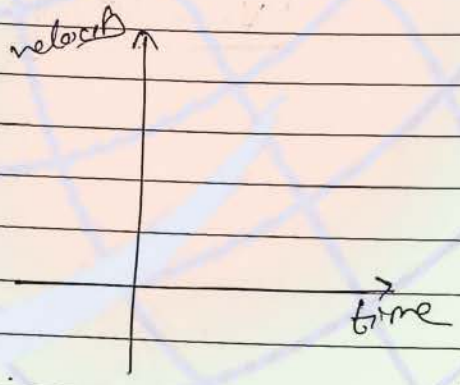
Note → Graph

(b)



Velocity = slope of tangent

(c)



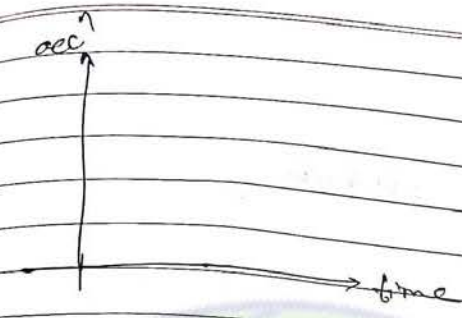
(a) acceleration → slope of tangent

(b) displacement = area under the curve

(d)

speed-time

distance travelled → area under the curve



change in velocity → Area under the curve.

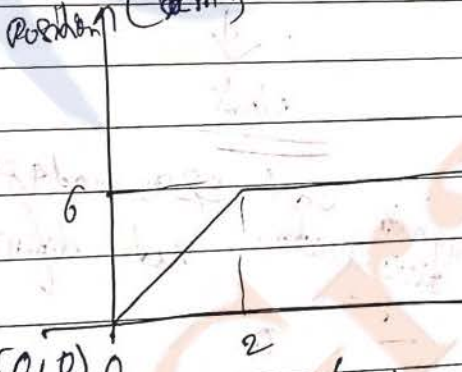
Note:

$$\int_{v_i}^{v_f} d\vec{v} = \int_{t_i}^{t_f} \vec{a}\, dt$$

$$\vec{v}_f - \vec{v}_i = \int_{t_i}^{t_f} \vec{a}\, dt$$

$$\boxed{\Delta \vec{v} = \int_{t_i}^{t_f} \vec{a}\, dt}$$

eg)



for a particle moving along x-axis.

(a) velocity at $t = 1$ sec, $t = 3$ sec, and $t = 5$ sec

* at $t = 1$ sec

(i) $(0,0)$, $(2,6)$
x_1, y_1 x_2, y_2

slope, $\frac{6}{2} = 3$ velocity

* at $t = 7$ sec

slope $= 0$

* at $t = 5$ sec

$(4,6)$, $(6,0)$
x_1, y_1 (x_2, y_2)

$\frac{-6}{2} = -3$ = vel

Note

(time, position)

$v_{t=1} = \frac{6-0}{2-0} = 3$ m/s

Note (0 से 2 sec के बीच)

Note

(2 से 4 sec के बीच)

$v_{t=3} = \frac{6-6}{4-2} = 0$

$v_{t=5} = \frac{0-6}{6-4} = -3$ m/s

Note

4 से 6 sec के बीच

Note

at 2 sec and 4 sec vel. is not defined.

Note

x → location
v → velocity
a → Acc^n →

जाने motion

+∞ ← 0 (origin से distance)

+ ← 0

+ दिशा को Relate करते हैं

+ (F लगाने) (F)

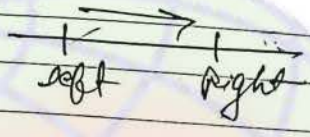
classmate
Date
Page

disp

+ location change left to right

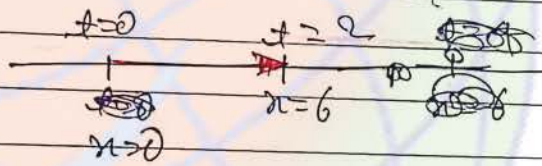
− location change from right to left

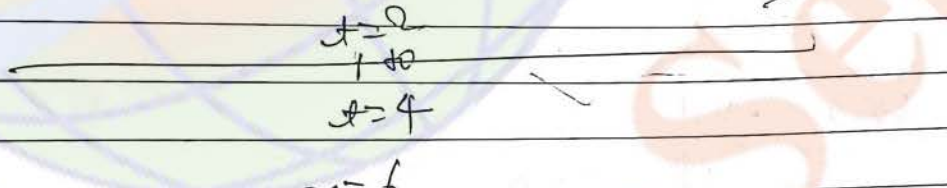
0

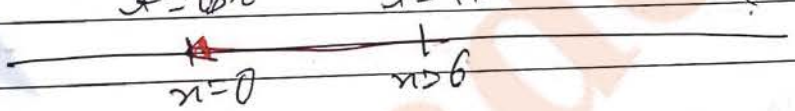


Note

i)







Note

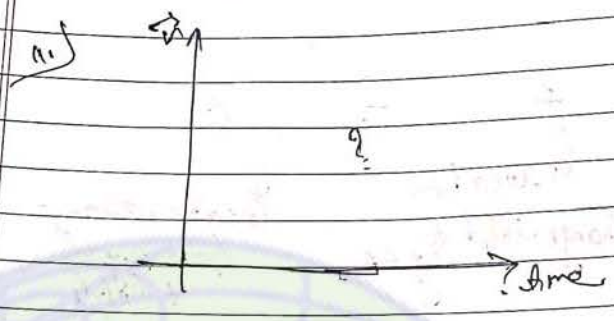
ii) in 6s

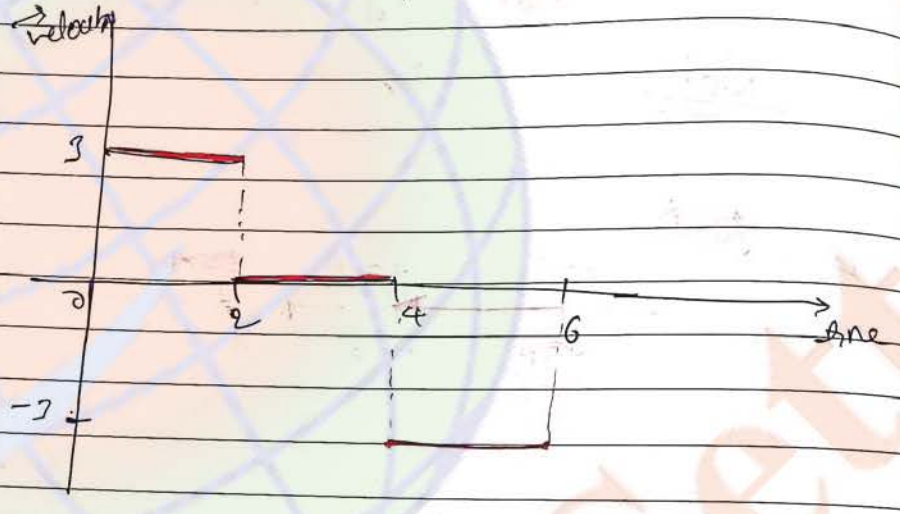
$$V_{avg} = \frac{12\ m}{6\ s} = 2\ m/s$$

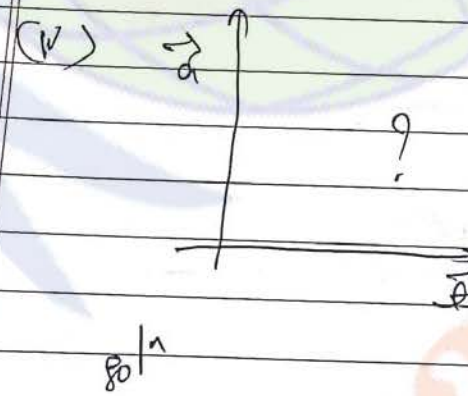
→ total distance

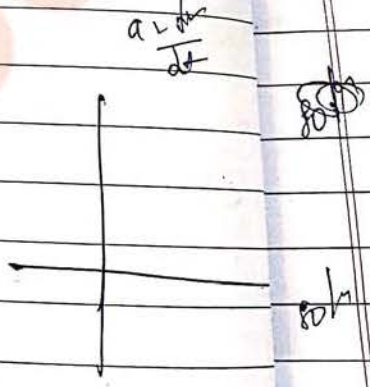
$$\text{ans } |\vec{V}_{avg}| = \frac{\vec{0}}{6} = 0$$

→ total displacement

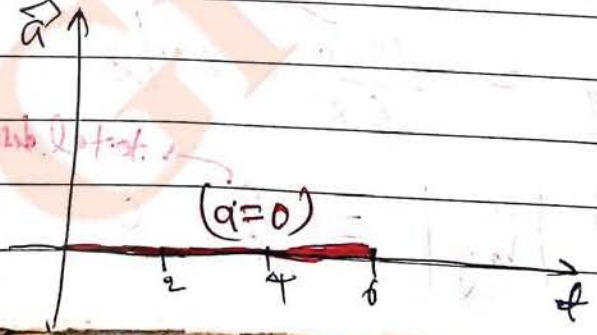








vel vs time graph का slope देखेंगे

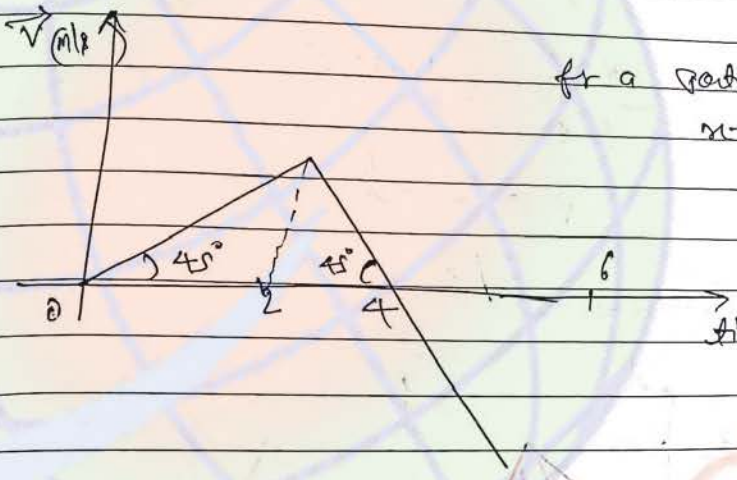


Here

$a = 0$

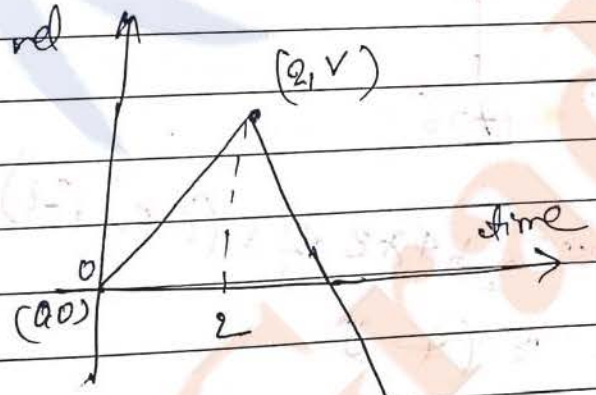
either at rest or in uniform motion.

Q2.

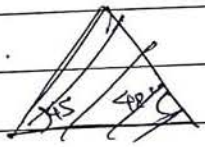


for a particle moving along x-axis.

