Progression

(2) Sum of n-terms of an A.P

$$S_n = T_1 + T_2 + T_3 + \ldots\ldots T_{n-1} + T_n$$

$$S_n = a + (a+d) + (a+2d) + \ldots [a+(n-2)d] + [a+(n-1)d] \quad \text{(i)}$$

$$S_n = [a+(n-1)d] + [a+(n-2)d] + \ldots (a+d) + a \quad \text{(ii)}$$

eq (i) + (ii)

$$2S_n = [2a+(n-1)d] + [2a+(n-1)d] + \ldots + [2a+(n-1)d]$$

$$\boxed{S_n = \frac{n}{2}[2a+(n-1)d]}$$

or

$$S_n = \frac{n}{2}[a + \underbrace{a+(n-1)d}_{T_n \text{ or } l}]$$

where,
S_n = sum of n-th term of an A.P

$$\boxed{S_n = \frac{n}{2}[a+l]}$$

Q1.) Find the sum $3+8+13+\ldots\ldots+$ up to 20th terms

Sol^n $a=3$, $d=5$, $n=20$

$$S_n = \frac{n}{2}(a+l) = \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{20}{2}[6+19\times5]$$

$$= 1010$$

Q2.) If the sum of 1st 10 terms of an A.P is equals to 4 times sum of its 1st five terms of an A.P find the ratio of first term : common difference

Sol^n $\quad n=10 \quad\quad n=5$

$$\frac{n}{2}(a+9d) = 4\times\frac{n}{2}(a+4d)$$

$$a-4a = 4d-9d$$

$$-3a = -5d$$

$$a = \frac{5}{3}$$

$$S_{10} = 4S_5$$

$$2a+9d = 4(2a+4d)$$

$$-6a = -5d$$

$$\boxed{\frac{a}{d} = \frac{5}{10} = \frac{1}{2}}$$

Properties:-

① If $\boxed{T_n = S_n - S_{(n-1)}}$ where, $S_n \Rightarrow$ sum of 1st n term of an A.P

$S_{n-1} \Rightarrow$ sum of 1st $(n-1)$ term of an A.P

② If $\boxed{d = S_2 - 2S_1}$,

$\because S_2 - 2S_1$

$\Rightarrow (T_1+T_2) - 2(T_1)$

$\Rightarrow (a + a+d) - 2(a)$

$= d$

③ • If the sum of infinite A.P is always infinite when $d>0$

• If the sum of infinite A.P is always

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eg:- find the sum

$3+6+9+12+ \cdots\cdots \infty$ terms

sol^n → ∞

(4) If the nth term of an A.P is in the form of $(An+B)$ then common difference is "A"

(common difference is equals to coefficient of n)

und:-

$T_1 + T_2 + \cdots\cdots + T_n$

$T_n = 2n+3$

so, method 1st

$d = 2$

coefficient of n

method 2nd

$T_1 = 2(1)+3 = 5$

$T_2 = 4+3 = 7$

$d = T_2 - T_1$

$= 7-5$

$(d = 2)$

(5) If the sum of nth term of an A.P is in the form of (An^2+Bn)

then

common difference (d) is $2a$

(common difference is equals to twice of coefficient of n^2)

und:-

$T_1 + T_2 + \cdots\cdots T_n$

$S_n = 4n^2 + 5n$

$(d = 4\times 2)$

$= 8$ twice of coeffi of n^2

$S_1 = 4(1)^2 + 5 = 9$

$S_2 = 16+10 = 26$

$d = S_2 - 2S_1$
$= 26 - 18$
$= 8$

If in two a.p's

$$\frac{S_{n_1}}{S_{n_2}} = \frac{f(n)}{g(n)}$$

then, the ratio of their nth term's of their two A.p's is

$$\boxed{\frac{T_{n_1}}{T_{n_2}} = \frac{f(2n-1)}{g(2n-1)}}$$

{ n is replaced by (2n-1) }

Both the a.p's having "n" terms or equal number of terms

Und:-

$S_{n_1} = T_1 + T_2 + \dots T_n$

$S_{n_2} = T_1' + T_2' + \dots T_n'$

$$\frac{S_{n_1}}{S_{n_2}} = \frac{f(n)}{g(n)}$$

Replace n by (n-1)

$$\frac{T_{n_1}}{T_{n_2}} = \frac{f(2n-1)}{g(2n-1)}$$

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(Reverse of property 6th)

7) If the ratio of their term's of 2 a.p are given

$$\frac{T_{n_1}}{T_{n_2}} = \frac{An+B}{Cn+D}$$

Then,

The ratio of their sum of two a.p's

$$\boxed{\frac{S_{n_1}}{S_{n_2}} = \frac{A\left(\frac{n+1}{2}\right)+B}{C\left(\frac{n+1}{2}\right)+D}}$$ Here, n is replaced by $\left(\frac{n+1}{2}\right)$

\# understanding

अगर दोनों ही a.p में equal no. of term मौजूद हो तो उस case में ऊपर लिखे हुए ये condition apply होगा।
और एक बात याद रखें अगर S_n देकर T_n मांगे तो S_n के "n" को "2n-1" से Replace करते है।

Q1.) If the nth term of an a.p is $\frac{1}{3}(2n+1)$ find the sum of its 1st 20 terms.

Sol^n

$T_n = \frac{2n}{3} + \frac{1}{3}$

$d = \frac{2}{3}$, $n = 20$

Put, $n=1$, $T_1 = \frac{2}{3} + \frac{1}{3} = 1$

$S_n = \frac{20}{2}\left(2\times1 + (20-1)\times\frac{2}{3}\right)$

$= 10\left(2 + \frac{38}{3}\right) = 10\left(\frac{44}{3}\right)$

$= \frac{440}{3}$

★ when equal no. of terms is not given in the Quest.

Quest If T_n be the nth term of an a.p and $\frac{T_1+T_2+\cdots T_p}{T_1+T_2+\cdots T_q} = \frac{p^2}{q^2}$ find $\frac{T_6}{T_{11}}$

Sol^n

$$\frac{\cancel{\frac{p}{2}}[2a+(p-1)d]}{\cancel{\frac{q}{2}}[2a+(q-1)d]} = \frac{p^{\cancel{2}}}{q^{\cancel{2}}}$$

Here, let, a = 1st term

d = common difference

$$\frac{2a+(p-1)d}{2a+(q-1)d} = \frac{p}{q}$$

$$2aq + \cancel{pqd} - qd = 2ap + \cancel{pqd} - pd$$

$$2aq - 2ap = pd + qd$$

$$2a(q-p) = -d(p-q)$$

$$2a(q-p) = d(q-p)$$

$$(2a = d)$$

Now

$$\frac{T_6}{T_{11}} = \frac{a+5d}{a+10d} = \frac{a+5(2a)}{a+10(2a)}$$

$$= \frac{11\cancel{a}}{21\cancel{a}} = \frac{11}{21}$$

Properties of Logarithm

① $\log_{10} 10 = 1$

② $\log_{10} a^n = n \log_{10} a$

③ $\log_b a = \dfrac{\log a}{\log b} = \dfrac{1}{\log_a b}$

④ $\log_b a \cdot \log_a b = 1$

⑤ $\log_{10} a + \log_{10} b = \log_{10} ab$

⑥ $\log_{10} \dfrac{a}{b} = \dfrac{\log_{10} a}{\log_{10} b}$

~~imp~~

⑦ ~~log~~ $\boxed{a^{\log_a b} = b}$

eg:- $7^{\log_7 x} = x$

⑦ if $\log_{10} x = \log_{10} y$

Then,

$x = y$

Imp (8) $\log_{10} a = 2$

$a = 10^2$

(9) $\log a = \log_e a$

(10) $\log_{b^y} a^x = \frac{x}{y} \log_b a$

eg → $\log_{25} 27 = \log_{5^2} 3^3 = \frac{3}{2} \log_5 3$

if $\log_a x > \log_a y$

Then,

(a) $x > y$

when

$a \geq 1$ → इसकी सहायता से मान करें।

(b) Then $x < y$

when

$0 < a < 1$

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Properties of A.P

① If we increase or decrease ~~to~~ by a constant term to every term of given ~~progr~~ a.p then the new progression will also in a.p

if

1, 3, 5, 7, 9, 11 are in a.p ✓

-2 → $-1, 1, 3, 5, 7, 9$ ⟶ a.p ✓

$+3$ → 4, 6, 8 ---- ⟶ a.p

② If we multiply or divide to every term of given a.p by a constant non-zero number so, the new progression will also in a.p

1, 3, 5, 7, 9 are in a.p.

$\div 7$ → $\frac{1}{7}, \frac{3}{7}, \frac{5}{7}, \frac{7}{7}, \frac{9}{7}$ ⟶ a.p

$\times 6$ → 6, 18, 30 ---- ⟶ a.p

③ If the sum of any two term of given a.p is equal to a constant ~~term~~ which are equal distance from 1st and last and is equal to the sum of 1st and last term.

if $T_1, T_2, T_3, \ldots\ldots T_{n-2}, T_{n-1}, T_n$ are in A.P

$$\boxed{T_2 + T_{n-1} = \text{Constant} = T_3 + T_{n-2} = T_1 + T_n}$$

Ex. If $\underline{2}, \underline{4}, \underline{6}, 8, \underline{10}, \underline{12}, \underline{14}, \longrightarrow$ are in a.p

$2+14 = 4+12 = 6+10$

$16 = 16 = 16$

Q1.) if a, b, c are in a.p then $b+c,\ c+a,\ a+b$ are in ______

Sol^n method 1st

if a, b, c are in A.P

$a-(a+b+c),\ b-(a+b+c),\ c-(a+b+c) \longrightarrow$ A.P

$-(b+c),\ -(a+c),\ -(a+b) \longrightarrow$ a.p

multiply by -1 to each ter.

$(b+c), (a+c), (a+b) \longrightarrow$ A.P

method 2nd

(And best method)

Put

$a=1,$

$b=2$

$c=3$

then,

Notes A.P में देने पर हम लगातार संख्या को a.p में मानकर भी बना सकते है।

Geometrical Progression (G.P)

If the ratio b/w two consecutive term is same or constant towards the series, this constant ratio is called common ratio and this series is called G.P

if,

$T_1, T_2, T_3, \ldots\ldots T_{n-1}, T_n$ are in G.P

Then,

$$\frac{T_2}{T_1} = \frac{T_3}{T_2} = \ldots\ldots \frac{T_n}{T_{n-1}} = \text{constant} = \text{common ratio}(r)$$

ex

(1) 1, 2, 4, 8, 16, ...

(2) 1, −3, 9, −27, 81, ...

Note

if a, b, c are in G.P

$$\frac{b}{a} = \frac{c}{b}$$

then $\boxed{b^2 = ac}$

(1) Find nth term of an G.P

$T_1 = a = ar^{1-1}$

$T_2 = ar = ar^{2-1}$

$T_3 = ar^2 = ar^{3-1}$

– – –

– – –

$\boxed{l = T_n = a r^{n-1}}$

where, a → first term
T_n → nth term of series or last term

$r \to$ common ratio
it may be +ve or -ve but
not equal to zero, or one

$r \neq 0,\ r \neq 1$

$1, 2, 3, 4, 5 \to$ A.P

$1, 2, 4, 8, 16 \to$ G.P

2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2

A.P / G.P

$d \neq 0$ $\qquad$ $r \neq 1$

Q1.) Find 12th term of series $1, 2, 4, 8 \ldots$

Sol^n

$\Rightarrow 1(2)^{11}$

$\Rightarrow 2^{11}$

$= 2048$

Q2.) which term of progression $18, -12, 8, \ldots$ is equal to $\frac{512}{729}$

$\frac{512}{729} = 18 \times \left(\frac{-2}{3}\right)^{n-1}$

G.M (Geometric mean).

If three or more than three terms are in G.P, the middle term b/w 1st and last is called G.m b/w them

if a, G, b are in G.P
Then

$$\frac{G}{a} = \frac{b}{G}$$

or

$$G^2 = ab$$

$$\boxed{G = \sqrt{ab}}$$

Note ⇒ "a" and "b" having the same sign.

