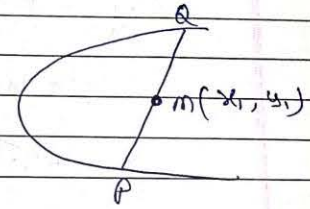
1. Parabola (continued)
2. ellipse
3. Hyperbola

4. Binomial theorem
5. Permutation and combination (contd)

Chord bisected at Given Point



Eqⁿ of chord of parabola whose mid-point is (x_1, y_1) is

$$\boxed{T = S_1}$$

Parabola :→ $y^2 = 4ax$
mid-point :→ (x_1, y_1)

Eqⁿ of chord :-

$$\boxed{yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1}$$

Example:→ Find the eqⁿ of chord of parabola $y^2 = 4x$ whose mid-point is $(4, 2)$

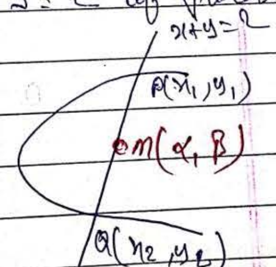
Ans:- $T = S_1$

$\Rightarrow yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$

$\Rightarrow y(2) - 2(x + 4) = (2)^2 - 4 \times a \times 4$

or, $x - y - 2 = 0$ Ans.

Example: Find the mid-point of chord $x + y = 2$ of parabola $y^2 = 4ax$



Ans:→ $x + y = 2$ —(1)

$y^2 = 4x$ —(2)

Now,

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Solving eq (i) & (ii)

$$y^2 = 4(2-y)$$

$$y^2 + 4y - 8 = 0 \quad \to y_1, y_2$$

$$\beta = \left(\frac{y_1+y_2}{2}\right) = \frac{-4}{2} = -2$$

(α, β) lies $x+y=2$

$$\alpha + (-2) = 2$$

$$\alpha = 4$$

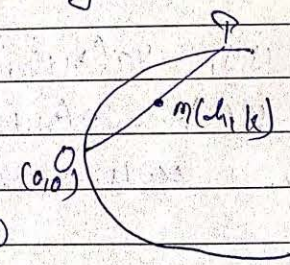
$$M(4, -2)$$

Example:→ Find the locus of mid point of chords of parabola $y^2 = 4ax$ (i) which passes through origin

(ii) whose slope is 'm'

(iii) which subtend the right angle at the vertex.

Ans:→ 1) OP :-



$$T = S_1$$

$$yk - 2a(x+h) = k^2 - 4ah \quad \text{—(i)}$$

Pass $(0,0)$

$$0 - 2a(0+h) = k^2 - 4ah$$

$$k^2 = 2ah$$

$$y^2 = 2ax$$

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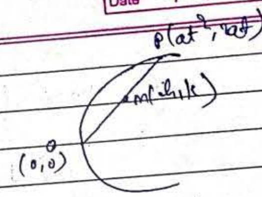


Alternate ⇒

$2h = at^2$ — (ii)
$2k = 2at$
$k = at$
$t = \frac{k}{a}$ — (iii)

$2h = a\left(\frac{k}{a}\right)^2$

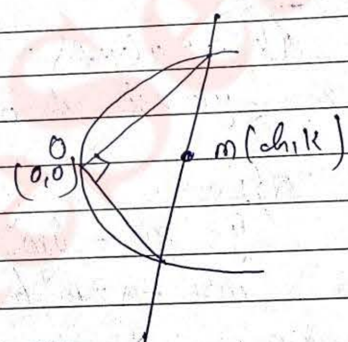
$y^2 = 2ax$

ii) Slope $= m$

$\frac{-(-2a)}{k} = m$ use ∴ slope $= \frac{-a}{b}$ in eq (i)

$$\boxed{y = 2a/m}$$

iii)



PQ ⇒
$T = S_1$

$yk - 2a(x+h) = k^2 - 4ah$

Homogenise the term —

$yk - 2ax = k^2 - 2ah$

$\left(\frac{yk - 2ax}{k^2 - 2ah}\right) = 1$ — (i)

Joint eqn of OP and OQ

$y^2 = 4ax\left(\frac{yk - 2ax}{k^2 - 2ax}\right)$

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$$y^2 - 4ax\left(\frac{yk - 2ax}{k^2 - 2ah}\right) = 0$$

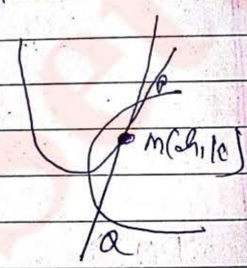
$OP \perp OQ$

Coeff of x^2 + Coeff of $y^2 = 0$ (In case of $\perp$ homogen. Teach (recop...

$$1 + \frac{8a^2}{k^2 - 2ah} = 0$$

$$\boxed{y^2 - 2ax + 8a^2 = 0}$$

Examples Find the locus of mid-point of chord of parabola $y^2 = 4ax$ which touches the parabola $x^2 = 4by$



Ans $T = S_1$

PQ :-

$\Rightarrow yk - 2a(x+h) = k^2 - 4ah$

$\Rightarrow 2ax - yk + k^2 - 2ah = 0$ — (1)

PQ :-

$y = mx - bm^2$

$mx - y - m^2 b = 0$ — (2)

Compare :-

$$\frac{2a}{m} = \frac{-k}{-1} = \frac{k^2 - 2ah}{-m^2 b}$$

$m = \frac{2a}{k}$ — (3)

$m^2 = \frac{k^2 - 2ah}{-kb}$ — (4)

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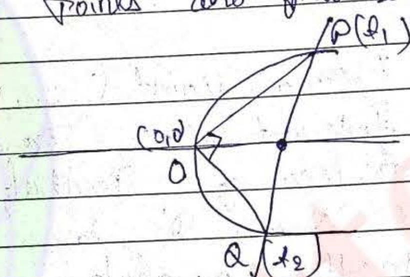
$$\left(\frac{2a}{k}\right)^2 = \frac{k^2 - 2ah}{-kb}$$

Example: To the vertex 'O' of parabola $y^2 = 4x$, chord 'OP' and 'OQ' are drawn at right angles to each other. Show that for all ~~portion~~ position of P, PQ cuts the axis of parabol at fixed points also find its co-ordinates.

Ans $y^2 = 4x$



PQ : $\rightarrow y(t_1+t_2) = 2x + 2t_1t_2$ — ①

$t_1t_2 = -4$ — ②

$y(t_1+t_2) = 2x - 8$

$\underbrace{2x-8} - \underbrace{(t_1+t_2)}y = 0$

$L_1 + \lambda L_2 = 0$

$\left.\begin{array}{l} 2x - 8 = 0 \\ y = 0 \end{array}\right\}$ (4, 0)

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Pole and Polar :-

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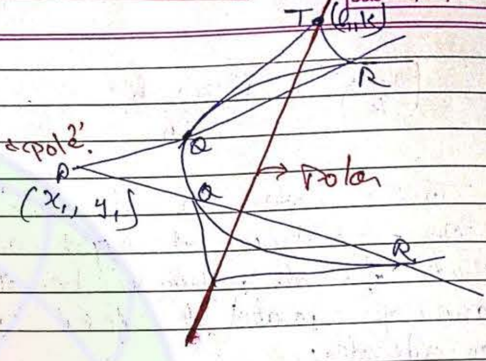


From a point (h_1, y_1) lines are drawn to Intersect the Parabola at 'Q' and 'R' the locus of point of Intersection of tangents at 'Q' and 'R' is polar of point 'P' with respect to the parabola.

Point 'P' is called Pole of the polar.

Note:

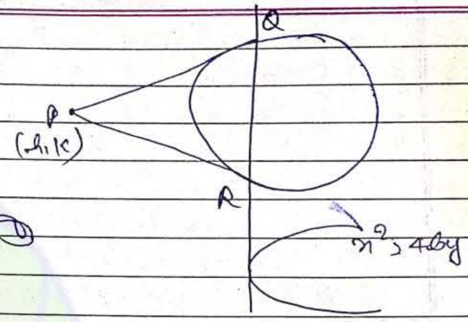
* If the eqn of parabola is $y^2 = 4ax$ then the eqn of polar of point 'P' w.r.t. the parabola is

$$\boxed{T = 0}$$

$$\boxed{yy_1 = 2a(x + x_1)}$$

Ex – Find the locus of pole If the polar w.r.t. the circle $x^2+y^2+2ax=0$ touches the parabola $x^2 = 4by$.

Ans: -



QR: -

$xh + yk + a(x+h) = 0$

$x(h+a) + yk + ah = 0$ —(1)

QR: -

$y = mx - bm^2$

$mx - y - bm^2 = 0$ —(2)

compare eq (1) and eq (2)

$$\frac{h+a}{m} = \frac{k}{-1} = \frac{ah}{-bm^2}$$

$$m = -\left(\frac{h+a}{k}\right) \quad —(3)$$

$$m^2 = \frac{ah}{bk} \quad —(4)$$

$$\left(\frac{h+a}{k}\right)^2 = \frac{ah}{bk}$$

$$(h+a)^2 = \left(\frac{a}{b}\right)hk$$

So Locus: -

$$(x+a)^2 = \left(\frac{a}{b}\right)xy$$ Ans.

Ex. If '⊥' drawn from point P on polar of 'P' ~~the point 'P'~~ w.r.t. $y^2 = 4ax$ touches the parabola $x^2 = 4by$ then find locus of P.

Ans: -

Ans: - $y(by + 2ax + 4a^2) = 0$

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Equation of Normal ⇒

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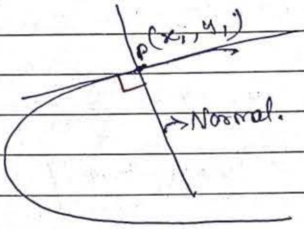
The line '⊥' to tangent and passing through its point of contact of is called 'Normal'.



Note: यह जरूरी नहीं की focus से pass करे।

1) Point form:-

Eqⁿ of Normal to parabola $y^2 = 4ax$ at the point (x_1, y_1) line on the parabola is

$$\boxed{y - y_1 = \frac{-y_1}{2a}(x - x_1)}$$

Hint:- ∵

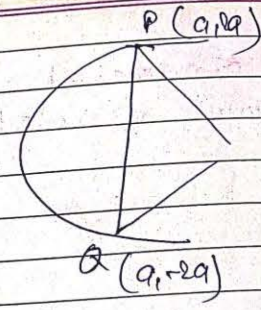
Tangent $yy_1 = 2a(x + x_1)$

Slope :⇒ $\frac{2a}{y_1}$

Slope of normal $= \frac{-y_1}{2a}$ }

Ex. Find the eqⁿ of normal to parabola $y^2 = 4ax$ at the end's of Ratus rectum.

Ans:> $y^2 = 4ax$



(i) $P(a, 2a)$

$\Rightarrow y - 2a = \frac{-2a}{2a}(x-a)$

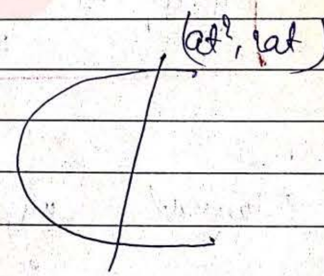
$\Rightarrow \boxed{x+y=3a}$

ii) $Q(a, -2a)$

$$y+2a = -\frac{(-2a)}{2a}(x-a)$$

$$\boxed{x-y=3a}$$

Ex:— Find the eq^n of Normal to Parabola $y^2=4ax$ at the point 't'



Ans) $(at^2, 2at)$

Eq^n of normal

$$y-2at = \frac{-2at}{2a}(x-at^2)$$

$$\boxed{y+tx = 2at+at^3}$$

2.

2:) Parametric form:→

The eqⁿ of normal to parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ is ~~$y + tx = 2at + at^3$~~

$$\boxed{y + tx = 2at + at^3}$$

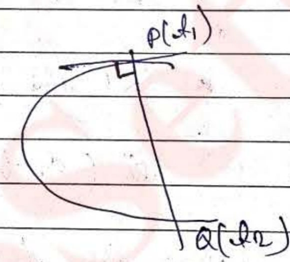
V.V.I. Imp.

Note:–

If the normal at point 't_1' to the parabola $y^2 = 4ax$ intersect the parabola again at point 't_2' then→

$$\boxed{t_2 = -t_1 - \frac{2}{t_1}}$$

Proof:–



PQ ⇒

$y + t_1 x = 2at_1 + at_1^3$

Pass

$Q(at_2^2, 2at_2)$

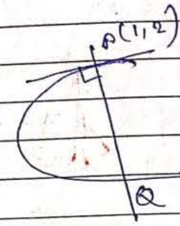
$2at_2 + t_1 \cdot at_2^2 = 2at_1 + at_1^3$

$t_1(t_2^2 - t_1^2) = 2(t_1 - t_2)$

$t_1(t_1 + t_2) = -2$

$t_1 + t_2 = \frac{-2}{t_1} \Rightarrow t_2 = -t_1 - \frac{2}{t_1}$

Ex:- If normal to parabola $y^2 = 4x$ at the point $(1,2)$ intersect the parabola again at 'Q' then find the co-ordinates of 'Q'.



Ans:- $(1, 2)$

$\left.\begin{array}{l} at_1^2 = 1 \\ 2at_1 = 2 \end{array}\right\} \boxed{t_1 = 1}$

$Q(t_2)$

$t_2 = -t_1 - \frac{2}{t_1}$

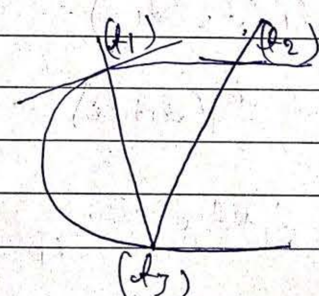
$\Rightarrow t_2 = -1 - 2$

$\Rightarrow t_2 = -3$

$Q(at_2^2, 2at_2)$

$Q(9, -6)$

Ex→ If the normals at point 't_1' and 't_2' to the parabola $y^2 = 4ax$ meet at the point t_3 on the parabola then show that ~~$t_1 t_2$~~
$t_1 t_2 = 2$



Ans.-

$t_3 = -t_1 - \frac{2}{t_1}$

$t_3 = -t_2 - \frac{2}{t_2}$

So,

$$\Rightarrow -t_1 - \frac{2}{t_1} = -t_2 - \frac{2}{t_2}$$

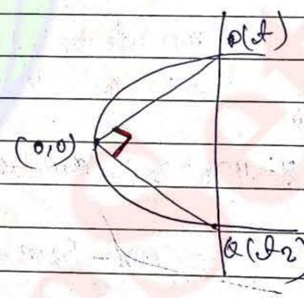
$$\Rightarrow t_2 - t_1 = \frac{2}{t_1} - \frac{2}{t_2}$$

$$\Rightarrow \cancel{(t_2 - t_1)} = \frac{2\cancel{(t_2 - t_1)}}{t_1 t_2}$$

$$\boxed{t_1 t_2 = 2}$$

Ex :- If Normal chord at point 't' on the parabola $y^2 = 4ax$ subtends right angle at the vertex, then show that $t^2 = 2$.

Ans)



OP ⊥ OQ

(S) $t\,t_2 = -4$

$t_2 = -t - \frac{2}{t}$

$t\,t_2 = -t^2 - 2$

$-4 = -t^2 - 2$

$$\boxed{t^2 = 2}$$

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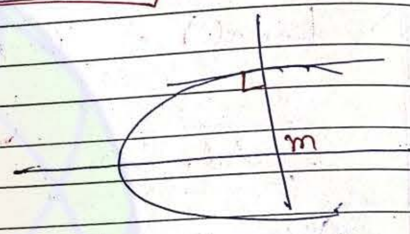
3.) Slope form

Eq^n of normal to parabola $y^2 = 4ax$ whose slope is 'm' is

$$\boxed{y = mx - 2am - am^3}$$



Hint! ->

$$y + tx = 2at + at^3$$

$$m = -t$$
$$t = -m$$

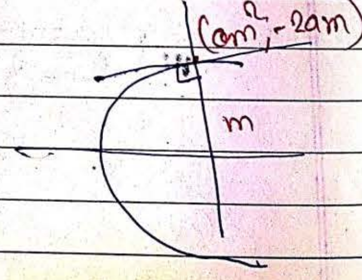
$$y - mx = -2am - am^3$$

$$y = mx - 2am - am^3$$

Note:—

1.) The point of Intersection of normal and Parabola is

$$(am^2, -2am)$$



ii) Maximum three normals can be drawn from a point to the parabola.

Q. Find the eqn of normal to the parabola $y^2 = 4ax$ whose Inclination is 30° with x-axis.

Ans.→ $m = \tan 30^\circ = \frac{1}{\sqrt{3}}$

$y = mx - 2am - am^3$

$y - mx = -2am - am^3$. Put value of m.

Q. Show that the line $y - x\sqrt{2} + 4a\sqrt{2} = 0$ is normal to parabola $y^2 = 4ax$

Ans. $y - x\sqrt{2} + 4a\sqrt{2} = 0$

$m = \sqrt{2}$

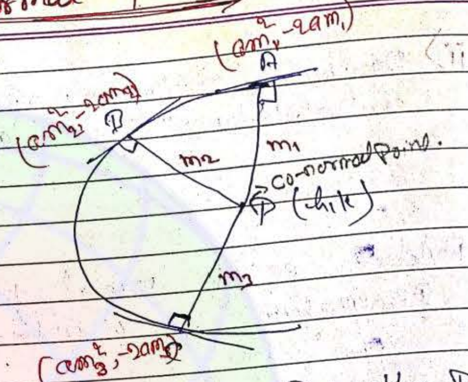
$\Rightarrow y = \sqrt{2}x - 2a\sqrt{2} - a(\sqrt{2})^3$

$\Rightarrow y - \sqrt{2}x = -2a\sqrt{2} - 2a\sqrt{2}$

~~$\Rightarrow y - \sqrt{2}x = -2a\sqrt{2}$~~

$$\boxed{y - \sqrt{2}x + 4a\sqrt{2} = 0}$$

Co- Normal Points



The feet of normals from the Point 'P' on the Parabola are called Co-Normal Point (A, B, C)

Let the eq^n of parabola is $y^2 = 4ax$ then the eq^n of normal is

$$y = mx - 2am - am^3$$

Let the co-ordinates of 'P' are (h, k) then

$$\boxed{k = mh - 2am - am^3}$$

$$\boxed{am^3 + m(2a - h) + k = 0}$$

याद रखें -> $ax^3 + bx^2 + cx + d = 0$

This eq^n is cubic equation in 'm' and roots of this equation are slopes of normal.

So,

$$\boxed{m_1 + m_2 + m_3 = 0} \quad -(1)$$

$$\boxed{m_1m_2 + m_2m_3 + m_3m_1 = \left(\frac{2a-h}{a}\right)} \quad \text{---} (2)$$

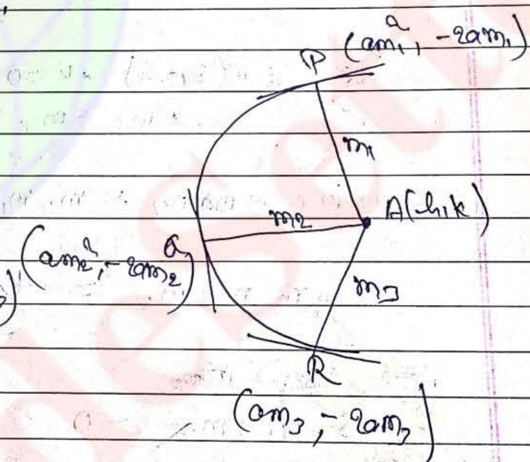
$$\boxed{m_1 m_2 m_3 = \frac{-k}{a}} \quad \text{---} (3)$$

Q:- If three normals at point 'P', 'Q' and 'R' are congruent then show that
(i) Sum of ordinates of (y-coord.) 'P' 'Q' and 'R' is zero.
(ii) The centroid of triangle P, Q, R lies on the axis of parabola.

Ans:

$am^2 + m(2a-h) + k = 0$

$m_1 + m_2 + m_3 = 0$



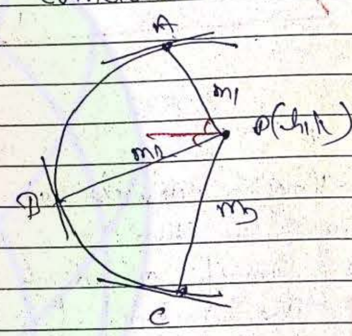
(i) $y_1 + y_2 + y_3 = -2a(m_1 + m_2 + m_3)$
$= 0$

(ii) $\frac{y_1 + y_2 + y_3}{3} = 0$

Example. Find the locus of point 'P' from which normals are drawn to parabola $y^2 = 4ax$ such that
(i) Two of them are equally inclined to axis in opp. direction.

ii) Two of them are '⊥' to each other.
iii) To obtain to satisfy the relation
$m_2 m_3 = \alpha$
iv) Two of them are coincident.

Ans)



Now,

$am^3 + m(2a-h) + k = 0$ — (1)

$m_1 + m_2 + m_3 = 0$ — (2)

$m_1 m_2 + m_2 m_3 + m_3 m_1 = \frac{2a-h}{a}$ — (3)

$m_1 m_2 m_3 = -k/a$ — (4)

i) $m_1 + m_3 = 0$

From eq (2)

$m_1 = 0$

m_1 is a root of eq^n (1)

$a(0) + 0 + k = 0$

$k = 0$

$\boxed{y = 0}$

ii) $m_2 m_3 = -1$

From eq (4)

$(-1)\, m_1 = -k/a$

$m_1 = k/a$

From (1)

$a\left(\frac{k}{a}\right)^3 + k/a\,(2a-h) + k = 0$

$y^2 = a(x-3a)$

iii)

iv) $m_2 = m_3$

$m_1 + 2m_2 = 0$ — (1)

From eq (4)

$m_1 \cdot m_2^2 = -k/a$

$-2m_2 \cdot m_2^2 = -k/a$

$m_2^3 = \left(\frac{k}{2a}\right)$

From eq (1)

$a\left(\frac{k}{2a}\right) + \left(\frac{k}{2a}\right)^{1/3} (2a-h) + k = 0$

$\left(\frac{k}{2a}\right)^{1/3} (2a-h) = -3k/2$

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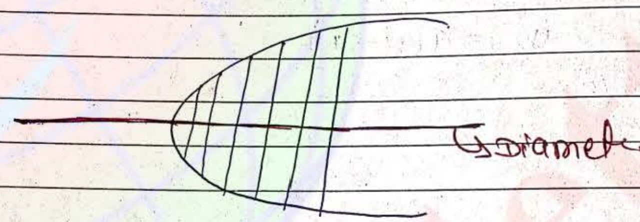
$$\frac{k}{2a}(2a-h)^3 = \frac{-27k^3}{8}$$

Diameter

Locus of middle point of system of parallel chords is called diameter.

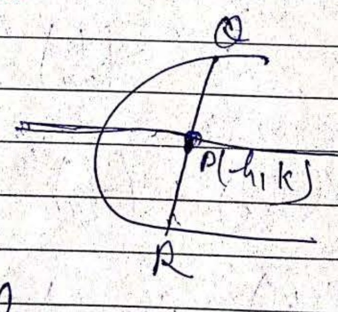


If the eq^n of parabola is $y^2 = 4ax$ then the eq^n of diameter corresponding to the system of parallel chords having slope 'm' is

$$\boxed{y = \frac{2a}{m}}$$

Hint :>

QR :>



$T = S_1$

$$4k - 2a(x+h) = k^2 - 4ah$$

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$$\text{Slope} = m = \frac{-(-2a)}{k}$$

$$m = 2a/k$$

$$k = 2a/m \Rightarrow \boxed{y = 2a/m}$$

Ex.) The eq^n of system of ~~Mchords~~ parallel chords on parabola $y^2 = \frac{2}{3}x$ is given by $y + 2x + \lambda = 0$. Find the eq^n of ~~the~~ corresponding diameter.

Ans.) $y = \frac{2a}{m}$ $\quad$ $y =$

$$4a = \frac{2}{3}$$

$$a = 1/6$$

$$m = -2$$

$$\boxed{y = -1/6} \text{ Ans}$$

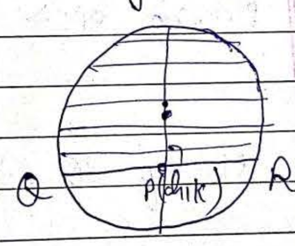
Q. Find the eq^n of diameter of circle $x^2+y^2=a^2$ corresponding to the system of parallel chords having slope 'm'.

Ans.) $x^2+y^2=a^2$

QR :-

$T = S_1$

$xh + yk = h^2 + k^2$



(Q) Slope $= m = \frac{-h}{k}$

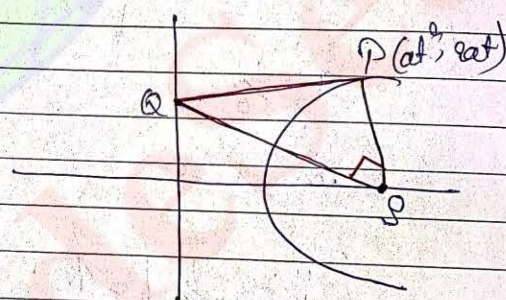
$mk + h = 0$

Locus :→

$my + x = 0$

* Properties of Parabola :⇒

i) The portion of tangent to a parabola intercepted b/w the directrix and the curve subtends right angle at the focus.



Proof :→

PQ :→

$ty = x + at^2$

$Q\left(-a, \frac{a(t^2-1)}{t}\right)$

Slope of SP :→

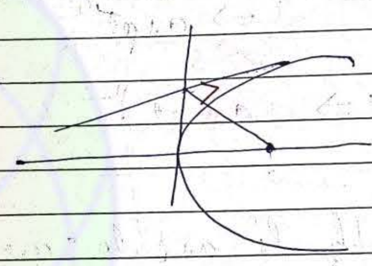
$\left(\frac{2at - 0}{at^2 - a}\right) = m_1$

Slope of SQ :→ $\frac{\frac{a(t^2-1)}{t} - 0}{-a-a} = m_2$

$m_1 m_2 = -1$

$\Rightarrow SP \perp SQ$

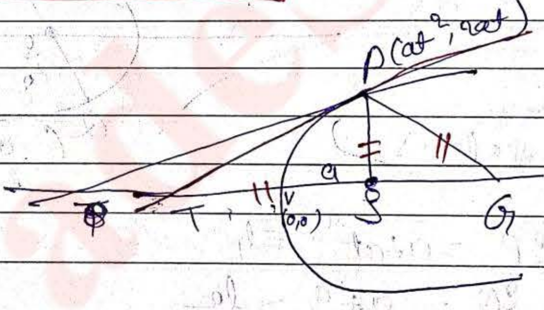
ii) The foot of '$\perp$' from ~~focus~~ focus upon any tangent to parabola lies on the tangent at vertex.



iii) If 'S' is the focus of parabola and ~~vertex~~ tangent and ~~at~~ normal at any point, point 'P' meet the axis of parabola at 'T' and 'G'.

Then

$$\boxed{ST = SP = SG}$$



PT! $\Rightarrow$

$yt = x + at^2$

$T(-at^2, 0)$

$ST = a + at^2$

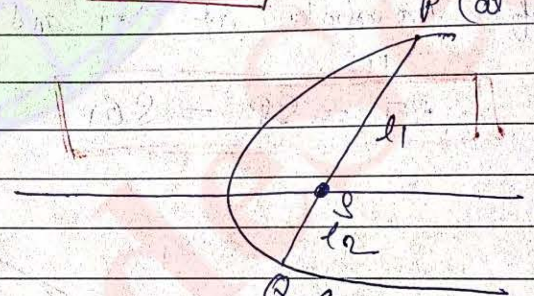
$PG :\Rightarrow \quad y + tx = 2at + at^3$

$\quad$ or $(2a + at^2, 0)$

$SG :\Rightarrow \quad VG - VS$
$= 2a + at^2 - a$
$\Rightarrow a + at^2$

$SP :\Rightarrow \quad a + at^2$

iv) If l_1 and l_2 are lengths of segments of a focal chord then the lactus rectum of Parabola is $\Rightarrow \boxed{\dfrac{4l_1l_2}{l_1+l_2}}$



Properties

$Q\left(\dfrac{a}{t^2}, \dfrac{-2a}{t}\right)$ ~> If PQ is focal chord then "$t_1t_2 =$

$SP = a + at^2 = l_1$

$SQ = a + \dfrac{a}{t^2} = l_2$

Now

$$\frac{4l_1l_2}{l_1+l_2} \Rightarrow \frac{4(a+at^2)\left(a+\frac{a}{t^2}\right)}{a+at^2+a+\frac{a}{t^2}}$$

$\Rightarrow 4a$

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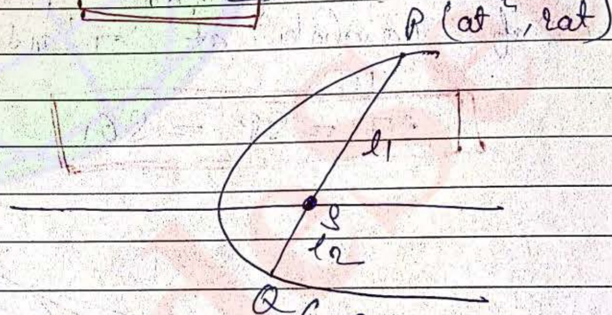
PG :⇒ $y + tx = 2at + at^3$

or $(2a + at^2, 0)$

SG :⇒ $VG - VS$
$= 2a + at^2 - a$
$\Rightarrow a + at^2$

SP :⇒ $a + at^2$

IV) If l_1 and l_2 are lengths of segments of a focal chord then the lactus rectum of parabola is ⇒ $\boxed{\dfrac{4l_1l_2}{l_1+l_2}}$



→ If PQ is focal chord then "$t_1 t_2 = -1$"

Properties

$SP = a + at^2 = l_1$

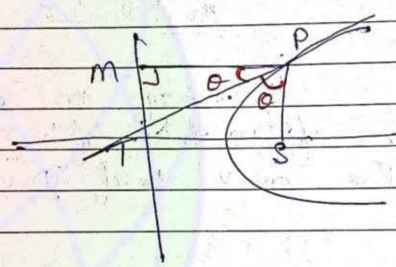
$SQ = a + \frac{a}{t^2} = l_2$

Now

$$\frac{4l_1l_2}{l_1+l_2} \Rightarrow \frac{4(a+at^2)(a+a/t^2)}{a+at^2+a+\frac{a}{t^2}}$$

$\Rightarrow 4a$

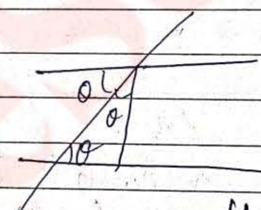
v) The tangent at any point 'P' lies on the parabola bisects the angle b/w focal distance and 'D' perpendicular (⊥) drawn from point 'P' to the directrix.



Proof :>
ΔSPT

$\boxed{ST = SP}$

$\angle SPT = \angle STP = \theta$ — ①



PM // TS
(∵ Alternate Interior angle)

so

②

$\angle TPM = \angle STP = \theta$ — ②

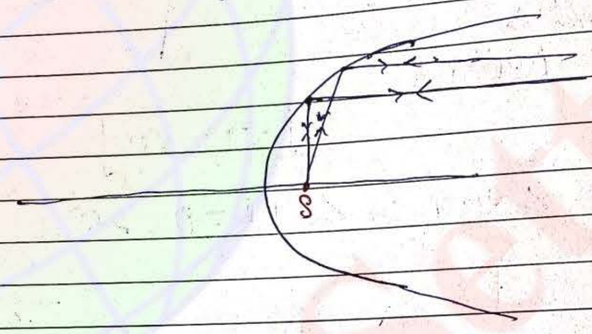
Now,

6.) Reflection Property:-

Any light ray parallel to axis of parabola after reflection passes through the focus of parabola.

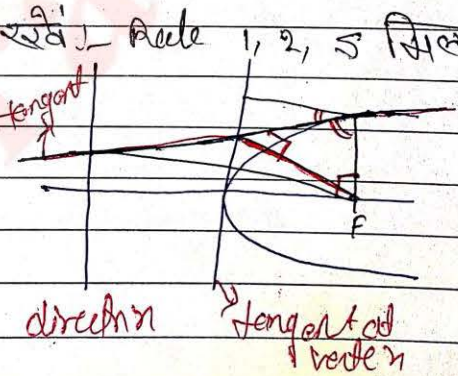
OR

Light ray emitted from the focus of parabola after reflection goes parallel to the axis of parabola.



7.) A circle passing through three co-normal points of a parabola passes through the vertex of parabola.

जीटक प्राप्त रखें। Rule 1, 2, 5 मिला है।



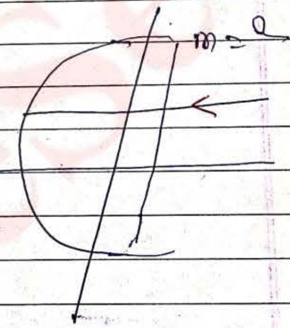
Ex.) A ray of light bisect the chord $y = 2x - 1$ and $y = 2x - 3$ and is Incident on parabola $y^2 = 4x$ (Integral)

Find the co-ordinate of point ~~with these~~ where the ray strikes the parabola.

i) First time
ii) IInd time
iii) IIIrd time

Ans: → (i) $(1/4, 1)$
(ii) $(4, -4)$
(iii)

Ans→

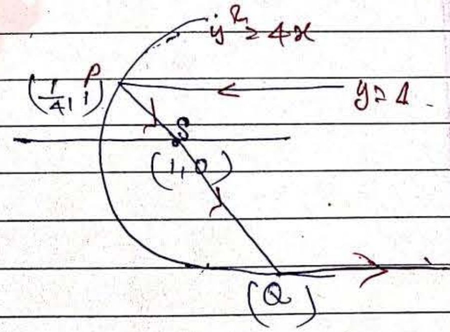


$m = 2$

$a = 1$

Incident ray: —

$y = \left(\frac{2a}{m}\right)$ → ordinate $= \frac{2}{2} = 1$



i) For point P

$y = 1$

$\Rightarrow x = \frac{1}{4}$

$P\left(\frac{1}{4}, 1\right)$

ii) PS: —

$$y - 0 = \left(\frac{1-0}{\frac{1}{4} - 1}\right)(x - 1)$$

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$4x + 3y = 4$

Now

$y^2 = 4x$

$4x + 3y = 4$

$y^2 = 4 - 3y$

$(y - 1)(y + 4) = 0$

$y = 1, -4$

$Q(4, -4)$ &.

Start → 1st Choice

Determinant

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* Determinant of a square matrix :→
corresponding to each square matrix :—

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}$$

There is associated an expression, called the determinant of "A", "denoted by det A or |A| or Δ" written as :—

$$\Delta = \det A = |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{vmatrix}$$

Note! →

1) For matrix A, |A| is read as determinant of A and not modulus of A.

2.) A matrix is an arrangement of number and so it has no fixed value, while each determinant has a fixed value.

3.) A determinant having 'n' rows and 'n' columns is known as a determinant of "order 'n'".

4.) Only square matrix have determinants.

1st Choice

Value of determinants

The value of determinant is the sum of the product of elements of a row (or a column) with their corresponding co-factors.

1) value of determinant of order 1 ⇒ The value of a determinant of a (1×1) matrix $[a]$ is defined as $|a| = a$

eg:-> $|4| = 4$ (determinant of order 1)

(Teacher may be +sign) → $|-4| = -4$ (" " ")

2) value of determinant of order 2 ⇒

Let $A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ be a matrix of order 2x2 then the determinant of 'A' is defined as:-

$$det(A) = |A| = \Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

eg:->

(i) $\begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} = 1.3 - 2.0 = 3$ Ans

(ii) $\begin{vmatrix} \sin\theta & \cos\theta \\ \cos(\pi-\theta) & \sin(\pi-\theta) \end{vmatrix} = \sin^2\theta + \cos^2\theta = 1$ Ans

Note:->

$$\begin{array}{ccl} \begin{vmatrix} a & b \\ c & d \end{vmatrix} & & \begin{array}{l} \to \text{First row } (R_1) \\ \to \text{Second row } (R_2) \end{array} \\ \text{First column } (C_1) & \text{IInd column } (C_2) & \end{array}$$

where a, b, c, d are called the element or entries of the matrix

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3.) Value of Determinant of order 3 or more :→

For finding the value of a determinant of order 3 or more, we need the following definition :→

minor of a_{ij} in $|A|$:→ The value of the determinant obtained by deleting the ith row and jth column of $|A|$, and it is denoted by M_{ij}.

Co-factor of a_{ij} in $|A|$:→ The co-factor C_{ij} of an element a_{ij} is defined

$$\boxed{C_{ij} = (-1)^{i+j} \cdot M_{ij}}$$

where :→ i → Row ~~no~~ number
j → column number

Q) eg :→ Find the minor and co-factor of the elements of the determinant :—

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Sol^n Let M_{ij} denoted the minor of a_{ij} in Δ

Now, a_{11} occurs in the 1st row and 1st column. So in order to find the minor of a_{11}, we delete the 1st row and 1st column of Δ, ~~the minor of~~ M_{11}.

So,

- minor of $M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = (a_{22}a_{33} - a_{32}a_{23})$

Similarly, we find remaining elements.

- If we denote the co-factor of a_{ij} by C_{ij} then :—

$$C_{11} = (-1)^{1+1} \cdot M_{11} = (-1)^2 \cdot (a_{22}a_{33} - a_{32}a_{23})$$

Similarly, we can find remaining.

Q.1) Find the value of determinant: — $\begin{vmatrix} -2 & 4 & -1 \\ 0 & 2 & 0 \\ 3 & 1 & -2 \end{vmatrix}$

Attention (+) Attention (−) Attention (+)

Sol) $\Rightarrow +\{-2[-4+0]\} - \{4[0]\} + \{-1[0-6]\}$

$\Rightarrow -2\times(-4) - 0 + -1\times(-6)$

$\Rightarrow 8+6 = 14$

By other side →

$\Rightarrow -2[-4-0] \pm 0(\;) + 3[4\times0-(-2)]$

$\Rightarrow 8+6 = 14$

Similarly we find same value by other sides

Q.2) Find the value of 'x' of following determinant is

$\begin{vmatrix} x & -6 & 1 \\ 2x & -3 & 4 \\ 0 & 1 & 2 \end{vmatrix} = 0$

Sol) $\Rightarrow +\{x[-6-4]\} - \{-6[2x\times2-0]\} + \{1[2x-0]\} = 0$

$\Rightarrow (-10x) - \{-24x\} + 2x = 0$

$\Rightarrow -10x + 24x + 2x = 0$

$\Rightarrow 12x = 0$

$x = 0$ Ans

1st Choice

Determinant

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$ad - bc$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

This expression $ad-bc$ can be written in the form

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

⇒ In a Determinant : ⇒

$\boxed{\text{No. of rows = The no. of colums}}$

No. of columns or rows is called the "order" ~~degree~~ of determinant.