(i) vel. after 2 sec. ?



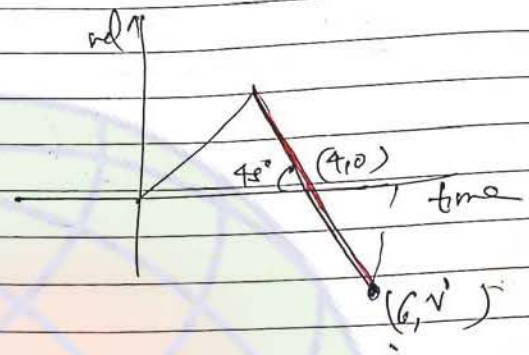
(time, velocity)



$$\frac{V-0}{2-0} = \tan 45^\circ$$

$$\Rightarrow V = +2 \text{ m/s}$$

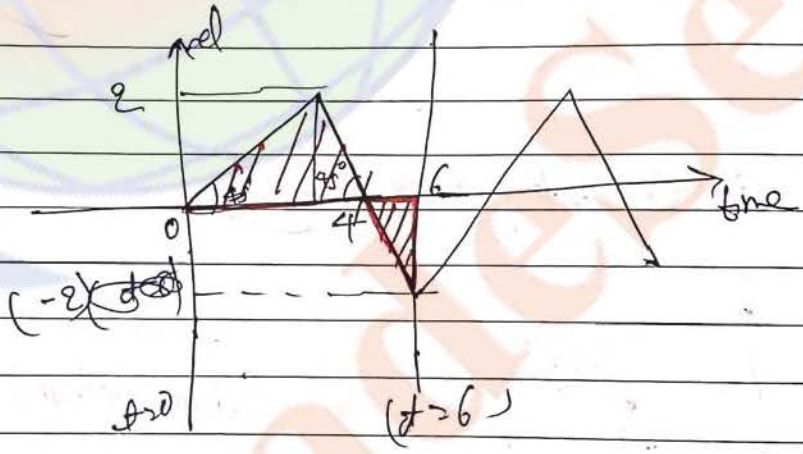
iv) vel at t = 6sec is



$$\frac{v'-0}{6-4} = \tan 135°$$

$$v' = -2 \text{ m/s}$$

(iii) Displacement in 6sec

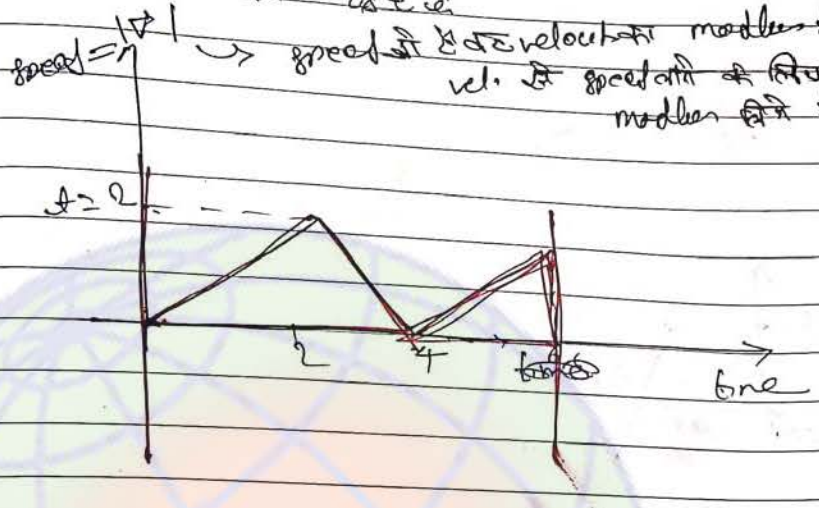


$$\text{Displacement} = \frac{1}{2} \times 4 \times 2 + \frac{1}{2} \times (6-4)(-2)$$

$$= 4 - 2$$

$$= 2 \text{ m}$$

iv) Distance travelled in 6 sec



speed में हम velocity का modulus लेते हैं।
vel. से speed पाने के लिए vel. का modulus लेते हैं।

Distance travelled = 4 + 2 = 6 m

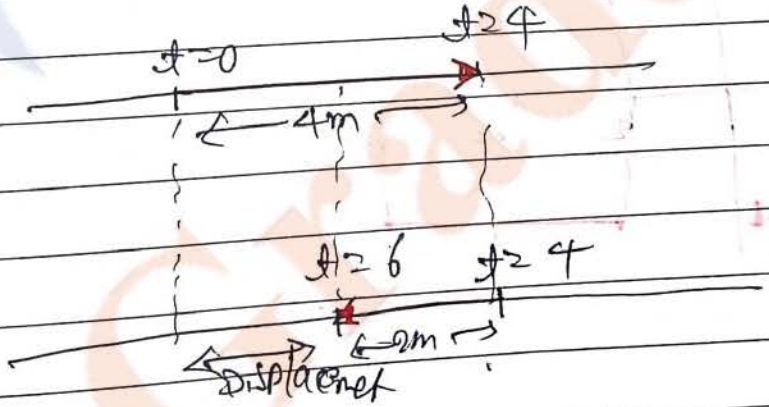
(v) $|\vec{v}_{avg}|$ and v_{avg} in 6 sec ज्ञात करो —

$$|\vec{v}_{avg}| = \frac{2}{6} = \frac{1}{3}\ m/s$$

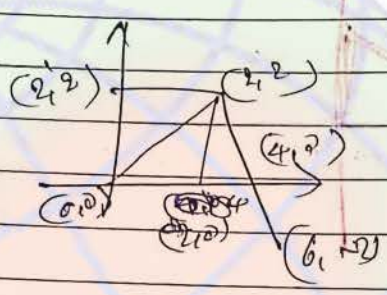
and

$$v_{avg} = \frac{6}{6} = 1\ m/s$$

vi)



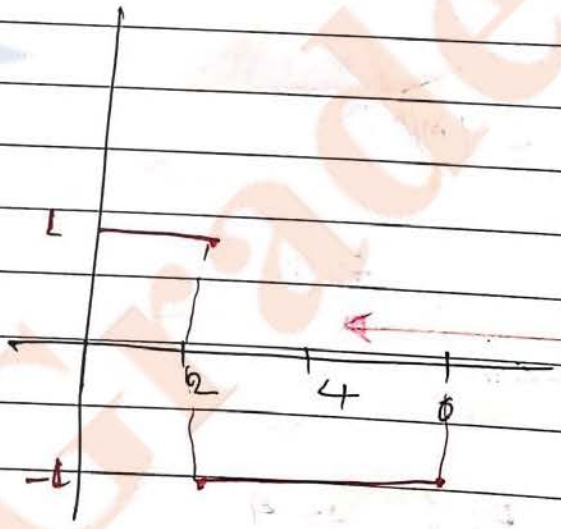
(11)

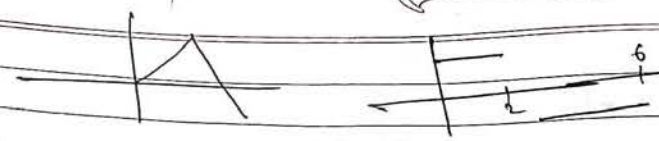


$m = \frac{1}{1}$

$= \frac{-2}{2} = -1$

$= \frac{-4}{4} = -1$





Note

time interval	velocity	acc^{n}
$0 < t < 2$	> 0 (greater than zero)	> 0 (greater than zero)
$2 < t < 4$	> 0 (greater than zero)	< 0 (less than zero)
$4 < t < 6$	< 0 (less than zero)	< 0 (less than zero)

→ In this case velocity and acc^{n} are in opp. sign so, deaccelerated or retarding motion ~~is~~ take place.

Note

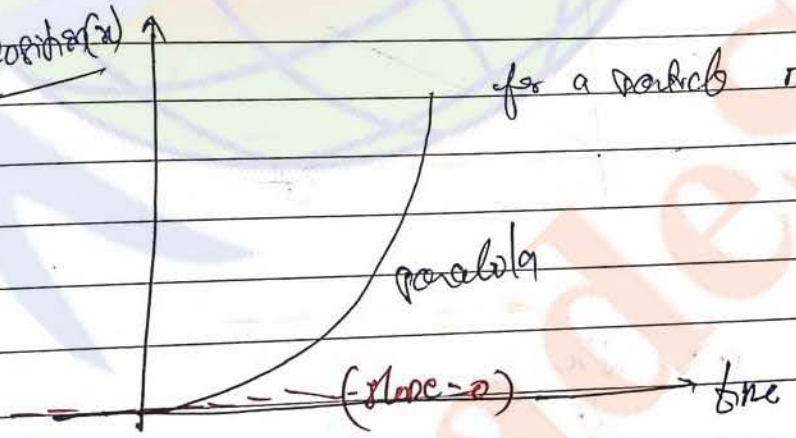
$a \downarrow$

$a > 0$ } ⇒ $\Delta a < 0$

$a < 0$ }

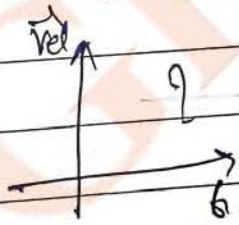
⇒ we say that retardation ~~is~~ sufficient condition vel. ~~is~~ greater than zero होना है।

eg: Position(x) for a particle moving along x-axis

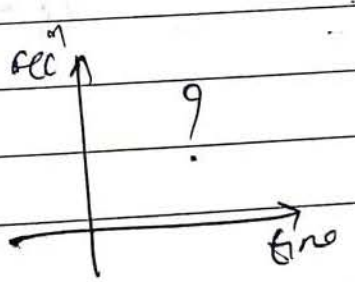


Draw its:

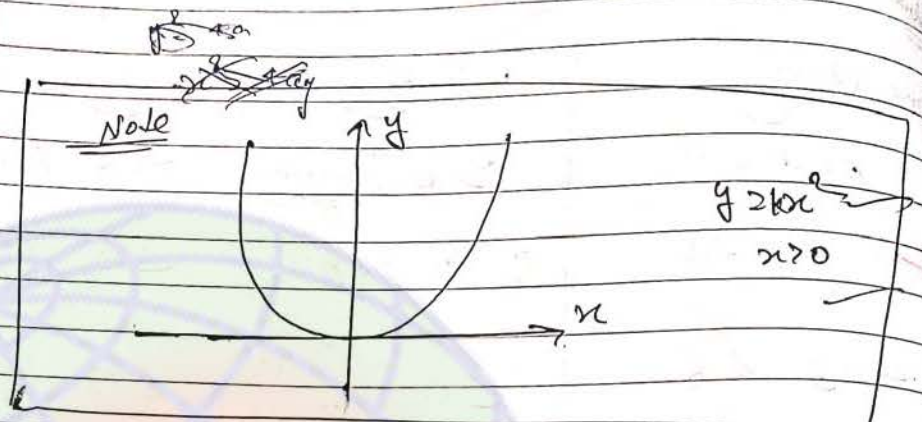
(I) Vel



(II) acc^{n}



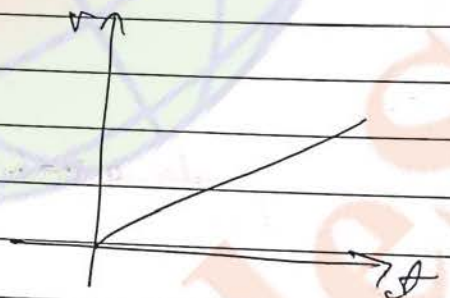
Ex:



$x = kt^2$

$\frac{dx}{dt} = 2kt$

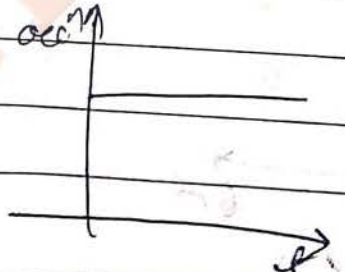
$v = 2kt$ → linear relation



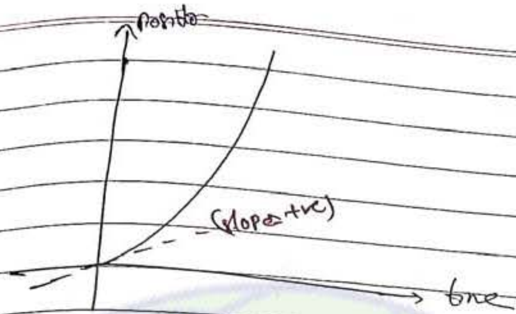
(ii)

$a = \frac{d^2x}{dt^2}$

$a = (2k)$ → constant

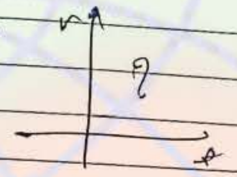


Q.)