Q.) Find G.m b/w −3 and −27

$\sqrt{3 \times 3 \times 3 \times 3} = 9$

* Point 1st

If we insert "n" G.m b/w "a" and "b"
Let

$G_1, G_2, G_3, \ldots G_n$ are "n" Geometrical means

$a, (G_1, G_2, G_3, \ldots G_n), b$ are in G.P

Here,

Ray

G.M (Geometric mean)

Total terms $\Rightarrow n+2$

$\because \quad T_n = ar^{n-1}$

$b = aR^{(n+2)-1}$

$\frac{b}{a} = R^{n+1}$

Ray

$$\boxed{R = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}}$$

so,

$G_1 = aR$

$G_2 = aR^2$

— — — —

— — — —

$G_n = aR^n$

* Point 2nd : The product of Insert these "n" G.M is equal to

$$= \boxed{G^n}$$

$= G_1 \cdot G_2 \cdot G_3 \cdot - - - \cdot G_n$

$= (aR)(aR^2)(aR^3) \cdot - - - - (aR^n)$

$= a^n R^{(1+2+3+ - - - - +n)}$

$$= a^n R^{\frac{n(n+1)}{2}}$$

$$= a^n \left[\left(\frac{b}{a}\right)^{\frac{1}{n+1}}\right]^{\frac{n(n+1)}{2}}$$

$$= a^n \left(\frac{b}{a}\right)^{n/2} = a^{n-n/2} = (ab)^{n/2}$$

$$= \left(\sqrt{ab}\right)^n = G^n$$

$\boxed{\text{Product of "n" G.m} = \left(\sqrt{ab}\right)^n = G^n}$

Q.) Insert 4 G.m b/w 5 and 160, find 3rd G.m

5, G_1, G_2, G_3, G_4, 160,

$$R = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} = (32)^{\frac{1}{5}} = 2$$

$$G_3 = aR^3$$

$$= 5(2^3) = 40$$

$$\begin{array}{r|l} 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$$

If we insert 2 G.m "P and "Q" b/w 2 given number and one a.m "A" insert b/w same given numbers then $p^3 + q^3$ is equal to

(I) $\frac{2A}{pq}$ (II) $\frac{pq}{2A}$ (III) 2APQ (IV) APQ

$a, P, q, b \to G.P$

$a, A, b \to A.P$

Soln

~~a, P, A, q, b~~ a, A, b

$2A = p+q$

$p^3+q^3 = (p+q)(p^2-pq+q^2)$

Try using this Let data

Let,

$a = 1$
$P = 2$
$q = 4$
$b = 8$

$A = \frac{a+b}{2} = \frac{9}{2}$

Then p^3+q^3 is equal to

$2^3+4^3 = 72$

check option C is $2 \times \frac{9}{2} \times 2 \times 4 = 72$

Properties of G.P

(i) If we multiply or divide to every term of given Progression by a constant non-zero number then the new Progression will also in G.P

$1, 2, 4, 8, 16 \to G.P$

$\times 3$: $3, 6, 12, 24, 48 \to G.P$

$\div 7$: $\frac{1}{7}, \frac{2}{7}, \frac{8}{7}, \frac{16}{7} \to G.P$

② If a, b, c are in G.P
~~then~~

~~$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in G.P~~

then

a^n, b^n, c^n are also in G.P

where

$n \rightarrow$ Integer.

eg 1

$1, 2, 4 \rightarrow$ G.P

$1^2, 2^2, 4^2 \rightarrow$ G.P

③ If a, b, c are in G.P

then

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P

④ If a, b, c are in G.P

then

$\log a$, $\log$, $\log c$, are in A.P

[For all possible base of log]

⑤ If $a_1, a_2, a_3, \ldots \ldots a_n$ are in G.P

and

$b_1, b_2, b_3, \ldots \ldots b_n$ are in G.P

then

$a_1b_1, a_2b_2, a_3b_3, \ldots \ldots a_nb_n$ are also in G.P

Supposition of terms in G.P

Case 1st

Total no. of terms = odd

$-----\frac{a}{r}, a, ar, ---$

$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2, ---$

middle term = a

common ratio = r

Case 2nd

Total no. of terms = even

middle term $= \frac{a}{r}, ar$

common ratio $= r^2$

$------\frac{a}{r^3}, \frac{a}{r}, ar, ar^3 -----$

$-----\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5 ----$

If three no. are in G.P and their sum is 38 and their product is 1728 find the terms

Let

$\frac{a}{r}, a, ar$ be the three terms of G.P

Harmonic Progression

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The reciprocal of A.P is called H.P

if

$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}$ are in HP

because $a, a+d, a+2d$ are in A.P

(i) Find the nth term of H.P

$$\boxed{T_n = \frac{1}{a+(n-1)d}}$$

because

$$\boxed{T_n = a+(n-1)d}$$

of A.P

Note → (i) There is no formula for sum of n-terms of H.P

(ii) It is totally depends on A.P

Q1.) If the value of 5th term and 11th term of H.P are $\frac{1}{45}$ and $\frac{1}{69}$ respectively. find their 16th term of H.P

Sol/n

"Binomial Theorem"

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Binomial Coeff :-

$\Downarrow$

$${}^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}$$

where

$\lfloor n = n \times (n-1) \times (n-2)(\dots$
7×2

$\Downarrow$

$\boxed{\lfloor n = n \times \lfloor n-1}$

$\Downarrow$

$n = 1$

$\lfloor 1 = 1 \times \lfloor 0$

$1 = \lfloor 0$

for ...

$\lfloor 0 = 1$

$\lfloor 1 = 1$

$\lfloor 2 = 2$

$\lfloor 3 \Rightarrow 6$

$\lfloor 4 = 24$

$\lfloor 5 = 120$

$\lfloor 6 = 720$

$\lfloor 7 = 720 \times 7 = 5040$

if $N \in N$

$0 \le r \le n, \; r \in W$

then $\boxed{{}^nC_r \in N}$

$\Downarrow$

ways

Properties :-

① $\boxed{{}^nC_r = {}^nC_{n-r}}$

eg

${}^5C_3 = {}^5C_2$

${}^6C_4 = {}^6C_2$

${}^7C_5 = {}^7C_2$

${}^8C_5 = {}^8C_3$

Reject करने को ही select भी कहना कहते है।

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(2) $\boxed{{}^nC_x = {}^nC_y} = \boxed{{}^nC_{n-y}}$

→ iff $\boxed{x=y}$ or $\boxed{x=n-y}$

→ iff $\boxed{x=y}$ or $\boxed{x+y=n}$

(3) $\boxed{{}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}}$

$\boxed{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r}$

eg $\quad {}^5C_3 + {}^5C_2 = {}^6C_3$

(4) $\boxed{\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-(r-1)}{r}}$

$$\frac{{}^5C_3}{{}^5C_2} = \frac{5-2}{3} = 1$$

$$\frac{{}^9C_5}{{}^9C_6} = \frac{1}{{}^9C_5/{}^9C_5} = \frac{1}{\frac{9-5}{6}} = \frac{6}{4}$$

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(3) $\boxed{{}^{n}C_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}}$

$${}^{n}C_r = \frac{n \times \lfloor n-1}{r \times \lfloor r-1 \times \lfloor (n-1) - (r-1)}$$

imp

$$\boxed{{}^{n}C_r = \frac{n}{r} \times {}^{n-1}C_{r-1}} \quad \# \#$$

$${}^{n}C_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1} = \frac{n(n-1)}{r(r-1)} \, {}^{n-2}C_{r-2} = \text{---}$$

eg:

$$\boxed{{}^{2n}C_n} = \frac{2n}{n} \times {}^{2n-1}C_{n-1}$$

$$= \boxed{2 \times {}^{2n-1}C_{n-1}}$$

$$= \boxed{2 \times {}^{2n-1}C_n}$$

short trick

$${}^{6}C_3 = \frac{6 \times 5 \times 4}{\lfloor 3} = 20$$

$${}^{7}C_2 = \frac{7 \times 6}{\lfloor 2} = 21$$

$${}^{8}C_3 = \frac{8 \times 7 \times 6}{\lfloor 3} = 56$$

$${}^{9}C_2 = \frac{9 \times 8}{\lfloor 2} = 36$$

Ra1
2

★ Binomial theorem →

$$\boxed{\begin{array}{l}\text{Bi - nomial} \\ \text{2 - terms}\end{array}}$$

eg: → $x+y$, $-y$, $y+x$

$\sqrt{x} + \frac{1}{\sqrt{y}}$, $x - \frac{1}{y}$, etc.

$$(x+y)^n = \underbrace{\left[{}^nC_0 (x)^n y^0\right]}_{T_1} + \underbrace{\left[{}^nC_1 x^{n-1} y^1\right]}_{T_2} + \underbrace{\left[{}^nC_2 x^{n-2} y^2\right]}_{T_3} \ldots$$

I II

$$\text{-----} + \underbrace{\left[{}^nC_r (x)^{n-r} y^r\right]}_{T_{r+1}} + \text{---} + \underbrace{\left[{}^nC_n x^0 y^n\right]}_{T_{n+1}}$$

$$= \sum_{r=0}^{n} {}^nC_r (x)^{n-r} (y)^r$$

I II

Note

$x+y$ = Base ✓

n → Index ✓

${}^nC_r$ → Binomial Coefficient ✓

Ra1 VVI

No. of terms = $n+1$ ✓

$$\boxed{T_{r+1} = \text{General term} = {}^nC_r (x)^{n-r} (y)^r}$$ ✓

Permutation and Combination

✡ multiplication law of Counting →

If there are two jobs job J_1 and job J_2, &

Job J_1 can be done in "m" ways and
Job J_2 " " " " n ways.

And according to multiplication law of countin both J_1 and J_2 can be done in $\boxed{m \times n}$ ways provided Job J_1 and Job J_2 are independent to each other.

Both job J_1 and job J_2 → $(m \times n)$ ways (provided J_1 and J_2 are independent to each other)

✡ Addition law of counting –

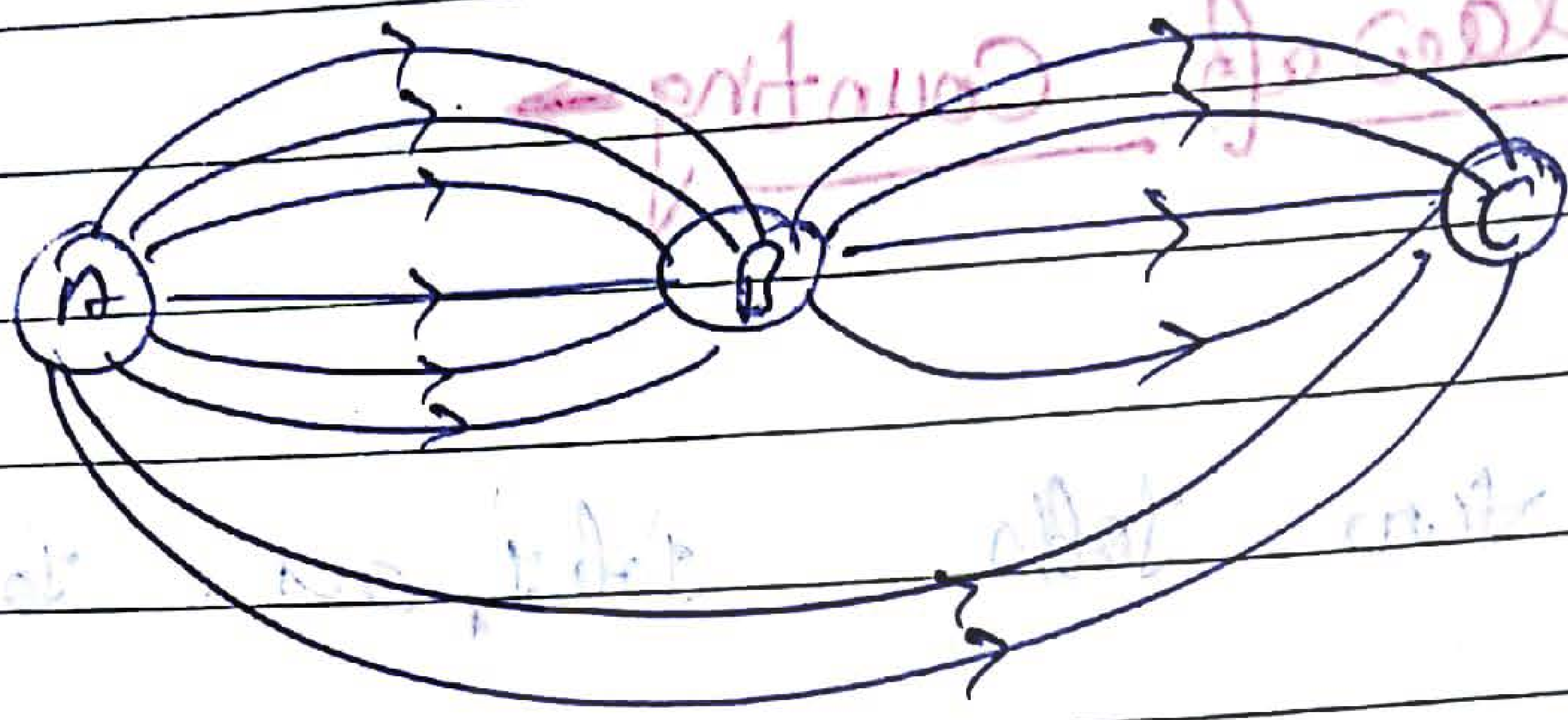
If there are two jobs job J_1 and job J_2,

Job J_1 can be done in m ways
Job J_2 " " " n ways

then according to addition law of counting either job J_1 or job J_2 can be done in m+n ways provided job J_1 and job J_2 are mutually exclusive.