⇒ minor of element a_{ij} : ⇒

In any determinant of ~~degree~~ we delete the row and column passing through the element a_{ij} and write the remaining element in the form of a determinant.

Then the determinant so obtained is called minor of element a_{ij},

where

$i \rightarrow$ Row number.

1) column matrix → $[a_{ij}]_{m\times 1}$
2) Row matrix → $[a_{ij}]_{1\times n}$
3) Square matrix → $[a_{ij}]_{m\times m}$

and

j → column number

Note:→ minor of element a_{ij} in $|A|$ is denoted by "m_{ij}".

eg 1)
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\text{minor of } a_{11} = M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$\text{'' '' } a_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$$

Co-factor of the element :⇒

~~or~~ In any determinant for any element a_{ij}

Co-factor $\boxed{C_{ij} = (-1)^{i+j} M_{ij}}$ → Remember this formula

eg 1)
$$C_{13} = (-1)^4 \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$C_{23} = (-1)^5 \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

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Expansion of Determinant :>

In any determinant If all the element of any of the row or column are multiplye their corresponding co-facto then sum of these products is equal to the value of determinant

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

e.g

$$\boxed{\begin{aligned} \Delta &= a_{11} C_{11} + a_{12} \cdot C_{12} + a_{13} C_{13} \\ &= a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} \\ &\quad + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \end{aligned}}$$

Note! :> Expansion of determinant of order three :>

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

Note :> Alternate '+' and '−' situation.

Q Find the value of $\begin{vmatrix} 3 & 6 & 9 \\ -1 & 4 & 9 \\ 3 & 4 & 5 \end{vmatrix}$

Ans $\Delta = 3|20-36| - 1(15-27) + 3(54-54)$

$\Rightarrow 3 \times -16 + 12 + 0$

$\Rightarrow 0$

f) Properties of Determinant :-

1) The value of determinant remains unchanged If Its rows and columns are ~~changed~~ Interchang.

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

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2.) If any two rows of determinant is interchange then their value of Determinant ~~remains unchange~~ ~~Then the value changes~~ change by negative sign only.

$$\boxed{D' = -D}$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= -\begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

3.) If any rows or columns of a determinant are same or Indentical then the value of Determinant is zero.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$

4.) If all the elements of any row or any column of a determinant are are zero then the value of determinant is zero.

$$\Delta = \begin{vmatrix} 0 & 1 & 8 \\ 0 & 2 & 12 \\ 0 & 3 & 18 \end{vmatrix} = 0$$

c) If each element of "a row or column" of a determinant is multiplied by a constant "k" then the value of new determinant is "k" times the value of original determinants.

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta K = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

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(i) In any determinant If elements both below or above the leading diagonal are zero then the value of determinant is equal to product of the elements of ~~the~~ leading diagonal.

$$\Delta = \begin{vmatrix} a & d & e \\ 0 & b & f \\ 0 & 0 & c \end{vmatrix} = abc$$

leading diagonal

very Important
V.V.I

If each element of a row ~~of~~ or column 'n' is multiplied by some constant and then added to corresponding elements of some other row or column then the value of determinant remains unchanged.

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

R → Row
C → column

$$R_1 \to R_1 + mR_2$$

$$\Delta = \begin{vmatrix} a_1 + ma_2 & b_1 + mb_2 & c_1 + mc_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$C_1 = C_1 + mC_2 + nC_3$$

$$\Delta \begin{vmatrix} a_1 + mb_1 + nc_1 & b_1 & c_1 \\ a_2 + mb_2 + nc_2 & b_2 & c_2 \\ a_3 + mb_3 + nc_3 & b_3 & c_3 \end{vmatrix}$$

Concept

Note.→
While applying operation It is necessary that at least one row should remain unchanged. So,
If determinant is of 'n' then we can apply 'n-1' operation

b) ~~S.T~~ ~~Show that~~ without expanding :-

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$$

Solⁿ C_3 changes to $C_3 + C_2$

$$C_3 \rightarrow C_3 + C_2$$

$$\Delta = \begin{vmatrix} 1 & a & b+c+a \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

~~S.T~~

$$\Rightarrow (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

$$\Rightarrow 0 \qquad (\because C_1 = C_3) \; (\text{Rule no. 3})$$

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eg 2)

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

sol)

$C_1 \to C_1 + C_2 + C_3$

(Rule no. 4)

$$\begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix} = 0$$

Proved

eg 3)

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$$

sol)

$C_3 \to C_3 + C_2$

$$\begin{vmatrix} 1 & bc & ab+bc+ca \\ 1 & ca & ab+bc+ca \\ 1 & ab & ab+bc+ca \end{vmatrix}$$

$R_1 \Rightarrow R_1 - R_2$

$R_2 \Rightarrow R_2 - R_3$

$$\Rightarrow (ab+bc+ca)\begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix} = 0$$

Q

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

Sol

$R_1 \Rightarrow R_1 - R_3$

$R_2 \Rightarrow R_2 - R_3$

~~$R_2 \Rightarrow R_2 - R_3$~~

$$\begin{vmatrix} 0 & a-c & a^2-c^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\Rightarrow (a-c)(b-c)\begin{vmatrix} 0 & 1 & a+c \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$= (a-c)(b-c)(b+c-a-c)$$

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5 Q)

$$\begin{vmatrix} a & bc & abc \\ b & ac & abc \\ c & ab & abc \end{vmatrix} = abc(a-b)(b-c)(c-a)$$

Soln)

$$\Rightarrow abc \begin{vmatrix} a & bc & 1 \\ b & ac & 1 \\ c & ab & 1 \end{vmatrix}$$

$$\Rightarrow R_1 = R_1 - R_3$$
$$R_2 = R_2 - R_3$$

$$\Rightarrow abc \begin{vmatrix} a-c & bc-ab & 1-1 \\ b-c & ac-ab & 1-1 \\ c & ab & 1 \end{vmatrix}$$

$$\Rightarrow abc \begin{vmatrix} a-c & b(c-a) & 0 \\ b-c & a(c-b) & 0 \\ c & ab & 1 \end{vmatrix}$$

$$\Rightarrow abc(a-c)(b-c) \begin{vmatrix} 1 & -b & 0 \\ 1 & -a & 0 \\ c & ab & 1 \end{vmatrix}$$

$\Rightarrow abc(a-b)(b-c)(b-a)$

$\Rightarrow abc(a-b)(b-c)(a-b)$ Ans.

6 Q) $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

A) $R_1 \to R_1 - R_3$

$R_2 \to R_2 - R_3$

$D = \begin{vmatrix} 1-1 & a-c & a^3-c^3 \\ 1-1 & b-c & b^3-c^3 \\ 1 & c & c^3 \end{vmatrix}$

$\Rightarrow (a-c)(b-c)\begin{vmatrix} 0 & 1 & a^2+ac+c^2 \\ 0 & 1 & b^2+bc+c^2 \\ 1 & c & c^2 \end{vmatrix}$

$\Rightarrow (a-c)(b-c)\cdot\left[b^2+bc+\cancel{c^2}-(a^2+\cancel{c^2}+ac)\right]$

$\Rightarrow (a-c)(b-c)\left[(b^2-a^2)+c(b-a)\right]$

$\Rightarrow (a-c)(b-c)(b-a)(b+a+c)$ A

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4) eg ⟹
$$\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (a^y+a^{-y})^2 & (a^y-a^{-y})^2 & 1 \\ (a^z+a^{-z})^2 & (a^z-a^{-z})^2 & 1 \end{vmatrix} = 0$$

Ans ⟹

$C_1 \to C_1 - C_2$

$\because 4a^x a^{-x} \Rightarrow 4 \times a^x \times \frac{1}{a^x} \Rightarrow 4$

$$\Rightarrow \begin{vmatrix} \cancel{(a^x)^2}+\cancel{(a^{-x})^2}+2a^x a^{-x}-\cancel{(a^x)^2}-\cancel{(a^{-x})^2}+2a^x a^{-x} & (a^x-a^{-x})^2 & 1 \\ \cancel{(a^y)^2}+\cancel{(a^{-y})^2}+2a^y a^{-y}-\cancel{(a^y)^2}-\cancel{(a^{-y})^2}+2a^y a^{-y} & (a^y-a^{-y})^2 & 1 \\ \cancel{(a^z)^2}+\cancel{(a^{-z})^2}+2a^z a^{-z}-\cancel{(a^z)^2}-\cancel{(a^{-z})^2}+2a^z a^{-z} & (a^z-a^{-z})^2 & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 4\times a^x \times \frac{1}{a^x} & (a^x-a^{-x})^2 & 1 \\ 4\times a^y \times \frac{1}{a^y} & (a^y-a^{-y})^2 & 1 \\ 4\times a^z \times \frac{1}{a^z} & (a^z-a^{-z})^2 & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 4 & (a^x-a^{-x})^2 & 1 \\ 4 & (a^y-a^{-y})^2 & 1 \\ 4 & (a^z-a^{-z})^2 & 1 \end{vmatrix}$$

$$\Rightarrow 4 \begin{vmatrix} 1 & (a^x-a^{-x})^2 & 1 \\ 1 & (a^x-a^{-x})^2 & 1 \\ 1 & (a^x-a^{-x})^2 & 1 \end{vmatrix} = 0$$

Q. Q.)
$$\begin{vmatrix} a & a & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3bc - a^3 - b^3 - c^3$$

$\Rightarrow a(bc-a^2) - b(b^2-ac) + c(ab-ac^2)$

$\Rightarrow abc - a^3 - b^3 + abc + abc - c^3$

$\Rightarrow 3bc - a^3 - b^3 - c^3$

VVI Q. Q.)
$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a+b+c)(a^2+b^2+c^2-ab-bc-ac)$$

Sol

$C_1 = C_1 + C_2 + C_3$

$$\begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$

$$\Rightarrow (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

$R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$\Rightarrow (a+b+c)\begin{vmatrix} 0 & b-a & c-b \\ 0 & c-a & a-b \\ 1 & a & b \end{vmatrix}$$

$\Rightarrow (a+b+c)\left[(a-b)(b-a) - (c-b)(c-a)\right]$

$\Rightarrow (a+b+c)\left(-a^2 - b^2 - 2ab - c^2 + ac + bc - ab\right)$

$\Rightarrow -(a+b+c)\left(a^2 + b^2 + c^2 + ab + bc + ac\right)$

So/n $\Rightarrow$

10. Q) $\Rightarrow$

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (ab+bc+ca)(a-b)(b-c)(c-a)$$

$R_1 \to R_1 - R_3$

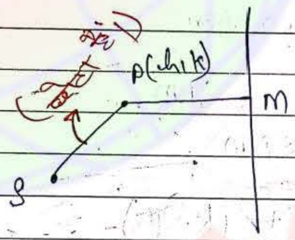
$R_2 \to R_2 - R_3$

Ellipse

Definition: $\Rightarrow$ It is the locus of a point which moves in a plane in such a way that the ratio of its distances from a fixed point to its distance from a fixed line is always constant.

The fixed point is called "focus" and fixed line is called "directrix" and the constant ratio is called "eccentricity(e)" of ellipse.

$$\boxed{\text{For ellipse: } \Rightarrow \quad 0 < e < 1}$$



So, By definition of ellipse: $\Rightarrow$

$$\boxed{\frac{SP}{PM} = e} \quad (0 < e < 1)$$

$$\boxed{SP = e\,PM}$$

Note: $\Rightarrow$

1) For parabola $e = 1$
2) If $e > 1$, The locus of point 'P' is a hyperbola.

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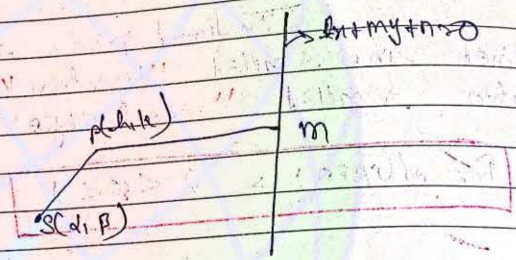
Ellipse :→

1) Def: General eqⁿ of ellipse

Let $S(\alpha, \beta)$ is the focus, line $lx+my+n=0$ is the directrix and eccentricity (e) of ellipse is "e".



By the definition

$$\frac{SP}{PM} = e$$

$$SP = e\,PM$$

$$\sqrt{\cancel{(h-\alpha)^2} + (k-\beta)^2} = \cancel{lx+my}\ \frac{lh+mk+n}{\sqrt{l^2+m^2}}$$

So,

$$\sqrt{(h-\alpha)^2 + (k-\beta)^2} = e\left|\frac{lh+mk+n}{\sqrt{l^2+m^2}}\right|$$

→ Locus (ellipse)

$$\boxed{(x-\alpha)^2 + (y-\beta)^2 = \frac{e^2(lx+my+n)^2}{l^2+m^2}}$$

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Q1 Find the e... directri...

Ans. Given,

$e = \frac{1}{2}$

focus ...

$x+y+$ ...

So, Eq...

$(x-\alpha$ ...

x^2+1 ...

Q Ident...

i) Def

ii)

iii)

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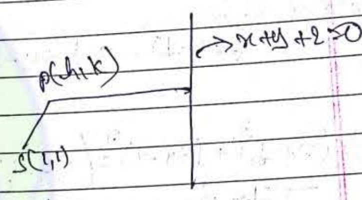
Q. Find the eqⁿ of ellipse whose focus is (1,1) directrix is $x+y+2=0$ and eccentricity (e) $= \frac{1}{2}$.

Ans. Given,

$e = \frac{1}{2}$

focus = (1,1)

$x+y+2$ = directrix



So, Eqⁿ of ellipse

$$(x-\alpha)^2 + (y-\beta)^2 = \frac{e^2(lx+my+n)^2}{l^2+m^2}$$

$|x| = \sqrt{x^2}$

$$\Rightarrow (x-1)^2 + (y-1)^2 = \left(\frac{1}{2}\right)^2 \frac{(x+y+2)^2}{2}$$

$$x^2+1-2x+y^2+1-2y = \frac{1}{4} \times (x^2 \quad)$$

$$\Rightarrow 7x^2+7y^2-20x-20y-2xy+12=0$$

Q. Identify the locus of point $P(x,y)$

i) If $(5x-1)^2 + (5y-2)^2 = \left(\frac{3x+4y+2}{5}\right)^2$

ii) If $(5x-1)^2 + (5y-2)^2 = (3x+4y+2)^2$

iii) $(5x-1)^2 + (5y-2)^2 = 25(3x+4y+2)^2$

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iv) $(5x-1)^2 (5y-2)^2 = \left(3x+4y-\frac{11}{5}\right)^2$

Ans:-

$$(5x-1)^2 + (5y-2)^2 = \left(\frac{3x+4y+2}{25}\right)^2$$

$$\Rightarrow (5x-1)^2 + (5y-2)^2 = \left(\frac{3x+4y+2}{5}\right)^2 = \frac{(3x+4y+2)^2}{3^2+4^2}$$

Take common 5 on L.H.S side

$$\Rightarrow 25\left[\left(x-\frac{1}{5}\right)^2 + \left(y-\frac{2}{5}\right)^2\right] = \left(\frac{3x+4y+2}{5}\right)^2$$

Taking root both side $\because \sqrt{x^2} = |x|$

$$\sqrt{\left(x-\frac{1}{5}\right)^2 + \left(y-\frac{2}{5}\right)^2} = \frac{1}{5}\left|\frac{3x+4y+2}{5}\right|$$

$$SP = \frac{1}{5} \cdot PM$$

$$e = \frac{1}{5}$$

So, It is ellipse.

$e = 1$ (Parabola)
$e < 1$ (ellipse)
$e > 1$ (hyperbola)

ii) Take 5 common on L.H.S side and then take root.

$$\sqrt{\left(x-\frac{1}{5}\right)^2 + \left(y-\frac{2}{5}\right)^2} = 1 \cdot \left|\frac{3x+4y+2}{5}\right|$$

v.v.i

Here we see that

$$e = 1$$

$\Rightarrow$ Parabola.

iii) $\sqrt{\left(x-\frac{1}{5}\right)^2+\left(y-\frac{2}{5}\right)^2} = 5\left|\frac{3x+4y+2}{5}\right|$

Here we see that,
$e=5 \Rightarrow$ Hyperbola.

iv) $(5x-1)^2+(5y-2)^2=\left(3x+4y-\frac{11}{5}\right)^2$ $\Rightarrow$

$\rightarrow$ This is not an eqn of any one

Attention:$\Rightarrow$ Here we solve Q'1' briefly, Conceptualize.

$(5x-1)^2+(5y-2)^2=\left(\frac{3x+4y+2}{5}\right)^2$

e=1 (Parabola)
e<1 (ellipse)
e>1 (hyperbola)

Our aim is to convert this eqn in the form of :-

$$\sqrt{(x-\alpha)^2+(y-\beta)^2} = e\left|\frac{lx+my+n}{\sqrt{l^2+m^2}}\right|$$

Accn of 'e' we decide Parabola ellipse or hyperbola.

Note:- i) we may take common in any term.
ii) $\sqrt{x^2}=|x|$.

General eqn of Second degree:-

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

This eqn represents an ellipse

If ->

i) $\Delta \neq 0$

$(abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0)$

ii) $h^2 < ab$

Note :- $\Delta \neq 0$ (Parabola, Ellipse and also hyperbola)
$h^2 = ab$ (Parabola), $h^2 < ab$ (ellipse), $h^2 > ab$ (hyperbola)

Eg:-
$13x^2 + 18xy + 37y^2 + 2x + 14y - 2 = 0$
check this eqn

Ans check

$2h = 18 \Rightarrow h = 9$

$a = 13, \; b = 37 \Rightarrow h^2 > ab$

so this is eqn of Ellipse, hyperbola]

eg2-> $\sqrt{ax} + \sqrt{by} = 1$
check this eqn

$(ax)^{1/2} + (by)^{1/2} = 1$

Ans) ~~By~~ squaring both side.

$$\Rightarrow \left(\sqrt{ax}+\sqrt{by}\right)^2 = 1$$

$$\Rightarrow ax+by+2\sqrt{ax\cdot by} = 1$$

$$\Rightarrow ax+by = 1-2\sqrt{axby}$$

$\Rightarrow$ squaring both side

$$\Rightarrow a^2x^2+b^2y^2+2axby = 1^2+4axby-4\sqrt{abxy}$$

$$\Rightarrow a^2x^2+b^2y^2+2abxy-4abxy-1+4\sqrt{abxy} = 0$$

$$\Rightarrow a^2x^2+b^2y^2-2abxy-1-4\sqrt{abxy} = 0$$

$\Rightarrow a = a^2$ (I) $\mid b = b^2$ (II) $\mid 2h = -2ab$, $h = -ab$, $h^2 = a^2b^2$ (III) — From eq (I), (II) and (III) $h^2 = ab$

2. Standard eq^n of ellipse :$\Rightarrow$

Let focus of ellipse be $S(ae, 0)$, directrix be the line $x = \frac{a}{e}$, where 'e' is the eccentricity of ellipse, then the eq^n of ellipse in standard form is given by

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$$

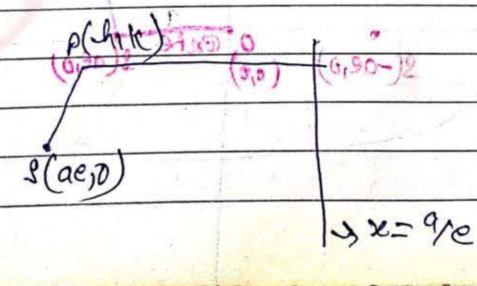
$\Rightarrow b^2 = K$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

M

where $b^2 = a^2(1-e^2)$

Proof:$\Rightarrow$



so,

By the definition :-

$$SP = e\,PM$$

$$\Rightarrow \sqrt{(h-ae)^2+(k-0)^2} = e\left|h-\frac{a}{e}\right|$$

$$\Rightarrow (h-ae)^2+k^2 = (eh-a)^2$$

$$\Rightarrow a^2e^2+h^2-\cancel{2aeh}+k^2 = e^2h^2+a^2-\cancel{2aeh}$$

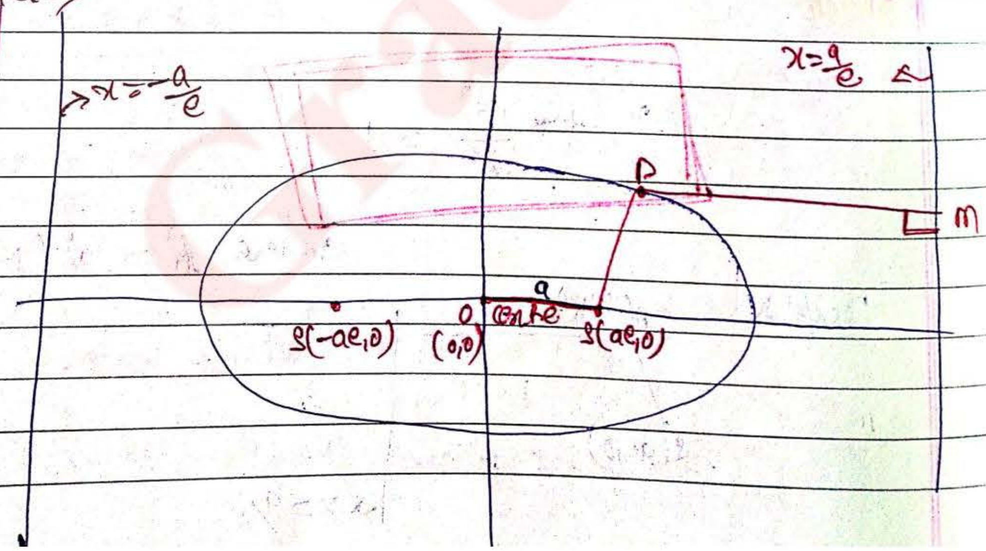
$$\Rightarrow h^2(1-e^2)+k^2 = a^2(1-e^2)$$

$$\Rightarrow \frac{h^2}{a^2}+\frac{k^2}{a^2(1-e^2)} = 1$$

Locus:-

$$\frac{x^2}{a^2}+\frac{y^2}{a^2(1-e^2)} = 1$$

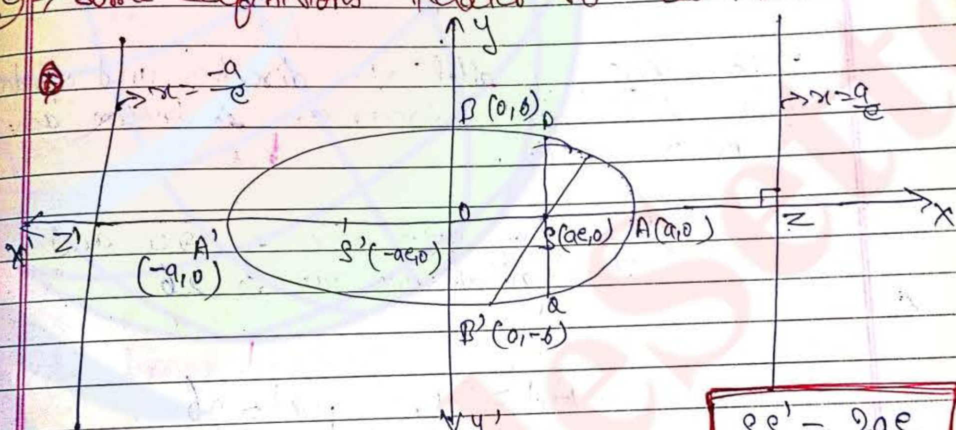
Note:⇒



If the centre of ellipse is (α, β) and axis are parallel to co-ordinate axis then the eqn of ellipse is

$$\boxed{\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1}$$

⇒ Some Definitions Related to Ellipse:→

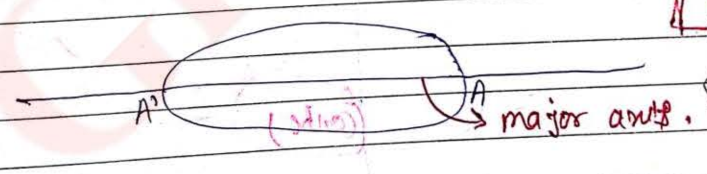


$$\boxed{\begin{aligned} SS' &= 2ae \\ ZZ' &= 2a/e \end{aligned}}$$

$$\boxed{\begin{aligned} AA' &= 2a \\ BB' &= 2b \end{aligned}}$$

1) Major Axis:→ "(AA')"

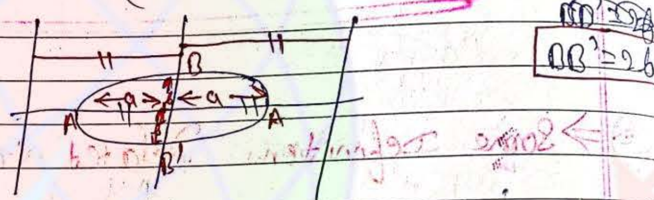
$$\boxed{AA' = 2a}$$



The line perpendicular to the directrix and passing through the focus of ellipse is called major axis.

ii) Minor axis :⇒

"(B B')"



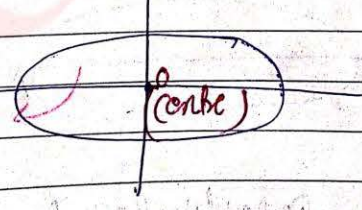
The line parallel to direction and equidistant from both the directrixes is known as minor axis.

Length of major axis is 2a and length of minor axis is (2b)

$AA' = 2a$, $BB' = 2b$

iii) Centre :→ "O"

The point of intersection of major and minor axis is called centre of ellipse



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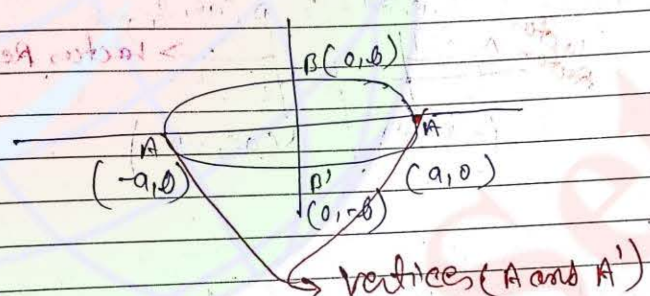
iv) vertices:→ (A and A')

Points of Intersection of ellipse and its major axis is called vertices

co-ordinate of vertices are

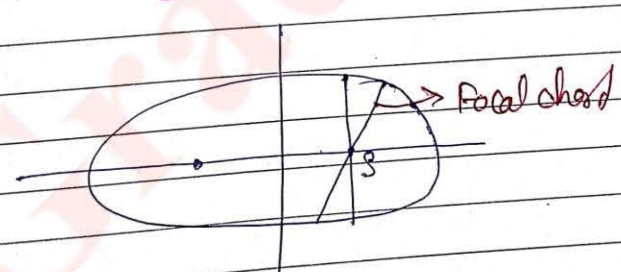
A (a, 0)

A' (−a, 0)



v) Focal chord:→

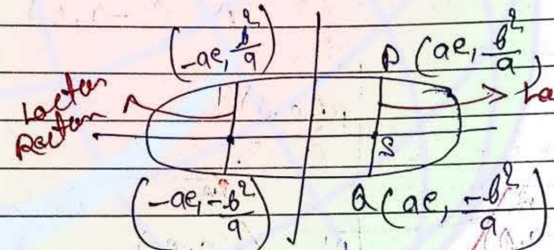
Any chord passing through the focus of ellipse is called focal chord,



vi) Lactus Rectum:-

The chord $\perp$ to major axis and passing through the focus of ellipse is called as lactus Rectum

Ends of lactus rectum are
$P\left(ae, \frac{b^2}{a}\right)$
$Q\left(ae, \frac{-b^2}{a}\right)$



And length of lactus Rectum is

$$\boxed{PQ = \frac{2b^2}{a}}$$

Q) Find centre, vertices, eccentricity, ~~focus~~ foci, directrix and ends of lactus rectum of ellipse

$$\frac{x^2}{3} + \frac{y^2}{2} = 1$$

Also draw its rough sketch.

Ans

a) centre $\equiv (0,0)$

(b) vertices $\equiv (\pm a, 0) \equiv (\pm\sqrt{3}, 0)$

(c) $b^2 = a^2(1-e^2)$

$$2 = 3(1-e^2)$$

$$e = \left(\frac{1}{\sqrt{3}}\right)$$

d) foci

$$(\pm ae, 0)$$

$$\left(\pm \sqrt{3}\cdot\frac{1}{\sqrt{3}}, 0\right)$$

$$(\pm 1, 0)$$

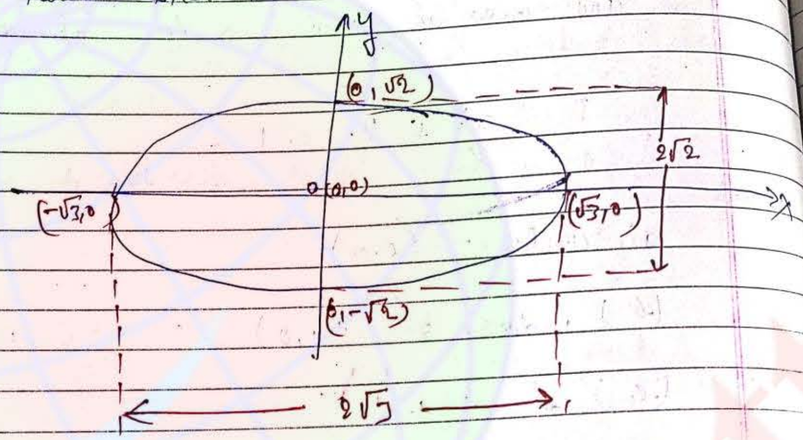
e) $x = \pm a/e$

$$x = \pm 3$$

f) $\left(\pm ae, \pm \frac{b^2}{a}\right)$

$$\left(\pm 1, \pm \frac{2}{\sqrt{3}}\right)$$

9) Rough sketch



eg:) Find the eqn of ellipse in standard form whose lactus rectum is '5' and $e = 2/3$

Ans)

Given :-

$$\frac{2b^2}{a} = 5$$

$$\Rightarrow 2b^2 = 5a \quad \text{—(1)}$$

$$e = 2/3$$

$$\boxed{b^2 = a^2(1 - e^2)}$$

$$b^2 = a^2\left(1 - \frac{4}{9}\right)$$

Now,

$$\frac{5}{2}a = a^2\left(\frac{5}{9}\right)$$

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$a = \frac{9}{2}$

$b^2 = \frac{45}{4}$

so,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

eqn

$$\boxed{\frac{4x^2}{81} + \frac{4y^2}{45} = 1}$$

Q) If $x^2 + 4y^2 + 2x + 16y + 13 = 0$, find all the points!

$\Rightarrow x^2 + 2x + 4(y^2 + 4y) + 13 = 0$

$\Rightarrow (x^2 + 2x + 1) + 4(y^2 + 4y + 4) + 13 = 1 + 16$

$\Rightarrow (x+1)^2 + 4(y+2)^2 = 4$

$\Rightarrow \frac{(x+1)^2}{4} + \frac{(y+2)^2}{1} = 1$

$\frac{X^2}{4} + \frac{Y^2}{4} = 0 \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

1) Centre :→

$X = 0$ $\qquad Y = 0$

$x + 1 = 0$ $\qquad y + 2 = 0$

$x = -1$ $\qquad y = -2$

centre $(-1, -2)$

ii) vertex :-

$A(a,0)$	$A'(-a,0)$
A' :-	A'
$x=a$; $y=0$	$x=-a$; $y=0$
$x+1=2$; $y+2=0$	$x+1=-2$; $y+2=0$
$x=1$; $y=-2$	$x=-3$; $y=-2$
$A(1,-2)$	$A'(-3,-2)$

iii) $b^2 = a^2(1-e^2)$

$1 = 4(1-e^2)$

$e = \frac{\sqrt{3}}{2}$

iv) focus

$(ae, 0)$ | $(-ae, 0)$

$\cancel{(2\times\frac{\sqrt{3}}{2}, 0)}$

$(\sqrt{3}, 0)$ | $\cancel{(-\sqrt{3}, 0)}$

iv) focus

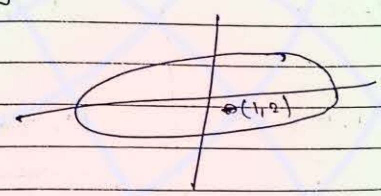
$S(ae,0)$	$S'(-ae,0)$
$x = 2\frac{\sqrt{3}}{2}$, $y=0$	$x=-2\frac{\sqrt{3}}{2}$, $y=0$
$x+1=\sqrt{3}$, $y+2=0$	$x+1=-\sqrt{3}$, $y+2=0$
$x=\sqrt{3}-1$, $y=-2$	$x=-\sqrt{3}-1$, $y=-2$
$S(\sqrt{3}-1, -2)$	$S'(-\sqrt{3}-1, -2$

v) Directrix

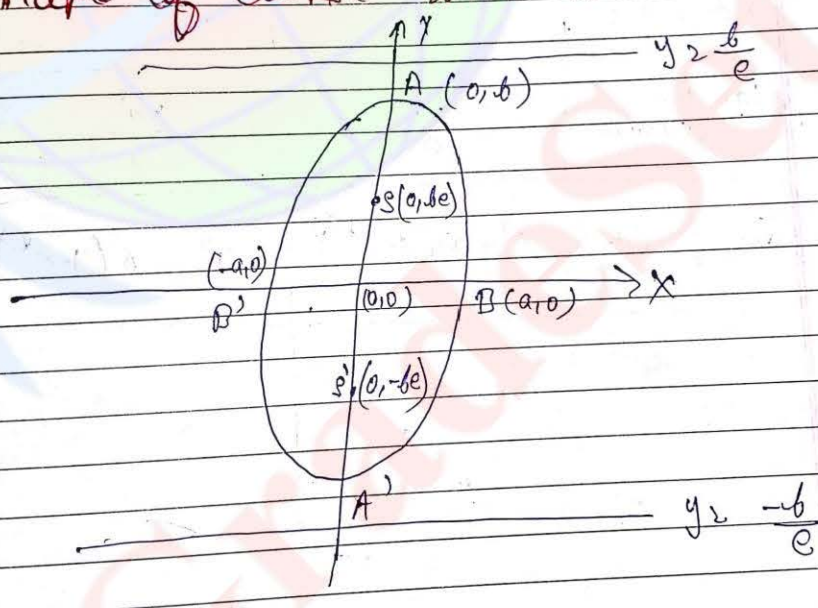
$x = \frac{a}{e}$	$x = \frac{-a}{e}$
$x+1 = \frac{2\times2}{\sqrt{3}}$	$x+1 = \frac{-2\times2}{\sqrt{3}}$

$x = \frac{4}{\sqrt{3}} - 1$ | $x = \frac{-4}{\sqrt{3}} - 1$

iv) Rough Sketch



(X) Shape of ellipse when $b^2 > a^2$:⇒



Comparision

	$a^2 > b^2$	$b^2 > a^2$
Centre	$(0,0)$	$(0,0)$
major axis	x-axis	y-axis
vertices	$(\pm a, 0)$	$(0, \pm b)$
~~focus~~ Focii	$(\pm ae, 0)$	$(0, \pm be)$
Length of major axis	$2a$	$2b$
Lactus Rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Eccentricity(e)	$b^2 = a^2(1-e^2)$	$a^2 = b^2(1-e^2)$

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Eg:1) Find Point, centre, vertices, eccentricity, foci, and directrix of the ellipse 1

$$3x^2+2y^2=6$$

Ans ⇒ $\frac{3x^2}{6}+\frac{2y^2}{6}=\frac{6}{6}$

$\frac{x^2}{2}+\frac{y^2}{3}=1$

i) Centre $\equiv (0,0)$

ii) vertices $\equiv (0,\pm\sqrt{3})$

iii) $a^2=b^2(1-e^2)$

$2=3(1-e^2)$

$e=\frac{1}{\sqrt{3}}$

iv) Focii $\equiv (0,\pm be)$

$\equiv (0,\pm 1)$

v) directrix

$y=\pm 3$

2.) Find point, centre, vertices, eccentricity, foci and direct of the ellipse.

$9x^2+5y^2-30y=0$

Ans ⇒ $9x^2+5y^2-30y=0$

⇒ $9x^2+5(y^2-6y+9)=45$

⇒ $9x^2+5(y-3)^2=45$

⇒ $\frac{x^2}{5}+\frac{(y-3)^2}{9}=1$

⇒ $\frac{X^2}{5}+\frac{Y^2}{9}=1$

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i) Centre :→

$x = 0$, $y = 0$

$x = 0$; $y - 3 = 0$

Centre : (0, 3)

ii) vertices :

Alternative

$A(0, 6)$

$x = 0$, $y = 3$

$x = 0$; $y - 3 = 3$

$x = 0$; $y = 6$

$A(0, 6)$

$A'(0, 0)$

$x = 0$, $y = -3$

$x = 0$, $y - 3 = -3$

$y = 0$

$A'(0, 0)$

iii) $a^2 = b^2(1 - e^2)$

$5 = 9(1 - e^2)$

$e = 2/3$

iv) Focus :→ $(0, \pm be)$

$\Rightarrow 0, \pm 3 \times \frac{2}{3}$

$\Rightarrow (0 \pm 2)$

cont.

Q) Find the co-ordinate of foci of ~~eqtion~~ ellipse $4x^2+9y^2=1$

Ans $4x^2+9y^2=1$

$\Rightarrow \frac{x^2}{1/4}+\frac{y^2}{1/9}=1$

$\Rightarrow a^2=1/4, \quad a=\frac{1}{2}$

$b^2=1/9, \quad b=\frac{1}{3}$

So, $b^2=a^2(1-e^2)$

$e=\sqrt{5}/3$

$S(\pm ae, 0)$

$S(\pm \sqrt{5}/6, 0)$

Q) Find the eqn of standard ellipse passing through (2, 1) and having eccentricity(e) = 1/2

Ans

Ans:- $3x^2+4y^2=16$

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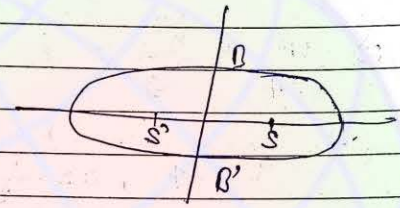
Q) Find the eqⁿ of standard ellipse whose minor axis is equal to the distance between foci and lactus Rectum is 10.

Ans)



$SS_1 = 2ae$
$BB' = 2b$

$SS' = BB'$

$2ae = 2b$

$b = ae$ — ①

$b^2 = a^2(1-e^2)$

from eqⁿ ①

$\cancel{a^2}e^2 = \cancel{a^2}(1-e^2)$

$e^2 = \frac{1}{2}$

$e = \frac{1}{\sqrt{2}}$

$b = \frac{a}{\sqrt{2}}$ — ①

$L.R = \frac{2b^2}{a} = 10$

$\frac{2\left(\frac{a}{2}\right)^2}{a} = 10$

$\boxed{a = 10}$

$b = \frac{10}{\sqrt{2}}$

$\frac{x^2}{100} + \frac{y^2}{50} = 1$

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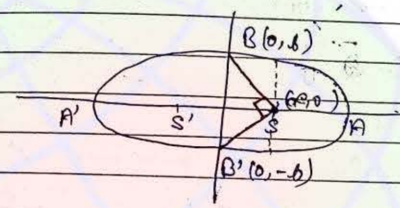
Q. Find the eccentricity of ellipse whose minor axis subtends right angle at the focus.