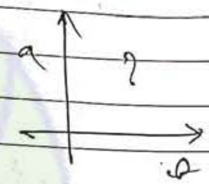


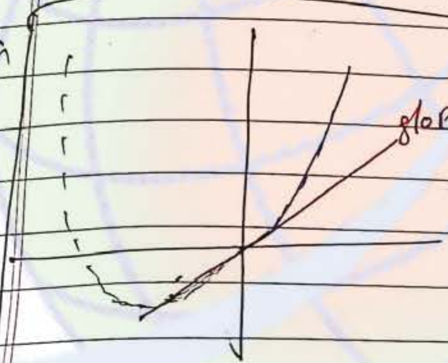
find

(i)



(ii)





$y = k_1 x + k_2 x^2$

~~$x^2 = k t^2$~~

~~$\frac{dx}{dt} = 2kt$~~

~~$v = 2kt$~~

Ans

$x = k_1 t + k_2 t^2$

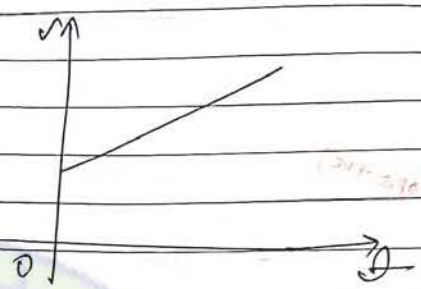
$k_1 > 0$ and $k_2 > 0$

(i) $v = \frac{dx}{dt}$

$v = 2k_2 t + k_1$ ⟷ linearly dependent

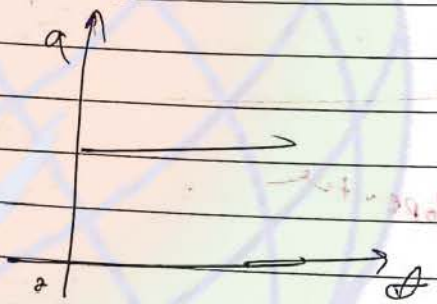
↳ intercept (+ve)

Represent

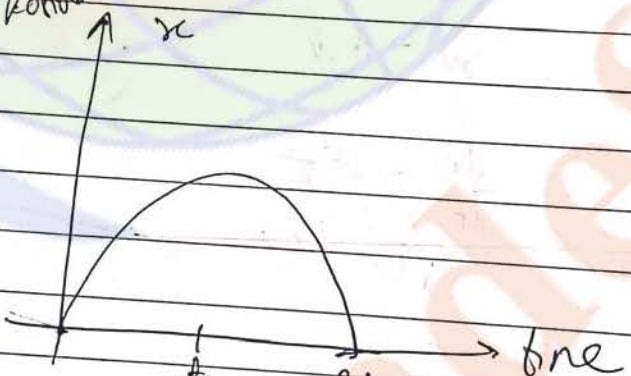


(ii) $a = \frac{d^2x}{dt^2}$

$a = 2k$

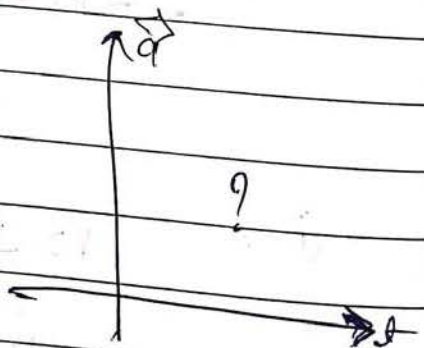


Eg.



(i)

(ii)

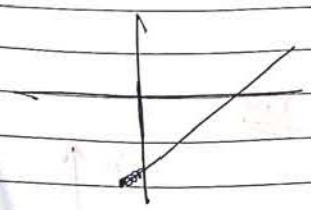


Q9

$x = ut - \frac{1}{2}at^2$

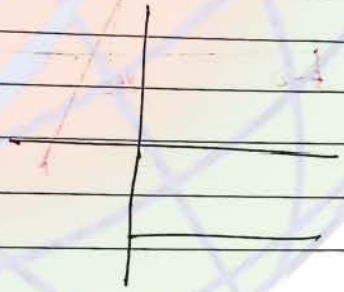
$\frac{dx}{dt} = u - \frac{1}{2} \times 2at$

$\frac{d^2x}{dt} = 0 - at$



(ii)