✡ Either job J_1 or job J_2 = $(m+n)$ ways → provided job J_1 and job J_2 are mutually exclusive

*



26 ways

~~A-C ways~~

$(A-C)$ or $([A-B)$ and $(B-C)]$

$2 + (6 \times 4] = 26$ ways

* $\cup$ = or

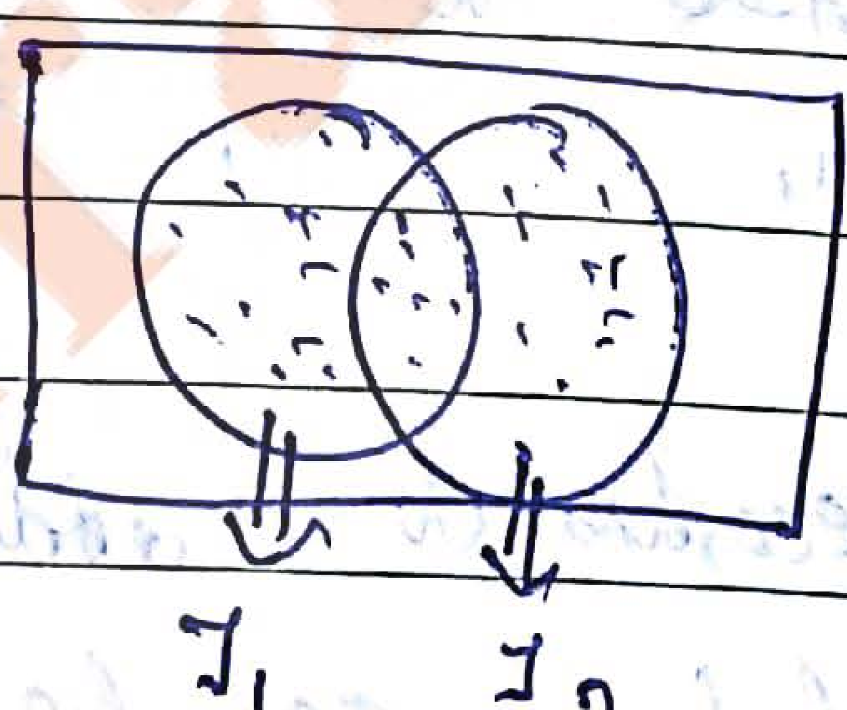
$\cap$ = and

C-1

$I_1, I_2 \to$ Not mutually exclusi

$\Downarrow$

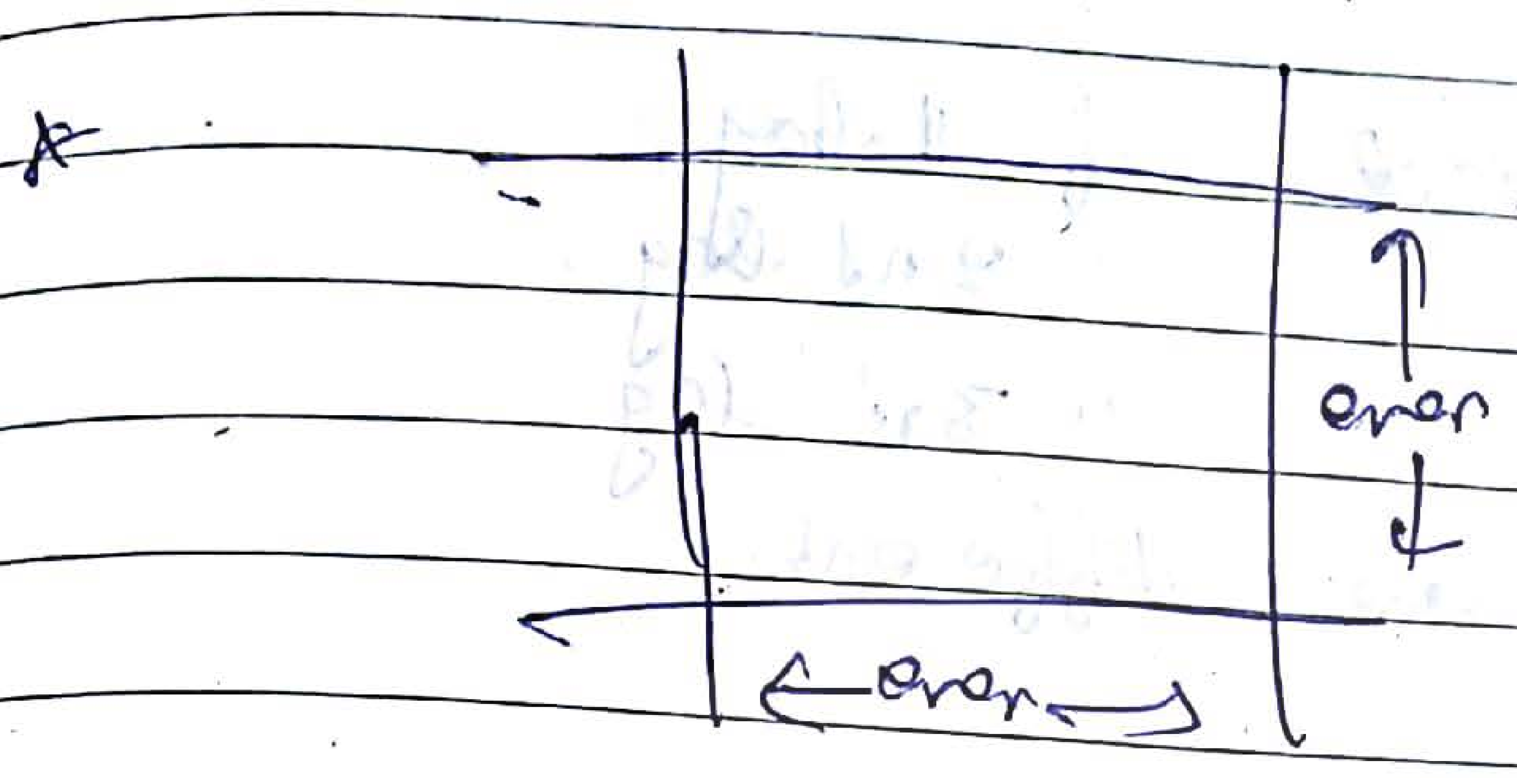
$I_1 \cap I_2$ $\qquad$ $I_1 \cap I_2 = \phi$



I_1 $\qquad$ I_2

$n(I_1 \cup I_2) = n(I_1) + n(I_2) - n(I_1 \cap I_2)$

or



$$\left({}^{m}C_2 + {}^{m}C_2\right) \times \left({}^{n}C_2 + {}^{n}C_2\right)$$

Combination of Identical object

The total no of combination of n-Identical objects taken any at a time is equal to $\boxed{(n+1)}$

A, B, C

$${}^{3}C_0 + {}^{3}C_1 + {}^{3}C_2 + {}^{3}C_3 = 2^3 = 8$$

{ }   {A}   {A, B}
       {B}   {A C}   {A, B, C}
       {C}   {B C}

$${}^{n}C_0 + {}^{n}C_1 + {}^{n}C_2 + \dots + {}^{n}C_r + \dots + {}^{n}C_n = 2^n$$

$$\Downarrow \quad \Downarrow \quad \Downarrow \qquad \Downarrow \qquad \Downarrow \quad \Downarrow$$

$$\boxed{1 + 1 + 1 + \dots + 1 + \dots + 1 = n+1}$$

when objects are Identical//

Articles —

(1) If ~~there~~ p identical object of 1 bag,
q " " " 2nd bag,
r " " " 3rd bag
and rest s things are different.

$(00000\ldots0)\ (\Delta\Delta\Delta\Delta\ldots\Delta)\ (\square\square\square\ldots\square)\ (x_1, x_2, x_3 \ldots)$
p q r s

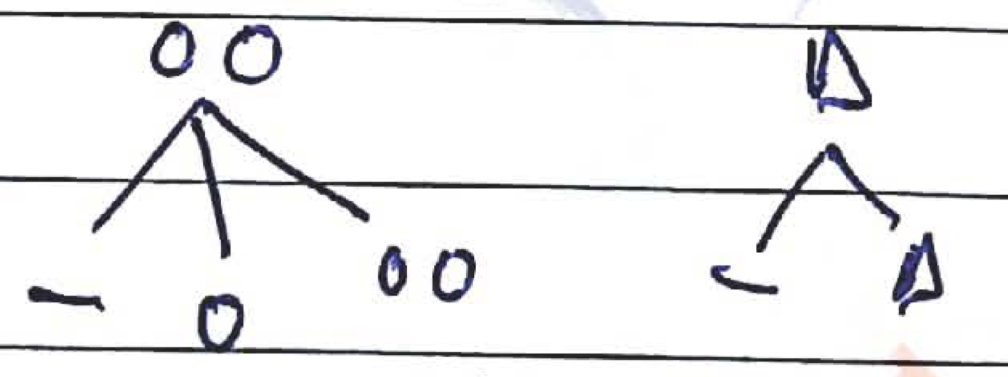
The total no of combination's of this p+q+r+s objects taken any at a time is equal to

Total combination taken any at a time $= \left[(p+1)(q+1)(r+1)\left[\frac{(1+1)(1+1)(1+1)\ldots(1+1)}{r}\right]\right]$ $\left[{}^sC_0 + {}^sC_1 + {}^sC_2 + \ldots + {}^sC_s\right]$

So

$$\boxed{\text{Total no of combination taken any at a time} = \left[(p+1)(q+1)(r+1)\,2^s\right]}$$

eg:-



$(2+1)\ (1+1) = 6$

(2) The total no of combination's of these p+q+r+s objects taken $\boxed{\text{at least one at a time}}$ is equal to

$$\boxed{\left[(p+1)(q+1)(r+1)(2^s)\right] - 1}$$

(3) The ~~total~~ no of combination of $p+q+r+s$ objects taken $\boxed{\text{at least one of each kind}}$

$$= (p+1-1)(q+1-1)(r+1-1)\{(1)(1)(1)(1)\cdots(1)\}$$

$$= (p)(q)(r)$$

$$= pqr$$

9) If a person have three identical coin of 1 rupee, 4 identical of 2 rupees and 4 identical coin of 5 rupees. in how many ways a person give donation to a beggar in the help of these coins

Sol

(1 1 1) (2 2 2 2) (5 5 5 5)

Total donation $= [(3+1)(4+1)(5+1)] - 1$

$$= \{(4\times5\times6) - 1\}$$

(donation न देने को तो घटाना होगा)

$$= 120 - 1$$

$$= 119$$

eg) In above question if at least one coin of 5 rupees have been given

Total donation $= [(3+1)(4+1)(5)]$

$$= 100$$

(Note - empty नहीं है इसलिए "1" नहीं होगा)

eg) In above ques. if at least one coin of each type has to be given

Total donation $= 3\times4\times5$

(1) Linear permutation (or Arrangement)

A-1 The total no. of linear arrangements (permutation) of "n" different objects taken all at a time $= \lfloor n$ or ${}^nP_n$

$x_1, x_2, x_3, x_4 ------ x_n$

$n \times (n-1) \times (n-2) ------ 1 \Rightarrow \lfloor n$ or ${}^nP_n$

Q) Out of 5 ladies and 5 Gent's —

In how many ways these 10 person can be seated on chair in a row such that —

(i) all ladies always seat together

(80|1)

Sol/

$x, x, (\lambda\lambda\lambda\lambda\lambda), x, x, x$

$= \lfloor 6 \times (\lfloor 5)$ Ans

Total arrange — Ladies also arrange

(ii) If all ladies always seat together as well as all gents also seat together

Sol/

$(\lambda\lambda\lambda\lambda\lambda)$ (22222)

$\Rightarrow (\lfloor 5 \; \lfloor 5) \times 2$ Ans

Ladies arrange — Gents arranged — (both arranged)

(iii) No any two ladies seat together.