Ans.



$SB \perp SB'$

So,

$$\left(\frac{b-0}{-ae}\right)\left(\frac{-b-0}{-ae}\right) = -1$$

$$b^2 = a^2e^2 \quad \text{--- (1)}$$

$$\Rightarrow b^2 = a^2(1-e^2)$$

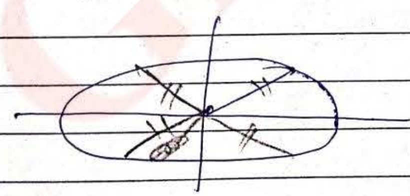
$$\Rightarrow a^2e^2 = a^2(1-e^2)$$

$$e^2 = \frac{1}{2}$$

$$e = \frac{1}{\sqrt{2}} \text{ Ans.}$$

Q. There are exactly two points on the ellipse whose distance from its centre is same and is equal to $\sqrt{\frac{a^2+2b^2}{2}}$, find the eccentricity of ellipse.

Ans.



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$$\sqrt{\frac{a^2+2b^2}{2}}$$

check major or minor axis! —

$$\sqrt{\frac{a^2}{2}+b^2} > b$$

$$d = a$$

$$\frac{a^2+2b^2}{2} = a^2$$

$$2b^2 = a^2 \quad \text{—} (1)$$

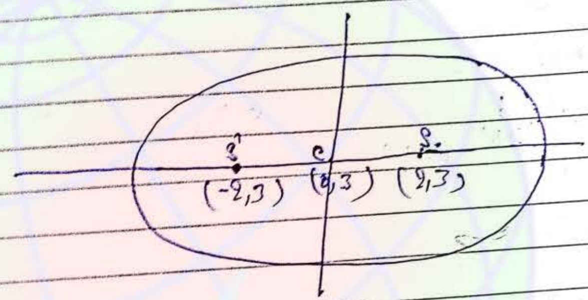
$$b^2 = a^2(1-e^2)$$

$$b^2 = 2b^2(1-e^2)$$

$$e = 1/\sqrt{2}$$

V.V.I Q1 Find the eq^n of ellipse whose foci are $(2,3)$ and $(-2,3)$ and semiminor axis is $\sqrt{5}$

A.



$b = \sqrt{5}$

$ae = CS$ (distance है (CS) का)

$ae = 2$ — (1)

$b = \sqrt{5}$ — (2)

$b^2 = a^2(1-e^2)$

$b^2 = a^2 - a^2e^2$ $\quad \because b^2 = 5,\ a^2e^2 = 4$

$5 = a^2 - 4$

$a^2 = 9$

Centre $\equiv (0,3)$

$a^2 = 9$

$b^2 = 5$

So,

Eq^n of ellipse :—

$$\frac{(x-0)^2}{9} + \left(\frac{y-3}{5}\right)^2 = 1$$

1st Choice

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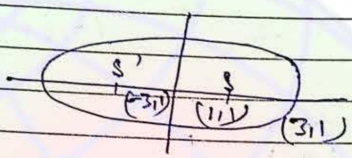
Q.) Find the eq^n of ellipse whose one vertex is $(3,1)$ and ~~the~~ ~~foc~~ fo~~r nearest~~ nearest focus is $(1,1)$ and $e = \frac{2}{3}$.

Ans: —



$e = \frac{2}{3}$

$$a - ae = 2$$

so,

$$a\left(1 - \frac{2}{3}\right) = 2$$

$$\boxed{a = 6}$$

$$b^2 = a^2(1 - e^2)$$
$$= 36\left(1 - \frac{4}{9}\right)$$
$$= 20$$

Centre $\equiv (-3, 1)$

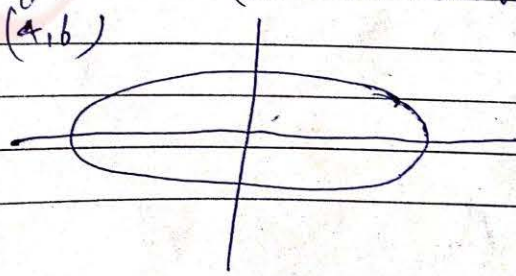
$$a^2 = 36$$
$$b^2 = 20$$

so, Eq^n of ellipse:—

$$\frac{(x+3)^2}{36} + \frac{(y-1)^2}{20} = 0$$

Q.) Find the eq^n of ellipse whose centre is $(1,2)$ and one focus is $(6,2)$ and ellipse passes through $(4,6)$

Ans:—



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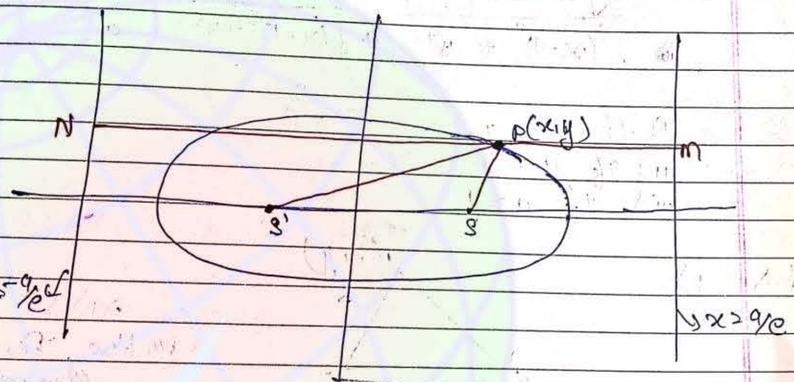
Another Definition of Ellipse :⇒



$$\boxed{\begin{aligned} SP &= ePM \\ S'P &= ePN \end{aligned}}$$

$$SP + S'P = e(PM + PN)$$
$$= e(MN)$$
$$= \cancel{e}\left(\frac{2a}{\cancel{e}}\right)$$

Hyperbola में minus होगा।

$$\boxed{SP + S'P = 2a} \qquad (2a > SS')$$

Definition :— It is the locus of a point which moves in such a way that sum of its distances from two fixed points remains constant

"$SP + S'P = 2a$"

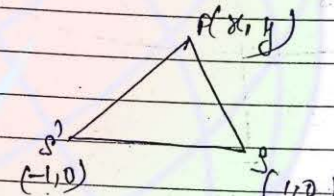
Two fixed points (S and S') are foci of ellipse and sum of its distances is length of major axis "2a".

eg:- Identify the locus of $P(x, y)$

$$\sqrt{(x-1)^2 + y^2} + \sqrt{(x+1)^2 + y^2} = a$$

i) If $a = 4$
ii) If $a = 2$
iii) If $a = 1$

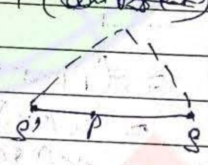
Ans:- i)



$2a > SS'$
So, the P forms ellipse

~~So, i) If a = 4 (ellipse)~~

ii)



~~Here, forms ellipse.~~
~~2a > SS'~~

So, $2a = SS'$
(So here form line segment (SS'))

iii) No form any locus.

Q:- A circle C_1 lies completely inside the circle C_2. A variable circle touches C_1 externally & C_2 internally. Identify the locus of centre of variable circle.

Ans:

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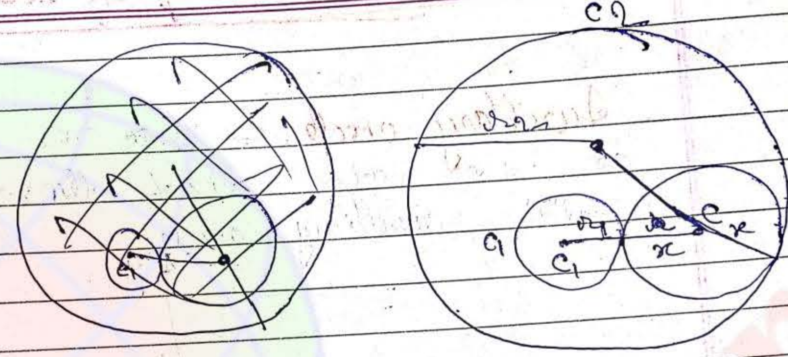
Parametric eqⁿ of ellipse



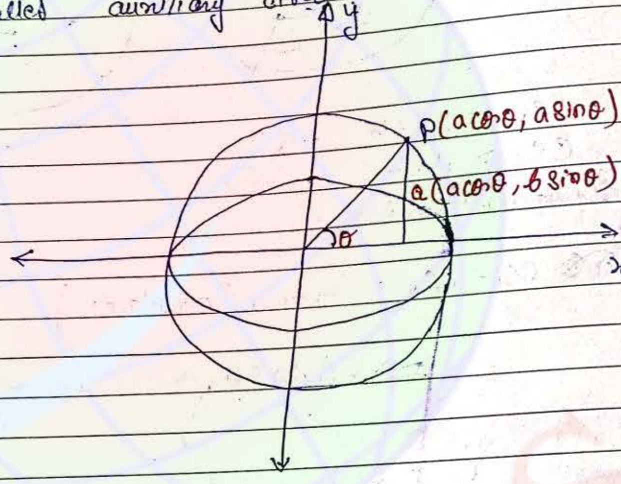
$CC_1 = r_1 + x$ —①
$CC_2 = r_2 - x$ —②
Add eq ① and eq ②

$CC_1 + CC_2 = r_1 + r_2$ (constant)
$\;\;SP \;\;\;\; S'P \;\;\;\; 2a$
$\Rightarrow$ Ellipse.

1st Choice

Paremetric eqⁿ of Ellipse :-

Auxillary circle :⇒ The circle described as the major axis of ellipse as its diameter is called auxiliary circle.



Eqⁿ :⇒ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Auxillary circle :⇒ $x^2 + y^2 = a^2$

Here,
Point P and Q are called "corresponding point"

So,
Paremetric eqⁿ of ellipse

$$\boxed{\begin{aligned} x &= a\cos\theta \\ y &= b\sin\theta \end{aligned}} \quad P(\theta)$$

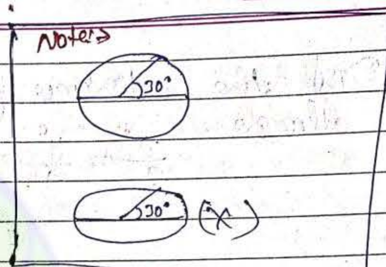
where, θ → Parameter.
(eccentric Angle)

1st Choice

Note:

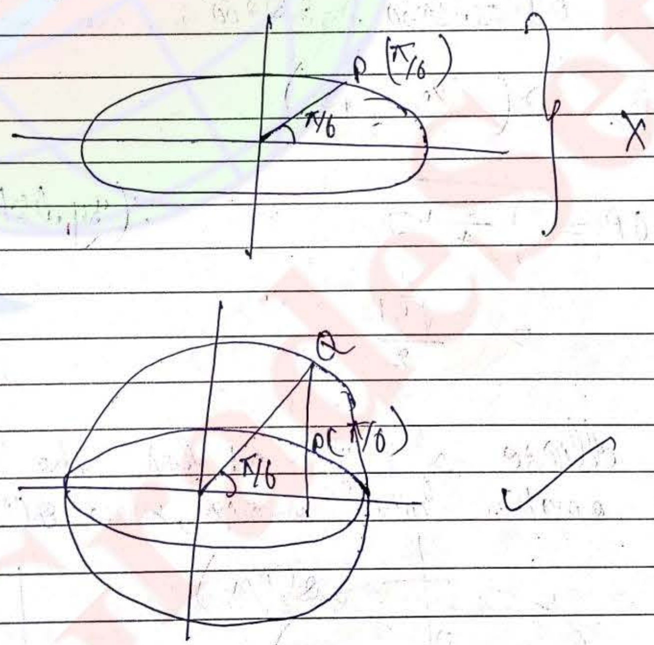
Attention:

Note:⇒



↳ जो circle का θ Angle होगा वह भी ellipse का θ Angle नहीं हो सकता

Note: ⇒



1) $\alpha \in [0, 2\pi]$

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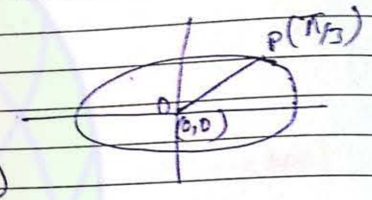
Q) Find the distance of point $P(\pi/3)$ on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ from its centre

Ans)



$x = a\cos\theta$
$y = b\sin\theta$

so $P(a\cos\theta,\ b\sin\theta)$

$P\left(a\cos\pi/3,\ b\sin\pi/3\right)$

⇐ [illegible]

$P\left(3\cdot\cos 60,\ 2\sin 60^\circ\right)$

$P\left(3/2,\ \sqrt{3}\right)$

$OP = \sqrt{\frac{9}{4}+3}$ ∵(By distance formula)

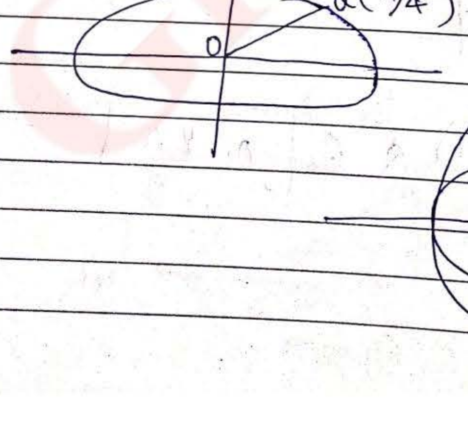
$= \frac{\sqrt{21}}{2}$

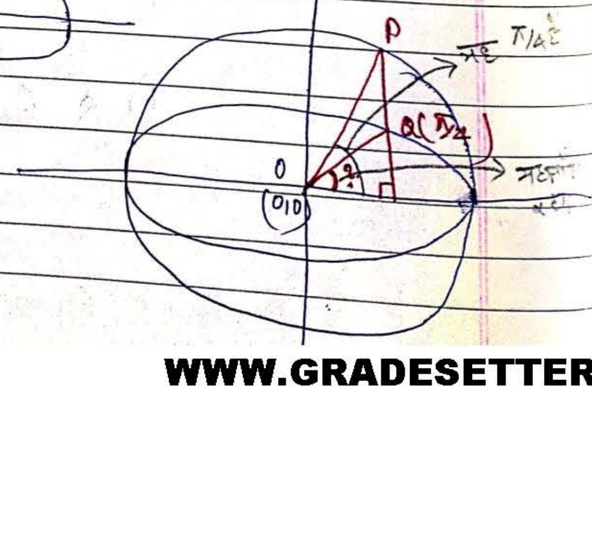
Diagram and again

imp

Q) For ellipse $\frac{x^2}{6}+\frac{y^2}{2}=1$ find the angle which OQ makes with x-axis, where $Q(\pi/4)$

Ans)





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$Q(a\cos\pi/4, b\sin\pi/4)$

$Q(\sqrt{3}, 1)$

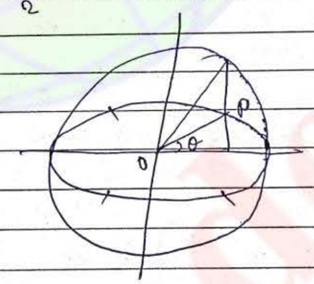
slope of $OQ = \tan\theta = \frac{1}{\sqrt{3}}$

$(\theta = 30^\circ)$ Ans.

Q-> Find the eccentric Angle of points on ellipse

$\frac{x^2}{6} + \frac{y^2}{2} = 1$ whose distance from centre is '2'.

Ans



(four points are possible.

Given , $OP = 2$

$\Rightarrow \sqrt{6\cos^2\theta + 2\sin^2\theta} = 2$

$\Rightarrow 6\cos^2 + 2(1-\cos^2\theta) = 4$

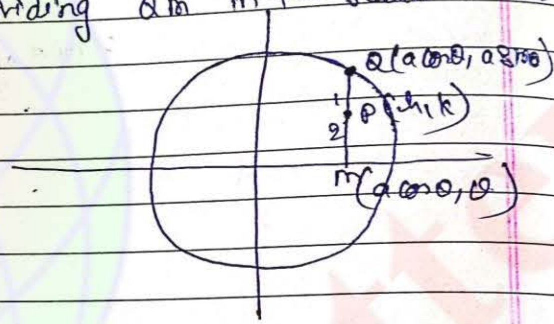
$\Rightarrow \cos^2\theta = 1/2$, $\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{\pi}{4}$

so, $\theta \Rightarrow \frac{\pi}{4}, \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

Q:> From a Point 'Q' on the circle $x^2+y^2=a^2$ Perpendicular QM is drawn to the x-axis find the locus of point 'P' dividing QM in the ratio 1 : 2

Ans:>



$$h = a\cos\theta \quad -①$$

$$k = \frac{2a\sin\theta}{3} \quad -②$$

Gen → 140

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Position of Point with respect to ellipse.

Ellipse :→ $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0$$

$$\boxed{S=0}$$

Point : (x_1, y_1)

$$S_1 = \frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$$

i) If "$S_1 < 0$"
Point lies "Inside" the ellipse.

ii) If "$S_1 > 0$"
Point lies "out side" the ellipse

iii) If "$S_1 = 0$"
Point lies "on" the ellipse.

Q:- check the position of $(4, -3)$ with respect to ellipse.
$5x^2+7y^2=140$

Ans :→ $S_1 : 5(4)^2+7(-3)^2-140$

$= 3 > 0$

$\Rightarrow$ out side

1st Choice **Line and Ellipse**

Ellipse :- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ —①

Line :- $y = mx + c$ —②

Solve eq ① and ②

$$\Rightarrow \frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$$

$$\Rightarrow x^2b^2 + a^2(m^2x^2 + 2mcx + c^2) = a^2b^2$$

$$\boxed{\Rightarrow x^2(b^2+a^2m^2) + 2ma^2cx + a^2(c^2-b^2) = 0}$$

$$\boxed{\Delta = f^2\left[a^2m^2+b^2-c^2\right]}$$

i) If $\Delta > 0$
$a^2m^2+b^2 > 0$ (line Intersect at two points on the ellipse.)

ii) If $\Delta < 0$
Neither touches nor Intersect.

iii) If $\Delta = 0$
line will make tangent of the ellipse.

Condition of tangency

"$\Delta = 0$"

$$\boxed{c^2 = a^2m^2 + b^2}$$

Q. Find the condition that line $lx + my + n = 0$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Ans. $lx + my + n = 0$

$$y = \frac{-lx - n}{m}$$

$$\Rightarrow \underbrace{\left(\frac{-l}{m}\right)}_{m}x + \underbrace{\left(\frac{-n}{m}\right)}_{c}$$

$$c^2 = a^2m^2 + b^2$$

$$\left(\frac{-n}{m}\right)^2 = a^2\left(\frac{-l}{m}\right)^2 + b^2$$

$$\boxed{n^2 = a^2l^2 + m^2b^2}$$

1st Choice

Equation of tangent

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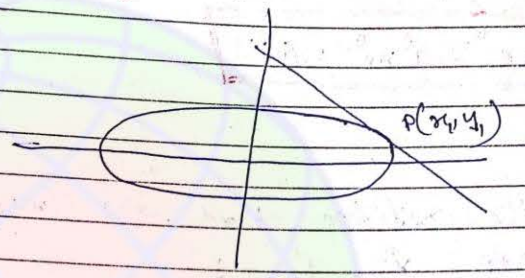
1.) Point form :⇒



Eqⁿ of tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point (x_1, y_1) lying on the ellipse is given by

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

$$\boxed{\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1=0}$$

$$\boxed{T=0}$$

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2.) Parametric form:

$$(x_1, y_1) \equiv (a\cos\theta, b\sin\theta)$$

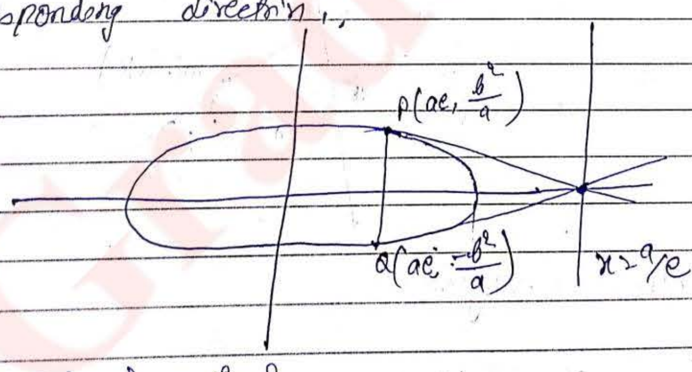
Eqⁿ of tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $a\cos\theta, b\sin\theta$ is

$$\frac{x(a\cos\theta)}{a^2}+\frac{y(b\sin\theta)}{b^2}=1$$

$$\boxed{\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1}$$

Eq. Show that tangents at the extremeties of latus rectum of an ellipse intersect on the corresponding directrix.

Ans



Ellipse :- $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

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Eqn of tangent at 'P'

$$T = 0$$

$$\frac{x(ae)}{a^2} + \frac{y(b^2/a)}{b^2} = 1$$

$$ex + y = a \quad \text{—(1)}$$

Eqn of tangent at 'Q'.

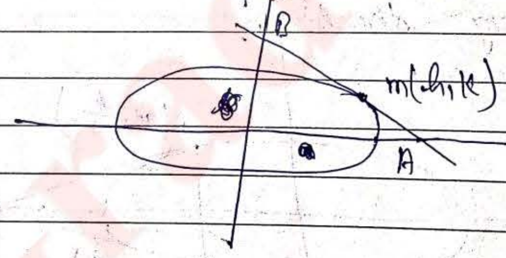
$$ex - y = a \quad \text{—(2)}$$

from (1) and eq (2)

$$x = \frac{a}{e}$$

Ex:- A tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Intersect the co-ordinate axis at point 'A' and 'B' Find the locus of mid-point of AB.

Ans:



AB : (tangent)

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

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$A\left(\frac{a}{\cos\theta}, 0\right)$

$B\left(0, \frac{b}{\sin\theta}\right)$

$h = \frac{a/\cos\theta}{2}$

$\cancel{\cos\theta \; \frac{2h}{a} = \cos\theta}$ — ①

$\frac{a}{2h} = \cos\theta$ — ①

$\frac{b}{2k} = \sin\theta$ — ②

$①^2 + ②^2$

$\frac{a^2}{4h^2} + \frac{b^2}{4k^2} = 1$

Locus: ⇒

$\cancel{\frac{x^2}{a^2} + \frac{b^2}{y^2}}$ $\boxed{\frac{a^2}{x^2} + \frac{b^2}{y^2} = 4}$

3.) Slope form:→

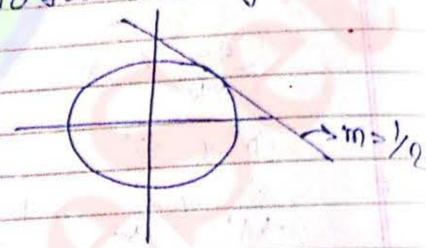
Eqⁿ of tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose slope is m is given by

$$\boxed{y = mx \pm \sqrt{a^2m^2+b^2}}$$

Q) Find the eqⁿ of tangent to ellipse $3x^2+4y^2=12$ which is perpendicular to the line $y+2x=4$

Ans. $3x^2+4y^2=12$

$y+2x=4$



so,

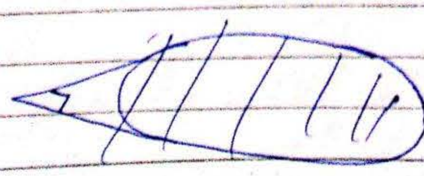
$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

Eqⁿ :⇒ $y = \frac{1}{2}x \pm \sqrt{4\cdot\frac{1}{4}+3}$

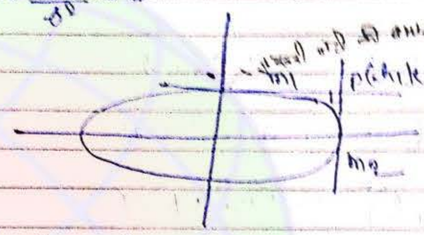
$$y = \frac{1}{2}x \pm 2$$

Q) Find the locus of point of intersection of ⊥ tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Ans



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Eqⁿ of tangent:-

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

Pass (h, k)

$$k = mh \pm \sqrt{a^2m^2 + b^2}$$

$$(k - mh)^2 = a^2m^2 + b^2$$

$$m^2(h^2 - a^2) - 2mkh + k^2 - b^2 = 0 \quad \begin{matrix} m_1 \\ m_2 \end{matrix}$$

$$m_1 m_2 = -1$$

$$\frac{k^2 - b^2}{h^2 - a^2} = -1$$

$$k^2 - b^2 = -h^2 + a^2$$

$$h^2 + k^2 = a^2 + b^2$$

Locus:- $\boxed{x^2 + y^2 = a^2 + b^2}$

→ "Director circle"!

→ more Important! → for any angle.

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Alt :→ PA :⇒ (Applicable for only 90°)

$$y = m_1 x + \sqrt{a^2 m_1^2 + b^2}$$

$$k - m_1 h = \sqrt{a^2 m_1^2 + b^2}$$

$$(k - m_1 h)^2 = a^2 m_1^2 + b^2 \quad \text{— (1)}$$

PB :-

$$y = \frac{-1}{m_1} x + \sqrt{\frac{a^2}{m_1^2} + b^2}$$

$$(k m_1 + h)^2 = a^2 + m_1^2 b^2 \quad \text{— (2)}$$

eq (1) + eq (2)

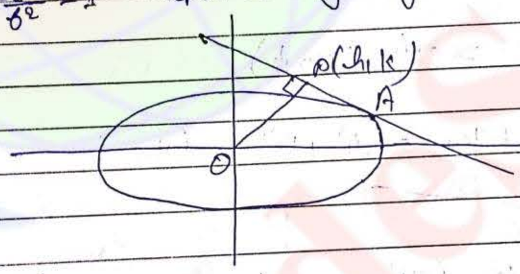
$$k^2(1 + m_1^2) + h^2(1 + m_1^2) = a^2(1 + m_1^2) + b^2(1 + m_1^2)$$

$$\boxed{h^2 + k^2 = a^2 + b^2}$$

Q ⇒ Show that locus of a point from which tangents to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ includes an angle "α" is given by $(x^2 + y^2 - a^2 - b^2)^2 = 4\cot^2\alpha\,(a^2y^2 + b^2x^2 - a^2b^2)$

Q:> Find the locus of foot of "⊥" from centre of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ upon its any tangent

Ans.



1st method:⇒

PA :-

$$y = mx \pm \sqrt{a^2m^2+b^2}$$

Now pass (h,k) $\quad k = mh \pm \sqrt{a^2m^2+b^2}$ —(1)

$OP \perp PA$

$$\left(\frac{k}{h}\right)m = -1$$

$$m = -\frac{h}{k} \quad —(2)$$

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$$k = \frac{-h^2}{k} \pm \sqrt{\frac{a^2h^2}{k^2} + b^2}$$

$$\left(k + \frac{h^2}{k}\right)^2 = \frac{a^2h^2}{k^2} + b^2$$

$$\left(h^2 + k^2\right)^2 = a^2h^2 + k^2b^2$$

~~Alter~~
Alternatively

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1 \quad \text{--- (1)}$$

PA :→

$$y - k = \frac{-h}{k}(x - h)$$

$$hx + ky = h^2 + k^2 \quad \text{--- (2)}$$

Compare

$$\frac{\cos\theta}{ah} + \frac{\sin\theta}{bk} = \frac{1}{h^2+k^2}$$

$$\cos\theta = \frac{ah}{h^2+k^2} \quad \text{--- (3)}$$

$$\sin\theta = bk / h^2 + k^2 \longrightarrow (4)$$

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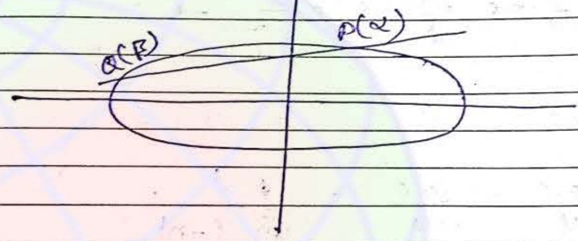
* Equation of chord: →



Eqⁿ of chord joining the points $P(\alpha)$ and $Q(\beta)$ lies on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

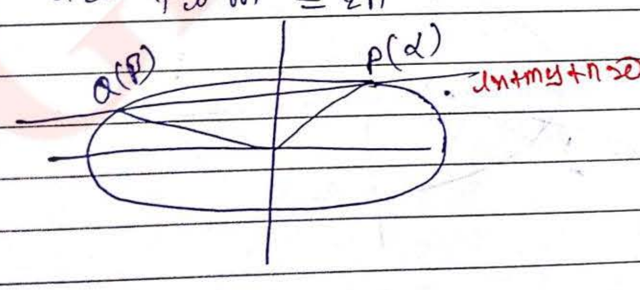
$$\boxed{\frac{x}{a}\cos\frac{\alpha+\beta}{2}+\frac{y}{b}\sin\frac{\alpha+\beta}{2}=\cos\frac{\alpha-\beta}{2}}$$

Hint ⇒ two point form.

Eg. If the line $lx+my+n=0$ this line intersect the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at two point whose ~~Intersect~~ eccentric angle differ by $\pi/2$ then show that

$$a^2l^2+b^2m^2=2n^2$$

Ans.



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$$lx + my = -n \quad \text{—(1)}$$

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = \frac{1}{\sqrt{2}} \quad \text{—(2)}$$

where, $\theta = \frac{\alpha+\beta}{2}$

Compare

$$\frac{\cos\theta}{al} = \frac{\sin\theta}{bm} = \frac{1}{\sqrt{2}\cdot(-n)}$$

$$\cos\theta = \frac{-al}{\sqrt{2}n} \quad \text{—(3)}$$

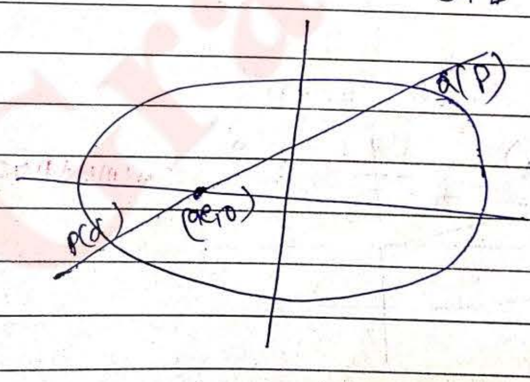
$$\sin\theta = \frac{-bm}{\sqrt{2}\,n} \quad \text{—(4)}$$

Q-> If chord joininng the point $P(\alpha)$ and $Q(\beta)$ is a focal chord then show that

i) $e = \dfrac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\frac{\alpha+\beta}{2}}$

ii) $\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2} = \frac{e-1}{e+1}$

Ans ⇒



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i) $\frac{x}{a}\cos\frac{\alpha+\beta}{2} + \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2}$

Pass $(ae, 0)$

$$\frac{ae}{a}\cos\frac{\alpha+\beta}{2} + 0 = \cos\frac{\alpha-\beta}{2}$$

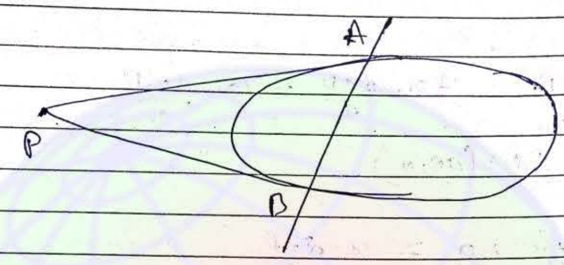
$$e = \frac{\cos\frac{\alpha-\beta}{2}}{\cos\frac{\alpha+\beta}{2}}$$

ii) $\frac{e-1}{e+1} = \frac{\cos\frac{\alpha-\beta}{2} - \cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2} + \cos\frac{\alpha+\beta}{2}}$

$$= \frac{\cancel{2}\sin\alpha/2\ \sin\beta/2}{\cancel{2}\cos\alpha/2\ \cos\beta/2}$$

$$\Rightarrow \tan\alpha/2 \cdot \tan\beta/2$$

Chord of Contact



From any external point two tangents can be drawn to a ellipse.
The chord joining ~~their~~ their point of contact is called chord of contact.

If eqⁿ of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

and external point is $P(x_1, y_1)$ then eqⁿ of chord of contact of tangents from point 'P' is:-

$$\boxed{\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1}$$

$$\frac{xx_1}{a^2}+\frac{yy_1}{b}-1=0$$

$$\boxed{T=0}$$

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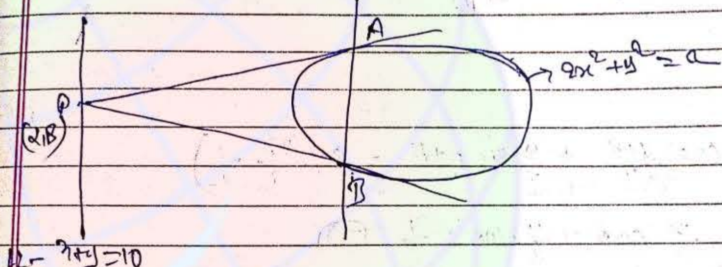
Q. Tangents are drawn from any point on the line $x+y=10$ to ellipse $2x^2+y^2=2$ then show that all chords of contact are congruent at $\left(\frac{1}{10}, \frac{1}{5}\right)$



(α, β) lies on "L"

$\alpha+\beta=10$ —(1)

Eq^n of AB

$T=0$

$2x\alpha + y\beta = 2$

$2x\alpha + y(10-\alpha) = 2$

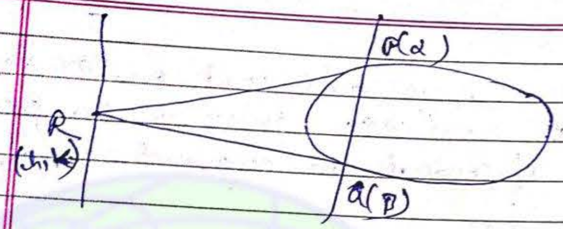
$\underbrace{(10y-2)}_{L_1} + \alpha\underbrace{(2x-y)}_{L_2} = 0$

$L_1 + \lambda L_2 = 0$

$\left.\begin{array}{l} 10y-2=0 \\ 2x-y=0 \end{array}\right\} \left(\frac{1}{10}, \frac{1}{5}\right)$

Q. If tangent at $P(\alpha)$ and $Q(\beta)$ intersect at point 'R', find the locus of point 'R' If $\alpha-\beta = \frac{2\pi}{3}$

A.



$$\frac{x}{a}\cos\frac{\alpha+\beta}{2} + \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\left(\frac{2\pi}{3}\cdot\frac{1}{2}\right)$$

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = \frac{1}{2} \quad \text{—(1)}$$

PQ:-

$$\frac{xh}{a^2} + \frac{yk}{b^2} = 1 \quad \text{—(2)}$$

Compare:-

$$\frac{\cos\theta\, a^2}{a\cdot h} = \frac{\sin\theta\, b^2}{bk} = \frac{1}{2}$$

$$\cos\theta = \frac{h}{2a}$$

Locus:-

$$\frac{x^2}{4a^2} + \frac{y^2}{4b^2} = 1$$

Q. chords of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Always touch the concentric ellipse $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$ than, find the locus of their pole.

A. $E_1 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b} = 1$

$E_2 \;\; \frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$

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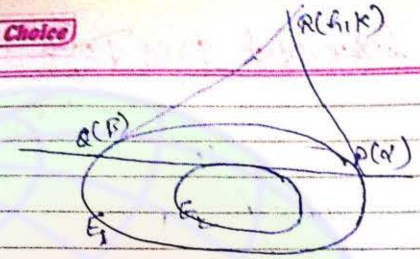
Ans:→



PQ :→ $\frac{xh}{a^2} + \frac{yk}{b^2} = 1$ —(1)

PQ :→ $\frac{x}{\alpha}\cos\theta + \frac{y}{\beta}\sin\theta = 1$ —(2)

$\frac{\cos\theta\, a^2}{\alpha \cdot h} = \frac{\sin\theta \cdot b^2}{\beta \cdot k} = 1$

$\cos\theta = \frac{\alpha h}{a^2}$

$\sin\theta = \frac{\beta k}{b^2}$

Locus→

$\frac{\alpha^2 x^2}{a^4} + \frac{\beta^2 y^2}{b^4} = 1$

Q.→ ~~cos of ellipse~~ chord of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touch ~~the~~ the parabola $ay^2 = -2b^2x$ then show that locus of their poles is the parabola $ay^2 = 2b^2x$

Ans

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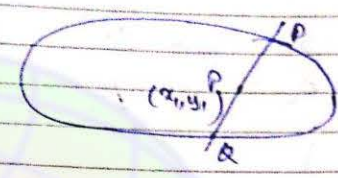
Chord bisected at given Point



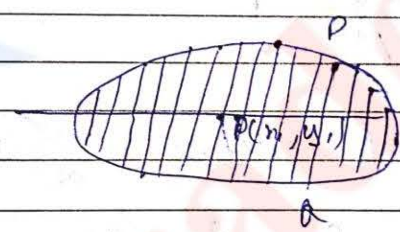
Eqn of chord of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose mid-point is (x_1, y_1) is

$$\boxed{T = S_1}$$

$$\boxed{\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}}$$

Q:> Find the locus of middle point of chord of standard ellipse if slope of chord is 'm'.

Ans:>



$$m = \frac{-x_1}{a^2} \cdot \frac{b^2}{y_1}$$

Locus:>

$$\boxed{y = \frac{-b^2}{a^2 m} x}$$ Diameter.

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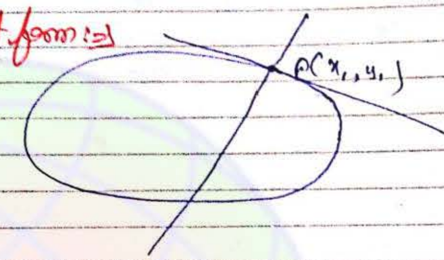
Eq^n of normal

1) Point form :-



The eq^n of normal to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ at the point (x_1, y_1) lying on the ellipse is

$$\boxed{\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2}$$

2) Parametric form :-

Eq^n of normal to standard ellipse at the point $P(a\cos\theta, b\sin\theta)$ is

$$\frac{a^2x}{a\cos\theta} - \frac{b^2y}{b\sin\theta} = a^2 - b^2$$

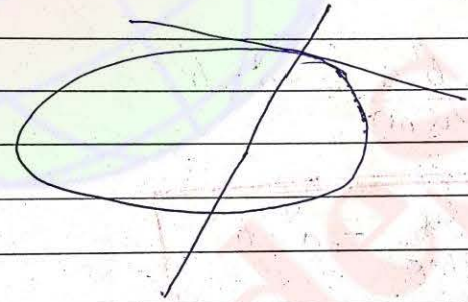
$$\boxed{\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2}$$

Imp

Q :) If the line $lx + my + n = 0$ is a normal to standard ellipse then show that

$$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$$

An.