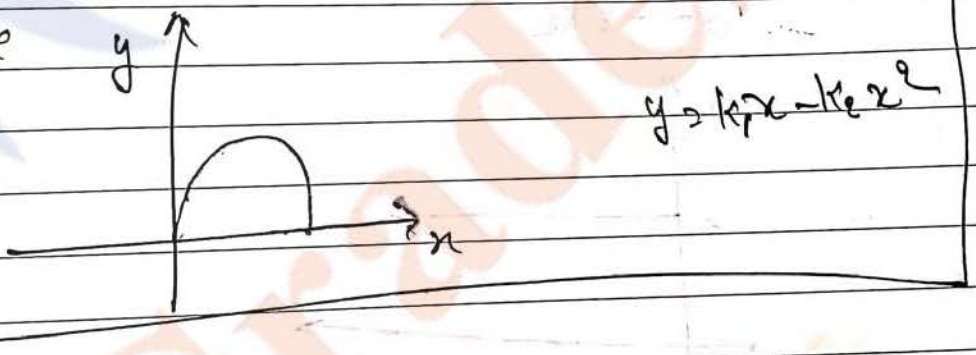
$\frac{d^2x}{dt^2} = -a$



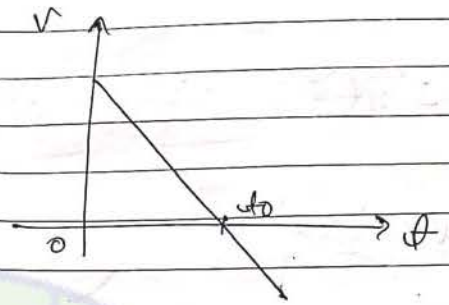
Q9

Yeah

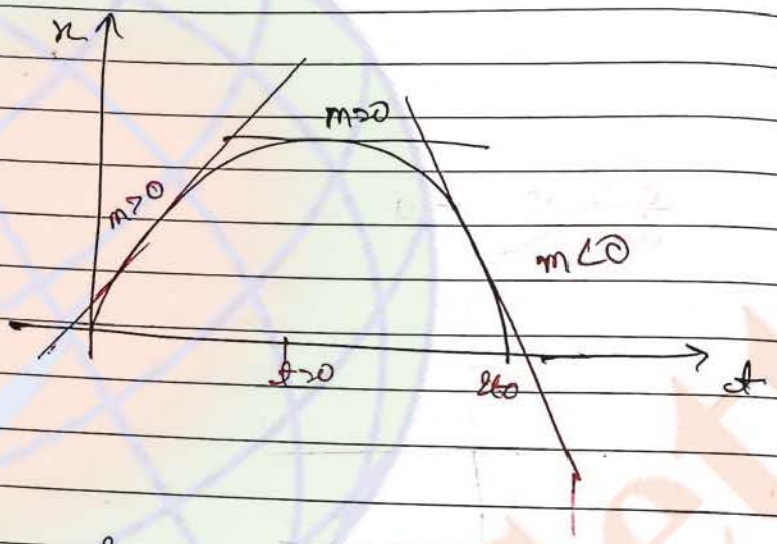
Nah



$v = \frac{dx}{dt}$ .Q4 $v = -2k_2t + k_1$

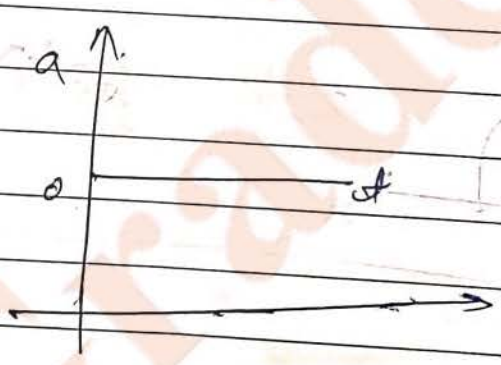


Note



(ii) $a = \frac{d^2x}{dt^2}$

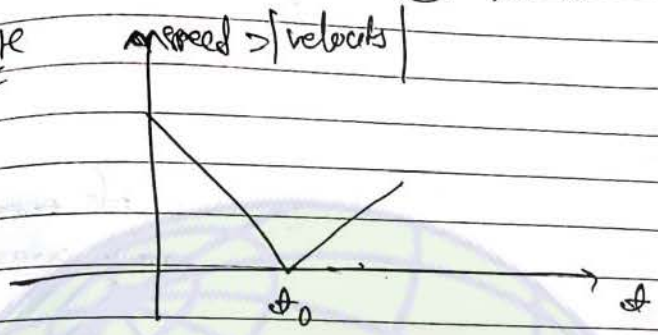
$a = -\omega^2 x$



(i) speed decrease

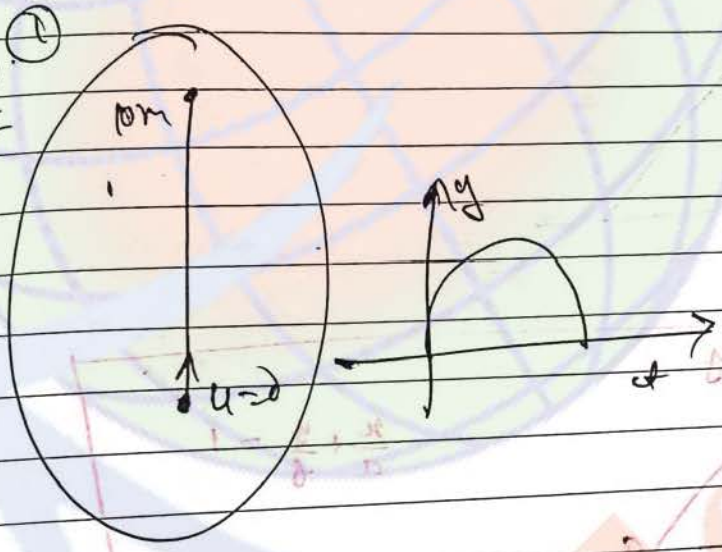
(ii) vel. or acc. same rahe

Note

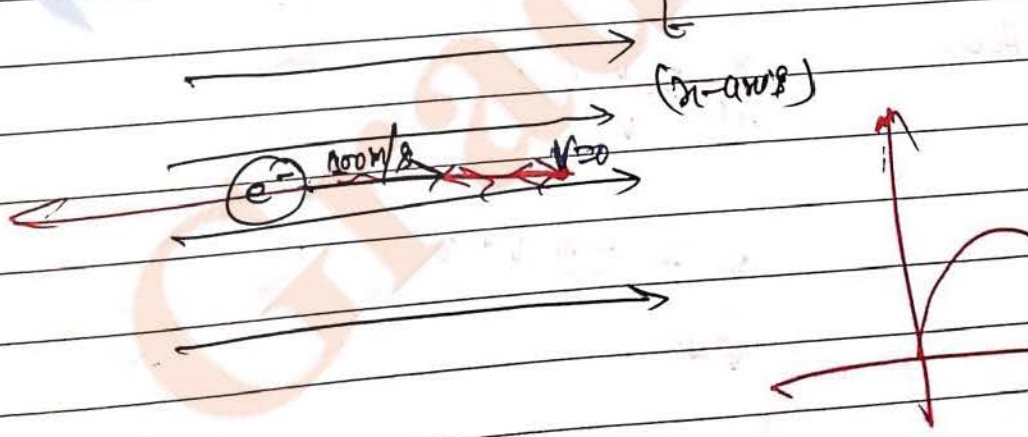


is se say that ऐसे जहाँ retardation motion हो

Note (i)



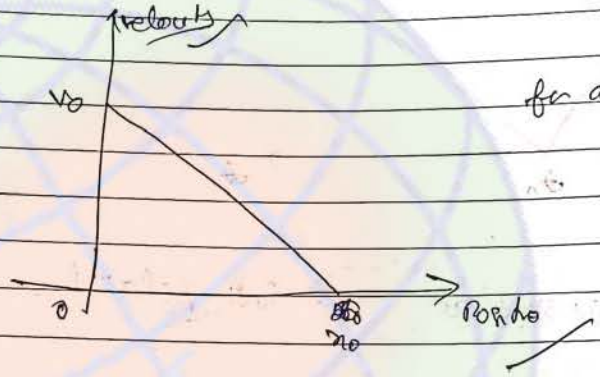
(ii)



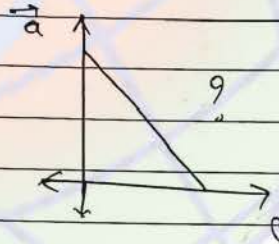
↳ L-1 → 1 to 7
↳ L-2 → 7 to 10, 15, 2 [illegible]
↳ L-3 → 3, 8, 15,
↳ L-4 → 9, 10, 11, 15

$x = a + bt + ct^2 + dt^3$

Eg.

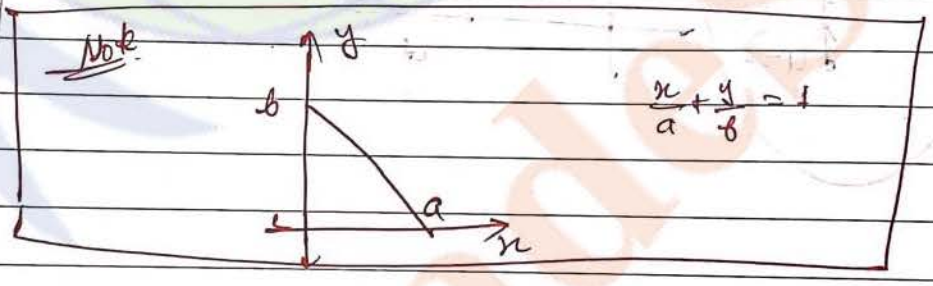


for a particle moving along x-axis :



Sol^n

Note

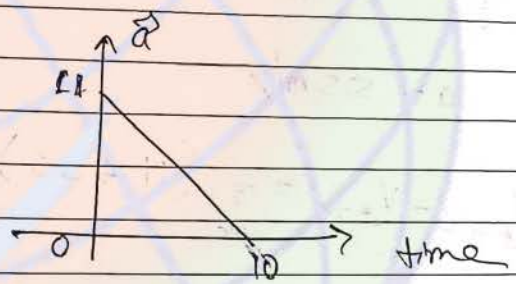


Here,

$$\frac{x}{x_0} + \frac{v}{v_0} = 1$$

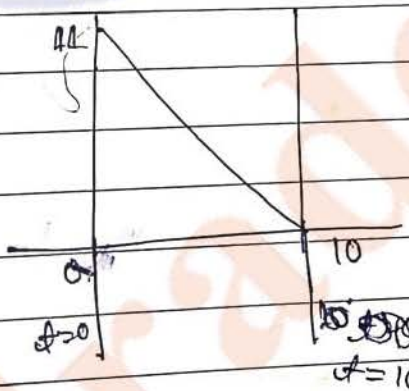
$$v = \frac{v_0}{x_0} x + v_0$$

$\frac{dv}{dn} \rightarrow \frac{F \cdot V_e}{x^0}$



Ans at $t = 5$ sec is

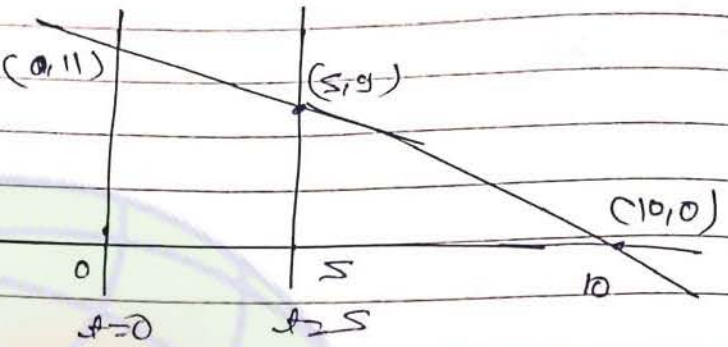
(i) find velocity of the particle at $t = 10$ sec its initial velocity zero



$V_{t=10} - V_{t=0} = \frac{1}{2}(10)(11)$

$V_{t=10} = 55$ m/s

ii) Find the above question velocity at $t = 5$ sec ?



$$\frac{a-11}{5-0} = \frac{0-9}{10-5}$$

$$a = 5.5 \text{ m/s}$$

$$V_{t=5} - V_{t=0} = \frac{1}{2}(11 + 5.6) \times 5$$

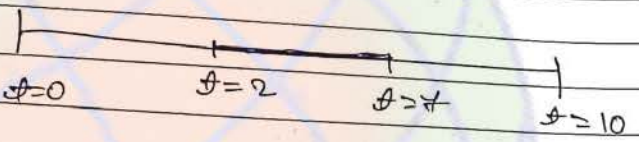
$$V_{t=5} = 41.25 \text{ m/s}$$

Uniformly accelerated motion along a straight line :-

Eg

$\vec{a}$ = Constant

u = initial velocity
v = final velocity
a = acc^n
s = displacement
t = time interval.



① First eq^n of motion

$$\frac{d\vec{v}}{dt} = \vec{a}$$

$$\int_{\vec{u}}^{\vec{v}} d\vec{v} = \int_0^t \vec{a}\, dt$$

$$\vec{v} - \vec{u} = \vec{a}\, t$$

$$\vec{a} = \frac{\vec{v} - \vec{u}}{t}$$

or,

$$\boxed{\vec{v} = \vec{u} + \vec{a}t}$$

② 2nd eq^n of motion :-

$$\frac{d\vec{s}}{dt} = \vec{v}$$

$$\int d\vec{s} = \int \vec{v}\, dt$$

$$\int_0^{\vec{s}} d\vec{s} = \int_0^t (\vec{u} + \vec{a}t)\, dt$$

$$s = \vec{u}t + \frac{\vec{a}t^2}{2}$$

or,

$$\boxed{s = \vec{u}t + \frac{1}{2}\vec{a}t^2}$$

Here,
s = change in position of motion taken place along x-axis

Eg →

$$\vec{s} = \Delta x$$
$$\vec{s} = x - x_0$$

(iii) Third eqn of motion :-

$$v_x \frac{dv_x}{dx} = a_x$$

$$\int_u^v v_x dv_x = \int_{x_0}^{x} a_x dx$$

$$\frac{v^2 - u^2}{2} = a_x (x - x_0)$$

$$v^2 - u^2 = 2 a_x \Delta x$$

or $$\boxed{v^2 = u^2 + 2 a_x s}$$

(iv) Average velocity ($\vec{v}_{avg}$)

$$\vec{v}_{avg} = \frac{\vec{s}}{t}$$

or

$$\vec{v}_{avg} = \frac{\vec{u}t + \frac{1}{2}\vec{a}t^2}{t}$$

$$\vec{v}_{avg} = \vec{u} + \frac{\vec{a}t}{2}$$

$$= \vec{u} + \frac{\vec{v} - \vec{u}}{2}$$

or $$\boxed{\vec{v}_{avg} = \frac{\vec{v} + \vec{u}}{2}}$$

This is applicable only when $\vec{a} = 0$

(v) Displacement

Speed

(vi)

(vii)

(viii)

Pg

(v) Displacement in nth second (S_n)

Disp. in 4th sec.

Disp in 4th sec.

$\vec{S_n}$ = displacement in n-sec − Disp. in (n-1) sec.

$$\vec{S_n} = \left[\vec{u}(n) + \frac{\vec{a}}{2}(n)^2\right] - \left[\vec{u}(n-1) + \frac{\vec{a}}{2}(n-1)^2\right]$$

$$\boxed{\vec{S_n} = \vec{u} + \frac{\vec{a}}{2}(2n-1)}$$

Special case:-

(A) If $u=0$ (Initially object is at rest)

(i) Velocity after time t ⇒

$$v = u + at$$

(ii) Distance travelled in time 't'

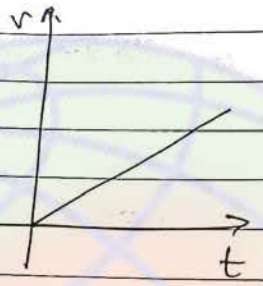
$$s = \frac{1}{2}at^2$$

(iii) velocity in travelling a distance 's'

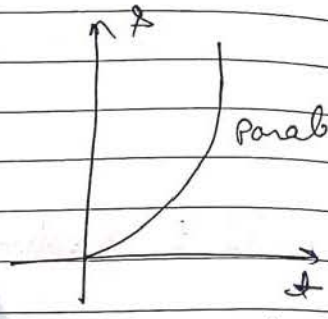
$$v^2 = 2as \quad \text{or} \quad v = \sqrt{2as}$$

(IV) Disp. in nth sec

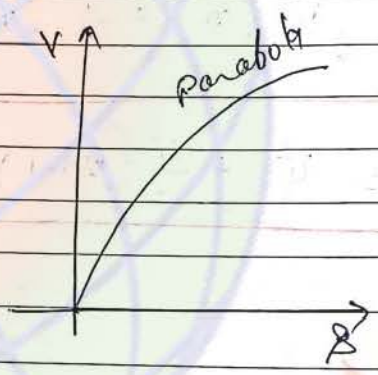
$$S_n = \frac{a}{2}(2n-1)$$



$v = at$



$s = \frac{1}{2}at^2$



$v^2 = 2as$

(B) If $v = 0$ (final velocity is Zero)

