Eqⁿ of tangent in parameter form→

Ex 1
Q-25

① If curve eqⁿ in parameter form :-

"Curve = Point"

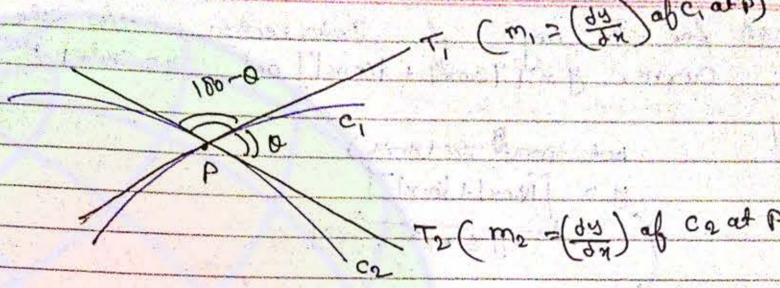
$$\frac{dy}{dx} = \frac{\phi \cdot \sin\theta}{\phi\,(1+\cos\theta)} = 1$$

$$= \sin\theta = 1+\cos\theta$$

$$\theta = \pi/2$$

$$\therefore \text{Point } \left(a\left(\frac{\pi}{2}+1\right), 0\right)$$

Angle of Intersection



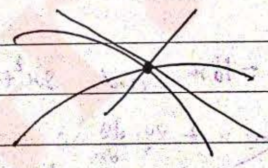
The angle of Intersection of two curves is the angle b/w their tangents (OR Normal) at their Point of Intersection.)

$$\boxed{\tan\theta = \left|\frac{m_1 - m_2}{1+m_1 m_2}\right|}$$

Always Acute Angle.)

$$\boxed{\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1+m_1 m_2}\right|}$$

* If $m_1 \to \infty$



$$\boxed{\theta = \tan^{-1}\left|\frac{1}{m_2}\right|}$$

* If $m_2 \to \infty$

∞

$$\boxed{\theta = \tan^{-1}\left|\frac{1}{m_1}\right|}$$

Q1) find the angle of intersection b/w the curves $y = [|\sin x| + |\cos x|]$ and $y = x^2+1$)

sol/ ~~let point be (x_1, y_1)~~

$y = \underbrace{[|\sin x| + |\cos x|]}_{[1 \leq (k) \leq \sqrt{2}]}$

How ?

$k = |\sin x| + |\cos x|$

$k^2 = \sin^2 x + \cos^2 x + 2|\sin x||\cos x|$

$k^2 = 1 + \underbrace{|\sin 2x|}_{0 \text{ to } 1}$

$\downarrow$

$1 \leq k^2 \leq 2$

$\underbrace{\sqrt{2} \leq k \leq -1}_{\text{Negative}}$ $\cup$ $\underbrace{1 \leq k \leq \sqrt{2}}_{\text{Positive}}$

(✗) (✓)

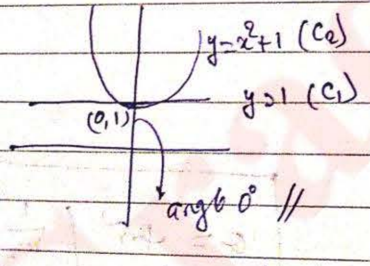
C_1: $y = 1$ → $\boxed{m_1 = 0}$

C_2: $y = x^2+1$ → $\boxed{m_2 = 2x = 0}$

] angle = 0°

So ∴ $m_1 = m_2$

$\theta = 0°$



Q2) Find the angle b/w $y = 16x$ and $2x^2+y^2 = 4$?

sol/ ~~$\frac{dy}{dx} = 16$~~ | ~~$2 \times 2x + 2y \cdot \frac{dy}{dx} = 0$~~

~~$2y \cdot x$~~ $= 2 \cdot \frac{dy}{dx}$

$y = 16x$, $2x^2 + y^2 = 4$

$C_1 : 2y \cdot y' = 16$; $C_2 : 4x + 2y y' = 0$

$y' = \frac{8}{y} = m_1$

As $m_1 . m_2 = -1$

$= \frac{8}{y} \cdot \left(\frac{-2x}{y}\right) = \frac{-16x}{y^2} = -1$

$m_1 \quad m_2$

$\theta = 90^\circ$ //

(Q.I) find the angle of intersection

$x^3 - 3xy^2 + 2 = 0$ and $-y^3 + 3x^2y - 2 = 0$

$\downarrow$

$C_1 \ \frac{dy}{dx} = \left(\frac{3x^2 - 3y^2}{-6xy}\right)$; $C_2 = -\left(\frac{2 \cdot 3yx}{-3y^2 + 3x^2}\right)$

$m_1 \qquad m_2$

as $m_1 \cdot m_2 = -1$

$\theta = 90^\circ$ //

eg 4) If $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ and $y^2 = 16x$ intersect orthogonally

then a^2 is ____ ?

sol^n C_1 $\frac{2x}{a^2} - \frac{2yy'}{4} = 0$

$y' = \frac{-x \cdot 4}{a^2 y} = m_1$

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C_2) $3y^2 . y' = 16$

$\therefore y' = \frac{16}{3y^2} = m_2$

As $\theta = 90°$

$\therefore m_1 m_2 = -1$

$\frac{\cancel{+4x}}{a^2 y} \cdot \frac{16}{3y^2} = 1$

$a^2 = \frac{4}{3}$

eg ⑤ findangle b/w the curve $y^2 = ax$, $x^2 = 4y$?

sol given

$y^2 = ax$

$2y . \frac{dy}{dx} = a$

$\frac{dy}{dx} = \frac{a}{2y} = m_1$

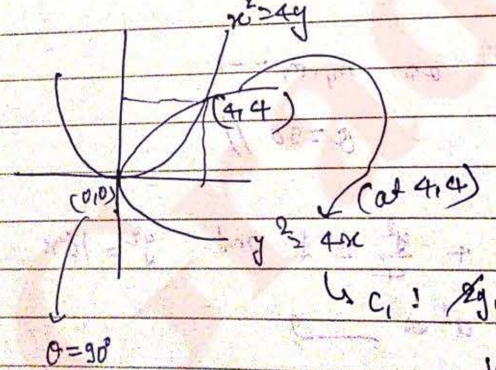
$x^2 = 4y$

$\cancel{2}x = \overset{2}{\cancel{4}} . \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{x}{2} = m_2$

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} \Rightarrow \frac{\frac{a}{2y} - \frac{x}{2}}{\quad}$

Touche



$\hookrightarrow C_1 : \cancel{2}y . y' = \cancel{4}^2$

$y' = \frac{2}{y} = \frac{1}{2} = m_1$ —(i)

$C_2 : 2x = 4 . y'$

$y' = \frac{x}{2} = 2 = m_2$ —(ii)

Board question + IIT

Theorem→

Proof:

$$\tan\theta = \left|\frac{m_1 - m_2}{1+m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{\frac{1}{2} - 2}{1+\frac{1}{2}\cdot 2}\right|$$

$$= \left|\frac{-3}{4}\right| = \frac{3}{4}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) \text{ Ans}$$

Board question + iit

Formula of orthogonal curve →

Theorem → If $a_1x^2+b_1y^2=1$ and $a_2x^2+b_2y^2=1$ intersect ~~orthog~~ orthogonally then,

$$\boxed{\frac{1}{a_1} - \frac{1}{b_1} = \frac{1}{a_2} - \frac{1}{b_2}}$$

Proof → C_1 : $2a_1x + 2b_1y\cdot y' = 0$

$$y' = \frac{-a_1x}{b_1y} = m_1$$

C_2, $m_2 = \dfrac{-a_2x}{b_2y}$

As $\theta = 90^\circ$,

$\Rightarrow m_1 m_2 = -1$

$\Rightarrow \dfrac{a_1a_2x^2}{b_1b_2y^2} = -1$

$$\frac{a_1 a_2}{b_1 b_2} = \frac{-y^2}{x^2} \quad \text{—(i)}$$

$\therefore$ C_1 and C_2 :

$$a_1x^2 + b_1y^2 = a_2x^2 + b_2y^2$$

$$\therefore x^2(a_1 - a_2) = y^2(b_2 - b_1)$$

$$\therefore \frac{y^2}{x^2} = \frac{a_1 - a_2}{b_1 - b_2} \quad \text{—(ii)}$$

Now

compare eq ① and eq (ii)

$$\frac{a_1 a_2}{b_1 b_2} = \frac{a_1 - a_2}{b_1 - b_2}$$

$$\therefore \frac{b_1 - b_2}{b_1 b_2} = \frac{a_1 - a_2}{a_1 a_2}$$

$$\frac{1}{b_2} - \frac{1}{b_1} = \frac{1}{a_2} - \frac{1}{a_1}$$

$$\therefore \boxed{\frac{1}{a_1} - \frac{1}{b_1} = \frac{1}{a_2} - \frac{1}{b_2}}$$

eg1) Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{l^2} - \frac{y^2}{m^2} = 1$ intersect orthogonally, then

(i) $a^2 - b^2 = l^2 - m^2$

(ii) $a^2 + b^2 = l^2 - m^2$

(iii) $a^2 - b^2 = l^2 + m^2$
(iv) $a^2 + b^2 = l^2 + m^2$

sol^n $\frac{1}{\frac{1}{a^2}} - \frac{1}{\frac{1}{b^2}} = \frac{1}{\frac{1}{l^2}} - \left(\frac{1}{\frac{1}{m^2}}\right)$

$a^2 - b^2 = l^2 + m^2$

(c)

Question based on hit and trial!

Q1.) Find angle b/w $y = 3^{x-1}\log x$ and $y = x^x - 1$?

sol^n $C_1 : y = 3^{x-1}\log x$; $C_2 ; y = x^x - 1$

$y = 3^0 \log 0$
$= 1 \times 0$

By hit and trial

(1,0) (satisfies eq^n)

$m_1 = \left(3^{x-1} \cdot \frac{1}{x} + 3^{x-1}\log 3 \cdot \log x\right)_{1,0}$

$= 1$

$m_2 = \left(x^x \log ex\right)_{1,0}$

$= 1$

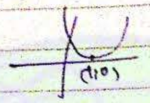
As $m_1 = m_2$

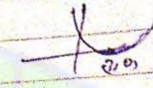
$\therefore \theta = 0^\circ$ or 180°

Question based on graph

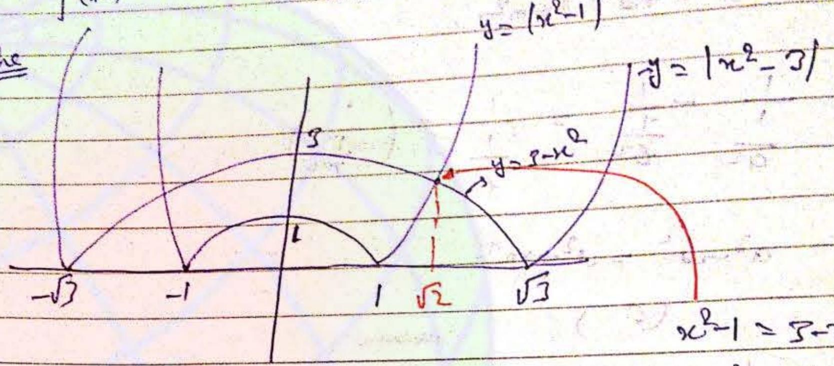
Q2) Draw $y = |x^2 - 1|$ and $y = |x^2 - 3|$ for $x > 0$, find Angle of Intersection ?

Sol^n





Trick



$x^2-1 = 3-x^2$

$2x^2 = 4$

$x^2 = 2$

$x = \sqrt{2}$

$C_1 : y = x^2 - 1$

$$\left(\frac{dy}{dx} = 2x\right)_{x=\sqrt{2}} = 2\sqrt{2} = m_1$$

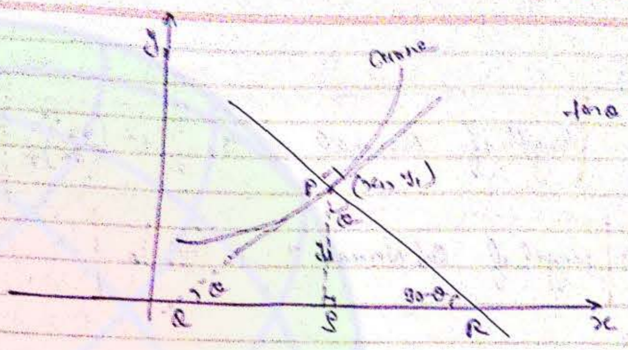
$C_2 : y = 3 - x^2$

$$\left(\frac{dy}{dx} = -2x\right)_{\sqrt{2}} = -2\sqrt{2} = m_2$$

$$\tan\theta = \left|\frac{4\sqrt{2}}{1+(-8)}\right|$$

$$\theta = \tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$$

★ Length of : Tangent, Normal, Subtangent, Subnormal



$\tan\theta = \left(\frac{dy}{dx}\right)_P$

PQ = tangent or (length of tangent)

PR = Normal or

QS : Subtangent or

SR = Sub Normal or

1) $\sin\theta = \frac{y_1}{PQ}$

$$PQ = \frac{y_1}{\sin\theta} = y_1 \operatorname{cosec}\theta = y_1\sqrt{1+\cot^2\theta}$$

$$= y_1\sqrt{1+\frac{1}{\tan^2\theta}}$$

$$\therefore \boxed{\text{Length of tangent} = y_1\sqrt{1+\frac{1}{\left(\frac{dy}{dx}\right)_P^2}}}$$

2.) $\tan\theta = \frac{y_1}{QS}$

$$\boxed{\therefore \text{Length of Subtangent} = \frac{y_1}{\left(\frac{dy}{dx}\right)_P}}$$

(L-1 → all) Lengths

(3.) $\boxed{\text{length of Normal} = y_1 \sqrt{1 + \left(\frac{dy}{dx}\right)_P^2}}$

④ $\boxed{\text{Length of Sub Normal} = y_1 \left(\frac{dy}{dx}\right)_P}$

Subtangent, ordinate, Subnormal at a point to curve are in G.P, whose common ratio is $\left(\frac{dy}{dx}\right)_P$.

Sheet Q81.

$by^2 = (x+a)^2$

$b\,\cancel{2}\,y \frac{dy}{dx} = 3\cancel{}(x+a)$

SN → Sub normal
ST → Sub tangent

~~Length of Sub~~

$\frac{dy}{dx} = \frac{3(x+a)}{2by}$ (x_1, y_1)

Length of Sub ~~nor~~ tangent $= \frac{y_1}{\left(\frac{dy}{dx}\right)_P} = \frac{y_1}{\frac{3}{2}\left(\frac{x+a}{by}\right)}$

Length of sub Normal $= y_1 \frac{3}{2}\left(\frac{x+a}{by}\right)$

$P\left(\frac{\cancel{y_1}}{\frac{3(x+a)}{2by}}\right) = q\left(\cancel{y_1}\frac{3}{2}\frac{x+a}{by}\right)$ $1 \to 3n$

$\frac{P}{q} = \frac{9\,\cancel{3}(x+a)^2}{4\cancel{2}b^2y^2} \cancel{} \frac{3(x+a)}{2by}$

Rate measure

c) $y = f(x)$, x and y are connected to each other

Ex
$A = \pi r^2$
$A = f(r)$
↳ Independent variable
↳ dependent variable.

$$\boxed{\frac{dy}{dx} = \text{Rate of change of } y \text{ w.r.t. } x}$$

* one rate is given other is to find relate these two variables.

Q1) The rate of change of Area of circular disc w.r.t its circumference with radius 3cm. will be

Sol. $\boxed{C = 2\pi r}$, $A = \pi r^2 = \frac{\pi C^2}{(2\pi)^2} = \frac{\cancel{\pi} C^2}{4\pi^{\cancel{2}}}$

$$\frac{dA}{dC} = \frac{d\, C^2/4\pi}{dC} = \frac{\cancel{2}C}{\cancel{4}\pi \, 2} = \frac{2\pi r}{2\pi} = r = 3 \text{ cm}$$

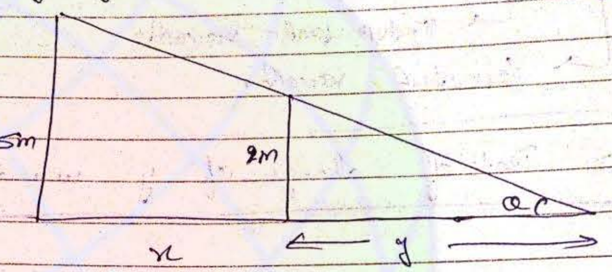
Q2) A stone is dropped into a quiet lake and wave moves in a circle ~~[illegible]~~ with vel = 3.5 cm/sec. At the Instant when radius is 7.5 cm then inclose area increases by.

Sol. $\frac{dr}{dt} = 7.5$, $\boxed{\text{Area} = \pi r^2}$

$$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt} = \pi \times 2 \times 7.5 \times 3.5$$
$$= \pi \times 15 \times 3.5 = 52.5\, \pi$$

Q2.) A man 2m high walks at a uniform speed of 6m/min away from the lamp post, 5m high. the rate at which his shadow is lengthing will be?

sol:



$\frac{dx}{dt} = 6 m/min, \quad \frac{dy}{dt} = ?$

$$\boxed{\tan\theta = \frac{5}{x+y} = \frac{2}{y}}$$ use trigonometrical concept

$5y = 2x + 2y$

$3y = 2x$

$3 \cdot \frac{dy}{dt} = 2 \cdot \frac{dx}{dt}$

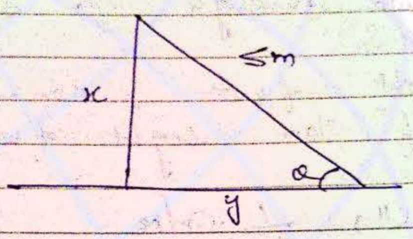
$\frac{dy}{dt} = \frac{2}{3} \cdot \frac{dx}{dt}$

$= \frac{2}{3} \times 6^2$

$= 4$ m/min.

Q.) A ladder of 5m length is leaning against a vertical wall the bottom of the ladder is pulled against the ground away from the wall at the rate 2m/s, How fast its height on the wall decreases, when foot of ladder is '4m' away from the wall?

Sol:



when $y = 4$
then $x = 3$

$\boxed{x^2+y^2 = 25} \longrightarrow$ use pythagoras theorem//

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = \frac{-y}{x} \cdot \frac{dy}{dx} = -\frac{4}{3} \cdot 2 = \frac{-8}{3} \text{ m/s}^2$$

$$\sin\theta = \frac{x}{5}$$

$$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{5} \frac{dx}{dt}$$

$$\frac{y}{5} \cdot \frac{d\theta}{dt} = \frac{1}{5} \times \frac{-8}{2}$$

$$\frac{d\theta}{dt} = \frac{1}{\cancel{5}} \times \frac{\cancel{5}}{4} \times \frac{-8}{3}$$

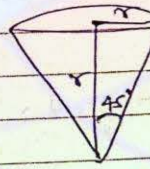
$$= -\frac{2}{3} \text{ rad/sec}$$

Q.) water is dripping out from conical funnel of semivertical angle $\pi/4$ at uniform rate $2cm^3/sec$, in its surface area through a tiny hole at vertex, when the slant height of water is 4cm. the rate of decrease of slant height of water is —

(i) $\frac{2}{\sqrt{2}\pi}$ cm/sec (ii) $\frac{1}{2\sqrt{2}\pi}$ cm/sec

(iii) $\frac{\sqrt{2}}{\pi}$ cm/sec (iv) $8\sqrt{2}\pi$ cm/s

Sol.



$$\frac{dv}{dt} = 2cm^3/sec.$$

$$\boxed{V = \frac{1}{3}\pi r^2 h} = \frac{1}{3}\pi r^3$$

$$= \frac{1}{3}\pi\frac{l^3}{2\sqrt{2}} = \frac{1}{6\sqrt{2}}\pi l^3$$

$$\therefore \tan 45^\circ = \frac{r}{h}$$

so, $r = h$

$l = \sqrt{r^2 + h^2} = \sqrt{2}r$

$r = l/\sqrt{2}$

$$\frac{dv}{dt} = \frac{1}{6\sqrt{2}}\pi 3l^2 \cdot \frac{dl}{dt}$$

$$2 \times 6\sqrt{2} = \pi 3 \times 16 \cdot \frac{dl}{dt}$$

$$\frac{dl}{dt} = \frac{6\sqrt{2}}{24\pi} = \frac{\sqrt{2}}{4\pi} = \frac{1}{2\sqrt{2}\pi}$$

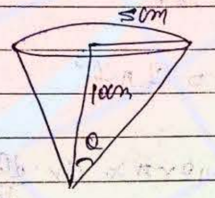
or $l^2 = r^2 + h^2$

$l^2 = 2r^2$

$2l \cdot \frac{dl}{dt} = 2.2r \cdot \frac{dr}{dt}$

eg3.) An invrated cone has dept of 10cm and radies of base 5cm water is filled into it at the rate of $\frac{3}{2}$ c.c./min the rate at which level of water in the cone is rising when dept is 4cm is ?

Sol



$\frac{dv}{dt} = \frac{3}{2} cm^3/min$

$\frac{dh}{dt} = ?$

$\boxed{V = \frac{1}{3} \pi r^2 h}$

$= \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \pi \cdot 4h^2 h$

$= \frac{4}{3} \pi h^2$

$\tan\theta = \frac{5}{10} = \frac{r}{4}$

$r = 2cm$

$\frac{r}{h} = \frac{\cancel{5}}{\cancel{10}}$

$\boxed{r = \frac{h}{2}}$

$\frac{dv}{dt} = \frac{4}{3} \pi \cdot \frac{3h^2}{4} \cdot \frac{dh}{dt}$

$\frac{3}{2} = \frac{4}{\cancel{3}} \pi \cancel{3} \times \frac{16}{4} \cdot \frac{dh}{dt}$

$$\frac{dh}{dt} = \frac{3}{2} \times \frac{1}{\frac{64\pi}{10}} = \frac{3}{8\pi}$$

eg. Sand is falling from a pipe at the rate 12 cm³/sec the falling sand forms a cone on the ground in such a way that the height of cone is always $\frac{1}{6}$ of radius of base. How fast is the height of sand cone is increase when h = 4cm

Sol) $h = \frac{r}{6}$, $\frac{dv}{dt} = 12\ cm^3/sec.$

$$\boxed{V = \frac{1}{3}\pi r^2 h} = \frac{1}{3}\pi\, 36\, h^3 = 12\pi h^3$$

$$\frac{dv}{dt} = 12 \times \pi \times 3h^2 \frac{dh}{dt} = 12 \times \pi \times 16 \times \frac{dh}{dt}$$

$$12 = 12 \times \pi \times 48 \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{48\pi}$$

Q4) A spherical Iron ball 10cm in radius is coated with a layer of ice of uniform thickness that melt at a rate of 50 cm³/min. when the thickness of ice is 5cm then the rate of which the thickness of ice decrease is

sol) $V = \frac{4}{3}\pi\left[(h+10)^3 - 10^3\right]$ $\frac{dv}{dt} = \frac{4}{3}\pi\left[3(h+10)^2 \cdot \frac{dh}{dt}\right]$

$$= \frac{4}{\cancel{3}}\pi \times \cancel{3}\left[\overset{9}{\cancel{225}}\right]\frac{dh}{dt} = \overset{2}{\cancel{50}}$$

$$\frac{dh}{dt} = \frac{1}{18\pi}$$

Q) on the curve $x^3 = 12y$, the absicca ~~of~~ changes at fastest rate than ordinate then ~~x~~
$x \in$?

sol: $y \to$ ordinate, abscissa $= x$

$$\frac{dy}{dx} < 1$$

$\leftarrow$ ordinate

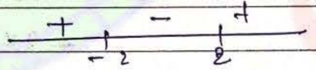
$\leftarrow$ abscissa

$$x^3 = 12y$$

$$\cancel{3x^2} = \cancel{12}\,4\,\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{x^2}{4} < 1$$

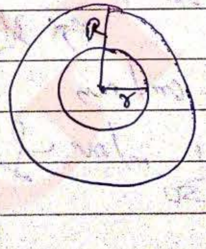
$$\frac{x^2}{4} < 1 \quad \Big| \quad \frac{x^2 - 4}{4} < 0$$



$x \in [-2, 2]$ Ans

Q) The volume of metal in hollow sphere is constant if the inner radius is 'r' at the rate of 1cm/sec than rate of increase of outer radius, when the radii are 4cm and 8cm resp. is —

sol/n



$$V = \frac{4}{3}\pi\left[R^3 - r^3\right]$$

$$\frac{dV}{dt} = 0 \quad [\because V = \text{constant}]$$

$$0 = \frac{4}{3}\pi\left[3R^2 \cdot \frac{dR}{dt} - 3r^2 \cdot \frac{dr}{dt}\right]$$

$$\frac{dR}{dt} = \frac{r^2}{R^2} \cdot \frac{dr}{dt} = \frac{16}{64} \times 1 = \frac{1}{4} \text{ cm/sec} \text{ //}$$

Q.) A spherical ballon is filled with 4500 π cm³ of helium gas. ~~if the~~ If a leak in the ballon causes the gas to escape at the rate of 72π cm³/min the rate (in m/min) at which the radius of the ballon decreases 49 min after the leakage begin.//

$\frac{dv}{dt} = 72\pi \ cm^3/min$ $\qquad \frac{dr}{dt} = ?$

$$\boxed{V = \frac{4}{3}\pi r^3}$$

$$\frac{dv}{dt} = \frac{4}{3}\pi \cdot 3 \cdot r^2 \cdot \frac{dr}{dt}$$

at $t=0$, $V = 4500\ \pi cm^3$

at $t = 49 min$, $V = 4500\pi - 72 \times 49 \pi$
$= 972\pi$

at
$t = 49 min$

$$\frac{\cancel{4}}{3}\cancel{\pi} r^3 = \overset{243}{\cancel{972}}\ \cancel{\pi}$$

$$r = \sqrt[3]{243 \times 3} = 9 cm$$

$$\frac{dr}{dt} = \frac{dv}{dt} \times \frac{1}{4\pi r^2} = \overset{\overset{2}{\cancel{18}}}{\cancel{72\pi}} \times \frac{1}{\cancel{4\pi}\ \cancel{81}_9}$$

$$= \frac{2}{9} cm/min$$

$$\frac{dr}{dt} = \frac{2}{900} m/min = \frac{1}{450} m/min$$

Q.) A man
41.6
of man
the
the
from

Q.) A man is moving away from ~~the~~ a tower of 41.6 m at a rate of 2m/s assuming eye level of man 1.6m.
The rate ~~of~~ at which angle of elevation of the top of the wate is changing when he is 30m away from the tower:

$$\tan\theta = \frac{40}{x}$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = -\frac{40}{x^2} \cdot \frac{dx}{dt}$$

Rolle's theorem

Suppose, $y = f(x)$ be a function such that

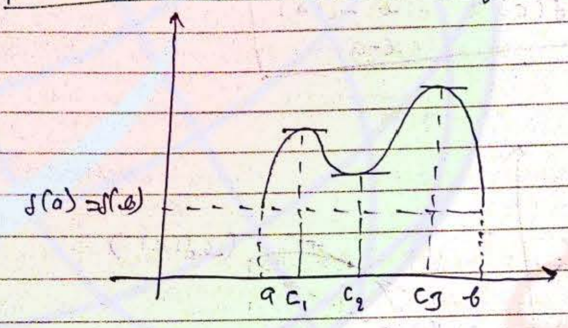
(i) $f(x)$ is conti on $[a, b]$
(ii) $f(x)$ is differentiable on (a, b)
(iii) $f(a) = f(b)$

Then

there is atleast one value of x (say c) in (a, b) such that $\boxed{f'(x) = 0}$

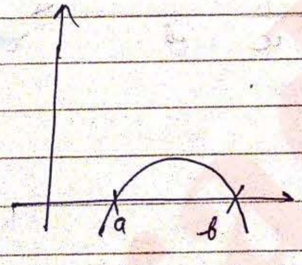
i.e.

"There is at least one real root" of $f'(x) = 0$ in (a, b)



$$f'(c_1) = f'(c_2) = f'(c_3) = 0$$

* Theorem Note



$f(a) = f(b) = 0$

$\therefore f(x) = 0 \begin{cases} \to a \\ \to b \end{cases}$

"Any two real roots of $f(x) = 0$ there is at least one root of its $f'(x) = 0$"

Note

(i) $y = \log x$, $x \in [1, 2]$ → Roll's theorem does not hold bec $f(a) \neq f(b)$

(ii) $y = |x|$, $x \in [-1, 1]$ → Rolle's theorem does not hold bec function is not diff ~~discontinuous~~ at $x = 0$

(iii) $y = [x]$, $x \in [1, 3]$ → " " " " because function is discont.

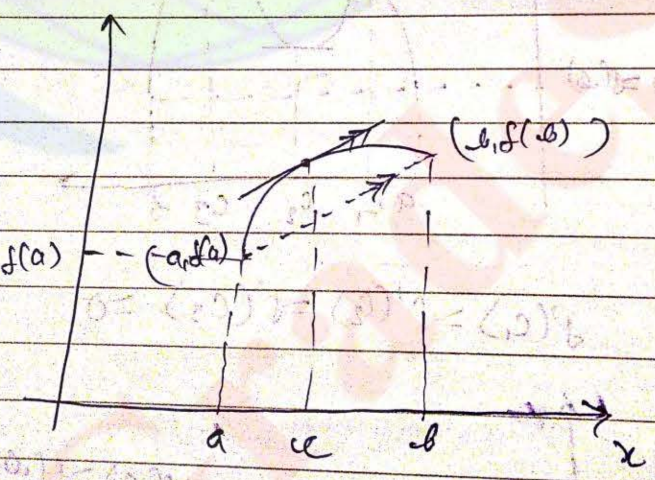
Lagrange's mean value theorem →

Suppose $y = f(x)$ be a function such that

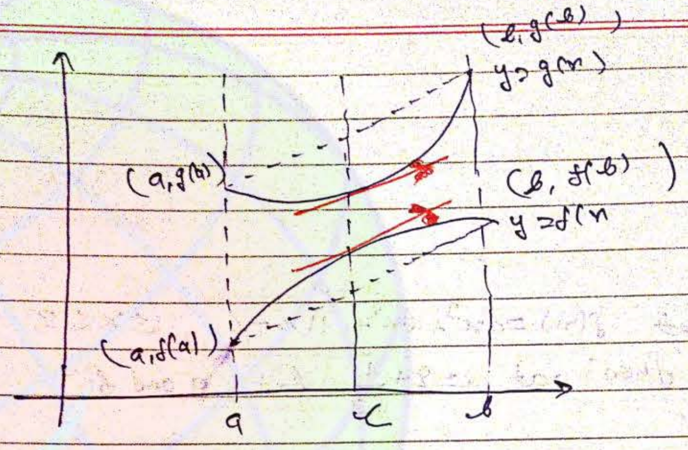
(i) $f(x)$ is cont' on $[a, b]$
(ii) $f(x)$ is differenciable on (a, b)
(iii) $f(a) \neq f(b)$

then,
there exist at least one value of "x" (say c) in (a, b) such that

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$



★ Cauchy's mean value theorem →



$$\frac{f'(c) = \dfrac{f(b) - f(a)}{\cancel{b-a}}}{g'(c) = \dfrac{g(b) - g(a)}{\cancel{b-a}}}$$

$$\boxed{\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}}$$

①

② Let $f(x) = ax^3 + bx^2 + 11x - 6$, $1 \leq x \leq 3$ hold's Rolle's then at $x = 2 + \frac{1}{\sqrt{3}}$ find a and b

So/n $f(x) = ax^3 + bx^2 + 11x - 6$ — (I) is a polynomial functio so it is continous at $1 \leq x \leq 3$ and

(II) it is also diff at $1 \leq x \leq 3$

③

(III) $f(1) = f(3)$

So

$\Rightarrow a + b + 11 - 6 = 27a + 9b + 27$

$\Rightarrow a + b + 5 = 27a + 9b + 27$

$\Rightarrow 26a + 8b + 22 = 0$

$\Rightarrow 13a + 4b + 11 = 0$ — (1)

diff:

$f'(x) = 3ax^2 + 2bx + 11$

$$f'\left(2 + \frac{1}{\sqrt{3}}\right) = 3a\left(2 + \frac{1}{\sqrt{3}}\right)^2 + 2b\left(2 + \frac{1}{\sqrt{3}}\right) + 11 = 0$$

$$= 3a\left(4 + \frac{1}{3} + \frac{4}{\sqrt{3}}\right) + 4b + \frac{2b}{\sqrt{3}} + 11 = 0$$

$$= 13a + 4a\sqrt{3} + 4b + \frac{2b}{\sqrt{3}} + 11 = 0$$

$$= \cancelto{0}{(13a + 4b + 11)} + \left(4a\sqrt{3} + \frac{2b}{\sqrt{3}}\right) = 0$$

by (from eq ①)

[illegible]

$4a\sqrt{3} + \frac{2b}{\sqrt{3}} = 0$

$2\left(\frac{6a+b}{\sqrt{3}}\right) = 0$

$\boxed{b = -6a}$

(1) $13a - 24a + 11 = 0$

$-11a + 11 = 0$

$\left.\begin{array}{l} a = 1 \\ b = -6 \end{array}\right\}$ Ans

③ Let $f(x) = x^3 + bx^2 + cx$, holds rolles theorem in $1 \le x \le 2$ at $\frac{4}{3}$. find ~~[illegible]~~ b and c ?