Eq^n of normal :—

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2-b^2 \quad \text{—} (1)$$

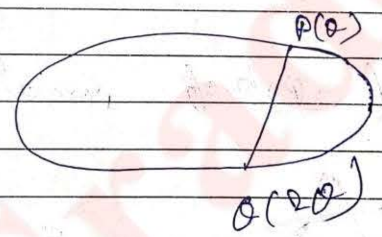
$$lx + my = -n \quad \text{—} (2)$$

Compare:>

$$\frac{l\cos\theta}{a} = \frac{-m\sin\theta}{b} = \frac{-n}{a^2-b^2}$$

Q If the normal at point $P(\theta)$ to the ellipse $\frac{x^2}{14}+\frac{y^2}{5}=1$ Intersect it again at Point $Q(2\theta)$. Then find the value of $\cos\theta$.

Ans)



eq^n of normal at "P"(θ)

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2-b^2$$

PQ :→

$$\frac{xh}{a^2} + \frac{ky}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2} \quad (2)$$

Compare :-

$$\frac{h\cos\theta}{a^3} = \frac{-k\sin\theta}{b^3} = \frac{\left(\frac{h^2}{a^2} + \frac{k^2}{b^2}\right)}{a^2 - b^2}$$

$$\frac{a^2}{x^2} + \frac{b^2}{y^2} = \left(\frac{a^2 - b^2}{\frac{x^2}{a^2} + \frac{y^2}{b^2}}\right)^2$$

H.W.

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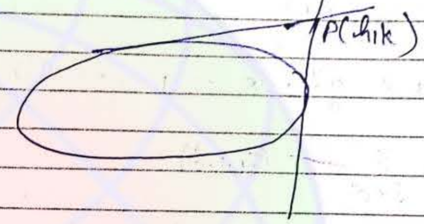
Director circle

Locus of point of intersection of Perpendicular (⊥) tangents.

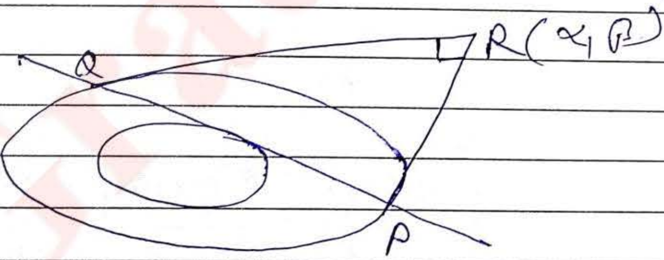


The eqn of director circle of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is :⇒

$$\boxed{x^2+y^2=a^2+b^2}$$

Ex:⇒ Show that the tangents to the ellipse $\frac{x^2}{a}+\frac{y^2}{b}=(a+b)$ drawn at the points where any tangent to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cuts at, Right angle.

Ans



PQ is chord of contact

$$\frac{x\alpha}{a(a+b)}+\frac{y\beta}{a(a+b)}=1$$

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$$\frac{x\alpha}{a} + \frac{y\beta}{b} = a+b \quad \text{—— (1)}$$

PQ is Tangent

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1 \quad \text{—— (2)}$$

$$\frac{\alpha}{\cos\theta} = \frac{\beta}{\sin\theta} = (a+b)$$

$$\alpha^2 + \beta^2 = (a+b)^2$$

Locus :-

$$x^2+y^2 = (a+b)^2$$

→ Direct circle.

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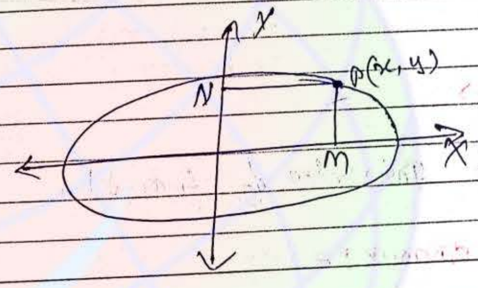
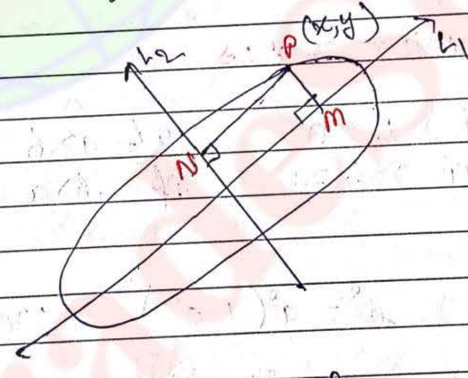
Eqⁿ of ellipse assuming any two Perpendicular lines as axis :→

Standard form :→

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$$\frac{(PN)^2}{a^2} + \frac{(PM)^2}{b^2} = 1$$



Ellipse :→ $\frac{(PN)^2}{a^2} + \frac{(PM)^2}{b^2} = 1$

L_1 :→ $a_1x + b_1y + c_1 = 0$

L_2 :→ $b_1x - a_1y + c_2 = 0$

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Ellipse

$$\boxed{\frac{\left(\frac{b_1x-a_1y+c_2}{\sqrt{a_1^2+b_1^2}}\right)^2}{a^2}+\frac{\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)^2}{b^2}=1}$$

1.) Centre :→

Point of Intersection of L_1 and L_2

2.) major axis :→

(i) $L_1 = 0$ If $a > b$
$L_2 = 0$ If $b > a$

3.) minor axis :→

(i) $L_2 = 0$ If $a > b$
(ii) $L_1 = 0$ If $b > a$

4.)

(i) $b^2 = a^2(1-e^2)$ ∵ when $(a > b)$

(ii) $a^2 = b^2(1-e^2)$ ∵ when $(a < b)$

5.) ~~Focus~~ - Foci :→

(i) $\frac{b_1x - a_1y + c_2}{\sqrt{a_1^2+b_1^2}} = \pm ae$ ∵ when $\{a > b\}$

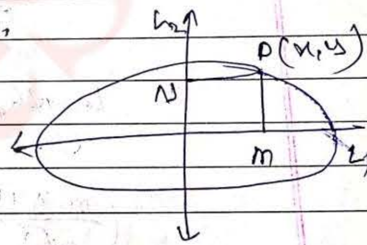
$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2+b_1^2}} = 0$$

(ii) If $b > a$

$$\frac{b_1x - a_1y + c_2}{\sqrt{a_1^2+b_1^2}} = 0$$

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2+b_1^2}} = \pm be$$

Q) Find the eqⁿ of ellipse whose axis are of length $(6, 2\sqrt{6})$ and their eqⁿ are $x-3y+3=0$ and $3x+y+1=0$ respectively.



Ans ⇒ L_1 ⇒ major axis ⇒ $x-3y+3=0$
L_2 ⇒ minor axis ⇒ $3x+y+1=0$

so,

Ellipse ⇒ $\frac{(PN)^2}{a^2} + \frac{(PM)^2}{b^2} = 1$

$$\boxed{\frac{\left(\frac{3x+y-1}{\sqrt{10}}\right)^2}{9} + \frac{\left(\frac{x-3y+3}{\sqrt{10}}\right)^2}{6} = 1}$$

$2a = 6$
$a = 3$
$2b = 2\sqrt{6}$
$b = \sqrt{6}$

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Eg 1-> Find major axis, minor axis, eccentricity and foci of ellipse, Length of major axis and length of minor axis.

$$\cancel{\frac{(x-y+1)}{3}} \quad \frac{(x-y+1)^2}{3} + \frac{(x+y-3)^2}{5} = 1$$

A-> $$\frac{(\overbrace{x-y+1}^{L_1})^2}{3} + \frac{(\overbrace{x+y-3}^{L_2})^2}{5} = 1$$

$$\frac{x^2}{3} + \frac{y^2}{5} = 1$$

$b^2 > a^2$

$L_1 = 0$

i) major axis L_1 -> $x-y+1=0$

ii) minor axis L_2 -> $x+y-3=0$

iii) $a^2 = b^2(1-e^2)$

$$\frac{3}{2} = \frac{5}{2}(1-e^2)$$

$$e = \sqrt{\frac{2}{5}}$$

iv) Length of major axis :->

$$\frac{\left(\frac{x-y+1}{\sqrt{2}}\right)^2}{\underbrace{(3/2)}_{a^2}} + \frac{\left(\frac{x+y-3}{\sqrt{2}}\right)^2}{\underbrace{5/2}_{b^2}} = 1$$

$$2b = 2\sqrt{\frac{5}{2}}$$

$$= \sqrt{10}$$

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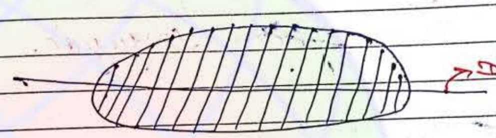
Diameter

It is the locus of midpoints of system of parallel chords



The slope of parallel chords is 'm' then the eqn of diameter corresponding to these system of parallel chords is

$$\boxed{y = \frac{-b^2}{a^2 m} x}$$

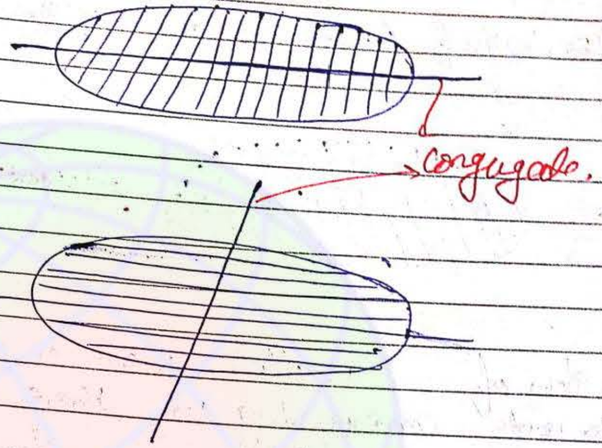
Hint :- $T = S_1$
slope $= m$

Congugate diameter -

Two diameters are said to be congugate ~~each~~ If, each bisect all chords parallel to the other.

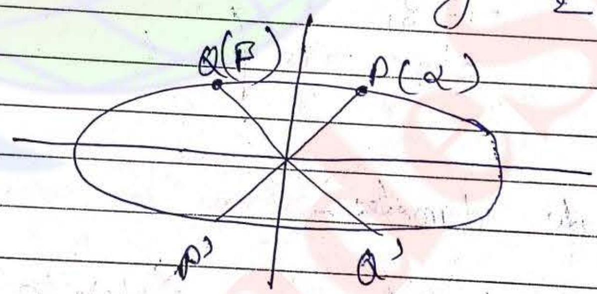
If $y = m_1 x$ and $y = m_2 x$ are equation of two congugate diameter then $m_1 m_2$ is

$$\boxed{m_1 m_2 = \frac{-b^2}{a^2}}$$

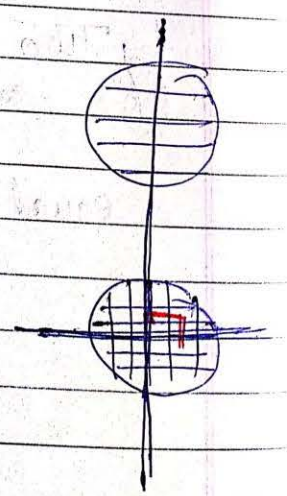


Note:>

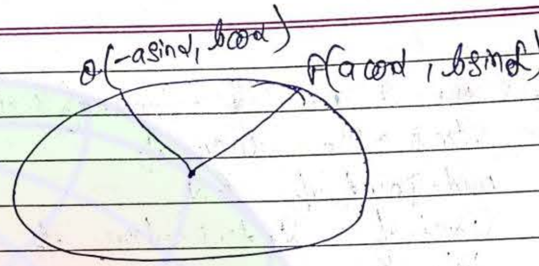
The eccentric angle of the the ends of pair of conjugate diameter of an ellipse differ by an Angle "$\frac{\pi}{2}$"



$$\boxed{|\beta - \alpha| = \frac{\pi}{2}}$$



Properties of ellipse

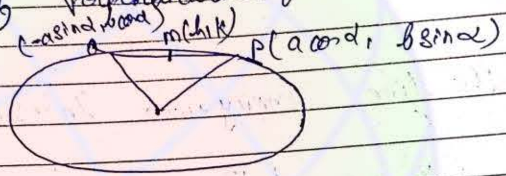


Q: If the line $lx+my+n=0$ Intersect the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the ends of congugate diamets, then show that $a^2l^2+b^2m^2=2n^2$

H.W :-> D.P.P.

Ex: If P and Q are the ends of semi conjugate diameter then find the locus of
i) mid-point of PQ
ii) Point of Intersection of tangents at 'P' and 'Q'.
iii) Foot of Perpendicular from centre O upon PQ

Ans :-> i)



$$2h = a\cos\alpha - a\sin\alpha$$

$$\frac{2h}{a} = \cos\alpha - \sin\alpha \quad -(1)$$

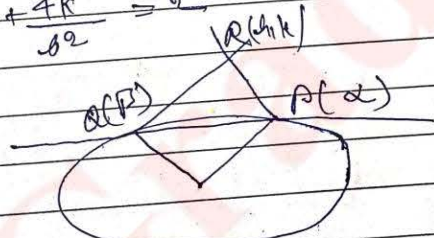
$$2k = b\cos\alpha + b\sin\alpha$$

$$\frac{2k}{b} = \cos\alpha + \sin\alpha \quad -(2)$$

$(1)^2$ and $(2)^2$

$$\frac{4h^2}{a^2} + \frac{4k^2}{b^2} = 2$$

ii)



$$\frac{x}{a}\cos\frac{\alpha+\beta}{2} + \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\left(\frac{\pi}{2}\cdot\frac{1}{2}\right)$$

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = \frac{1}{\sqrt{2}} \quad -(1)$$

PQ: $\Rightarrow \frac{xh}{a^2} + \frac{yk}{b^2} = 1$ —②

Compare:-

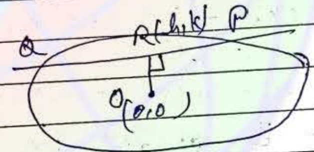
$\frac{a\cos\theta}{h} = \frac{b\sin\theta}{k} = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{h}{\sqrt{2}a}$, $\sin\theta = \frac{k}{\sqrt{2}b}$

Locus: $\Rightarrow$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

iii)



PQ:-

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = \frac{1}{\sqrt{2}}$ —①

$\theta = \frac{\alpha+\beta}{2}$

PQ: $\Rightarrow$

$y-k = \frac{-h}{k}.(x-h)$

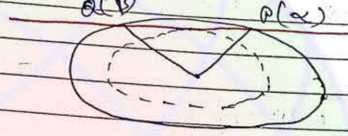
$hx + ky = h^2+k^2$ —②

Compare-

$\frac{\cos\theta}{ah} = \frac{\sin\theta}{bk} = \frac{1}{\sqrt{2}(h^2+k^2)}$

Q:- Show that the chord which joins the ends of a pair of semi-conjugate diameters of an ellipse always touches the similar ellipse.

Ans.



PQ :→ $$\frac{x}{a}\cos\frac{\alpha+\beta}{2} + \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\pi}{2}\cdot\frac{1}{2}\right)$$

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = \frac{1}{\sqrt{2}}$$

$$\frac{x}{(a/\sqrt{2})}\cos\theta + \frac{y}{(b/\sqrt{2})}\sin\theta = 1$$

Ellipse :→

$$\frac{x^2}{(a/\sqrt{2})^2} + \frac{y^2}{(b/\sqrt{2})^2} = 1.$$

Q: Circles are drawn on two semi conjugate diameters of an ellipse as diameter. Show that the locus of their second point of intersection is

$$2(x^2+y^2)^2 = a^2x^2 + b^2y^2$$

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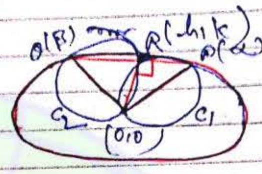
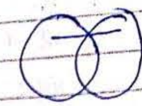
Ans





$P(a\cos\alpha,\ b\sin\alpha)$

$Q(-a\sin\alpha,\ b\cos\alpha)$

$C_1 :\to x(x-a\cos\alpha)+y(y-b\sin\alpha)=0$

$x^2+y^2 = ax\cos\alpha + by\sin\alpha$ — (1)

$C_2 :\to x(x+a\sin\alpha)+y(y-b\cos\alpha)=0$

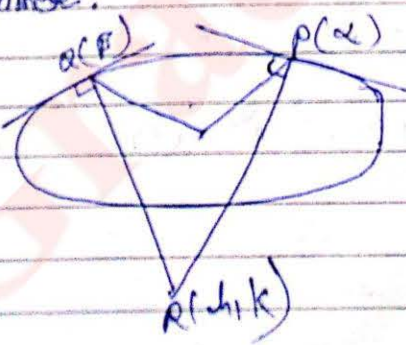
$x^2+y^2 = by\cos\alpha - ax\sin\alpha$ — (2)

$(1)^2+(2)^2$

VVI program

Q(9) Find the locus of point of intersection of normal at two points which are ends of extremeties of conjugate diameter.

Ans



Now :→

PR! → (Normal) :→

$$\frac{ax}{\cos\alpha} - \frac{by}{\sin\alpha} = a^2 - b^2$$

Pass through point (h, k)

$$\frac{ah}{\cos\alpha} - \frac{bk}{\sin\alpha} = a^2 - b^2 \quad (1)$$

QR! →

$Q(-a\sin\alpha, b\cos\alpha)$

Normal! -

$$\frac{a^2x}{-a\sin\alpha} - \frac{b^2y}{b\cos\alpha} = a^2 - b^2$$

$$\frac{ax}{\sin\alpha} + \frac{by}{\cos\alpha} = b^2 - a^2$$

Pass (h, k)

$$\frac{ah}{\sin\alpha} + \frac{bk}{\cos\alpha} = b^2 - a^2 \quad (2)$$

$$\frac{1}{\sin\alpha} = p$$

$$\frac{1}{\cos\alpha} = q$$

$$ahp - bkq = a^2 - b^2 \quad (3)$$

$$ahq + bkp = b^2 - a^2 \quad (4)$$

Solve eq (3) and (4)

$p =$ ___

$q =$ ___

Locus: -

$$\cancel{\frac{1}{p^2}} = \cancel{\quad} \quad \boxed{\frac{1}{p^2} + \frac{1}{q^2} = 1}$$

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Tripley Order

Ex. ~~Show~~ Prove that the straight "lines" joining centre of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the intersection of straight line $y = mx + \sqrt{\frac{a^2m^2+b^2}{2}}$ with the ellipse are conjugate diameter.

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Properties of ellipse

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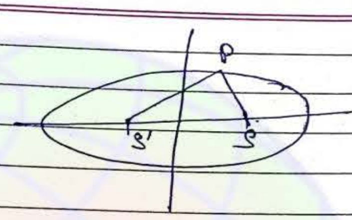
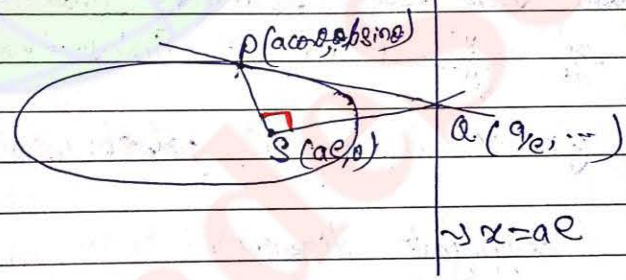
1.)



$$\boxed{SP + S'P = 2a}$$

2.) The portion of the tangent to any ellipse b/w point of contact and directrix subtends right angle at corresponding focus.

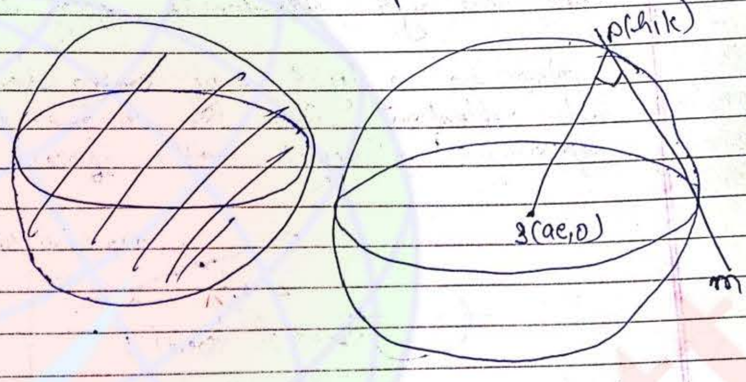


Proof :-

3.) ~~The focus~~

7.) The locus of foot of perpendicular from the focus upon any tangent to ellipse is auxiliary circle.

Proof: —



Tangent is

$$y = mx + \sqrt{a^2m^2+b^2}$$

$$(y-mx)^2 = a^2m^2+b^2 \quad \text{—— (1)}$$

SP: —

$$y-0 = \frac{-1}{m}(x-ae)$$

$$(my+x)^2 = a^2e^2 \quad \text{—— (2)}$$

eq (1) + eq (2)

$$y^2(1+m^2) + x^2(1+m^2) = a^2m^2+b^2+a^2e^2$$

$$\cancel{(1+m^2)}(x^2+y^2) = a^2m^2 + b^2(1-e^2) + \cancel{a^2e^2}$$

$$= a^2\cancel{(1+m^2)}$$

so, $\boxed{x^2+y^2 = a^2}$

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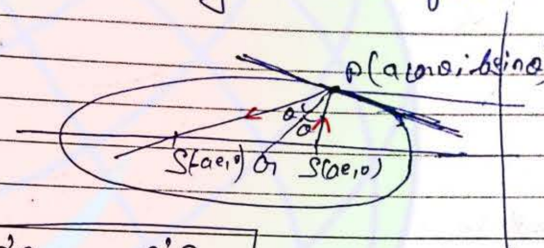
Reflection properties of ellipse :>

Tangent and normal at any point Point 'P' of ellipse are the angle bisectors of focal distances of point 'P'.

~~Stops~~ so a light ray coming from one focus after reflection from the concave side of ellipse ~~foc~~ passes through another focus.



Hint

$$\boxed{\frac{S'G}{SG} = \frac{S'P}{SP}}$$

$$SP = e\left|\frac{a}{e} - a\cos\theta\right|$$

$$= a - ae\cos\theta$$

$$S'P = a + ae\cos\theta$$

Normal :>

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$$

$$G\left(\frac{a^2-b^2}{a}\cos\theta, 0\right)$$

$$G\left(ae^2\cos\theta, 0\right)$$

$$GS' = ae^2\cos\theta + ae$$

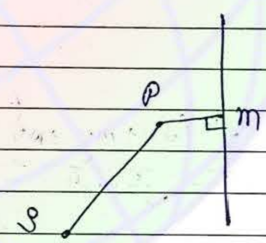
$$GS = ae - ae^2\cos\theta$$

Hyperbola

It is the locus of a point which move in such a way that ratio of its distances from a fixed point and ~~its~~ a fixed line remains constant.

The fixed line is called directrix and fixed point is called focus and the ratio is called eccentricity.

For Hyperbola :→ $\boxed{e > 1}$



$$\frac{SP}{PM} = e$$

$$\boxed{SP = ePM} \quad (e > 1)$$

* General equation of hyperbola :⇒

Let $S(\alpha, \beta)$ is the focus.

and line $lx + my + n$ is the directrix and 'e' is the eccentricity and $e > 1$.

so,

By the definition of hyperbola :-

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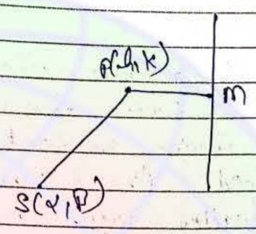


$$SP = e\,PM$$

$$\sqrt{(h-\alpha)^2 + (k+\beta)^2} = e\left|\frac{lh + mk + n}{\sqrt{l^2+m^2}}\right|$$

So,

Locus:-

$$(x-\alpha)^2 + (y-\beta)^2 = \frac{e^2\,(lx + my + n)^2}{l^2+m^2}$$

Q) Find the eqⁿ of hyperbola whose directrix is $2x+y-1=0$ and focus is $(1,2)$ and $e=\sqrt{3}$

A.

$$(x-1)^2 + (y-2)^2 = (\sqrt{3})^2\,\frac{(2x+y-1)^2}{\sqrt{5}}$$

2)

General eq^n of second degree :-

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a hyperbola. If

(i) $\Delta \neq 0$

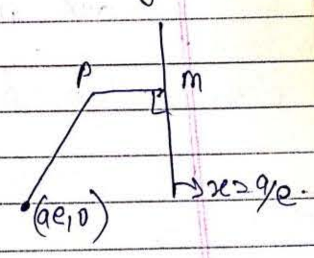
$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

(ii) $h^2 > ab$

Standard eq^n of ~~ellipse~~ Hyperbola :-

Let S $(ae, 0)$ is the focus lying $x = a/e$ is the directrix of hyperbola then the eq^n of hyperbola is

$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$



where,

$$\boxed{b^2 = a^2(e^2 - 1)}$$

Then the eq^n of hyperbola is standard form.

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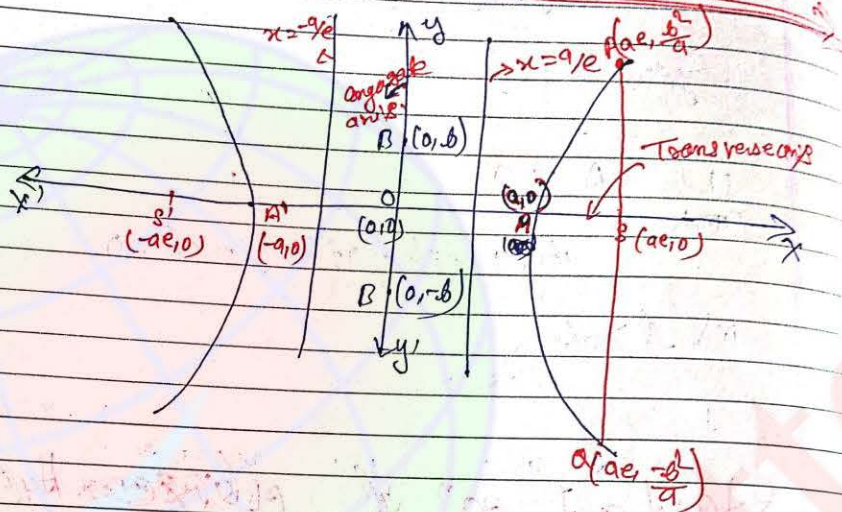
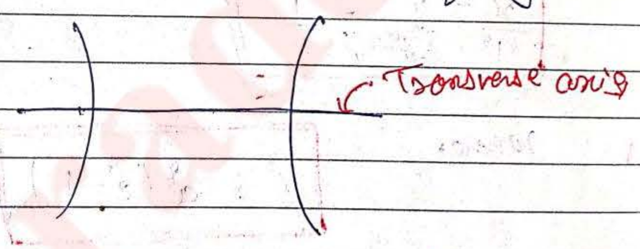
Some definition Related to hyperbola



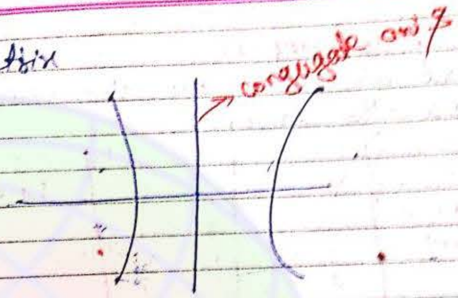
i) Transverse Axis :- (AA')

The line perpendicula to the direction and passing through the focus of hyperbola.



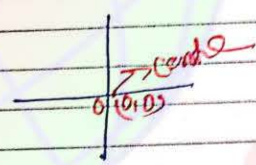
ii) Conjugate Axis :- (BB')

The line parallel to the direction and equi-distant ~~fo~~ from both the directrix



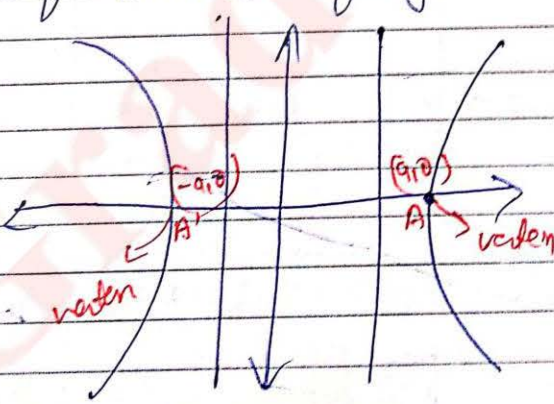
3.) Centre :- (O)

A point of intersection of transverse and conjugate axis is known as centre.



4.) Vertex (A and A')

Point of intersection of hyperbola and its major axis.



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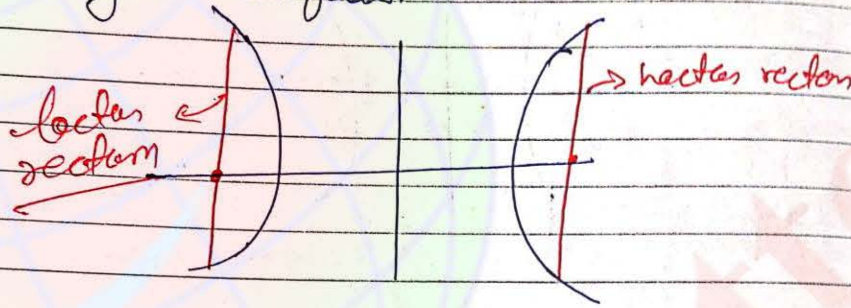
5:) Lactus Rectum:→

The chord '⊥' to the transverse axis passing through the focus.



length of lactus rectum is "$\frac{2b^2}{a}$" and its ends are $P\left(ae, \frac{b^2}{a}\right)$ and $Q\left(ae, \frac{-b^2}{a}\right)$

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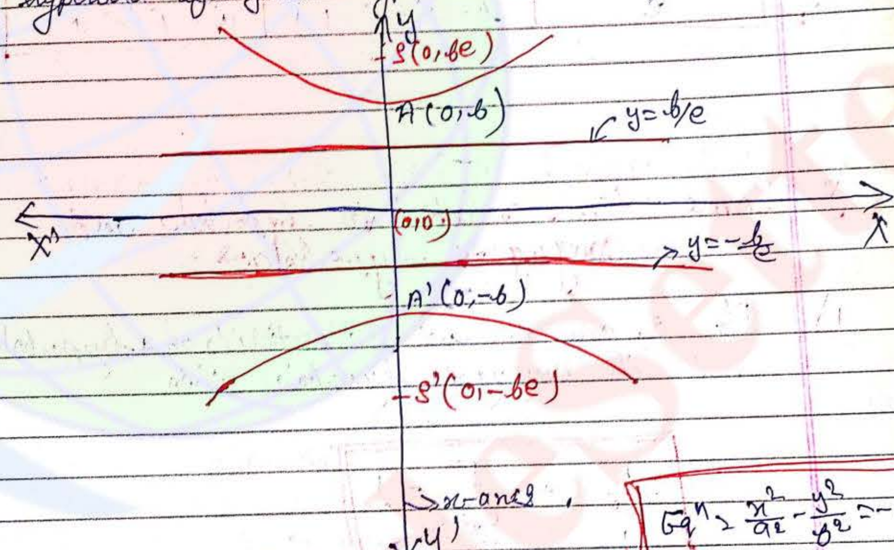
Congugate hyperbola :→

The hyperbola whose transverse and conjugate axis are respectively the conjugate and transverse axis of given ~~hyperbola~~ hyperbola is called conjugate hyperbola of given hyperbola.



$$\boxed{Eq^n:\ \frac{x^2}{a^2}-\frac{y^2}{b^2}=-1}$$

* Comparision :→

Equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$
Centre	$(0,0)$	$(0,0)$
Vertices	$(\pm a, 0)$	$(0, \pm b)$

foci	$(\pm ae, 0)$	$(0, \pm be)$
Direction	$x = \pm a/e$	$y = \pm b/e$
Lactus Rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
e	$b^2 = a^2(e^2-1)$	$a^2 = b^2(e^2-1)$

Some terms related to hyperbola and conjugate hyperbola:-

1) If e_1 and e_2 are eccentricity of a hyperbola and its conjugate hyperbola then

$$\boxed{\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1}$$

Proof:→

$$b^2 = a^2(e_1^2 - 1)$$

$$e_1^2 = \frac{a^2 + b^2}{a^2}$$

$$\frac{1}{e_1^2} = \frac{a^2}{a^2+b^2} \quad (1)$$

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Also:-

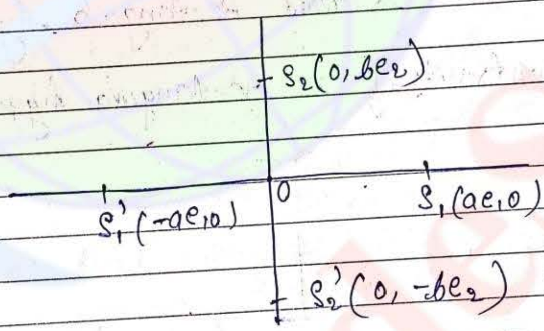
$$a^2 = b^2(e^2-1)$$

$$\frac{1}{e_2^2} = \frac{b^2}{a^2+b^2} \quad \text{—(ii)}$$

eq (i) + eq (ii)

$$\boxed{\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1}$$

2.) Foci of a hyperbola and its conjugate hyperbola are concilic.



$$\boxed{OS_1 \cdot OS_1' = OS_2 \cdot OS_2'}$$

$$\boxed{a^2e_1^2 = b^2e_2^2}$$

Now:-

Proof:-

$$\cancel{a^2}\left(\frac{a^2+b^2}{\cancel{a^2}}\right) = \cancel{b^2}\left(\frac{a^2+b^2}{\cancel{b^2}}\right)$$

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Standard eqⁿ of Hyperbola at Point (α, β)

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If the centre of hyperbola is at point (α, β) and axis are parallel to co-ordinate axis then the eqⁿ of hyperbola is

$$\boxed{\frac{(x-\alpha)^2}{a^2} - \frac{(y-\beta)^2}{b^2} = 1}$$

Note :⇒

If length of transverse axis and conjugate axis of a hyperbola are equal then the hyperbola is called Rectangular hyperbola.

eccentricity of a rectangular hyperbola is "$\sqrt{2}$".

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~~Points~~ ~~Find~~

Find, ~~point~~, centre, vertex, eccentricity, foci, directrix and lactus rectum of hyperbola.

(i) $16x^2 - 9y^2 = 144$

(ii) $9x^2 - 16y^2 - 18x + 32y - 151 = 0$

Ans (i). $\frac{16x^2}{144} - \frac{9y^2}{144} = \frac{144}{144}$

$\Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$

~~Point~~ :-

ii) Centre : $\Rightarrow (0,0)$

iii) vertices : $\rightarrow (\pm 3, 0)$ ~~$(\pm 25, 0)$~~

iv) eccentricity : $\Rightarrow \frac{5}{3}$

$b^2 = a^2(e^2-1)$

$16 = 9(e^2-1)$

$16+9 = 9e^2$

$e^2 = \frac{25}{9}$

$e = \frac{5}{3}$

v) foci = $(\pm 5, 0)$

vi) Directrix = $x = \pm \frac{9}{5}$

$x = \frac{3/3}{5/3}$

vii) lactus rectum : $\rightarrow \frac{2b^2}{a}$

$\Rightarrow \frac{2\times16}{3} = \frac{32}{3}$

ii) $9x^2 - 16y^2 - 18x + 32y - 151 = 0$

$9(x^2 - 2x) - 16(y^2 - 2y) = 151$

$9(x^2 - 2x + 1) - 16(y^2 - 2y + 1) = 151 + 9 - 16$

$\frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1$

$\boxed{\frac{X^2}{16} - \frac{Y^2}{9} = 1} \Rightarrow \frac{x^2}{4} - \frac{y^2}{3} = 1$

1) Centre $\Rightarrow x = 0, \quad y = 0$

$x - 1 = 0 \qquad y - 1 = 0$

$x = 1 \qquad y = 1$

centre $(1, 1)$

ii) vertices $\Rightarrow$

$x = \pm 9 \qquad y = 0$

$y - 1 = 0$

$x - 1 = \pm 16 \qquad y = 1$

take,

$x - 1 = +4$

$x = 1 + 4$

$\Rightarrow 5$

So, vertex $(5, 1)$ and $(-3, 1)$

-ve

$x - 1 = -4$

$x = -1 - 4 \Rightarrow -3$

$\Rightarrow -15$

vertex $\Rightarrow (5, 0)$ and $(-3, 0)$

So, ~~vertex (17, 0) and (-15, 0)~~

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iii) eccentricity :->

$b^2 = a^2(e^2-1)$

$\Rightarrow 3\cancel{9} = 4(e^2-1)$ $\Rightarrow 9 = 16(e^2-1)$

$\Rightarrow \cancel{\frac{3}{4}+1 = e^2}$ $\Rightarrow e^2 = \frac{7}{4}$ $\frac{9}{16}+1 = e^2$

$e = \cancel{\frac{\sqrt{7}}{2}}$ $\frac{25}{16} = e^2$

$e = \frac{5}{4}$

iv) foci :->

$(\pm ae, 0) \Rightarrow \pm(\cancel{4} \times \frac{5}{\cancel{4}}, 1)$

$= (\pm 5, 1)$.

$y-1 = 0$
$y = 1$

vi) direction :->

$x-1 = \pm \frac{a}{e}$

$\Rightarrow \pm \frac{4 \times 4}{5}$

$\Rightarrow \pm \frac{16}{5}$

So,
$x-1 = +\frac{16}{5}$
$x = \frac{16}{5}+1$
$x = \frac{21}{5}$

$x-1 = -\frac{16}{5}$
$x = -\frac{16}{5}+1$
$= -\frac{11}{5}$

vii)

~~$x-1 = +\frac{16}{5}$~~
~~$x = \frac{16}{5}+1$~~
~~$\Rightarrow \frac{16+5}{5}$~~
~~$\Rightarrow \frac{21}{5}$~~

~~$x-1 = -\frac{16}{5}$~~
~~$x = -\frac{16}{5}+1$~~
~~$\Rightarrow \frac{-16+5}{5}$~~
~~$\Rightarrow -\frac{11}{5}$~~

So, $x = \frac{21}{5}$ and $x = -\frac{11}{5}$

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vii) Lactus Rectum:

$$\frac{2b^2}{a} \Rightarrow \frac{\cancel{2} \times 9}{\cancel{16}\,8} = \frac{9}{8}$$

Q. Find the eqⁿ of standard hyperbola whose conjugate axis is '5' and distance b/w foci is "13".

A.
$2b = 5$
$2ae = 13$
squaring both side
$4(a^2+b^2) = 169$
$(2a)^2 + (2b)^2 = 169$
$(2a)^2 + 25 = 169$
$2a = 12$
$a = 6$
$$\frac{x^2}{36} - \frac{4y^2}{25} = 1$$

Q. Foci of a hyperbola coincide with the foci of ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. find the equation of hyperbola. if $e = 2$

Ans.
$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$
$\hookrightarrow S(\pm 4, 0)$
Hyperbola :-
$2ae = 8$
$a = 2$
$b^2 = a^2(e^2 - 1)$
$= 12$
$$\frac{x^2}{4} - \frac{y^2}{12} = 1.$$

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Q. Find the eqn of hyperbola whose foci are (0,3) and (8,3) and e = 4/3

Ans. Centre (4,3)

$2ae = 8$

$ae = 4$

$a = 3$

$$\frac{(x-4)^2}{9} - \frac{(y-3)^2}{7} = 1$$

Q. Find the eqn of standard hyperbola If distance of one vertex from foci are 1 and 9

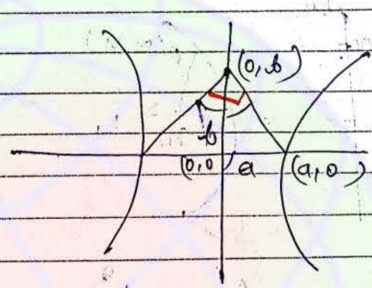
Ans.