Let retarding $= a$

(i) Velocity after time "t"

$$v = u - at$$

(ii) Displacement in time "t"

$$s = ut - \frac{1}{2}at^2$$

(iii) velocity after displacement s is:

$$v^2 = u^2 - 2as$$

(iv) displacement in n^{th} sec

$$s_n = u - \frac{a}{2}(2n-1)$$

(v) stopping time

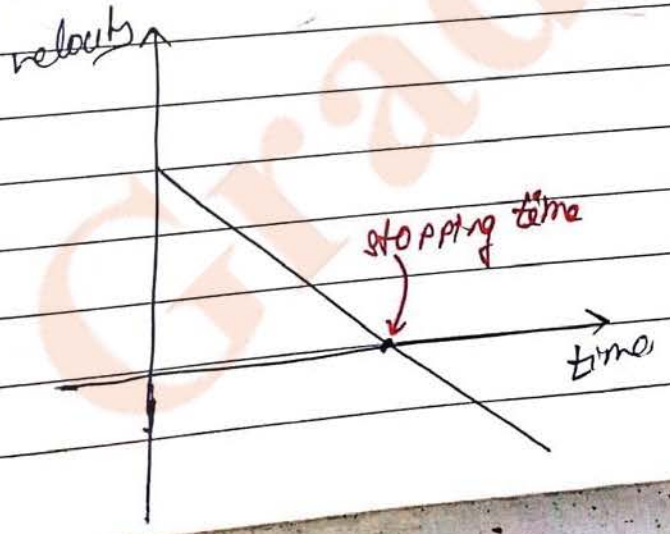
$$0 = u - at$$

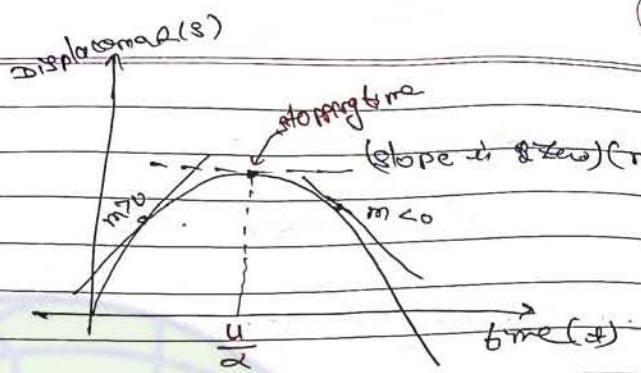
$$t = \frac{u}{a} = \frac{\text{Initial velocity}}{\text{Retardation}}$$

(vi) Stopping distance:

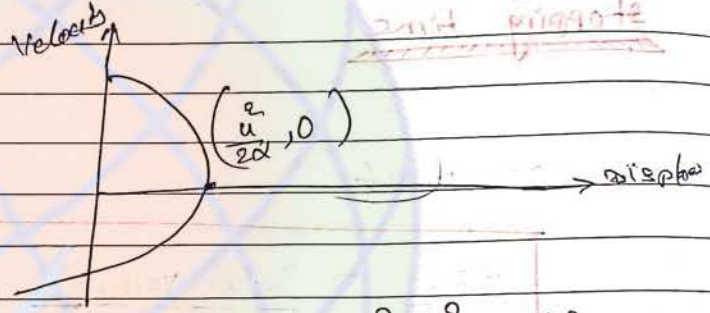
$$0 = u^2 - 2as$$

$$s = \frac{u^2}{2a} = \frac{(\text{Initial velocity})^2}{2(\text{retardation})}$$





$$\vec{s} = ut - \frac{1}{2}at^2$$

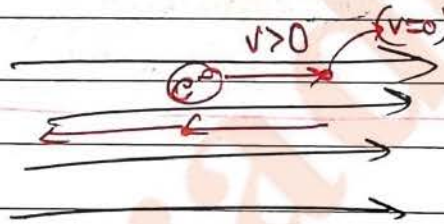


$$v^2 = u^2 - 2as$$

$$v^2 = -2a\left(s - \frac{u^2}{2a}\right)$$

$$(v-0)^2 = -2a\left(s - \frac{u^2}{2a}\right)$$

understand

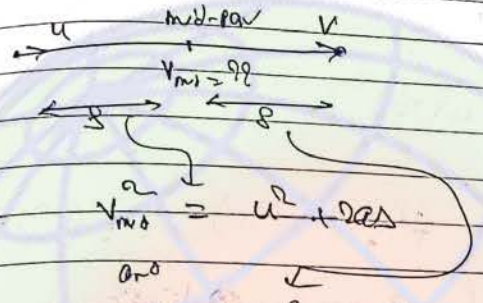


classmate
Date
Page

vel. at mid-point

Q1 For a particle moving along a straight line under uniform acc^n initial and final velocities are "u" and "v" resp. Find vel. of the particle at the mid-point of its path

Sol:



V, a → I
s, a → II
V, s → III

$$v_{mid}^2 = u^2 + 2as$$

or

$$v^2 = v_{mid}^2 + 2as$$

$$v^2 - v_{mid}^2 = u^2 - v_{mid}^2$$

Imp

$$v_{mid} = \sqrt{\frac{u^2+v^2}{2}}$$

Q2 A particle initially at rest moves under uniform acc^n for such a time as distance travelled by it in last 10 sec is next 10 sec

Sol:

$s = ut + \frac{1}{2}at^2$

$s_2 = 0 + \frac{1}{2} \times a \times (10)^2$

$s_2 = 10 \times s$

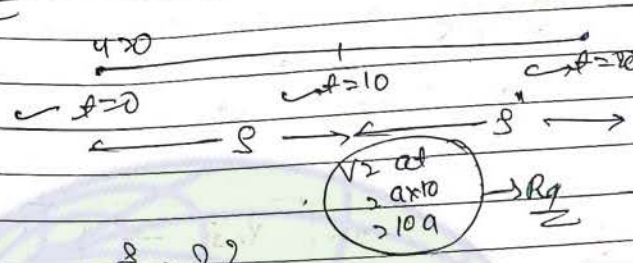
$s = 5a$

$s_2 = 10 \times 10 + \frac{1}{2} \times 10 \times$

u = 0
acc^n const
(a → u)
s_2 4

Q4

Tanhe



$\frac{s}{s'} = ??$

$s = \frac{1}{2} a(10)^2$

and

$s' =$ (10a) · (10) $+ \frac{1}{2} a(10)^2$

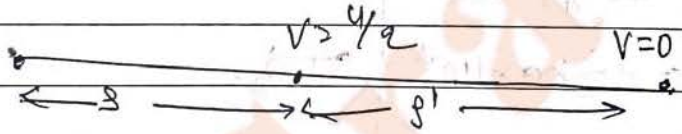
vel. at t = 10 sec.

$\frac{s}{s'} = \frac{1}{3}$

Note

$s + s' = \frac{1}{2} a(20)^2$

Q3 Initial vel. of a particle is "u" it moves along straight line under uniform retardation. In loosing 50% of its initial vel. it travels distance "s". find distance travelled by it further to come to rest



$u = u$

$a = a$

$u / v = \frac{1}{2} u$

$s = s$

$v = 0$

$s = ?$

$\left(\frac{u}{2}\right)^2 = u^2 - 2as$

$a \rightarrow$ retardation

u, v, a, s

u, a, s

$$2\alpha s = \frac{I}{t} u^2 \quad \text{---(i)}$$

and

$$0 = \left(\frac{u}{2}\right)^2 - 2\alpha s'$$

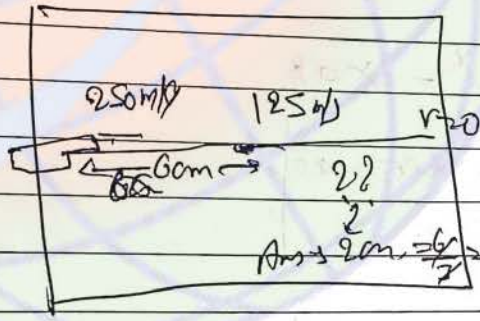
$$2\alpha s' = \frac{u^2}{4}$$

$$\frac{s}{s'} = \frac{I}{t}$$

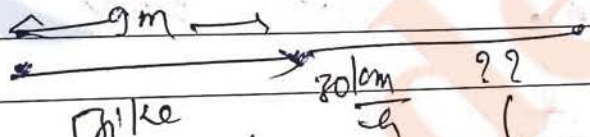
$$\boxed{s' = s/3}$$

Application :-

(i)



(ii)



60km/h

$$s' = \frac{9}{3} = 3m$$

(iii)

$$\boxed{\frac{t}{t'} = 1}$$

Next

$\frac{u}{2} = u - \alpha t$

$\alpha t = \frac{u}{2}$ —(i)

and $0 = \frac{u}{2} - \alpha t'$

$\alpha t' = \frac{u}{2}$ —(ii)

Q) Two identical cars A and B are moving along straight lines with vel. 60 km/hr and 30 km/hr resp. They are stopped by applying the same brakes find ratio of stopping distance A to B.

$0 = u^2 - 2as$

$u^2 = 2as$

$(60)^2 = 2as$

$\frac{s_1}{s_2} = \frac{(60)^2}{(30)^2} = \frac{60 \times 60}{30 \times 30} = 4$

$\frac{s_1}{s_2} = 4$

Teacher

$0 = u^2 - 2as$

$s = \frac{u^2}{2a}$

$a = \frac{F}{m}$

$S \propto u^2$

$\frac{S_A}{S_B} = \left(\frac{u_A}{u_B}\right)^2 = \left(\frac{60}{20}\right)^2 = 9$

Q. A truck and a car are moving along straight line they are stopped by applying the same breaking which will come to rest at what distance

$m_{truck} > m_{car}$

(i) Both have same initial vel.
(ii) Both have same initial K.E.
(iii) Both have same initial momentum

$0 = u^2 - 2aS$

$S = \frac{u^2}{2a}$

$F = ma$

$a = \frac{F}{m}$

$S = \frac{mu^2}{2F}$

(i) $S = \frac{mu^2}{2F}$ same / same

$S \propto m$

$S_T > S_{car}$

(Car will stop at short distance)

(ii) $s = \frac{mu^2}{2F}$

$s = \frac{(ki)}{(\delta)}$ → Same

$s_T = s_{con}$

(iii) $s = \frac{ki}{F}$

$s = \frac{(p_i)^2}{2mF}$ → same, same

$K.E = \frac{p^2}{2m}$

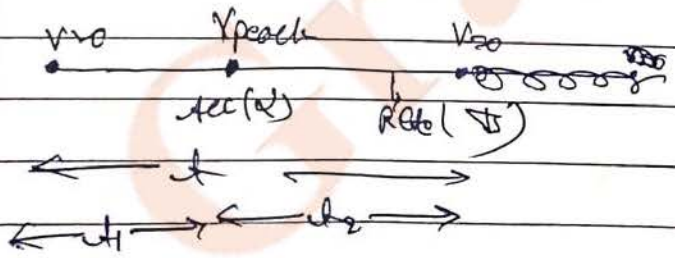
$s \propto \frac{1}{m}$

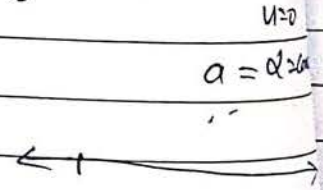
$s_T < s_{con}$

Ex. A body initially at rest acc" at constant rate "α" then retards at constant rate "β" to come to rest motion is along straight line and total time is "t".