× G × G × G × G × G ×

$(\lfloor 5) \times ({}^{6}C_{5} \times \lfloor 5)$

Arrangem of Gents / selection of ladies / Arrangement of ladies

पहले Gents को बिठा दिया फिर Ladies को Select करके बैठाएँगे

(iv) Neigher any two ladies togeth Nor any two gents together

Sol^n

M-1

— — — — —

$(LG\,LG\,LG\,LG\,LG)$ or $(GL\,GL\,GL\,GL\,GL)$

$(\lfloor 5 \times \lfloor 5)$ + $(\lfloor 5 \times \lfloor 5)$ Ans

$= (\lfloor 5 + \lfloor 5) + (\lfloor 5 \times \lfloor 5)$ Ans

M-2

• — ⊙ — ⊙ — ⊙ — ⊙ — •

$\lfloor 5 ({}^{4}C_{4} \times {}^{2}C_{1}) \times \lfloor 5$

Date
Page

INTERMEDIATE

Vowel = I, I, E, E, E, A

Const = T, T, R, M, N, D

Q) By using all the alphabets of the given word at a time

a) How many different words other than itself can be found.

$$\Rightarrow \left(\frac{\lfloor 12}{\lfloor 2 \lfloor 3 \lfloor 2}\right) - 1$$

I E T

[other than itself or Rearrange से शब्द की संख्या आएं तो −1 होगा]

(b) If vowel always remain together

T, T, (I, I, E, E, E, A) R, M, N, D

$$\frac{\lfloor 7}{\lfloor 2} \left(\frac{\lfloor 6}{\lfloor 2 \lfloor 3}\right)$$

(T) I E

(c) No any two vowels remain together

. T . T . R . M . N . D .

$$\frac{\lfloor 6}{\lfloor 2}\left({}^{7}C_{6} \times \frac{\lfloor 6}{\lfloor 2 \lfloor 3}\right.$$

(1) sol.

$$\underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{2}\ \underline{3} = \frac{\lfloor 7}{\lfloor 5}$$

or

$$\underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{2}\ \underline{2}\ \underline{2} = \frac{\lfloor 7}{\lfloor 3}$$

Circular permutation

(A1) The total no. of c.p of n. diff objects taken all at a time is equal to $\lfloor n-1$

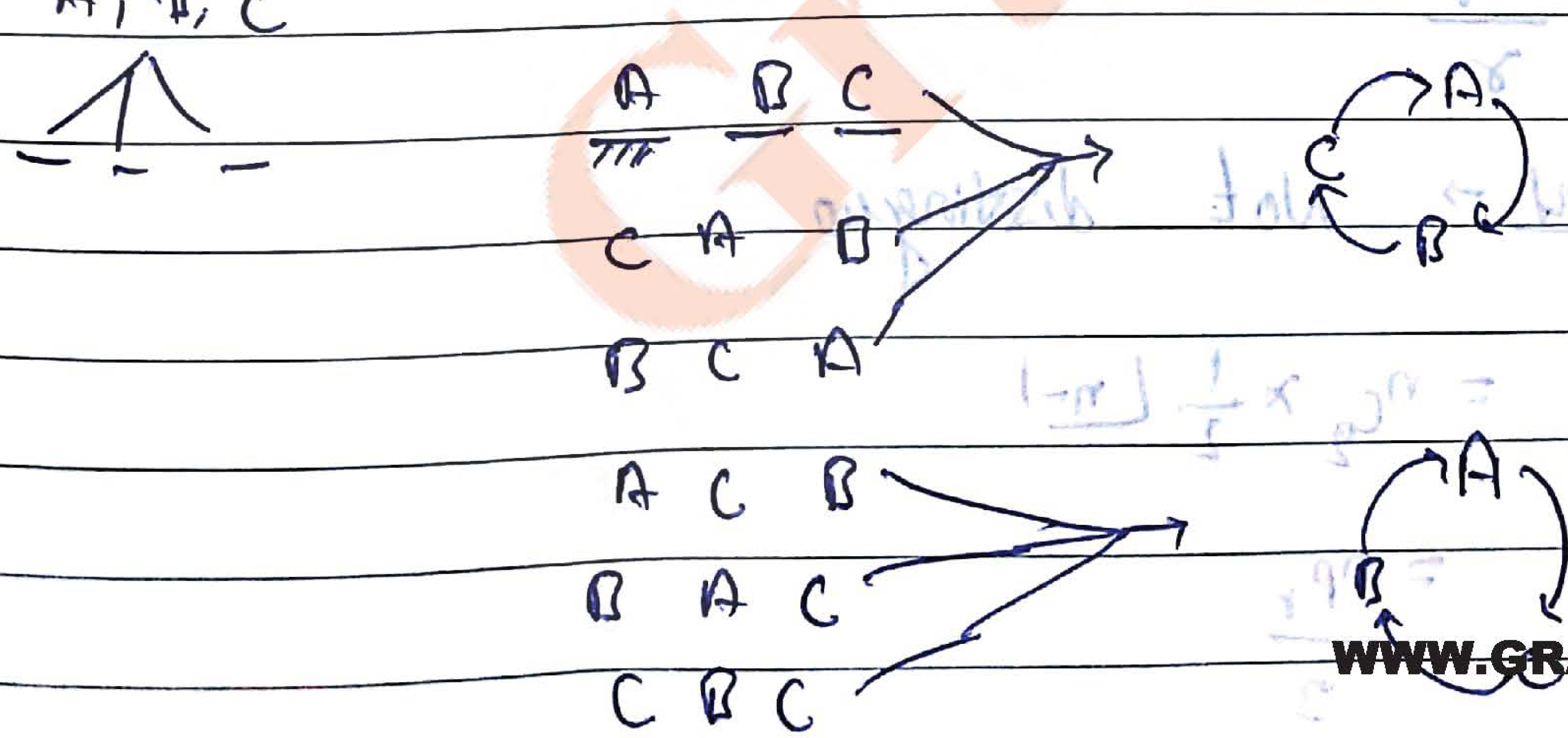
n-objects
↓

[No. of linear arrangement] = (n) times (No. of circular arrangement)

$$\lfloor n = n x$$

$$x = \frac{\lfloor n}{n} = \lfloor n-1$$

A, B, C



Note 1.) In these $\lfloor n-1$ circula arrangement clockwise arrangement and its anticlockwise arrangement are considered as diff arrangement. (119 / L-1)

(2) some time we can't be distinguished clockwise and anticlockwise the case of neck and lass then clockwise arrangement and its anticlockwise arrangement will be considered as single arrangement therefore in this case the total no. of circula permutation to n-diff. objects taken all at a time is equal to $\frac{1}{2}\lfloor n-1$ (120 / L-1)

[A-2] The total no of circula permutation of n-diff objects taken 'r' at a time is equal to (L-1 / 124)

case 1st → clockwise and its anticlockwise arrangement is distinguisable.

$= {}^nC_r \cdot \lfloor r-1$

$= {}^nC_r \times \frac{\lfloor r}{r}$ (121 / L-1)

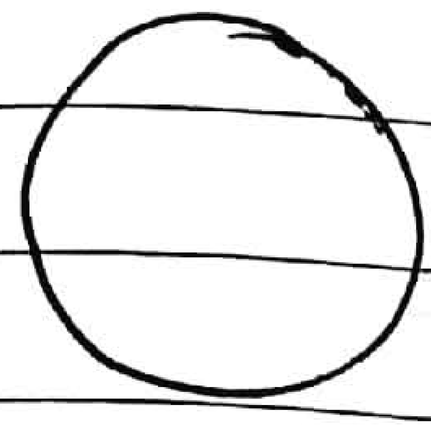
$= \frac{{}^nP_r}{r}$

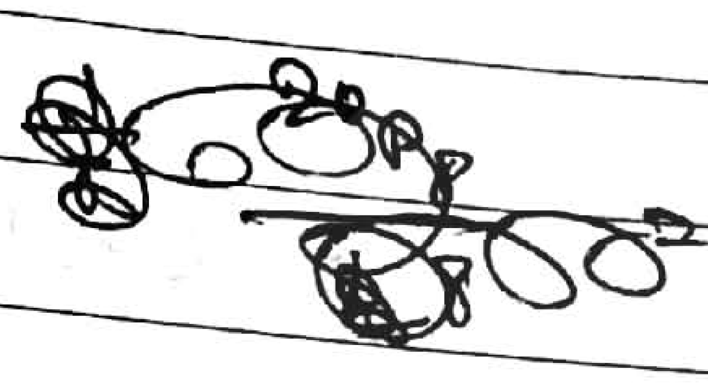
case 2nd → Not distingue

$= {}^nC_2 \times \frac{1}{2}\lfloor n-1$ (Q12 / L-1)

$= \frac{{}^nP_r}{2}$

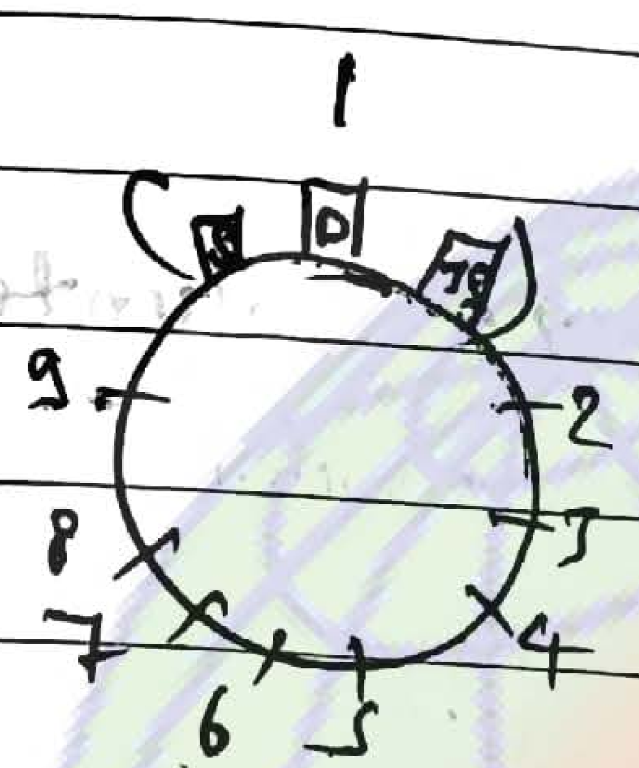
(119 L-1)





$\Rightarrow \frac{1}{8} \times {}^{20}P_8$

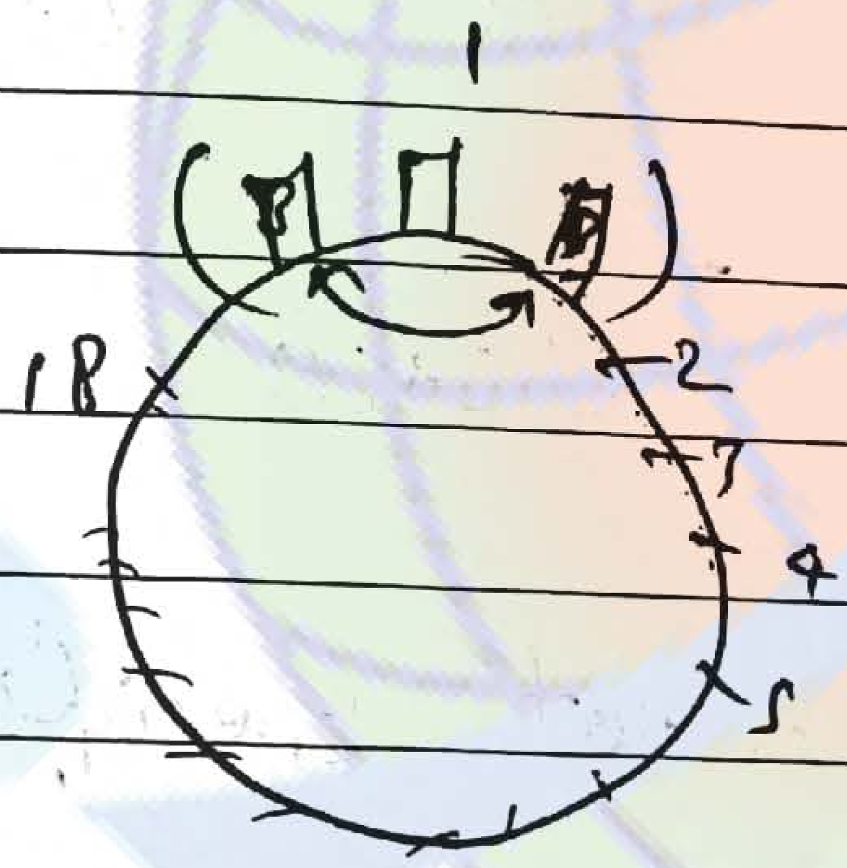
(120 L-1)



$= \lfloor 9-1 \times \lfloor 2$

$= \lfloor 8 \times \lfloor 2$

(L-1 124)