$ae - a = 1$

$ae - a = 9$

Q.→ Find the axis of hyperbola whose transverse axis subtends a right angle at the ends of conjugate axis.

Ans.



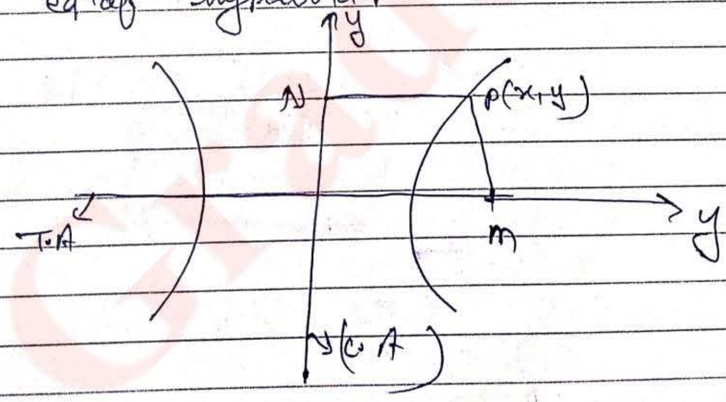
$\boxed{a = b}$

Rectangle hyperbola so,

$(e = \sqrt{2})$

Q.→ The eq^n of transverse axis of hyperbola is $x - y + 2 = 0$ and conjugate axis is $x + y - 1 = 0$. If length of transverse and conjugate axis are same and hyperbola passes through the point (1, 2) then find the eq^n of hyperbola.

Ans



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

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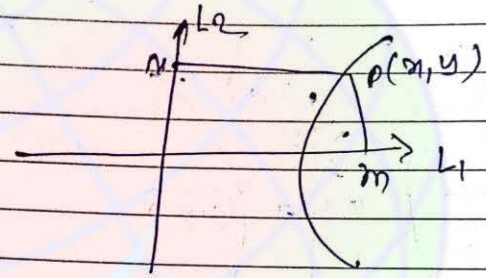
$$\frac{(PN)^2}{a^2} - \frac{(PM)^2}{b^2} = 1$$

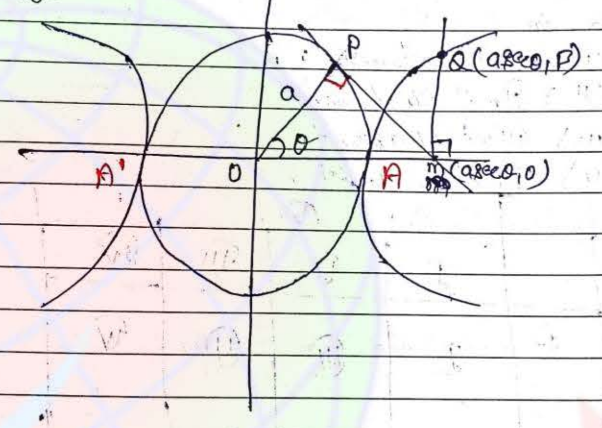


$$\frac{\left(\frac{x+y-1}{\sqrt{2}}\right)^2}{a^2} - \frac{\left(\frac{x-y+2}{\sqrt{2}}\right)^2}{a^2} = 4$$

Pass (1, 2)

Parametric equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



P (a cos θ, a sin θ)

P, Q are corresponding points.

Q lies on hyperbola

$$\frac{(a\sec\theta)^2}{a^2} - \frac{\beta^2}{b^2} = 1$$

$$\beta = b\tan\theta$$

Q (a sec θ, b tan θ)

$$\boxed{\begin{aligned} x &= a\sec\theta \\ y &= b\tan\theta \end{aligned}}$$

$$\theta \in [0, 2\pi)$$

Ex) Find the quadrant in which point 'P' lies if 'Q' lies in

(I) 1st quadrant,
(II) IInd quadrant
(III) IIIrd quadrant.
(IV) IVth quadrant.

Q	I	II	III	IV
P	I	III	II	IV

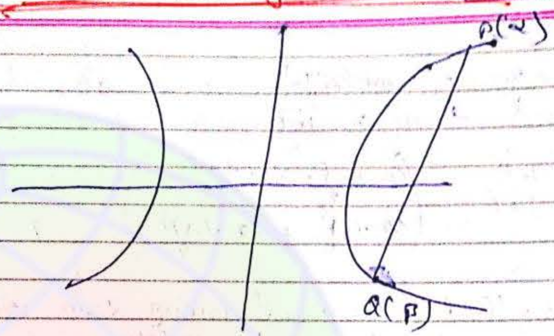
'x' और 'y' का साइन और (-)ve से यह decide होगा।

$x = a\sec\theta$

$y = b\tan\theta$

1st Choice

Chord joining two Points



Eqⁿ of chord joining two points P(α) and Q(β) on hyperbola is given by:

$$\boxed{\frac{x}{a}\cos\frac{\alpha-\beta}{2} - \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha+\beta}{2}}$$

Q. If the chord joining two point P(α) and Q(β) is a focal chord of hyperbola then show that

$$\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2} = \left(\frac{1-e}{1+e}\right)$$

Ans. P(α), Q(β)

$$\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2} = \left(\frac{1-e}{1+e}\right)$$

so,

$$\frac{x}{a}\cos\frac{\alpha-\beta}{2} - \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha+\beta}{2}$$

Pass (ae, 0)

$$e\cos\frac{\alpha-\beta}{2} = \cos\frac{\alpha+\beta}{2}$$

$$\frac{e}{1} = \frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\frac{\alpha-\beta}{2}}$$

$$\frac{1-e}{1+e} = \left(\frac{\cos\frac{\alpha-\beta}{2} - \cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2} + \cos\frac{\alpha+\beta}{2}}\right)$$

Fixed point

Show that the chord joining two points whose sum of eccentric angles is $\pi/2$ passes through a fixed point of hyperbola $\frac{x^2}{8} - \frac{y^2}{4} = 1$

Sol:—

$$\frac{x}{a}\cos\frac{\alpha-\beta}{2} - \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha+\beta}{2}$$

$$\frac{x}{a}\cos\frac{\alpha-\beta}{2} - \frac{y}{2}\sin\frac{\pi}{4} = \cos\frac{\pi}{4}$$

$$\underbrace{\left(\frac{y}{2\sqrt{2}} + \frac{1}{\sqrt{2}}\right)}_{L_1} - \underbrace{\left(\cos\frac{\alpha-\beta}{2}\right)}_{\lambda}\underbrace{\frac{x}{a}}_{L_2} = 0$$

$$\left.\begin{aligned} \frac{x}{a} &= 0 \\ \frac{y}{2\sqrt{2}} + \frac{1}{\sqrt{2}} &= 0 \end{aligned}\right\} (0, -2)$$

$$\frac{e}{1} = \frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\frac{\alpha-\beta}{2}}$$

$$\frac{1-e}{1+e} = \left(\frac{\cos\frac{\alpha-\beta}{2} - \cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2} + \cos\frac{\alpha+\beta}{2}}\right)$$

fixed method

$\sqrt{10}$ Show that the chord joining two points whose sum of eccentric angles is $\pi/2$, passes through a fixed point for hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$

Ans:—

$$\frac{x}{a}\cos\frac{\alpha-\beta}{2} - \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha+\beta}{2}$$

$$\frac{x}{3}\cos\frac{\alpha-\beta}{2} - \frac{y}{2}\sin\frac{\pi}{4} = \cos\frac{\pi}{4}$$

$$\underbrace{\left(\frac{y}{2\sqrt{2}} + \frac{1}{\sqrt{2}}\right)}_{L_1} - \underbrace{\left(\cos\frac{\alpha-\beta}{2}\right)}_{\lambda}\underbrace{\frac{x}{3}}_{L_2} = 0$$

$L_1 \quad \lambda \quad L_2 = 0$

$$\left.\begin{aligned} \frac{x}{3} &= 0 \\ \frac{y}{2\sqrt{2}} + \frac{1}{\sqrt{2}} &= 0 \end{aligned}\right\} (0, -2)$$

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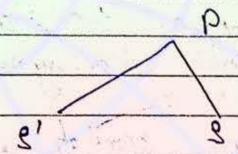
Other Definition of Hyperbola

It is the locus of a point whose difference of distances from two fixed points remains constant



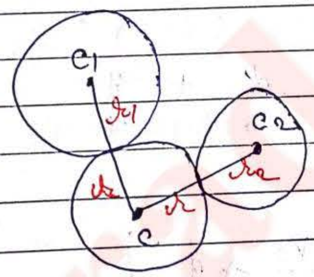
$$\boxed{|S'P - SP| = 2a}$$

→ Length of Transverse axis.

Two fixed points are foci of hyperbola

Q:- Show that the locus of centre of a circle ~~which~~ touching two given circle externally is a hyperbola.

Ans:-



$CC_1 \Rightarrow r + r_1$

$CC_2 \Rightarrow r + r_2$

$$CC_1 - CC_2 = \underbrace{r_1 - r_2}_{\text{constant}}$$

1st Choice

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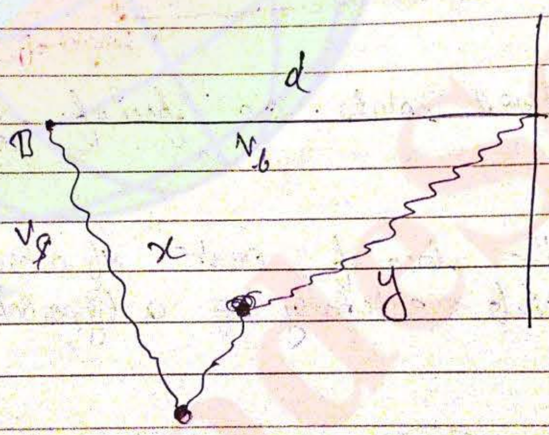
Ex.:- On a plane a crack of a rifle and the that of a bullet ~~strck~~ striking the target ~~as~~ ~~of~~ is

heard at the same time. then show that the locus of the listener is a hyperbola.

Assume speed of bullet is greater than the speed of sound.

As.



$$\frac{x}{v_s} = \frac{d}{v_b} + \frac{y}{v_s}$$

$$x - y = \left(\frac{v_s}{v_b} \times d\right)$$

1st Choice

Position of a Point with respect to hyperbola

$$\boxed{S: \Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0}$$

$P(x_1, y_1)$

so,

$$\boxed{S_1 \Rightarrow \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1 = 0}$$

1) Point 'P' lies Inside the hyperbola If $\boxed{S_1 > 0}$.

2) 'P' lies outside the hyperbola if $\boxed{S_1 < 0}$,

3) Point 'P' lies on the hyperbola if $\boxed{S_1 = 0}$.

Q:) check the position of point (1,2) and (3,-4) w.r.t. hyperbola $2x^2 + 5xy + 2y^2 + 4x + 5y = 0$

Ans $S_1 \Rightarrow 2x_1^2 + 5x_1y_1 + 2y_1^2 + 4x_1 + 5y_1 = 0$

$\Rightarrow 2 \times 1 + 5 \times 2 + 2 \times 4 + 4 + 10 \angle 0$

$\Rightarrow 2 + 10 + 8 + 4 + 10 > 0$

$\Rightarrow 30 > 0$ (Inside the hyperbola.)

1st Choice

ii) S.A

Equation Tangent of Hyperbola

1.) Point form

$$\boxed{\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1}$$

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1 = 0$$

$$\boxed{T = 0}$$

(chord of contact)

2.) Parametric form :⇒

$$\boxed{\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1}$$

1st Choice

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→ Slope form:→

$$\boxed{y = mx \pm \sqrt{a^2m^2 - b^2}}$$

Notes:→

Condition of tangency:→

$$\boxed{c^2 = a^2m^2 - b^2}$$

• Tangent should be real only if

$$\boxed{a^2m^2 > b^2}$$

Q. If line $lx + my + n = 0$ is a tangent to standard hyperbola then show that $a^2l^2 - b^2m^2 = n^2$

Ans→ Given $lx + my + n = 0$

$$y = \left(\frac{-l}{m}\right)x + \left(\frac{-n}{m}\right)$$

$$\left(\frac{-n}{m}\right)^2 = a^2\left(\frac{-l}{m}\right)^2 + b^2$$

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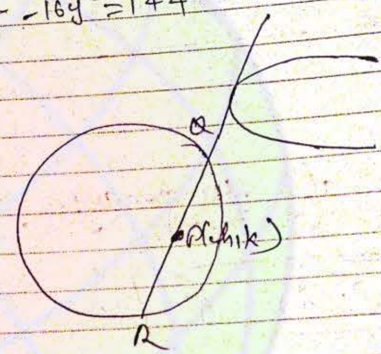
Q. Find the locus of mid-point of chord of circle $x^2+y^2=16$ ~~is~~ ~~tan~~ which are tangent to hyperbola $9x^2-16y^2=144$

Ans.



QR : $\boxed{T = S_1}$

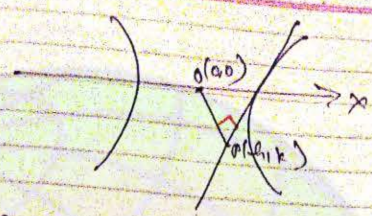
$$xh + yk = h^2+k^2$$

$$y = \left(\frac{-h}{k}\right)x + \left(\frac{h^2+k^2}{k}\right)$$

$$\rightarrow \left(\frac{h^2+k^2}{k}\right) = 16\frac{h^2}{k^2} - 9$$

Q. Find the locus of foot of perpendicular from centre on any tangent to hyperbola $x^2-y^2=4$

A



$x^2 - y^2 = 4$

~~$y = mx + \sqrt{a^2m^2}$~~

$y = mx + \sqrt{a^2m^2 - b^2}$

Pass (h, k)

$(k - mh)^2 = a^2m^2 - b^2$

$(k - mh)^2 = 4(m^2 - 1)$ — (1)

$m\left(\frac{k}{h}\right) = -1$

$m = \frac{-h}{k}$ — (2)

from eq (1)

$\left(k + \frac{h^2}{k}\right)^2 = 4\left(\frac{h^2}{k^2} - 1\right)$

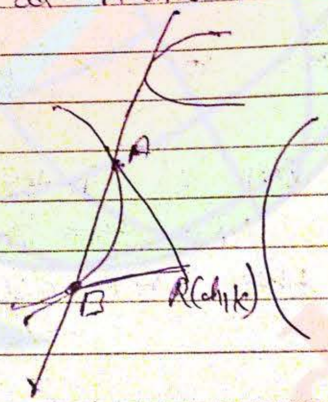
Q. Find the locus of point of intersection of tangents drawn at two points whose parametric angle differ by $\pi/2$

A.

Ans $\Rightarrow a^2y^2 = b^2(2x^2 - a)$

Q.) If a tangent to parabola $y^2 = 4ax$ Intersect the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at A and B then find the locus of point of Intersection of tangents at A and B.

A.



AB: →

$\frac{xh}{a^2} - \frac{yk}{b^2} = 1$ —(1)

$y = mx + \frac{a}{m}$

$-mx + y = \frac{a}{m}$ — (1)

$\frac{h}{a^2(-m)} = \frac{-k}{b^2} = \frac{m}{a}$

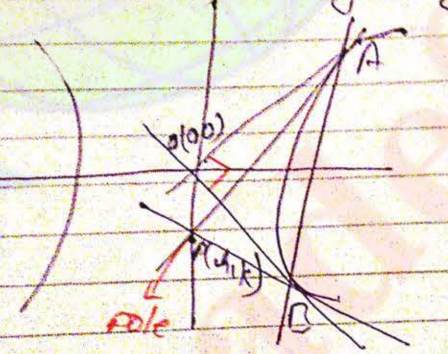
$m = \frac{hb^2}{ka^2}$ — (2)

$m = \frac{-ka}{b^2}$ — (3)

$\frac{hb^2}{ka^2} = \frac{-ka}{b^2}$

Q. Find the locus of poles of chords of standard hyperbola which subtend right angle at the centre.

Ans



$AB: \Rightarrow \frac{xh}{a^2} - \frac{yk}{b^2} = 1$ — (1)

OA and OB

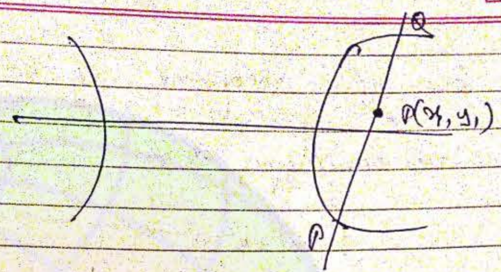
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \left(\frac{xh}{a^2} - \frac{yk}{b^2}\right)^2$

1st Choice

coeff. of x^2 + coeff. of $y^2 = 0$

$\frac{1}{a^2} - \frac{h^2}{a^4} - \frac{1}{b^2} - \frac{k^2}{b^4} = 0$

Chord bisected at given Points

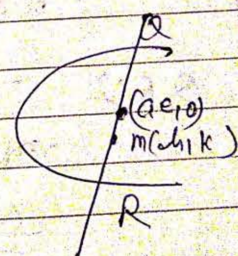


$$QR = \boxed{T = S_1}$$

$$\boxed{\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2}}$$

Q. Find the locus of mid-point of focal chord of standard hyperbola

Ans.



$T = S_1$

$$\frac{xh}{a^2} - \frac{yk}{b^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2}$$

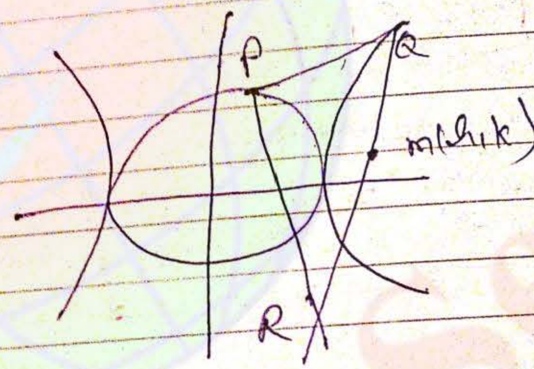
Pass $(ae, 0)$

$$\frac{aeh}{a^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2}$$

1st Choice

Q. Find the locus of midpoint of chord of contact of tangents to hyperbola $x^2-y^2=a^2$ from the points on auxiliary circle.

Ans. $\underset{\text{Hyperbola}}{\underline{x^2-y^2=a^2}}$; $\underset{\text{Auxiliary circle}}{\underline{x^2+y^2=a^2}}$



$P\,(a\cos\theta,\ a\sin\theta)$

QR :-

$x(a\cos\theta)-y(a\sin\theta)=a^2$

$x\cos\theta-y\sin\theta=a$ — (1)

QR :- $T=S_1$

$xh-yk=h^2-k^2$ — (2)

$\frac{\cos\theta}{h}=\frac{\sin\theta}{k}=\frac{a}{h^2-k^2}$

25/11/2017

Equation of Normal

(i) Point form:-

$$\boxed{\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2 = a^2e^2}$$

(ii) Parametric form:-

$$\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$$

Q. Find the value of 'λ' such that line $x+y=\lambda$ is a normal to hyperbola $\frac{x^2}{4}-y^2=1$

Ans $x+y=\lambda$ —(1) $\quad \frac{x^2}{4}-y^2=1$

$$\frac{2x}{\sec\theta}+\frac{y}{\tan\theta}=5 \quad —(2)$$

$$\frac{\sec\theta}{2}=\tan\theta=\frac{\lambda}{5}$$

$$\sec^2\theta-\tan^2\theta=1 \qquad \lambda=\pm\frac{5}{\sqrt{3}}$$

Q. A normal to hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ meets the axis in P and Q respectively PR and QR are ⊥' to axis the find the locus of point R'.

1st Choice

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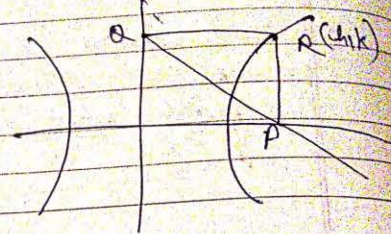
Sol/n ⇒ $\frac{ax}{\sec\theta} - \frac{by}{\tan\theta} = a^2+b^2$



$P\left(\frac{a^2+b^2}{a}\sec\theta, 0\right)$

$Q\left(0, \frac{a^2+b^2}{b}\tan\theta\right)$

$h = \frac{a^2+b^2}{a}\sec\theta$

$ah = (a^2+b^2)\sec\theta$ — (1)

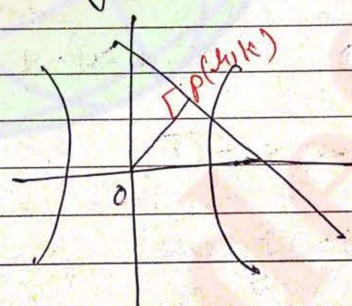
$bk = (a^2+b^2)\tan\theta$ — (2)

$(1)^2 - (2)^2$

$a^2h^2 - b^2k^2 = (a^2+b^2)^2$

Q. Find the locus of foot of perpendicular from centre upon any normal to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Ans.



$\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2+b^2$ — (1)

$y-k = \frac{-h}{k}(x-h)$

$xh + yk = h^2+k^2$ — (2)

$\frac{h}{a}\sec\theta = \frac{k}{b}\tan\theta = \frac{h^2+k^2}{a^2+b^2}$

use $\sec^2\theta - \tan^2\theta = 1$

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(x) If normal at any point 'Q' meets the axis in Q and R' then show that PQ = PR = PC where 'C' is the centre of hyperbola.

1st Choice

Asymptodes

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An Asymptodes to a curve is a straight line which tends to become a tangent as the point of contact approaches to Infinity.

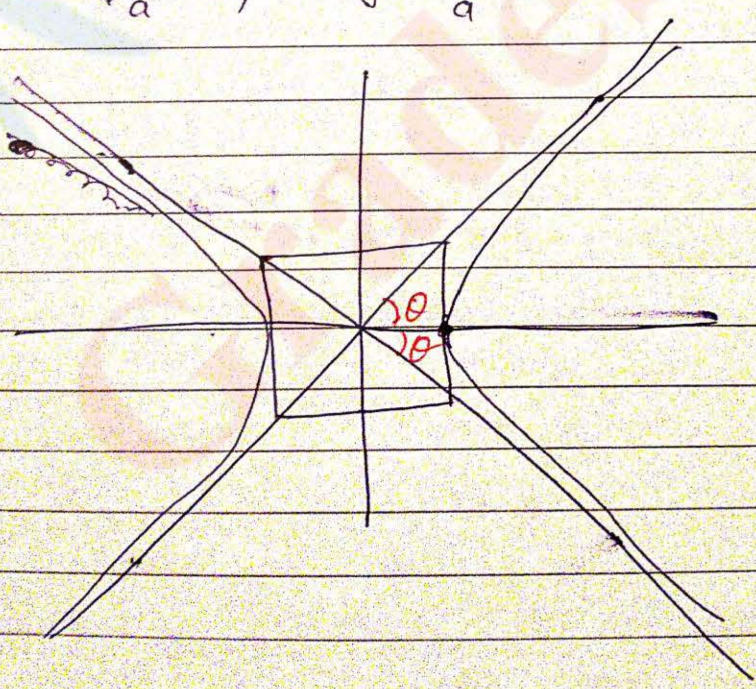
(That is tangent at Infinity.)

The eqⁿ of Asymptodes of hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are

$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0}$$

$y = \frac{b}{a}x$, $y = \frac{-b}{a}x$



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Properties :→

i) Assymptodes of an hyperbola always passes through a centre.

ii) Asymptodes of a hyperbola and its congugate hyperbola are same.

iii) The angle b/w assymptods of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is :

$$\boxed{\theta = 2\tan^{-1}\left(\frac{b}{a}\right)}$$

iv) The angle bisector of Assymptodes of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are co-ordinate axis

v) The eq^n of pair of Assymptodes of any hyperbola differ the eq^n of hyperbola and its congugate hyperbola by the same constant. That is If the eq^n of hyperbola is $S=0$ then the eq^n of Assymptodes will be

$S+\lambda=0$ and eq^n of congugate hyperbola will be

$S+2\lambda=0$

hyperbola — $H : S=0$ → $H = \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$

Asymptodes — $A : S+\lambda=0$ → $A = \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

congugate hyperbola — $C : S+2\lambda=0$ → $C = \frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$

so, $\boxed{C+H = 2A}$

1st Choice

Properties :→

i) Assymptodes of an hyperbola always passes through a centre.

ii) Assymptodes of a hyperbola and its conjugate hyperbola are same.

iii) The angle b/w assymptodes of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$$\boxed{\theta = 2\tan^{-1}\left(\frac{b}{a}\right)}$$

iv) The angle bisector of Assymptodes of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are co-ordinate axes.

v) The eq^n of pair of Assymptodes of any hyperbola differ the eq^n of hyperbola and its conjugate hyperbola by the same constant. That is if the eq^n of hyperbola is $S=0$ then the eq^n of Assymptodes will be $S+\lambda=0$ and eq^n of conjugate hyperbola will be

$$S + 2\lambda = 0$$

hyperbola — H : $S=0$ → $H = \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$

Assymptodes — A : $S+\lambda=0$ → $A = \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

conjugate hyp. — C : $S+2\lambda=0$ → $C = \frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$

so,

$$\boxed{C + H = 2A}$$

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Note: ⇒
The value of "λ" can be obtained by using the condition $\boxed{\Delta = 0}$ for Asymptotes.

Ex) Find the eqn of Asymptotes and conjugate hyperbola of hyperbola $xy - 2x + 3y + 4 = 0$.

Ans.

$$H :\to xy - 2x + 3y + 4 = 0$$
$$A :\to xy - 2x + 3y + \lambda = 0$$

$$\Delta = 0$$

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$
$$0 + 2\left(\frac{3}{2}\right)(-1)\left(\frac{1}{2}\right) - 0 - 0 - \lambda\left(\frac{1}{2}\right)^2 = 0$$

$$A :\to xy - 2x + 3y - 6 = 0$$

So

$$C :\to xy - 2x + 3y - 16 = 0$$

Ex:) Eqn of hyperbola is $2x^2 + 5xy + 2y^2 + 4x + 5y = 0$ find Asymptotes and conjugate hyperbola.

Ans) $H :\to 2x^2 + 5xy + 2y^2 + 4x + 5y = 0$

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A: $\Rightarrow 2x^2 + 5xy + 2y^2 + 4x + 5y + \lambda = 0$

$\Delta = 0$

abc

$\lambda = 2$

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Rectangular Hyperbola

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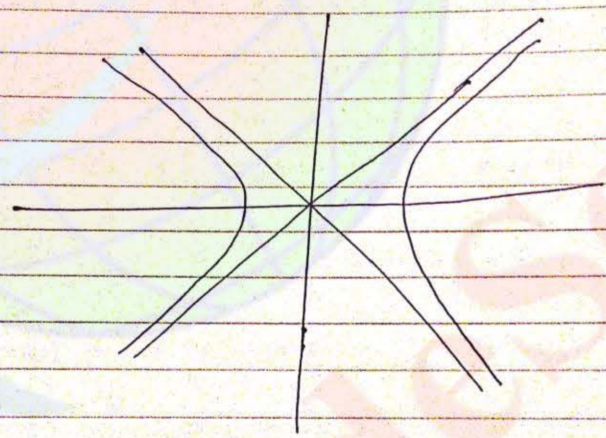
i) Assymptodes are perpendicular.
ii) $a = b$
iii) $e = \sqrt{2}$

sol

Hyperbola :⇒(H)⇒ $\boxed{x^2 - y^2 = a^2}$

Assymptodes(A) ⇒ $x^2 - y^2 = 0$



✱ Special case ⇒ when co-ordinates axis are Asymptodes

Rotate the axis by "45°"

	x	y
X	$\cos\theta$	$\sin\theta$
Y	$-\sin\theta$	$\cos\theta$

$x = X\cos\theta - Y\sin\theta$

$x = \left(\frac{X+Y}{\sqrt{2}}\right)$ — ①

$y = X\sin\theta + Y\cos\theta$

$y = \left(\frac{Y-X}{\sqrt{2}}\right)$ — ②

Eqⁿ of new systems

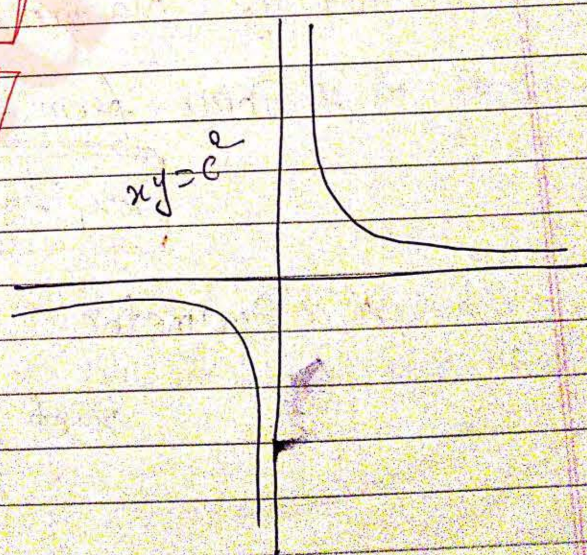
$\left(\frac{X+Y}{\sqrt{2}}\right)^2 - \left(\frac{Y-X}{\sqrt{2}}\right)^2 = a^2$

$\frac{(X+Y)^2 - (Y-X)^2}{2} = a^2$

$2XY = a^2$

$\boxed{XY = a^2/2}$

$\boxed{XY = c^2}$



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Properties of Rectangular hyperbola, $xy = c^2 \Rightarrow$

(1) Parametric :-

$$\left(ct, \frac{c}{t}\right)$$

2.) Eqⁿ of tangent :⇒

* Point form :⇒

$$\boxed{xy_1 + yx_1 = 2c^2}$$

* Parametric form :⇒

$$x \cdot \frac{c}{t} + y \cdot ct = 2c^2$$

$$\boxed{\frac{x}{t} + yt = 2c}$$

3.) Eqⁿ of Normal :⇒

* Point form :⇒

$$\boxed{xx_1 - yy_1 = x_1^2 - y_1^2}$$

* Parametric form :⇒

$$\Rightarrow \quad x \cdot ct - y \cdot \frac{c}{t} = c^2t^2 - \frac{c^2}{t^2}$$

$\pi \Rightarrow$ Product of root
$\Sigma \Rightarrow$ Sum of root

$$\Rightarrow \frac{xt^2-y}{t} = c\left(\frac{t^4-1}{t^2}\right)$$

$$\boxed{xt^3-yt = ct^4-c}$$

Q.) A circle with centre at origin Intersect the hyperbola $xy=c^2$ at four points t_1, t_2, t_3 and t_4 then show that:→
(i) $\pi t_i = 1$
(ii) $\Sigma t_i = 0$

Ans → Circle :→
$x^2+y^2 = r^2$
H :→ $x = ct$
$y = \frac{c}{t}$

$$(ct)^2 + \left(\frac{c}{t}\right)^2 = r^2$$

$$c^2t^4 - t^2r^2 + c^2 = 0$$

(from hyperbola)

(i) $t_1+t_2+t_3+t_4 = 0$
(ii) $t_1t_2t_3t_4 = \frac{c^2}{c^2} = 1$

Q.) If the normal at point "t_1" on hyperbola $xy=c^2$ Intersect it again at point "t_2" then show that

$$t_2 = \frac{-1}{t_1^3}$$

Ans: $xt_1^3 - yt_1 = ct_1^4 - c$

pass $\left(ct_2, \frac{c}{t_2}\right)$

$$ct_2 t_1^3 - \frac{c}{t_2} \cdot t_1 = ct_1^4 - c$$

$$t_2 t_1^3 - t_1^4 = \left(\frac{t_1}{t_2} - 1\right)$$

$$t_1^3 (t_2 - t_1) = \left(\frac{t_1 - t_2}{t_2}\right)$$

$$t_1^3 = -\frac{1}{t_2}$$

$$t_2 = \frac{-1}{t_1^3}$$

Ex Find the eqn of chord joining the points t_1 and t_2 of hyperbola $xy = c^2$

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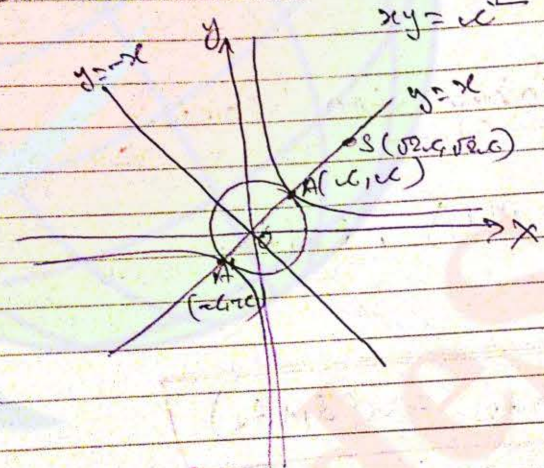
(Asymptote concept)

Q. For hyperbola $xy = c^2$ find :-

i) Asymptotes
ii) 'e'
(iii) Eqn of Axis (transverse or conjugate)
iv) Centre
v) vertices
(vi) foci
vii) ~~Lactus~~ Length of latus rectum
viii) Auxiliary circle
ix) Director circle.

Ans



(i) $xy = 0$

(ii) $e = \sqrt{2}$

(iii) $y = x$ | $y = -x$

$$y^2 - x^2 = 0$$

iv) Centre $= (0, 0)$

v) vertices :- (c, c)
$(-c, -c)$

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vi) $S(\sqrt{2}c, \sqrt{2}c)$

$S'(-\sqrt{2}c, -\sqrt{2}c)$

vii) $c^2 = \frac{a^2}{2}$

$L\cdot R = \frac{2b^2}{a} = 2a = 2\sqrt{2}c$

viii) $x^2 + y^2 = 2c^2$

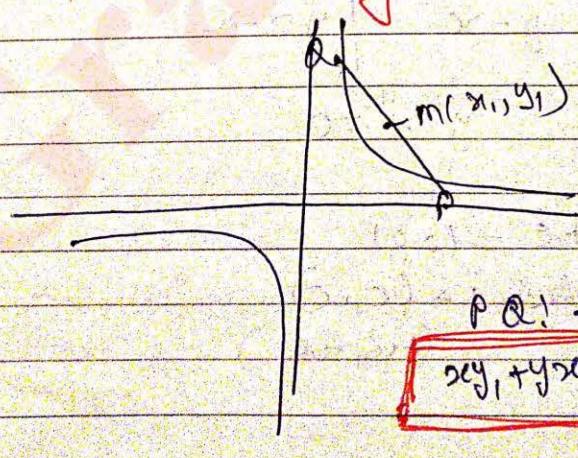
Chord joining two Points:→

$P(t_1)$, $Q(t_2)$

PQ: →

$$\boxed{x + y t_1 t_2 = c(t_1 + t_2)}$$

Chord bisected at given Points:→



PQ: →

$$\boxed{x y_1 + y x_1 = 2 x_1 y_1}$$

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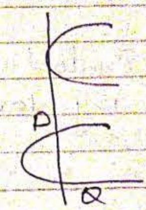
Q. A tangent to parabola $x^2 = 4ay$ meets the hyperbola $xy = c^2$ in two points 'P' and 'Q' show that locus of mid-point of 'PQ' is a parabola.

Ans :⇒



$$y = mx - am^2$$

$$xk + yh = 2hk \quad \text{—(1)}$$

$$mx - y = am^2 \quad \text{—(2)}$$

$$\frac{k}{m} = \frac{h}{-1} = \frac{2hk}{m^2}$$

$$m = \frac{-k}{h}$$

$$m = \frac{2h}{a}$$

$$\Rightarrow \frac{2h}{a} = \frac{-k}{h}$$

$$-ak = 2h^2$$

$$\boxed{ay = -2x^2}$$

Q. If a variable chord of slope = 4, Intersect the hyperbola $xy = 1$ at two point 'P' and 'Q' find the locus of point which divides PQ in 1:2

Ans

$P(t_1, \frac{1}{t_1})$

$Q(t_2, 1/t_2)$

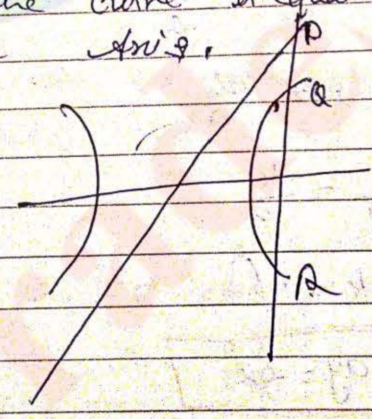
1st Choice Properties ⇒

1.) The locus of foot of perpendicular drawn from the focus of hyperbola upon any tangent is auxiliary circle.

ii) Product of length of ⊥ from foci upon any tangent is "b^2"

III) The portion of tangent b/w point of contact and directrix subtends right angle at the focus.

iv) If from any point on asymptotes, straight line is drawn ⊥ to transverse axis the product of the segments of this line intercepted b/w the point and the curve is equal to square of semi conjugate axis.



$$\boxed{PQ \cdot PR = b^2}$$

V) Confocal ellipse and hyperbola cut orthogonally!

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Properties ⇒

1.) The locus of foot of perpendicular drawn from the focus of hyperbola upon any tangent is Auxiliary circle.