Sol^n $f(x) = \underbrace{~~a~~x^3 + bx^2 + cx}$

(i) function is polynomial so ~~[illegible]~~ it is continuous ~~[illegible]~~ in $1 \le x \le 2$

(ii) differentiable in $1 \le x \le 3$

(iii) $f(1) = f(2)$

$1 + b + c = 8 + 4b + 2c$

$-3b - c - 7 = 0$

$3b + c + 7 = 0$ —(1)

Differentiate

$f'(x) = 3x^2 + 2bx + c$

~~[illegible]~~ $f'\left(\frac{4}{3}\right) = \frac{16}{3} + \frac{8b}{3} + c = 0$

$\therefore \ 8b + 3c = -16$ —(ii)

Now
Solving eq ① and eq ⓘⓘ we get

$b = -5$
$c = 8$ Ans

* Problem's based on Inverse of Rolle's theorem:-

(1*) If $2a + 3b + 6c = 0$ then atleast one root of $ax^2 + bx + c = 0$ is in

i) $(-1,1)$ (ii) $(1,2)$ (iii) $(-1,0)$ (iv) $(2,3)$

Sol^n According to the Rule of Rolle's theorem that there is at least one value of $f'(x) = 0$ in (a, b)

so $f'(x) = ax^2 + bx + c$

So to find $f(x)$ we Integerate

$$f(x) = \underbrace{\frac{ax^3}{3} + \frac{bx^2}{2} + cx + k}_{}$$

poly) → contn + diff
(1) ②

$f(0) = k$ — (i)

$f(1) = \frac{a}{3} + \frac{b}{2} + c = k$

$$= \frac{\cancel{2a + 3b + 6c}}{6} + k \quad \xrightarrow{} 0 \text{ (given)}$$

$= k$

$f(1) = k$ —(ii)

So from eq ⓘ and ⓘⓘ

$f(0) = f(1)$

Integrate करने पर
are और operable
Satisfy करता

② Let $27a + 9b + 3c + d = 0$ than at least one root of $4ax^3 + 3bx^2 + 2cx + d = 0$ is in the Interval ——?

Sol^n Acc^n to the rule of Rolle's theorem that

there is at least one root of $f'(x) = 0$ in (a, b)

so $f'(x) = 4ax^3 + 3bx^2 + 2cx + d$

so to find $f(x)$ we Integrate it

$$f(x) = \underbrace{ax^4 + bx^3 + cx^2 + dx + k}_{\text{poly: conti' ① + diff ②}}$$

$f(0) = k$

$f(3) = 81a + 27b + 9c + 3d + k$

$= 3(27a + 9b + 3c + d) + k$ $\xrightarrow{} 0$

& $f(0) = f(3)$

At least one root in $(0, 3)$

sheet (iib) ③ If the eq^n $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x = 0$

$a_1 \neq 0$, $n \geq 1$ has a +ve root $x = \alpha$ than the eq^n

$$\boxed{n a_n x^{n-1} + a_{n-1} (n-1) x^{n-2} + \cdots + a_1 = 0}$$

has a +ve root which is

(i) $> \alpha$ ✓(ii) $< \alpha$ (iii) $\geq \alpha$ (iv) $= \alpha$

L-2 (all)

Sol: According to Rolle's theorem

$$f'(x) = n.a_n\, x^{n-1} + (n-1)\, a_{n-1}\, x^{n-2} + \dots + a_1$$

So, to find $f(x)$ we integrate it

$$f(x) = \underbrace{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + K}_{}$$

Poly: Contn + Diff
(I) (II)

$$\left.\begin{array}{l} f(0) = K \\ f(\alpha) = K \end{array}\right] \quad f(0) = f(\alpha)$$

Soln
(Q-8)

$$f(x) = \begin{vmatrix} \sin x & \sin a & \sin b \\ \cos x & \cos a & \cos b \\ \tan x & \tan a & \tan b \end{vmatrix}$$

$f'(x) > 0$

$\longrightarrow$ $f(x)$ is conti + Diff

$f(a) = 0 = f(b)$

$\therefore$ so, $f'(x) = 0$ has at least one root in (a, b)

L-1
[Q-90] If $\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + \cdots \frac{a_{n-1}}{2} + a_n = 0$, then the eqn

$a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = 0$ has in the Interval $(0, 1)$

(A) exactly one root
(B) at least one root
(C) atmost one root
(D) No root

Sol/n

$f'(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n$
Integrate

$$f(x) = \underbrace{\frac{a_0x^{n+1}}{n+1} + \frac{a_1}{n}x^n + \cdots + a_nx + K}_{\text{poly! conti + diff}}$$

① ②

$f(0) = K$

$f(1) = \left(\frac{a_0}{n+1} + \frac{a_1}{n} + \cdots a_n\right)^{\to 0} + K$

$= K$

$f(0) = f(1)$
atleast one

L-1
[Q2]

sol^n $f'(x) = \frac{1}{\sin x} \cdot \cos x = \cot x = 0$

$\downarrow$

$x = \frac{\pi}{2}$

$\therefore c = \frac{\pi}{2}$

↳ "c" is a point where derivative of function is zero.//

Question based on Cauchys mean value theory

[Q-96] Let $f, g \rightarrow$ diff , $0 \leq x \leq 1$

$f(0) = 2,\ g(0) = 0,\ f(1) = 6$

$c \in (0,1)$, st $f'(c) = 2g'(c)$ } $g(1) = ?$

CMVT) $\rightarrow$ $\frac{f'(c)}{g'(c)} = \frac{f(1) - f(0)}{g(1) - g(0)}$

$2 = \frac{6-2}{g(1)-0}$

$g(1) = \frac{4}{2} = 2$ //

L-2
Q17.

sol^n

L-2
[Q19]

L-2
[Q2

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L-2
Q17.

$$f(x) = \begin{cases} x\cos\left(\frac{1}{x}\right) & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$$ in the interval [-1,1]

Sol^n

[illegible] concept → $x = 1$, so f is conti but not differenciable

so,

L.M.V.T is Not applicable.

so

c = does not exist

L-2
[Q19]

f, g → diff , $0 \leq x \leq 1$

$f(0) = 2$, $g(0) = 0$, $f(1) = 6$, $g(1) = 2$

(0,1)

According to cauchy schwart Rule//

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)} = \cancel{\frac{g(1) - g(0)}{f(1)}}$$

$$= \frac{f(1) - f(0)}{g(1) - g(0)} = \frac{6 - 2}{2 - 0} = \frac{4}{2} = 2$$

$f'(c) = 2g'(c)$ Ans

L-2
[20]

$f(x) = (x-4)(x-5)(x-6)(x-7)$ then $f'(x) = 0$ has "3" real roots , each one in (4,5) , (5,6) , (6,7)

How?

$f(x) = \underbrace{(x-4)(x-5)(x-6)(x-7)}_{\text{polyl} \to \text{cont} + \text{diff}}$ ← degree 4 so 4 root's
(1) ②

But $f'(x)$ has 3 real root's //
↳ because $f'(x)$ has 3 degree //

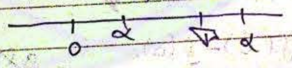
$f(4) = f(5) = f(6) = f(7) = 0$

So
exactly one root is in (4,5),
in (5,6)
in (6,7) Ans

L-2
[21]

The no of values of k for which the eq^n $x^3-3x+k=0$ has two distinct roots lying in the interval (0,1) are

sol^n differentiat
$3x^2-3=0$
$x=\pm 1$ ← roots of $f'(x)=0$



No value of "k".

Note f(n) का value x = ±1 है की कि (0,1) के बिच में lie नहीं कर रहा है इसलिए कोई भी f'(n) का value (0,1) में lie नहीं करेगा

~~[16] If, a, b, c be non-zero real numbers, such that~~

~~$\int_0^1 (1+\cos^8 x)(ax^2+bx+c)dx$~~

~~$\int_0^2 (1+\cos^8 x)(ax^2+bx+c)dx = 0$~~

~~than~~

L-2
[16] Let

sol^n Let

[18]

L-2

[16] Let $a, b, c \in R_0$ s.t

$$\int_0^1 (1+\cos^8 x)(ax^2+bx+c)\,dx = \int_0^2 (1+\cos^8 x)(ax^2+bx+c)=0$$

then

$(ax^2+bx+c)dx = 0$ will be

sol^n Let

$$f(x) = \int_0^x \underbrace{(1+\cos^8 t)}_{\text{Conti + Diff}} \underbrace{(at^2+bt+c)}_{\text{Conti + diff}}\, dt$$

Conti + Diff

① ⑪

$f(0) = f(1) = f(2) = 0$

So, exactly one root in $(0,1)$

exactly one root in $(1,2)$

$$f'(x) = \underbrace{\cancel{(1+\cos^8 x)}}_{⊕}(ax^2+bx+c)=0$$

$$\Downarrow$$

$ax^2+bx+c=0$

$$\boxed{\begin{aligned} &(x-1)=0 \Rightarrow x=1 \\ &(1+x^2)(x-1) \Rightarrow x=1 \\ &(1+\cos^8 x)(x-1) \Rightarrow x=1 \end{aligned}}$$

[18] $f(2) = -4$

$f'(x) \geq 6$, $[2,6]$

By L.M.V.T

$$f'(x) = \frac{f(4)-f(2)}{4-2}$$

Int of logn x log(-x)

classmate
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$f'(n) = \frac{f(4)+2}{2} \geq 6$

$f(4) \geq 8$

☆ Some extra example :-1

Q1) Prove that
eq $x \log n = 3 - x$ has atleast one root in (1,3)?

Sol^n

$\Rightarrow x \log x = 3 - x$

$\Rightarrow \underbrace{x \log n - 3 + n = 0}_{f'(n)}$

$\Rightarrow f'(n) = \underbrace{x \log n - 3 + n = 0}$

$\underbrace{\log n - \frac{3}{x} + 1 = 0}_{f'(n)}$

$f(n) = x \log n - x - 3 \log n + x + k$

$f(x) = \underbrace{(\log n)(x-3) + k}_{\text{Conti + Diff}}$

① ⑪

$f(1) = f(3)$

$\therefore x \log = 3 - x$ has atleast one root in (1,3] eg2.) Eqⁿ $\sin x + x\cos x = 0$ has at least one real root in

(i) $\left(-\frac{\pi}{2}, 0\right)$ (ii) $(0, \pi)$ (iii) $\left(\pi, \frac{7\pi}{2}\right)$ (iv) ____

Sol/ⁿ $f'(x) = \sin x + x.\cos x = 0$

$f(x) = -\cos x + x \cdot \sin x - \int \sin x \, dx$

$f(x) = -\cancel{\cos x} + x \sin x + \cancel{\cos x} + c$

$$\therefore f(x) = \underbrace{x \sin x + c}_{\text{cont.} \ \text{①} \ + \ \text{Diff} \ \text{②}}$$

$f(0) = f(\pi)$

∴ At least one root in $(0, \pi)$

eg3.)

$$f(x) = \begin{cases} x^{\alpha} \log x & ; 0 < x \le 1 \\ 0 & ; x = 0 \end{cases}$$

, Rolle's theorem is applicable to this function in $[0,1]$ if α is

(i) -2,
(ii) -1
(iii) 0
(iv) $1/2$

Sol/ⁿ $f(0) = 0$ $\Rightarrow f(0) = f(1)$

$f(1) = 0$ (iii)

(ii) Differentiability in $(0,1)$; Yes it is diff

① Continuity in $[0,1]$

$\downarrow$

$f(0^+) = f(0)$

$\Rightarrow \lim_{x \to 0^+} x^{\alpha} \log x = 0$

$\lim_{h \to 0} h^{\alpha} \log h = 0$

(i) $\alpha = 0$ ✗

(ii) $\alpha < 0$ ✗

(iii) $\alpha > 0$ ✓

$\Rightarrow \lim_{h \to 0} h^{\alpha} \log h$

$\Rightarrow \frac{\log h}{(1/h^{\alpha})} \Rightarrow \frac{1/h}{\frac{-\alpha}{h^{\alpha+1}}}$

$\Rightarrow -\frac{1}{\alpha} h^{\alpha}$

$\Rightarrow 0$

for value greater 0

~~Here~~ Here,

(i) $\alpha = 0$, $\log_{>1} \infty = -\infty$ (X)

(ii) $\alpha < 0$, $\frac{-\infty}{0} = -\infty$ (X)

(iii) $\alpha > 0$ (✓)

↳ $\lim_{h\to 0} h^{\alpha} \cdot \log h =$

(e4) find value of "C" for which $f(x) = \sqrt{25-x^2}$ holds L.M.V.T in [1, 5]

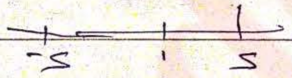
Sol^n ~~own~~ $f'(x) = \frac{1}{\cancel{2}}(25-x^2) - \cancel{2}x$

$$\cancel{x = 0 \quad x^2 - 25}$$

$$= x^2 - 25 = 0$$

$$x^2 = 25$$

$$\boxed{x = \pm 5}$$



$f'(x) =$

Take $f'(c) = \frac{f(5) - f(1)}{5-1}$; $f'(x) = \frac{-x}{\sqrt{25-x^2}}$

$$\frac{-x}{\sqrt{25 - c^2}} = \frac{0 - \sqrt{24}}{4}$$

$$16c^2 = 24 \cdot 25 - 24c^2$$

$\Rightarrow 40c^2 = 24.25$

$c^2 = 15$

$c = \pm\sqrt{15}$

$c = \sqrt{15} \in (1, 5)$

chal
Co.J

The value of "c" for which conclusion of mean value theorem holds for the function $f(x) = \log^{x}_{e}$ on the interval $[1, 7]$ is

(i) $2\log^{e}_{7}$ (ii) $\frac{1}{2}\log^{7}_{e}$ (iii) $\log^{e}_{7}$ (iv) $\log^{7}_{e}$

Sol/n

$f'(x) =$ ~~log x~~ $\frac{1}{x}$.

$f'(c) = \frac{1}{c}$ —(i)

$f'(c) = \frac{f(7) - f(1)}{7-1}$

$= \frac{f(7) - f(1)}{2}$ —(ii)

from eq (i) and (ii)

$\frac{1}{c} = \frac{\log^{7}_{e}}{2}$

$c = \frac{2}{\log^{7}_{e}} = 2\log^{e}_{7}$

chal
(Q)

~~$y = \int_0^x |t|\, dt, \ x \in R$~~

L-2
Q-6

L-4B
Q13

~~$x \in [0, 1]$~~

~~$f'(x) = -1 = \frac{f(1) - f(0)}{1}$~~

~~$-1 = \frac{-\frac{1}{2} - \frac{1}{2}}{}$~~

(L-4B)
Q-13 (A) $x \in [0,1]$

$f(x) = \begin{cases} \frac{1}{2} - x & ; x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2 & ; x \geq \frac{1}{2} \end{cases}$

→ LHL = 0

→ R.H.D = value = 0

so, cont

$f'(x) = \begin{cases} -1 & ; x < \frac{1}{2} \rightarrow L.H.D = -1 \\ 2\left(\frac{1}{2} - x\right) & ; x > \frac{1}{2} \rightarrow R.H.D = 0 \end{cases}$

non - diff

L.M.V.T not applicable.

L-2
Q-8

~~$[0,1] \rightarrow C$~~

~~$(0,1) \rightarrow$ diff~~

(B) $\frac{\sin x}{x}$; $x \neq 0$

1 ; $x = 0$

$[0,1] \rightarrow C$

$(0,1) \rightarrow$ diff

L-2

Q-6

$$y^2 = 4a\left(x + a\sin\frac{x}{a}\right)$$

$$2y \cdot y' = 4a\left(1 + \cos\frac{x}{a}\right)$$

$$y' = \frac{\cancel{4}a\left(1 + \cos\frac{x}{a}\right)}{\cancel{2y}} = 0$$

$$\cos\frac{x}{a} = -1$$

$$\therefore x = \pi$$

$$x = a\pi$$

$$y^2 = 4a(x)$$

parabola

L-2

Q-8

$$x^m \cdot y^n = a^{m+n}$$

$$x^m \cdot y^n \cancel{a} - c = 0$$

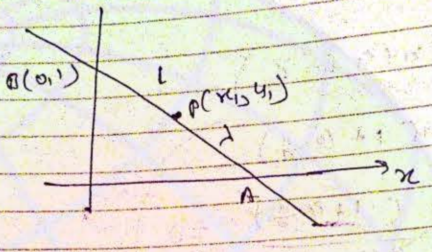
$$\frac{dy}{dx} = -\left(\frac{m \cdot \cancel{x^{m-1}} \cdot \cancel{y^n}}{n \cdot \cancel{x^m} \cdot \cancel{y^{n-1}}}\right)$$

$$\frac{dy}{dx} = -\left(\frac{m}{n} \cdot \frac{y}{x}\right)$$

Eq^n of tangent:

$$y - y_1 = -\frac{m}{n} \cdot \frac{y_1}{x_1} \cdot (x - x_1)$$

$$\boxed{\frac{nx_1}{m} + x_1}$$



$$\frac{n}{m}x_1 + x_1$$

$$x_1 = \frac{\lambda \cdot 0 + \left(\frac{n}{m}+1\right)x_1}{\lambda+1}$$

$$\lambda + 1 = \frac{n}{m} + 1$$

$$\frac{\lambda}{1} = \frac{n}{m}$$

$\lambda : 1 :: n : m$ //

$\boxed{\frac{L-2}{14}}$

$$x^{2/3} + y^{2/3} = a^{2/3}$$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3} \cdot y' = 0$$

$$\therefore \quad y' = \frac{-y^{1/3}}{x^{1/3}} = \pm 1$$

$-y^{1/3} = x^{1/3}$; $-y^{1/3} = x^{1/3}$

$y = -x$; $y = x$

$x^{2/3} + x^{2/3} = a^{2/3}$

$2x^{2/3} = a^{2/3}$

$x = \pm a$

$(+, +), (-, -)$

$$2x^{2/3} = a^{2/3}$$

$$x^{2/3} = \frac{a^{2/3}}{2}$$

$$x = \pm \frac{a}{2^{3/2}}$$

$$\left(\frac{-a}{2^{3/2}}, \frac{a}{2^{3/2}}\right); \left(\frac{a}{2^{3/2}}, \frac{-a}{2^{3/2}}\right)$$

four poma

Monotonic function

Increasing — Decreasing

Increasing	Strictly Increasing	Decreasing	strictly Decreasing
or (monotonically Increasing)	(Increasing)	(monotonically decreasing)	~~strictly~~ Decreasing
$\frac{dy}{dx} \geq 0$	$\frac{dy}{dx} > 0$ = holds at some points	$\frac{dy}{dx} \leq 0$	$\frac{dy}{dx} < 0$ = holds at some points

likes $y = x^3 \longrightarrow$ Domain R

$\frac{dy}{dx} = 3x^2 > 0$

$\therefore x^3$ is strictly ↑

① If function is constant ⇒ it is non-monotonic

② If function is both Increasing as well as decreasing ⇒ it is non-monotonic

likes.

$y = x^2$ is non-monotonic in "R"

But in $(-\infty, 0) \Rightarrow f(x) = x^2$ is strictly "↓" and in $(0, \infty) \Rightarrow$ $f(x)$ is strictly "↑"

③ If $f(x)$ is "↑" then $\underbrace{-f(x)}$ is "↓"

$\Downarrow$ — $\downarrow$

when: $f'(x) > 0$ — $-f'(x) < 0$

④ if $f(x)$ is "↑" then $\frac{1}{f(x)}$, $f(x) \neq 0$ is ↓

$\Downarrow$ — $\Downarrow$

$f'(x) > 0$ — $-\frac{1}{(f(x))^2} f'(x)$

(–ve)

T→ "Composite function"

⑤ If $f(x)$ is ↑ and $g(x)$ ↑ then $f(x)+g(x)$ is "↑"

⑥

$f(x)$	$g(x)$	fog and gof
↑	↑	+ ↑
↑	↓	− } ↓
↓	↑	−
↓	↓	+ ↑

Like: →
$f(x) = 2\tan^{-1}(e^x) - \frac{\pi}{2}$ (test)

↓ $\tan^{-1}x$ ↓ e^x

$2\tan^{-1}(e^x)$ "↑"

↓ $2\tan^{-1}(e^x) - \frac{\pi}{2}$ "↑"

like: → Let $f'(x) > 0$ and $g'(x) < 0$ then

✓(i) $fog(3) > fog(5)$
✗(ii) $fog(x-5) > fog(x-6)$
✗(iii) $fog(3) < gof(5)$
✓(iv) $gof(x-1) > gof(x+1)$

Sol^n $f(x)$ ↑ , $g(x)$ ↓

fog , gof ↓

CAREER POINT

Q.1 Which of
decreasing f
(1) e^{-x}
(3) tan x

Q.2 At x = 0, f

(1) Monot
(2) Monot
(3) Not m
(4) Const

Q.3 If f(x) =
increasi
(1) k <
(3) k >

Q.4 Functi
increa
(1) λ
(3) λ

Q.5 Func
when
(1)
(2)
(3)
(4)

Q.6 In
no
(1
(3

CAREE

Ra1 2 ✓

eg1) Let $f(x) = \frac{x}{x^2-6x-16}$, find the Interval in which it is "↓"?

So/n $f'(x) = \frac{(x^2-6x-16) - x\,(2x-6)}{(x^2-6x-16)^2}$

$$f'(x) = \frac{\cancel{x^2}-\cancel{6x}-16-2x^2+\cancel{6x}}{(x^2-6x-16)^2} = \frac{-x^2-16}{(x^2-6x-16)^2} = \frac{-\overset{\oplus}{(x^2+16)}}{\underset{\oplus}{(x^2-6x-16)^2}} < 0$$

So

f is decreasing on R(X)

Domi → $x^2-6x+16 \neq 0$

$\therefore (x+2)(x-8) \neq 0$

$\therefore x \neq -2, 8$

(लेकिन नीचे वाला does not equal to zero होना चाहिए)

→ Ans $R - \{-2, 8\}$

eg2) find $f(x) = \frac{x}{\log x}$, find the set in which "f" is "↓"?

So/n

$$f'(x) = \frac{\log x - \cancel{x}\cdot\frac{1}{\cancel{x}}}{(\log x)^2} < 0$$

$$= \frac{\log x - 1}{\underset{\oplus}{(\log x)^2}} < 0$$

$\log x - 1 < 0$

$\therefore \log x < 1$

$\therefore x < e$

$(-\infty, e)$ ✗

$(0, e)$ ✗

~~$(0, e) - \{1\}$~~

$(0, e) - \{1\}$ ✓

Ra1 2

(ध्यान दे $(0,e) - 1$ को consider किया गया है)

eg3:) Let $f(x) = \begin{cases} 3x^2+12x-1 & ; [-1,2] \\ 37-x & ; (2,3] \end{cases}$

$6x+12$ (

$-1 \le x \le 2$

$-6 \le 6x \le 12$

$\boxed{6 \le 6x+12 \le 24}$

Here,
(i) $f(x)$ is ↑ on $[-1,2]$
(ii) f is cont on $[-1,3]$
(iii) f has great at $x=2$
at val also

LC
$L=V=35$
$R=35$

---

cont. $[-1,3]$

eg4:) Let $f(x)=x^3+bx^2+cx+d$, $0<b^2<c$

P.T.
f is strictly "↑", everywhere ?

sol $f'(x) = \underbrace{3x^2+2bx+c}_{\hookrightarrow \text{Poly} \to \text{cont} + \text{ve}} > 0 \ \forall \ R$

$a = 3 \oplus$

and
$D = 4b^2 - 4 \cdot 3 \cdot c$
$= 4(b^2-3c) \ominus$

so
f is ↑ on "R"

eg5) Let $h(x) = f(x) - f(x)^2 + f(x)^3$
then,
when f "↑", h "↑" (T/F)

sol $h'(x) = f'(x) + 2f(x) \cdot f'(x) + 3f(x)^2 \cdot f'(x)$

$= f'(x) \underbrace{\left[3f(x)^2 - 2f(x) + 1 \right]}_{\text{Quadratic} \Rightarrow a = 3 \oplus}$

$D < 4 - 4 \cdot 3 \cdot 1$ (−ve)

CAREER POINT

Ta

Q.1 Consider the fun

(1) f(x) increase

(2) f(x) is decre

(3) the interva

transforms

(4) all above

Q.2 Function f(x) =

increasing whe

(1) x < 0

(3) x ∈ R

Q.3 The interval

less rapidly t

(1) (−∞, −1)

(3) (−1, 5)

Q.4 The functio

(1) R

(3) (0, ∞)

Q.5 If S is the

in S then

(1) (−1/4

(3) (−∞,

Q.6 If f(x)

(1) f(x

(2) f(

(3) f(

(4) f

CA

CAREER

when $f'(x)$ ⊕ $\Rightarrow h'(x)$ ⊕

when

$f \uparrow \Rightarrow h \uparrow$

eg 5) Let $f(x) = \sin^4 x + \cos^4 x$ then f is "$\uparrow$" on

(i) ✗ $\left(0, \frac{\pi}{4}\right)$ (ii) ✓ $\left(\frac{\pi}{4}, \frac{3\pi}{8}\right)$ (iii) ✗ $\left(\frac{7\pi}{8}, \frac{5\pi}{8}\right)$ (iv) ✗ $\left(\frac{5\pi}{6}, \frac{7\pi}{c}\right)$

Sol/n $f'(x) = 4\sin^3 x \cdot \cos x - 4\cos^3 x \sin x$

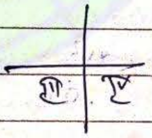
$\Rightarrow 4\sin x \cos x \left(\sin^2 x - \cos^2 x\right)$

~~$\Rightarrow 4\sin x \cos x (\sin x + \cos x)(\sin x - \cos x)$~~

$= 2\sin 2x \left(-\cos 2x\right)$

$f'(x) = -\sin 4x > 0$

$\therefore \sin 4x < 0$



$\pi < 4x < 2\pi$ $x - x^2$

$\frac{\pi}{4} < x < \frac{\pi}{2}$

eg) Let $f(x) = x \cdot e^{x(1-x)}$ find the interval of "$\uparrow$"?

Sol/n $f'(x) = x \cdot e^{x(1-x)} (1 - 2x) + e^{x(1-x)}$

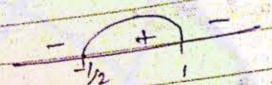
$= e^{x(1-x)} \left(x(1-2x) - 1 \right)$

$= e^{x(1-x)} \left[x - 2x^2 + 1 \right)$

$= -e^{x(1-x)} \left(2x^2 - x - 1 \right)$

RA1

$= -e^{x(1-x)}(2x+1)(x-1)$



$\Rightarrow \left(-\frac{1}{2}, 1\right)$

eg 7) Let $f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$

find the Interval in which f is "↑"

Sol^n

$$\boxed{\text{Leibnitz formula:} \quad \frac{d}{dx}\int_{\phi x}^{\psi x} f(t)\,dt = f(\psi(x))\,\psi'(x) - f(\phi(x)).\phi'(x)}$$

$f'(x) = e^{-(x^2+1)^2} .2x - e^{-(x^2)^2} .2x$

$f'(x) = 2x\left(e^{-(x^2+1)^2} - e^{-(x^2)^2}\right) > 0$

$\ominus$

[less − more]

$e^3 < e^5$

$e^{-3} > e^{-5}$

$\downarrow$

R^-

$\Rightarrow (-\infty, 0)$ RA1

eg 8) Let $f''(x) < 0$ and $g(x) = f(2-x) + f(4+x)$ find the Interval in which $g(x)$ is "↑"?

Sol^n

sol^n

$$g(x) = f(2-x) + f(4+x)$$

$$\downarrow$$

$$g'(x) = -f'(2-x) + f'(4+x) > 0$$

$$\Rightarrow f'(4+x) > f'(2-x)$$

$4+x > 2-x$

$2x > -2$

$\boxed{x > -1}$

(X)

$4+x < 2-x$

$2x < -2$

$\boxed{x < -1}$

(✓)

egg) Let $f(x) = x^{1/x}$; $x > 0$

f is ↑ on $(0,e)$ and ↓ (e,∞)

farthe:

select bigger e^π and π^e ?

sol^n

$$\log f(x) = \frac{\log x}{x}$$

$$f'(x) = f(x)\left(\frac{x\cdot\frac{1}{x} - \log x}{x^2}\right)$$

$$\therefore f'(x) = \underset{⊕}{f(x)}\left(\frac{1-\log x}{\underset{⊕}{x^2}}\right)$$

$1-\log x > 0$	$1-\log x < 0$
$\log x < 1$	$\log x < 1$
$x < e$	$x > e$

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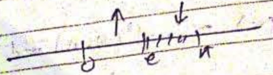
$e < \pi$

$f(e) > f(\pi)$

$e^{1/e} > \pi^{1/\pi}$

$e^{\pi} > \pi^{e}$



(10) Let $f(x) = kx + 3\sin x$ is ↑ on R then find set of value of "k"?

Sol^n $f'(x) = k + 3\cos x > 0$

$k > \underset{\text{value from } -3 \text{ to } 3}{\boxed{-3\cos x}}$

$\Rightarrow k > 3$ Ans

sheet (11) The value of "a" in order that $f(x) = \sin x - \cos x - ax + b$ ↓ $\forall x \in R$ is given by

(i) $a \geq \sqrt{2}$ (ii) $a < \sqrt{2}$ (iii) $a \geq 1$ (iv) $a < 1$

Sol^n $f'(x) = \cos x + \sin x - a < 0$

$a > \underbrace{\cos x + \sin x}_{\text{from } -\sqrt{2} \text{ to } \sqrt{2}}$

$a > \sqrt{2}$ Ans

eg (12) If $f(x) = -\dfrac{p^2-1}{p^2+1}x^3 - 7x + \log 2$ is a decreasing function of x in "R" then set of possible value of "p".

(a) (

Sol^n $f'(x$

sheet (Q13)

Sol.

$b^2 - 4ac$

$4a^2 - 4 \times$

$\boxed{4a^2 < 16}$

(a) $(-1,1)$ (b) $[1,\infty)$ (c) $(-\infty,-1]$ (d) none.

Sol/ $f'(x) = \left(\frac{p^2-1}{p^2+1}\right)\cdot \cancel{3}x^2 - \cancel{3} < 0$

$$\frac{p^2-1}{p^2+1}x^2 < 1$$

$$\therefore \frac{p^2-1}{p^2+1} < \boxed{\frac{1}{x^2}}$$

$0 \to \infty$

$$\frac{p^2-1}{p^2+1} < 0$$

$$\therefore (p-1)(p+1) < 0$$

$1+x < 2-x$

$2x < -2$

$\boxed{x < -1}$

$+ \quad - \quad +$

$-1 \quad 1$

check (Q13) $f(x) = x^3 + ax^2 + bx + 5 \cdot \sin^2 x$ is an "↑" function in "R" in "a" and "b" satisfied

(i) $a^2 - 3b - 15 > 0$ (ii) $a^2 - 3b + 15 > 0$ (iii) $a^2 - 3b + 15 < 0$

(iv) $a > 0$, $b > 0$

Sol/ $f'(x) = 3x^2 + 2ax + b + 5\sin 2x > 0$

~~$3x^2 + 2ax + b$ ... $< \sin x$~~

$$\boxed{3x^2 + 2ax + b} > \boxed{-5\sin 2x}$$

$[-5, 5]$

Quadratic ⇓

$\frac{-D}{4a} > 5$

* $b^2 - 4ac$

$4a^2 - 4 \times 3 \times b$

$\boxed{4a^2 - 12b}$

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Tangent and Normals
monotonicity L-2

$$\Rightarrow -\frac{[4a^2-4\cdot 7\cdot 5]}{4\cdot 7} > \cancel{0}\ 5$$

$a^2 - 7\cancel{0} < +5$

$\therefore a^2 - 7\cancel{0} + 15 < 0$

that ⑭ if $f(x) = (ab - b^2 - 2)x + \int_0^x (\cos^4\theta + \sin^4\theta)\, d\theta$

is ↓ on R, $b \in R$, then

(a) $a \in (0, \sqrt{6})$ (b) $a \in (-\sqrt{6}, \sqrt{6})$

(c) $a \in (-\sqrt{6}, 0)$ (d) $a \in (-2, 2)$

sol^n $f'(x) = \underbrace{ab - b^2 - 2}\ + \underbrace{\cos^4 x + \sin^4 x} < 0$

$+ 2\sin^2 x \cdot \cos^2 x$

$- 2\sin^2 x \cdot \cos^2 x$

$$ab - b^2 \cancel{-} 2 + 1 < \boxed{\frac{(\sin 2x)^2}{2}}$$

$a \to 1/2$

$ab - b^2 - 1 < 0$

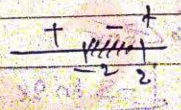
$\therefore$ ① $b^2 - ab + 1 > 0$

↓

$D < 0$

⊕

$a^2 - 4 < 0$



eg15 Let $f(x)=$

(1) $(-$

sol^n $f'(x)$

$\therefore 3x$

$\Rightarrow$

eg16 Le

sol^n

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Q15) Let $f(x) = x^3 + \lambda x^2 + 5x + \sin 2x$ is increasing on R then $\lambda \in$

(i) $(-\infty, 3)$ (ii) $(-3, 3)$ (iii) $(3, \infty)$ (iv) ——

Sol^n

$f'(x) = 3x^2 + 2\lambda x + 5 + 2\cos 2x > 0$ or < 0

$\therefore 3x^2 + 2\lambda x + 5 > -2\cos 2x$ or $< -2\cos 2x$

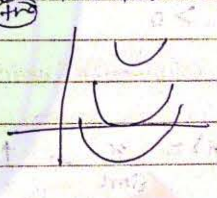
$\Rightarrow 3x^2 + 2\lambda x + 5 > 0$ or < -2

$3x^2 + 2\lambda x + 3 > 0$ or $\underline{3x^2 + 2\lambda x + 7 < 0}$
$\downarrow$
$\textcircled{+ve}$ Not possible

$\therefore D < 0$

$\therefore 4\lambda^2 - 4 \cdot 3 \cdot 3 < 0$

$\lambda \in (-3\ 3)$



Q16) Let $f(x) = \dfrac{\log(\pi + x)}{\log(e + x)}$ $\downarrow$ in

(i) $(-\infty, \infty)$ (ii) $(0, \infty)$

(iii) $(-\infty, 0)$ (iv) None

Sol^n

$$f'(x) = \frac{\log(e+x) \cdot \frac{1}{\pi + x} - \log(\pi + x) \cdot \frac{1}{(e+x)}}{(\log(e+x))^2}$$

$$\therefore f'(x) = \frac{(e+x) \cdot \log(e+x) - (\pi + x) \cdot \log(\pi + x)}{\underset{\textcircled{+}}{(\pi + x)}\ \underset{\textcircled{+}}{(\log(e+x))^2}\ \underset{\textcircled{+}}{(e+x)}} < 0$$

Test: $(0, \infty)$
$e < \pi$

$e+x < \pi+x$

$\log(e+x) < \log(\pi+x)$

$\log(e+x)\log(e+x) < (\pi+x)\log(\pi+x)$

Question based on Increasing and decreasing functions

eg) $f(x) = \frac{a\sin x + b\cos x}{c\sin x + d\cos x}$

Increasing (↑) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} > 0$

Decreasing (↓) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} < 0$

eg) P.T. $f(x) = \frac{x}{\sin x}$ is "↑" while $g(x) = \frac{x}{\tan x}$ is ↓ on $(0,1)$?