Find pick value of its velocity.

Sol[n]





$t = t_1 + t_2$

$V_{peak} = 0 + \alpha t_1$

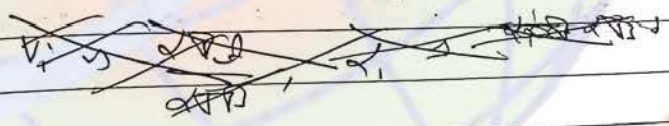
and

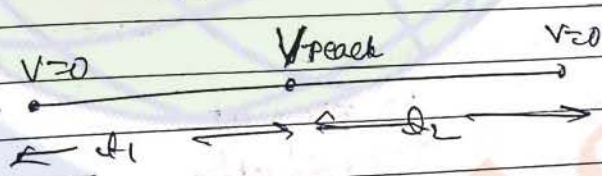
$0 = V_{peak} - \beta t_2$

$\frac{V_{peak}}{\alpha} + \frac{V_{peak}}{\beta} = t$

$$V_{peak} = \frac{\alpha\beta t}{(\alpha+\beta)}$$

(iii) Distance travelled by the object =

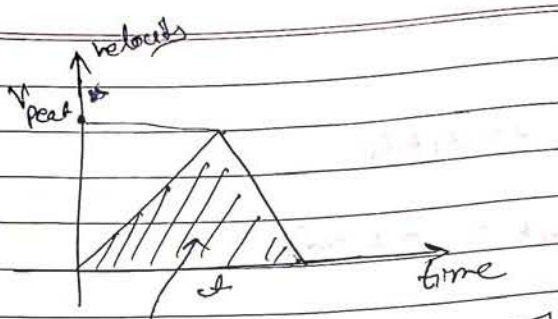




Total distance $= \left(\frac{0+V_{peak}}{2}\right)t_1 + \left(\frac{V_{peak}+0}{2}\right)t_2$

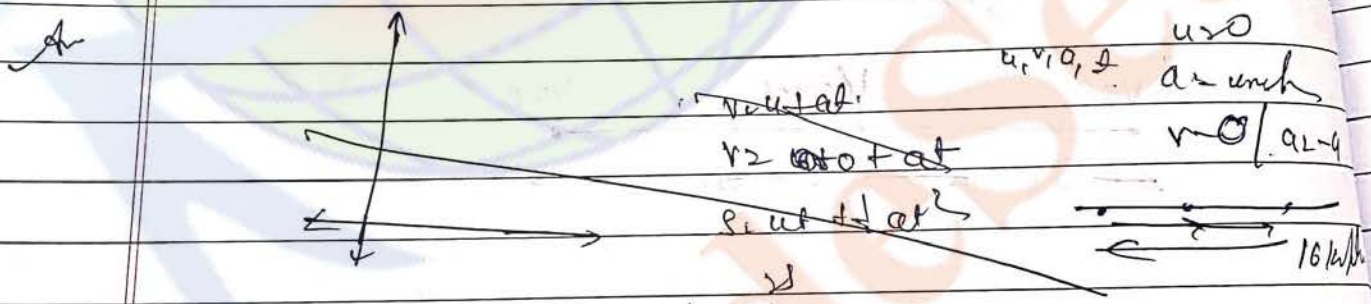
$= \frac{V_{peak}(t_1+t_2)}{2} = \frac{V_{peak}\,t}{2}$

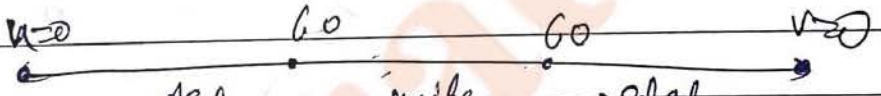
$$S = \frac{\alpha\beta t^2}{2(\alpha+\beta)}$$

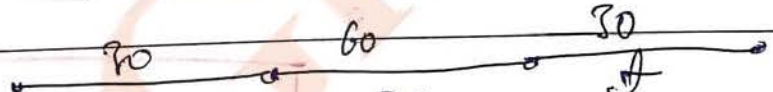


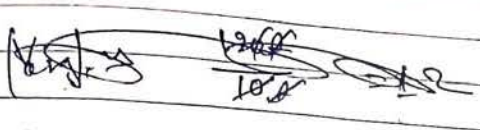
$$s = \frac{1}{2} v_{peak} \cdot t$$

eg.) A body initially at rest moves under uniform acc^n then with uniform velocity and finally comes to rest under uniform retardation. motion is along straight line ratio of time taken are 1:8:1 peak velocity is 16km/hour Average speed of the complete journey is.



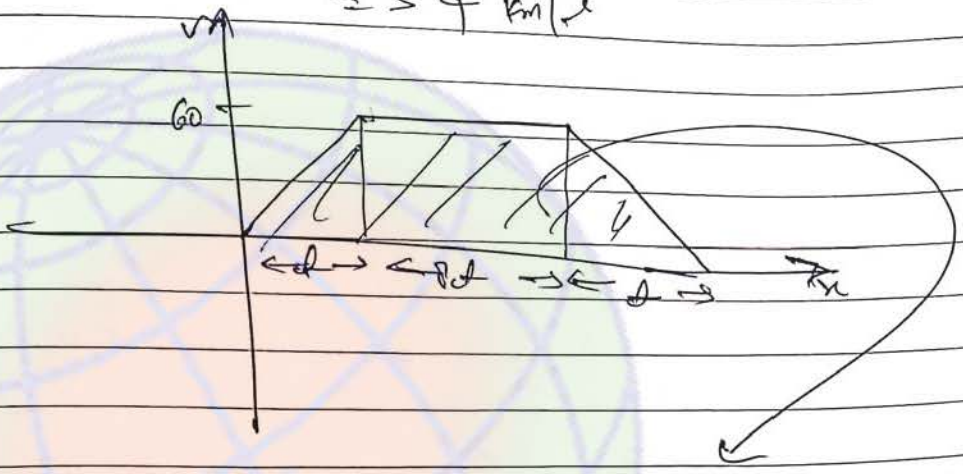






$$\{V_{avg}\} = \frac{30(\theta) + 60(8\theta) + 30(\theta)}{10\theta}$$

$$= 54 \text{ km/h}$$



$$V_{avg} = \frac{\frac{1}{2}\theta(0) + (8\theta)(60) + \frac{1}{2}(\theta)(60)}{10\theta}$$

$$\Rightarrow 54 \text{ km/h}$$

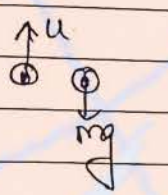
Motion under Gravity

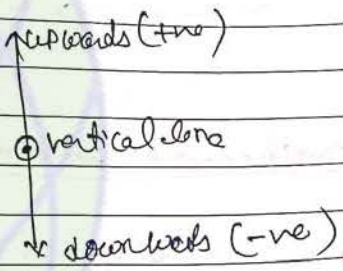
Note

(i) object is near the earth's surface
$g = 9.8 m/s^2 \simeq 10 m/s^2$

(ii) air friction force is neglected.

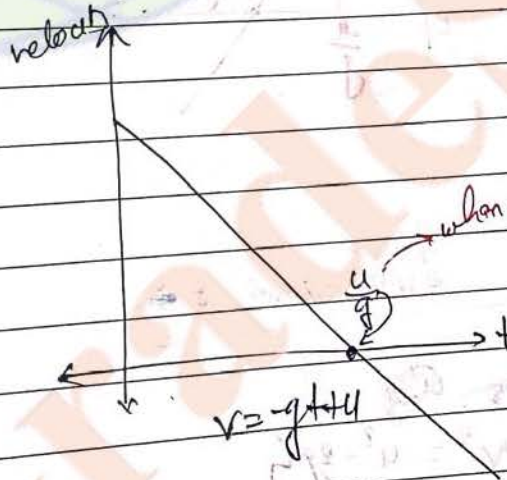
(A) If object is thrown vertically upwards:-

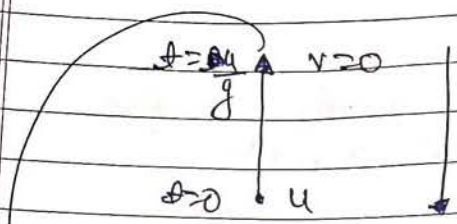




(1) Velocity after time t :-

$v = u - gt$





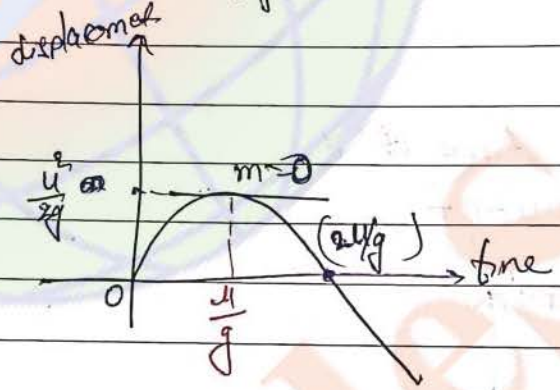
Note: stopping time

$$0 = u - gt$$

$$\rightarrow t = \frac{u}{g}$$

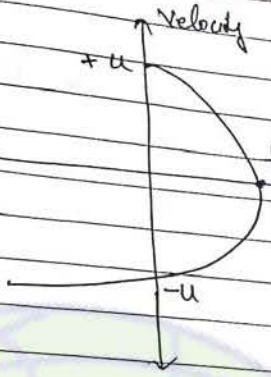
iii) Displacement in time "t":

$$s = ut - \frac{1}{2}gt^2$$



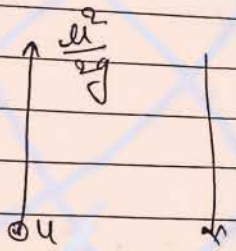
iv) Velocity w/r displacement s →

$$v^2 = u^2 - 2gh$$

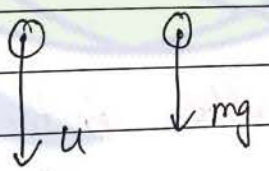


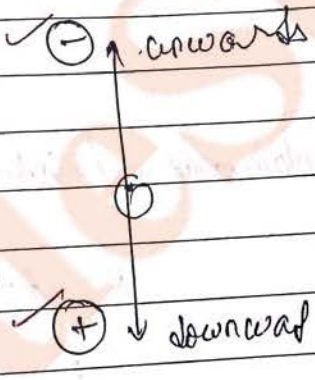
$$v^2 = -2g\left(s - \frac{u^2}{2g}\right)$$

$$(v-0)^2 = -2g\left(s - \frac{u^2}{2g}\right)$$



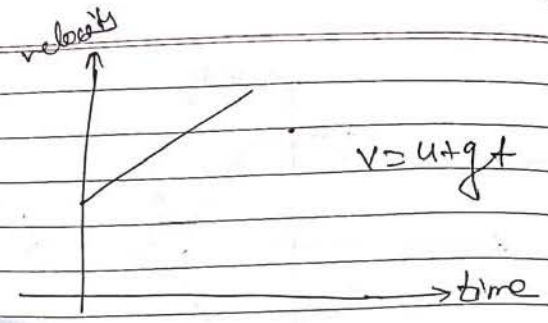
(1) If object is drown vertically downwards from some height (H),