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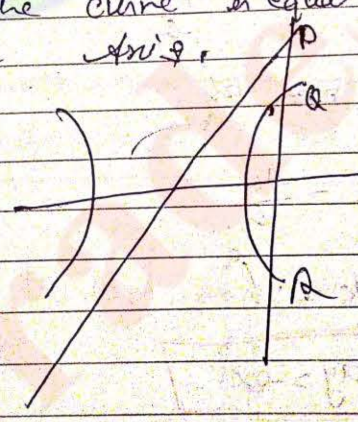
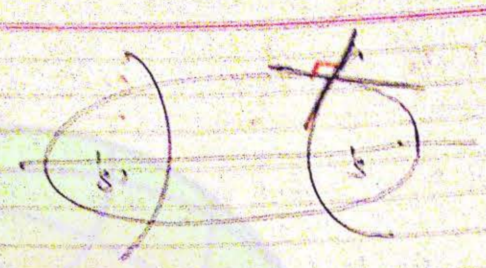


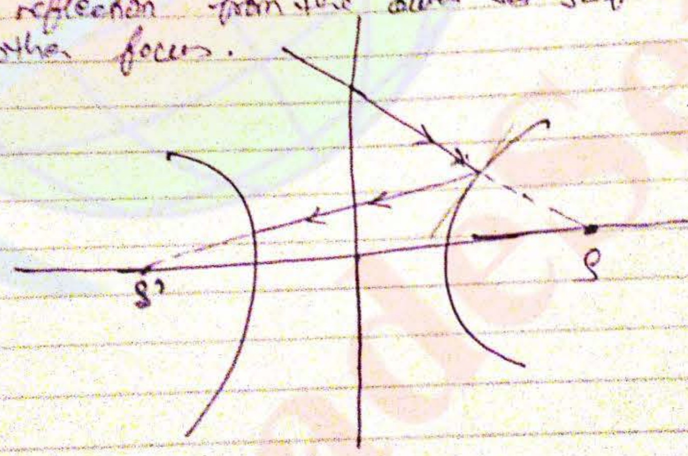
$\boxed{PQ \cdot PR = b^2}$

v) Confocal ellipse and hyperbola cut orthogonally!



vi) The tangent and normal at any point on the hyperbola bisect the angle b/w the focal distances of that point.

So an incoming ray light ray directed towards one focus of hyperbola after reflection from the outer surface goes towards another focus.



H.W ⇒ D.P.P. 3,7,3 | Ex-1 (Hyperbola) | Conic ⇒ 32

1st Choice

Centre of Any Conic ⇒

$$f(x,y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

The centre of this conic is the solution of ~~the~~ this equation

$$\frac{\partial f}{\partial x} = 0 \Rightarrow 2ax + 2hy + 2g = 0$$

and

$$\frac{\partial f}{\partial y} = 0 \Rightarrow 2by + 2hx + 2f = 0$$

Vol:->

1st Choice

Binomial theorem

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Some definitions:-

I) Factorial Notation:- Product of first 'n' natural numbers is written as "$\lfloor n$". or $n!$ (factorial 'n') ($n \in N$)
so,

$$\lfloor n = n(n-1)(n-2) - - - - - 2 \cdot 1.$$

Properties:>

(a)

$$\lfloor n = n\lfloor n-1$$

(b) Factorial of ~~negati~~ '-ve' ~~no~~ number is not defined

(c) $\lfloor 0 = 1 = \lfloor 1$

$$\boxed{\lfloor 1 = 1 \lfloor 1-1}$$

(d) $\left(\because {}^nC_n = 1 \right)$

Binomial Co-efficient:->

$${}^nC_r \text{ or } \binom{n}{r} \text{ or } C(n,r)$$

$$\boxed{{}^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}}$$

where ~~is~~ ~~non~~

$$(0 \leq r \leq n)$$

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eg → $^4C_2 = \frac{\lfloor 4}{\lfloor 2 \lfloor 2} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 6$

Trick

$\left\{ \frac{4 \cdot 3}{1 \cdot 2} = 6 \right\}$

eg $^6C_3 = \frac{\lfloor 6}{\lfloor 3 \cdot \lfloor 3}$

$= \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3} \cdot 2 \cdot 1}{1 \cdot 2 \cdot \cancel{3} \cdot 1 2 \cdot \cancel{3}} = 20$

Trick

$\left(\frac{6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3} = 20 \right)$

eg → $^{10}C_2 = \frac{\cancel{10}^5 \times 9}{1 \cdot \cancel{2}} = 45$ $\quad \frac{\cancel{10}^5 \times 9}{1 \times \cancel{2}}$

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Properties:>

(i) $^nC_r \in N$

ii) $\boxed{^nC_0 = {^nC_n} = 1}$

Proof:> $\frac{\lfloor n}{\lfloor 0 \lfloor n-0} = \frac{\lfloor n}{\lfloor 0 \cancel{\lfloor n}} = 1.$

iii) $\boxed{^nC_r = {^nC_{n-r}}}$

Proof:
RHS:>

$$\frac{\lfloor n}{\lfloor n-r \lfloor n-(n-r)} \Rightarrow \frac{\lfloor n}{\lfloor n-r \lfloor r}$$

$$\Rightarrow \frac{\lfloor n}{\lfloor r \lfloor n-r} = {^nC_r}$$

eg:> $^{20}C_{17} = {^{20}C_{20-17}}$

$= {^{20}C_3}$

$= \frac{20 \cdot 19 \cdot 18}{1 \cdot 2 \cdot 3} \Rightarrow 1140$

$\frac{38 \times 3}{84}$

1st Choice

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Properties:

i) $^nC_r \in N$

ii) $\boxed{^nC_0 = {^nC_n} = 1}$

Proof: $\frac{\lfloor n}{\lfloor 0 \lfloor n-0} = \frac{\lfloor n}{\lfloor 0 \lfloor n} = 1.$

iii) $\boxed{^nC_r = {^nC_{n-r}}}$

Proof:
RHS:

$$\frac{\lfloor n}{\lfloor n-r \lfloor n-(n-r)} = \frac{\lfloor n}{\lfloor n-r \lfloor r}$$

$$\Rightarrow \frac{\lfloor n}{\lfloor r \lfloor n-r} = {^nC_r}$$

eg: $^{20}C_{17} = {^{20}C_{20-17}}$

$= {^{20}C_3}$

$= \frac{20 \cdot 19 \cdot 18}{1 \cdot 2 \cdot 3} \Rightarrow 1140$

$\frac{38}{84}$

eg $^{100}C_{98} = {}^{100}C_{100-98}$

$= {}^{100}C_2$

$= \frac{\cancel{100} \cdot 99}{1 \cdot \cancel{2}}$ (50) $= 4950$

iv) $\boxed{\begin{array}{l} ^nC_x = {}^nC_y \\ \text{then} \\ \quad \text{either (a) } x = y \\ \quad \quad \text{or (b) } x + y = n \end{array}}$

v) $\boxed{^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}} = {}^nC_{n-r}$

L.H.S :->

$\frac{\lfloor n}{\lfloor r \lfloor n-r} = \frac{n \lfloor n-1}{r \lfloor r-1 \lfloor n-r}$

$= \frac{n}{r} \cdot \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-1-(r-1)}$

$= \frac{n}{r} \cdot {}^{n-1}C_{r-1}$

vi) $$\boxed{\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}}$$

LHS

$$\frac{\lfloor n}{\lfloor r \, \lfloor n-r} \times \frac{\lfloor r-1 \cdot \lfloor n-r+1}{\lfloor n}$$

$$(\because \lfloor n = n \lfloor n-1)$$

$$\frac{\cancel{\lfloor r-1} \cdot (n-r+1) \cancel{\lfloor n-r}}{r \cancel{\lfloor r-1} \cdot \cancel{\lfloor n-r}}$$

$$= \frac{n-r+1}{r}$$

Imp vii) $$\boxed{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r}$$

Notes:

viii) $r \cdot {}^nC_r = n \cdot {}^{n-1}C_{r-1}$

ix) $\frac{1}{r+1} {}^nC_r = \frac{1}{n+1} \cdot {}^{n+1}C_{r+1}$

x) $r^2 = r(r-1) + r$

xi) $r^3 = r(r-1)(r-2) + 3r(r-1) + r$

xii) $${}^nC_r = \frac{r+1}{n+1} \, {}^{n+1}C_{r+1}$$

Ex → If $^nC_{r-1} = 36$

$^nC_{r+1} = 126$

$^nC_r = 84$

find the value of r.

Ans →

i) $\frac{^nC_r}{^nC_{r-1}} = \frac{\cancel{84}^{7}}{\cancel{36}_{3}}$

$\frac{n-r+1}{r} = \frac{7}{3}$ — (1)

ii) $\frac{^nC_{r+1}}{^nC_r} = \frac{126}{84}$

$$\boxed{\frac{^nC_{r+1}}{^nC_r} = \frac{n-(r+1)+1}{r+1}}$$

$\frac{n-(r+1)+1}{r+1} = 3/2$

$\frac{n-r}{r+1} = 3/2$ — (2)

solving eq (1) and eq (2)

$$\boxed{r = 3}$$

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Ex) ${}^{n+1}C_{r+1} : {}^{n}C_{r} : {}^{n-1}C_{r-1} = 11 : 6 : 3$

find value of 'n'.

Ans) from IInd and IIIrd term

$$\frac{\frac{n}{r} \cdot \cancel{{}^{n-1}C_{r-1}}}{\cancel{{}^{n-1}C_{r-1}}} = \frac{6}{3}$$

n = 10 Ans.

$$\frac{n}{r} = 2$$

$$\boxed{n = 2r} \text{ — (I)}$$

Now

By using formula

$$\boxed{{}^{n}C_{r} = \frac{n}{r} \cdot {}^{n-1}C_{r-1}}$$

इसको उपयोग करने का तरीका।

$$\Rightarrow \frac{{}^{n+1}C_{r+1}}{{}^{n}C_{r}} = \frac{11}{6}$$

$$\Rightarrow \frac{\frac{n+1}{r+1} \cdot {}^{n+1-1}C_{r+1-1}}{{}^{n}C_{r}} = \frac{11}{6}$$

$$\Rightarrow \frac{\frac{n+1}{r+1} \cdot \cancel{{}^{n}C_{r}}}{\cancel{{}^{n}C_{r}}} = \frac{11}{6}$$

$$\Rightarrow \frac{n+1}{r+1} = \frac{11}{6} \text{ — (II)}$$

Now solving eq (I) and (II) we get

n = 10 Ans.

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Deed VII Imp (IIT) are asked mostly :—

$$\boxed{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r}$$

$$\frac{n!}{r!\,(n-r)!} + \frac{n!}{(r-1)!\cdot(n-r+1)\cdot(n-r)!}$$

$$\Rightarrow \frac{n!}{r\,(r-1)!\,(n-r)!} + \frac{n!}{(r-1)!\cdot(n-r+1)\,(n-r)!}$$

$$\Rightarrow \frac{n!}{(r-1)!\cdot(n-r)!}\cdot\left(\frac{1}{r} + \frac{1}{n-r+1}\right)$$

$$\Rightarrow \frac{(n+1)\,n!}{(r\,(r-1)!)\,(n-r+1)\,(n-r)!}$$

$$\Rightarrow \frac{(n+1)!}{r!\,(n+1-r)!}$$

$$\Rightarrow {}^{n+1}C_r$$

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Binomial Expression

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An Algebric expression ~~consisting~~ consisting of two terms with '+ve' or '-ve' sign b/w them is called binomial Expression.

eg:-

$(a+b)^2$, $(\sqrt{7}+1)^{10}$, . . . etc.

Binomial Expansion (for positive index n) ($n \in N$)

If $n \in N$, then

$$(x+a)^n = {}^nC_0\, x^n \cdot a^0 + {}^nC_1\, x^{n-1} \cdot a^1 + {}^nC_2\, x^{n-2} a^2 + - - - - \cdots + {}^nC_{n-1}\, x^1 \cdot a^{n-1} + {}^nC_n\, x^0 \cdot a^n$$

Note:- जब हम binomial expression को लिखते हैं तो generally x अपने maximum power से शुरू होकर minimum power तक जाता है

where ${}^nC_r$ is binomial coefficient.

so,

$$\boxed{(x+a)^n = \sum_{r=0}^{n} {}^nC_r\, x^{n-r} \cdot a^r}$$

For $(1+x)^3 = {}^3C_0\, 1^3 x^0 + {}^3C_1\, 1^2 x^1 + {}^3C_2\, 1^1 x^2 + {}^3C_3\, 1^0 x^3$

$\Rightarrow 1 + 3x + 3x^2 + x^2$

(b) $(x-3)^5 = {}^5C_0 x^5(-3)^0 + {}^5C_1 x^4(-3)^1 + {}^5C_2 x^3(-3)^2 + {}^5C_3 x^2(-3)^3 + {}^5C_4 x(-3)^4 + {}^5C_5 x^0(-3)^5$

$\Rightarrow x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243$

(c) $(2x-y)^5$

$\Rightarrow {}^5C_0(2x)^5(-y)^0 + {}^5C_1(2x)^4(-y)^1 + {}^5C_2(2x)^3(-y)^2 + {}^5C_3(2x)^2(-y)^3 + {}^5C_4(2x)^1(-y)^4 + {}^5C_5(2x)^0(-y)^5$

$\Rightarrow 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$

(e) $(1+\sqrt{2})^7 + (1-\sqrt{2})^7$

Ans $\boxed{2\{\text{sum of terms at odd place}\}}$ → If $(x+y)^n + (x-y)^n$

$\Rightarrow 2\{{}^7C_0 1^7(\sqrt{2})^0 + {}^7C_2 1^5(\sqrt{2})^2 + {}^7C_4 1^3(\sqrt{2})^4 + {}^7C_6 1^1(\sqrt{2})^6\}$

$\Rightarrow 2\{1 + 21\times 2 + 35\times 4 + 7\times 1\times 8\}$

$\Rightarrow 2\{1 + 42 + 140 + 56\}$

$\Rightarrow 2\{239\}$

$\Rightarrow 478$ Ans.

(c) $(x-y)^5 = {}^5C_0\, x^5(-y)^0 + {}^5C_1\, x^4(-y)^1 + {}^5C_2\, x^3(-y)^2$
$+ {}^5C_3\, x^2(-y)^3 + {}^5C_4\, x^1(-y)^4 + {}^5C_5\, x^0(-y)^5$

$\Rightarrow x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

(d) $(2x-y)^5$

$\Rightarrow {}^5C_0(2x)^5(-y)^0 + {}^5C_1(2x)^4(-y)^1 + {}^5C_2(2x)^3(-y)^2$
$+ {}^5C_3(2x)^2(-y)^3 + {}^5C_4(2x)^1(-y)^4 + {}^5C_5(2x)^0(-y)^5$

$\Rightarrow 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$

(e) $(1+\sqrt{2})^7 + (1-\sqrt{2})^7$

Ans $\boxed{2\{\text{Sum of terms at odd place}\}}$ If $(x+y)^n + (x-y)^n$

$\Rightarrow 2\{{}^7C_0\, 1^7(\sqrt{2})^0 + {}^7C_2\, 1^5(\sqrt{2})^2 + {}^7C_4\, 1^3(\sqrt{2})^4 + {}^7C_6\, 1^1(\sqrt{2})^6\}$

$\Rightarrow 2\{1 + 21\times 2 + 35\times 4 + 7\times 1\times 8\}$

$\Rightarrow 2\{1 + 42 + 140 + 56\}$

$\Rightarrow 2\{239\}$

$\Rightarrow 478$ Ans.

1st Choice

Properties of Binomial

$(x+a)^n =$ $r+1$

1) No. of terms in the expansion of $(x+a)^n$ is $(n+1)$ where r is varying from 0 to 1

2) Sum of exponents of x and a is constant and equal to 'n'.

3) The binomial coefficient of the terms equidistant from the begning and the end are equal.

General term :-

Imp: $\boxed{T_{r+1} = {}^nC_r\, x^{n-r} \cdot a^r}$

Q. Find the fourth term in $\left(2x - \frac{y}{2}\right)^7$.

Ans $T_{3+1} = \cancel{{}^4C_3\, x^{4-3} \cdot a^3}$ $\quad r = 3$

$\Rightarrow \cancel{4 \times x \times a^3} \Rightarrow {}^7C_3 \cdot (2x)^4 \cdot \left(\frac{-y}{2}\right)^3$

$\Rightarrow \cancel{4 \times 2x \times \frac{y}{2}} \Rightarrow -\left(\frac{7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3}\right) 2^4 \cdot x^4 \cdot \frac{y^3}{2^3}$

$= -70\, x^4 \cdot y^3$

Q. Find no. of terms in $(1-3x+3x^2-x^3)^{20}$

Ans) $\Rightarrow \left[(1-x)^3\right]^{20}$

$\Rightarrow (1-x)^{60}$

so terms $60+1 = 61$

Q. Find the term involving x^3 in $\left(2x^2-\frac{1}{3x}\right)^6$

Ans. $$\boxed{{}^6C_0(2x^2)^6\left(\frac{-1}{3x}\right)^0 + {}^6C_1(2x^2)^5\left(\frac{-1}{3x}\right)^1 + {}^6C_2(2x^2)^4\left(\frac{-1}{3x}\right)^2 + {}^6C_3(2x^2)^3\left(\frac{-1}{3x}\right)^3 + {}^6C_4(2x^2)^2\left(\frac{-1}{3x}\right)^4 + {}^6C_5(2x^2)^1\left(\frac{-1}{3x}\right)^5 + {}^6C_6}$$

$$T_{r+1} = {}^6C_r(2x^2)^{6-r}\left(\frac{-1}{3x}\right)^r$$

$$\Rightarrow {}^6C_r \cdot 2^{6-r}\left(\frac{-1}{3}\right)^r (x^2)^{6-r}\cdot\left(\frac{1}{x}\right)^r$$

$$\Rightarrow \underbrace{{}^6C_r \cdot 2^{6-r}\cdot\left(\frac{-1}{3}\right)^r}\cdot x^{12-3r}$$

$\Rightarrow 12-3r = 3$

$r = 3$

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$\Rightarrow {}^{6}C_{3} \cdot 2^{3} \left(\frac{-1}{3}\right)^{3} x^{3}$

$\Rightarrow -\frac{6 \cdot 5 \cdot 4}{1 \cdot 2 \cdot 3} \cdot \frac{8}{27} \cdot x^{3}$

$\Rightarrow \frac{-160}{27} x^{3}$

Q which term in expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$ is free of x' or (Independent of x)

A. $T_{r+1} = {}^{12}C_{r} (x^2)^{12-r} \cdot \left(\frac{1}{x}\right)^{r}$

$= {}^{12}C_{r} \cdot x^{24-3r}$

$\Rightarrow 24 - 3r = 0$

$\boxed{r = 8}$

so, 9th term is Independent of x'

Q Find the co-efficient of x^{32} and x^{-17} in $\left(x^4 - \frac{1}{x^3}\right)^{15}$

A

$T_{r+1} = {}^{n}C_{r}\, x^{n-r} a^{r}$

$T_{r+1} = {}^{15}C_{r}\, x^{15-r} a^{r}$

so, A/a

$\Rightarrow {}^{15}C_{r} (x^4)^{15-r} \left(\frac{-1}{x^3}\right)^{r}$

$\Rightarrow {}^{15}C_{r}\, x^{60-4r} \cdot -x^{-3r}$

$\Rightarrow {}^{15}C_{r} - x^{60-7r}$

so,

Ans $\Rightarrow (1365, -1365)$

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11) ~~find the coefficient of x^{-17} in $(1+3x^2+x^4)\left(1+\frac{1}{x}\right)^8$~~
Ans: 232

So,

$60-7r = 32$

$\cancel{7}r = \cancel{28}\,4$

$r = 4$

Now

coefficient: ${}^{15}C_4 = \frac{15\times14\times13\times12}{1\times2\times3\times4}$

$= 1365$ Ans

Now so,

$60-7r = -17$

$\cancel{60}\ \cancel{7}r = \cancel{77}$

$r = 11$

~~15C~~ So, coefficient of x^{-17} is

${}^{15}C_{11} = 15\times$

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11) ~~find the coefficient of x^{-7} in~~ $(1+3x^2+x^4)(1+\frac{1}{x})^8$

Ans:- 232

so,

$60-7r = 32$

$\cancel{7}r = \cancel{28}\,4$

$r = 4$

Now

coefficient $= {}^{15}C_4 = \frac{15\times\cancel{14}\times13\times\cancel{12}}{1\times\cancel{2}\times\cancel{3}\times\cancel{4}}$

$= 1365$ Ans

Now so:

$60-7r = -17$

$\cancel{60} \cancel{7}r = \cancel{77}$

$r = +11$

so, coefficient of x^{-17} is

${}^{15}C_{11} = 15\times$

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11) find the coefficient of x^{-1} in $(1+3x^2+x^4)\left(1+\frac{1}{x}\right)^8$

Ans) $(1+3x^2+x^4)\left(1+\frac{1}{x}\right)^8$

$(1+3x^2+x^4)\left(1+{}^8C_1\left(\frac{1}{x}\right)+{}^8C_2\left(\frac{1}{x}\right)^2 \cdots\right)$

coeff of x^{-1}

${}^8C_1 + 3\cdot{}^8C_3 + {}^8C_5$

$\Rightarrow$ 232

12) Find the term with x^9 in the expansion of $\left(2x^2-\frac{1}{x}\right)^{20}$

Ans

$T_{r+1} = {}^nC_r\, x^{n-r} a^r$

$T_{r+1} = {}^{20}C_r\,(2x^2)^{20-r}\cdot\left(\frac{-1}{x}\right)^r$

$T_{r+1} = {}^{20}C_8\, x^{20-8} a^8$

$\Rightarrow {}^{20}C_8\,(2x^2)^{20-8}\left(\frac{-1}{x}\right)^8$

$\Rightarrow {}^{20}C_r\, 2^{20-r}\, x^{40-3r}$

$\Rightarrow {}^{20}C_8\, \cancel{2x^{24}\cdot x^{-8}}$

$40-3r = 9$

$\Rightarrow$

$r = \frac{31}{3}$ X

13) Find coeff of x^{15} in the $(x-x^2)^{10}$

Ans) $(x-x^2)^{10}$

$x^{10}(1-x)^{10}$

$\Rightarrow x^{10}\left(1-{}^{10}C_1 x + {}^{10}C_2 x^2 - \cdots\right)$

$\Rightarrow x^{10}\ \ {}^{10}C_5$

(Imp)

Q. In the expansion of $(1+x)^{10}$ binomial coefficient of $(4r+5)^{th}$ and $(2r+1)^{th}$ term is equal. then find r?

Ans- Given→

$${}^{10}C_{4r+4} = {}^{10}C_{2r}$$

By using→ ${}^{n}C_{x} = {}^{n}C_{y}$
i) $x = y$
ii) $x + y = n$

(i) $2r = 4r + 4$ ~~X~~

Imp (ii) $4r + 4 + 2r = 10$

$\boxed{r = 1}$

(the Tth term उपस्थित term given expansion का है coeff equal करता है equal होता है)

Ex:→ Find the coefficient of x^{37} in the expansion

$$\sum_{r=0}^{100} {}^{100}C_r (x-3)^{100-r} \cdot 5^r$$

Ans: $(x - 3 + 5)^{100}$

$\Rightarrow (x+2)^{100}$

$T_{r+1} = {}^{100}C_r\, x^{100-r} \cdot 2^r$

$r = 63$

$\Rightarrow$ Coeff ~~of~~ ${}^{100}C_{63} \cdot 2^{63}$

(Q.) Find the no. of rational terms in $(3^{1/5} + 2^{1/3})^{15}$

Ans: $T_{r+1} = {}^{15}C_r\, 3^{\frac{15-r}{5}} \cdot 2^{r/3}$

so,

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$\frac{15-r}{5} \in$ Integer , $\frac{r}{3} \in$ Integer

Rational terms $\Rightarrow r = 0, 15$, $(0 \le r \le 15)$

So, Rational terms $= 2$

(i) Find the no. of rational terms in $\left(9^{1/4} + 8^{1/6}\right)^{1000}$

Ans $T_{r+1} = {}^{1000}C_r \cdot 3^{\frac{1000-r}{2}} \cdot 2^{r/2}$

$r = 0, 2, 4, \ldots\ldots 1000$

terms $\Rightarrow$ ~~500~~ $\boxed{501}$ Ans

(ii) Find the term independent of x of

$$\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$$

Ans $\Rightarrow$

$$\left(\frac{(x^{1/3})^3 + (1)^3}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - \sqrt{x}}\right)$$

$$\Rightarrow x^{1/3} + 1 - \frac{\cancel{(\sqrt{x}-1)}(\sqrt{x}+1)}{\sqrt{x}\,\cancel{(\sqrt{x}-1)}}$$

$$\Rightarrow \left(x^{1/3} - x^{-1/2}\right)$$

$$T_{r+1} = {}^{10}C_r \cdot x^{\frac{10-r}{3}} \cdot x^{-r/2}$$

1st Choice

Middle terms

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$(x+a)^n$ → Even, → odd

1) If 'n' belongs to even then the no. of terms $(n+1)$ is odd so middle term will be unique.

$$\boxed{\text{And middle term is } T_{\frac{n}{2}+1}}$$

ii) In 'n' belongs to odd then

The no. of terms $(n+1)$ will be even so, there will be two middle terms

$$\boxed{T_{\frac{n+1}{2}} \text{ and } T_{\frac{n+3}{2}}}$$

Q) Find the middle term of i) $\left(1-\frac{x^2}{2}\right)^{14}$

ii) $\left(3a - \frac{a^3}{6}\right)^9$

Ans. ~~T~~ $T_{\frac{14}{2}+1} = T_8$

so,

$$\boxed{r = 7}$$

$\Rightarrow {}^{14}C_7 \left(\frac{-x^2}{2}\right)^7$

$\Rightarrow \frac{-429}{16}x^{14}$

1st Choice

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ii) $\left(\cancel{T_{\frac{9+1}{2}}}, \cancel{T_{\frac{9+3}{2}}}\right)$ $\cancel{\left(T_5, T_6\right)}$

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$T_{\frac{9+1}{2}}$ | $T_{\frac{9+3}{2}}$

$r = 4$ | $r = 5$

$\Rightarrow {}^9C_4 (3a)^5 \cdot \left(\frac{-a^2}{6}\right)^4$

$\Rightarrow \frac{189}{8} a^{17}$ | $\Rightarrow -\frac{21}{6} a^{19}$

Q) Find $x:y$ If the ratio of middle terms in $(2x+3y)^{11}$ is '4'.

Ans) $\frac{11+1}{2}$, $\frac{11+3}{2}$

Given

$$\frac{{}^{11}C_5 (2x)^6 \cdot (3y)^5}{{}^{11}C_6 (2x)^5 \cdot (3y)^6} = \frac{4}{1}$$

$\frac{2x}{3y} = 4$

$\frac{x}{y} = 6$

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~~In the expansion of $(x+y)^n$~~

Note:→

The middle term in the expansion of $(x+y)^n$ as the greatest binomial expansion.

Ex In the expansion of $(2x+2y)^{20}$. find the greatest coefficient of $x^r \cdot y^{20-r}$

Ans→

$$2^{20}(y+x)^{20}$$

$$T_{r+1} = 2^{20}\, {}^{20}C_r\, y^{20-r} \cdot x^r$$

max at $r=10$

$$2^{20} \cdot {}^{20}C_{10}$$

Ex) In the expansion of $(x+y)^n$ coefficient of 11th term is greater, than find the value of n.

Ans→ n ∈ even | n ∈ odd

$$\frac{n}{2}+1 = 11$$

$$n = 20$$

$$\frac{n+1}{2} = 11$$

$$\frac{n+3}{2} = 11.$$

1st Choice

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Numeraically and Algebrically greater value ⇒

$(x+a)^n$

$n \Rightarrow$ given
$a \Rightarrow$ given
$x \Rightarrow$ given

i) First calculate the value of $\frac{n+1}{1+\left|\frac{x}{a}\right|} = K$ (let)

Note:- "a" तब consider करना है जब original expression के अन्दर coeff "x" का लिया गया हो

ii) ~~If K belongs~~ If $K \in I$ (Integer) then

T_K, T_{K+1} are numerically greatest terms.

iii) If $K \notin I$ (Integer) then

$$[K] = m \text{ (let)}$$

Than is

T_{m+1} is the numerically greatest term.

Q) Find numerically greater term in $(7-5x)^{11}$ when $x = 2/3$.

Sol) $K = \frac{n+1}{1+\left|\frac{x}{a}\right|}$

1st Choice

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$$= \frac{11+1}{1+\left|\frac{7\times 7}{5\times 2}\right|}$$

very Impt concept. होता वाले ज्यादा

$$k = \left(\frac{120}{71}\right)$$

$[k] = 7 = m$

$T_{7+1} = T_4$ is numerically greatest.

Notes: If no. obtained in fraction then we take only one term. (one more than least value of k)

Q) Find numerically greatest term in $(7-2x)^9$ when $x = 1$.

A) $$k = \frac{9+1}{1+\left|\frac{3}{2}\right|} = 4$$

T_4 and T_5

(If k is Integer then we take two value one obtained + 1 + obtained)

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Doing again.

Q) Find i) numerically greatest
ii) Algebrically greatest
iii) Algebrical least term in the expansion of $(3-5x)^{15}$ at $x = \frac{1}{5}$

A) 1) $$k = \frac{n+1}{1+\left|\frac{K}{9}\right|}$$

$$\Rightarrow \frac{15+1}{1+\left|\frac{3}{5}\times 5\right|} \Rightarrow \frac{16}{1+3} \Rightarrow \frac{16}{4} = 4$$

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ii) T_4 and T_5 are numerically greatest.

T_5 (greatest) +ve

T_7

T_1 T_7 T_9 T_{11}

T_2 T_{10}

T_6 T_8

T_4 T_5 (smallest) -ve

Algebrically greater = T_5
Algebrically least = T_4 Ans.

Q6 Find i) numerically great.
ii) Algebrically least.
iii) Algebrically second greater in the expansion
of $(3x-5)^{17}$ at $x=2$

Sol.

$\cancel{k = \frac{|17+1|}{1+}}$ $(-5+3k)^{15}$ $\cancel{k=2}$

$\cancel{k = \frac{13+1}{1+\left|\frac{-5}{3}\times\frac{1}{2}\right|} = \frac{14}{1+\frac{5}{6}} = \frac{14}{11}\times 6 = \frac{84}{11} = 7}$

T_7 and ~~T_8 terms.~~

$(3x-5)^{17}$ at $x=2$

$k = \dfrac{17+1}{1+\left(\frac{6}{5}\right)} = 6.7$

$K = 6 = m$

$T_{6+1} = T_7$

$(3x-5)^{17}$

$\cancel{{}^{n}C_0 (3x)^{17}(-5)^0}$, $\cancel{{}^{n}C_0 (3x)^{12}(-5)^1}$

T_3 T_5 T_7 T_9

T_1 T_{11}

T_2 T_{12}

T_4 T_6 T_8 T_{10}

$T_6 = -{}^{17}C_5\,(6)^8\,(5)^5$

$T_8 = -{}^{17}C_7\,(6)^6 \cdot (5)^7$

$\dfrac{T_6}{T_8} = \dfrac{{}^{17}C_5}{{}^{17}C_7} \cdot \dfrac{6^2}{5^2}$

$= \dfrac{\lfloor 6 \lfloor 7}{\lfloor 5 \lfloor 8} \cdot \dfrac{36}{25}$

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$$= \frac{6\cancel{\lfloor 5}\;\cancel{\lfloor 7}}{\cancel{\lfloor 5}\;8\cancel{\lfloor 7}} \cdot \frac{36}{25}$$

$$\Rightarrow \frac{^{3}\cancel{6}}{\cancel{8}_{4}} \cdot \frac{\cancel{36}^{9}}{25}$$

$$\Rightarrow \frac{27}{25} > 1 \to T_6 \quad \text{(} T_6 \text{ में बड़ा है इसलिए वास्तव में छोटा होगा)}$$
$$\to T_5$$

So,
T_6 is Algebrically least.

Now -

$$T_5 = {}^{13}C_4 (6)^9 (5)^4$$

$$T_9 = {}^{13}C_8 (6)^5 (5)^8$$

$$\frac{T_5}{T_9} = \frac{{}^{13}C_4}{{}^{13}C_8} \cdot 6^4 \cdot 5^{-4}$$

$$\Rightarrow \frac{{}^{13}C_4}{{}^{13}C_8} \cdot \frac{6^4}{5^4}$$

$\Rightarrow$

1st Choice

$(1+a)^n$, $a \to x$

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Summation of Series using Substitution/ calculus.

General expansion:

$(x+a)^n$, $x \to 1$, $a \to x$

$$(1+x)^n = {}^nc_0 + {}^nc_1 \cdot x + {}^nc_2 \cdot x^2 - \cdots - {}^nc_n x^n$$

$$\boxed{(1+x)^n = c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n}$$

Note:

1\. Show that $c_0 + c_1 + c_2 + c_3 + \cdots + c_n = 2^n$

Ans (Hint: use $x = 1$ in eq ①

$(1+1)^n = c_0 + c_1 + c_2 + \cdots c_n$)

$2^n = c_0 + c_1 + c_2 + \cdots c_n$

So,

$$\boxed{c_0 + c_1 + c_2 + \cdots c_n = 2^n}$$

1st Choice

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(ii) $c_0 - c_1 + c_2 - c_3 + \ldots + (-1)^n c_n$

Hint:

use $x = -1$

$$\boxed{(1-1)^n = c_0 - c_1 + c_2 - c_3 \ldots (-1)^n c_n}$$

$$\boxed{\cancel{c_0 - c_1 + c_2 - c_3 \ldots (-1)^n c_n = 0}}$$

sol

(ii) $c_0 + c_2$

$$\boxed{c_0 - c_1 + c_2 - c_3 \ldots (-1)^n c_n = 0}$$

(iii) $$\boxed{c_0 + c_2 + c_4 + \ldots = 2^{n-1}}$$

add (i) + (ii)

$$2(c_0 + c_2 + \ldots) = 2^n + 0$$

$$c_0 + c_2 + c_4 \ldots \ldots 2^{n-1}$$

Atleta

(iv) $$\boxed{c_1 + c_3 + c_5 + \ldots = 2^{n-1}}$$

(v) $$\boxed{3c_1 + 3^2 c_2 + 3^3 c_3 + \ldots 3^n c_n = 4^n - 1}$$

Atleta

1st Choice

Use of Differentiation

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Steps : ⇒

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

I) If the coefficient of last term of series is 'm' then divide 'm' by n.

let 'q' is the quotient and 'r' is the remainder,

that is $m = nq + r$

II) Replace 'x' by "x^q" with the general expansion and multiply both the sides by "x^r"

III) Differenciate both the sides and use suitable values of "x" (1, -1, and i)

IV) If there is product of two numbers then differenciate twice and show on.

Q. Show that $C_1 + 2C_2 + 3C_3 + 4C_4 \cdots + nC_n = n \cdot 2^{n-1}$

A. $n \overline{)\,n\,}\ 1 - q$, $\ \underline{n}$, $\ 0 - r$

$$(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots C_nx^n$$

$$n(1+x)^{n-1} = 0 + C_1 + 2C_2x + \cdots nC_nx^{n-1}$$

use $x = 1$.

$$n(n+1)^{n-1} = c_1 + 2c_2 + \cdots nc_n = n \cdot 2^{n-1}$$

Q6. Show that

$$c_0 + 2c_1 + 3c_2 + \cdots (n+1)c_n = 2^{n-1}(n+2)$$

$$(1+x)^n = c_0 + c_1x + c_2x^2 + c_3x^3 \cdots + c_nx^n$$

$$x(1+x)^n = c_0x + c_1x^2 + \cdots c_nx^{n+1}$$

$$(1+x)^n \cdot 1 + x \cdot n(1+x)^{n-1} = c_0 + 2c_1x + 3c_2x^2 + \cdots (n+1)c_nx^n$$

use $x = 1$.

$$(1+1)^n + n(1+1)^{n-1} = c_0 + 2c_1 + \cdots (n+1)c_n$$

$$2^{n-1}(2+n) = c_0 + 2c_1 + \cdots (n+1)c_n$$

Q) $2c_1 + 4c_2 + 6c_3 + \cdots + 2nc_n = 2n \cdot 2^n$

A) $(1+x)^n = c_0 + c_1x + c_2x^2 + c_3x^3 \cdots c_nx^n$

Replace $x \xrightarrow{by} x^2$

$$(1+x^2)^n = c_0 + c_1x^2 + c_2x^4 + \cdots + c_nx^{2n}$$

diff.

$$n(1+x^2)^{n-1} \cdot 2x = 2xc_1 + 4c_2x^3 + \cdots 2nc_nx^{2n-1}$$

use $x = 1$.

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$n(1+1)^{n-1} \cdot 2 = 2C_1 + 4C_2 + \ldots 2nC_n$

$\Rightarrow n \cdot 2^n = 2C_1 + 4C_2 + \ldots - 2nC_n$

Q) $C_0 + 3C_1 + 5C_2 + \ldots (2n+1)C_n = S$

Ans) $(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 - \ldots C_nx^n$

$x \to x^2$ (multiply by x^1)

$x(1+x^2)^n = C_0x + C_1x^3 + C_2x^5 + \ldots + C_nx^{2n+1}$

$(1+x^2)^n \cdot 1 + x \cdot n(1+x^2)^{n-1} \cdot 2x = C_0 + 3C_1x^2 + \ldots$
$\ldots + (2n-1)x^{2n}$

use $x = 1$

$2^n + n2^{n-1} 2 = S$

$\Rightarrow S = 2^n(n+1)$

Q) $2 \cdot 1C_2 + 3 \cdot 2C_3 + 4 \cdot 3C_4 + \ldots n(n-1)C_n = S$

A.