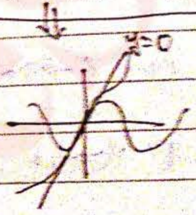
Sol $f'(x) = \frac{\sin x - x\cos x}{(\sin^2 x)}$ ⊕

$f'(x) = \frac{\cos x\left(\tan x - x\right)}{(\sin^2 x)}$ ⊕

∴

$f(x) = \frac{x}{\sin x}$ is "↑" (Increasing function)

Note
$0<\theta<1$
$\tan\theta > \theta$
$\sin\theta < \theta$
⇓



Now

$g(x) = \frac{x}{\tan x}$

$g'(x) = \frac{\tan x - x\cdot\sec^2 x}{(\tan^2 x)}$

$= \frac{\sec^2 x(\sin x\cdot\cos x - x)}{\tan^2 x}$

$= \frac{\sec^2 x(\sin 2x - 2x)}{2\tan^2 x} < 0$

Note

$P(x) = \sin x - x\cos x$ ↑

$P'(x) = \cancel{\cos x} - \cancel{\cos x} + x\sin x$

$P'(x) = \underset{\oplus}{x} \cdot \underset{\oplus}{\sin x} \quad \oplus$

$x > 0$

$P(x) > P(0)$

$\sin x - x\cos x > 0$

✱ Comparsion of two function →

In a given Interval, to know $f(x) > g(x)$ or $g(x) > f(x)$

method: →

Take a new function $H(x) = f(x) - g(x)$

then

differenciate $H(x)$ to know whether H is "↑" or "↓"

then apply $H(x)$ to given Inequality.

If $H(x)$ ↑, then Inequality remains same otherwise change.

eg.) Let $x > 0$

then

$$\log(1+x) < \frac{x}{1+x} \quad (T/F)$$

Sol $H(x) = \log(1+x) - \dfrac{x}{1+x} \quad (↑)$

$$H'(x) = \frac{1}{1+x} - \frac{(1+x) - x}{(1+x)^2}$$

$$H'(x) = \frac{1}{1+x} - \frac{1}{(1+x)^2}$$

$$H'(x) = \frac{\cancel{1}+x-\cancel{1}}{(1+x)^2} = \frac{x}{(1+x)^2} \quad \oplus$$

$x > 0$

$\Rightarrow H(x) > H(0)$

$\Rightarrow \log(1+x) - \frac{x}{1+x} > 0$

$\therefore \log(1+x) > \frac{x}{1+x}$

eg) eg Let $x \geq 0$
then
$1 + x\log(x + \sqrt{x^2+1} \geq \sqrt{1+x^2}$. (T/F)

so/n $H(x) = 1 + x\log(x + \sqrt{x^2+1} - \sqrt{1+x^2}$

$\frac{1}{2}-1$
$-\frac{1}{2}$

~~$H'(x) = x \log \frac{x\left(1+\frac{2x}{2\sqrt{x^2+1}}\right)}{(x+\sqrt{x^2+1})} + \log(x+\sqrt{x^2+1}) - \frac{2x}{2\sqrt{1+x^2}}$~~

$H'(x) = x \cdot \frac{1}{\sqrt{1+x^2}} + \log(x + \sqrt{x^2+1}) - \frac{x}{\sqrt{1+x^2}}$

$H'(x) = \log(x + \sqrt{x^2+1}) > 0$

$\therefore H(x)$ "↑"

$x \geq 0$

$H(x) \geq H(0)$

$\Rightarrow 1 + x\log(x + \sqrt{x^2+1}) - \sqrt{1+x^2} \geq 0$

$\therefore 1 + x\log(x + \sqrt{x^2+1}) \geq \sqrt{1+x^2}$

eg) In $(0,1)$

(i) $e^x < 1+x$

(iii) $\sin x > x$ (ii) $\log(1+x) < x$

(iv) $\log x > x$

Sol^n

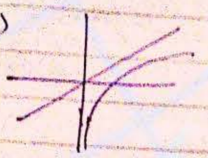
(i)

(ii) $H(x) = \log(1+x) - x$ so "$\downarrow$"

$H'(x) = \frac{1}{1+x} - 1 = \frac{-x}{1+x} < 0$

$\therefore x > 0$

(iii)



$H(x) < H(0)$ { छोटे वाले Argument पर छोटा Value प्राप्त होता है

$\therefore \log(1+x) - x < 0$

$\therefore \log(1+x) < x$

(1-2, 0-15)

eg 4) $f_1(x) = 2x$, $f_2(x) = 3\sin x - x\cos x$; $x \in (0, \frac{\pi}{2})$

(i) $f_1(x) < f_2(x)$ (ii) $f_1(x) > f_2(x)$

(iii) $f_1(-x) < f_2(x)$ (iv) None of these.

Sol^n

$H(x) = f_1 - f_2 = 2x - 3\sin x + x\cos x$, so function is "$\uparrow$"

$H'(x) = 2 - 3\cos x + [-x\sin x + \cos x]$

$= 2 - 3\cos x + \cos x - x\sin x$

$= 2 - 2\cos x - x\sin x$

$H'(x) = 2(1-\cos x) - x.\sin x$

$H'(x) = 2.2\sin^2\frac{x}{2} - x.2\sin\frac{x}{2}.\cos\frac{x}{2}$

$H'(x) = 2\underset{+}{\sin\frac{x}{2}}\left(2\sin\frac{x}{2} - x\cos\frac{x}{2}\right)$

$H'(x) = \underset{+}{2\sin\frac{x}{2}}.\underset{+}{2\cos\frac{x}{2}}\underbrace{\left(\tan\frac{x}{2} - \frac{x}{2}\right)}_{+}$

$$x > 0$$

$$H(x) > H(0)$$

$$d_1 - d_2 > 0$$

$$d_1 > d_2$$

(Ex-1) (monotone function)

Q17) $f'(x) = \underset{\oplus}{100x^{99}} + \underset{\oplus}{\cos x} + 0 \geq 0$! f ↑

A (0,1)

read 36)

(18) $f'(x) = \underset{\oplus}{g(x)} \ \underbrace{(x-\lambda)^2}_{\oplus}$

A $(x=\lambda, \ g(x) \to +ve \to f$ ↑

20) $f'(x) = 3x^2 - 12x - 36$

$f'(x) = 3(x^2-4x-12)$

$= 3(x+2)(x-6)$

$+ \quad - \quad +$

↑ -2 ↓ 6 ↑

(29) i) ✗ $(e^x - e^{-x}) = e^x + e^{-x}$

ii) e^{-x} ↑

e^x ↑ $-\frac{1}{x}$ ↑

iii) $(e^x+e^{-x}) = e^x - e^{-x} \begin{cases} \to +ve; & x>0 \\ \to -ve; & x<0 \end{cases}$

No-m

31) $f' = x \cdot e^{4-x}(-1) + e^{4-x}$

$= -e^{4-x}(x-1)$

$+ \quad -$

↑ 1 ↓

44) (L-3 → Diff Q-22)

44) ; $x + \cos x \uparrow$

$\downarrow$

$\underline{1 - \sin x}$

$\downarrow -1 \text{ to } 1$

$\underbrace{\qquad}_{0 \to 2}$

(45) $x + \cos x \; \boxed{-a}$

$\uparrow$

Level ⇒ 2

(5) $f' = (6 \cdot s^2 - 6 \cdot s + 12)\, c$

$= 6\underbrace{(s^2 - s + 2)}_{\ominus} \cdot c \; \oplus$

$\underbrace{D = 1 - 8}_{f \uparrow}$

(6) $\uparrow f : [0, \infty) \to [0, \infty)$

$\downarrow g : [0, \infty) \to [0, \infty)$

$f \circ g : [0 \to \infty)$

$h(x) = f(g(x))$

$p(x) = h(x) - h(1)$

$x \geq 0$

$\Rightarrow p(x) \leq p(0)$

$h(x) - \cancel{h(1)} \le \cancel{h}(0) - \cancel{h(1)}$

$h(x) \le 0 \Rightarrow \boxed{h(x) = 0}$

and 26)

09) $f'(x) = (a^2 - 2a - 2) - \sin x > 0$ or < 0

$\therefore a^2 - 2a - 2 > 1$ or $a^2 - 2a - 2 < -1$

(17) $f' > 0,\ g' < 0$

$\Rightarrow fog \cdot gof \downarrow$

(18) Invertible ⇒ one-one onto

↓

either) or (

(19) Non-Invertible

= R − (Invertible)

(20) $0 < x < \frac{\pi}{2}$

$0 < \sin x < 1$

$1 > \cos(\sin x) > \underbrace{\cos 1}_{(1/2)}$

$0 < x < \pi/2$

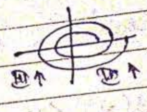
$1 > \cos x > 0$

$\underbrace{\sin 1}_{\left(\frac{\sqrt{3}}{2}\right)} > \sin(\cos x) > 0$

$x = 2$

Level → 3

1) $\cos\left(\frac{\pi}{x}\right)$



$\downarrow$

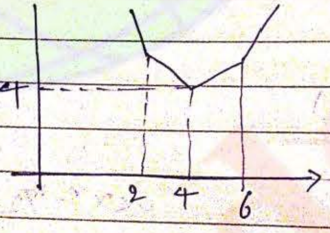
$\cancel{\pi} < \frac{\cancel{\pi}}{x} < 2\cancel{\pi} \longrightarrow \frac{1}{1} > x > \frac{1}{2}$

$3\cancel{\pi} < \frac{\cancel{\pi}}{x} < 4\cancel{\pi} \longrightarrow \frac{1}{3} > x > \frac{1}{4}$

$\frac{1}{\text{even}} < x < \frac{1}{\text{odd}}$

(C) Ans

Q11



$$\begin{tikzcd} & f(x) - a = 0 \arrow[dl] \arrow[dr] & \\ y = f(x) & & y = a \end{tikzcd}$$

$\boxed{a < 4}$

Maxima ~ minima

Critical point

Let $x = x_0$ is a point of Domain, such that

(i) $f(x)$ is ~~not~~ non-differenciable at $x = x_0$ or ($f'(x_0)$ = does not exist)

or (ii) $f'(x_0) = 0$

then

$x = x_0$ is called a critical point.

eg.) $f(x) = \frac{1}{x}$, critical point is $x = 0$ (X)

$\frac{1}{3} - 1$

$-\frac{2}{3}$

eg2.) Find critical point of $f(x) = x^{1/3}(x-4)$?

Sol: $\cancel{f'(x) = x^{1/3} + \frac{(x-4)}{3} x^{-2/3}}$

$f'(x) = x^{4/3} - 4x^{1/3}$

$f'(x) = \frac{4}{3} x^{1/3} - 4 \cdot \frac{1}{3} x^{-2/3}$

$f'(x) = \frac{4}{3}\left(x^{1/3} - \frac{1}{x^{2/3}}\right)$

$f'(x) = \frac{4}{3}\left(\frac{x-1}{x^{2/3}}\right)$ → Not defined at $x = 0$; → $= 0$ at $x = 1$

eg3.) Find critical point of $f(x) = (x-2)^{2/3}(2x+1)$

Sol: ~~f(x)~~ $f(x) = \cancel{2x^2 + x} - \frac{1}{2}$

~~f(x) = (x-2)~~

$f'(x) = (x-2)^{2/3} \cdot 2 + (2x+1) \cdot \frac{2}{3} \cdot \frac{1}{(x-2)^{1/3}}$

$= 2\left[\frac{3x-6+2x+1}{3(x-2)^{1/3}}\right]$

$= \frac{2(5x-5)}{3(x-2)^{1/3}}$ → $x = 1$; → Not def $x = 2$

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Q3.) Find ~~value of~~ set of value of a for which $f(x) = (a^2 - 3a + 2)\cos\frac{x}{2} + (a-1)x + 2$

has no critical point

Sol. $\underbrace{f(x) = (a^2-3a+2)\cos\frac{x}{2} + (a-1)x+2}_{\text{Everywhere differentiable.}}$

$$f'(x) = -(a^2-3a+2)\sin\frac{x}{2}\cdot\frac{1}{2} + (a-1) \neq 0$$

Note

$A\cdot B = 0 \Rightarrow A = 0$ $\boxed{\text{or}}$ $B = 0$

$A\cdot B \neq 0 \Rightarrow A \neq 0$ ~~or~~ $\boxed{\text{and}}$ $B \neq 0$

$$= (a-1)\left[-\frac{1}{2}(a-2)\sin\frac{x}{2} + 1\right] \neq 0$$

$a - 1 \neq 0$ AND $-\left(\frac{a-2}{2}\right)\sin\frac{x}{2} + 1 \neq 0$

$\boxed{a \neq 1}$ AND $\sin\frac{x}{2} \neq \frac{2}{a-2}$

$$\underbrace{\frac{a}{a-2} < -1 \quad \cup \quad \frac{a}{a-2} > 1}$$

~~$\frac{a}{a-2}+1<0$~~ $\boxed{(0,4) - \{2\}}$

$\therefore (0,4) - [1,2]$

Ta

EER POINT

Let the tangent t
point x = a b
f '(a – h) > 0 and
small positive nu
point is-
(1) a maximum
(2) a minimum
(3) both maximu
(4) neither maxi

If $f(x) = x^3 + ax$
and maxima at
(1) a = – 3, b =
(2) a = 3, b = 9
(3) a = – 3, b =
(4) none of the

If y = a log|x|
at x = – 1 and
(1) a = 2, b =
(3) a = –2, b

Let f(x) = {

f has ?
(1) a local m
(2) no local
(3) a local
(4) no extre

CAREER
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R POINT, C

+ 6(a − 7) x + 3(a
on is α & β and α
α)(x − β)
. & β is min.
s +ve then β is also
−7) / 3 > 0 and α.β
get a∈(−∞, −3) ∪

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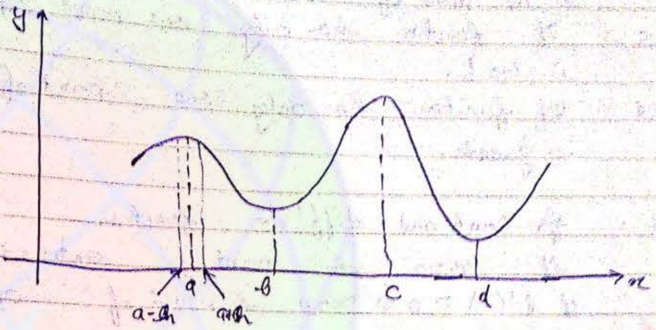
[L-4]
monotonocity → (L-1)

Local maxima and Local minima →



(1) At $x = a$;

If $f(a) > f(a+h)$

and $f(a) > f(a-h)$

then

$x = a$ is called point of local maxima

(2) At $x = b$;

If $f(b) < f(b+h)$

and $f(b) < f(b-h)$

then

$x = b$ is called point of local minima

Points :→

(I) A function may have more than one points of minima and maxima

(II) Combindly, point of minima } "point of extrema"
point of maxima }

(III) minima may be greater than maxima.

classmate
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(4) If a function is monotonic in open Interval th
is no point of extrema.

(5) If function has only one point of min ⇒ that
is least.

(6) If function has only one point of max ⇒ that
is greate.

(7) for conti and diff'ble function
if $x=a$ is point of extrema ⇒ $f'(a)=0$ But
if $f'(a)=0 ⇒ x=a$ is point of extrema.

(8) Let $f(x) = \log(g(x))$ or $e^{g(x)}$ the point of g
and $f(x)$ will be same.

(9) If at $x=a$, $f(x)$ is extrema than at the same point $x=a$, $\log f(x)$ and $e^{f(x)}$ will be extremum.

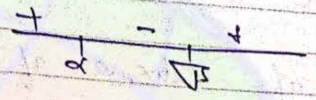
First Derivative test

1) If at $x=a$, $f'(x)$ changes it's sign from −ve to +ve ⇒ $x=a$ is a point of local minima

(2) If at $x=a$, $f'(x)$ changes it's sign from +ve to −ve ⇒ $x=a$ is point of maximum

(3) If at $x=a$, $f'(x)$ does not changes it's sign ⇒ $x=a$ is neither a point of min nor a point of max.

Like : $f'(x) = (x-\alpha)^{odd}(x-\beta)^{even}(x-\gamma)^{odd}$

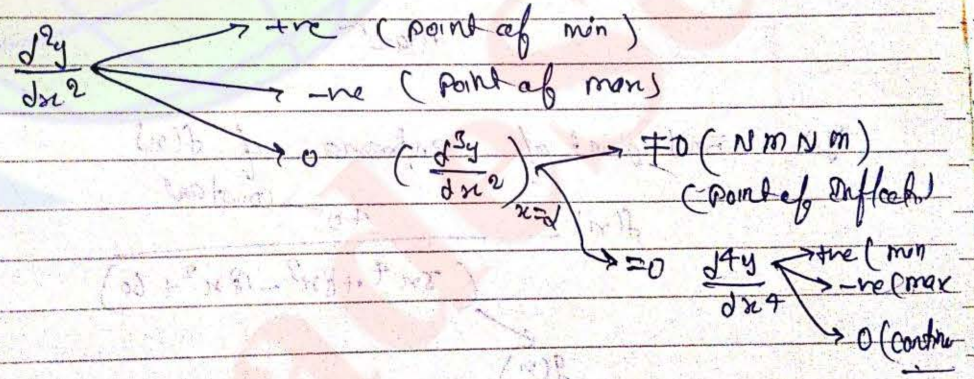


But $x = \beta$ = N M N M

Higher Derivative test :-

Let $y = f(x)$ be a function

Such that $f'(x) = 0 \to$ gives $x = \alpha$



eg.) Let $f'(a) = f''(a) = \text{———} f^{n-1}(a) = 0$

and $f^n(a) \neq 0$ if $x=a$ is the point of max. then

① n = even and ② $f^n(a) < 0$

q1→ let $f'(c) < 0$ and $f''(c) > 0$

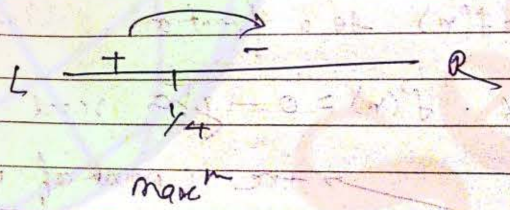
then at $x = c$ the point is NMNM

q→ Find point of extrema of $f(x) = x^{25}(1-x)^{75}$

$$f'(x) = x^{25} \cdot 75(1-x)^{45}(-1) + 25x^{24}(1-x)^{75}$$

$$f'(x) = 25 \cdot x^{24}(1-x)^{74}[-3x + 1 - x]$$

$$f'(x) = 25 \cdot x^{24}(1-x)^{74}\left[x - \frac{1}{4}\right](-4)$$



q.) Find point of extrema of $f(x)$

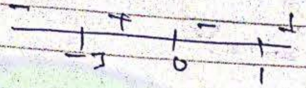
$$f(x) = \frac{\overset{\xrightarrow{\text{constant}}}{40}}{\underbrace{(3x^4 + 8x^3 - 18x^2 + 60)}_{g(x)}}$$

$$g(x) = 3x^4 + 8x^3 - 18x^2 + 60$$

$$g'(x) = 12x^3 + 24x^2 - 36x$$

$$= 12x(x^2 + 2x - 3)$$

$$= 12x(x+3)(x-1)$$



g(x) ; min max min

f(x) ; max min max

eg:-> Let $0 < a_1 < a_2 < a_3 \text{ --- } < a_n$

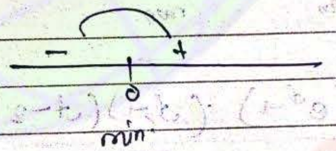
then

$$f(x) = a_0 + a_1 x^2 + a_2 x^4 + \text{ --- } + a_n x^{2n}$$

has a point of local ?

$$f'(x) = 0 + 2a_1 x + 4a_2 x^3 + \text{ --- } + 2n\, a_n x^{2n-1}$$

$$f'(x) = 2x\left(a_1 + 2a_2 x^2 + \text{ --- } + n a_n x^{2n-2}\right)$$



eg.) Let $f(x) = \dfrac{x}{1+x\tan x}$, $x \in \left(0, \dfrac{\pi}{2}\right)$

has a point of local ?

$$f'(x) = \frac{(1+x\tan x)\cdot 1 - x\,(0 + x\sec^2 x + \tan x)}{(1+x\tan x)^2}$$

$$= \frac{1 + x\tan x - x^2\sec^2 x - x\tan x}{(1+x\tan x)^2}$$

$$= \frac{1 - x^2\sec^2 x}{(1+x\tan x)^2}$$

$$f'(x) = \frac{\cos^2 x - x^2}{\cos^2 x\,(1 + x\tan x)^2}$$

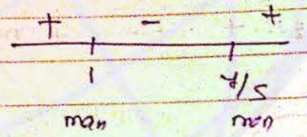
$$\left(0 \longrightarrow \pi/2\right)$$

⊕ ⊖

max

eg.) Find extreme point of $f(x) = \int_1^x 2(t-1)(t-2)^3 + 3(t-1)^2(t-2)^2\,dt$

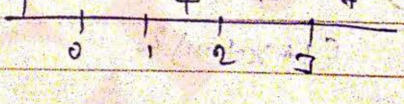
$$f'(x) = 2(x-1)(x-2)^3 + 3(x-1)^2(x-2)^2$$
$$f'(x) = (x-1)(x-2)^2\,[2x-4+3x-3]$$
$$f'(x) = (x-1)(x-2)^2\,(5x-7)$$

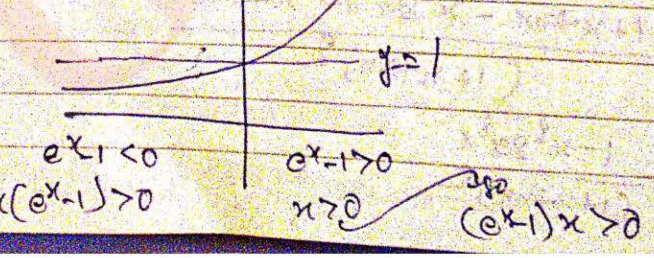


eg.) Let $f(x) = \int_{-1}^{x} t\,(e^t-1)\,(t-1)(t-2)^3(t-3)^5\,dt$

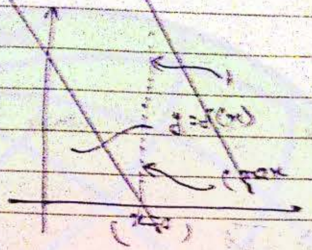
find extreme points;

$$f'(x) = \underbrace{x(e^x-1)}_{+ve}\,(x-1)(x-2)^3(x-3)^5$$





* ~~Maximum and minimum of Discontinuous function~~



ho

—

eg.) Let m(b) be the min value of $f(x) = (1+b^2)x^2 + 2bx + 1$

find Range of $m(b) = 1$

$$m(b) = \frac{-D}{4a}$$

$$m(b) = \frac{-(4b^2 - 4(1+b^2))}{4(1+b^2)}$$

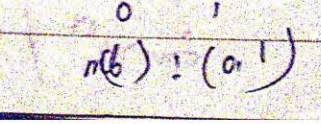
$$m(b) = \frac{1}{1+b^2}$$

$$y + yx^2 = 1$$

$$yx^2 = 1-y$$

$$x^2 = \frac{1-y}{y} \geq 0$$

$$\frac{y-1}{y} \leq 0$$



m(b) : (0, 1]

eg)
$f(x) = x^2 - 2bx + 2c^2$
$g(x) = x^2 - 2cx + b^2$

If the min of "f" is always great than max of "g" then

(1) $|c| > \sqrt{2}|b|$ (2) $c > \sqrt{2}b$
(3) $c < -\sqrt{2}b$ (4) $|c| < \sqrt{2}|b|$

$$\left(\frac{-D}{4a}\right)^f > \left(\frac{-D}{4a}\right)^g$$

$$-\left(\frac{4b^2 - 8c^2}{\cancel{4}}\right) > +\left(\frac{4c^2 + 4b^2}{\cancel{4}}\right)$$

$$-4b^2 + 8c^2 > 4c^2 + 4b^2$$

$$4c^2 > 8b^2$$
$$c^2 > 2b^2$$
$$|c| > \sqrt{2}|b|$$

eg) Let $f(x) = 2^{(x^2-3)^2+27}$ the min of "f" is 2^{27}?

$2^{(x^2-3)^2+27}$ will be minm when Power will be minm.

$(x^2-3)^2 + 27 \rightarrow min$
min $= 0$
when $x = \pm\sqrt{3}$
$f_{min} = 2^{27}$

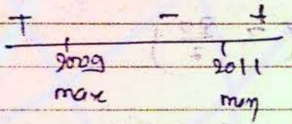
of g

Q) Let 'f' be a function defined on "R" s.t that

$f'(x) = 2010 (x-2009)(x-2010)^2 (x-2011)^3 (x-2012)^4 \ \forall x \in R$

If g is a function defined on R with value in $(0, \infty)$ st $f(x) = \ln g(x) \ \forall x \in R$ then no. of in "R" points at which g has local max. is _

(I) 0 (II) 1 (III) 2 (IV) 5



eg if $f(x) = x^3 - 3(a-7)x^2 + 3(a^2-9)x - 1$ has a the point of max. than a ∈

(1) $(3, \infty) \cup (-\infty, -3)$ ② $(-\infty, -3) \cup (3, \frac{29}{7})$

(3) $(-\infty, 7)$ (4) $(-\infty, 29/7)$

$f'(x) = 3x^2 + 6(a-7)x + 3(a^2-9)$

(1) $D > 0$ ② $\alpha + \beta > 0$ (C) $\alpha\beta > 0$

$D = 6^2(a-7)^2 - 4 \cdot 3 \cdot 3(a^2-9) > 0$

$= 6^2(a^2 - 49 - 14a) - 36(a^2-9) > 0$

$= a^2 + 49 - 14a - a^2 + a > 0$

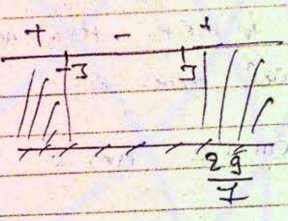
$= 14a < 58$

$a < \frac{29}{7}$

(ii) $\frac{-6(a-7)}{3} > 0$

$a < 7$

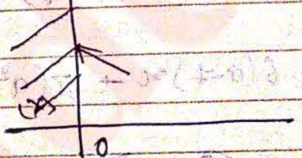
(iii) $\frac{3(a^2-9)}{3} > 0$



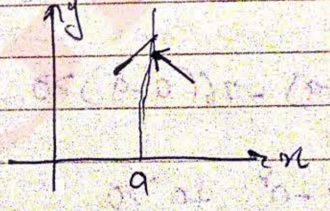
$(-\infty, -3) \cup (3, \frac{29}{7})$

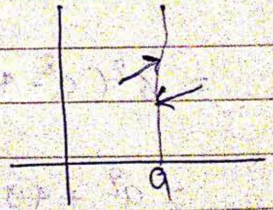
eg.) Let $f(x) = \begin{cases} \cos\frac{\pi}{2}x & ; x > 0 \\ x+a & ; x \leq 0 \end{cases}$

has local max at $x=0$ if $a \in$ —?



min/max of Disc. funcho





(i) Let $f(x) = \frac{\log x}{x}$. find extreme values
i.e. x only

(ii) Let $y = x^x$, $x > 0$ find extreme values?

Q.) find CP of $f(x) = \frac{|x-2|}{x^2}$

$$f(x) = \begin{cases} \frac{x-2}{x^2} & x \geq 2 \\ \frac{-(x-2)}{x^2} & x < 2 \end{cases}$$

$$f'(x) = \begin{cases} \frac{x^2-(x-2)\,2x}{x^2} & x > 2 \quad \text{R.H.D} \\ = \frac{-x^2+4x}{x^2} & \\ \frac{-(-x^2+4x)}{x^2} & x < 2 \quad \text{L.H.D} \end{cases}$$

L.H.D ≠ R.H.D Non-diff

$x = 2$ C.P

$-x^2+4x=0$

$4x = x^2$

$x^2-4x=0$

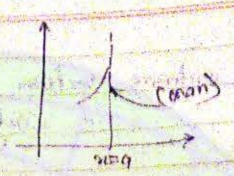
$x(x-4)=0$

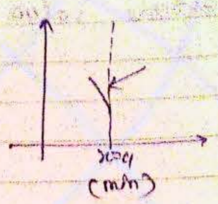
$x=0$ or $x=4$ ✓

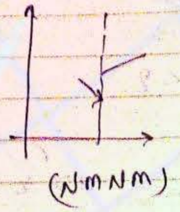
(×)

C. Point = 2, 4

→ Maxima and minima of disc. function →

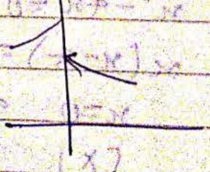






Q.) $\begin{cases} \cos\frac{\pi}{2}x \downarrow & , x > 0 \\ x+a \uparrow & , x \leq 0. \end{cases}$

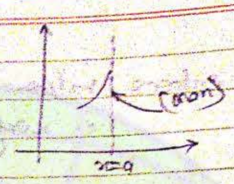
Sol $\begin{cases} \underbrace{-\sin\frac{\pi}{2}x\cdot\frac{\pi}{2}}_{-ve} & , x > 0 \\ \overset{1}{⊕} & , x < 0 \end{cases}$

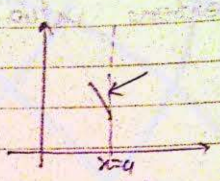


x∈R

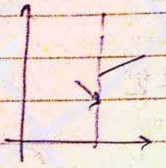
a≥1

* Maxima and minima of disc. function →





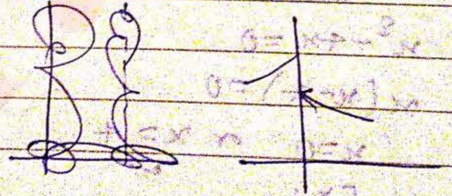
(min)



(NMNM)

Q) $\begin{cases} \cos\frac{\pi}{2}x \downarrow & ; x>0 \\ x+a \uparrow & ; x\le 0. \end{cases}$

Sol $\begin{cases} \underbrace{-\sin\frac{\pi}{2}x\cdot\frac{\pi}{2}}_{-ve} & ; x>0 \\ \underset{⊕}{1} & ; x<0 \end{cases}$



a≥R

a≥1

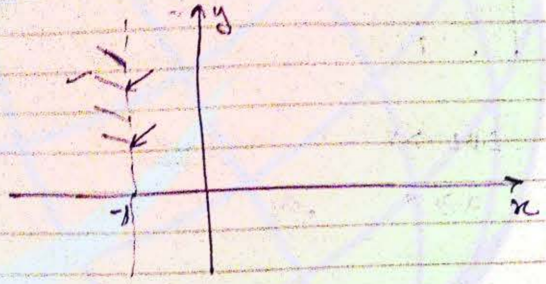
(LAQ) (8.9) Let f: R → R be defined by

$$f(x) = \begin{cases} k-2x, & \text{if } x \le -1 \\ 2x+3, & \text{if } x > -1 \end{cases}$$

If 'f' has a local minima at x = -1, then a possible value of k is?