(1) Velocity after time "t".

$$V = u + gt$$

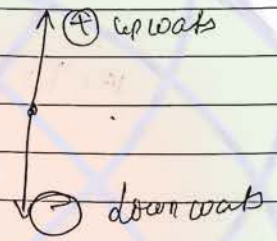


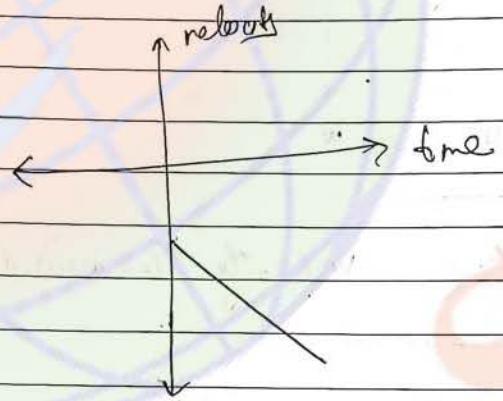
$v = u + gt$

Note

more points

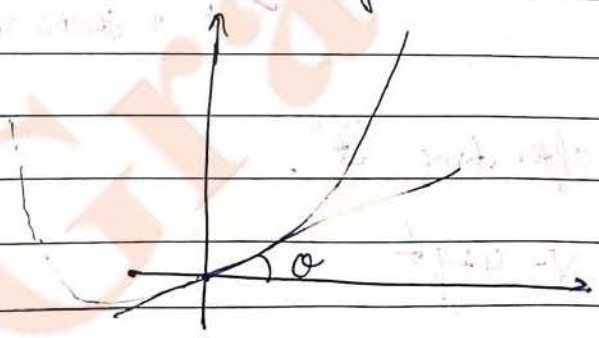
first time is mentioned





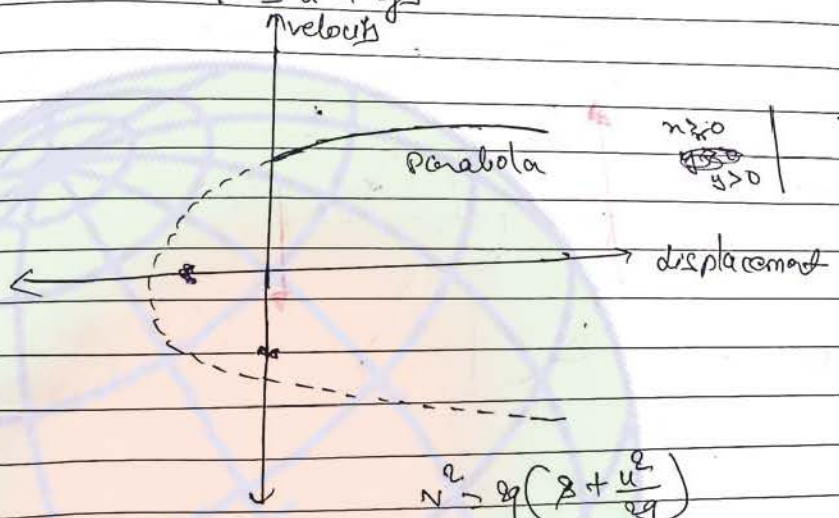
(ii) Displacement / distance travelled in time "t"

$s = ut + \frac{1}{2}gt^2$



velocity after displacement (s) :⇒

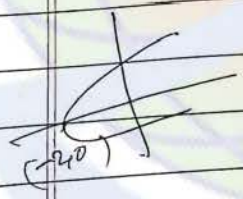
$$v^2 = u^2 + 2gs$$



$$v^2 = 2g\left(s + \frac{u^2}{2g}\right)$$

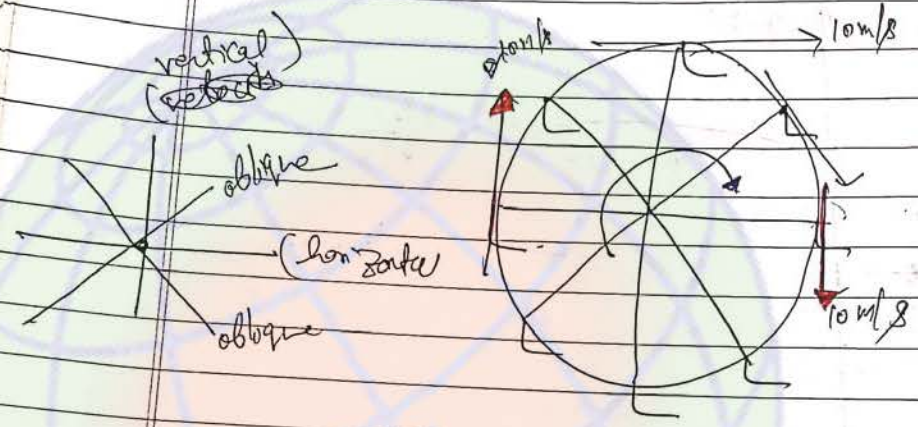
$$(y-0)^2 = 2(x+0)$$

$$(v-0)^2 = 2g\left(s + \frac{u^2}{2g}\right)$$



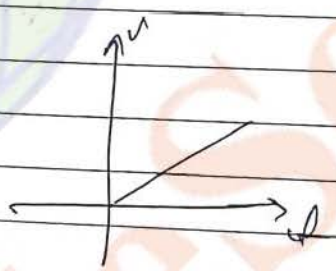
Special cases

① If object is released or dropped from rest from a body or platform

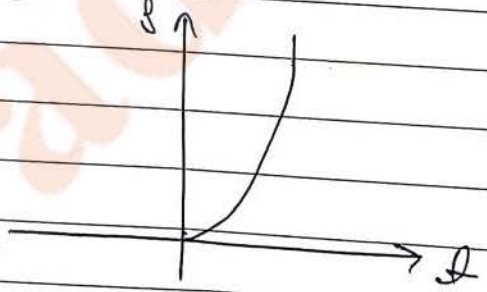


$u = 0$

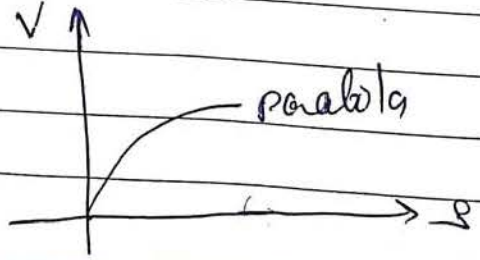
(i) $v = gt$



(ii) $s = \frac{1}{2} g t^2$

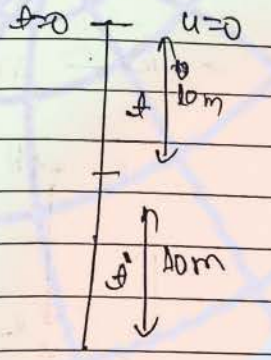


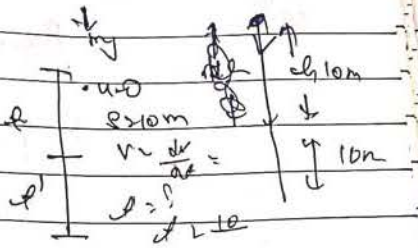
(iii) $v^2 = 2gs$ =



Q5 A body is released from some height find ratio of time taken in travelling 1st 10 metre and next 10 metre

$$\frac{t}{t'} = \frac{1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{1}$$





$$10 = \frac{1}{2} g t^2$$

and

$$20 = \frac{1}{2} g (t+t')^2$$

$$\frac{1}{2} = \frac{t^2}{(t+t')^2}$$

$$\frac{1}{\sqrt{2}} = \frac{t}{(t+t')}$$

$$t' = (\sqrt{2}-1)\, t$$

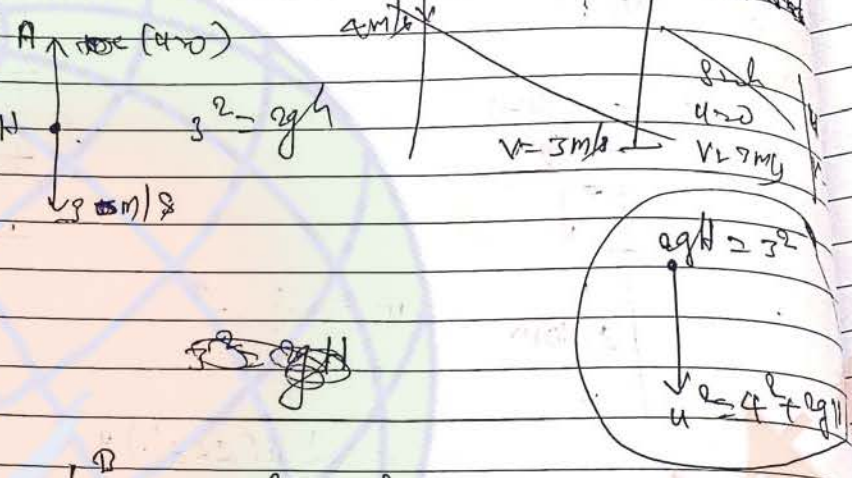
Now

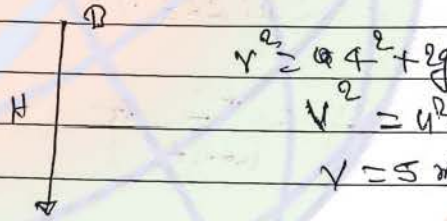
$$\frac{t}{t'} = \frac{t}{\sqrt{2}-1}$$

$$= \frac{\sqrt{2}+1}{1} = \frac{2.4}{1}$$

Q) A body released from height 'h' reaches the ground with speed of 4 m/s. another body is thrown vertically downwards with speed of 4 m/s from the same height find its speed when it reaches the ground

Sol/n





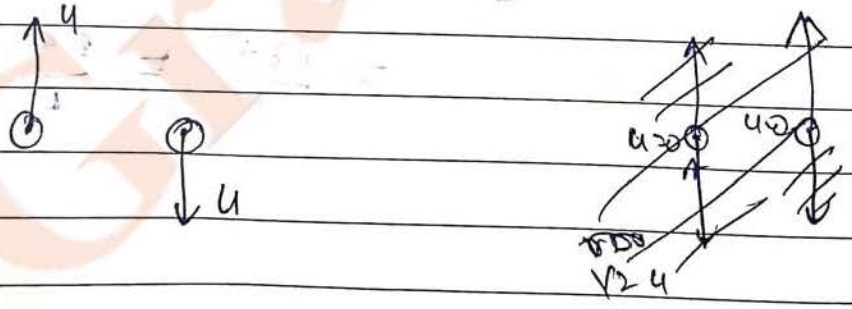
$v^2 = u^2 + 2gH$

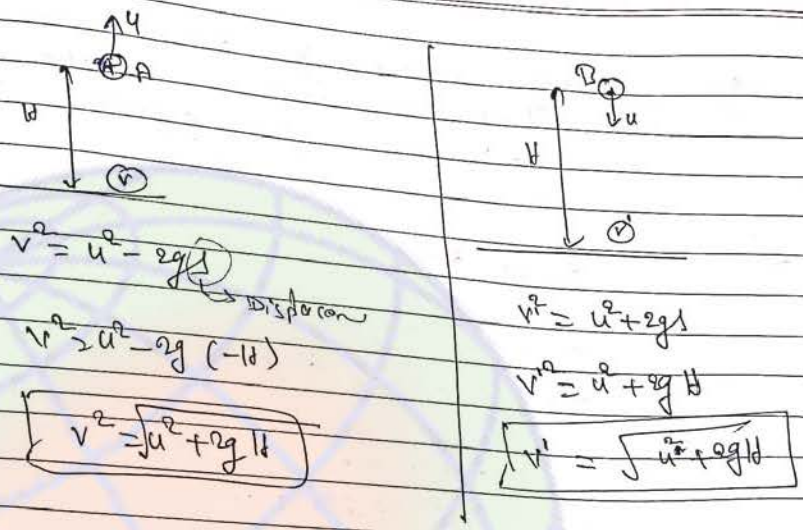
$v^2 = 4^2 + 3^2$

$v = 5$ m/s

Q) Two objects "A" and "B" are thrown with same speed along vertical line from the same height but in opposite direction find ratio of their velocities when they hit the ground.