$n \boxed{\cancel{n(n-1)}}$

$(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$

diff:-

$n(1+x)^{n-1} = 1 \cdot C_1 + 2 \cdot C_2x^1 + 3C_3x^2 \ldots + nC_nx^{n-1}$

diff:-

$n(n-1)(1+x)^{n-2} = 2 \cdot 1C_2 + 3 \cdot 2C_3x + 4 \cdot 3x_4x^2 \ldots$
$\ldots n(n-1)C_nx^{n-2}$

use $x = 1$

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$n(1+1)^{n-1} \cdot 2 = 2c_1 + 4c_2 + \text{---} 2nc_n$

$\Rightarrow n \cdot 2^n = 2c_1 + 4c_2 + \text{---} 2nc_n$

Q) $c_0 + 3c_1 + 5c_2 + \text{---} (2n+1)c_n = S$

Ans) $(1+x)^n = c_0 + c_1x + c_2x^2 + c_3x^3 \text{---} c_nx^n$

$x \to x^2$ (multiply by x^1)

$x(1+x^2)^n = c_0x + c_1x^3 + c_2x^5 + \text{---} + c_nx^{2n+1}$

$(1+x^2)^n \cdot 1 + x \cdot n(1+x^2)^{n-1} \cdot 2x = c_0 + 3c_1x^2 + \text{---}$
$\text{---} + (2n+1)x^{2n}$

use $x = 1$

$2^n + n2^{n-1} \cdot 2 = S$

$\Rightarrow S = 2^n(n+1)$

Q) $2 \cdot 1c_2 + 3 \cdot 2c_3 + 4 \cdot 3c_4 + \text{---} n(n-1)c_n = S$

A) $\frac{n(n-1)}{n}$

$(1+x)^n = c_0 + c_1x + c_2x^2 + \text{---} + c_nx^n$

diff:-

$n(1+x)^{n-1} = 1 \cdot c_1 + 2 \cdot c_2x^1 + 3c_3x^2 \text{---} + nc_nx^{n-1}$

diff:-

$n(n-1)(1+x)^{n-2} = 2 \cdot 1c_2 + 3 \cdot 2c_3x + 4 \cdot 3x_4x^2$
$\text{---} n(n-1)c_nx^{n-2}$

use $x = 1$

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$n(n-1)(1+1)^{n-2} = S$

$\Rightarrow S = n(n-1) \cdot 2^{n-2}$

V.V.I. Q. $1^2 C_1 + 2^2 C_2 + 3^2 C_3 + \cdots + n^2 C_n$

$(1+x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 \cdots C_n x^n$

diff

$n(1+x)^{n-1} = 1 C_1 x^0 + 2 C_2 x + 3 C_3 x^2 \cdots C_n n x^{n-1}$

Now multiply both side by x;

$\Rightarrow x n (1+x)^{n-1} = C_1 x + 2 C_2 x^2 + 3 C_3 x^3 \cdots x C_n n x^{n-1}$

$\Rightarrow n x (n-1)(1+x)^{n-2} + (1+x)^{n-1} \cdot n$

$x \to 1$

Now

$n(n-1)(1+1)^{n-2} + (1+1)^{n-1} \cdot n$

$\Rightarrow n(n-1)(2)^{n-2} + (2)^{n-1} \cdot n$

Note Above example से ज्ञात है, कि According to different

$n \,\overline{)\, n^2 \,}\, (n$
$\underline{n^2}$
$\times$

Rule $q = n$ प्राप्त हुआ it means $x \to x^q$ से अर्थात् $x \to x^n$ से Replace करना है किसी भी question को lengthy बना देगा इसलिए जब भी question कोई no. के अलावा variable में आ जाएँ तब उस case में हम direct diff करके Answer पाने का प्रयास करेंगे।

1st Choice

Use of Integration

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$$\int x^n \cdot dx = \frac{x^{n+1}}{n+1} + c$$

$$(1+x)^n = C_0 + C_1 x + C_2 x^2 + \cdots C_n x^n$$

1.) Integrate both the side b/w suitable limits.

2.) If all the terms are positive than Integrate b/w the limits of '0' to '1'.

3.) If there is Alternate +ve, −ve sign than Integrate b/w limits '−1' to '0'.

4.) If there is product of two numbers in the denominator than Integrate twice and so on.

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \cdots \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$$

$$\int_0^1 (1+x)^n dx = \int_0^1 \left(C_0 + C_1 x + C_2 x^2 + C_3 x^3 \cdots C_n x^n\right)$$

Integrate

$$\left[\frac{(1+x)^{n+1}}{n+1}\right]_0^1 = \left[C_0 x + C_1 \frac{x^2}{2} + C_2 \frac{x^3}{3} + C_3 \frac{x^4}{4} \cdots C_n \frac{x^{n+1}}{n+1}\right]_0^1$$

$$\frac{(1+1)^{n+1}}{n+1} - \frac{1}{n+1} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \cdots + \frac{C_n}{n+1}$$

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7.) (x) $c_0 - \frac{c_1}{2} + \frac{c_2}{3} + \frac{c_3}{4} + \cdots = \frac{1}{n+1}$

Ans

$\Rightarrow \int_{-1}^{0} (1+x)^n \, dx = \int_{-1}^{0} (c_0 + c_1 x + c_2 x^2 + \cdots$

$\Rightarrow \left[\frac{(1+x)^{n+1}}{n+1} \right]_{-1}^{0} = \left[c_0 x + \frac{c_1 x^2}{2} \right.$

$\Rightarrow \frac{1}{n+1} - 0 =$

(xi) $\frac{c_1}{2} + \frac{c_3}{4} + \frac{c_5}{6} + \cdots =$

$\cancel{S = \int_{-1}^{1} (1+x)^n \, dx}$

(i) − (ii)

(i)+(ii)

Ex.) $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + \cdots = \frac{1}{n+1}$

Ans $\Rightarrow \int_{-1}^{0} (1+x)^n dx = \int_{-1}^{0} (C_0 + C_1 x + C_2 x^2 + \cdots$

$\Rightarrow \left[\frac{(1+x)^{n+1}}{n+1}\right]_{-1}^{0} = \left[C_0 x + \frac{C_1 x^2}{2}\right.$

$\Rightarrow \frac{1}{n+1} - 0 =$

Ex. $\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \cdots =$

$\cancel{S = \int_{-1}^{0} (1+x)^n dx}$ (i) − (ii)

$\cancel{(i) - (ii)}$

Ex:- $\frac{C_0 3^2}{1\cdot 2} + \frac{C_1 3^3}{2\cdot 3} + \frac{C_2 3^4}{3\cdot 4} + \cdots$

$$\Rightarrow \int_0^x (1+x)^n = \int_0^x (C_0 + C_1 x + C_2 x^2 \cdots C_n x^n)$$

$$\Rightarrow \left[\frac{(1+x)^{n+1}}{n+1}\right]_0^x = \left[\frac{C_0 x}{1} + \frac{C_1 x^2}{2} + \cdots \frac{C_n x^{n+1}}{n+1}\right]_0^x$$

$$\Rightarrow \frac{(1+x)^{n+1}}{n+1} - \frac{1}{n+1} = C_0 x + \frac{C_1 x^2}{2} + \frac{C_2 x^3}{3} + \cdots \cdots \frac{C_n x^{n+1}}{n+1}$$

$$\Rightarrow \int_0^3 \left(\frac{(1+x)^{n+1}}{n+1} - \frac{1}{n+1}\right) dx = \int_0^3 \left(C_0 x + \cdots \frac{C_n x^{n+1}}{n+1}\right)$$

$$\left[\frac{(1+x)^{n+2}}{(n+1)(n+2)} - \frac{x}{n+1}\right]_0^3 = \left(\frac{C_0 x^2}{2} + \frac{C_1 x^3}{2\cdot 3} + \cdots + \frac{C_n x^{n+2}}{(n+2)(n+1)}\right)$$

$$\Rightarrow \frac{4^{n+2}}{(n+1)(n+2)} - \frac{3}{n+1} - \frac{1}{(n+1)(n+2)} = S$$

^nC_r lower suff. 1st Choice

Sequence Containing multiplication of two Binomial coefficients.

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(1) If the difference of lower suffix is same

^nC_r lower suffix

Step:-

1) write:-

$$(1+x)^n = C_0 + C_1x + C_2x^2 \cdots\cdots C_{n-1}x^{n-1} + C_nx^n$$

$$(x+1)^n = C_0x^n + C_1x^{n-1} + C_2x^{n-2} \cdots C_{n-2}x^2 + C_{n-1}x + C_n$$

IInd step

$\boxed{\text{Then multiply both the eq}^n \text{ and Compare suitable power of } x \text{ on both the side}}$

पहले इनको देखकर समझ लें

Ex) $S = C_0C_1 + C_1C_2 + C_2C_3 + \cdots\cdots C_{n-1}C_n$

$S =$ coeff of x^{n-1} in eq (1) $\times$ eq (2) (RHS)

$=$ " " " " " (L.H.S)

$=$ coeff of x^{n-1} in $(1+x)^n(x+1)^n$

$=$ coeff of x^{n-1} $(1+x)^{2n}$.

$-\,-$ x^{n-1} $-\,-$ in $= \left({}^{2n}C_0 + {}^{2n}C_1x + {}^{2n}C_2x^2 \cdots\cdots {}^{2n}C_{n-1}x^{n-1} + {}^{2n}C_{2n}x^{2n}\right)$

$= {}^{2n}C_{n-1}$

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[10.] $C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = \frac{\lfloor 2n}{(\lfloor n)^2}$

$S =$ coeff of x^n in $(1+x)^{2n}$

$= {}^{2n}C_n$

[11.] $\sum_{r=0}^{n-3} C_r C_{r+3} = \frac{\lfloor 2n}{\lfloor n-3 \lfloor n+3}$

Ans. we can also write this equation as:

$C_0C_3 + C_1C_4 + C_2C_5 + C_3C_6 \dots$

1) Step 1:→

$(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + C_4x^4 + C_5x^5 \dots C_{n-1}x^{n-1} + C_nx^n$

and

$(x+1)^n = C_0x^n + C_1x^{n-1} + C_2x^{n-2} + C_3x^{n-3} + C_4x^{n-4} + C_5x^{n-5} \dots$

$\dots C_{n-2}x^2 + C_{n-1}x + C_n$

Note, Here we see that:→ when we multiply this two than we get

$\Rightarrow C_0C_3x^{n-3} + C_1C_4x^{n-4} + C_2C_5x^{n-5}$

$\Rightarrow C_0C_3x^{n-3} + C_1C_4x^{n-3} + C_2C_5x^{n-3}$

Here, we see that:— x^{n-3} is common in each term. so,

Now:— we multiply both the eq and compare suitable power of x on both the sides:—

$(1+x)^n \cdot (x+1)^n \Rightarrow (1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1x + {}^{2n}C_2x^2 \dots {}^{2n}C_{n-3}x^{n-3}$

Now Coefficient:— of x^{n-3} is ${}^{2n}C_{n-3} = \frac{\lfloor 2n}{\lfloor n-3 \lfloor n+3}$

Type :> II =>

If the sum of lower suffix is same :-

write :—

1) $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \cdots + C_n x^n$

$(1+x)^n = C_0 + C_1 x + C_2 x^2 + \cdots + C_{n-2} x^{n-2} + C_{n-1} x^{n-1} + C_n x^n$

Than multiply both the equation and compare suitable power of 'x'.

Q. $C_0 C_n + C_1 C_{n-1} + C_2 C_{n-2} + \cdots + C_n C_0$

S = Coefficient of x^n in $(1+x)^n \times (1+x)^n$

= —— —— $(1+x)^{2n}$

$\Rightarrow {}^{2n}C_n$

Ex $^mC_r \cdot {}^nC_0 + {}^mC_{r-1} \cdot {}^nC_1 + {}^mC_{r-2} \cdot {}^nC_2 + \cdots$

$\cdots + {}^mC_0 \cdot {}^nC_r = S$

$\boxed{\text{where } r < m,\ r < n}$

Av

$(1+x)^m = {}^mC_0 + {}^mC_1 x + {}^mC_2 x^2 + \cdots {}^mC_{r-1} x^{r-1} +$

${}^mC_r x^r + \cdots {}^mC_m x^m$

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \cdots + {}^nC_r x^r + \cdots$

$\cdots + {}^nC_n x^n$

$S =$ coeff of x^r in $(1+x)^m \times (1+x)^n$

$=$ " " " $(1+x)^{m+n}$

$= {}^{m+n}C_r$

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when constant are multiplied or divided with Binomial Coefficients than

*) First Differenciate and/or Integrate than multiply the equation and compare suitable Power of 'x'.

Ex:- $c_1^2 + 2c_2^2 + 3c_3^2 + \cdots + {}^nc_n^2 = S$

Ans $(1+x)^n = c_0 + c_1x + c_2x^2 + c_3x^3 \cdots c_nx^n$

~~$(x+1)^n = c_0x^n + c_1x^{n-1} + c_2x^{n-2} \cdots$~~

Diff. b/s

$n(1+x)^{n-1} = c_1 + 2c_2x + 3c_3x^2 + \cdots nc_nx^{n-1}$

$(x+1)^n = c_0x^n + c_1x^{n-1} + c_2x^{n-2} + \cdots + c_nx^0$

$S =$ coeff of x^{n-1} in $n(1+x)^{n-1} \times (x+1)^n$

$= \quad n(1+x)^{2n-1}$

$= n \cdot {}^{2n-1}C_{n-1}$

$= n \cdot \dfrac{\lfloor 2n-1}{\lfloor n-1 \lfloor n}$

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Ex:-> $C_0^2 + \frac{C_1^2}{2} + \frac{C_2^2}{3} + \frac{C_3^2}{4} + \text{------} + \frac{C_n^2}{n+1} = \frac{\lfloor 2n+1}{(\lfloor n+1)^2}$

Ans $(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 - - - - C_nx^n$

$\Rightarrow \int_0^x (1+x)^n = \int_0^x (C_0 + C_1x + C_2x^2 + C_3x^3 - - C_nx^n)$

$\Rightarrow \cancel{\left[\frac{(1+x)^{n+1}}{n+1}\right]_0^1} = \cancel{\left[C_0 + \frac{C_1x^2}{2} + \frac{C_2x^3}{3}\right.}$

$\Rightarrow \left[\frac{(1+x)^{n+1}}{n+1}\right]_0^x = \left[C_0 + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + \text{---} \frac{C_nx^{n+1}}{n+1}\right]_0^x$

$\Rightarrow \frac{(1+x)^{n+1} - 1}{n+1} = C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + \text{---} \frac{C_nx^{n+1}}{n+1}$ (1)

Now.

$(x+1)^n = C_0x^n + C_1x^{n-1} + C_2x^{n-2} + \text{---} C_nx^0$

$S =$ Coefficient of x^{n+1} in $\left(\frac{(1+x)^{n+1} - 1}{n+1}\right) \times (x+1)^n$

$= \text{--} \frac{(1+x)^{2n+1}}{n+1} - \frac{(1+x)^n}{(n+1)}$

$= \frac{1}{n+1} \cdot {}^{2n+1}C_{n+1}$

$= \frac{\lfloor 2n}{(n+1)\lfloor n \lfloor n+1} = \frac{\lfloor 2n+1}{(\lfloor n+1)^2}$

Prove: ${}^{n}C_n + {}^{n+1}C_n + {}^{n+2}C_n + \dots + {}^{n+k}C_n = {}^{n+k+1}C_{n+1}$

A:

$$\boxed{{}^{n}C_n + {}^{n}C_{n-1} = {}^{n+1}C_n}$$

$\left({}^{n+1}C_{n+1} + {}^{n+1}C_n\right) + {}^{n+2}C_n + \dots {}^{n+k}C_n$

$\left({}^{n+2}C_n + {}^{n+2}C_n\right) + {}^{n+3}C_n + \dots {}^{n+k}C_n$

$\left({}^{n+3}C_{n+1} + {}^{n+3}C_n\right) + {}^{n+4}C_n + \dots {}^{n+k}C_n$

$= {}^{n+k+1}C_{n+1}$

Alt: — coeff of x^n in

$(1+x)^n + (1+x)^{n+1} + (1+x)^{n+2} + \dots$

$\dots + (1+x)^{n+k}$

$\Rightarrow$ Coeff of x^n in $\dfrac{(1+x)^n \cdot \left[(1+x)^{k+1} - 1\right]}{1+x-1}$

$= \dfrac{(1+x)^{n+k+1} - (1+x)^n}{x}$

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Q. $C_0 + 2C_1 + 3C_2 + \text{------} + (n+1)C_n$

Ans. $\sum_{r=0}^{n} (r+1)\,{}^nC_r$

$\left(\sum(a+b) = \sum a + \sum b \right)$

$\sum r\,{}^nC_r + \sum {}^nC_r$

$n \cdot 2^{n-1} + 2^n$

Q. ~~$C_0 + 3C_1 + 5C_2 +$~~

$C_0 + 3C_1 + 5C_2 + \text{------} + (2n+1)C_n$

A. $\sum_{r=0}^{n} (2r+1)\,{}^nC_r$

$\Rightarrow 2\sum_{r=0}^{n} r \cdot {}^nC_r + \sum_{r=0}^{n} {}^nC_r$

$\Rightarrow 2 \cdot n \cdot 2^{n-1} + 2^n = 2^n(n+1)$

Q. $2 \cdot 1\, C_2 + 3 \cdot 2\, C_3 + \text{------} + n(n-1)C_n$

Ans:- $\sum_{r=2}^{n} r \cdot (r-1)\,{}^nC_r$

$\sum n \cdot (r-1)\,{}^{n-1}C_{r-1}$

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Q: $C_0 + 2C_1 + 3C_2 + \text{——} + (n+1)C_n$

Ans: $\sum_{r=0}^{n} (r+1)\, {}^nC_r$

$\left(\sum(a+b) = \sum a + \sum b \right)$

$\sum r \cdot {}^nC_r + \sum {}^nC_r$

$n \cdot 2^{n-1} + 2^n$

Q ~~$C_0 + 3C_1 + 5C_2 + \text{——}$~~

$C_0 + 3C_1 + 5C_2 + \text{——} + (2n+1)C_n$

A $\sum_{r=0}^{n} (2r+1)\, {}^nC_r$

$\Rightarrow 2\sum_{r=0}^{n} r \cdot {}^nC_r + \sum_{r=0}^{n} {}^nC_r$

$\Rightarrow 2 \cdot n \cdot 2^{n-1} + 2^n = 2^n(n+1)$

Q:- $2\cdot1\, C_2 + 3\cdot2\, C_3 + \text{——} + n(n-1)\, C_n$

Ans:- $\sum_{r=2}^{n} r\cdot(r-1)\, {}^nC_r$

$\Rightarrow \sum n \cdot (r-1)\, {}^{n-1}C_{r-1}$

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$$\sum_{r=2}^{n} n \cdot (r-1) \frac{(n-1)}{\cancel{(r-1)}} \, {}^{n-2}C_{r-2}$$

$$n(n-1) \sum_{r=2}^{n} {}^{n-2}C_{r-2} = n(n-1)\, 2^{n-2}$$

Q. $1^2 C_1 + 2^2 C_2 + 3^2 C_3 + \cdots\cdots n^2 C_n$

A. $\sum_{r=1}^{n} r^2 \, {}^{n}C_r$

$$n \sum_{r=1}^{n} r \, {}^{n-1}C_{r-1}$$

$$n \left[\sum \left[(r-1)+1\right] {}^{n-1}C_{r-1} \right.$$

$$n \left(\sum (r-1)\, {}^{n-1}C_{r-1} + \sum {}^{n-1}C_{r-1} \right]$$

$$n \left((n-1)\, 2^{n-2} + 2^{n-1} \right)$$

$$2^{n-2} \cdot 2$$

The diff :-
$n(n+1) \cdot 2^{n-2}$

Ex! $\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} + \cdots\cdots + \frac{C_n}{n+1}$

Ans $\sum_{r=0}^{n} \frac{{}^{n}C_r}{r+1}$

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$$\frac{1}{n+1} \sum_{r=0}^{n} (n+1) \frac{{}^{n}C_{r}}{r+1}$$

$$= \frac{1}{n+1} \sum_{r=0}^{n} {}^{n+1}C_{r+1}$$

$$= \frac{1}{n+1} \left[{}^{n+1}C_{1} + {}^{n+1}C_{2} + \text{——} {}^{n+1}C_{n+1} \right]$$

$$= \frac{1}{n+1} \left[{}^{n+1}C_{0} + {}^{n+1}C_{1} + \text{——} {}^{n+1}C_{n+1} - {}^{n+1}C_{0} \right]$$

$$= \left(\frac{2^{n+1} - 1}{n+1} \right) \text{Ans}$$

Dooggy [illegible]

~~Ans)~~ $\cancel{\sum_{r=0}^{n} \frac{{}^{n}C_{r}}{(r+1)(r+2)}}$

~~Ans)~~ $\cancel{\frac{1}{n+1} \sum \frac{(n+1)}{(r+1)} \frac{{}^{n}C_{r}}{(r+2)}}$

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(v) $\sum_{r=0}^{n} \frac{{}^nC_r}{(r+1)(r+2)}$

$\Rightarrow \frac{1}{n+1} \sum \frac{(n+1)}{(r+1)} {}^nC_r$

$\Rightarrow \frac{1}{n+1} \sum \frac{(n+2)\, {}^{n+1}C_{r+1}}{(n+2) \times (r+2)}$

$\Rightarrow \frac{1}{(n+1)(n+2)} \sum_{r=0}^{n} {}^{n+2}C_{r+2}$ → निश्चित check कर लें।

ध्यान रहे की निश्चित व्याख्या

$\rightarrow \frac{1}{(n+1)(n+2)} \left[2^{n+2} - \left({}^{n+2}C_0 + {}^{n+2}C_1 \right) \right]$

Double S Note:→

$\frac{1}{(n+1)(n+2)} \underbrace{\sum_{r=0}^{n} {}^{n+2}C_{r+2}}$

इसे तोड़ने पर

$\left[{}^{n+2}C_2 + {}^{n+2}C_3 + {}^{n+2}C_4 \ldots \right]$

हम यह पाते हैं।

लेकिन कोई formula उपयोग करने के लिए हमें ${}^{n+2}C_0 + {}^{n+2}C_1$ भी लाना होगा इसलिए हमें आगे में ${}^{n+2}C_0 + {}^{n+2}C_1$ लिख देते हैं और फिर इसे balance करने के लिए इतना ही minus भी कर देते हैं।

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Double Sigma

1) Independent term :⇒

(There is No relation b/w these two Sigma operator.)

(These two sigma operator are Independent to each other.)

(i)
$$\sum_{i=0}^{n}\sum_{j=0}^{n} C_i C_j$$

$$\Rightarrow \left(\sum_{i=0}^{n}\left(C_i \sum_{j=0}^{n} C_j\right)\right)$$

$$\sum_{i=0}^{n} C_i \left(2^n\right)$$

$$\Rightarrow 2^n \sum_{i=0}^{n} C_i$$

$$\Rightarrow 2^n \cdot 2^n = 4^n$$

ii)
$$\sum_{i=0}^{n}\sum_{j=0}^{n} i\cdot j\, C_i C_j$$

लिख सकते है

$$\Rightarrow i\cdot j\, C_i C_j$$

$$\Rightarrow \cancel{i}\cdot\frac{n}{\cancel{i}}\,{}^{n-1}C_{i-1}\ \cancel{j}\cdot\frac{n}{\cancel{j}}\,{}^{n-1}C_{j-1}$$

$$\Rightarrow n^2\,{}^{n-1}C_{i-1}\,{}^{n-1}C_{j-1}$$

[illegible] '0' [illegible] value -ve [illegible] '1' [illegible]

$$\sum_{i=1}^{n}\sum_{j=1}^{n} n^2\,{}^{n-1}C_{i-1}\,{}^{n-1}C_{j-1}$$

$$\Rightarrow n^2 \sum_{i=1}^{n}\left({}^{n-1}C_{i-1} \sum {}^{n-1}C_{j-1}\right)$$

$$\Rightarrow n^2 \sum_{i=1}^{n} 2^{n-1}\,{}^{n-1}C_{i-1} \quad \Rightarrow n^2\cdot 2^{n-1}\cdot 2^{n-1}$$

Ex: $\sum_{i=0}^{n} \sum_{j=0}^{n} (i+j)\, c_i c_j = S$

Ans:—

$\sum \sum i\, c_i c_j + \sum \sum j\, c_i c_j$

$\sum_{i=0}^{n} i\, c_i\, 2^n + \sum_{i=0}^{n} c_i\, n \cdot 2^{n-1}$

$\left(n \cdot 2^{n-1}\right) 2^n + n \cdot 2^{n-1} \cdot 2^n$

$= 2^1 n \cdot 2^{2n-1}$

$= n \cdot 2^{2n}$

$= n \cdot 4^n$

Alt:→

Trick

$S = \sum \sum (n-i+n-j)\, c_{n-i}\, c_{n-j}$

$S = \sum \sum \left(2n-(i+j)\right) c_i c_j$

$S = \sum \sum 2n\, c_i c_j - S$

$\cancel{2} S = \cancel{2} n\, 4^n$

$S = n\, 4^n$

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ii) Dependent term :→

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Ex) $\sum_{m=0}^{n} \sum_{p=0}^{m} {}^nC_m \, {}^mC_p = 3^n$ [There is relation b/w both sigma operator (Summation sign)]

$\left({}^nC_0 \cdot {}^0C_0\right) + {}^nC_1\left({}^1C_0 + {}^1C_1\right) + {}^nC_2\left({}^2C_0 + {}^2C_1 + {}^2C_2\right) +$

$\ldots \; {}^nC_n\left({}^nC_0 + {}^nC_1 + \ldots {}^nC_n\right)$

${}^nC_0 + {}^nC_1 2^1 + {}^nC_2 \, 2^2 + {}^nC_3 \cdot 2^3 + \ldots \; {}^nC_n 2^n$

$(1+2)^n$

$\Rightarrow 3^n$

Ex. $\sum\sum_{0 \le i < j \le n} C_i C_j = S$

$C_0\left(C_1 + C_2 + \ldots C_n\right) + C_1\left(C_2 + \ldots C_n\right) + C_2\left(C_3 + C_4 \ldots C_n\right)$

$\ldots + C_{n-1}C_n = S.$

$\left(C_0 + C_1 + \ldots C_n\right)^2 = \underbrace{C_0^2 + C_1^2 + C_2^2 + \ldots C_n^2} + 2S$

$\left(2^n\right)^2 = {}^{2n}C_n + 2S$

$S = \left(\dfrac{4^n - {}^{2n}C_n}{2}\right)$

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(vi) $\sum\sum_{0\le i<j\le n} (C_i + C_j)^2 = (n-1)^{2n}C_n + 2^{2n}$

$\Rightarrow \left[(C_0+C_1)^2 + (C_0+C_2)^2 + (C_0+C_3)^2 + \cdots + (C_0+C_n)^2\right]$

$\Rightarrow \left[(C_1+C_2)^2 + (C_1+C_3)^2 + \cdots \cdots (C_1+C_n)^2\right] +$

$(C_{n-1}+C_n)^2$

$\Rightarrow \left\{ n\,C_0^2 + n\,C_1^2 + n\,C_2^2 + \cdots\cdots n\,C_n^2 \right\} + 2\sum\sum_{0\le i<j\le n} C_i C_j$

$= n \cdot {}^{2n}C_n + \cancel{2}\left[\frac{4^n - {}^{2n}C_n}{\cancel{2}}\right]$

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Divisibility Problems:-

$\boxed{x^n + y^n \text{ is always divisible by } (x+y) \text{ If } n \in odd}$

Ex) Show that :- (i) $9^n + 7$ is divisible by 8.

ii) $6^{2n} - 35n - 1$ is divisible by 1225

iii) $3^{2n+1} + 2^{n+2}$ is divisible by 7

$(n \in N)$

Ans

(i) $(8+1)^n + 7$

$({}^nC_0 8^n + {}^nC_1 8^{n-1} + \cdots {}^nC_{n-1} 8 + {}^nC_n) + 7$

$\Rightarrow ({}^nC_0 8^n + {}^nC_1 8^{n-1} + \cdots {}^nC_{n-1} 8 + 8)$

$\Rightarrow 8[{}^nC_0 \cdot 8^{n-1} + \cdots {}^nC_{n-1} + 1]$

ii) $6^{2n} - 35n - 1$

$36^n - 35n - 1$

$\Rightarrow (35+1)^n - 35n - 1$

$\Rightarrow {}^nC_0 \cdot 35^n + {}^nC_1 \cdot 35^{n-1} + \cdots + {}^nC_{n-2}(35)^2 + \cancel{{}^nC_{n-1} 35 + 1}) - \cancel{35n} - \cancel{1}$

$(\because {}^nC_{n-1} = {}^nC_1 = n)$

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Divisibility Problems :→

$\boxed{x^n + y^n \text{ is always divisible by } (x+y) \text{ If } n \in odd}$

Q. Show that :- (i) $9^n + 7$ is divisible by 8.

ii) $6^{2n} - 35n - 1$ is divisible by, 1225

iii) $3^{2n+1} + 2^{n+2}$ is divisible by 7

$(n \in N)$

An

(i) $(8+1)^n + 7$

$({}^nC_0 8^n + {}^nC_1 8^{n-1} + \cdots {}^nC_{n-1} 8 + {}^nC_n + 7)$

$\Rightarrow ({}^nC_0 8^n + {}^nC_1 8^{n-1} + \cdots {}^nC_{n-1} 8 + 8)$

$\Rightarrow 8 [{}^nC_0 \cdot 8^{n-1} + \cdots {}^nC_{n-1} + 1]$

ii) $6^{2n} - 35n - 1$

$36^n - 35n - 1$

$\Rightarrow (35+1)^n - 35n - 1$

$\Rightarrow {}^nC_0 \cdot 35^n + {}^nC_1 \cdot 35^{n-1} + \cdots + {}^nC_{n-2}(35)^2 + \cancel{{}^nC_{n-1} 35} + \cancel{1}) - \cancel{35n} - \cancel{1}$

$(\because {}^nC_{n-1} = {}^nC_1 = n)$

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Q. Show that $9^7 + 7^9$ is divisible by 64.

Ans:— $(8+1)^7 + (8-1)^9$ (82) महत्वपूर्ण है ∵ 9^7+7^9 आया है।

$\Rightarrow {}^7C_0 \cdot 8^7 + \ldots {}^7C_5 8^2 + {}^7C_6 \cdot 8 + \cancel{1}) + {}^9C_0 8^9 - {}^9C_1 8^8 + \ldots\ldots$

${}^9C_7 \cdot 8^2 + {}^9C_8 \cdot 8 - \cancel{1})$

$\Rightarrow \left({}^7C_0 \cdot 8^7 + \ldots {}^7C_5 8^2\right) + \left({}^9C_0 \cdot 8^9 - {}^9C_1 8^8 \ldots {}^9C_7 \cdot 8^2\right)$

$+ \underbrace{{}^7C_6 \cdot 8 + {}^9C_8 \cdot 8}_{128 = 64 \times 2}$

So, Here we see that we take common '8'.

Q. Find the remainder when 5^{98} is divided by 12 and 13.

Ans. $(25)^{49} = (24+1)^{49}$

$\Rightarrow \left(\underbrace{{}^{49}C_0 \, 24^{49} + \ldots {}^{49}C_{48} \cdot 24}_{\text{12 is common so divisible by 12}} + \underbrace{{}^{49}C_{49}}_{\text{Remainder}}\right)$

Now,

$(26-1)^{49}$

$\Rightarrow \left({}^{49}C_0 \, 26^{49} - \ldots + {}^{49}C_{48} \cdot 26 - 1\right)$ Attention Negative Remainder are Not Possible

$\Rightarrow \left({}^{49}C_0 \cdot 26^{49} \ldots + {}^{49}C_{48} \cdot 26 - 13\right) + \underbrace{12}_{\text{Remainder}}$

Q. Find the last two digit of the number $(71)^{10}$

A. $(279)^5 = (280-1)^5$

$\Rightarrow ({}^5C_0(280)^5 - \cdots - {}^5C_3(280)^2 + {}^5C_4(280) - 1)$

$\Rightarrow \ldots 00 + (1400 - 1)$

$\ldots 00$

$+ 1399$

$\overline{\ldots 99}$

Q. Find the last two digit of the number (17^{52})

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Nature of Integral part of $(a+k\sqrt{b})^n$

Step :-

1) For first write $(a+k\sqrt{b})^n = I + f$ — ①

where I is the Integral part and f is the fractional part so :-

$$\boxed{0 < f < 1}$$

2.) Let $(a-k\sqrt{b})^n = f'$

where $\boxed{0 < f' < 1}$

III) So,

$$\boxed{0 < f + f' < 2}$$

and

$$\boxed{-1 < f - f' < 1}$$

IV) Add or substract equation one(1) and two (2), show, that the resultant is a "rational" number

V) use step 'I' to find the nature of Integral part of $(a+k\sqrt{b})^n$.

1st Choice

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Ex:- If $n \in N$ than show that Integral Part of $(3+\sqrt{7})^n$ is an odd number.

Ans:- Step I :->

$$(3+\sqrt{7})^n = I + f \quad \text{—(1)}$$

Step II :->

$$(3-\sqrt{7})^n = f' \quad \text{—(2)}$$

Now eq(1) + eq(2)

$$\underbrace{2\left[{}^nC_0 \cdot 3^n + {}^nC_2 \cdot 3^{n-2}(\sqrt{7})^2 - - -\right]}_{\text{Integer}} = \underbrace{I}_{\text{Intege}} + \underbrace{(f+f')}_{\text{Theis also make an Integer}}$$

But

$$0 < f + f' < 2$$

$$\Rightarrow f + f' = 1$$

Now

$$\underbrace{2(\quad)}_{\text{even}} = I + 1$$

$$= \underbrace{I}_{\text{odd.}} + 1$$

$(\because \text{odd} + \text{odd} = \text{even})$

1st Choice

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Q. If n is +ve Integer then show that Integral Part of $(7+5\sqrt{2})^{2n+1}$ is an even number.

Ans. Step I.

Step-1 ~~$(7+5\sqrt{2})^{2n+1} = I+f$ — (i)~~

$$(5\sqrt{2}+7)^{2n+1} = I+f \quad \text{— (i)}$$

Step 2:

$$(5\sqrt{2}-7)^{2n+1} = f' \quad \text{— (ii)}$$

(~~and~~ is not Possible so we do this $(7-5\sqrt{2})^{2n+1}$ no is Reverse.)

Step 3:—

Substract eq (i) − eq (ii)

$$\underbrace{\left[{}^{2n+1}C_1 (5\sqrt{2})^{2n}(7) + {}^{2n+1}C_3 (5\sqrt{2})^{2n-2} \cdot 7^3 \ldots\right]}_{\text{Integer}} = \underbrace{I}_{\text{Intege}} + \underbrace{(f-f')}_{\text{So this Also G.I.}}$$

$\Rightarrow -1 < f - f' < 1$

$\Rightarrow f - f' = 0$

{ क्योंकि (−1 और 1 के बीच एकमात्र Intege Zero(0) ही है

$$2(\text{Integer}) = I + 0$$

Q.: $N = (2+\sqrt{3})^6 \begin{cases} I \\ f \end{cases}$ show $N(1-f) = 1$

Ans. Step-1.

$$(2+\sqrt{3})^6 = I+f \quad \text{— (i)}$$

Step 2:

$$(2-\sqrt{3}) = f' \quad \text{— (ii)}$$

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Now.→

eq (I) + eq (II)

$2\left[{}^6C_0 \cdot 2^6 + {}^6C_1 \cdot 2^4 (\sqrt{3})^2 - \ldots \right] = I + (f - f')$

$\Rightarrow \quad f + f' < 2$

$0 < f + f' < 2$

$\Rightarrow \quad f + f' = 1$

$\Rightarrow \quad f' = 1 - f$ —— (v)

So,

$N \cdot (1 - f) = N \cdot f'$

$\Rightarrow (2+\sqrt{3})^6 \cdot (2-\sqrt{3})^6$

$\Rightarrow (2+\sqrt{3}) \cdot (2-\sqrt{3})^6$

$\Rightarrow (4-3)^6$

$= 1$

1st Choice

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Binomial Expansion for negative or fractional Indices.

If 'n' is negative or fraction than the expansion of binomial is possible if :—

1) It's first term is '1'.
2) It's second term is numerically less than '1'

$$\boxed{(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2} x^2 + \frac{n(n-1)(n-2)}{\lfloor 3} x^3 + \cdots \quad (|x| < 1)}$$

Q) Expand $(1-x)^{-1} = (1+(-x))^{-1}$

$((-x)$ of Put of -1 attention)

$(1-x)^{-1} = 1 + x + \frac{-1(-1-1)}{\lfloor 2}(-x)^2 + (-x)$

$\Rightarrow 1 + x + \frac{(-1)(-2)}{\lfloor 2}(-x)^2 + \frac{(-1)(-2)(-3)}{\lfloor 3}(-x)^3$

$\Rightarrow 1 + x + x^2 + x^3 + x^4 + \cdots$

Q) $(1-x)^{-2}$

A) $(1-x)^{-2} = 1 + 2x + \frac{(-2)(-3)}{\lfloor 2}(-x)^2 +$

$= 1 + 2x + 3x^2 + 4x^3 + \cdots$

Q. Find the coefficient of x^{100} in $\frac{3-5x}{(1-x)^2}$

Ans -197

Ans $(3-5x)(1-x)^{-2}$

$(3-5x)\left(1+2x+3x^2+ \cdots 100x^{99}+101x^{100} \cdots\right)$

coefficient of $x^{100} = 3\times 101 - 5\times 100$
$= -197$

Q. $S = 1-\frac{1}{2}\cdot\frac{1}{2}+\frac{1\cdot 3}{2\cdot 4}\cdot\frac{1}{2^2}-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{1}{2^3}+ \cdots$

$(1+x)^n = 1+nx+\frac{n(n-1)}{\lfloor 2}x^2 \cdots$

$nx = -\frac{1}{4}$ — (1) $\quad \frac{n(n-1)}{2}x^2 = \frac{3}{32}$ — (2)

$n = -\frac{1}{2}$

$x = \frac{1}{2}$

$\left(1+\frac{1}{2}\right)^{-1/2} = \sqrt{2/3}$

Q. If x is so small that ~~that~~ it's square and higher powers may be neglected then find the value of

$$\frac{(1-3x)^{1/2}+(1-x)^{5/3}}{(4+x)^{1/2}}$$

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Ans:— $\left\{(1-3x)^{\frac{1}{2}} + (1-x)^{\frac{5}{3}}\right\} \cdot (4+x)^{-\frac{1}{2}}$

$\Rightarrow \left(1-\frac{3}{2}x - -\right) + \left(1-\frac{5}{3}x - -\right) 4^{-1/2}\left(1+\frac{x}{4}\right)^{-1/2}$

$\Rightarrow \frac{1}{2}\left(2-\frac{3}{2}x-\frac{5}{3}x\ldots\right)\left(1-\frac{x}{8} - -\right)$

$= \frac{1}{2}\left(2-\frac{19}{6}x\right)\left(1-\frac{x}{8}\right)$

$\Rightarrow \frac{1}{2}\left(2-\frac{x}{4}-\frac{19}{6}x - -\right)$

$\Rightarrow \frac{1}{2}\left(2-\frac{41}{12}x\right]$

$= \left(1-\frac{41}{24}x\right)$ Ans

1st Choice

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Multinomial Theorem (for +ve Index)

If 'n' is a +ve Integer and $a_1, a_2, a_3, \ldots a_m \in$ C (complex no.)
Then

$$\boxed{(a_1 + a_2 + a_3 + \cdots\cdots a_m)^n = \sum \frac{\lfloor n}{\lfloor n_1 \lfloor n_2 \ldots \lfloor n_m} \cdot a_1^{n_1} \cdot a_2^{n_2} \cdots a_m^{n_m}}$$

where $n_1, n_2 \ldots n_m$ are non-negative Integer

Such that

$$\boxed{n_1 + n_2 + n_3 + \cdots n_m = n}$$

* 1.) The no. of terms in the expansion of $(a_1 + a_2 + a_3 + \cdots + a_m)^n$ is $\boxed{{}^{n+m-1}C_{m-1} \text{ or } {}^{n+m-1}C_n}$

2.) The coefficient of $a_1^{n_1} \cdot a_2^{n_2} \cdots a_m^{n_m}$ in the expansion $(a_1 + a_2 + a_3 + \cdots + a_m)^n$ is

$$\boxed{\frac{\lfloor n}{\lfloor n_1 \lfloor n_2 \ldots \lfloor n_m}}$$

1st Choice

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Q. Find the no. of terms in the expansion

$(a+b+c+d+e+f)^{10}$

Ans $^{10+6-1}C_{6-1}$

$^{15}C_5$ A.