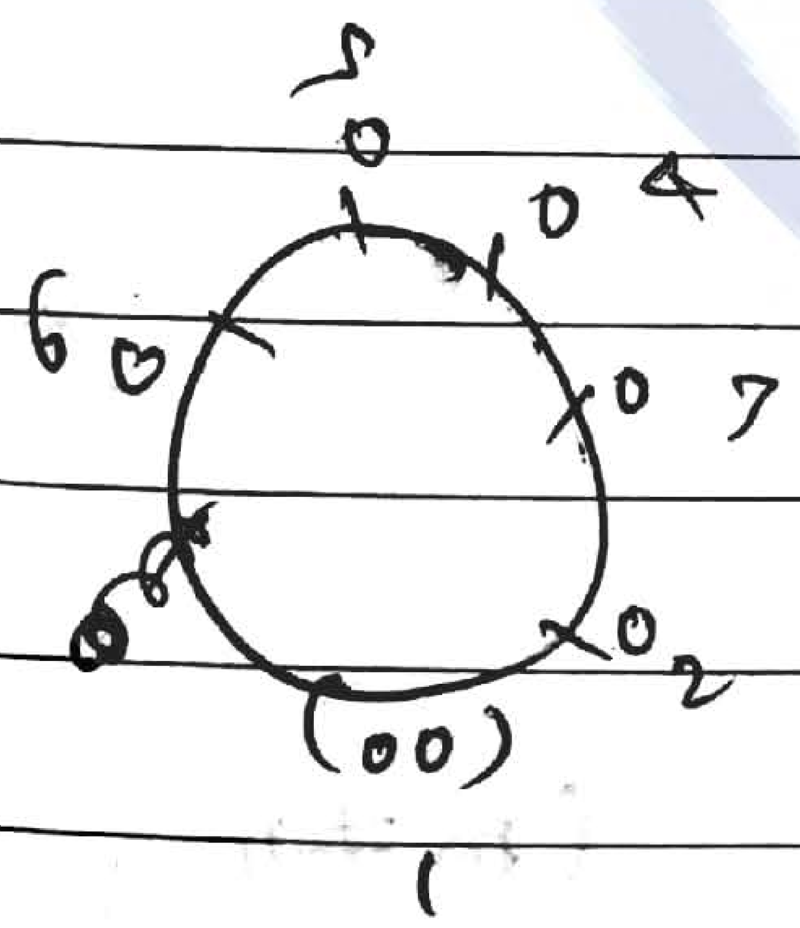


$= {}^{18}C_1 \times \lfloor 1 \times \lfloor 2 \, \lfloor 18-1$

$= 18 \times \lfloor 2 \, \lfloor 17$

$= \lfloor 18 \times 2$

(21 -1)



$= \lfloor 6-1 \times \lfloor 2$

$= \lfloor 5 \times \lfloor 2 \times \frac{1}{2}$

$= 120$

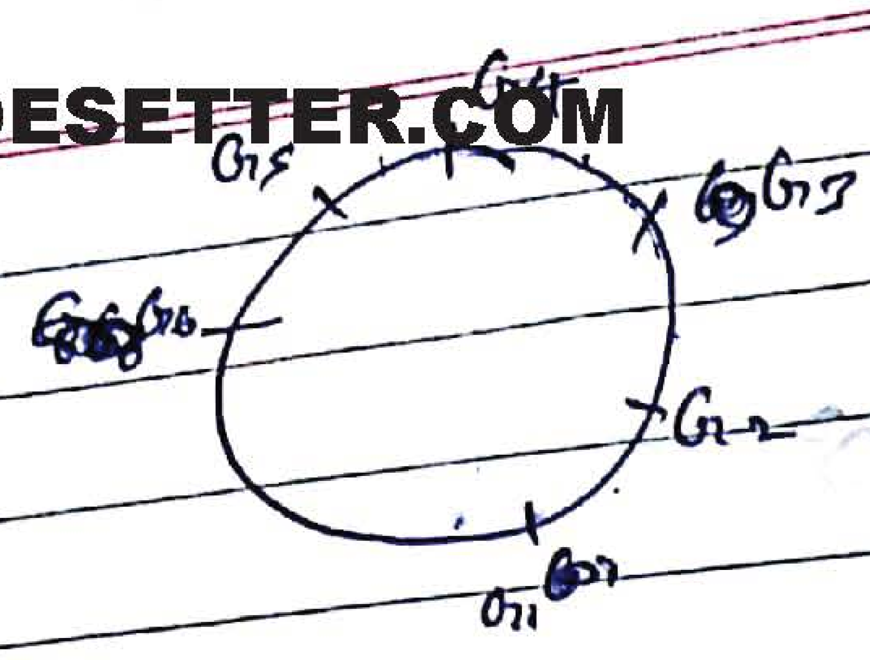
(clockwise and anticlockwise No of distinct(10) in Ring)

6 beads and 5 beads

${}^{6}C_4 \times {}^{5}C_4 \left[\frac{1}{2} \lfloor 8-1 \right]$

(6G, 5L)

$\begin{pmatrix} L-4n \\ G-5 \end{pmatrix}$



$= (6-1)! \; {}^6C_5 \times 5!$

$= 5!\, 5!\, 6!$

★ Division into groups →

Q) The no. of ways in which 20 volunteers can be divided into groups of 4, 7 and 9 persons

Sol^n Total ways $= {}^{20}C_4 \times {}^{16}C_7 \times {}^{9}C_9 = \dfrac{20!}{4!\,16!} \times \dfrac{16!}{7!\,9!}$

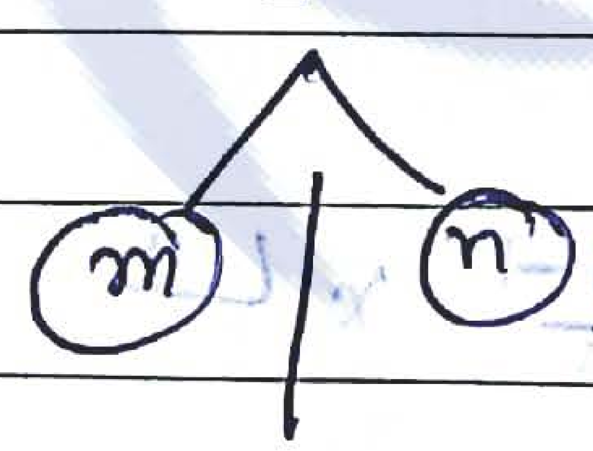
$= \dfrac{20!}{4!\,7!\,9!}$

Case 1st →

when name of the group is not specified

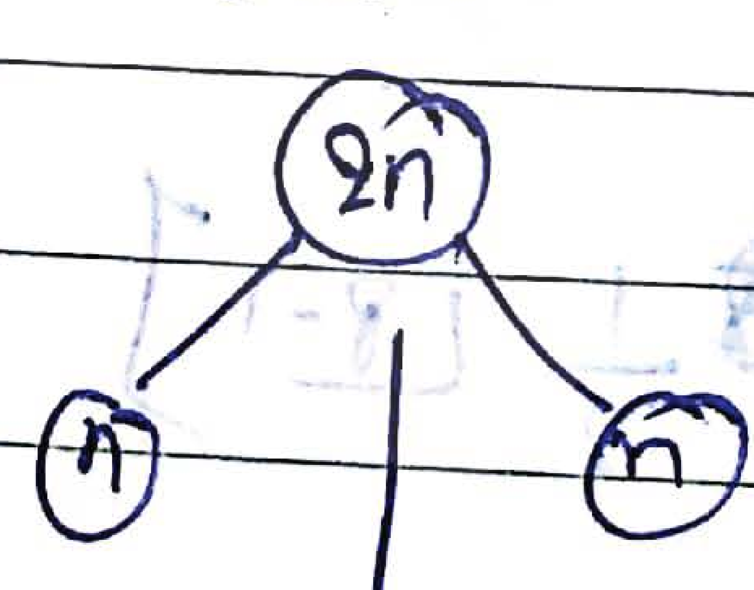
(a) unequal distribution

(m+n) diff objects



Total ways $= \dfrac{(m+n)!}{m!\,n!}$ $(m \neq n)$

(b) equal distribution $(m=n)$



Total ways $= \frac{(2n)!}{n!\,n!} \times \frac{1}{2!}$

Case 2nd) -

when name of the group is specified

(a) unequal distribution

(i) The total no. of ways in which (m+n) diff objects can be distributed in to ~~two~~ two group "A and B" such that group "A" will get "m" things and group "B" will get n-things ($m \neq n$)

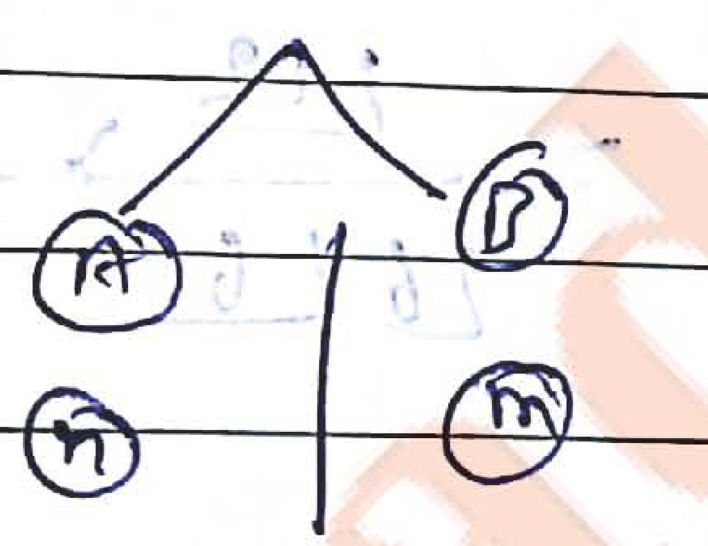
$$\boxed{= \frac{(m+n)!}{m!\,n!}}$$

(ii) The total no. of ways in which (m+n) diff object can be distributed in to two groups "A" and "B" such that any one group will get "m" things and other will get "n" things ($m \neq n$)

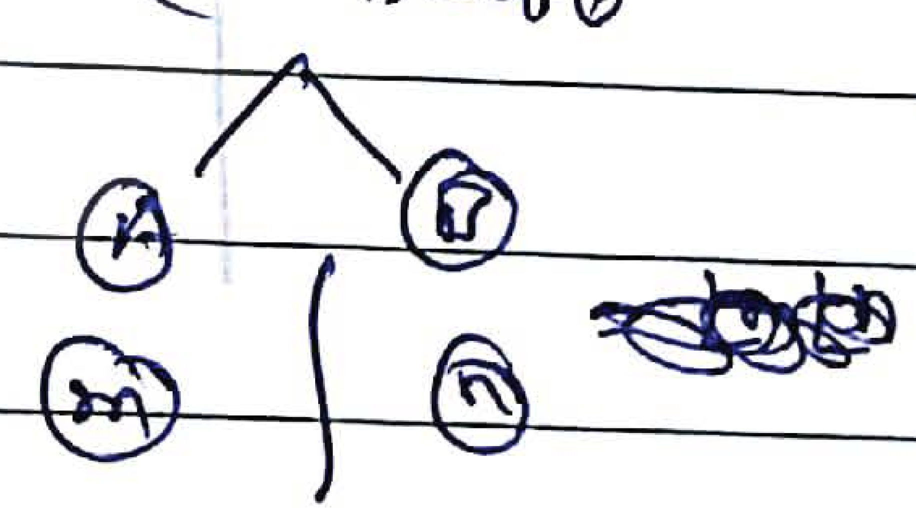
(m+n) diff

or

(m+n) diff



(m+n) diff



$= \frac{(m+n)!}{m!\,n!} \times 2!$

So

Total ways $= \frac{(m+n)!}{m!\,n!} + \frac{(m+n)!}{m!\,n!}$

Rank of word in Dictionary

Note
1) words before given word = Rank − 1
2) words after given word = Total words − RANK
3) Rank in Inverse order = [total words] − [Rank − 1]

Find the Rank of the following words in the Dictionary
i) RANDOM
ii) MOTHER
iii) SACHIN

iv) SPOON
v) COCHIN
vi) CRICKET

start what line

$(5\times L5)\ (0\times L4)\ (2\times L3)\ (0\times L2)\ (1\times L1)\ (1\times L0)$ — exceptional

R A N D O M

⇓

A D M N O R

$\cancel{A}$ $\cancel{D}$ $\cancel{M}$ N O $\cancel{R}$ — 5, 2

$600 + 0 + 12 + 0 + 1 + 1 = 614$ Ans

Note ⇒ 1) Right Side से L1, L2 ... लिखना शुरू करें
2) Right Side का पहला word L1 के साथ 1 का multiply करें

iii)

Exponent of any Prime number in $\lfloor n$

$\lfloor 5 = 5 \times 4 \times 3 \times 2 \times 1$

$\lfloor 5 = (2)^3 (3)^1 (5)^1$

$\lfloor 100 = (2)^a \times (3)^b \times (5)^c \times (7)^d \times \cdots \times 97$

$a = \left[\frac{100}{2}\right] + \left[\frac{100}{2^2}\right] + \left[\frac{100}{2^3}\right] + \left[\frac{100}{2^4}\right] + \left[\frac{100}{2^5}\right] + \left[\frac{100}{2^6}\right]$

$= 50 + 25 + 12 + 6 + 3 + 1$

$= 97$

(b) $b = \left[\frac{100}{3}\right] + \left[\frac{100}{3^2}\right] + \left[\frac{100}{3^3}\right] + \left[\frac{100}{3^4}\right] + \cdots$

$= 33 + 11 + 3 + 1$

$= 48$

short trick

$a = 50 + 25 + 12 + 6 + 3 + 1 = 97$

Divide 100 by 2, Divide 50 by 2, Divide 25 by 2, divide 12 by 2, divide 6 by 2, Divide ... by ...