(A) 1, (B) 0 (C) $-\frac{1}{2}$ (D) -1

Sol^n



$v \le e$

$k+2 \le 1$

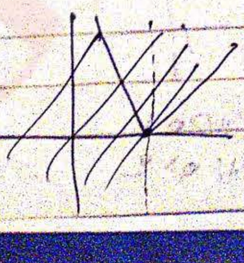
$[k \le -1]$ Ans

9.) Let $f(x) = \begin{cases} |x-1|+a & ; x<1 \\ 2x+3 & ; x \ge 1 \end{cases}$

at x = 1, local min if

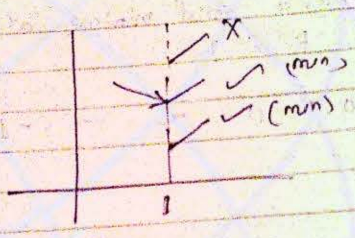
(i) $a \ge 5$ (ii) $a > 5$
(iii) $a \le 5$ (iv) —

Sol



Sol^n

$$\begin{cases} 1-x+a \quad (-1\downarrow) & \because x<1 \\ (2\uparrow) & x\geq 1 \end{cases}$$



L.H.L $\geq$ V

$a \geq 3$ Ans

$\begin{bmatrix}(L-2)^n \\ 0-25\end{bmatrix}$

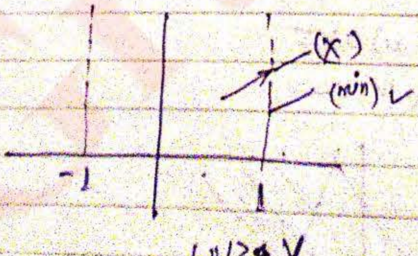
$$f(x) = \begin{cases} \sin^{-1}\lambda + x^2, & 0<x<1 \\ 2x, & x\geq 1 \end{cases}$$

f(x) can have a minimum at $x=1$ then value of λ is

<A> 1 <B> -1 <C> 0 <D> None of these.

Sol^n

$$f'(x) \begin{cases} 2x\uparrow \\ 2\uparrow \end{cases}$$



L.H.L > V

$\sin^{-1}\lambda+1 > 2$

$3n^2\lambda > 1$

$\lambda > 3n1$

~~[illegible]~~

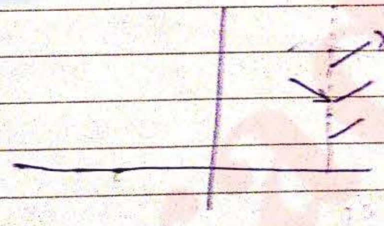
$1 \geq \lambda > 3n1$ Ans

Q) Let $f(x) = \begin{cases} -x^3 + \dfrac{b^3 - b^2 + b - 1}{b^2 + 3b + 2} & ;\ x < 1 \\ 2x - 3 & ;\ x \geq 1 \end{cases}$

1 is local minima?

Sol ~~$f'(x) = \{ -3x^2 + 3b^2 - 2b + 1$~~

$f'(x) = \begin{cases} -3x^2 \ \downarrow & ;\ x < 1 \\ 2 \ \uparrow & ;\ x \geq 1 \end{cases}$



L.H.L $\geq$ V

$\Rightarrow -1 + \dfrac{b^3 - b^2 + b - 1}{(b^2 + 5b + 1)} \geq -1$

$\Rightarrow \dfrac{b^2(b-1) + 1(b-1)}{(b+1)(b+2)} \geq 0$

$$= \frac{(b^2+1)(b-1)}{(b+1)(b+1)} \geq 0$$

★ Global maxima (Greatest) and Global minima (least) →

Let $y = f(x)$ be defined on $[a, b]$

↳ $\frac{dy}{dx} = f'(x) = 0 \Rightarrow x = c_1, c_2, c_3, \ldots c_n \; (a, b)$

$\{f(c_1), f(c_2), f(c_3) \ldots f(c_n), f(b)\}$

select which is higher

select the least one = least

If $f'(x) \neq 0$

$f'(x) > 0$ | $f'(x) < 0$

$f(x) \uparrow$ | $f(x) \downarrow$

Least $= f(a)$ | Least $= f(b)$

Greatest $= f(b)$ | Greatest $= f(a)$

$[f(a), f(b)]$ = Range = Image set //

Q Let $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$, $x \in [0,2]$

find greatest, least and Image set

Sol $f'(x) = 12x^3 - 6x^2 - 12x + 6$

$f'(0) = 0 - 0 - 0 + 6 = 6$

~~$f'(2) = 48 - 16 - 24 + 12$~~

$f'(2) = 96 - 24 - 24 + 6$

$= 96 - 48 + 6$

$= 48 + 6$

$= 54$

So,

$f'(x) = 6x^2(2x+1) - 6(2x-1)$

$= 6(x-1)(x+1)(2x-1)$

~~$12 - 6 - 12 + 6$~~

$12x^2 + 6x - 6$

$x-1 \overline{) 12x^3 - 6x^2 - 12x + 6}$

$-12x^3 - 12x^2$
$\quad +$

$6x^2 - 12x$

$-6x^2 - 6x$
$\quad +$

$-6x + 6$

$-6x + 6$

0

$(x-1)(12x^2 + 6x - 6)$

$x = -1, 1, \frac{1}{2}$

$f(1) = 2$

$f(\frac{1}{2}) = 3/16 - \frac{2}{8} - \frac{6}{4} + \frac{6}{2} + 1 = \frac{29}{16}$

$f(0) = 1$

$f(2) = 21$

$L = 1$, $G = 21$

Image set $[1, 21]$ Ans

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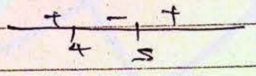
Q.) Find Greatest Value of $f(x) = 2x^3 - 15x^2 + 36x - 48$ in the Interval

$$A = \{x : x^2 + 20 \le 9x\}$$

Sol^n Interval :- $x^2 + 20 \le 9x$

$x^2 - x + 20 \le 0$

$(x-4)(x-5) \le 0$

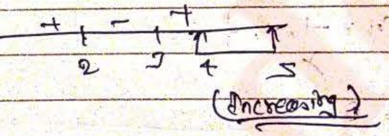


therefore $A = [4, 5]$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6)$

$= 6(x-2)(x-3)$



$f(5) = (\text{greatest}) = 2 \times 125 - 15 \times 25 + 36 \times 5 - 48$

$$\begin{array}{r} = 430 \\ -423 \\ \hline 7 \end{array}$$ Ans

$f(2) = 16 - 60$

Q2) on $[1, e]$ find g and l of $f(x) = x^2 \ln x$?

Sol: $f'(x) = x^2 \cdot \frac{1}{x} + \ln x \cdot 2x$

$= x + 2x \ln x$

$= x(1 + 2\ln x) = 0$

$x = 0$ / $x = e^{-1/2}$

(✗) $x = \frac{1}{\sqrt{e}}$

(✗)

Least $= f(1) = 0$

Great $= f(e) = e^2$

Q3.) Find the difference b/w greatest and least of $f(x) = \sin 2x - x$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$?

Sol: $f'(x) = 2 \cdot \cos 2x - 1 = 0$

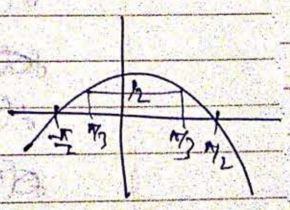
$\frac{\pi}{6}$

$2\cos 2x = 1$

$\cos 2x = \frac{1}{2}$

~~$2x = \cos^{-1}(\frac{\pi}{2})$~~

~~$= \frac{\pi}{6}$~~



$2x = \pm \frac{\pi}{6}$

$x = \pm \frac{\pi}{6} \in [\frac{-\pi}{2}, \frac{\pi}{2}]$

$f(-\frac{\pi}{2}) = -\frac{\sqrt{3}}{2} + \frac{\pi}{6}$

$f(\frac{\pi}{6}) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$

$f(-\frac{\pi}{2}) = \frac{\pi}{2}$

$f(\frac{\pi}{2}) = -\pi/2$

$L = -\frac{\pi}{2}$

$G = \frac{\pi}{2}$

Diff $= \pi$

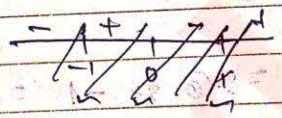
eg.) Find Image set of $[-1,3]$ under the map, functn

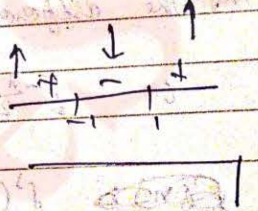
$f(x) = 4x^3 - 12x$?

Sol/ $f'(x) = 12x^2 - 12 = 0$

$= 12(x^2-1) = 0$

$= 12(x-1)(x+1) = 0$





$f(-1) = -4 + 12 = 8$

~~$f(0) = 0$~~

$f(1) = 4 - 12 = -8$ ✓

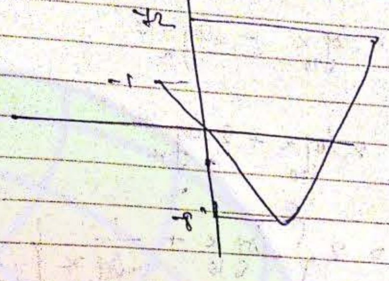
~~$f(0)$~~

$f(3) = 108 - 36 = 72$ ✓

$f(1), f(3) = (-8, 72)$

27×4
108
-36
72

a^b, ab



☆ Use of $A.M \geq G.M$

Least of A.M is G.M
Greatest of G.M is A.M

1.) if $x>0$, $x+\frac{1}{x} \geq 2$

2> So $e^x+e^{-x} \geq 2$

$e^x+\frac{1}{e^x} \geq 2 \longleftarrow$ least

3.) So if $x \in (0, \frac{\pi}{2})$

then
$(\sin x+\operatorname{cosec} x)$ least $=2$

As $\sin x+\frac{1}{\sin x} \geq 2$

(4) if $x>1$, then find least value of

$f(x) = 2\log_{10} x - \log_x (0.01)$

sol. $2\log_{10} x + \log_{x} 0.01$

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$$f(x) = 2\log_{10} x - \log_x 10^{-2}$$

$$= 2\log_{10} x + 2\log_x 10$$

$$= 2\left\{\underset{\oplus}{\log_{10} x} + \underset{\oplus}{\frac{1}{\log_{10} x}}\right\} \geq \underset{\text{i.e. } 4}{\underbrace{2\cdot 2}}$$

5) ⓐ Let α $(0, \frac{\pi}{2})$, then least value of $\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}$ is

Sol A.m $\geq$ G.m

$$\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}} \geq 2\sqrt{\tan^2\alpha}$$

$$f(x) \geq 2\,|\underset{\oplus}{\tan\alpha}|$$

$$f(x) \geq 2\tan\alpha$$

(L-3 Q-2)

Concept →

Greatest of G.m is A.m

$a^2x^4 + b$

monotonocity (1-1,2,3)

Note

① $f(x) = \underbrace{a\sin x + b\cos x}$

Least $= -\sqrt{a^2+b^2}$, Greatest $= \sqrt{a^2+b^2}$

② $f(x) = \underbrace{a\sec x + b\operatorname{cosec} x}$

$x \in \left(0, \frac{\pi}{2}\right)$

Least $= \left(a^{2/3} + b^{2/3}\right)^{3/2}$

③ $f(x) = a^2\sec^2 x + b^2\operatorname{cosec}^2 x$; $x \in \left(0, \frac{\pi}{2}\right)$ and $a>0, b>0$

Least $= (a+b)^2$ / Ans

eg1) Least of $(\sec x + \operatorname{cosec} x) = 2^{3/2} = 2\sqrt{2}$

eg2) Least of $(64\sec x + 27\operatorname{cosec} x) = \left(64^{2/3} + 27^{2/3}\right) = (16+9)^{3/2}$

$= 5^3 = 125$

eg3) Greatest value of $f(x) = 5\cos x + 3\cos\left(x + \frac{\pi}{3}\right) + 3$ is ——?

$f(x) = 5\cos x + 3\left(\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x\right) + 3$

$= \frac{13}{2}\cos x - \frac{3\sqrt{2}}{2}\sin x + 3$

$= \sqrt{\frac{169}{4} + \frac{27}{4}} + 3 = \frac{14}{2} + 3 = 10$ Ans

(iv) Find Greatest value of $f = \dfrac{\sin 2x}{\sin(x+\frac{\pi}{4})}$; $x \in (0, \frac{\pi}{2})$ So|n

So|n
$$f = \frac{2\sin x \cdot \cos x}{\frac{1}{\sqrt{2}}(\sin x + \cos x)}$$

$$f_g = \frac{2\sqrt{2}}{(\sec x + \operatorname{cosec} x)_L}$$

$$\therefore f_g = \frac{\cancel{2\sqrt{2}}}{\cancel{2\sqrt{2}}} = 1$$

(v) Let $a>0$, $b>0$ and $\dfrac{c^4}{r^2} = \dfrac{a^2}{\cos^2\theta} + \dfrac{b^2}{\sin^2\theta}$ find greatest value of r?

So|n
$$\frac{c^4}{r^2} = (a^2\sec^2\theta + b^2\operatorname{cosec}^2\theta)$$

$$r_g^2 = \frac{c^4}{(a^2\sec^2\theta + b^2\operatorname{cosec}^2\theta)}$$

$$r_g^2 = \frac{c^4}{(a+b)^2}$$

$$r_g = \frac{c^2}{a+b} \quad \text{Ans}$$

Q.) A wire of length P is cut into two pieces. A circle and square are formed from these two parts. If sum of areas of circle and square is min then find ratio of diameter of circle and side of square

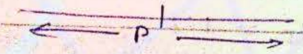
8/4

1.) The Quantity, which we have to minimise or maximise first ~~[illegible]~~ write its expression.

2.) Conversion of single variable

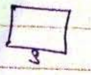
3.) $\frac{d(\text{Quantity})}{d(\text{that variable})} = 0$

Ques



$2\pi r = \frac{P}{2}$

$r = \frac{P}{4\pi}$



$4s = P$

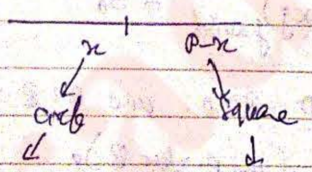
$s = \frac{P}{4}$

Area $= \pi r^2 = \pi\left(\frac{P^2}{16\pi^2}\right)$

Area $= \frac{P^2}{16}$

Sum $= \frac{\pi P^2}{16\pi^2} + \frac{P^2}{16}$

Teacher



$2\pi r = x$

$r = \frac{x}{2\pi}$

$4 \cdot \text{side} = P-x$

$\text{side} = \frac{P-x}{2}$

$S = \quad A_1 = \cancel{\pi}\frac{x^2}{4\pi^2} + A_2 = \frac{(P-x)^2}{16} \leftarrow$ to min

$\frac{dS}{dx} = 0$

$$\frac{2x}{4\pi} - \frac{\cancel{2}(p-x)}{\cancel{16}_4} = 0$$

$$4x = p\pi - \pi x$$

$$x = \frac{p\pi}{4+\pi}$$

Diameter ⇒ $2r = \frac{x}{\pi} = \frac{p}{4+\pi}$

Side ⇒ $\frac{p-x}{4} = \frac{1}{4}\left(p - \frac{p\pi}{4+\pi}\right) = \frac{1}{\cancel{4}}\left(\frac{\cancel{4}p}{4+\pi}\right) = \frac{p}{4+\pi}$

∴ Ratio = 1:1

Note

1) * If sum of two +ve number is given and we require their greatest multiplication

⇒ Divide sum into two equal parts and multiply them

Likes -

1) $x+y = 12$, $x>0$, $y>0$

then $(xy)_{greatest} = 36$

(ii) Let $A>0$, $B>0$ and $A+B = \frac{\pi}{3}$

find $(\tan A \cdot \tan B)_g$

$$\boxed{A = \frac{\pi}{6} = B}$$

then $\tan A \cdot \tan B = \frac{1}{3}$ /

iii) If perimeter of Rectangle is given

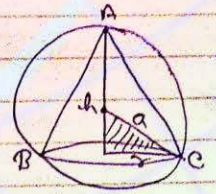
$$2(l+b) = P$$

$$l+b = \frac{P}{2}$$

and
we require its max. Area $\Rightarrow$ Rectangle = Area.

Eg.) Find ratio of height ~~a~~ of a right ~~rectangle~~ circular ~~cone~~ cone of greatest volume Inscribed in a sphere of radius "a" with diameter of sphere?

Sol)



$$r^2 = a^2 - (h-a)^2$$
$$= 2ah - h^2$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(2ah-h^2)h$$

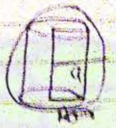
$$\frac{dV}{dh} = \frac{1}{3}\pi(4ah - 3h^2) = 0$$

$$\Rightarrow h = 0 \quad \Big| \quad h = \frac{4a}{3}$$
$$\times$$

$$\frac{\text{Height}}{\text{Diameter}} = \frac{\frac{4a}{3}}{2a} = \frac{2}{3}$$

eg) Find the height of right circular cylinder of max. volume ~~Sec~~ Inscribed in a sphere of radius "A".

Sol



$\frac{1}{3}\pi r^2 h$

~~$V = 2\pi r h$~~

$V = \pi r^2 h$

~~$\frac{dv}{dh} = \pi r^2$~~

~~$\frac{dr}{dr} = 2\pi$~~

Tarike

$r^2 = a^2 - \frac{h^2}{4}$

$V = \pi r^2 h$

$V = \pi\left(a^2 h - \frac{h^3}{4}\right)$

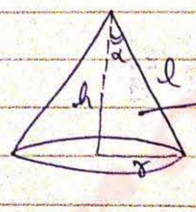
$\frac{dv}{dh} = \pi\left(a^2 - \frac{3h^2}{4}\right) = 0$

$4a^2 = 3h^2$

$h = \frac{2a}{\sqrt{3}}$

Q52 If volume of right circula cone of given slant height is maximum then prove that semi vertical angle $\boxed{= \tan^{-1}\sqrt{2}}$

Sol



$l^2 = r^2 + h^2$

$r^2 = l^2 - h^2$ —(ii)

$r^2 = 2h^2$ — from eq(i) and eq(ii)

$\frac{r}{h} = \sqrt{2}$

$\tan\alpha = \sqrt{2}$

$\alpha = \tan^{-1}\sqrt{2}$

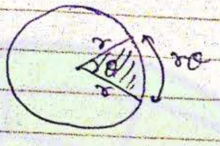
$V = \frac{1}{3}\pi r^2 h$

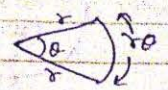
$V = \frac{1}{3}\pi(l^2 - h^2).h$

$\frac{dv}{dh} = \frac{1}{3}\pi(l^2 - 3h^2) = 0$

$l^2 = 3h^2$ —(i)

Sector →





is called sector

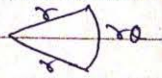
$$\boxed{\text{Area} = \frac{1}{2} r^2 \theta}$$

The max. possible ~~area~~ area that can be enclosed by a wire of length 20cm by making it into a sector is —?

Sol^n

$$\text{Area} = \frac{1}{2} r^2 \theta$$

$$A = \frac{1}{2} r^2 \left(\frac{20-2r}{r}\right)$$



$$r + r + r\theta = 20$$

$$2r + r\theta = 20$$

$$\theta = \left(\frac{20-2r}{r}\right) \text{—(ii)}$$

$$\frac{dA}{dr} = 0$$

$$\frac{dA}{dr} = \frac{1}{2}(20-4r) = 0$$

$$\therefore r = 5 \text{ —(i)}$$

so from eq (i) and eq (ii)

$$\theta = 2$$

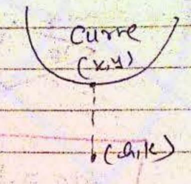
so

$$\text{Area}_{max} = \frac{1}{\cancel{2}} \cdot 25 \cdot \cancel{2}$$

$$= 25 \text{cm}^2 //$$

★ Finding minimum Distance →

1) Curve and a Point :→



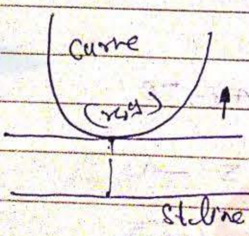
$d = \sqrt{(x-h)^2 + (y-k)^2}$

$d^2 = (x-h)^2 + (y-k)^2$

using the help of curve, conversion to single variable.

$\frac{df}{d(\text{that variable})} = 0$

2) Curve and a line →



A point on the straight line which is nearest to given St. line, at which the drawn point is || to given line eg.) Find the point on the curve

$y = x^2 + 7x + 2$, which is nearest to straight line $y = 3x + 3$?

Sol: $\frac{dy}{dx} = 2x + 7 = 3$

$x = -2$

Curve → $y = -8$

$(-2, -8)$

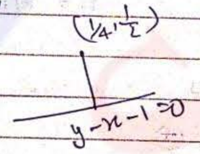
eg2) find min distance b/w $x = y^2$ and $y - x = 1$

Soln $\frac{dy}{dx} = \frac{1}{2y} = 1$

$\therefore y = \frac{1}{2}$

Curve : $x = \frac{1}{4}$

so,



$d = \left| \frac{\frac{1}{2} - \frac{1}{4} - 1}{\sqrt{2}} \right|$

$= \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$ A

eg3) Find the point on the curve $\frac{x^2}{24} - \frac{y^2}{18} = 1$ which is closest to $3x + 2y + 1 = 0$?

Sol: $\frac{2x}{\cancel{24}} - \frac{\cancel{2} \cdot y \cdot y'}{\cancel{18}\,3} = 0$

$y' = \frac{3x}{\cancel{4y}\,2} = \frac{-\cancel{3}}{\cancel{2}}$ $\boxed{x = -2y}$

Curve 1 –

$\frac{4y^2}{24} - \frac{y^2}{18} = 1$ $(6, -3)$, $(-6, 3)$

$\sqrt{13}$ $\left(\frac{11}{\sqrt{13}}\right)$

$\frac{2y^2}{18} = 1$ $\frac{13}{\sqrt{9}}$

$(y = \pm 3)$

eg 4) $f(x) = (x^2-4)^{n+1} \underset{\oplus}{(x^2-x+1)}$

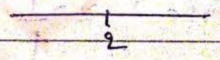
$x = 2$ point of extrema

$n = E/O$?

sol $f(2) = 0$

$f(2^+) = \oplus$

$f(2^-) = (\ominus)^{n+1} \cdot \oplus = \oplus$



$n+1 = E$

$n = \text{odd}$ //

(L-1) (maxima and minima)

(15) ~~10~~ 19)

$f' = 3x^2 - 6x + 3 = 0$ ✓

$f'' = 6x - 6 = 0$ ✓

$f''' = 6 \neq 0$ (Point of Inflection)

25) $f'(x) = 2kx + \frac{2k^2-81}{2}$

$= 2k\frac{9}{4} + \frac{2k^2-81}{2} = 0$

$k = \underset{✓}{-9}, \underset{\times}{\frac{9}{2}},$

$f''(x) = 2k$

24) $x^2 \log\left(\frac{1}{x}\right) = -x^2 \log x$

$f' = -x(1 + 2\log x)$

$\downarrow$ $x = 0$ $\quad \Downarrow$ $x = e^{-1/2}$

(26) $\cos\underbrace{\left[x e^{[x]} + 2x^2 - x\right]}_{x=0 ✓}$, $-1 < x < \infty$

cos → -1, ①

↑ max value / $\therefore \cos 0 = 1$

28) $\cosh x = \frac{e^x + e^{-x}}{2}$

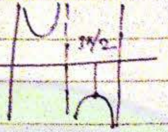
$(\cosh x)' = \frac{e^x - e^{-x}}{2}$ → ⊕ : $x > 0$; → ⊖ : $x < 0$

$-\ |\ +$

0

(min

31)

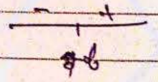


cosec x

$\swarrow \quad \searrow$

(min = 1) (max = -1)

(40) $\cancel{(x-a)^{2n}}\,(x-b)^{2n+1}$



54) $\log_a x + \frac{1}{\log_a x} \geqslant 2$

(43) $y = x(1-x)^2$

$y' = x \cdot 2(1-x)(-1) + (1-x)^2$

$= (1-x)(-2x+1-x)$

$= (x-1)(3x-1)$

$\Rightarrow x = 1, \frac{1}{3}$

$\underbrace{0, \frac{1}{3}, 1, 2}$

44) $y = \cos x + \frac{1}{2} \cdot \sin 2x$

$y' = -\sin x + \cos 2x > 0$

$-\sin x + 1 - 2\sin^2 x > 0$

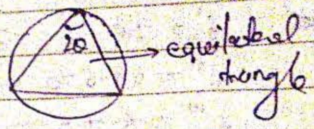
$\therefore 2\sin^2 x + \sin x - 1 < 0$

$= (2\sin x - 1)(\sin x + 1) < 0$

$s = \frac{1}{2}, -1$

$x = \frac{\pi}{6}, \boxed{\frac{7\pi}{2}}$ $f(\pi/6) = 2$

65.)



$2\theta = 60^\circ$

$\theta = 30^\circ$

67)

(a, a, c) , $(a, 0, 0)$

$d = \sqrt{b^2+c^2}$ Ans

Level-2

Q1) s^3+c^3

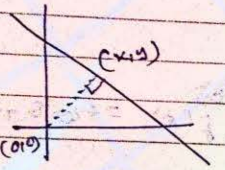
$3s^2 \cdot c - 3c^2 \cdot s = 0$

$s=0$, $c=0$, $s=c$

$\underbrace{x=0 \, , \, x=\pi/2}_{①\ \text{max}}$ $\underbrace{x=\pi/4}$

$\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\cancel{2}}{\cancel{2}\sqrt{2}} = \underline{\underline{\frac{1}{\sqrt{2}}}}$ min

Q4) $\frac{x}{a}+\frac{y}{b}=1$ $\underline{\underline{x^2+y^2}}$



$d = \sqrt{x^2+y^2} = \left| \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} \right|$

$x^2+y^2 = \frac{a^2b^2}{a^2+b^2}$

7) $a^2\sec^2x + b^2\operatorname{cosec}^2x$

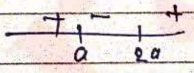
$\cancel{2}a^2 \cdot \sec^2x \cdot \tan x = \cancel{2}b^2 \cdot \operatorname{cosec}^2x \cdot \cot x$

$a^2 \cdot \frac{\sin x}{\cos^3 x} = b^2 \cdot \frac{\cos x}{\sin^3 x}$

$\tan^4 x = \frac{b^2}{a^2}$

$x = \tan^{-1}\sqrt{\frac{b}{a}}$

(9) $f'(x) = 6(x-a)(x-2a)$

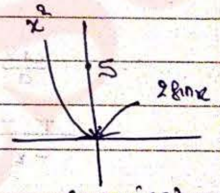


$p^2 = q$

$a^2 = 2a$

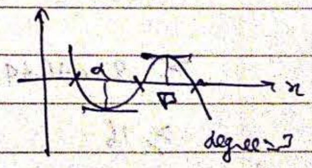
$a=0$, $a=2$
✗ ✓

10)



(maxima)

12)



$f'(\alpha)=0$, $f'(\beta)=0$

$\underbrace{f(\alpha) \cdot f(\beta)}_{\text{opp. sign}} < 0$

Contin -

12) continue

$$\begin{cases} x^3 - 3x + a = 0 \\ 3x^2 - 3 = 0 \end{cases}$$

$x = \pm 1$

~~f(1)·f~~

$f(1) \cdot f(-1) < 0$

$(a-2)(a+2) < 0$

$+ \quad - \quad +$
$-2 \quad 2$

15) Hint:

$f'' = 0$ and $f''' \neq 0$

$ax^2 + 2(a+2)x$

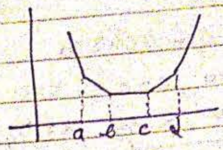
$2ax + 2(a+2) = 0$

$x = -\dfrac{(a+2)}{a} < 0$

$\therefore \dfrac{x+2}{a} > 0$

$+ \quad - \quad +$
$-2 \quad 0$

20)



$f = |x-a| + \cancel{|x-b|} + |x-c| + |x+d|$

$= \cancel{b} - a + c - \cancel{b} + d - b$

$= c + d - a - b$

21) $f = 7x^2 + 4xy + 3y^2$, $\underbrace{x^2 + y^2 = 1}$
$x = \cos\theta$
$y = \sin\theta$

$f(\theta) = 4\cos^2\theta + 4\sin\theta \cdot \cos\theta + 3$

$\dfrac{d f(\theta)}{d\theta} = 0$

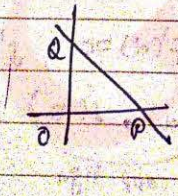
Q16) $= (1+a)(1+b)(1+c)(1+d)$

$= 1 + (a+b+c+d)(ab+bc+cd+da+ac+bd) + (abc+bcd+cda+abd) + abcd$

$\geq 2 + 4 + 6 + 4$

≥ 16

18)



$y - b = m(x-a)$

$OP^2 = a - \dfrac{b}{m}$

$OQ = b - ma$

$S = a - \dfrac{b}{m} + b - ma$

$\dfrac{dS}{dm} = 0 \Rightarrow$

$\dfrac{b}{m^2} - a = 0$

$m = \pm\sqrt{\dfrac{b}{a}}$

Start

Indefinite Integration

Definition. –

Let $\frac{d}{dx} f(x) = F(x)$

then Indefinite Integerals of $F(x)$ is

$$\underbrace{\int}_{\text{symbol of Integerals}} \underbrace{F(x)}_{\text{Integrant}} \underbrace{dx}_{\text{w.r.t which variable}} = \underbrace{f(x) + \overset{\text{Constant of Integeration}}{c}}_{\text{Integral}}$$

Formula :→

1) $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad , \quad n \neq -1$

$\int \frac{1}{\sqrt{x}} \, dx = 2\sqrt{x} + c$

$\int \sqrt{x} = \frac{2}{3} x^{3/2} + c$

$\int \frac{1}{x^2} \, dx = \frac{-1}{x} + c$

2) $\int e^x dx = e^x + c$

3) $\int a^x dx = \frac{a^x}{\log a} + c$

4) $\int \sin x \, dx = -\cos x$

5) $\int \cos x dx = \sin x$

6) $\int \sec^2 x dx = \tan x$

7) $\int \sec x \cdot \tan x = \sec x$

8) $\int \tan x \, dx = \log|\sec x|$

9) $\int \cot x \, dx = \log|\sin x|$

10) $\int \sec x dx = \log|\sec x + \tan^2 x|$

$= \log \tan\left(\frac{x}{2} + \frac{\pi}{4}\right)$

11) $\int \operatorname{cosec} x dx = \log|\operatorname{cosec} x - \cot x|$

$= \log\left|\tan \frac{x}{2}\right|$

12) $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$

13) $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right|$

14) $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right|$

15) $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \frac{x}{a}$

16) $\int \frac{1}{\sqrt{x^2+a^2}} dx = \log |x+\sqrt{x^2+a^2}|$

17) $\int \frac{1}{\sqrt{x^2-a^2}} dx = \log (x+\sqrt{x^2-a^2})$

18) $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log (x+\sqrt{x^2+a^2})$

19) $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log (x+\sqrt{x^2-a^2})$

20) $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$

21) $\int \mathrm{cosec}^2 x\, dx = -\cot x\, dx$

22) $\int \mathrm{cosec}\, x \cdot \cot x\, dx = -\mathrm{cosec}\, x$

1) $\sin^2 x = \frac{1-\cos 2x}{2}$

2) $\cos^2 x = \frac{1+\cos 2x}{2}$

3) $\tan^2 x = \sec^2 x - 1$

4) $\cot^2 x = \mathrm{cosec}^2 x - 1$

16) $\int \frac{1}{\sqrt{x^2+a^2}} dx = \log|x+\sqrt{x^2+a^2}|$

17) $\int \frac{1}{\sqrt{x^2-a^2}} dx = \log(x+\sqrt{x^2-a^2})$

18) $\int \sqrt{x^2+a^2} dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log(x+\sqrt{x^2+a^2})$

19) $\int \sqrt{x^2-a^2} dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log(x+\sqrt{x^2-a^2})$

20) $\int \sqrt{a^2-x^2} dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$

21) $\int \operatorname{cosec}^2 x \, dx = -\cot x \, dx$

22) $\int \operatorname{cosec} x \cdot \cot x \, dx = -\operatorname{cosec} x$

1) $\sin^2 x = \frac{1-\cos 2x}{2}$

2) $\cos^2 x = \frac{1+\cos 2x}{2}$

3) $\tan^2 x = \sec^2 x - 1$

4) $\cot^2 x = \operatorname{cosec}^2 x - 1$

*) $x^{\log_x(\text{Some})} = \text{Some}.$

* $\sec^2 x \cdot \operatorname{cosec}^2 x = \sec^2 x + \operatorname{cosec}^2 x$

$\Downarrow$

$\cancel{\sin^2 + \cos^2}$

$\frac{1}{\sin^2 x \cos^2 x}$

$$\frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} = \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$$

eg1) Integral x^3 w.r.t. x^2 ?

sol/ $\int x^3 \, d(x^2)$

$= \int x^3 \cdot 2x\,dx$

$= 2 \int x^4\,dx = \frac{2}{5} x^5 + c$

eg2) Integral $\sin x$ w.r.t $\cos x$

$\int \sin x \, d(\cos x)$

$= \int \sin x(-\sin x)\,dx$

$= -\int \sin^2 x\,dx \quad \neq \cancel{\frac{\sin^3 x}{3}} + c$

T_1

$$= \frac{-1}{2} \int 1 - \cos 2x \, dx$$

$$= \frac{-1}{2} \left[x - \frac{\sin 2x}{2} \right] + c$$

Q4) $\int \frac{\sin^3 x}{(1+\cos x)} dx$

$\boxed{\sin^2 x = (1-\cos x)(1+\cos x)}$

Soln $= \int \sin x \cdot (1-\cos x) \, dx$

$= \int \sin x - \sin x \cdot \cos x \, dx$

$= \int \sin x - \frac{\sin 2x}{2} dx$

$= \frac{\cos 2x}{4} - \cos x + c$

Q5) $\int \frac{\sec x}{\sec x + \tan x} dx$

~~$\int \frac{\sec x}{\sec x}$~~ $\int \frac{\sec x(\sec x - \tan x)}{1} dx$

$\tan x - \sec x + c$

Q6) $\int 5^{\log x} dx$

$\left(a^{\log_c b} = b^{\log_c a} \right)$

$$\int x^{\log 5}\,dx \quad \searrow x^{n}$$

$$= \frac{x^{\log 5+1}}{\log 5+1} + c$$

$$= ~~\log 5x~~ \frac{x^{\log 5e}}{\log 5e} + c$$

eg) $$\int \frac{\sin 2x}{\sin 3x \cdot \sin 5x}\,dx$$

eg

Soln $$\int \frac{\sin(5x-3x)}{\sin 3x \cdot \sin 5x}$$

$$= \int \frac{\sin 5x \cdot \cos 3x - \cos 5x \cdot \sin 3x}{\sin 3x \cdot \sin 5x}$$

$$= \int \cot 3x - \cot 5x\,dx = \frac{\log \sin 3x}{3} - \log \frac{\sin 5x}{5} + c$$

eg

eg) $$\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}\,dx$$

Soln $$\int \frac{2\cos^2 x - \cancel{1} - 2\cos^2\alpha + \cancel{1}}{\cos x - \cos\alpha}$$

$$2\int \cos x - \cos\alpha\,dx$$

$$= 2(\sin x - x\cos\alpha) + c$$

n integrati

$\int \cos\sqrt{x}dx$

$\int x\tan^2 x$

$\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}$

$\int x^3\tan^{-1}$

$\int \sin^{-1}$

$\int e^x\left(\frac{1}{x}\right.$

$\int e^x\left(\frac{1}{1}\right.$

$\int e^x$

$\int e^x$

$$\int x^{\log 5}\,dx \quad \searrow x^{n}$$

$$= \frac{x^{\log 5+1}}{\log 5+1} + c$$

$$= \cancel{x \log 5x}\ \frac{x^{\log 5e}}{\log 5e} + c$$

eg

eg) $\int \frac{\cancel{\sin 2x}}{\sin 3x \cdot \sin 5x}\,dx$

Sol $\int \frac{\sin(5x-3x)}{\sin 3x \cdot \sin 5x}$

$$= \int \frac{\sin 5x \cdot \cos 3x - \cos 5x \cdot \sin 3x}{\sin 3x \cdot \sin 5x}$$

$$= \int \cot 3x - \cot 5x\,dx = \frac{\log \sin 3x}{3} - \log \frac{\sin 5x}{5} + c$$

eg

eg 7) $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}\,dx$

Sol $\int \frac{2\cos^2 x - \cancel{1} - 2\cos^2\alpha + \cancel{1}}{\cos x - \cos\alpha}$

$$2\int \cos x - \cos\alpha\,dx$$

$$= 2(\sin x - x\cos\alpha) + c$$

Ta

n integratio

$\int\cos\sqrt{x}dx$

$\int x\tan^2 xdx$

a^3+b^3

eg 8) $$\int \frac{(x^3+8)\ (x+1)}{(x^2+4-2x)}\,dx$$

$$= \int \frac{(x+2)\cancel{(x^2+4-2x)}\ (x+1)}{\cancel{x^2+4-2x}}\,dx$$

$$= \int x^2+3x+2\,dx$$

$$= \frac{x^3}{3}+\frac{3}{2}x+2x+c$$

eg 9) $$\int \frac{\sin^4 x+\cos^4 x}{\sin^2 x\cdot\cos^2 x}\,dx$$

Seperation—

$$= \int \tan^2 x+\cot^2 x\,dx$$

$$= \int \sec^2 x+\operatorname{cosec}^2 x-2\,dx$$

$$= \tan x-\cot x-2x+c$$

eg 10) $$\int \frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}\,dx$$

$$\int \frac{x^5-x^4}{x^3-x^2}\,dx$$

$$\int x^2\,dx = \frac{x^3}{3}+c$$

eg 11:) $\int \tan x \tan 2x \cdot \tan 3x \, dx$

Sol: ~~$\int(\tan x + \tan 2x + \tan 3x)$~~

$\left[\because \tan x \cdot \tan 2x \tan 3x = \tan 3x - \tan 2x - \tan x\right]$

Hence

$\tan 3x = \tan(2x + x)$

$= \dfrac{\tan 2x + \tan x}{1 + \tan 2x \cdot \tan}$

$\dfrac{\log \sec 3x}{3} - \dfrac{\log \sec 2x}{2} - \log \sec x + c$

eg 12) $\int \dfrac{x^2 + \cos^2 x}{1+x^2} \operatorname{cosec}^2 x \, dx$?