Q. Find the coefficient of $x^2y^2z^4$ in $(ax-by+cz)^8$

Ans $\frac{\lfloor 8}{\lfloor 4 \lfloor 2 \lfloor 2} \cdot (ax)^2(-by)^2 \cdot (zc)^4$

$\frac{\lfloor 8}{\lfloor 4 \lfloor 2 \lfloor 2} a^2b^2c^4 \; (x^2y^2z^4)$

$\Rightarrow 420\, a^2b^2c^4$

Q. Find the coefficient of $a^5 b^4 c^7$ in the expansion of

$(bc+ca+ab)^8$ Ans.

$\frac{\lfloor 8}{\lfloor 7 \lfloor 4 \lfloor 5}$

Ans. $\frac{\lfloor 8}{\lfloor n_1 \lfloor n_2 \lfloor n_3} \cdot (bc)^{n_1} \cdot (ca)^{n_2} (ab)^{n_3}$

$\frac{\lfloor 8}{\lfloor n_1 \lfloor n_2 \lfloor n_3} \cdot a^{n_2+n_3} \cdot b^{n_1+n_3} c^{n_2+n_1}$

Now

$n_1 + n_3 = 5$
$n_1 + n_3 = 4$
$n_1 + n_2 = 7$

$2(n_1 + n_2 + n_3) = 16$

$\Rightarrow n_1 = 3$
$n_2 = 4$
$n_3 = 1$

How shortly solve this (Optional)

$$\Rightarrow \left(\frac{\lfloor 8}{\lfloor 3 \lfloor 4 \lfloor 1}\right) = 280 \text{ Ans}$$

Distribution of Power: ⇒

Ex. Find the Coefficient of x^4 in the expansion,

$$(1+x-2x^2)^7$$

Ans ⇒ $$\frac{\lfloor 7}{\lfloor n_1 \lfloor n_2 \lfloor n_3}(1)^{n_1}(x)^{n_2}(-2x^2)^{n_3}$$

⇒ $$\frac{\lfloor 7}{\lfloor n_1 \lfloor n_2 \lfloor n_3}(-2)^{n_3} \cdot x^{n_2+2n_3}$$

$$n_2 + 2n_3 = 4$$

$$n_1 + n_2 + n_3 = 7$$

Distribution of Power

n_1	n_2	n_3
3	4	0 → We can take zero also.
4	2	1
5	0	2

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coeff of x^4

$$\frac{\lfloor 7}{\lfloor 3\lfloor 4\lfloor 0}(-2)^0 + \frac{\lfloor 7}{\lfloor 4\lfloor 2\lfloor 1}(-2)^1 + \frac{\lfloor 7}{\lfloor 5\lfloor 0\lfloor 2}\cdot(-2)^2$$

$= -91$ Ans

(Q) Find the no. of rational terms in $(1+3^{1/3}+5^{1/7})^{10}$?

Ans. $\rightarrow$ $$\frac{\lfloor 10}{\lfloor n_1\lfloor n_2\lfloor n_3}\cdot(1)^{n_1}(3)^{\frac{n_2}{3}}(5)^{\frac{n_3}{7}}$$

$n_1+n_2+n_3=10$

Distribution of power.

n_1	n_2	n_3
10	0	0
7	3	0
4	6	0
1	9	0
3	0	7
0	3	7

$\Rightarrow 6$ (term)

Note: $\Rightarrow$ If n' is a positive integer $a_1, a_2, a_3 \dots a_m \in C$ then the coefficient of "x^r" in the expansion of $(a_1+a_2x+a_3x^2+\dots a_mx^{m-1})^n$ is

$$\boxed{\sum \frac{\lfloor n}{\lfloor n_1\lfloor n_2\lfloor n_3 \dots \lfloor n_m} a_1^{n_1}\cdot a_2^{n_2}\cdot a_3^{n_3}\dots a_m^{n_m}}$$

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where $n_1, n_2, n_3, \ldots n_m$ are all non-negative Integer Subject to the condition.

$$\left(n_1 + n_2 + n_3 + \ldots + n_m = n\right)$$

and $\left(n_2 + 2n_3 + 3n_4 + \ldots (m-1)n_m = r\right)$

3.) The greatest coefficient in the expansion of $(a_1 + a_2 + a_3 + \ldots a_m)^n$ is

$$\frac{\lfloor n}{(\lfloor q)^{m-r}\left[\lfloor q+1\right]^r}$$

where 'q' is the quotient and 'r' is the remainder when 'n' is divided by 'm'.

1st Choice

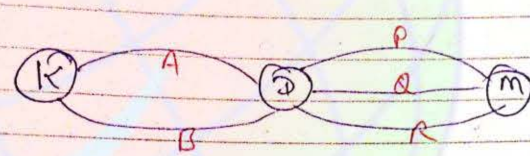
Permutation and Combination

Page No. 1

A B C
A C B
B C A
B A C
C A B
C B A

A, B, C
A B
B C
C A

1.) Law of multiplication: ⇒



2 × 3 = 6

(m×n) = 6

2.) ~~Law of mult~~ If An operation can be performed in 'm' different ways following another operation in 'n' different ways than the two operation in succession can be perform in "m×n" different ways.

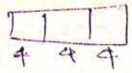
"And" ⇒ "X"

3.) Law of Addition: ⇒

If an operation can be performed in 'm' different ways and another operation in 'n' different ways than either of two operation can be performed in "m+n" different ways.

2 + 3 = 5 ways.

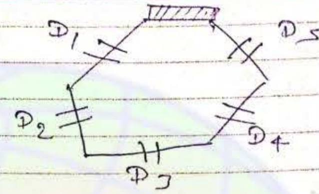
"OR" ⇒ "+"
"either"



$4 \times 3 \times 2 = 24$

$\frac{24}{24}$

Ex:)



i) The no. of ways in which an person can ~~Date~~ ente and leave the hall by a different door.

$5 \times 4 = 20$

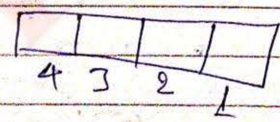
ii) He can ente by D_1 and D_2 and leave by D_3, D_4 or D_5:

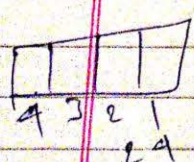
$2 \times 3 = 6$

Ex: In how many ways can three ring be worn in four ring If any no. of ring can be worn in any finger.

$4 \times 4 \times 4 = 64$ Ans.

Ex: In how many ways 4 books can be arranged in four blocks.





So, ~~total~~ $4 \times 3 \times 2 \times 1 = 24$

1st Choice

$\frac{\lfloor 10}{\lfloor 7} \Rightarrow 10 \times 9 \times 8$ (720)

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Ex) 10 person participate in race in how many ways they can occupy first three places.

method Ist:-

$^{10}P_3 = \frac{10 \times 9 \times 8 \lfloor 7}{\lfloor 7} = 720$

method IInd:-

10	9	8

$10 \times 9 \times 8 = 720$

Ex) How many three digit no can be form using the digits 1, 2, 3, 4, 5 if the:-

i) Repitition allowed.
ii) Repitition not allowed

Ans) i)

5	5	5

$5 \times 5 \times 5 = 125$

ii)

5	4	3

$5 \times 4 \times 3 = 60$

Ex) How many four digit no. can be form using 0, 1, 2, 3, 4

i)

4	4	3	2

$= 96$

ii) How many even no.

i)

4	4	3	2

$5 \times 4 \times 4 \times 3$

80×3 240

$\frac{\lfloor 10}{\lfloor 7} \Rightarrow 10\times9\times8$ (720)

Ex) 10 person participate in race in how many ways they can occupy first three place.

method Ist :-

$$^{10}P_3 = \frac{10\times9\times8\cancel{\lfloor 7}}{\cancel{\lfloor 7}} = 720$$

method IInd :-

10	9	8

$10\times9\times8 = 720$

Q) How many three digit no. can be form using the digits 1, 2, 3, 4, 5 if the :-

i) Repetition allowed.
ii) Repetition not allowed

Ans) i)

5	5	5

$5\times5\times5 = 125$

ii)

5	4	3

$5\times4\times3 = 60$

Q) How many four digit no. can be form using 0, 1, 2, 3, 4

i)

4	4	3	2

$= 96$

ii) How many even no.

4	4	3	2

0, 1, 2, 3, 4

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iii) Find the ~~no.~~ ~~no~~ number divisible by 4.

(v.) How many four digit ~~no~~ ~~no~~ numbers can be formed using 0, 1, 2, 3, 4 no. repitation is allowed.

i) ⇒ 4×4×3×2 = 96

4 4 3 2

ii) How many of these are ~~g~~ even numbers.

iii) Find the no. ~~of~~ ~~not~~ divisible by 4.

Ans)

Ans (i)

4 4 3 2

ii)

2 3 4 3 ⇒ 72 (zero is also Included)

0

1 2 3 2 ⇒ 12

72 − 12 = 60 of (4 digit number)

iii) 0, 1, 2, 3, 4

a) | | | 0 | 4 | = 6

2 3

b) | | | 4 | 0 | = 6

2 3

(अंतिम को दो अंक 4 से भाग लगाना चाहिए)

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$\boxed{\ \ |\ \ |2|0} = 6$

$\boxed{\ \ |\ \ |1|2} = 4$

$\boxed{\ \ |\ \ |?|4} = 4$

$\boxed{\ \ |\ \ |3|2} = 4$

(V.S)

Permutation ⇒

It is all possible arrangements of ~~them~~ some of all things ~~all although~~ out of 'n' different things.

(10) (*) Permutation of 'r' things out of 'n' different things is denoted by ^nP_r or $P(n,r)$

where

$$^nP_r = \frac{\lfloor n}{\lfloor n-r}$$

$$= \frac{\lfloor n}{\lfloor r \lfloor n-r} \times \lfloor r$$

$$= {}^nC_r \times \lfloor r$$

~~Reflection~~ So,

$$\boxed{^nP_r = \frac{\lfloor n}{\lfloor n-r} = {}^nC_r \lfloor r}$$

1st Choice

(^nC_r is
Selection of 'r' different things out of n different things.)

($\lfloor r$ is Arrangement of r different things)

$$\boxed{^nP_r = n\left(^{n-1}P_{r-1}\right) = n(n-1)\left(^{n-2}P_{r-2}\right) = n(n-1)(n-2)\left(^{n-3}P_{r-3}\right) = \cdots}$$

⇒ Properties :→

(i) $^nP_n = \lfloor n = {}^nP_{n-1}$

(ii) $^nP_1 = n$

(iii) $^nP_r = n \cdot {}^{n-1}P_{r-1}$

(iv) $\lfloor 0 = \lfloor 1 = 1$

(v) $\lfloor n = n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1$

(vi) $\lfloor 2n = 2^n \cdot \lfloor n \left[1 \cdot 3 \cdot 5 \cdot 7 \ldots\ldots (2n-1)\right]$

(vii) $^nP_0 = 1$

Proof (iii)

L.H.S :- $\dfrac{\lfloor n}{\lfloor n-r} = \dfrac{n \cdot \lfloor n-1}{\lfloor n-r}$

$$\Rightarrow \frac{n \lfloor n-1}{\lfloor (n-1)-(r-1)} = n \cdot {}^{n-1}P_{r-1}$$

Ex:- Prove that $\boxed{^nP_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}}$

R.H.S :-

$$\Rightarrow \frac{\lfloor n-1}{\lfloor n-1-r} + \frac{r \lfloor n-1}{\lfloor n-r}$$

$$\Rightarrow \frac{\lfloor n-1}{\lfloor n-r-1} + \frac{r \cdot \lfloor n-1}{n-r \lfloor n-r-1}$$

$$\Rightarrow \frac{\lfloor n-1}{\lfloor n-r-1}\left(1+\frac{r}{n-r}\right)$$

$$\Rightarrow \frac{\lfloor n-1}{\lfloor n-r-1} \times \left(\frac{n}{n-r}\right)$$

$$\Rightarrow \frac{n\lfloor n-1}{(n-r)\lfloor n-r-1} = \frac{\lfloor n}{\lfloor n-r} = {}^{n}P_{r}$$

Ex $\boxed{1\cdot{}^{1}P_1 + 2\cdot{}^{2}P_2 + 3\cdot{}^{3}P_3 \ldots n\cdot{}^{n}P_n = {}^{n+1}P_{n+1} - 1}$

$$\Rightarrow 1\lfloor 1 + 2\lfloor 2 + 3\lfloor 3 + \ldots n\lfloor n$$

$$(2-1)\lfloor 1 + (3-1)\lfloor 2 + (4-1)\lfloor 3 + \ldots ((n+1)-1)\lfloor n$$

$$(2\lfloor 1 - \lfloor 1) + (3\lfloor 2 - \lfloor 2) + (4\lfloor 3 - \lfloor 3) + \ldots (n+1\lfloor n - \lfloor n)$$

$$\Rightarrow (\lfloor 2 - \lfloor 1) + (\cancel{\lfloor 3} - \cancel{\lfloor 2}) + (\cancel{\lfloor 4} - \cancel{\lfloor 3}) + \ldots (\lfloor n+1 - \cancel{\lfloor n})$$

$$\lfloor n+1 - \lfloor 1 = {}^{n+1}P_{n+1} - 1$$

Q.) If $^{56}P_{r+6} : {}^{54}P_{r+3} = 30800$ find 'r'

$$\frac{\lfloor 56}{\lfloor 50-r} \times \frac{\lfloor 51-r}{\lfloor 54} = \frac{30800}{1}$$

$$\frac{56 \cdot 55 \lfloor 54}{\lfloor 50-r} \times \frac{(51-r)\lfloor 50-r}{\lfloor 54} = 30800$$

Q6 How many words can be formed using all letter of the word. GREAT. such that

i) vowels are together
ii) vowels are separated.

प्रकार से

A.) i) GR(EA)T

→ 1st

$~~\lfloor 4 = 4\times3\times2\times1 = 24~~$

$\lfloor 4 \times \lfloor 2 = 48$

ii) GR(EA)T → जहाँ साथ है घटा दो

$\lfloor 5 - \lfloor 4 \cdot \lfloor 2 = 72$

1st Choice

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Q6 i) How many words can be formed using letter of word MIRACLE
ii) How many of these words begins and ends with vowel.

A i) $\lfloor 7$ =

ii) (vowels A, I, E)

3 5 4 3 2 1 2 = 720

IInd method ⇒

$^3P_2 \cdot {}^5P_5 = 720$ A

Alt ⇒

Q11 ⇒ How many ways 10 persons can be arranged on a rectangular table If three(3) person always sit on a particular side and 2 persons opposite to them.

A

i) $^5P_3 \cdot {}^5P_2 \lfloor 5$

or

ii) $\left({}^5C_3 \lfloor 3\right)\left({}^5C_2 \cdot \lfloor 2\right) \lfloor 5$

Q) How many 4 letter words can be formed from the letters of word ARTICLE. Each containing two vowels and two consonants.

A) A, I, E $\quad$ $R\ T\ C\ L$

$^3C_2$ $\quad$ $^4C_2$

(पहले selection फिर arrange)

80

$^3C_2\ ^4C_2\ \lfloor 4 = 288$ Ans.

or $^3P_2\ ^4P_3\ \lfloor 2 = 288$

(Q.) No. of words which can be formed from the letters of word MACHINE

If vowels occupy i) odd position.
ii) Even position

Ans i) MACHINE

A, I, E $\quad$ M, C, H, N

$\left(^4C_3\ \lfloor 3\right)\ \lfloor 4$

arrangement of vowel.

selection of 3 odd position

ii) $\lfloor 3 \times \lfloor 4$

Gap method :->

Page-3 पे देखें

Sol.-> Example sheet - 1 (Permutation and combination) Q. No => 3)

Ans.->

i) $\lfloor 10 = 8.$ (No. of boys and girls can be arranged.) GBB

X - X - X - X - X - X

ii)

iii) a) $\overline{B}$ X - X - X - X - X

$\lfloor 5 . \lfloor 5$

b) $\overline{G}$ X - X - X - X - X

$\lfloor 5 \times \lfloor 5$

so, Addition to 'a' and 'b'

$\Rightarrow 2(\lfloor 5)^2$ 8.

iv) B . . . R 5

$\underbrace{{}^5C_3}_{\text{Girls select}} \; \underbrace{\lfloor 3}_{\text{Girls arrangement}} \; \underbrace{\lfloor 2}_{\text{boys arrangement}} \quad \lfloor 6$

पाँच के Group में से 3 girls को select किये फिर उसको arrange किये। तथा 2 boy को भी arrangement किये। तथा बचे हुए '5' और '5' के group को '1' मान कर total "6" का arrangement किये।

v)

$\left({}^{5}C_{2}\, \underline{|2}\right) \underline{|8}$

combinat / selection. arrangement of boys

पाँच में से दो को select किये, फिर उनको arrangement किये तथा किया की 8' को भी arrangement कर दिये।

vi)

A > B > C

$\left({}^{10}C_{7}\, \underline{|7}\right)$

may be any place but order is fixed

A _ _ _ _ B _ _ C _ _

$\left({}^{10}C_{7}\, \underline{|7}\right)(1)$ → fixed order होने के कारण अब एक ही तरह से बैठेंगी। ↑order fixed. (Not arranged)

vii) यह आपस में दो प्रकार से arrange होंगे।

A > B > C
B > A > C

$\left({}^{10}C_{7}\, \underline{|7}\right) \underline{|2}$

viii)

$\left({}^{10}C_{5}\, \underline{|5}\right)(1)$ → fixed order होने के कारण आपस में arrangement नहीं किया जा सकता।

→ selection of boys → arrangement of boys

10) (1) → fixed ↑ order में height होने के कारण उन्हें आपस में arrange नहीं किया जा सकता।

v)

$$\left({}^{5}C_{2}\ \lfloor 2\right)\ \lfloor 8$$

combination / selection — arrangement of boys

पाँच में से दो को select किये, फिर उसको arrangement किया, फिर बाकी 8 को भी arrangement कर दिये।

vi)

$A > B > C$ — may be any place but order is fixed

$$\left({}^{10}C_{7}\ \lfloor 7\right)$$

A _ _ _ _ B _ _ C _ _ _

$$\left({}^{10}C_{7}\ \lfloor 7\right)(1)$$

→ Fixed order होने के कारण सब एक ही तरह से बैठेंगे। order fixed (Not arrange)

vii) यह आपस में दो प्रकार से arrange होंगे।

$A > B > C$

$B > A > C$

$$\left({}^{10}C_{7}\ \lfloor 7\right)\ \lfloor 2$$

viii)

$$\left({}^{10}C_{5}\ \lfloor 5\right)(1)$$

→ fixed order होने के कारण आपस में arrangement नहीं किया जा सकता।

→ selection of boys → arrangement of boys

10) (A) → fixed order में height होने के कारण उन्हें आपस में arrange नहीं किया जा सकता।

1st Choice

नियंत्रित / प्रतिबंधित (limited or small in size or amount)

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Restricted Permutations:

The no. of permutation of 'r' different things out of 'n' different things when :–

i) 'p' particular things are always Included :–

$$\boxed{{}^{n-p}C_{r-p} \, \lfloor r}$$

nCr के द्वारा सिर्फ Included / Excluded तय करेगा / नहीं।

10 में से 8 fixed वाली है। बता दी है।

10 → 8 → 2-special

8 में से सिर्फ 6 को ही select करना है क्योंकि दो को fixed selection है।

$({}^{8}C_{6} \, \lfloor 8)$ लेकिन arrangement total का करेंगे।

ii) when 'q' particular things are always excluded

$$\boxed{{}^{n-q}C_{r} \, \lfloor r}$$

दो को बाहर निकाल दिया।

10 → 8

10 → 6

8 में से 6 की सिर्फ select कर arrangement करेंगे क्योंकि दो को पहले ही बाहर निकाला गया है।

${}^{8}C_{6} \, \lfloor 6$ → जिसे निकाला गया उसे arrange नहीं कर सकते।

iii) when 'p' particular things are always Included and 'q' things are always excluded

$$\boxed{{}^{n-p-q}C_{r-p} \, \lfloor r}$$

iv) when 'n' particular things occupy 'm' particular places.

$$\boxed{{}^{n-m}C_{r-m} \, \lfloor r-m}$$

1st Choice

Q) How many 4 letter words can be form using the letter of the word DAUGHTER
i) If each word is to Include G.

Ans $\left({}^{7}C_{3}\ \lfloor 4\right)$ Ans.

$8 \rightarrow 4$

${}^{8}C_{4}\ \lfloor 4$

8 में से G को रखने के लिए बचे 7 में से तीन का select Again 4 are Arrangement

Dictionary Problem :>

Q) If word's formed using all letter of word are RAVI are arranged alphabetically find the position of RAVI

A

A I R V

A → 6
I → 6
RA → 2
RAI → 1
RAVI → 1

A, I, R, V

i) | A | | | | $\lfloor 3 = 6$

ii) | I | | | | $= 6$

iii) | R | A | I | V | $\Rightarrow 1$

iv) | R | A | V | I | $\rightarrow 1$ Ans 14 Rank

24
24
2
2
52

Q.) ROHIT

H I O R T

H → 24
I → 24
O → 24
RH → 6

RI → 6
ROHIT → 1

24
24
24
12
84

Teche! →

R O H I T

H I O R T

i) H I O | | | |

4! + 4! + 4! = 72

ii) R | H | | |

$\underbrace{\quad}_{3!}$ = 6

iii) R | I | | |

$\underbrace{\quad}_{3!}$ = 6

iv) R | O | H | I | T ⇒ 1

85th Rank.

Ex TOUGH

Ans GHOTU

G → 24
H → 24
O → 24
TG → 6
TH → 6

TOG → 2
TOH → 2
TOUGH → 1

1
24
24
24
14
89

A Teacher

G H O T U

i) | G,H | | | | |

$\lfloor 4 + \lfloor 4 + \lfloor 4 = 72$

ii) | T | G,H | | | |

$\lfloor 3 + \lfloor 3 \rightarrow 6 + 6 = 12$

iii) | T | O | G,H | | |

$\lfloor 2 + \lfloor 2 = 2 + 2 = 4$

iv) | T | O | U | G | H |

$\rightarrow$ 89th word

Q) If All the letter of words NUMBER are arranged in Dictionary then find 260th word.

Q a) How many four letter words can be formed from the letter of word HISTORY.

b) How many of them contain only consonant.

c) How many of them begin and end with a consonant.

A.) NUMBER ! →

MNUBERN

B E M N R U

$\boxed{B}\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{R}$

$\lfloor 5 + \lfloor 5 = 120 + 120$
$= 240$

$\boxed{M}\boxed{B}\boxed{E}\boxed{\ }\boxed{\ }\boxed{\ }$

$\lfloor 3 + \lfloor 3 + \lfloor 3 = 6 + 6 + 6$
$= 18$

$\boxed{M}\boxed{B}\boxed{U}\boxed{E}\boxed{R}\boxed{N}$ — 1

$\boxed{M}\boxed{B}\boxed{U}\boxed{E}\boxed{R}\boxed{N}$

Q →

i) $^{7}C_4 \lfloor 4$

ii) $^{5}C_4 \times \lfloor 4 = 120$

7 में से 4 lette लेगे।

iii) $\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }$

$(^{5}C_2 \lfloor 2)$ $(^{5}C_2 \lfloor 2)$

7 lette के word में 2 का selection हो गया अब 5 lette (consonant + vowel) बचा उनके लिए दो बार / का selection + arrangement

5 consonant में 2 का selection फिर arrangement किये

7/12/2018

1st Choice

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Permutation of Alike object

(1) The no. of permutations of 'n' things when 'p' things are alike of one kind 'q' things are atleast alike of 2nd kind and rest are different.

$$\boxed{\frac{\lfloor n}{\lfloor p \lfloor q}}$$

A A B
A B A
B A A

$$\frac{\lfloor 3}{\lfloor 2}$$

Q How many words can be formed using all the letter of word BANANA

S $\frac{\lfloor 6}{\lfloor 3 \lfloor 2} = 60$ Ans.

$$\frac{\lfloor 6}{\lfloor 3 \lfloor 2 \lfloor 1}$$

$\rightarrow 60$

~~How many words can be~~

In the word "MISSISSIPPI"

i) How many words can be formed

$$\frac{\lfloor 11}{\lfloor 3}$$

1st Choice

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ii) In how many word all 'S' are together.
iii) In how many words NO two "S" are together

A i) $\dfrac{\lfloor 11}{\lfloor 4 \lfloor 4 \lfloor 2} =$

ii) SSSS MIIII PP

$\left(\dfrac{\lfloor 8}{\lfloor 4 \lfloor 2} \cdot \dfrac{\lfloor 4}{\lfloor 4} \right)$ → 4 'S' का आपस में arrangement, → लेकिन चारो 'S' same है इसलिए $\lfloor 4$ का भाग।

iii) Gap-method :→

x — x — x — x — x — x — x — x

$\left(\left(\dfrac{\lfloor 7}{\lfloor 4 \lfloor 2} \right) {}^{8}C_{4} \dfrac{\lfloor 4}{\lfloor 4} \right)$ → 4 are simila

→ 8 से 4 की selection

Q. 21 white and 19 black ball are arranged in a line find the no. arrangement If black balls are seperated. (Assume all balls are Identical.)

A. Gaps ⇒ 22 → white ball make gap. (By gap method.)

${}^{22}C_{19} \times \dfrac{\lfloor 19}{\lfloor 19}$ → all balls are same as (Identical)

→ 1

8/12/

14.) Exemplar Shelf (1)

ex.) SURITI

IIRSTU

i) | I | | | | | | → $\lfloor 5 = 120$

ii) | R | | | | | | → $\dfrac{\lfloor 5}{\lfloor 2} = 60$

iii) | S | I | | | | | → $\lfloor 4 = 24$

iv) | S | R | | | | | → $\dfrac{\lfloor 4}{\lfloor 2} = 12$

v) | S | T | | | | | → $\dfrac{\lfloor 4}{\lfloor 2} = 12$ → student

vi) | S | U | I | | | | → 6

vii) | S | U | R | I | I | T | → 1

viii) | S | U | R | I | T | I | → 236th

8/12/2012

1st Choice

Circular Permutation

Arrangement around a circular table:-

The no. of circular permutations of 'n' different things taken all at a time is

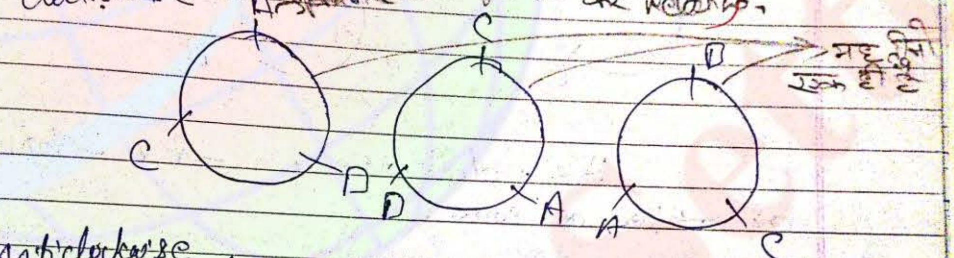
$\boxed{|\underline{n-1}}$

(If clockwise and Anticlockwise are taken as different.)

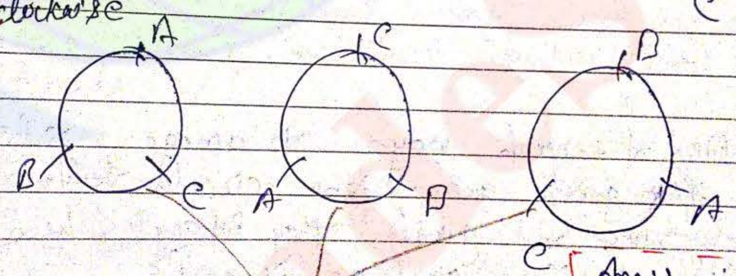
'1' इसलिए घटा रहा है क्योंकि किसी एक को fix मानकर ही हम दूसरे की तुलना करते हैं।

clockwise

किसी एक को fix मान कर Relative.



Anticlockwise



If seats are numbered in circular Permutation then it is case of linear arrangement.

तीनों एक ही

Ans is $|\underline{3-1} = 2$ A

Note:⇒

1) No. of arrangement of 'n' distinct object in a circle is equal "$|\underline{n-1}$".

II) In "$|\underline{n-1}$" is clockwise and is Anticlockwise counted.

If clockwise and Anticlockwise are not to be distinguishing then Answer will be equal to $\boxed{\frac{|\underline{n-1}}{2}}$

8/12/2012

1st Choice

Circular Permutation

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* Arrangement around a circular table:–

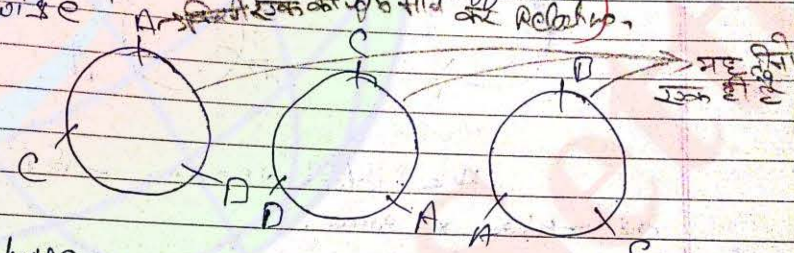
The no. of circular permutations of 'n' different things taken all at a time is

$\boxed{|\underline{n-1}}$

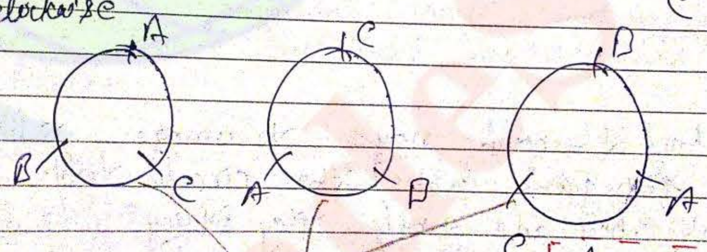
'1' इसलिए घटा रहा है क्योंकि किसी एक को fix मानकर ही हम दूसरे की तुलना करते हैं।

(If clockwise and Anticlockwise are taken as different.)

clockwise — A को fix मान कर Relation.



Anticlockwise



* If seats are numbered in Circular Permutation then it is case of linear arrangement.

Ans:- $|\underline{3-1} = 2$ Ans

Note:⇒

1) No. of arrangement of 'N' distinct object on a circle is equal "$|\underline{n-1}$".

11) In "$|\underline{n-1}$" in clockwise and in Anticlockwise counted.

If clockwise and Anticlockwise are not to be distinguishing then Answer will be equal to $\boxed{\frac{|\underline{n-1}}{2}}$

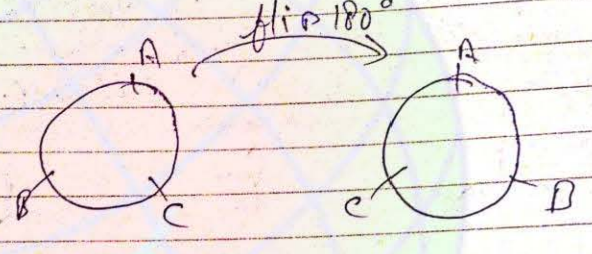
1st Choice

Date / /

Note:>

ii) Arrangement of beads around a circular Necklace.

The circular permutation of 'n' beads taken all at a time to form a necklace is $\boxed{\frac{1}{2}\lfloor n-1}$.



Here clockwise and And Anticlockwise are taken as same.

Q) Find the no. of ways to arrange 5 boys and four girls around the circular table
i) boys and girls they sit any where.
ii) All the girls are together. $\lfloor 5$
iii) All the girls are together and boys are also together
iv) No two girls are together.

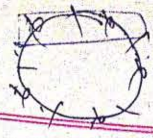
A. i) $\lfloor 9-1 = \lfloor 8$ $\boxed{\lfloor 6 \cdot \lfloor 4}$

ii) $\lfloor 6 \lfloor 4 =$

iii) (4G) (5B) $\lfloor 5 \lfloor 4$

$\lfloor 6-1 \times \lfloor 4$

$\lfloor 5 \times \lfloor 4$



$\lfloor 4 \lfloor 4$

iii) 4 G

5 B

$^{6}C_4$

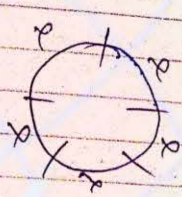
$\lfloor 2-1 \; \lfloor 5 \; \lfloor 4$

$\Rightarrow \lfloor 5 \; \lfloor 4$

$^{5}C_4 \lfloor$

$\cancel{\lfloor 7 \lfloor 2}$

iv)



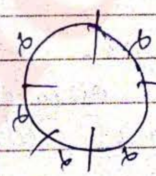
$\underbrace{\lfloor 5-1}_{\text{arrangement of boy}} \; \underbrace{^{5}C_4}_{\text{selection of girl}} \; \lfloor 4$

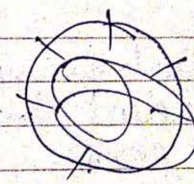
Q) In How many different ways 5 boys and 5 girls form a circle, such that boys and girls are alternate.

A) $\cancel{\lfloor 5 \lfloor 4}$

$\lfloor 5-1 \; \lfloor 5$

$\lfloor 4 \; \lfloor 5$ Ans.





Ex) Find no. of circular permutation of 'n' persons If two specified persons are Never together.

S.

$n-2$

2

$\lfloor (n-1)-1 \times \lfloor 2$

So) $\underbrace{\lfloor n-1}_{\text{total}} - \underbrace{\lfloor 2 \lfloor n-2}_{\text{together}}$

$80+1$

$80+1$

180

$\lfloor n \lfloor 2$

$\cancel{\lfloor n-3 / \lfloor 2}$

1st Choice

Q. out of 10 flowers of different colours how many different garlands can be made if each garland consist of six flower.

A. $\dfrac{{}^{10}C_6 \, \lfloor 6-1}{2}$ Ans

Garland

Q. Consider 21 pearls in a neckles in how many ways the person can be replaced in this neckles such that 3 specific pearls remain together.

A. $\dfrac{\lfloor 19-1 \, \lfloor 3}{2}$

(XXX) 19

$\dfrac{\lfloor 18 \, \lfloor 3}{2}$

Q. Find the no. of arrangement of 20 persons around the circular table if 2 particular persons are always neighbor to the host.

A. 17

(·H·)

$\lfloor 21 \, \lfloor 2$

$\lfloor 18-1 \times \lfloor 2$

$\Rightarrow \lfloor 17 \, \lfloor 2$

1st Choice

Q. Find the no. of ways in which 5 boys and 5 girls can be sited on a circle circle are alternatively. If a particula boys and a particula girls are none adjacent to each other.

Ans.

Ans 1728

Note! →

I) The no. of circula permutation of 'n' different things taking 'r' at a time $\boxed{\frac{^nP_r}{r}}$ when clockwise and Anticlockwise orders are treated as different.

II) The no. of circular permutation of 'n' different things taking altogether $\boxed{\frac{^nP_n}{n} = \lfloor n-1}$ when clockwise and Anti-clockwise orders are treated as different.

1st Choice

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Combination

(^nC_r)

No. of ways of selecting 'r' things out of 'n' different things.

$$^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}$$

Note — In a "combination" only selection is made, whereas in a "permutation" not only a selection is made but also an arrangement in a definite order is considered.

1.) The No. of combination of 'n' different things taking some or all at a time is

$$\boxed{^nC_1 + {^nC_2} + {^nC_3} + {^nC_4} - - - - {^nC_n} = 2^n - 1}$$

ii) No. of ways of selecting 'r' consecutive things from 'n' different things is

a) when they are arranged in a "row" is

$$\boxed{n - r + 1}$$

b) when they are arranged in a "circle" is

$$\boxed{\begin{array}{l} n, \text{ If } r \in \{1, 2, - - - - n-1\} \\ 1, \text{ If } r = n \end{array}}$$

Note :-> In ... of s... whereas ... orderi...

A, B a... Combina...

eg:- a b c $(3-2+1) = 2$ (when two consecutive)

a b c d $(4-2+1) = 3$ (")

1st Choice

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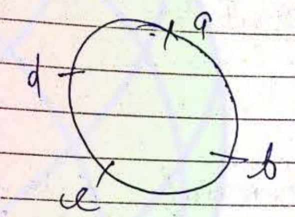
abcd $\quad (4-3+1) = 2$ (when 3 are consecutive

Eg:→



$r = 2$

ab, bc, cd, da

$r = 3$

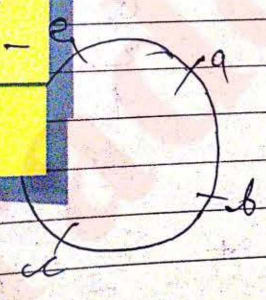
abc, bcd, cda, dab $\quad r \in (1, 2, \ldots (n-1))$

$r = 4$

$(r = n)$

Note:→ In a combination, the ordering of selected objects is Immaterial whereas in a Permutation, the ordering is essential. For eg:—

A, B and B, A are same as combination but different as Permutation.



$r = 2$

ab, bc, cd, de, ea

Eg In how many ways a team of 3 boys and 4 girls can be ~~fo~~ formed from 5 boys and 6 girls.

A. $^5C_3 \times {}^6C_4$

selection of boys — selection of girls

Eg i) How many committee of 6 persons can be formed from 5 boys and 5 girls such that boys are in majority.

Ans 55

ii) How of committee are possible if at least 2 boys are always there

A. i)

B	G
4	2
5	1

$$\left({}^5C_4 \cdot {}^5C_2 + {}^5C_5\,{}^5C_1\right) = 55 \text{ Ans.}$$

ii)

B	G
2	4
3	3
4	2
5	1

1st Choice

Now $\left({}^5C_2 \cdot {}^5C_4 + {}^5C_3\,{}^5C_3 + {}^5C_4\,{}^5C_2 + {}^5C_5\,{}^5C_1 \right)$

$\Rightarrow 205$ A.

Ex→ In how many ways a team of 4 boys and ~~three~~ 3 girls can be selected out of 5 boys and 3 girls If a particular boy and a particular girls ~~which~~ refuse to work together.

A. ~~इनकार करना~~

5 B   5 G

$\underbrace{{}^5C_4\,{}^5C_3}_{\text{total}} - \underbrace{{}^4C_3 \cdot {}^4C_2}_{B \cdot G \rightarrow A,\ A \text{ दोनों पहले आ गए हैं}}$

alt!→

$\underbrace{{}^4C_3 \cdot {}^4C_3}_{B\,G} \quad \underbrace{{}^4C_4 \cdot {}^4C_3}_{B\,G} \quad \underbrace{{}^4C_4 \cdot {}^4C_3}_{B\,G}$

Example sheet ⇒ 1

Q1→ (i) ${}^{19}C_{11}$

ii) ${}^{19}C_{10}$

iii) $\underbrace{{}^{21}C_{11}}_{\text{total}} - \underbrace{{}^{18}C_{19}}_{S,D}$