$b = 33 + 11 + 3 + 1$

$c = 20 + 4 = 24$

$d = 14 + 2 = 16$

$$\lfloor 100 = (2)^{97} \times (3)^{48} \times (5)^{24} \times (7)^{16} \times \text{---} (97)^{1}$$

* find no. of zero's at the end of $\lfloor n$ after simplification

$$\lfloor 100 = [\text{---}\ \text{---}\ \underbrace{(00000 \ldots\ldots 0)}_{24}]$$

$$\boxed{\text{No. of Zero at end} = \text{Exponent of 5 in } \lfloor n}$$

Q) find digit at tens place in expansion of $[\lfloor 1 + \lfloor 2 + \lfloor 3 + \text{------} + \lfloor 50]$

sol

$\lfloor 1$ = --- 01

$\lfloor 2$ = -- 02

$\lfloor 3$ = --- 06

$\lfloor 4$ = --- 24

$\lfloor 5$ = --120

$\lfloor 6$ = --720

$\lfloor 7$ = -5040

$\lfloor 8$ = --880

$\lfloor 9$ = -- 00

$\lfloor 10$ = --00

$\lfloor 11$ = --00

$\lfloor$

--

T U

13

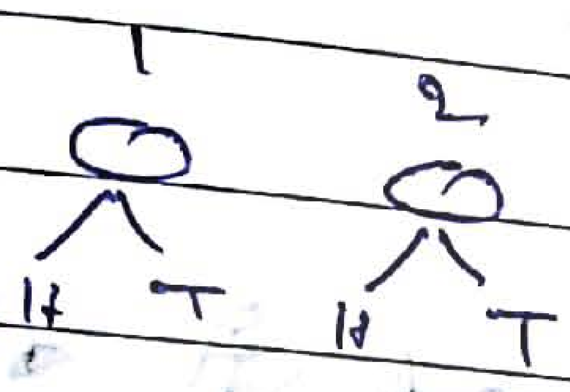
Ans 1

Probability

It is a ~~chance~~ $\boxed{\text{chance}}$ of an occurence of an event

Experiments → Total no. of exhaustive events ; All possible outcomes of an event

eg. →

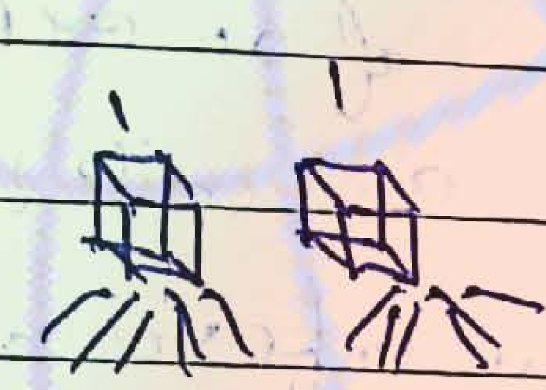


← Non - Favourable events

[Total exhaustive events → $2 \times 2 = 4$]

Sample space = $S = \{(H, H), (H, T), (T, H), (T, T)\}$

eg 2.)



Total = $6 \times 6 = 36$

Sample space → (1,1) (2,1) (3,1) (4,1) (5,1) (6,1)
(1,2) (2,2) (3,2) (4,2) (5,2) (6,2)
(1,3) (2,3) (3,3) (4,3) (5,3) (6,3)
(1,4) (2,4) (3,4) (4,4) (5,4) (6,4)
(1,5) (2,5) (3,5) (4,5) (5,5) (6,5)
(1,6) (2,6) (3,6) (4,6) (5,6) (6,6)

Three language of Probability

1) If two cards are drawn at random from 52 playing cards then total no. of exhaustive events is equal to

$$= {}^{52}C_2$$

(2) If two cards are drawn one by one (without Replacement) from a bag of 52 playing cards, then total no. of exhaustive events

$$= 52 \times 51 = {}^{52}C_2 \times \lfloor 2 = {}^{52}P_2$$

(iii) If two cards are drawn one-by-one with replacement then total no. of exhaustive events is equal to

$= 52 \times 52$

Maa → Favourable event's →

Total तरीके में event को जितनों favour करे।

So,

favourable + unfavourable = 1

(event का होना)

All those particular outcomes of an experiment in which a given event may happen are called favourable events ~~of that~~ otherwise they are unfavourable

Therefore

* $\boxed{\text{favourable} + \text{unfavourable} = \text{Total Exhaustive events}}$

eg: एक dice में even नंबर आने का favour $= \frac{3}{6}$ (2, 4, 6)

* Sum of two dice →

Sum of two dice	favourable events	events
2	1	(1,1)
3	2	(2,1), (1,2)
4	3	(2,2), (3,1), (1,3)
5	4	
6	5	
7	6 (max	
8	5	
9	4	
10	3	
11	2	(6,5), (5,6)
12	1	(6,6)

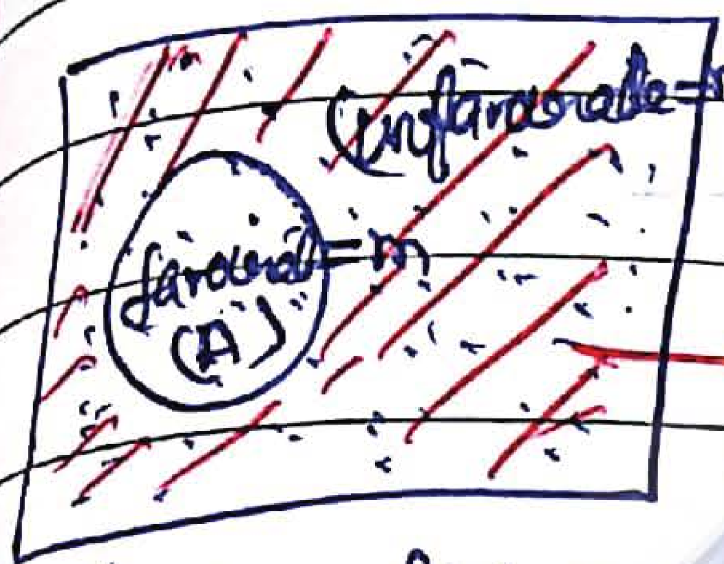
mathematical Definition of Probability →

Experiment

⇓

Samplespace

⇓



$n(S) = n$ (let)

$\to \bar{A}$ ("A complement")

(Venn digram)

so,

$$P(A) = \frac{\text{favourable}}{\text{favourable} + \text{unfavourable}} = \frac{\text{fav.}}{\text{total}}$$

$$\boxed{P(A) = \frac{m}{n}} \times 100 = \%$$

$$P(\bar{A}) = \frac{n-m}{n}$$

$$P(A) + P(\bar{A}) = \frac{m}{n} + \frac{n-m}{n}$$

$$P(A) + P(\bar{A}) = \frac{n}{n}$$

$$= 1$$

So $\boxed{P(A) + P(\bar{A}) = 1}$ ← (i)

$$\boxed{P(\bar{A}) = 1 - (P(A))}$$

$\boxed{0 \le m \le n}$

min max

$\frac{0}{n} \le \frac{m}{n} \le \frac{n}{n} \Rightarrow \boxed{0 \le P(A) \le 1}$

0

(min) (max)

$\boxed{P(A) = 0} \Rightarrow$ Event "A" is Impossible

$\boxed{P(A) = 1} \Rightarrow$ Event "A" is sure events.

★ ODD's in favour and odd's in against of any events →

(1) odds in $\boxed{\text{favour}}$ of event $\boxed{\text{"A"}}$ $= \frac{P(A)}{P(\bar{A})}$ or

$m : (n-m)$

↓ ↓

(fav) (unfav)

(2) odds in $\boxed{\text{against}}$ of event $\boxed{\bar{A}}$ =

(3) odd in $\boxed{\text{favour}}$ of event $\boxed{\bar{A}}$ $= \frac{P(\bar{A})}{P(A)} = \underbrace{n-m}_{\text{unfav}} : \underbrace{m}_{\text{favou}}$

(4) odd in $\boxed{\text{against}}$ of event $\boxed{A}$ ⇒

Q1 $P(A) = \frac{3}{7}$

$P(A) = \frac{3}{3+4} = \frac{\text{fav}}{\text{fav} + \text{unfav}}$

Set-theory ⇒ (Three language of set theory)

$\boxed{\text{or} = \cup = +}$ $\boxed{= \text{या}}$

$\boxed{\text{and} = \cap = \times}$

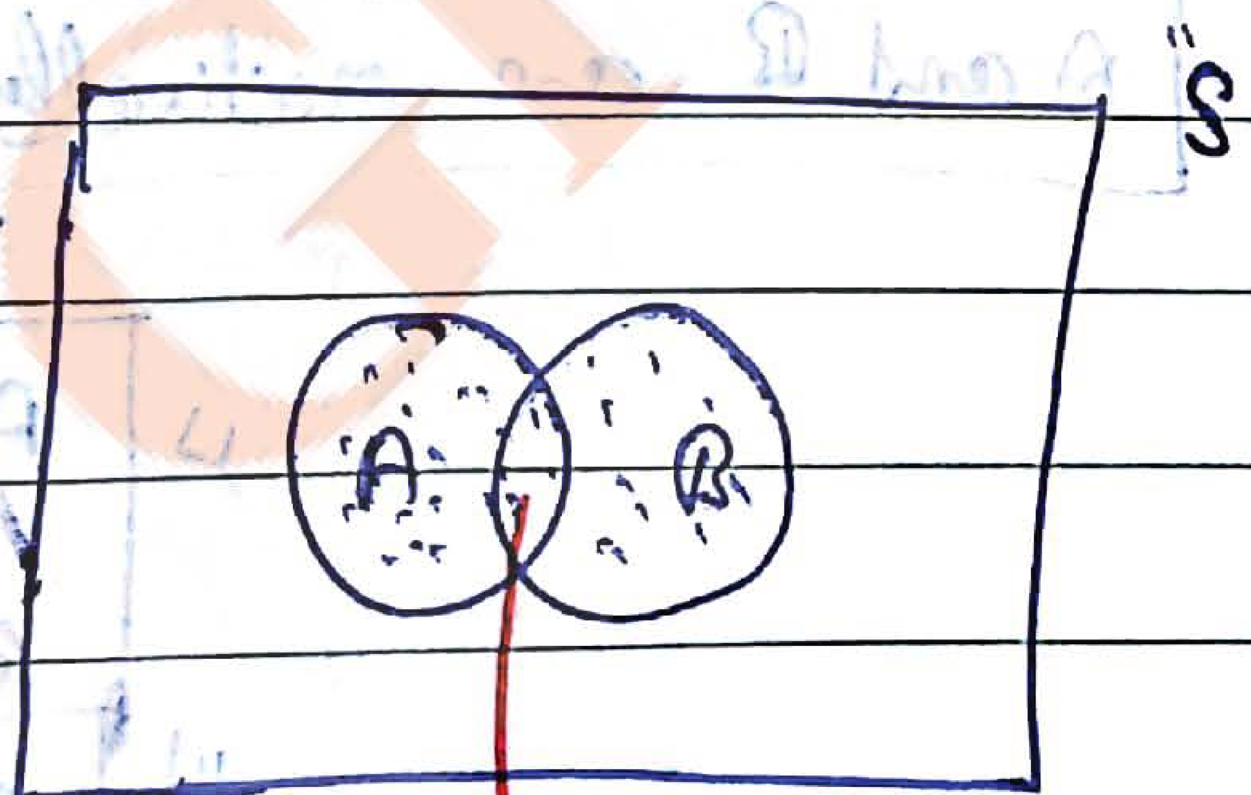
Probability of occurrence of $\boxed{\text{either}}$ (कम से कम एक) event "A" or event "B"

or

Probability of occurrence of $\boxed{\text{atleast one}}$ event out of two events "A" and "B"