Sol) $\int \dfrac{1+x^2 - \sin^2 x}{1+x^2} \cdot \operatorname{cosec}^2 x \, dx$

$= \int \operatorname{cosec}^2 x - \dfrac{1}{1+x^2} \, dx$

$= -\cot x - \tan^{-1} x \, dx$

Based on

Q.1

Q.3

Q.5

Q.7

Q.9

Q.11

Q.13

Q.15

$+\frac{1}{4}\Big| + C$

eg 13)

$$\int \frac{1+\cos 4x}{\cot x - \tan x}\, dx$$

$$= \cot x - \tan x$$

$$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$$

$$= \frac{\cos 2x}{\sin x \cdot \cos x}$$

$$= \int \frac{2\cos^2 2x}{\cancel{\cos 2x}} \sin x \cdot \cos x \, dx$$

$$= \frac{1}{2} \int 2\cos 2x \cdot \sin 2x \, dx$$

$$= \frac{1}{2} \int \sin 4x \, dx$$

$$= \frac{-1}{8} \cos 4x + C$$

eg 14)

$$\int \left(1 + 2\tan x \left(\sec x + \tan x\right)\right)^{1/2} dx$$

$$= \int \left(1 + 2\tan x \cdot \sec x + 2\tan^2 x\right)^{1/2} dx$$

$$= \int \left(\sec^2 x + 2\tan x \cdot \sin x + \tan^2 x\right)^{1/2} dx$$

$$= \int \left(\sec x + \tan x\right)^{\cancel{2}\,1/2} dx$$

$$= \log(\sec x + \tan x) + \log \sec x + \log C$$

$$= \log\left(C \cdot \sec x(\sec x + \tan x)\right)$$

(maxima - 1.1 to 1.4)

classmate
Date
Page

Eg 15) Let $\int \frac{1}{x+x^5}\,dx = f(x)$ then find $\int \frac{x^4}{x+x^5}\,dx$?

So|n $\int \frac{1+x^4-1}{x+x^5}$

$= \int \frac{\cancel{1+x^4}}{x(\cancel{1+x^4})} - \int \frac{1}{x+x^5}\,dx$

$= \log(x) - f(x) + C$ Ans

Eg 16) $\int \frac{2x-1}{(x+1)^2} + \frac{8x+15}{\sqrt{4x+3}}\,dx$?

~~$\int \frac{2+2x+1-2}{(x+1)2} + \frac{1+8x+15-1}{\sqrt{4x+3}}$~~

~~$\int \frac{(x+1)^2}{(x+1)^2} - \int \frac{2}{(x+1)^2} +$~~

$\int \frac{2(x+1)}{(x+1)^2} - \frac{3}{(x+1)^2} + 2\frac{\sqrt{4x+7}}{\sqrt{4x+7}} - \frac{1}{\sqrt{4x+7}}\,dx$

$2\log(1+x) + \frac{5}{(x+1)} + \frac{4}{3}\frac{(4x+7)^{3/2}}{\cancel{4}} - \cancel{2}\frac{\sqrt{4x+7}}{\cancel{4}2} + C$

HW 1) $\int$

Q 2) $\int$

3) $\int$

Q4)

So|n

So

1) $\int \frac{1}{\sqrt{3x+4} - \sqrt{3x+1}}\, dx$

Hint → Rationalisation of

2) $\int \sec^2 x \cdot \cos^2 2x\, dx$

Hint $\cos^2 2x = (2\cos^2 x - 1)^2$

3) $\int \sin\left(\tan^{-1} \frac{x}{\sqrt{1-x^2}}\right) dx$

Hint $\int x\, dx$

4) $\int \operatorname{cosec}\left(x - \frac{\pi}{3}\right) \cdot \operatorname{cosec}\left(x - \frac{\pi}{6}\right) dx$

sol^n

$$\boxed{\text{Remarks} \rightarrow \quad \int \frac{1}{\sin(x-a) \cdot \sin(x-b)}\, dx}$$

~~$\int \operatorname{cosec}\left(\frac{1}{\sin(x-\frac{\pi}{3})}\right)$~~

$$= \frac{1}{\sin(b-a)} \int \frac{\sin(x-a)-(x-b)}{\sin(x-a) \cdot \sin(x-b)} \longrightarrow \frac{\sin A \cos B - \cos A \sin B}{\sin A \cdot \sin B} \rightarrow (\cot B - \cot A)$$

$$= \frac{1}{\sin(b-a)} \int \cot(x-b) - \cot(x-a)\, dx$$

$$= \frac{1}{\sin(b-a)} \log \left(\frac{\sin(x-b)}{\sin(x-a)}\right) + c$$

eg 5) $\int \frac{1}{\cos(x-a)\cdot\cos(x-b)}dx$

Hints

$= \frac{1}{\sin(b-a)}\int \frac{\sin((x-a)-(x-b))}{\cos(x-a)\cdot\cos(x-b)}dx$

$= \frac{1}{\sin(b-a)}\int \tan(x-a) - \tan(x-b)$ ✓

$= \frac{1}{\sin(b-a)}\log\left(\frac{\sec(x-a)}{\sec(x-b)}\right)+c$ A_2

eg 6) $\int \frac{1}{\sin(x-a)\cdot\cos(x-b)}dx$

$= \frac{1}{\cos(b-a)}\int \frac{\cos(x-a)-(x-b)}{\sin(x-a)\cdot\cos(x-b)}dx$

$= \frac{1}{\cos(b-a)}\int \cot(x-a) + \tan(x-b)\,dx$

$= \frac{1}{\cos(b-a)}\left[\log(\sin(x-a)\cdot\sec(x-b)\right]+c,$

Note

1) $\int \frac{1}{\sin\cdot\sin}dx$ or $\int \frac{1}{\cos\cdot\cos}dx$

then $\boxed{\sin(A-B)}$

2) $\int \frac{1}{\sin\cdot\cos}dx$ then $\boxed{\cos(A-B)}$

eg 7) $\int \sin$

Sol^n $\frac{1}{16}\int$

$\frac{1}{16}\int$

$\frac{1}{64}$

$\frac{1}{64}$

✱ N

eg*) $\int \sin^4 x \cos^4 x\,dx$

Sol^n $\frac{1}{16}\int (\sin 2x)^4\,dx$

$\frac{1}{16}\int \frac{(1-\cos 4x)^2}{4}\,dx$

$\frac{1}{64}\int 1+\cos^2 4x - 2\cos 4x\,dx$

$\frac{1}{64}\int \{ 1+\frac{1+\cos 8x}{2} - 2\cos 4x\,dx$

$\frac{1}{64}\left[\frac{3}{2}x + \frac{\sin 8x}{16} - \frac{1}{2}\sin 4x\right] + C$

☆ method of substitution →

$$\int \underbrace{f(\phi(x))\ \phi'(x)\,dx} = \int f(t)\,dt$$

Put $\phi(x) = t$

$\phi(x)\,dx = dt$

↓ Desired formula

1) $\int \frac{f'(x)}{f(x)}\,dx = \log f(x) + C$

(2) $\int f(x)^n \cdot f'(x)dx = \frac{(f(x))^{n+1}}{n+1} + c$

(3) $\sqrt{\frac{a+x}{a-x}}$ or $\sqrt{\frac{a-x}{a+x}}$ $x = a\cos 2x$

(4) $\sqrt{\frac{x}{a-x}}$ or $\sqrt{x(a-x)}$ $x =$ ~~[illegible]~~ $a\sin^2\theta$

(5) $\sqrt{\frac{x}{a+x}}$ or $\sqrt{x(a+x}$ $x = a\tan^2\theta$

(6) a^2-x^2 : $x = a\sin\theta$

(7) a^2+x^2 = $x = a\tan\theta$

(8) x^2-a^2 $x = a\sec\theta$.

eg1) $\int \frac{1-\tan x}{1+\tan x} dx = \log(\sin x + \cos x) + c'$

How?

$\int \frac{\cos x - \sin x}{\cos x + \sin x} dx = \log(\sin x + \cos x) + c'$

eg2) $\int \cos^3 x \sqrt{\sin x}\, dx$

$\int (1-\sin^2 x) \sqrt{\sin x}$ $\cos x dx$

$\sin x$

Sol) $\int \sqrt{t} - t^{5/2}\, dt$

$\frac{2}{3}(\sin x)^{3/2} - \frac{2}{7}(\sin x)^{7/2} + c$

eg 3.) $\int \frac{\sin x}{\sin(x-\alpha)}\, dx$

Sol Let ~~$\sin(x-\alpha)$~~

$x-\alpha = t$

$dx = dt$

$\int \frac{\sin(t+\alpha)}{\sin(t)}\, dt$

$\int \cos\alpha + \sin\alpha \cdot \cot t\, dt$

$(x-\alpha)\cos\alpha + \sin\alpha \log \sin(x-\alpha) + c$

eg 4) $\int \frac{x^2}{a+bx}\, dx$!

Sol

$a+bx = t$

$b\,dx = dt$

$\frac{1}{b^3}\int \frac{(t-a)^2}{t}\, dt$

$= \frac{1}{a^4}\int t\,dt \quad \frac{a^4}{t} = 2x\,dt$

$= \frac{1}{a^4}\left(\frac{(a+bx)^2}{2} + a^2 \log(a+bx) - 2a(a+bx)\right) + c$

eg.) $\int \frac{e^x(x+1)}{\cos^2(x.e^x)}\,dx$

Let $x.e^x = t$

$(x+1)e^x = dt$

$= \int \frac{1}{\cos^2\theta}\,d\theta$

$\int \sec^2\theta\,d\theta$

$\tan(x.e^x) + c$

eg.) $\int \frac{\sin x.\cos x}{a^2\sin^2x + b^2\cos^2x}\,dx$

$\Rightarrow a^2\sin^2x + b^2\cos^2x = t$

$\Rightarrow \left[a^2 . 2\sin x.\cos x + b^2 2\cos x.-(\sin x)\right]dx = dt$

$\Rightarrow 2\sin x.\cos x\left(a^2 - b^2\right)dx = dt$

$\Rightarrow 2(a^2-b^2)\sin x.\cos x\,dx = dt$

$\Rightarrow \frac{1}{2(a^2-b^2)}\log(a^2\sin^2x + b^2\cos^2x) + c$

Q7) $\int \frac{\tan x}{1+2\tan^2 x}\,dx$?

Sol $~~1+2\tan^2 x = t~~$

$~~2 \cdot 2\tan x \cdot \sec^2 x\, dx = dt~~$

$$\int \frac{\sin x \cdot \cos x}{\boxed{\cos^2 x + 2\sin^2 x}}$$

$\hookrightarrow 1+\sin^2 x$

$\frac{1}{2}\log\left(1+\sin^2 x\right)+c$ /

Q8) $\int 2^{2^{2^{x}}} \cdot 2^{2^{x}} \cdot 2^{x}\, dx$?

Sol ~~Let~~

$$2^{2^{2^{x}}} = t$$

$$2^{2^{2^{x}}} \cdot 2^{2^{x}} \cdot 2^{x}\,(\log 2)^3 \cdot dx = dt$$

$$= \frac{1}{(\log 2)^3}\int 2^{t} \cdot dt$$

$$= \frac{2^{2^{2^{2^{x}}}}}{(\log 2)^4} + C$$ /

Q9.) $\int \frac{a}{b+ce^{x}}\,dx$ multiply by e^{-x} /

Sol/ $a\int \frac{e^{-x}}{be^{-x}+c}\,dx$

Let $be^{-x}+c=t$

$-b\,e^{-x}\,dx=dt$

$= \frac{-a}{b}\log(be^{-x}+c)+k$ /

Q10) $\int \frac{\cos^2 x}{\sin^6 x}\,dx$?

Sol/ $\int \cot^2 x(1+\cot^2 x)\operatorname{cosec}^2 x\,dx$ $\sin^4 x$

$\cot x = t$

$= -\int t^2+t^4\,dt$ /

$= -\left(\frac{\cot^3 x}{3}+\frac{\cos^5 x}{5}\right)+c$ /

Q11.) $\int \frac{1}{\sqrt{\sin^2 x.\cos x}}\,dx$

Sol/ $\int \frac{\operatorname{cosec}^2 x}{\sqrt{\cot x}}\,dx$

Q12)

Sol/

$$= -2\sqrt{\cot x} + c$$

$\cot x = t$

$-\operatorname{cosec}^2 x \, dx = dt$

Eg12) $$\int \frac{1}{\sqrt{\sin^3 x \cdot \sin(x+\alpha)}} \, dx$$

So/n $$\int \frac{1}{\sqrt{\sin^3 x(\sin x \cos\alpha + \cos x \cdot \sin\alpha)}} \, dx$$

$$\int \frac{\operatorname{cosec}^2 x}{\sqrt{\cos\alpha + \cot x \cdot \sin\alpha}} \, dx$$

Let

$\cos\alpha + \cot x \cdot \sin\alpha = t$

$-\operatorname{cosec}^2 x \, dx = dt$

$$-\frac{1}{\sin\alpha} 2\sqrt{\cos\alpha + \cot x \cdot \sin\alpha} + c$$

eg13) $$\int \sqrt{\frac{x}{a^3 - x^3}} \, dx$$

$$\int \frac{\sqrt{x}}{\sqrt{(a^{\frac{3}{2}})^2 - (x^{3/2})^2}} \, dx$$

Let

$\cancel{(a^{3/2})^2 - (x^{3/2})^2} = t$

$x^{3/2} = t$

$\cancel{-2 \cdot x^{3/2-1}}$

$\frac{3}{2} x^{1/2}\, dx = dt$

$\cancel{-0\, x^{\frac{1}{2}}}$

$$\frac{2}{3}\int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}}$$

$$= \frac{2}{3}\sin^{-1}\left(\frac{t}{a^{3/2}}\right) + c$$

eg14.) $\int \sqrt{\frac{\cos x - \cos^3 x}{1-\cos^3 x}}\, dx$?

$\cancel{\cos x \left(\frac{1-\cos^2 x}{1-\cos^3 x}\right)}$

So/ $\int \frac{\sqrt{\cos x}\ \sin x}{\sqrt{1-(\cos^{3/2} x)^2}}\, dx$

$\cancel{1-\cos^3 x = t}$

$\hookrightarrow \cos^{3/2} x = t$

$\cancel{3\cos^2 x \cdot \sin x\, dx = dt}$

$$-\frac{2}{3}\int \frac{dt}{\sqrt{1-t^2}}$$

$$\frac{2}{3}\cos^{-1}(\cos^{3/2} x) + C$$

Sheet eg15) $\int \frac{\cos x + x\sin x}{x^2 + \cos^2 x}\, dx$

So/ Divide by $\cos^2 x$

$$= \int \frac{\sec x + x.\tan x . \sec x}{(x\sec x)^2 + 1} dx$$

$$x.\sec x = t$$

$$\int \frac{1.\ dt}{t^2+1}$$

$$= \tan^2(x.\sec x) + c$$

Ncert eg16)

$$\int \frac{\cos x - \sin x}{\sqrt{\sin 2x}} dx$$

Sol^n

$$\int \frac{\boxed{\cos x - \sin x}}{\sqrt{1 + \sin 2x - 1}} \longrightarrow \frac{d}{dx}(\sin x + \cos x)$$

$$\hookrightarrow \underbrace{\sin^2 x + \cos^2 x}_{1} + 2\sin x \cos x$$

$$1 + \sin 2x$$

$$\int \frac{\cos x - \sin x}{\sqrt{(\sin x + \cos x)^2}} dx$$

$$\sin x + \cos x = t$$

$$\int \frac{1}{\sqrt{t^2 1}} dt$$

$$= \log(\sin x + \cos x - \sqrt{\sin 2x}) + c$$

eg 17)

$$= \int \frac{ax^2-b}{x\sqrt{c^2x^2-(ax^2+b)^2}}\,dx$$

$$\boxed{\left(ax+\frac{b}{x}\right)^2 = a-\frac{b}{x^2}}$$

$$= \int \frac{\cancel{ax^2-b}}{\cancel{x}\sqrt{c^2x^2-\cancel{(ax^2+b)}}}$$

$$= \int \frac{ax^2-b}{x^2\sqrt{c^2-\left(ax+\frac{b}{x}\right)^2}}$$

$$= \int \frac{a-\frac{b}{x^2}}{\sqrt{c^2-\underbrace{\left(ax+\frac{b}{x}\right)}_{\to t}{}^2}}\,dx$$

$$= \int \frac{dt}{\sqrt{c^2-t^2}} = \sin^{-1}\left(\frac{ax^2+b}{cx}\right) + k$$

eg 18)

$$= \int \sqrt[3]{\frac{\sin^n x}{\cos^{n+6} x}}\,dx$$

$$= \int \frac{\sin^{\frac{n}{3}} x}{\cos^{\frac{n}{3}} x \cdot \cos^2 x}\,dx$$

$$= \int \tan^{\frac{n}{3}} x \cdot \sec^2 x\,dx$$

$$= \frac{\tan^{\frac{n}{3}+1} x}{\left(\frac{n}{3}+1\right)} + c$$

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eg19) $\int \frac{\sec^4 x}{\sqrt{\tan x}} dx$

Hint : $\tan x = t$

eg20) $\int \sec^4 x . \tan x dx = \frac{\sec^4 x}{4} + c$

eg21) $\int \frac{x}{\sqrt{1-x^2} . \cos^2(\sqrt{1-x^2})} dx$

Hint, $\sqrt{1-x^2} = t$

★ Integeration By - Parts

Def: →

$$\int \underset{I}{f(x)}\cdot \underset{II}{g(x)}\,dx = f(x)\cdot \int g(x)\,dx - \int \frac{d}{dx}f(x)\cdot \int g(x)\,dx + c$$

To select first function -

I L A T E

- Inverse functi
- Logarithmic functio
- Algebric functi
- Trigonometic functi
- Exponential functions

One(1) as a II functions -

↳ Required By-Part.

i) log function

ii) Inverse trigonometric functi.

eg 1) $\int \underset{II}{1}\cdot \log x\,dx$

2) $\int \underset{II}{1}\cdot \log(\log x)\,dx$

3) $\int \underset{II}{1}\cdot \log(x+\sqrt{x^2+a^2}\,dx$

4) $\int \underset{II}{1}\cdot \sin^{-1}x\,dx$

sheet (L-1 to L-4)
↳ Partial fracti

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5) $\int \underbrace{1}_{II} \cdot \tan^{-1}x\,dx$

eg) $\int \sin\sqrt{x}\,dx$ or $\int \cos\sqrt{x}\,dx$

$\sqrt{x} = t$

$\frac{1}{2\sqrt{x}}\,dx = dt$

$2\int \underbrace{t}_{I} \cdot \underbrace{\sin t}_{II} \cdot dt$

eg.) $\int \cos\sqrt{x}\,dx$ or $\int \sin\sqrt{x}$ or $\int \sin^{-1}\sqrt{x}\,dx$ or $\int \cos^{-1}\sqrt{x}\,dx$ or $\int \tan^{-1}\sqrt{x}\,dx$

$\downarrow$

$\sqrt{x} = t$

$\frac{1}{2\sqrt{x}}dx = dt$

$= 2\int \underbrace{t}_{I} \cdot \underbrace{\cos t}_{II}\,dt$

$= 2\,[t \cdot \sin t + \cos t] + c$

$= 2\,[\sqrt{x} \cdot \sin\sqrt{x} + \cos\sqrt{x}] + c$

eg 5.) $\int \underbrace{\frac{1}{x^2}}_{II} \cdot \underbrace{\log(x^2+a^2)}_{I}\,dx$

$= \log(x^2+a^2)\left(\frac{-1}{x}\right) + \int \frac{2\cancel{x}}{x^2+a^8} \cdot \frac{1}{\cancel{x}}\,dx + c$

$$= \frac{-1}{x}\log(x^2+a^2) + \frac{2}{a} \cdot \tan^{-1}\frac{x}{a} + c$$

eg4) $\int x \cdot \sin x \cdot \sec^3 x \, dx$

$$= \int \underbrace{x}_{I} \cdot \underbrace{\tan x \cdot \sec^2 x}_{II} \, dx$$

Reason : $\int f(x) \cdot f'(x) \, dx$

$$= \frac{f(x)^2}{2}$$

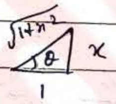
$$= x \cdot \frac{\tan x^2}{2} - \frac{1}{2}\int \tan^2 x \, dx \, dc$$

$$= \frac{x}{2} \tan^2 x - \frac{1}{2}(\tan x - x) + c$$

eg5) $\int \frac{x \cdot \tan^{-1} x}{(1+x^2)^{3/2}}$

first substitution then by parts

So/ $x = \tan\theta \longrightarrow$



~~sec θ dθ~~

$$= \int \frac{\tan\theta \cdot \theta}{\sec^3\theta} \cdot \sec^2\theta \, d\theta$$

$$= \int \underbrace{\theta}_{I} \cdot \underbrace{\sin\theta}_{II} \, d\theta$$

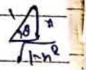
$$= \theta \cdot \int \sin\theta - \int (-\cos\theta)$$

$$= \theta(-\cos\theta) + \sin\theta + c$$

$$= \frac{\tan^{-1} x}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$$

eg6) $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$

So/ⁿ $\sin^{-1} x = \theta$ $\quad x = \sin\theta$



$\frac{1}{\sqrt{1-x^2}} dx = d\theta$

$\int \frac{\theta}{\cos^3\theta} \cdot \cos\theta \, d\theta$

$$= \int \underbrace{\theta}_{1} \cdot \underbrace{\sec^2\theta}_{II} \, d\theta$$

$$= \theta \cdot \tan\theta - \log(\sec x) + c$$

$$= \frac{x \sin^{-1} x}{\sqrt{1-x^2}} - \log\frac{1}{\sqrt{1-x^2}} + c \text{ Ans}$$

eg 5) $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$

Let
$x = a\tan^2\theta$

$dx = a\,2\tan\theta\cdot\sec^2\theta\,d\theta$

$= \int \sin^{-1}\sqrt{\frac{\cancel{a}\tan^2\theta}{\cancel{a}\sec^2\theta}}\cdot 2a\tan\theta\cdot\sec^2\theta\,d\theta$

$= \int \sin^{-1}(\sin\theta)\cdot 2a\cdot\tan\theta\cdot\sec^2\theta\cdot d\theta$

$= 2a\int \theta\cdot\tan\theta\cdot\sec^2\theta\cdot d\theta$

$= 2a\left[\frac{\theta}{2}\cdot\tan^2\theta - \frac{1}{2}(\tan\theta-\theta)\right]+c$

$= 2a\left[\frac{1}{2}\frac{x}{a}\tan^{-1}\sqrt{\frac{x}{a}} - \frac{1}{2}\left(\sqrt{\frac{x}{a}} - \tan^{-1}\sqrt{\frac{x}{a}}\right)\right]+c$

eg 6) $\left(\begin{array}{l}\int e^{ax}\cdot\sin bx\,dx \\ \int e^{ax}\cdot\cos bx\,dx\end{array}\right)$

$\int \sin(\log x)\,dx$ or $\int \cos(\log x)\,dx$ or $\int e^{\sin^{-1}x}\,dx \to \int e^{t}\cdot\cos t\,dt$

Note

$$\boxed{\int f(\log x)\,dx = \int e^{t}\cdot f(t)\,dt}$$

$\log x = t$

Sol

S/:

$$I = \int \underbrace{e^t}_{II} \cdot \underbrace{\sin t}_{I} \cdot dt$$

$$= \sin t\, e^t - \int \left(\underbrace{\cos t}_{I} \cdot \underbrace{e^t}_{II} \right)$$

$$= e^t \cdot \sin t - \left\{ \cancel{\sin t}\, \cos t \cdot e^t + \int \sin t \cdot e^t \right\}$$

$$= e^t \sin t - e^t \cos t - I$$

$$2I = e^t (\sin t - \cos t)$$

$$\boxed{I = \frac{x}{2} \left(\sin(\log x) - \cos(\log x) \right) + c}$$

eg.) $$\int \frac{x^2}{(x \sin x + \cos x)^2} dx \;?$$

Hint:

$$(x \cdot \sin x + \cos x)' = x \cdot \cos x + \cancel{\sin x} - \cancel{\sin x}$$
$$= x \cdot \cos x$$

&

$$\int \frac{f'}{f^2} dx = \frac{-1}{f}$$

$$\int \underbrace{\frac{x}{\cos x}}_{I} \cdot \underbrace{\frac{x \cdot \cos x}{(x \sin x + \cos x)^2}}_{II} dx$$

$$\int \frac{x}{\cos x} \cdot \frac{-1}{x \sin x + \cos x} + \int \frac{\cos x \cancel{+ x \cdot \sin x}}{\cos^2 x} \cdot \frac{1}{(x \sin x \cancel{+ \cos x})} dx$$

$= \frac{-x \cdot \sec x}{(x \sin x + \cos x)} + \tan x + c$ Ans

eg 7) $\int \underbrace{x^3}_{I} \cdot \underbrace{\sin 2x}_{II}\, dx$

Sol^n $-x^3 \frac{\cos 2x}{2} - \int 3x \left(-\frac{\cos 2x}{2}\right)$

$= -x^3 \frac{\cos 2x}{2} + \frac{3}{2} \int x \cdot \cos 2x$

form → $\int x^n \cdot f(x)\, dx$

↳ $e^{ax}, e^{-ax}, \sin ax, \cos ax$

$+x^3\left(-\frac{\cos x}{2}\right) - (3x^2)\left(-\frac{\sin 2x}{4}\right) + (6x)\left(\frac{\cos 2x}{8}\right) - (6)\left(\frac{\sin 2x}{10}\right) + c$

eg 8) $= \int x^3 \cdot e^{3x}\, dx$

$= x^3 \cdot \frac{e^{3x}}{3} - (3x^2)\left(\frac{e^{3x}}{9}\right) + (6x)\left(\frac{e^{3x}}{27}\right)$

eg 9) $\int (\log x)^4\, dx$

$= \int t^4 \cdot e^t \cdot dt$

$= t^4 e^t - 4t^3 \cdot e^t + 12t^2 \cdot e^t - 24t e^t + 24 e^t + c$

$= x(\log x)^4 - 4x(\log x)^3 + 12x(\log x)^2 - 24x\log x + 24x + c$

eg10) $\int (\sin^{-1}x)^3\,dx$

$= \int t^3 \cdot \cos t \cdot dt$

H.W

g) $= \int \frac{\sin x}{\sin 4x} dx$

$= \int \frac{\sin x}{4 \sin x \cdot \cos 2x \cdot \cos x} dx$

$= \frac{1}{4} \int \frac{1}{(\cos x)(\cos 2x)} dx$

$= \frac{1}{4} \int \frac{\cos x}{(\cos^2 x)(\cos 2x)} dx$

$\sin x = t$

$= \frac{1}{4} \int \frac{dt}{(1-t^2)((1-2t^2)}$

$= \frac{1}{4} \int \frac{1}{(t^2-1)(2t^2-1)} dt \longrightarrow \frac{1}{(P-1)(2P-1)} = \frac{1}{P-1} - \frac{2}{2P-1}$

$= \frac{1}{4} \int \frac{1}{t^2-1} - \frac{1}{t^2 - 1/2} dt$

$= \frac{1}{4} \left[\frac{1}{2} \log \left| \frac{t-1}{t+1} \right| - \frac{1}{\sqrt{2}} \log \left| \frac{\sqrt{2}t-1}{\sqrt{2}t+1} \right| + c \right.$

$t = \sin x$ //

eg2) $= \int \frac{1}{\sin x + \sin 2x} dx$

$= \int \frac{1}{\sin x(1+2\cos x)} dx$

$= \int \frac{\sin x}{(1-\cos^2 x)(1+2\cos x)} dx$

$\cos x = t$

$= \int \frac{dt}{(t-1)(t+1)(2t+1)}$

$= \int \frac{1}{6(t-1)} + \frac{1}{2(t+1)} - \frac{4}{3(2t-1)} \, dt$

$= \frac{1}{6}\log(\cos x-1)+\frac{1}{2}\log(\cos x+1)-\frac{2}{3}\log(2\cos x+1)+c$

✱ close relation :

i) $\sin 2x$
ii) $\sin x \cos x$
iii) $\sin x \pm \cos x$
iv) $\sec x + \operatorname{cosec} x$
v) ~~$\frac{d}{dx}(1+e^x)$~~
v) $\frac{d}{dx}(x\cdot e^x) = e^x(x+1)$
vi) $\frac{d}{dx}(x\sin x+\cos x) = x\cos x$

Q) $\int \frac{x+1}{x(1+x\cdot e^x)^2}\,dx$

$= \frac{\cancel{x}}{\cancel{x}(1+x\cdot e^x)^2} + \frac{1}{x(1+xe^x)}$

$\frac{1}{x}$

$\int \frac{e^x(x+1)}{x\cdot e^x(1+x\cdot e^x)^2}\,dx$

$1+x\cdot e^x = t$

$$= \int \frac{dt}{(t-1)(t^2)}$$

$$= \frac{1}{(t-1)(t^2)} = \frac{A}{t-1} + \frac{B}{t} + \frac{C}{t^2}$$

$1. \quad 1 = At^2 + Bt^2 - Bt + Ct - C$

$A + B = 0$

$-B + C = 0$

$C = -1$

$B = -1$

$A = 1$

$$\int \frac{1}{t-1} - \frac{1}{t} - \frac{1}{t^2}\,dt \longrightarrow \frac{\log(x\cdot e^x)}{(1+x\cdot e^x)} + \frac{1}{1+x\cdot e^x} + c$$

★ Standard Integration

1) $\int \frac{1}{a\sin x + b\cos x}\,dx$ ★

$$\boxed{\cancel{\sin x = \frac{2\tan x/2}{1+\tan x/2}}}$$

$\sqrt{a^2+b^2}$ = multiply + Divide

then change to single T-ratio

like: - $\int \frac{1}{\sin x + \cos x}\,dx$

$$\frac{1}{\sqrt{2}} \int \operatorname{cosec}\left(x + \frac{\pi}{4}\right) dx \longrightarrow = \frac{1}{\sqrt{2}} \log \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$$

like:- $\int \frac{1}{\underbrace{\sin x+\sqrt{3}\cos x}} dx$

$\sqrt{1+(\sqrt{3})^2} = \sqrt{4} = 2$

$= \int \frac{1}{2\left(\sin x \cdot \frac{1}{2} + \cos x \cdot \frac{\sqrt{3}}{2}\right)} dx$

$= \frac{1}{2}\int \frac{1}{\sin\left(x+\frac{\pi}{3}\right)} dx = \frac{1}{2}\int \operatorname{cosec}\left(x+\frac{\pi}{3}\right) dx$

$= \frac{1}{2} \log \tan\left(\frac{x}{2}+\frac{\pi}{6}\right) + c$

good
eg 1) $\int \frac{1}{\sec x + \operatorname{cosec} x} dx$

S/n $= \frac{1}{2}\int \frac{2\sin x \cdot \cos x}{(\sin x + \cos x)} dx$

$= \frac{1}{2}\int \frac{1+\sin 2x - 1}{\sin x + \cos x} dx$

$= \frac{1}{2}\int \frac{(\sin x + \cos x)^2 - 1}{\sin x + \cos x}$

$= \frac{1}{2}\int \sin x + \cos x - \frac{1}{\sin x + \cos x} dx$

$= \frac{1}{2}\left[\sin x - \cos x - \frac{1}{\sqrt{2}} \log \tan\left(\frac{x}{2}+\frac{\pi}{8}\right)\right] + c$

<2> $\int \frac{1}{ax^2+bx}$

$\int \frac{1}{\sqrt{ax^2}}$

$\int \sqrt{ax^2+}$

3) $\int \frac{px}{(a}$

$\int \frac{px}{\sqrt{}}$

$\int (px+$

eg 1) $\int$

S/n

$= \frac{1}{2}$

<2> $\int \frac{1}{ax^2+bx+c}dx$

$\int \frac{1}{\sqrt{ax^2+bx+c}}dx$

$\int \sqrt{ax^2+bx+c}$

method:-

$ax^2+bx+c \rightarrow$ Perfect square

How!