Sol/n





$v^2 = u^2 - 2gh$ → displacement

$v^2 = u^2 - 2g(-h)$

$\boxed{v^2 = \sqrt{u^2 + 2gh}}$

$v^2 = u^2 + 2gh$

$\boxed{v' = \sqrt{u^2 + 2gh}}$

$$\frac{v}{v'} = \frac{\sqrt{u^2+2gh}}{\sqrt{u^2+2gh}} = 1$$

Q) A body is thrown vertically downwards from a tower of height 20m with speed of 20m/s find time taken to reach the ground.

Sol:

$v^2 = u^2 + 2as$ $\quad v = u + at$

$0 = 20 \times 20 + 2 \times 10 \times 20$

$s = ut + \frac{1}{2}gt^2$

$20 = 20 \times t + \frac{1}{2} \times 10 \times t^2$

$t^2 + 4t - 4 = 0$

$t = \frac{-4 \pm \sqrt{4^2 + 4(4)}}{2}$

$t = \frac{-4 + 4\sqrt{2}}{2} = -2 + 2\sqrt{2} \simeq 0.8 \text{ sec}$

u, v, a, s
u, v, a, s
a, a, t, s
v

$u = 20 \text{ m/s}$ ↓
$s = 20 \text{ m}$
$t = ?$
$a = +10$

20m

Q) A body is vertically upward with speed of 20m/s from top of the tower of height 20m find time taken to reach the ground

Sol:-

$s = ut + \frac{1}{2}gt^2$

$-20 = 20t - \frac{1}{2}10t^2$

$-20 = 20t - 5t^2$

$-t^2 + 4t - 4 = 0$

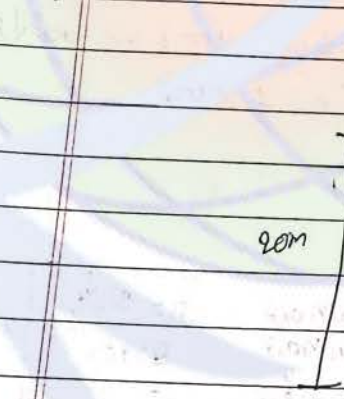
$t^2 - 4t + 4 = 0$

$t^2 - 2t - 2t + 4 = 0$

$t(t-2) - 2(t-2) = 0$

$(t-2)(t-2)$

Note:-



displacement

$s = ut - \frac{1}{2}gt^2$

$-20 = 20t - 5t^2$

$t^2 - 4t - 4 = 0$

$t = \frac{4 \pm \sqrt{4^2 + 4(4)}}{2}$

$t = \frac{4 + 4\sqrt{2}}{2}$

$= 2 + 2\sqrt{2}$ or 4.8sec

Q) A body

20m

Sol:-

Q) A body is released from top of a tower of height 2020m. Find time taken by it to reach the ground.

Sol^n

$s = \frac{1}{2}gt^2$ (u=0)

$t = \sqrt{\frac{2020}{10}} = 2\sqrt{202}$ sec

Note

$(-2+2\sqrt{2})(2+2\sqrt{2}) = 8-4 = 4$

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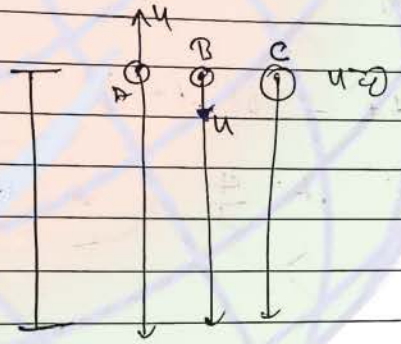
Note



$t_c = t_A + t_B$

where, t = time taken to reach the ground.

Q) A body released from some height travels $\frac{7}{16}$ part of its total height in last second of its journey. Find its total time of journey.

No. of t

$\frac{7}{16}$ | $h = 7/16$

L-1 → 3 4 5 6 7 8 9

L-2 → 1, 2, 1 to 14, 16

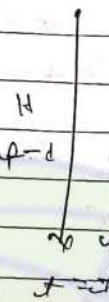
L-3 → 1, 3, 6, 8, 11, 12, 14, 1

L-4 → 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 15

Class

Date

Page



$H = \frac{1}{2}gt^2$ → (1)

and

$\frac{9}{10}H = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2$

$\frac{9}{10}H = \frac{g}{2}(2t-1)$ → (2)

$\frac{9}{10}\left(\frac{1}{2}gt^2\right) = \frac{g}{2}(2t-1)$

$9t^2 = 10(2t-1)$

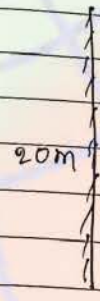
$t = \frac{9}{3}$, $t = 3s$

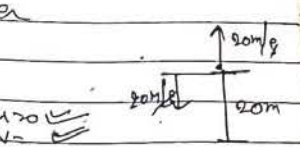
Q. A body ... time ... with ... what ...

eg.

Q.) A body is thrown vertically upwards with speed of 20 m/s from foot of a tower of height 20 m, at the same time another body is thrown vertically downwards with speed of 20 m/s from top of the tower. At what time will they cross each other

Sol:



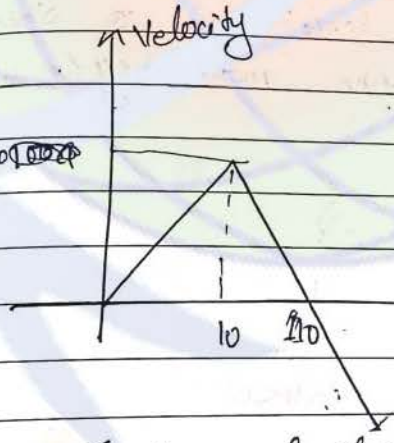


$$S_A + S_B = 20$$

$$\left(20t - \frac{1}{2}gt^2\right) + \left(20t + \frac{1}{2}gt^2\right) = 20$$

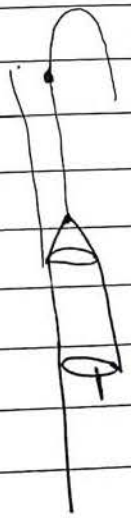
$$\boxed{t = \frac{1}{2} \text{ sec.}}$$

Eg.)

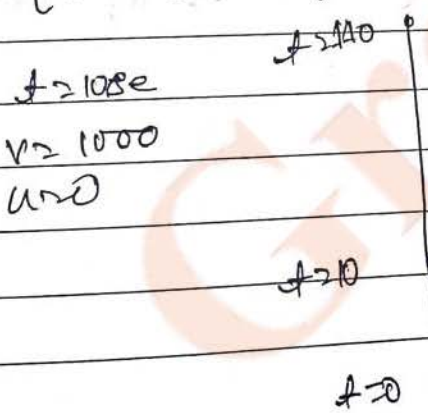


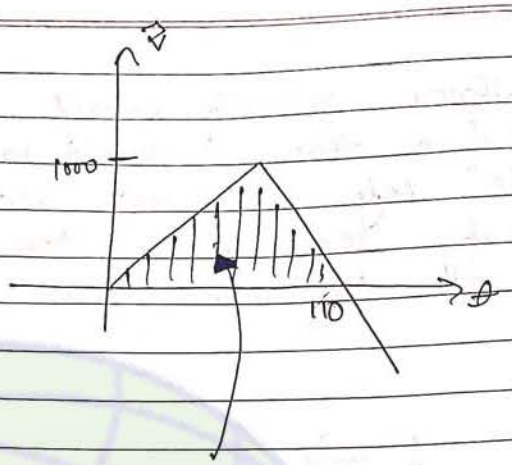
[v-t graph of a rocket]

A rocket is launched from ground level maximum height attained by the rocket is



Sol:





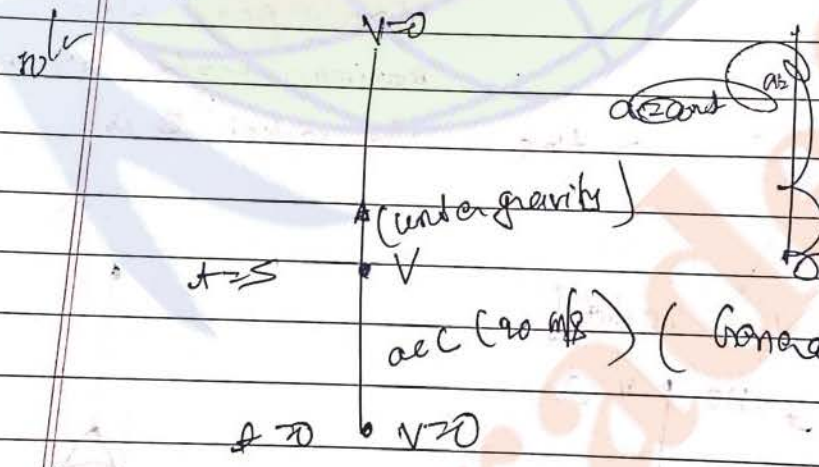
$$\text{max height} = \frac{1}{2} \times 110 \times 1000$$

$$= 55{,}000 \text{ m}$$

$$= 55 \text{ km}$$

Q9

A rocket is launched along a vertical line from ground level. It starts its motion from rest with the acc^n of 20 m/s^2. In 5 sec its fuel is exhausted find max. height attained by the rocket



Now

(i) velocity at $t = 5$ sec =

$V = at$

$= (20)(5) = 100$ m/s

(ii) Distance travelled by the rocket in 5 sec

$s = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2}(20)(5)^2 = 250$

(iii) $0 = (100)^2 - 2hg$

$h = \frac{(100)^2}{20} = 500$ m

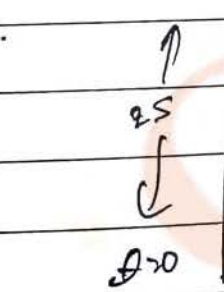
maximum height $= 250 + 500$

Q4) A baloon starts raising vertically upwards from ground level with zero

After 5 sec and 60

Find time taken by the body in reaching the ground after 5 sec

Sol (i)



$V = at = 10$ m/s

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 10 \times 5 \times 5$

$= 25$ m

(ii)

(iii)

$$s = ut - \frac{1}{2}gt^2$$

$$-25 = 10t - 5t^2$$

$$t^2 - 2t - 5 = 0$$