$$\boxed{\Rightarrow P(A \cup B)}$$

Case No 1 :→



$(A \cap B)$ ⇒ A and B

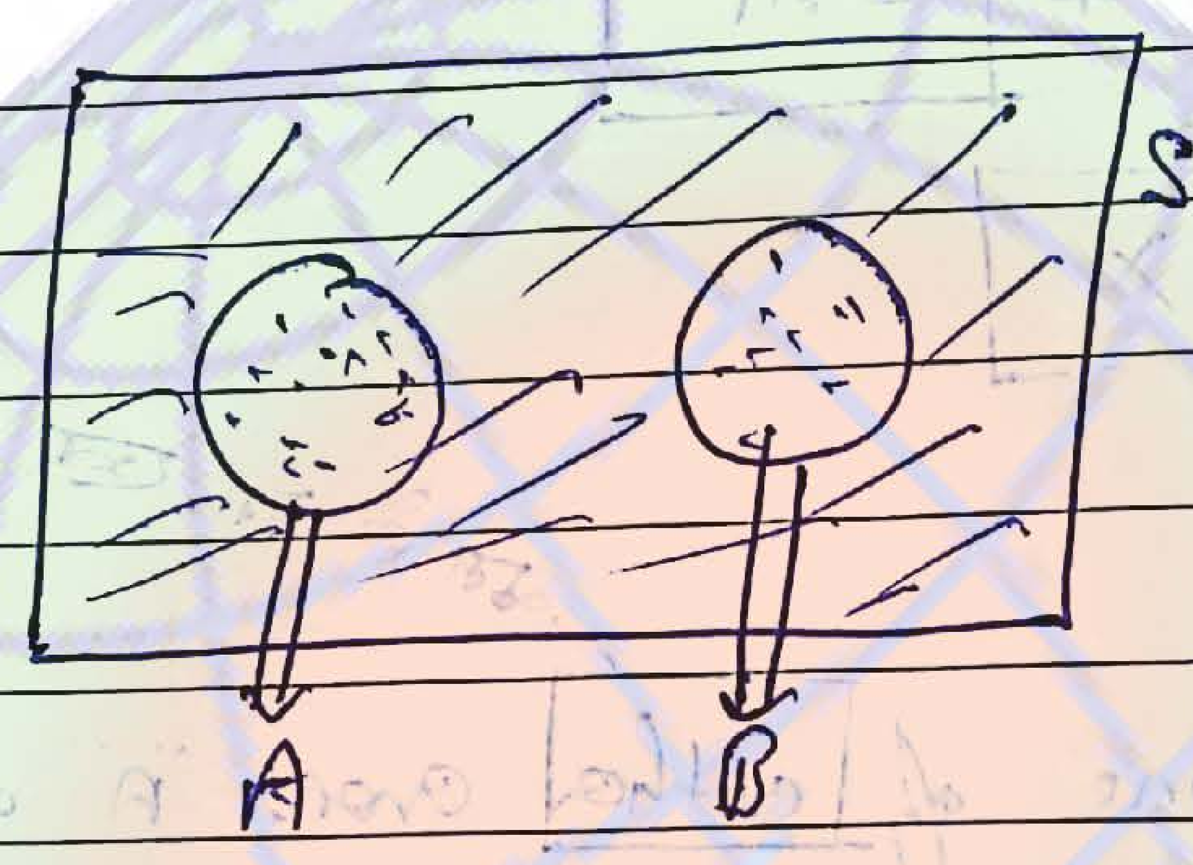
$$\frac{n(A\cup B)}{n(S)} = \frac{n(A)+n(B)-n(A\cap B)}{n(S)}$$

$$\boxed{P(A\cup B) = P(A)+P(B)-P(A\cap B)}$$

* $\boxed{C-2}$ ⇒ If $\boxed{\text{A and B are mutually exclusive}}$

⇓

$$\boxed{A\cap B=\phi}$$



$$\frac{n(A\cup B)}{n(S)} = \frac{n(A)+n(B)-0}{n(S)}$$

$$\boxed{P(A\cup B) = P(A)+P(B) \le 1}$$

↳ addition theorem of Probability

* $\boxed{C-3}$ ⇒ If $\boxed{\text{A and B are mutually exclusive and exhaustive}}$

⇓

i) $A\cap B=\phi$ → mutually exclusive

~~[illegible]~~

ii) $A\cup B=S$ → exhaustive

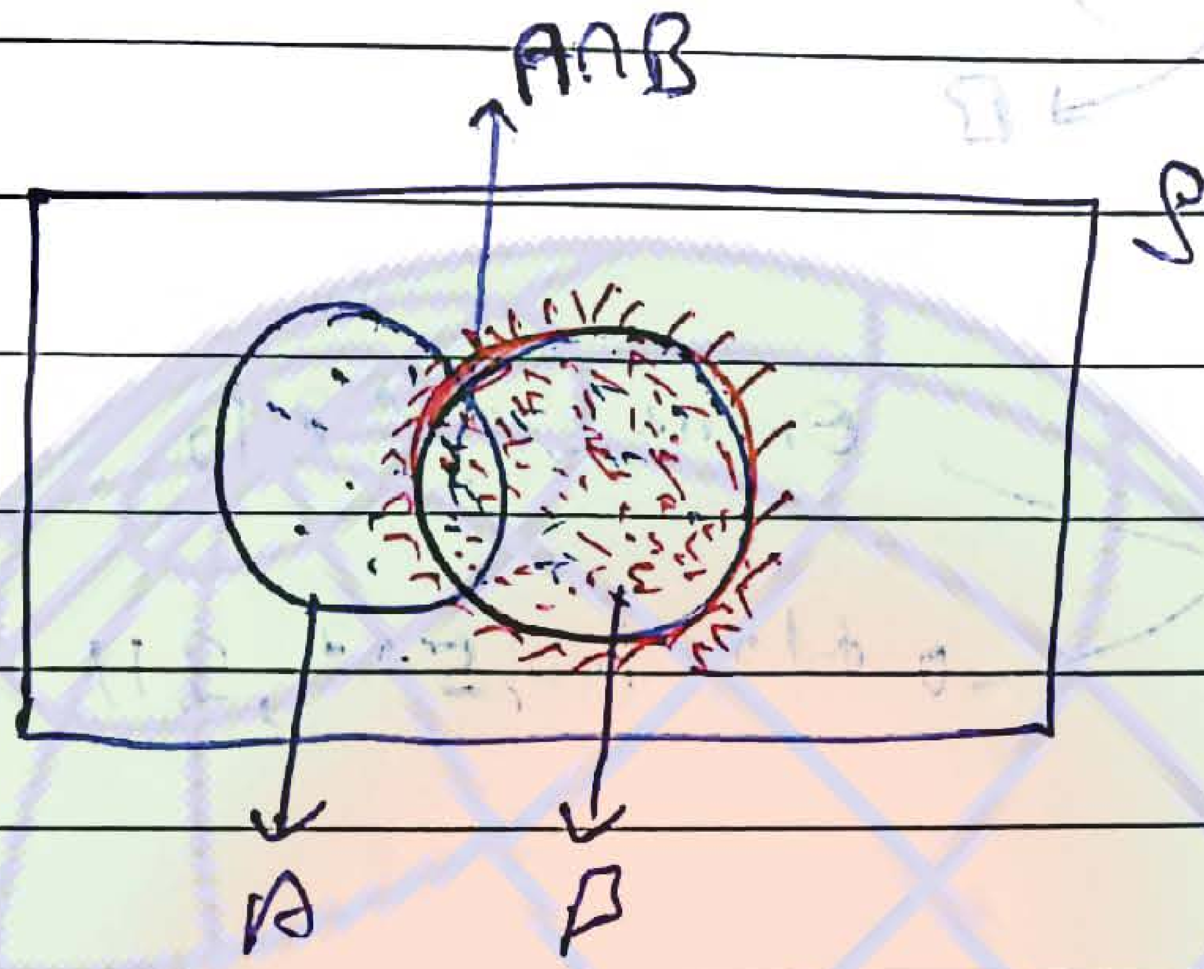
 (Probability)

classmate Date ______ Page ______

Conditional probability –

(i) $P\left(\frac{A}{B}\right)$ = Probability of occurrence of event "A" if "B" has already occured.

(ii) $P\left(\frac{B}{A}\right)$ = probability of occurrence of event "B" if "A" has already occured.



$$\boxed{P\left(\frac{A}{B}\right) = \frac{n(A\cap B)}{n(B)}}$$

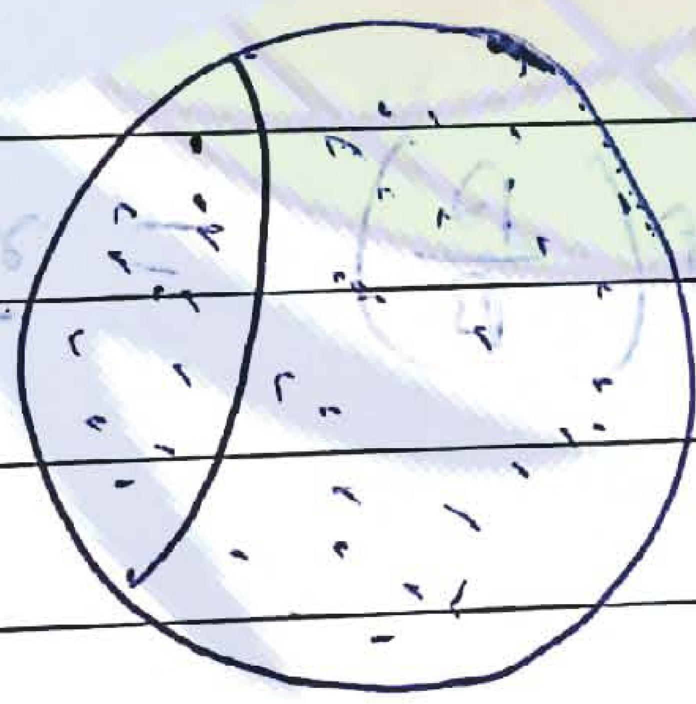
or

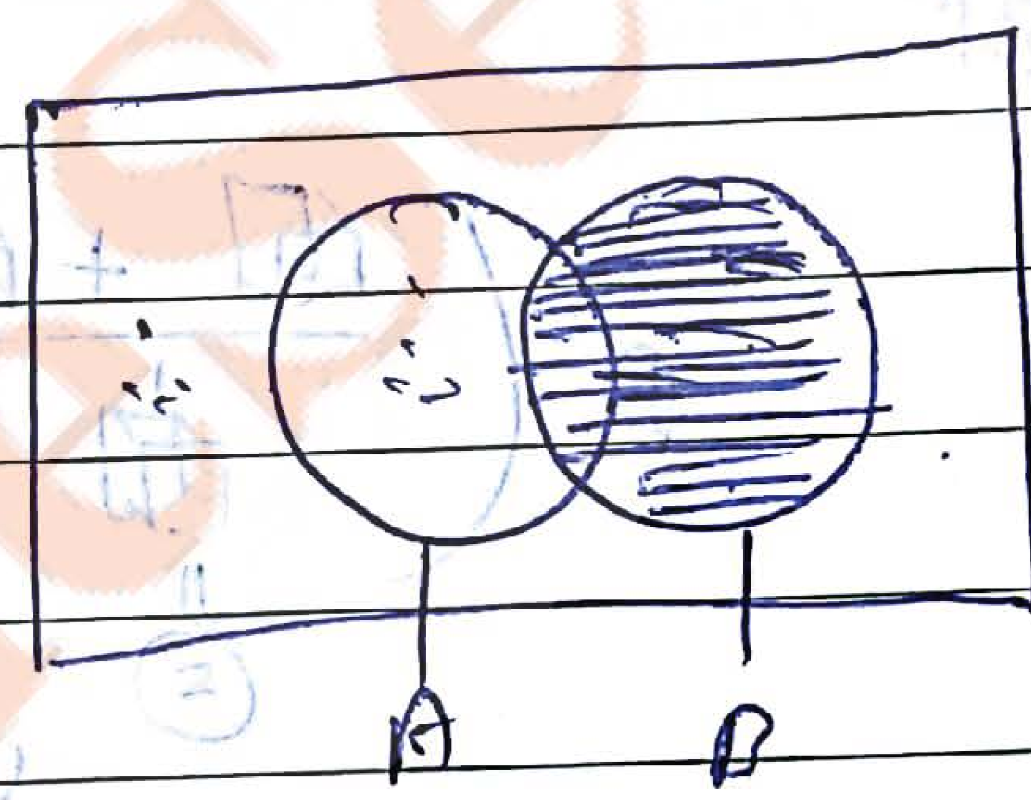
$$\boxed{P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}}$$