$= a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right]$

$= a\left[x^2+2x.\frac{b}{2a}+\frac{b^2}{4a^2}-\frac{b^3}{4a^3}+\frac{c}{a}\right]$

$= \left(x+\frac{b}{2a}\right)^2$

3) $\int \frac{px+q}{(ax^2+bx+c)}dx$

$\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$

$\int (px+q)\sqrt{ax^2+bx+c}\,dx$

method:-

Linear = A. d.c of Quadratic + B

~~By comparision~~ ~~find~~

By comparision, find A and B.

eg 1) $\int \frac{x}{\sqrt{1-x^2-x^4}}dx$

Sol $x^2 = t$

$= \frac{1}{2}\int \frac{dt}{\sqrt{1-t-t^2}}$

$1-t-t^2$

$= -(t^2+t-1)$

$= -\left((t)^2 + 2t.\frac{1}{2}. + \frac{1}{4} - \frac{1}{4} - 1\right)$

$$-\left(\left(t+\frac{1}{2}\right)^2-\left(\sqrt{\frac{5}{2}}\right)^2\right)$$

$$=\frac{1}{2}\int\frac{dt}{\sqrt{\left(\frac{\sqrt{5}}{2}\right)^2-\left(t+\frac{1}{2}\right)^2}}\,dt$$

$$=\frac{1}{2}\sin^{-1}\left(\frac{2x^2+1}{\sqrt{5}}\right)+c \text{ A}$$

Note

i) $\int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\frac{x}{a}$

than $\int\frac{1}{\sqrt{a^2-(x^2)^2}}dx=\sin^{-1}\left(\frac{x^2}{a}\right)$ (X)

ii) $\int\frac{1}{1+x^2}dx=\tan^{-1}x$

than $\int\frac{1}{1+(x^2)^2}dx=\tan^{-1}(x^2)$ (X)

➡ Both are wrong

eg1) $=\int\frac{}{\sqrt{}}$

eg2)

eg1) $\int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}}\, dx$

$\sin x = t$

$\int \frac{dt}{\sqrt{t^2 - 2t - 3}}$

$t^2 - 2t - 3$

$= t^2 - 2t \cdot 1 + 1 - 1 - 3$

$(t-1)^2 - 2^2$

$\int \frac{dt}{\sqrt{(t-1)^2 - 2^2}}$

$\therefore \log(\sin x - 1) + \sqrt{(\sin^2 x - 2\sin x - 3)} + c$

eg2) $\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}}\, dx$

Sol^n let $e^x = t$

$e^x \cdot dx = dt$

$\int \frac{dt}{\sqrt{5 - 4t - t^2}}\, dt$

$= \int \frac{1}{\sqrt{-(t^2 + 4t - 5)}}$

$= \int \frac{1}{\sqrt{-\{(t)^2 + 2t \cdot 2 + 4 - 4 - 5)}}$

$2m = 4$
$m = 2$

$= \int \frac{dt}{\sqrt{3^2 - (t+2)^2}}$

$= \sin^{-1}\left(\frac{e^x + 2}{3}\right) + c$

eg3) $\int \frac{e^{-2x}}{2 + 3e^{-x} + e^{-2x}}\, dx$

$e^{-x} = t$

$-\int \frac{t}{2 + 3t + t^2}\, dt$

$t = A(3 + 2t) + B$

$\therefore 2A = 1$

$A = \frac{1}{2}$

$3A + B = 0$

$B = -\frac{3}{2}$

$-\int \frac{\frac{1}{2}(3 + 2t) - \frac{3}{2}}{2 + 3t + t^2}\, dt$

$\frac{3}{2}\int \frac{1}{t^2 + 3t + 2}\, dt - \frac{1}{2}\log(2 + 3t + t^2) + c$

(P.S)

$t^2 + 3t + 2$

$= (t)^2 + 2t \cdot \frac{3}{2} + \frac{9}{4} - \frac{9}{4} + 2$

$= \left(t + \frac{3}{4}\right)^2 - \frac{1}{4}$

q3)

$$= \frac{5}{2}\int \frac{1}{\left(t+\frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2} - \frac{1}{2}\log(t^2+5t+2)+c$$

$$= \frac{5}{2}\log\left|\frac{t+1}{t+2}\right| - \frac{1}{2}\log(t^2+5t+2)+c$$

where $t = e^{-x}$ Ans

q2) $$\int \frac{2\sin\phi - \cos\phi}{6-\cos^2\phi - 4\sin\phi}\,d\phi$$

sol. $$\int \frac{(4\sin\phi - 1)\cdot\cos\phi\,d\phi}{6-(1-\sin^2\phi)-4\sin\phi}$$

$6-(1-\sin^2\phi)-4\sin\phi$
$6-1+\sin^2\phi-4\sin\phi$
$\sin^2\phi-4\sin\phi+5$

$\sin\phi = t$

$$\int \frac{4t-1}{t^2-4t+5}\,dt$$

$4t-1 = A(2t-4)+B$

$A = 2$

$-4A+B = -1$

$B = 7$

93)

$$\int \frac{1}{t^2+2xt+1}\,dx, \quad (x^2>1)\ ?$$

$$t^2+2xt+1+x^2-x^2+1 = (t+x)^2+\left(\sqrt{1-x^2}\right)^2$$

$$\int \frac{1}{(t+x)^2+\sqrt{1-x^2}}\,dt$$

$$\therefore \frac{1}{\sqrt{1-x^2}}\tan^{-1}\left(\frac{t+x}{\sqrt{1-x^2}}\right)+c$$

(X)

Reason : $x^2>1$

$$(t+x)^2-\left(\sqrt{x^2-1}\right)^2$$

$$\int \frac{1}{(t+x)^2-\left(\sqrt{x^2-1}\right)^2}\,dx$$

$$\therefore \frac{1}{2\sqrt{x^2-1}}\log\left|\frac{t+x-\sqrt{x^2-1}}{t+x+\sqrt{x^2-1}}\right|+c$$

(4)

$$\int \frac{1}{(x+a)^p(x+b)^q}\,dx$$

where $p+q=2$

method -

$$= \int \frac{1}{(x+a)^p(x+b)^q}\,dx$$

$$= \int \frac{1}{\frac{(x+a)^p}{(x+b)^p}(x+b)^2}\,dx$$

$$\therefore \int \frac{1}{\underbrace{\left(\frac{x+a}{x+b}\right)^p}_{=t}\cdot(x+b)^2}\,dx$$

//

eg1) $= \int \frac{\cos x}{(\sin x-1)^{3/4}\cdot(\sin x+2)^{5/4}}dx$!

$p+q=2$

$= \int \frac{dt}{(t-1)^{3/4}\cdot(t+2)^{5/4}}$

$= \int \frac{1}{\left(\frac{t-1}{t+2}\right)^{3/4}\cdot(t+2)^{2}}dt$

$\hookrightarrow \frac{t-1}{t+2} = p$

$= \frac{3}{(3+2)^{2}}dt = dp$

$\frac{1}{3}\int p^{-3/4}\cdot dp$

$= \frac{4}{3}\left(\frac{\sin x-1}{\sin x+3}\right)^{1/4} + c$ Ans

eg2) $\int \frac{1}{(x-p)\sqrt{(x-p)(x-q)}}dx$

so/n

$= \int \frac{1}{(x-p)^{3/2}\cdot(x-q)^{1/2}}dx$

$= \int \frac{1}{\left(\frac{x-p}{x-q}\right)^{3/2}(x-q)^{2}}dx$

$\hookrightarrow t$

$\frac{p-q}{(x-q)^{2}}dx = dt$

$\frac{1}{(p-q)}\int t^{-3/2}\cdot dt$

$= \frac{-2}{(p-q)}\left(\frac{x-q}{x-p}\right)^{1/2} + c$

(5) a) $\int$

b)

c)

eg)

(5) a) $\int \frac{x^2+1}{x^4+\lambda x^2+1} dx$

b) $\int \frac{x^2-1}{x^4+\lambda x^2+1} dx$

c) $\int \frac{10R\, x^2}{x^4+\lambda x^2+1} dx$

Method :-

a & b : Divide by x^2

$$1+\frac{1}{x^2} \longrightarrow \left(x-\frac{1}{x}\right)=t$$

$$1-\frac{1}{x^2} \longrightarrow \left(x+\frac{1}{x}\right)=t$$

For this : Add and subtract "2" in denominate

(C) $1=\frac{1}{2}\left(\underset{a}{(x^2+1)}-\underset{b}{(x^2-1)}\right)$

$$x^2=\frac{1}{2}\left(\underset{a}{(x^2+1)}+\underset{b}{(x^2-1)}\right)$$

eg) $\int \frac{1}{x^4+1} dx$

$$= \frac{1}{2} \int \underset{(a)}{\frac{x^2+1}{x^4+1}} - \underset{(b)}{\frac{x^2-1}{x^4+1}} dx$$

$$= \frac{1}{2}\left(\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-2+2} dx - \int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}+2-2} dx\right)$$

$$= \frac{1}{2}\left(\int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+\left(\sqrt{2}\right)^2} dx - \int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-\left(\sqrt{2}\right)^2} dx\right)$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right| + c\right]$$

eg2.) $\int \frac{x^2+1}{x^4+x^2+1}\,dx$

$\frac{1}{2}\cancel{\left(\frac{x^2+1}{(x^4+x^2+1)(x^2+1)}\cdot x^2+1\right.} - \int\frac{x^2-1}{x^4+x^2}$

eg1)

Sol^n

Sol^n $\int \frac{1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}+2-2}\,dx$

$\int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+(\sqrt{3})^2}\,dx$

$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right)+c$

eg3.) $\int \frac{1}{\sin^4x+\cos^4x}\,dx$

$\int \frac{(1+\tan^2x)\sec^2x\,dx}{(\tan^4x+1)}$

Let
$\tan x = t$
$\sec^2x\cdot dx = dt$

$\int \frac{(1+t^2)}{(t^2+1)}\,dt$

$\int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}-2+1}\,dt$

$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right)+c$

$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x-\cot x}{\sqrt{2}}\right)+c$

eg1) $\int \sqrt{\tan x}\,dx$ or $\int \sqrt{\cot x}\,dx$ or $\int \sqrt{\tan x} + \sqrt{\cot x}\,dx$

Sol^n → $\sqrt{f(x)}$ ∴ Put $f(x) = t^2$

$\tan x = t^2$

$\sec^2 x\,dx = 2t\,dt$

$$\int \sqrt{t^2} \cdot \frac{2t\,dt}{\sec^2 x} = \int \frac{t \cdot 2t\,dt}{\sec^2 x} = \int \frac{2t^2\,dt}{1+\tan^2 x} = \int \frac{2t^2\,dt}{1+t^4}$$

$$= \int \frac{t^2+1}{t^4+1}\,dt \quad \int \frac{t^2-1}{t^4+1}\,dt$$

$$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}}\right) + \frac{1}{2\sqrt{2}} \log \left|\frac{\sqrt{\tan x} + \sqrt{\cot x} + \sqrt{2}}{\sqrt{\tan x} + \sqrt{\cot x} + \sqrt{2}}\right| + c$$

$$= \frac{1}{\sqrt{2}} \tan^{-1}\Big($$

✡ Standard Integration →

eg 6) $\int \frac{1}{a+b\sin^2 x}\,dx$

$\int \frac{1}{a+b\cos^2 x}\,dx$

$\int \frac{1}{a+b\sin^2 x+c}\,dx$

$\int \frac{1}{(a\sin x+b\cos x)^2}\,dx$

$\int \frac{1}{a\sin^2 x+b\sin x\cos x+c\cos^2 x}\,dx$

method:-

Then divide by $\cos^2 x$ then $\tan x = t$

eg 1) $\int \frac{\sin x}{\cos x}\,dx$, $= \int \frac{\sin x}{\sin 3x}\,dx$

$\Downarrow$

$= \int \frac{\sin x}{3\sin x - 4\sin^3 x}\,dx$

$= \int \frac{1}{3-4\sin^2 x}\,dx$

$= \int \frac{1}{3\cos^2 x - \sin^2 x}\,dx$

$= \int \frac{\sec^2 x}{(\sqrt{3})^2 - \tan^2 x}\,dx$

$\tan x = t$

$$= \int \frac{dt}{(\sqrt{3})^2 - t^2}$$

$$= \frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x} \right| + c$$

eg $\displaystyle\int \frac{1}{(2\sin x + 3\cos x)^2} dx$

$$\int \frac{\sec^2 x}{(2\tan x + 3)^2} dx$$

$2\tan x + 3 = t$

$$\frac{1}{2} \int \frac{dt}{t^2} = \frac{-1}{2(2\tan x + 3)} + c$$

$\displaystyle\int \frac{1}{a + b\sin x} dx$

$\displaystyle\int \frac{1}{a + b\cos x} dx$

$\displaystyle\int \frac{1}{a + b\sin x + c\cos x} dx$

method;

$$\sin x = \frac{2\tan x/2}{1 + \tan^2 \frac{x}{2}}$$

$$\cos x = \frac{1 - \tan^2 x/2}{1 + \tan^2 \frac{x}{2}}$$

$\tan \frac{x}{2} = t$

- $\frac{1}{1+\sin x} \curvearrowleft (1-\sin x)$
- $\frac{1}{1-\sin x} \curvearrowleft (1+\sin x)$
- $\frac{1}{1+\cos x} \curvearrowleft (1-\cos x)$
- $\frac{1}{1+\cos x} \curvearrowleft (1+\cos x)$

eg.∫ $\int \frac{1}{5+7\cos x+\sin x}dx$

$$\int \frac{1}{5+7\left(\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right)+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}$$

$$\int \frac{\left(1+\tan^2\frac{x}{2} = \sec^2\frac{x}{2}\right)}{\left(2-2\tan^2\frac{x}{2}+2\tan\frac{x}{2}\right)}dx$$

$$2\int \frac{dt}{-2t^2+3t+12}$$

$$= -\int \frac{1}{t^2-t-6}$$

$$= \int \frac{1}{(t-3)(t+2)}dx$$

$$= \frac{-1}{5}\int \left(\frac{1}{t-3}-\frac{1}{t+2}\right)$$

$= \frac{-1}{5} \log$

(8) $\int \frac{a \sin x + b \cos x}{p \sin x + q \cos x} \, dx$

method:

Numerator :⇒ A(den) + B(den)'

* $\int \frac{ae^{x} + be^{-x}}{ce^{x} + d \cdot e^{-x}} \, dx$

methods

Numerator = A(den) + B(den)'

* $\int \frac{a \sin x + b \cos x}{p \sin x + q \cos x + r} \, dx$

or $\int \frac{a \sin x + b \cos x + c}{p \sin x + q \cos x + r} \, dx$

method :⇒ Numerators →

Q1) $\int \frac{\sin x + 2\cos x}{2\sin x + \cos x}\,dx$

$\sin x + 2\cos x = A(2\sin x + \cos x) + B(2\cos x - \sin x)$

$(2A - B = 1)^2$

$\underline{A + 2B = 2}$

$5A = 4$

$A = \frac{4}{5}$

$2B = 2 - \frac{4}{5} = \frac{6}{5}$

$B = \frac{5}{5}$

$\int \frac{\frac{4}{5}(\text{den}) + \frac{5}{5}(\text{den})'}{\text{den}}\,dx$

$\cancel{2\sin x + \cos x} = \frac{4}{5}x + \frac{5}{5}\log(2\sin x + \cos x) + c$

Q2) $\int \frac{1}{1-\cot x}\,dx$; $\int \frac{1}{1+\cot x}\,dx$, $\int \frac{\cot x}{1+\cot x}$, $\tan x$ in place of $\cot x$ /

~~$\sin x = A(\sin x - \cos x) + B(\cos x + \sin x)$~~

$\int \frac{\sin x}{\sin x - \cos x}\,dx$

$\sin x = A(\sin x - \cos x) + B(\cos x + \sin x)$

$A + B = 1$

$-4x + 4e^{x}$

$-A + B = 0$

$A = B = \frac{1}{2}$

$$\int \frac{\frac{1}{2}\,\text{den} + \frac{1}{2}\,\text{den}'}{\text{den}}\,dx$$

I. $\frac{1}{2}x + \frac{1}{2}\cdot\log|\sin x - \cos x| + c$

Q12) $$\int \frac{4e^{x} + 6e^{-x}}{9e^{x} - 4e^{-x}}\,dx = Ax + B\log(9e^{2x} - 4) + c$$

find A and B?

$\frac{N}{D}$

Sol) $4e^{x} + 6e^{-x} = k(9e^{x} - 4e^{-x}) + l(9e^{x} + 4e^{-x})$

$4e^{x} + 6e^{-x} = A(9e^{x} - 4e^{-x}) + B(9e^{x} + 4e^{-x})$

$4e^{x} + 6e^{-x} = A9e^{x} + B9e^{x} - 4e^{-x} + 4e^{-x}$

$9k + 9l = 4 \quad -4k$

$\cancel{k} + l = \frac{4}{9}$

$-\cancel{k} + l = \frac{3}{2}$

$2l = \frac{8 + 27}{18} = \frac{35}{18}$

$l = \frac{35}{36}$

$-k = \frac{3}{2} - \frac{35}{36} \Rightarrow k = \frac{-19}{36}$

$\frac{-19x}{36}x + \frac{35}{36}\log(9e^{2x} - 4)$

$\frac{-35}{36}x + c$

$A = \frac{54}{36} = \frac{-3}{2}$

$B = \frac{35}{36}$

$$\int \frac{\frac{-19}{36}\,\text{den} + \frac{35}{36}\,\text{den}'}{\text{den}}\,dx$$

$$= \frac{-19}{36}x + \frac{35}{36}\log\left(\frac{9e^{2x} - 4}{e^{x}}\right) + c$$

92)

(9) $\int \sin^m x \cdot \cos^n x \, dx$

1) when m = even, n = odd ⇒ put $\sin x = t$

(2) when m = odd, n = even ⇒ put $\cos x = t$

(3) If Both m and n are odd = Put any of the $\sin x$ or $\cos x = t$

(4) If Both m and n are even ⇒ use double angle formula of $\cos x$

(5) (Imp)

If $m + n = \mathrm{Ie}^{-}$

then change Integrand in term ~~of~~ of $\tan x$ and $\sec x$

put $\tan x = t$

like

$$\int \frac{\sin^{4} x}{\cos^{8} x} dx$$

$$(m + n = 4 + (-8) = -4 = \mathrm{Ie}^{-})$$

$$\int \tan^4 x (1 + \tan^2 x) \sec^2 x \, dx$$

$\tan x = t$

$$\therefore \int t^4 + t^6 \, dt$$

$$\frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + c \quad \text{Ans}$$

Q2) $\int \sec^{4/3}x \cdot \csc^{8/3}x\,dx$

$\left(m+n = -\frac{4}{3} - \frac{8}{3} = -4\right)$

$\cancel{= \int \sin^{-8/3}x \cdot \cos^{-4/3}x\,dx}$

$= \int \sin^{-8/3}x \cdot \cos^{-4/3}x\,dx$

$\int \frac{\sin^{-8/3}x}{\cos^{-8/3}x} \cos^{-4/3}x \cdot \cos^{-8/3}x\,dx$

$= \int \tan^{-8/3}x\,(1+\tan^2x)\sec^2x\,dx$

$\tan x = t$

$= \int t^{-8/3}(1+t^2)\,dt$

$= \int t^{-8/3} + t^{-2/3}\,dt$

$= \frac{-3}{5}\cos^{5/3}x + 3\tan^{1/3}x + C$

Q3) $\int \sqrt{\frac{\cos^3 x}{\sin^{11}x}}\,dx$

Sol^n $\int \cos^{3/2}x \cdot \sin^{-11/2}x\,dx$

~~m+n = 3+11 = 14~~

$m+n = \frac{3}{2} + \left(\frac{-11}{2}\right) = -4$

~~$\int \tan^{-11/2}$~~ $\int \tan^{-11/2} x(1+\tan^2 x)\sec^2 x\,dx$

$\tan x = t$

$\int t^{-11/2} + t^{-7/2}\,dt$

$= -\frac{2}{9}\cot^{9/2} - \frac{2}{5}\cot^{5/2} x + c$

(10) $\int \frac{\phi x}{P\sqrt{Q}}\,dx \longrightarrow \phi(x) =$ linear or constant

(a) when $P =$ linear, and $Q =$ linear

method $\Rightarrow$ put $Q = t^2$

like $\int \frac{1}{(x-3)\sqrt{x+1}}\,dx$

$x+1 = t^2$

$dx = 2t\,dt$

$= \int \frac{2t\,dt}{(t^2-4)\cancel{t}}$

$= \cancel{2}\frac{1}{2^2}\log\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + c$

Q) $\int \frac{x}{(x+1)\sqrt{x}}\,dx$?

Sol/n $x = t^2$

$\int \frac{t^2 \cdot 2t\,dt}{(t^2+1)\cancel{t}}$

$$= 2\int \frac{t^2+1-1}{t^2+1}\, dt$$

$$= 2\int 1 - \frac{1}{t^2+1}\, dt$$

$$= 2\left[\sqrt{x} - \tan^{-1}\sqrt{x}\right] + c$$

eg.) $$\int \frac{\sqrt{\sec x}}{(\sin x - 3\cos x)\sqrt{\sin x + \cos x}}\, dx$$

$$= \int \frac{\sqrt{\sec x}}{(\tan x - 3)\cos x \sqrt{\cos x} - \sqrt{\tan x + 1}}\, dx$$

$$= \int \frac{\sec^2 x}{(\tan x - 3)\sqrt{\tan x + 1}}\, dx$$

$$\int \frac{dt}{(t-3)\sqrt{t+1}}$$

$$= \frac{1}{2}\log\left|\frac{\sqrt{\tan x + 1} - 2}{\sqrt{\tan x + 1} + 2}\right| + c \;//$$

(10)(d) when, P = quadratic and Q = linea

method: I

1st put $Q = t^2$ /

eg.) $$\int \frac{1}{(x^2 - 4)\sqrt{x+1}}\, dx$$

$$x + 1 = t^2$$

$$\int \frac{2\cancel{t}\, dt}{\left((t^2-1)^2 - 2^2\right)\cancel{t}}$$

$$\xrightarrow{\quad} (A+B)(A-B)$$

<10> CC

$$= 2\int \frac{dt}{(t^2+1)(t^2-3)}$$

$$= \cancel{2}\cdot\frac{1}{\cancel{4}_{2}} \int \frac{1}{t^2-3} - \frac{1}{t^2+1}\, dt$$

$$= \frac{1}{2}\left[\frac{1}{2\sqrt{3}} \log\left|\frac{\sqrt{x+1}-\sqrt{3}}{\sqrt{x+1}+\sqrt{3}}\right| - \tan^{-1}\sqrt{x+1}\right] + c$$

eg 1) $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}} \cdot dx$

sol^n

$$x+1 = t^2$$
$$dx = 2t\,dt$$

$$\int \frac{(t^2+1)\ \cancel{2t}\,dt}{\left((t^2-1)^2+3(t^2-1)+3\right)\cancel{(t)}}$$

$$\int \frac{t^2+1}{t^4+1-2t^2+3t^2\cancel{-3}+3}\,dt$$

$$= \int \frac{t^2+1}{t^4+t^2+1}\,dt$$

$$\therefore\ 2\cdot\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{3}}\right) + c$$

$$\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right) + c$$

Let $x+1 = t^2-1$
$dx = 2t\,dt$

$$\int \frac{(t^2-1+2)}{(x^2+3x-3)\,t} \cdot 2t\,dt$$

$$\int \frac{t^2+1}{(t^2-1)^2+3(t^2-1)+3(t)}$$

$$\int \frac{t^2+1}{}$$

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Page

(10) (c) when P = linear, and Q = Quadratic

method:-

Put, $P = \frac{1}{t}$

eg. $\int \frac{1}{(x+1)\sqrt{x^2-1}}\,dx$

$x+1 = \frac{1}{t}$

$dx = -1 \cdot t^{-2}$

$dx = \frac{-1}{t^2}$

$\int \frac{\left(\frac{-1}{t^2}\right)}{\left(\frac{1}{t}\right)\sqrt{\left(\frac{1}{t}-1\right)^2-1}}\,dt$

$\int \frac{1}{1/t\sqrt{\frac{1}{t^2}-\frac{2}{t}+\cancel{1}-\cancel{1}}}\left(\frac{-1}{t^2}\right)dt$

$= \int \frac{1}{1/\cancel{t^2}\sqrt{1-2t}}\left(\frac{-1}{\cancel{t^2}}\right)dt$

$= \frac{(\cancel{-2}\sqrt{1-2t})}{\cancel{-2}} + C$

$= \sqrt{1-\frac{2}{x+1}} + C$

$= \sqrt{\frac{x-1}{x+1}} + C$

e.g.) $$\int \frac{1}{(1+\sqrt{x})\sqrt{x}(\sqrt{1-x})}dx$$

$$\sqrt{x}=t$$

$$\frac{1}{2\sqrt{x}}dx=dt$$

$$2\int \frac{1}{(1+t)\sqrt{1-t^2}}dt$$

Put $1+t=\frac{1}{p}$

$$=2\int \frac{1}{\frac{1}{p}\sqrt{1-\left(\frac{1}{p}-1\right)^2}}\left(\frac{-1}{p^2}\right)dp$$

$$=\int \frac{1}{\frac{1}{p}\sqrt{\frac{2}{p}-\frac{1}{p^2}}}\left(\frac{-1}{p^2}\right)dp$$

$$=2\int \frac{1}{1/p^2\sqrt{2p-1}}\left(\frac{-1}{p^2}\right)dp$$

$$=\frac{-2\cdot2\sqrt{2p-1}}{2}+c$$

$$=-2\sqrt{\frac{2}{1+t}-1}+c$$

$$=-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+c$$

(10) (d) when $P =$ Quadratic, $Q =$ Quadratic

★ $\int \frac{x}{(ax^2+b)\sqrt{cx^2+d}}\,dx$ or $\int \frac{1}{(ax^2+b)\sqrt{cx^2+d}}\,dx$

method:
$cx^2+d = t^2$

method:-
Put $x = \frac{1}{t}$

eg.) $\int \frac{1}{(1+x^2)\sqrt{1-x^2}}\,dx$?

Sol^n $\cancel{\int \frac{1}{(1+x^2)\sqrt{1-x^2}}}$

$x = \frac{1}{t}$

$$\int \frac{1}{\frac{(t^2+1)}{t^2}\sqrt{\frac{t^2-1}{t^2}}}\left(\frac{-1}{t^2}\right)dt$$

$$= -\int \frac{t}{(t^2+1)\sqrt{t^2-1}}$$

$t^2-1 = p^2$

$$= -\int \frac{p.\,dp}{(p^2+1).\cancel{p}}$$

$$= \frac{-1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{t^2-1}}{\sqrt{2}}\right)+C \quad \text{when } t = \frac{1}{x}$$

(11) $\int f\left(x^{\frac{1}{m}}, x^{\frac{1}{n}} \ldots\right) dx$

or

$$\int f\left((ax+b)^{\frac{1}{m}}, (ax+b)^{\frac{1}{n}} \ldots\right) dx$$

method: 1

find L.C.m of $(m, n \ldots)$

L.C.m (m, n)

Put $x = t$

like $\displaystyle\int \frac{x^{1/2}}{x^{1/2} - x^{1/3}}\, dx$

$x = t^6$

$$= \int \frac{t^3 \cdot 6t^5}{t^3 - t^2}$$

$$= 6 \int \frac{t^6}{t-1}\, dt$$

$$= 6 \int t^5 + t^4 + t^3 + t^2 + t + \frac{1}{t-1}\, dt$$

(Imp)

Definite Integration

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1) Definitions
let $\int f(x)dx = f(x) + C$
then
definite Integral of $f(x)$ from $x=a$ to $x=b$

$$\int_a^b f(x)dx = [f(x)]_a^b = \underbrace{f(b) - f(a)}_{\text{constant number } (\mathbb{R})}$$

(fundamental theorem of calculus)

2) * $$\boxed{\frac{d}{dx}\left(\int_a^b f(x)\,dx\right) = 0}$$

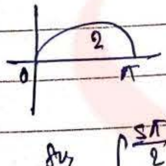
3) * $$\boxed{\int 0\,dx = c}$$ * $$\boxed{\int_a^b 0\,dx = 0}$$

likes

$$\int_0^{\pi} \sin x\,dx = 2$$

$= [-\cos x]_0^{\pi}$

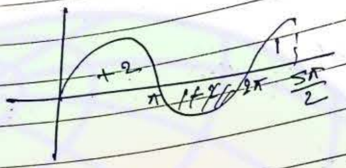
$= -(-1-1) = 2$



So $\int_{2\pi}^{3\pi} \sin x\,dx = 2$

So $\int_0^{\frac{5\pi}{2}} \sin x\,dx = 2 + (-2) + 1 = 1$

But what is the Area with x-axis,
Ans → 5 Area



* In the graph of $y=f(x)$ above x-axis then $\boxed{A = D}$

* If the graph of $y=f(x)$ is below x-axis then A and D are of opp. sign.

D = Algebric sum

A = Absolute sum

* $\int_{-a}^{a} \sqrt{a^2-x^2}\, dx \, . \left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a} \right]_{-a}^{a}$

CAREER POINT

Question base

(1) $\int_0^a f(x)$

(2) $\int_{-a}^{a} f(x)$

(3) $\int_0 f(x)$

Q.1 f(x)

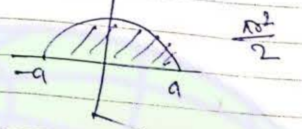
then

(1)

(3

Q.2 T

Q.3



$\frac{\pi a^2}{2}$

eg.) $\int_0^5 |x-1| + |x-2| dx$

Sol. [illegible] $= \int_0^1 (\cancel{x}-1) - \cancel{x} + 2 + \int_0^4 (x-1) + x - 2\, dx$

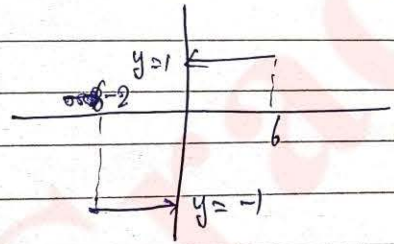
$= [x]_0^1 + \int_0^2 (2x-3)\, dx$

$= 1 + \left[\frac{2x^2}{2} - 3x\right]_0^2$

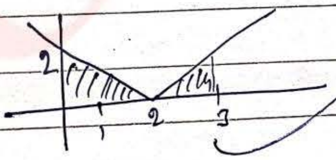
$= 1+4+12$

eg.) $\int_{-2}^{6} sgn(x)\, dx = 6+(-2) = 4$

A



eg.) $\int_0^3 |x-2| dx$



$\frac{1}{2}\cdot 2\cdot 2 + \frac{1}{2}\cdot 1\cdot 1 = \frac{5}{2}$ A

Eg.) $\frac{1}{2}\int_0^{\pi/2} 2\sin\theta \cdot \sin 3\theta \, d\theta$

Soln $\boxed{2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)}$

$= \int_0^{\pi/2} \left(\cos(-2\theta) - \cos 4\theta\right) d\theta$

$= \left[\frac{\sin 2\theta}{2} - \frac{\sin 4\theta}{4}\right]_0^{\pi/2}$

$= 0 - 0$

$= 0$

Eg.) $\int_0^{e^{37}} \frac{\pi}{x} \cdot \sin(\pi \cdot \ln x) \, dx$

$\pi \ln x = t \Rightarrow x = 1 \Rightarrow t = 0$

$\frac{1}{x} dx = \frac{dt}{\pi} \Rightarrow x = e^{37} = \pi \times 37$

$\int_0^{37\pi} \sin t \, dt$

$\left[-\cos t\right]_0^{37\pi}$

$= -\left(\cos 37\pi - \cos 0\right)$

$= -(-1-1) = 2$

$\boxed{\cos(\text{odd } \pi) = -1}$

CAREER POINT

Question bas

(1) $\int_0^{2a} f(x)\,dx =$

(2) $\int_0^{a} f(x)\,d$

(3) $\int_0^{na} f(x)\,dx$

(4) Summat

Q.1 $\lim_{n \to \infty}$

(1)

(3

Q.2

Q.3

eg) $\int_0^\infty \frac{x\tan^{-1}x}{(1+x^2)^2}\,dx$

$x=0,\ t=0$
$x\to\infty,\ t\to\pi/2$

Sol $\tan^{-1}x = t \quad = \frac{1}{1+x^2}\,dx = dt$

$\int_0^\infty \frac{\tan t}{1+\tan^2 t}\,dt$

$= \frac{1}{2}\int_0^{\pi/2} \underset{I}{t}\cdot\underset{II}{\sin 2t}\cdot dt$

$= \frac{1}{2}\left[t\left(-\frac{\cos 2t}{2}\right)\right]_0^{\pi/2} + \int\left[\left(\frac{\cos 2t}{2}\right)dt\right]$

$= \frac{1}{2}\left(\frac{\pi}{2}\cdot\frac{1}{2}\cdot(-1) - 0\right) + \frac{1}{2}\left[(\sin 2t)\right]_0^{\pi/2}$

$= \frac{1}{2}\left[\frac{\pi}{4} + \frac{1}{4}(0-0)\right]$

$= \frac{\pi}{8}$

eg) $\int_0^\infty \left(\frac{1}{(x+\sqrt{x^2+1})^3}\right)dx$

$x = \tan\theta \qquad , dx = \sec^2\theta\, d\theta$
$x=0,\ \theta=0$
$x=\infty,\ \theta=\frac{\pi}{2}$

$\int_0^\infty \frac{\sec^2\theta\, d\theta}{(\tan\theta+\sec\theta)^3}$

$\int_0^{\pi/2} \frac{1}{(\sin\theta+1)^3}\cdot\frac{\cancel{\sec^2\theta}}{\sec^3\theta}\,d\theta$

$$\int_0^{\pi/2} \frac{\cos\theta}{(\sin\theta+1)^3}\,d\theta \quad ; \quad \sin\theta+1 = t$$

$$\int_1^1 \frac{1}{t^3}\,d\theta$$

$$= -\frac{1}{2}\left[t^{-2}\right]_1^2$$

$$= \frac{-1}{2}\left(\frac{1}{4}-1\right) = \frac{3}{8}$$

9 $$\int_0^{\log 5} \frac{e^x\sqrt{e^x-1}}{e^x+6}\,dx$$

fu $$e^x-1 = t^2$$

$$\int_0^2 \frac{t\cdot 2t\cdot dt}{t^2+4}\,dt$$

$$2\int_0^2 \frac{t^2+4-4}{t^2+4}\,dt$$

$$2\int_0^2 1-\frac{4}{t^2+4}\,dt$$

$$2\left[\times 2 - \frac{1}{2}\times 4\left(\tan^{-1}\left(\frac{t}{2}\right)_0^2\right]\right.$$

$$= 4 - \cancel{4}\cdot\frac{\pi}{\cancel{4}}$$

$$= 4-\pi$$ Ans

q.) Let $I_n = \int_0^{\pi/4} \tan^n x\,dx$

then

$n(I_{n-1} + I_{n+1})$ is ——?

$n\left(\int_0^{\pi/4} \tan^{-1} n-1\, x\,dx + \int_0^{\pi/4} \tan^{n+1} x\,dx\right.$

$n\int_0^{\pi/4} \tan^{n-1} x\,(1+\tan^2 x)\,dx$

$\tan x = \theta$

$n\int_0^1 t^{n-1}\,d\theta$

$= \cancel{n} \cdot \left(\frac{t^n}{\cancel{n}}\right)_0^1$

$1 - 0 = 1$

eg Let $\frac{d}{dx} f(x) = \frac{e^{\sin x}}{x}, \ x > 0$

and

$\int^{4} \frac{3}{x} e^{\sin x^3}\,dx = \underline{f(k) - f(1)}$

$f(x)]_1^k$

find k.

30/k

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$$\int_0^4 \frac{e^{\sin x}}{x} dx = f(x)$$

$$x^3 = t$$

$$\int_1^{64} \frac{3x^2 \cdot \sin x^3}{x^3}$$

$$\int^{64} \frac{e^{\sin t}}{t} \cdot dt$$

$$, \quad f(t)\Big|_1^{64} = f(64) - f(1)$$

$$K = 64$$

★ ~~Wallis~~ Walli's formula.

<A>

$$\int_0^{\pi/2} \sin^n x \text{ or } \int_0^{\pi/2} \cos^n x\, dx$$

$$= \frac{(n-1)(n-3) \text{ ---- } \times 1 \text{ or } \frac{\pi}{2}}{n(n-2) \text{ ---- }}$$

even

$n \in N$ even

(stop at 1 or 2)

e.g.)

$$\int_0^{\pi/2} \cos^4 x\, dx$$

$$\frac{3 \quad 1}{4 \quad 2} \times \frac{\pi}{2}$$

$$\frac{3}{8} \times \frac{\pi}{2} = \frac{3\pi}{16}$$

eg.) $$\int_0^{k} \sqrt{\frac{x^3}{k-x}}\, dx$$

$x = k\sin^2\theta$

$$= \int_0^{\pi/2} \sqrt{\frac{k^3 \cdot \sin^6\theta}{k(1-\sin^2\theta)}} \cdot 2k\sin\theta \cdot \cos\theta d\theta$$

$$= 2k \cdot k \cdot \int_0^{\pi/2} \sin^4\theta d\theta \hookrightarrow \frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}$$

$$= 2k^2 \cdot \frac{3\pi}{16}$$

$$= \frac{3k^2\pi}{8}$$

SN

$1-x^2 = x = \sin\theta$

$k-x^2 = x = \sqrt{k}\sin\theta$

$1-x = x - \sin^2\theta$

$k-x = x = k\sin^2\theta$

eg.) $$\int_0^1 \frac{x^3}{\sqrt{1-x^2}}\, dx$$

$x = \sin\theta$

$$\int_0^{\pi/2} \frac{\sin^3\theta}{\cos\theta}\, dx \cdot \cos\theta$$

$$\frac{2}{3} \cdot 1 = \frac{2}{3}$$

9.) $\int_0^{\pi/4} (\cos 2x)^{3/2}$

$= \int_0^{\pi/4} (1-2\sin^2 x)^{3/2} \cos x \, dx$

$= \int_0^{\pi/4} \left(1-(\sqrt{2}\sin x)^2\right)^{3/2} \cos x \, dx$

$\sqrt{2}\sin x = \sin t$

$\sqrt{2} \cos x \, dx = \cos t \, dt$

$= \frac{1}{\sqrt{2}} \int_0^{\pi/2} \cos^3 t \cdot \cos t \, dt$

$= \frac{1}{\sqrt{2}} \cdot \frac{3\pi}{16}$

2) * $\int_0^{\pi/2} \sin^m x \cdot \cos^n x \, dx = \frac{(m-1)(m-3)(m-5) \ldots (n-1)(n-3) \ldots}{(m+n) \cdot (m+n-2) \cdot (m+n-4) \ldots} \times$

else

both

(stop at 2 or 1)

* $\int_0^{\pi/2} \sin^m x \cdot \cos x \, dx = \frac{1}{m+1}$

* $\int_0^{\pi/2} \sin^5 x \cdot \cos x \, dx = \frac{1}{6}$

eg.) $\int_0^{\pi/2} \sin^3 x \cdot \cos^4 x \, dx$

sol^n $\begin{bmatrix} m=3 \\ n=4 \end{bmatrix}$

$$\boxed{\frac{(3-1)\,(4-1)\,(4-3)}{(7)(7-2)\,(7-4)\,(7-6)} = \frac{2\times3\times1}{7\times5\times3\times1} = \frac{2}{35}}$$

eg2) $\displaystyle\int_0^{\infty} \frac{x^3}{(1+x^2)^{9/2}}$

Let $x = \tan\theta$

$\sec^2-\tan^2=1$

$$\int_0^{\pi/2} \frac{\tan^3\theta}{(1+\tan^2\theta)^{9/2}} = \int_0^{\pi/2} \frac{\tan\theta \cdot \tan^2\theta}{(\sec^2\theta)^{9/2}} = \int_0^{\pi/2} \frac{\tan^3\theta \cdot \cancel{\sec^2\theta}}{\cancel{\sec^9\theta} \to 7}$$

$$= \int_0^{\pi/2} \sin^3\theta \cdot \cos^4\theta \cdot d\theta = \frac{2}{35} \text{ Ans}$$

eg3) $\displaystyle\int_0^1 x^2(1-x^2)^{3/2}\,dx$?

$x = \sin\theta$

$$= \int_0^{\pi/2} \sin^2\theta \cdot \cos^3\theta \cdot \cos\theta\, d\theta$$

$$= \int_0^{\pi/2} \sin^2\theta \cdot \cos^4\theta \cdot d\theta$$

$$= \frac{1\cdot\cancel{3}\cdot1}{\cancel{6}\cdot4\cdot2} \cdot \frac{\pi}{2}$$

$$= \frac{\pi}{32} \text{ Ans}$$

<3> Use of Lebnitz formula →

[(when we have to diff'ate integral (S))]

$$\boxed{\frac{d}{dx}\int_{\phi(x)}^{\psi(x)} f(t)\,dt = f(\psi(x))\,\psi'(x) - f(\phi(x))\cdot\phi'(x)}$$

✓ like ↓

$$\lim_{x\to\frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)\,dt}{x^2-\frac{\pi^2}{16}} \quad \frac{\left(\int_2^2 = 0\right)}{0}$$

⇓ L' Hospital

$\psi\ \sec^2 x \cdot \tan x$

$f(x^2 - \frac{\pi^2}{16})$

sol^n $$\lim_{x\to\frac{\pi}{4}} \frac{f(\sec^2 x)\cdot \cancel{2}\sec^2 x\cdot\tan x - 0}{\cancel{2}x}$$

$$= \frac{f(2)\cdot 2\cdot 1}{\frac{\pi}{4}} = \frac{8\,(f(2))}{\pi}$$

✓ like $$\lim_{x\to 0} \frac{\int_0^x \cos t^2\,dt}{x}$$

$$\lim_{x\to 0} \frac{\cos x^2\cdot 1 - 0}{1}$$

$= 1$

eg ①) Let f(x) be a differenciable function such that

$f(2)=6$, $f'(2)=\frac{1}{48}$ than $\lim_{x\to 2} \int_6^{f(x)} \frac{4t^3}{x-2}\,dt$ is ---?,

$$= \lim_{n\to 2} \frac{\int_6^{f(n)} 4t^3\cdot dt}{x-2}$$

$$= \lim_{n\to 2} \frac{4\,(f(n))^3\cdot f'(n)}{1}$$

$$\lim_{n\to 2} \frac{f(4t^3\cdot dt)\cdot 12t^2}{12t^2}$$

$$= \cancel{4}\cdot\cancel{6}\cdot\underset{3}{\cancel{6}}\cdot 6\cdot \frac{1}{\frac{\cancel{48}}{\cancel{2}}}$$

$$= 18 \text{ Ans}$$

eg) Let $f:(0,\infty)\to R$ be a diff'ble function such that

$$\underbrace{F(x^2) = \int_0^{x^2} f(t)\,dt}$$ and ~~f(x)~~ $\underbrace{F(x^2) = x^2(1+x)}$ find $f(4)$?

Sol^n $F'(x^2)\cdot\cancel{2x} = f(x^2)\cdot\cancel{2x}$

~~$f(x^2)$~~

$2x\cdot F'(x^2) = 2x + 3x^2$

$\therefore F'(x^2) = 1 + \frac{3}{2}x$

$f(x^2) = F'(x^2)$

$f(x^2) = 1 + \frac{3}{2}x$

$f(4) = 1 + \frac{3}{\cancel{2}}\cdot\cancel{2}$

$= 4$ Ans

eg3) Let $\int_0^{t^2} x\cdot f(x)\cdot dx = \frac{2}{5}t^5,\ \underbrace{t>0}_{0,\infty}$ find $f\left(\frac{4}{25}\right)$?

Sol^n $t^2\cdot f(t^2)\cdot 2t = \frac{2}{\cancel{5}}\cdot\cancel{5}t^4$

$f(t^2) = t$

$f\left(\frac{4}{25}\right) = \frac{2}{5}$

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Let

eg4) $\int_{\sin x}^{1} t^2(f(t))dt = 1-\sin x$, $x \in (0, 2\pi)$

find $f\left(\frac{1}{\sqrt{3}}\right)$?

~~$\therefore 1\cdot(1)\cdot dx - 2tf'(t) -$~~
~~$\sin^2 x\,(f(\sin^2 x))\cdot$~~
~~$= 2 - 2tf'$~~

sol: $0 - \sin^2 x \cdot f(\sin x) \cdot \cos x = 0 - \cos x$

$\therefore f(\sin x) = \frac{1}{\sin^2 x}$

$\Rightarrow f\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{(1/3)} = 3$

★ $f(10)$ = Not defined

q5) Let $y = \int_0^x f(t)\cdot \sin(k(x-t))dt$ then $\frac{d^2y}{dx^2} + k^2y = k\cdot f(x)$ T/F

sol: $y = \int_0^x f(t)(\sin kx \cdot \cos kt - \cos kx \cdot \sin kt)\,dt$

$y = \sin kx \cdot \int_0^x f(t)\cos kt \cdot dt - (\quad)$

$\frac{dy}{dx} =$

★ Properties -

1) $\int_a^b f(x)dx =$

(

2.) $\int_a^b f(x)\,d$

3.) (Imp)

$\int_0^a f(x$

Step

★ Properties →

1) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

(Name of Variable can be changed at any Stage of Integeration)

2.) $\int_a^b f(x)\,dx = -\int_b^a f(x)dx$

3.) (Imp)

$$\underbrace{\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx}_{\text{Sum = Integrable}}$$

Steps: -

$$I = \int_0^a f(x)\,dx \quad \text{—(i)}$$

$$I = \int_0^a f(a-x)dx \quad \text{—(ii)}$$

Add:

$$2I = \int_0^a f(x) + f(a-x)dx$$

★ if $f(x) + f(a-x) = 1$

$$I = \frac{a}{2}$$

$$\therefore I = \frac{\text{upper limit}}{2} \quad //$$

eg1) $\int_0^\pi \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}\,dx = \frac{\pi}{2}$

How?

Sol/ $I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx$ —(i)

$I = \int_0^{\pi} \frac{e^{-\cos x}}{e^{-\cos x}+e^{\cos x}}dx$ —(ii)

$2I = \int_0^{\pi} \frac{\cancel{e^{\cos x}+e^{-\cos x}}}{\cancel{e^{\cos x}+e^{-\cos x}}}dx$

$I = \frac{\pi}{2}$

Q.) $\int_0^1 \log\left|1-\frac{1}{x}\right|dx$

Sol/ $\cancel{I = \int_0^1 \log\left|\frac{x-1}{x}\right|dx}$ —(i)

$\cancel{I = \int_0^1 \log\left|\frac{1-x}{x-1}\right|dx}$ —(ii) $\leftarrow$ $= \int_0^1 \log\left|1-\frac{1}{1-x}\right|dx$

$= \int_0^1 \log\left|\frac{1-x-1}{1-x}\right|dx$

$= \int_0^1 \log\left|\frac{-x}{1-x}\right|dx$

$= \int_0^1 \log$

~~Add~~ $2I = \int_0^1$

$I = \int_0^1 \log\left|1-\frac{1}{x}\right|dx$

$I = \int_0^1 \log\left|\frac{x-1}{x}\right|dx$ —(i)

$I = \int_0^1 \log\left|\frac{1-x-1}{1-x}\right|dx$

$I = \int_0^1 \log\left|\frac{x}{x-1}\right|dx$ —(ii)

$2I = \int_0^1 \boxed{\log\left|\frac{x-1}{x}\right| + \log\left|\frac{x}{x-1}\right|dx} \rightarrow \log 1 = 0 \Rightarrow \boxed{I=0}$ Ans

eg 6) $I = \int_0^{\pi/4} \log($

$I = \int_0^{\pi/4}$

$I = \int_0^{\pi/4}$

$I = \int_0^{\pi/4}$

$I =$

Add

eg7)

eg 6) $I = \int_0^{\pi/4} \log(1+\tan x)\,dx = \frac{\pi}{8}\log 2$

$I = \log(1+\tan x)\,dx$ —(1)
$I = \log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$
$= \log(1+$

$I = \int_0^{\pi/4} \log(1+\tan x)\,dx$ —(1)

$I = \int_0^{\pi/4} \log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$

$I = \int_0^{\pi/4} \log\left(1+\frac{1-\tan x}{1+\tan x}\right)dx$

$I = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right)dx$ —(1)

Add: I

$2I = \int_0^{\pi/4} \log(1+\tan x) + \log\frac{2}{(1+\tan x)}\,dx$

$2I = \int_0^{\pi/4} \log 2\,dx = \frac{\pi}{4}\log 2$

$I = \frac{\pi}{8}\cdot\log 2.$

eg 7) $8\int_0^1 \frac{\log(1+x)}{(1+x^2)}\,dx$

$\hookrightarrow x = \tan\theta$

$8\int_0^{\pi/4} \frac{\log(1+\tan\theta)}{(1+\tan^2\theta)}\cdot\sec^2\theta\cdot d\theta$

$= \cancel{8}\cdot\frac{\pi}{\cancel{8}}\cdot\log 2 = \pi\cdot\log 2$

eg.) $\int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x\cdot\cos x}\,dx$

Soln $I = \int_0^{\pi/2} \frac{\sin x-\cos x}{1+\sin x\cdot \cos x}\,dx$ —(i)

$I = \int_0^{\pi/2}\frac{\cos x - \sin x}{1+\sin x\cdot\cos x}\,dx$ —(ii')

$2I = \int_0^{\pi/2}\frac{\sin x-\cos x+\cos x-\sin x}{1+\sin x\cdot\cos x}\,dx$

$2I = 0 \Rightarrow \boxed{I=0}$

~~eg.) $\int_0^{\pi/2}$~~

① Note (formula) →

(i) $\int_0^{\pi/2}\frac{\sin^n x}{\sin^n x+\cos^n x}\,dx = \int_0^{\pi/2}\frac{\cos^n x}{\sin^n x+\cos^n x}\,dx = \frac{\pi}{4}$

(ii) $\int_0^{\pi/2}\frac{\tan^n x}{\tan^n x+\cot^n x}\,dx = \int_0^{\pi/2}\frac{\cot^n x}{\tan^n x+\cot^n x}$

(iii)

(iv) $\int_0^{\pi/2}\frac{1}{\tan^n x+1}\,dx = \int_0^{\pi/2}\frac{\tan^n x}{\tan^n x+1}\,dx = \frac{\pi}{4}$

eg1) $\int_0^{\infty} \frac{x}{(1+x)(1+x^2)}\,dx$

$x = \tan\theta$

$\int_0^{\pi/2} \frac{\tan\theta}{(1+\tan\theta)\cdot\cancel{(1+\tan^2\theta)}} \cdot \cancel{\sec^2\theta}\, d\theta$

$= \frac{\pi}{4}$

eg2) $\int_0^{a} \frac{1}{x+\sqrt{a^2-x^2}}\,dx$

$\xrightarrow{} x = a\sin\theta$

$\int_0^{\pi/2} \frac{1 \cdot \cancel{a}\cos\theta\, d\theta}{\cancel{a}\sin\theta+\cancel{a}\cos\theta}$

$= \frac{\pi}{4}$

<2> (a) $\boxed{\int_0^{\pi/2} \log(\sin x)\,dx = \int_0^{\pi/2} \log(\cos x)\,dx = -\frac{\pi}{2}\log 2}$

(b) $\boxed{\int_0^{\pi/2} \log(\tan x)\,dx = \int_0^{\pi/2} \log(\cot x)\,dx = 0}$

(c) $\boxed{\int_0^{\pi/2} \log(\sec x)\,dx = \int_0^{\pi/2} \log(\operatorname{cosec} x)\,dx = \frac{\pi}{2}.\log 2}$

~~$\int_0^{\pi/4} \log(\sin 2x)\,dx$~~

$$= \int_0^{\pi/4} \log(\sin 2x)\, dx$$

$$= \frac{1}{2} \int_0^{\pi/2} \log \sin(x)\, dx$$

$$= \frac{1}{2}\left(-\frac{\pi}{2} \log 2\right) = -\frac{\pi}{4} \log 2$$

$$\int_a^b f(x)\, dx = k \int_{a/k}^{b/k} f(kx)\, dx$$

$$\int_a^b f(x)\, dx = \int_{a-k}^{b-k} f(x+k)\, dx$$

$$\int_a^b f(x)\, dx = \int_{a+k}^{b+k} f(x-k)\, dx$$

eg.1)
$$\int_0^{\pi/2} \underbrace{\sin 2x}_{} \cdot \boxed{\log(\tan x)}\, dx$$

$$\Downarrow$$

$$= \sin 2\left(\frac{\pi}{2} - x\right)$$

$$= \sin(\pi - 2x)$$

$$= \sin 2x$$

(Ans = 0)

eg.2)
$$\int_0^{\pi/2} \frac{\sin 8x \cdot \boxed{\log(\cot x)}}{\cos 2x}\, dx$$

so/n

su

$$= \frac{\sin 8\left(\frac{\pi}{2}-x\right)}{\cos 2\left(\frac{\pi}{2}-x\right)}$$

$$\Rightarrow \frac{\sin(4\pi - 8x)}{\cos(\pi - 2x)}$$

$$= \frac{\cancel{-}\sin 8x}{\cancel{-}\cos 2x} = f(x)$$ Ans → 0

(3) Removal of "x" →

$$\boxed{\int_0^a x\cdot f(x)\,dx = \frac{a}{2}\int_0^a f(x)\,dx \quad \downarrow \quad \boxed{\text{if } f(a-x) = f(x)}}$$

eg.) Let $I_1 = \int_0^{\pi} x\cdot \log(\sin x)\,dx$ and

$I_2 = \int_0^{\pi} \log(\sin x)\,dx$ then

$2I_1 = I_2$ (T/F)?

so/n $I_1 = \int_0^{\pi} x\cdot \log(\sin x)\,dx$

$I_1 = \frac{\pi}{2}\int_0^{\pi} \log(\sin x)\,dx$

$2I_1 = \pi \cdot I_2$

so false /

$\frac{\pi}{2}\log(\sin x) \to \frac{\pi}{2}\log(\sin x)$

eg 2) $\int_0^{\pi} \frac{x}{1+\sin x}\,dx$ (1-sinx multiply)

Sol^n $= \frac{\pi}{2}\int_0^{\pi} \frac{1}{(1+\sin x)}\,dx$

$= \frac{\pi}{2}\left(\tan x - \sec x\right)\Big|_0^{\pi} = \frac{\pi}{2}\left(\left(0-(-1)\right)-\left(0-1\right)\right)$

$= \frac{\cancel{\pi}}{\cancel{2}}\cdot\cancel{2}$

$= \pi$ Ans/

$\Rightarrow \frac{\pi}{2}\int_0^{\pi}\frac{1}{1+\sin x}\,dx$

$= \frac{\pi}{2}\int \frac{1}{1+\sin(\frac{\pi}{2}...}$

$= \frac{\pi}{2}\int \frac{1}{1+\cos x}\,dx$

$\frac{\pi}{2}$

① eg 8)

eg 3.) $\int_0^{\pi/2} \frac{x\cdot\sin x\cdot\cos x}{\sin^4 x+\cos^4 x}\,dx$

Sol^n $\frac{\pi}{4}\int_0^{\pi/2} \frac{\sin x\cdot\cos x}{\sin^4 x+\cos^4 x}\,dx$

Divide by $\cos^4 x$

$\tan^2 x = t$ /

<2>

$\boxed{P-4}$ $\int_a^b f(x)\,dx = \int_a^{c_1} f(x)\,dx + \int_{c_1}^{c_2} f(x)\,dx + \int_{c_2}^{c_3} f(x)\,dx + \cdots + \int^{b} f(x)\,dx$

when

$f(x) = [\ ]\ ,\ \{\ \}\ ,\ |\ |\ ,\ \mathrm{sgn}(x)$, multiple definition/

like $\int_0^{n\in N} [x]\,dx$

$= \cancel{\int_0^1 0\,dx} + \int_1^2 1\,dx + \int_2^3 2\,dx + \cdots + \int_{n-1}^{n} (n-1)\,dx$

$\quad 1 \quad + 2 \quad + \cdots + (n-1)$

$$= \sum (n-1)$$

$$= \frac{(n-1)}{2}\,(n-1+1) \quad = \frac{n(n-1)}{2}$$

like:- $\int_0^5 [x]\,dx = \frac{5\cdot 4}{2} = 10$

① eg) $\int_0^{5.2} [x]\,dx = \int_0^5 [x]\,dx + \int_5^{5.2} [x]\,dx$

$$= \cancel{10}\ \frac{5\cdot 4}{2} + \int_5^{5.2} 5\,dx$$

$= \frac{5\times 4}{2}$:

=

$$= 10 + 5\times(0.2)$$

$$= 10+1 = 11$$

<2> $n\in N$

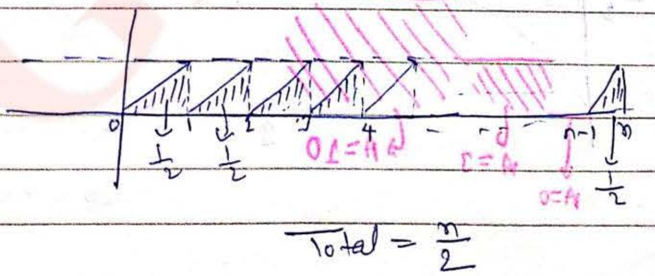
$$= \int_0^n \{x\}\,dx = \int_0^n x-[x]\,dx$$

$$= \int_0^n x\cdot dx - \int_0^n [x]\,dx$$

$$= \left(\frac{x^2}{2}\right)_0^n - \frac{(n-1)n}{2}$$

$$= \frac{\cancel{n^2}}{2} - 0 - \frac{\cancel{n^2}}{2} + \frac{n}{2}$$

$$= \frac{n}{2}$$



$$\text{Total} = \frac{n}{2}$$

eg $\int_{1}^{4} \{x\}^{[x]} \, dx \xrightarrow{\quad} (x-[x])^{[x]}$

$\int_{01}^{4}$

eg 4.) $\int_{0}^{3/2}$

Sol^n

Sol^n $= \int_{1}^{2} (x-1)\, dx + \int_{2}^{3} (x-2)^2 dx + \int_{3}^{4} (x-3)^3 dx$

$= \left(\frac{(x-1)^2}{2}\right)_1^2 + \left(\frac{(x-2)^3}{3}\right)_2^3 + \left(\frac{(x-3)^4}{4}\right)_3^4$

$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}$

eg 3.) $\int_{0}^{9} [\sqrt{x}] \, dx$

$0 < x < 9$

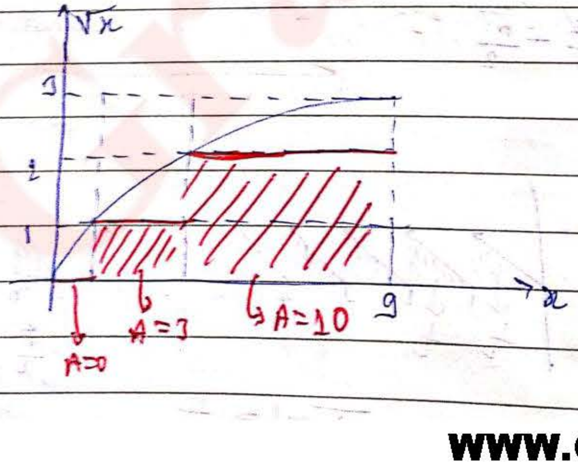
$0 < \sqrt{x} < 3$

$0 < \sqrt{x} < 1 \; , \; 1 < \sqrt{x} < 2 , \quad 2 < \sqrt{x} < 3$

$[\sqrt{x}] = 0 \; ; \; [\sqrt{x}] = 1 \; ; \; [\sqrt{x}] = 2$

$\cancelto{0}{\int_{0}^{1} 0\, dx} + \int_{1}^{4} 1\, dx + \int_{4}^{9} 2\, dx$

$3 + 2 \times 5 = 13$



eg) $\int_{0}^{3/2} [x^2]\,dx$

$0 < x^2 < \frac{9}{4}$

$0 < [x^2] < \underline{\underline{2}}$

sol) $0 < x < \frac{3}{2}$

$0 < x^2 < \frac{9}{4}$

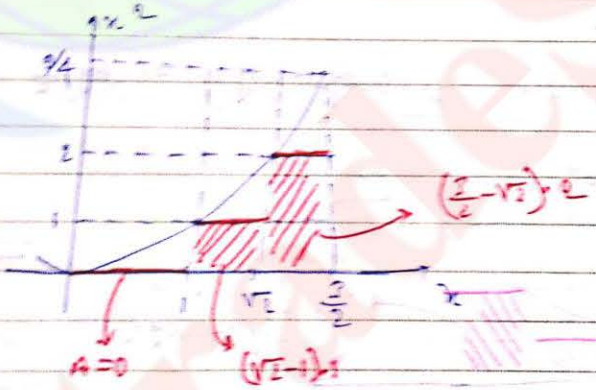
$0 < x^2 < 1$; $1 < x^2 < 2$; $2 < x^2 < \frac{9}{4}$

$[x^2] = 0$ $[\ ] = 1$ $[\ ] = 2$

$= \int_{0}^{1} 0\,dx + \int_{1}^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{3/2} 2\,dx$

$= \sqrt{2} - 1 + 2\left(\frac{3}{2} - \sqrt{2}\right)$

$= 2 - \sqrt{2}$



eg) $\int_{-\infty}^{\log 2} [e^x]\,dx$

$-\infty < x < \log 2$

sol) $-\infty < x < \log 2$

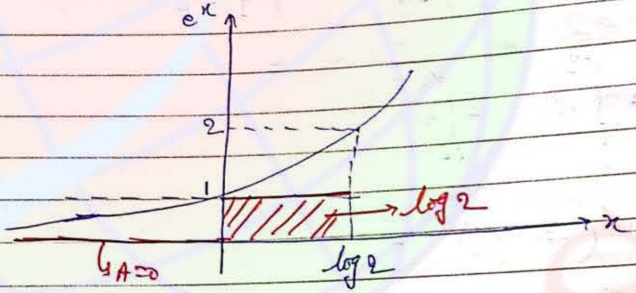
$0 < e^x < 2$

$0 < e^x < 1 \quad ; \quad 1 < e^x < 2$

$[e^x] = 0 \; , \quad [e^x] = 1$

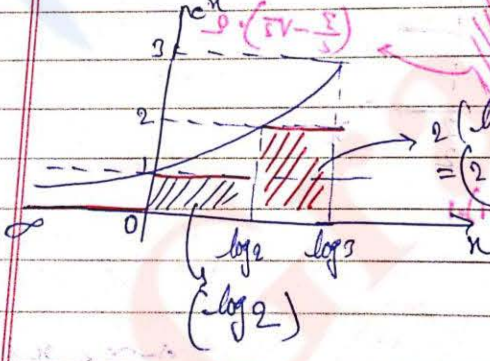
$$\int_{-\infty}^{0} 0\,dx + \int_{0}^{\log 2} 1\,dx$$

$$= \log 2$$

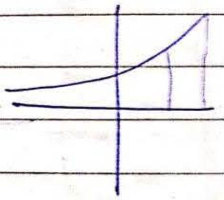


q6)

$$\int_{-\infty}^{\log 3} [e^x]\,dx = 2\log 3 - \log 2 = \log \frac{9}{2}$$

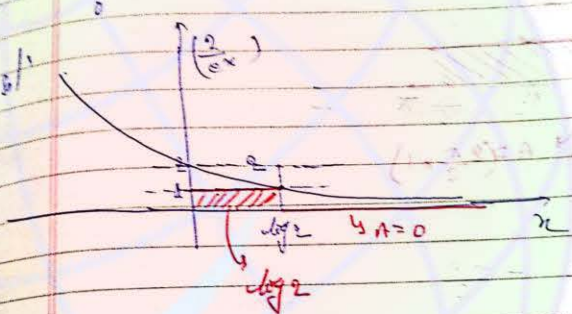


$2(\log 3 - \log 2)$
$= (2\log 3 - 2\log 2)$



q1) $\int_0^{\infty} \left[\frac{2}{e^x}\right] dx$? $= \log 2$

Sol



$\frac{2}{e^x} = 1$

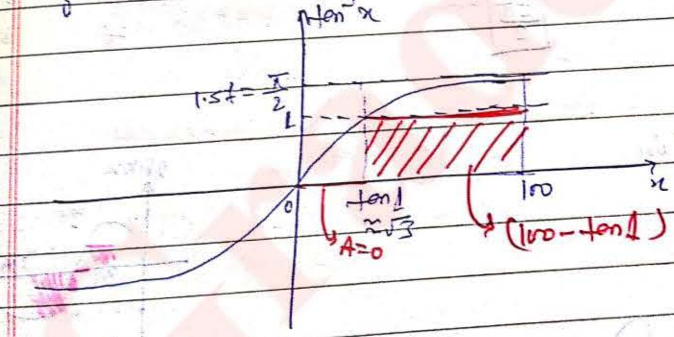
$\therefore e^x = 2$

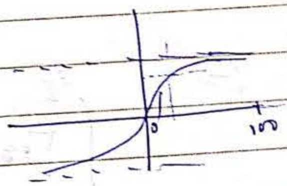
$x = \log 2$

Q2) $\int_0^{100} [\tan^{-1} x]\, dx$? $= 100 - \tan 1$

1.57

Sol



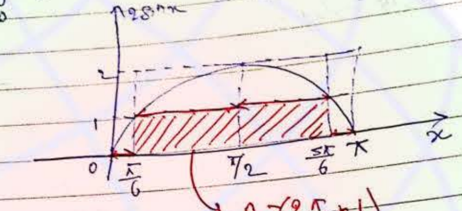


$\tan^{-1} x = 1$

$x = \tan 1$

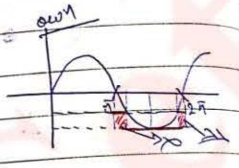
Eg 2) $\int_0^{\pi} [2\sin x]\, dx = \frac{2\pi}{3}$