Note

If event B has already occurs

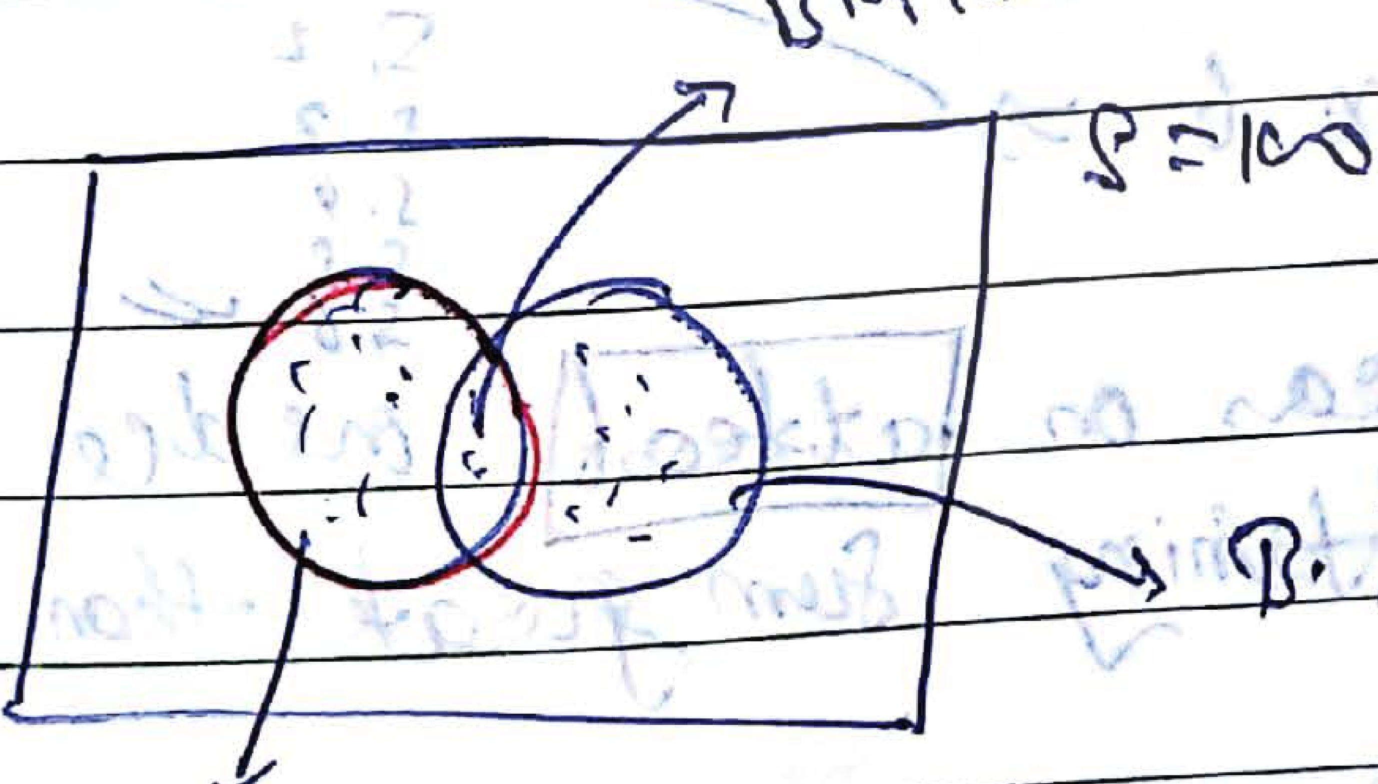
If B has not occures





$$\boxed{P\left(\frac{A}{B}\right) + P\left(\frac{\bar{A}}{B}\right) = 1}$$

$$\boxed{P\left(\frac{A}{\bar{B}}\right) + P\left(\frac{\bar{A}}{\bar{B}}\right) = 1}$$



$$P\left(\frac{BE}{BH}\right) = \frac{{}^{15}C_1}{{}^{40}C_1}$$

Dependent events

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$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = \boxed{P(B) \times P\left(\frac{A}{B}\right)} \quad \text{or} \quad \boxed{P(A) \times P\left(\frac{B}{A}\right)}$$

$$P(A \cap B \cap C) = P(A) \times P\left(\frac{B}{A}\right) \times P\left(\frac{C}{A \cap B}\right)$$

Independent events:

Two or more than two events are said to be independent if the occurrence or non-occurrence of any one event does not affect the occurrence or non-occurrence of any other events.

If A and B are independent events: —

(i) $P\left(\frac{A}{B}\right) = P(A)$

(ii) $P\left(\frac{B}{A}\right) = P(B)$

(iii)(a) $P(A \cap B) = P(A) \times P(B)$

(b) $P(A \cap \overline{B}) = P(A) \times P(\overline{B})$

(c) $P(\overline{A} \cap B) = P(\overline{A}) \times P(B)$

(d) $P(\overline{A} \cap \overline{B}) = P(\overline{A}) \times P(\overline{B})$

Multiplication theorem of probability.

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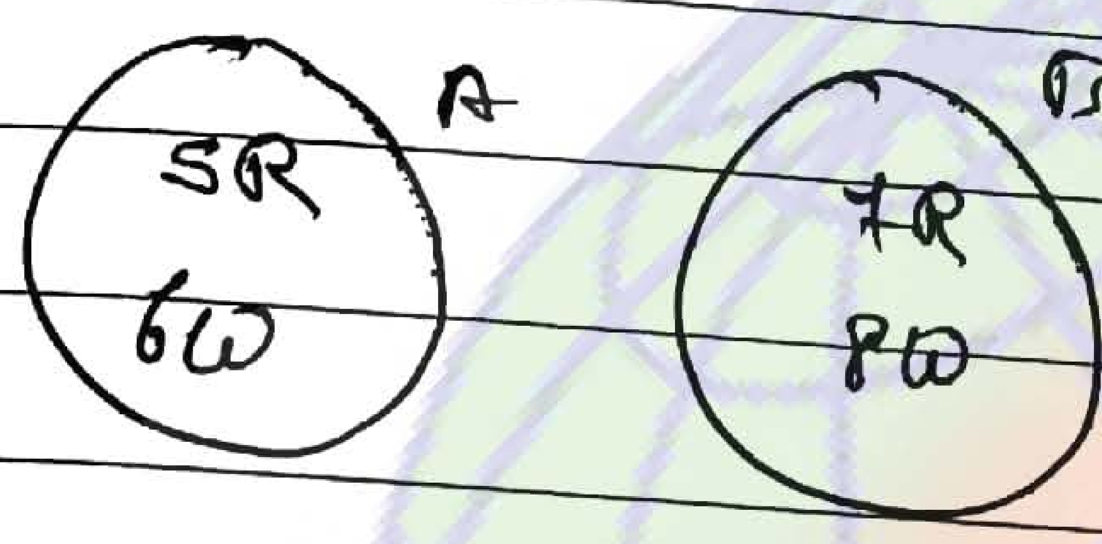
when in comp. event prob. of 1st event is asked and outcome of 2nd event is given

पहला (Prob) = ?
दूसरा परिणाम → पता है

(1)



E-1 E-2

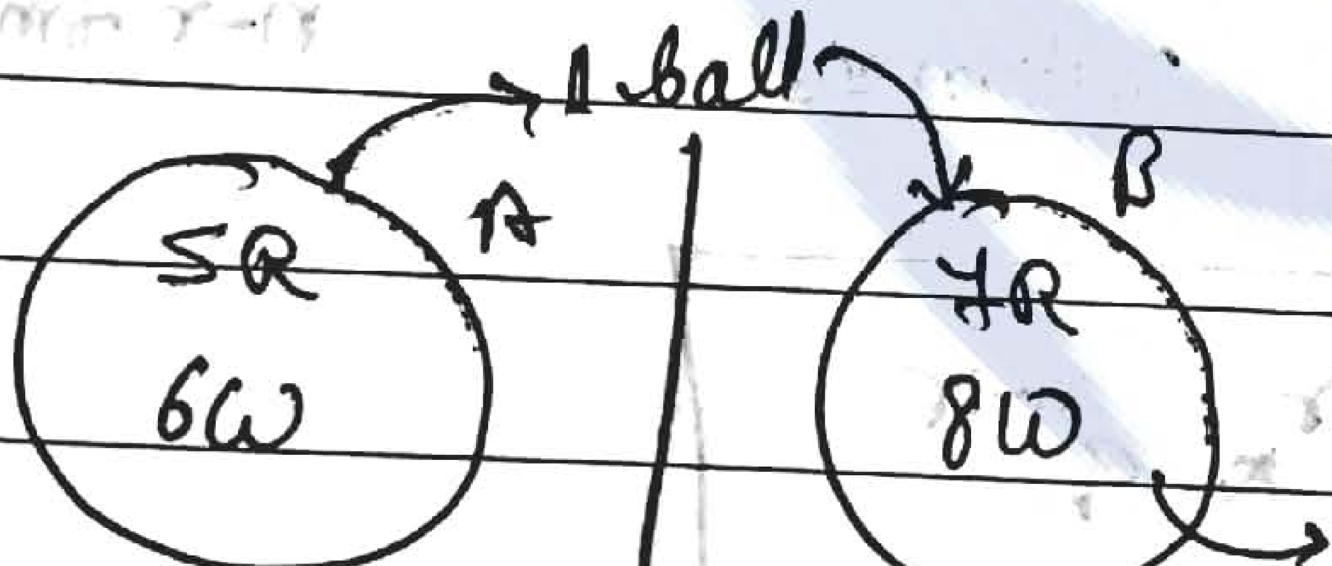
$\boxed{\text{A bag select}}$ then $\boxed{\text{a ball is}}$ $\boxed{\text{select}}$ which come red find the prob. that ball was drawn from bag B.

दूसरा का परिणाम E2

$$P\left(\frac{B}{R}\right) = \frac{P(B\cap R)}{P(R)} = \frac{P(B)\cdot P\left(\frac{R}{B}\right)}{P(B)\cdot P\left(\frac{R}{B}\right) + P(A)\cdot P\left(\frac{R}{A}\right)}$$

$$= \frac{\frac{1}{2}\cdot\frac{7}{15}}{\frac{1}{2}\cdot\frac{7}{15} + \frac{1}{2}\cdot\frac{5}{11}}$$

(E-1)



(E-2)

परिणाम

1 ball drawn
↳ which comes red.

Find the Prob. that, ~~which~~ white ball has been transfer.

→ Prob = ? that white transfer

?

Binomial Prob. distribution

Let the experiment is n-times under similar condition and if occurance of any one ~~particle~~ Particular event is called a success whose probability is "p", then its non-occurance is called a failure whose probability is "q".

$P(\text{success}) = p$

$P(\text{failure}) = q$

$\boxed{p+q=1}$ vvi

Experiment
⇓
n-times (Independently)
⇓

$P(r\ \text{success}) = {}^nC_r \times [\underbrace{(p \times p \times p \cdots p)}_{r\text{-times}} \times \underbrace{(q \times q \cdots \times q)}_{n-r\text{-times}}]$

$= \boxed{{}^nC_r \times p^r \times q^{n-r}}$

...of success "r"	0	1	---	r
...(r)	${}^nC_0 \times p^0 \times q^n$	${}^nC_1\, p^1 q^{n-1}$	---	${}^nC_r\, p^r q^{n-r}$

	---	n
	---	${}^nC_n\, p^n q^0$

Random variable Distribution $\xRightarrow{\text{Replace}}$ Binomial Probability Distribution.

"x_i"

x_i	x_1	x_2	x_3	---	x_i	--- x_n
F_i	F_1	F_2	F_3		F_i	F_n
P_i	$P_1 = \frac{F_1}{N}$	$P_2 = \frac{F_2}{N}$			$P_i = \frac{F_i}{N}$	$P_N = \frac{F_N}{N}$

r	0
$P(r)$	${}^nC_0\, p^0 q^n$

(i) $x_i = r$

(ii) $P_i = P(r) = {}^nC_r\, p^r q^{n-r}$

(iii) $p+q = 1$ ← Note

$f_1 + f_2 + \cdots + f_n = N$ = total exhaustive event

$P_1 + P_2 + P_3 + \cdots P_n = 1$

(i) Mean $(\bar{x}) = \sum_{i=1}^{n} \frac{x_i f_i}{N} = \sum_{i=1}^{n} x_i P_i \Rightarrow \sum_{r=0,1}^{n} r\, {}^nC_r\, p^r q^{n-r} = np$

(ii) variance $= \sum_{i=1}^{n} \frac{f_i (x_i - \bar{x})^2}{N} \Rightarrow \sum_{r=0}^{n} {}^nC_r\, p^r q^{n-r} (r-np)^2 = npq$

$= \sum_{i=1}^{n} P_i (x_i - \bar{x})^2$

(iii) standard deviation $= S.D = \sigma \Rightarrow \sqrt{npq}$

$= \sqrt{\text{variance}}$

Q.) The mean and variance of a binomial distribution are 4 and 2 resp. Then the probability of 2 success is –

Sol^n mean $= 4 \Rightarrow np = 4$

variance $= 2 \Rightarrow npq = 2$

$\frac{npq}{np} = \frac{2}{4} = \frac{1}{2}$

$q = \frac{1}{2}$

$p = \frac{1}{2}$

$n = 8$

Prob. of success

$P(r=2) = {}^8C_2 \left(\frac{1}{2}\right)^8$