Sol^n

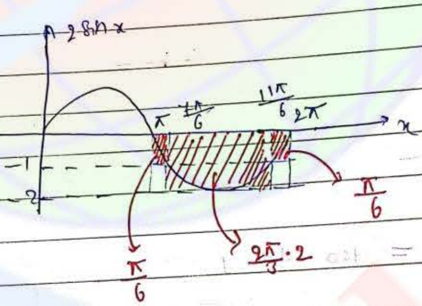


$A = \left(\frac{2\pi}{3} \times 1\right)$

Eg 3) $\int_{\pi}^{2\pi} [2\sin x]\, dx = -\frac{5\pi}{3}$



Sol^n



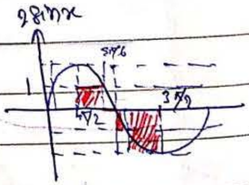
$2\sin x = -1$

$\pi + \frac{\pi}{6}$ $\sin x = -\frac{1}{2}$

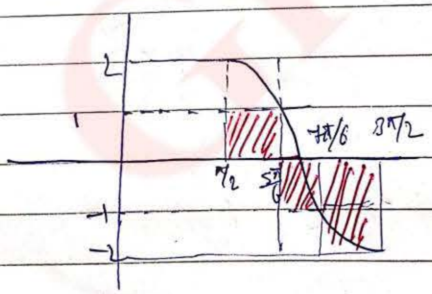
$\frac{2\pi}{6}$ $\sin x, \sin\frac{\pi}{6}$

$\boxed{\frac{5\pi}{3}}$

Eg 4) $\int_{\pi/2}^{3\pi/2} [2\sin x]\, dx$



Sol^n



$1\left(\frac{5\pi}{6} - \frac{\pi}{2}\right) + \frac{\pi}{6}$

Ans → $-\frac{\pi}{6}$

★ modulus funct

$\int_a^b |f(x$

If

break

like

$\int_0^1$

★ modlus function

$$\int_a^b |f(x)|\,dx$$

If f(x) changes its sign at x=c in (a,b) then break integral at x=c.

like

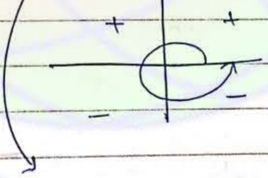
$$\int_0^1 |\sin 2\pi x|\,dx$$

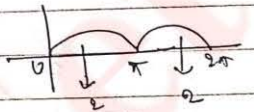
$2\pi x = t$

$$\frac{1}{2\pi}\int_0^{2\pi} |\sin t|\,dt$$

$0 < x < 1$

$0 < 2\pi x < 2\pi$





$\frac{1}{2\pi}(4) = \boxed{\frac{2}{\pi}}$

$0 < 2\pi x < \pi$, $\pi < 2\pi x < 2\pi$

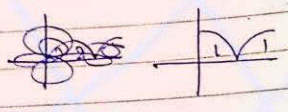
⊕ ⊖

$$\int_0^{\frac{1}{2}} \sin 2\pi x\,dx + \int_{1/2}^{1} -\sin 2\pi x\,dx$$

$$= -\left(\frac{\cos 2\pi x}{2\pi}\right)_0^{\frac{1}{2}} + \left(\frac{\cos 2\pi x}{2\pi}\right)_{1/2}^{1} = \boxed{\frac{2}{\pi}}$$ Ans

Eg 2) $\int_0^\pi \sqrt{\frac{1+\cos x}{2}}$

$\int_0^\pi |\cos x|\, dx \quad = 2$

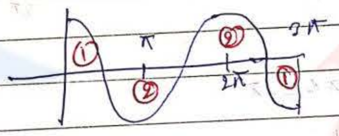


$\Rightarrow 2$ Ans

Eg 3) $\int_0^{3/2} |\cos 2\pi x|\, dx$

$2\pi x = t$

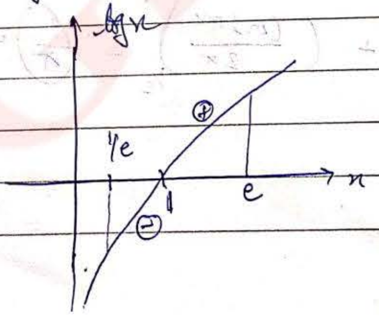
$\frac{1}{2\pi}\int_0^{3\pi} |\cos t|\, dt$



$\frac{1}{2\pi} \times 6 = \frac{3}{\pi}$ Ans

Eg 4) $\int_{e^{-1}}^{e} |\log x|\, dx$

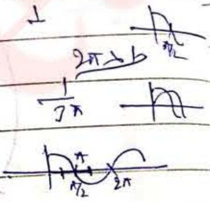
Sol^n



$\cos 2\pi x > \frac{3}{2}$

$\frac{1}{2\pi}$

$2\pi > b$



Eg 5)

−2.

$$= \int_{1/e}^{1} -\log x\, dx + \int_{1}^{e} \log x\, dx$$

$$= -\left(x\log x - x\right)_{\frac{1}{e}}^{1} + \left(x\log x - x\right)_{1}^{e}$$

$$= -\left((-1) - \left(-\frac{1}{e} - \frac{1}{e}\right)\right) + \left((\cancel{e} - \cancel{e}) - (0-1)\right)$$

$$= 1 - \frac{2}{e} + 1 = 2 - \frac{2}{e} \text{ Ans}$$

g) $\displaystyle\int_{e^{-}}^{e^2} \left|\frac{\log x}{x}\right| dx$

$$\int_{e^{-1}}^{e^2} \frac{|\log x|}{x}\, dx$$

$$\int_{\frac{1}{e}}^{1} -\frac{\log x}{x} + \int_{1}^{e^2} \frac{\log x}{x}\, dx$$

Let $\log x = t$

$$\Rightarrow -\int_{-1}^{0} t\, dt + \int_{0}^{2} t\, dt$$

$$\Rightarrow \frac{5}{2} \text{ A}$$

$\boxed{P-5}$

$$\int_{-a}^{a} f(x) = \begin{cases} 2\int_0^a f(x)\,dx & ; f(x) = \text{even} \\ \text{Ans} = 0 & ; f(x) = \text{odd} \end{cases}$$

likes

$$\int_{-\pi/2}^{\pi/2} \underbrace{\log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)}_{\downarrow} d\theta = 0$$

$$\log\left(\frac{2+\sin\theta}{2-\sin\theta}\right) = -f(\theta)$$

$$= \text{odd}$$

like

$$\int_{-1}^{1} \underbrace{\sqrt{1+x+x^2} - \sqrt{1-x+x^2}}_{\downarrow}\, dx = 0$$

$$f(-x) = \sqrt{1-x+x^2} - \sqrt{1+x+x^2}$$

$$= -f(x) \text{ (odd)}$$

likes -

$$\int_{-\pi/2}^{\pi/2} \frac{(f(x)+f(-x))^3 \cdot (g(x)-g(-x))^5}{(h(x)-h(-x))^4}\, dx = 0$$

$$\frac{E \cdot O}{E} = \text{odd}$$

likes -

$$\int_{-1/2}^{1/2} [x] + \cancel{\underbrace{\log\left(\frac{1-x}{1+x}\right)}_{\text{odd}}} \xrightarrow{} 0 \; dx$$

$$= \int_{-1/2}^{0} -1\, dx + \cancel{\int_0^{1/2} 0\, dx} \xrightarrow{} 0$$

$$= -1\left(0 - \left(-\frac{1}{2}\right)\right) = -\frac{1}{2} \text{ Ans}$$

$\boxed{P-5}$

$$\int_{-a}^{a} f(x) = \begin{cases} 2\int_{0}^{a} f(x)\,dx & ;\ f(x) = \text{even} \\ \text{Ans} = 0 & ;\ f(x) = \text{odd} \end{cases}$$

likes

$$\int_{-\pi/2}^{\pi/2} \underbrace{\log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)}_{x} d\theta = 0$$

$$\log\left(\frac{2+\sin\theta}{2-\sin\theta}\right) = -f(\theta)$$
$$= \text{odd}$$

like

$$\int_{-1}^{1} \underbrace{\sqrt{1+x+x^2} - \sqrt{1-x+x^2}}\,dx = 0$$

$$f(-x) = \sqrt{1-x+x^2} - \sqrt{1+x+x^2}$$
$$= -f(x)\ (\text{odd})$$

likes -

$$\int_{-\pi/2}^{\pi/2} \frac{\left(f(x)+f(-x)\right)^3 \cdot \left(g(x)-g(-x)\right)^5}{\left(h(x)-h(-x)\right)^4}\,dx = 0$$

$$\frac{E \cdot O}{E} = \text{odd}$$

likes -

$$\int_{-1/2}^{1/2} [x] + \underbrace{\cancel{\log\left(\frac{1-x}{1+x}\right)}}_{\text{odd}} \xrightarrow{} 0 \; dx$$

$$= \int_{-1/2}^{0} -1\,dx + \cancel{\int_{0}^{1/2} 0\,dx} \rightarrow 0$$

$$= -1\left(0 - \left(-\frac{1}{2}\right)\right) = -\frac{1}{2} \text{ Ans}$$

eg. $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\, dx$

eg1) $= \int_{-\pi/2}^{\pi/2} \underbrace{\sqrt{\cos x} \cdot |\sin x|\, dx}_{\text{even}}$

$= 2\int_0^{\pi/2} \sqrt{\cos x}\, [\sin x]\, dx$

$= -2\int_1^0 \sqrt{t} \cdot dt$

$= 2\int_0^1 \sqrt{t}\, dt$

$= 2 \cdot \frac{2}{3} \left(t^{3/2}\right)_0^1$

$= \frac{4}{3}$

eg3) $\int_{-\pi/2}^{\pi/2} \underbrace{\underset{\textcircled{E}}{\cos^3\theta} \cdot (1+\sin\theta)^2 d\theta}_{\text{separate}}$

$\int_{-\pi/2}^{\pi/2} \underset{E}{\cos^3\theta} + \underset{E}{\cos^3\theta \cdot \sin^2\theta} + \underset{\text{odd}}{\cancel{2\cos^3\theta \cdot \sin\theta}}^{\,0}\, d\theta$

$= 2\left(\int_0^{\pi/2} \cos^3\theta\, d\theta + \int_0^{\pi/2} \cos^3\theta \cdot \sin^2\theta\, d\theta\right)$

$= 2\left(\frac{2}{3} + \frac{2 \cdot 1}{5 \cdot 3 \cdot 1}\right)$

$= 2 \cdot \frac{2}{3}\left(1 + \frac{1}{5}\right)$

$= 2 \cdot \frac{2}{\cancel{3}} \cdot \frac{\cancel{6}\,2}{5}$

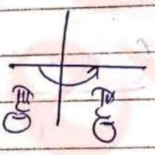
$= \frac{8}{5}$ Ans

eg2) $= \int_{-1}^{3/2} |x \sin \pi x|\, dx$

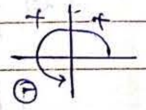
$= \int_{-1}^{3/2} |x|\, |\sin \pi x|\, dx$

$= \int_{-1}^{0} -x\, |\underset{\ominus}{\sin \pi x}|\, dx + \int_0^{3/2} x\, |\sin \pi x|\, dx$

$= -1 < x < 0$
$-\pi < \pi x < 0$



$0 < x < \frac{3}{2}$
$0 < \pi x < \frac{3\pi}{2}$



$= \int_{-1}^{0} x \cdot \sin \pi x\, dx + \int_0^1 x \cdot \sin \pi x\, dx + \int_1^{3/2} -x \sin \pi x\, dx$

$= 2\int_0^1 \underset{E}{x \sin \pi x}\, dx - \int_1^{3/2} x \cdot \sin \pi x\, dx$

eg4) let

so|n

eg5)

so|n

classmate

Date
Page

eg 4) Let $f(x) = \begin{cases} e^{\cos x} \sin x \, dx & ; \boxed{|x| < 2} \\ 10 & ; \text{else} \end{cases}$ find $\int_{-2}^{3} f(x)\,dx$

$-2 < x < 2$

sol^n

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{2} \underbrace{\cancel{e^{\cos x} \sin x\,dx}}_{\text{odd}} + \int_{2}^{3} 10\,dx$$

$$= 10 \text{ A/}$$

eg 5) $\displaystyle\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\,dx$?

sol^n $$= \int_{-\pi}^{\pi} \cancel{\frac{2x}{1+\cos^2 x}\,dx} \xrightarrow{} 0 + \int_{-\pi}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\,dx$$

odd

$$= 2 \cdot 2 \int_{0}^{\pi} \underbrace{\frac{x\sin x}{1+\cos^2 x}\,dx}_{\text{Removal of } x}$$

$$= 2 \cdot \frac{2\pi}{\cancel{2}} \int_{0}^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$$

$$= 2\pi \int_{1}^{-1} \frac{-dt}{1+t^2}$$

$$= 2\pi \int_{-1}^{1} \frac{1}{1+t^2}\,dt$$

$$= 2\pi \left(\tan^{-1} t\right)_{-1}^{1}$$

$$= 2\pi\left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right) = \cancel{2}\pi \cdot \frac{\pi}{\cancel{2}} = \pi^2 \text{ A}$$

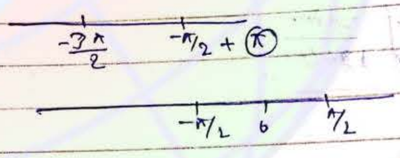
Note

i) $\int_a^b f(x)\,dx = \int_{a+k}^{b+k} f(x-k)\,dx$

ii) $\int_a^b f(x)\,dx = \int_{a-k}^{b-k} f(x+k)\,dx$

iii) $\int_a^b f(x)\,dx = \frac{1}{k}\int_{ak}^{bk} f\left(\frac{x}{k}\right)dx$ //

eg1) $\int_{-\frac{3\pi}{2}}^{-\pi/2} (x+\pi)^3 + \cos^2(3\pi+x)\,dx$



$= \int_{-\pi/2}^{\pi/2} \underset{odd}{x^3} + \underset{E}{\cos^2 x}\,dx$

$= 2\int_0^{\pi/2} \cos^2 x\,dx = \cancel{2}\cdot\frac{1}{\cancel{2}}\cdot\frac{\pi}{2} = \pi/2$

eg2) $\int_{-2}^{0} \underbrace{x^3 + 3x^2 + 3x + 7} + (x+1)\cos(x+1)\,dx$

$(x+1)^3 = x^3 + 3x^2 + 3x + 1$

Sol: $\int_{-2}^{0} (x+1)^3 + 2 + (x+1)\cos(x+1)\,dx$

$\int_{-1}^{1} \underset{odd}{(x)^3} + 2 + \underset{odd}{x\cos x}\,dx \rightarrow = 2(1-(-1)) = 4$ Ans

eg3) Let

Sol

classmate Date Page

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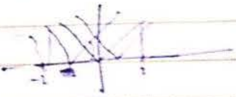
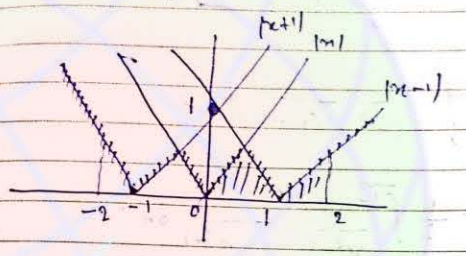
Q5) Let $f(x) = \min\{|x-1|, |x|, |x+1|$

find $\int_{-2}^{2} f \, dx$?



sol



$$\int_{-2}^{2} \cancel{\text{even}} \, dx = 2\int_{0}^{2} f(x)\,dx$$

$$= 2\left(\frac{1}{2}\cdot1\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot1\cdot1\right)$$

$$= 2\left(\frac{1}{4}+\frac{1}{2}\right) = \frac{3}{2}$$

Q) $$= \int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \left(\frac{x^4}{1-x^4}\right) \cos^{-1}\left(\frac{2x}{1+x^2}\right) dx$$

$$= \int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{x^4}{1-x^4}\left(\frac{\pi}{2} - \sin^{-1}\frac{2x}{1+x^2}\right) dx$$

$$= \int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{\pi}{2}\cdot\frac{x^4}{1-x^4}\,dx - \int_{-1/\sqrt{3}}^{\frac{1}{\sqrt{3}}} \cancel{\frac{x^4}{1-x^4}\cdot\sin^{-1}\left(\frac{2x}{1+x^2}\right)}dx$$

(even) odd

$$= \frac{\pi}{2}\cdot 2\int_{0}^{1/\sqrt{3}} \frac{x^4}{1-x^4}\,dx$$

$$= -\pi \int_0^{\frac{1}{\sqrt{3}}} \frac{1-x^4+1}{1-x^4}\, dx$$

$$= -\pi \left[\int_0^{1/\sqrt{3}} 1\, dx + \int_0^{1/\sqrt{3}} \frac{1}{1-x^4}\, dx \right]$$

$$= -\pi \left[\frac{1}{\sqrt{3}} + \int_0^{1/\sqrt{3}} \frac{1}{(x^2-1)(x^2+1)}\, dx \right]$$

$$= -\pi \left[\frac{1}{\sqrt{3}} + \frac{1}{2} \int_0^{\frac{1}{\sqrt{3}}} \frac{1}{x^2-1} - \frac{1}{x^2+1}\, dx \right.$$

$$= -\pi \left[\frac{1}{\sqrt{3}} + \frac{1}{2} \left[\frac{1}{2} \left(\log \left| \frac{x-1}{x+1} \right| \right)_0^{\frac{1}{\sqrt{3}}} - \left(\tan^{-1} x \right)_0^{\frac{1}{\sqrt{3}}} \right] \right]$$

$$= -\pi \left[\frac{1}{\sqrt{3}} + \frac{1}{2} \left[\frac{1}{2} \left(\log \frac{\sqrt{3}-1}{\sqrt{3}+1} - 0 \right) - \frac{\pi}{6} \right] \right]$$

$$= -\pi \left[\frac{1}{\sqrt{3}} + \frac{1}{4} \log \left(\frac{\sqrt{3}-1}{\sqrt{3}+1} \right) - \frac{\pi}{12} \right]$$

P-6

$$\int_0^{2a} f(x)\, dx = \begin{cases} 2\int_0^{a} f(x)\, dx & ; f(2a-x) = f(x) \\ \text{Ans} = 0 & ; f(2a-x) = -f(x) \end{cases}$$

Proof:-

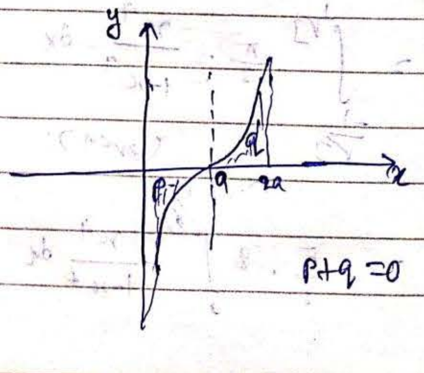
Case-1

$f(2a-x) = -f(x)$

$\therefore x = a-x$

~~$f(a+x)$~~ $= -f(a-x)$

$f(a+x) = -f(a-x)$



$P+q = 0$

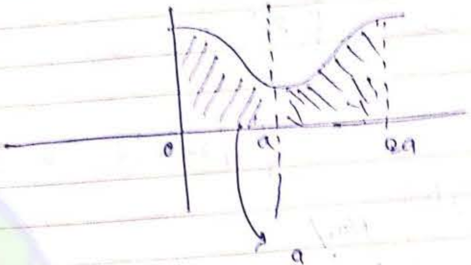
Cases –

$f(2a-x) = f(x)$

$\therefore x = a-x$

$= f(a-x)$

$f(a+x) = f(a-x)$



$$2\int_0^a f(x)\,dx$$

Note

$2\pi, 4\pi, 6\pi ---- 2n\pi$

$$\boxed{(1)\ \int_0^{} \sin^{odd} x \text{ or } \cos^{odd} x\,dx = 0}$$

Hence :-

$$\int_0^{2\pi} \sin^5 x\,dx = 0 \quad \text{Ans}$$

$\hookrightarrow = \sin^5(2\pi - x)$

$= -\sin^5 x\,dx = -f(x)$

Hw:- $\int_0^{2\pi} \cos^5 x\,dx$

$\hookrightarrow = \cos^5(2\pi - x)$

$= \cos^5 x = f(x)$

$$= 2\int_0^{\pi} \cos^5 x\,dx$$

$\hookrightarrow \cos^5(\pi - x)$

$= -\cos^5 x = -f(x)$

So $2.0 = 0$ //

eg.) $\int_0^{2\pi} \sin^3\theta \, d\theta = 0$

$\therefore \int_0^{2\pi} |\sin^3\theta| \, d\theta = 0$? Yes/No

$= [\sin(2\pi-\theta)]^3 \cdot d\theta$
$= -\sin$

Sol.) No./

$|\sin^3(2\pi-\theta)| = |-\sin^3\theta| = |\sin^3\theta|$

$$2\int_0^{\pi} \sin^3\theta \, d\theta \quad \textcircled{+} \longrightarrow \sin^3(\pi-\theta) = \sin^3\theta$$

$$= 2\cdot 2\int_0^{\pi/2} \sin^3\theta \, d\theta$$

$$= 2\cdot 2\cdot \frac{2}{3} = \frac{8}{3}$$

eg 5.) Let $a, b \in R$, then $\int_0^{2\pi} \frac{\sin 2\theta}{a-b\cos\theta} \, d\theta = $ ———— ?

Sol.)
$$= \frac{\sin(2\pi-2\theta)}{a-b\cos(2\pi-\theta)}$$

$$2\int_0^{\pi} \frac{\cos(2\pi-2\theta)}{a-b\cos(2\pi-\theta)}$$

$$= \cancel{\frac{-\sin 2\theta}{a-b\cos\theta}} \; (\times) \longrightarrow \text{so} \quad \frac{\sin 2(2\pi-\theta)}{a-b\cos(2\pi-\theta)}$$

$$= \frac{-\sin 2\theta}{a-b\cos\theta} = -f(\theta) \qquad = \frac{-\sin 2\theta}{a-b\cos\theta}$$

$$= -f(\theta)$$

Reframe formula

$* \int_0^{2a} f(x)$

eg.) $\int_0^{\pi}$

(Q4) eg 4

Sol.

$x+1$ (Pa$_1$)

Definite formula

$$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^{a} f(x)\,dx & ;\ f(2a-x)=f(x) \\ Ans=0 & ;\ f(2a-x)=-f(x) \end{cases}$$

Q) $\int_0^{\pi} e^{\sin^2 x}\cdot\cos^3(2n+1)x\,dx$; $n\in N$

$\downarrow$

$= e^{\sin^2(\pi-x)}\cdot\cos^3\underset{odd}{(2n+1)}(\pi-x)$

$= e^{\sin^2 x}\left(-\cos^3(2n+1)x\right)$

$= -f(x)$

Ans $= 0$

(Pa$_1$) Q4) $\int_0^{\pi/2}\log(\sin x)\,dx = -\frac{\pi}{2}\log 2$

~~$\int_0^{\pi/4}\log\left(\sin\left(\frac{\pi}{2}-x\right)\right)$~~

So/ $I = \int_0^{\pi/2}\log(\sin x)\,dx$ — (1)

$I = \int_0^{\pi/2}\log\cos x\,dx$ — (ii)

$2I = \int_0^{\pi/2}\log(\sin x\cdot\cos x)\,dx$

$2I = \int_0^{\pi/2}\log\sin 2x\,dx - \int_0^{\pi/2}\log 2\,dx$

$2x = t$

So,

$$2I = \frac{1}{2}\int_0^{\pi} \underbrace{\log \sin\theta \, d\theta}_{[P-6]} - \frac{\pi}{2}\log 2$$

$$2I = \frac{1}{2}\cdot 2\underbrace{\boxed{\int_0^{\pi/2}\log\sin\theta\,d\theta}}_{I} - \frac{\pi}{2}\log 2$$

$$I = -\frac{\pi}{2}\log 2 \quad Ay$$

$\boxed{P-7}$ Solve periodic function–

1.) $$\int_0^{nT} f(x)\,dx = n\int_0^{T} f(x)\,dx$$

period = T

2.) $$\int_{nT}^{mT} f(x)\,dx = \int_0^{(m-n)T} f(x)\,dx$$

3.) $$\int_0^{nT+\alpha} f(x)\,dx = \int_0^{nT} f(x)\,dx + \int_0^{\alpha} f(x)\,dx$$

4.) $$\int_{nT}^{nT+\alpha} f(x)\,dx = \int_0^{\alpha} f(x)\,dx$$

where
m, n → Integer

5.) $$\int_a^{a+T} f(x)\,dx = \int_0^{T} f(x)\,dx$$

eg1) $\int_0^{1000\pi} \sin^4 x \, dx$

$= 1000 \int_0^{\pi} \underbrace{\sin^4 x \, dx}_{\to (P-6)}$

$= 1000 \cdot 2 \int_0^{\pi/2} \sin^4 x \, dx$

$= 1000 \times 2 \times \frac{3 \cdot 1}{4 \cdot 2} = 375\pi$ Ans/

$\boxed{\text{Even} \Rightarrow \pi}$ ✓

$\boxed{\text{Odd} \longrightarrow 2\pi}$

eg2) $\int_0^{1000\pi} \sin^5 x \, dx$

$= \cancel{1000 \int_0^{\pi} \sin^5 x \, dx} = 500 \cdot \int_0^{2\pi} \sin^5 x \, dx$

$\cancel{= 1000 \cdot 2 \int_0^{\pi/2} \sin^5 x \, dx}$ $= 500 \cdot 0 = 0$ /

$\cancel{= 1000}$

eg3) $\int_0^{10} e^{x-[x]} \, dx$

$\int_0^{10 \cdot 1} e^{\{x\}} \, dx$

$10 \int_0^1 e^{\{x\}} \, dx$

$= 10 \int_0^1 e^{x} \, dx$

$= 10 \left(e^x\right)_0^1$

$= 10(e-1)$ Ans/

(eg4) $\sum_{n=1}^{1000} \int_{n-1}^{n} e^{x-[x]} \, dx$

Sol^n

$= \int_0^1 + \int_1^2 + \text{---} + \int_{999}^{1000}$

$= \int_0^{1000} e^{\{x\}} \, dx$

$= 1000(e-1)$ //

Q5) $\int_0^{2n\pi} |\sin x| - \left|\frac{1}{2}\sin x\right| dx, \; n \in N$

Sol $= \int_0^{2n\pi} \frac{|\sin x|}{2} dx$

$= \frac{1}{\cancel{2}} \cdot \cancel{2}n \boxed{\int_0^{\pi} |\sin x| dx}$ ②

$= 2n$

Q6) $\int_0^{100\pi} \sqrt{1-\cos 2x}\, dx$

Sol $= \int_0^{100} \sqrt{2}\, |\sin x|\, dx$

$= 100 \cdot \sqrt{2} \boxed{\int_0^{\pi} |\sin x| dx}$ ②

$= 200\sqrt{2}$ Ans

Q7) $\int_{5\pi}^{16\pi/3} |\sin x|\, dx$

$\frac{16\pi}{3} - 5\pi$

$= \frac{16\pi - 15\pi}{3}$

$= \frac{\pi}{3}$

Sol $\int_{5\pi}^{5\pi + \frac{\pi}{3}} |\sin x|\, dx$

$\int_0^{\pi/3} \sin x dx$

$= -(\cos x)_0^{\pi/3} = -\left(\frac{1}{2} - 1\right)$

$= \frac{1}{2}$ Ans

Q8) $\int_0^{16\pi/3} |\sin x|\,dx$

$= \int_0^{5\pi+\frac{\pi}{3}} |\sin x|\,dx$

$= \int_0^{5\pi} |\sin x|\,dx + \int_0^{\pi/3} |\sin x|\,dx$

$= 5\cdot 2 + \frac{1}{2} = \frac{21}{2}$ Ans

Q9.) $\int_0^{[x]} \frac{2^x}{2^{[x]}}\,dx$

Sol) $= \int_0^{[x]\cdot 1} 2^{\{x\}}\,dx$

$= [x]\int_0^1 2^x\,dx$

$= [x]\left(\frac{2^x}{\log 2}\right)_0^1$

$= \frac{[x]}{\log 2} = [x]\log_2 e$

Q10.) $\int_0^{n\pi+\alpha} |\sin x|\,dx,\ n\in N,\ \alpha\in(0,\pi)$

1.) $\int_0^{nT} f(x)\,dx = n\int_0^T f(x)\,dx$

2.) $\int_{nT}^{mT} f(x)\,dx = \int_0^{(m-n)\cdot T} f(x)\,dx$

3.) $\int_0^{nT+\alpha} f(x)\,dx = \int_0^{nT} f(x)\,dx + \int_0^{\alpha} f(x)\,dx$

4.) $\int_{nT}^{nT+\alpha} f(x)\,dx = \int_0^{\alpha} f(x)\,dx$

5.) $\int_a^{a+T} f(x)\,dx = \int_0^T f(x)\,dx$

eg1.) Let $f(x) = \int_0^x \cos^4 t\,dt$

then

$f(x+\pi) = f(x) + f(\pi)$!

How? -

$$= \int_0^{x+T} \underbrace{\cos^4 t}_{\therefore T=\pi} \cdot dt$$

$$= \int_0^{\pi} \cos^4 t\,dt + \int_0^{x} \cos^4 t\,dt$$

$$= f(\pi) + f(x) = \text{R.H.S}/$$

eg2.) $\int_{-\pi}^{n\pi} |\sin x|\,dx =$

$\int_0^{2k} |\sin x|\,dx$

$$= 11 \cdot \boxed{\underset{2}{\int_0^{\pi} |\sin x|\, dx}}$$

$$= 22$$

eg7) $$\int_{-3}^{12} e^{x-[x]}\, dx$$

$$\int_0^{15} e^{\{x\}}\, dx$$

$$= 15 \int_0^{1} e^{x}\, dx$$

$$= 15(e-1)$$

eg4) Let $f(x)$ be a periodic function with fendomental period "T" s.t.

$$\int_0^{T} f(x)\, dx = I$$

then

$$= 1 \int_3^{3+3T} f\left(\frac{x}{2}\right) dx \qquad 2 \int_3^{3+3T} f(2x)\, dx \ ?$$

$$= 2 \int_{3/2}^{3/2+\frac{3}{2}T} f(x)\, dx \qquad \frac{1}{2} \int_6^{6+6T} f(x)\, dx$$

$= \text{can not be calculate}$ $$= \frac{1}{2} \int_0^{6T} f(x)\, dx$$

$$= 3 \int_0^{T} f(x)\, dx = 3 \cdot I$$

eg 5:) Let $f : [-10, 10] \to R$ and defined by

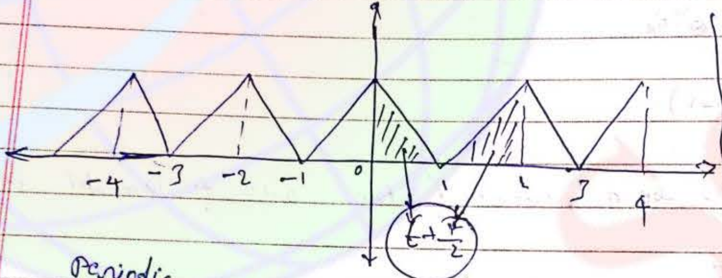
(even/odd period = ?)

$$f(x) = \begin{cases} x - [x] \; ; & [x] = \text{odd} \\ 1 - (x - [x]) \; ; & [x] = \text{even} \end{cases}$$

i) graph ii) even/odd iii) periodic, period iv) $\int_{-10}^{10} f(x)$

Sol^n

$$f(x) = \begin{cases} f(x) : & [x] = \pm 1, \pm 3, \pm 5 \dots \\ 1 - \{x\} : & [x] = 0, \pm 2, \pm 4 \dots \end{cases}$$



$= 2\int_{0}^{10} f(x)dx$

$= 2 \cdot 5 \int_{0}^{2} f(x)dx$

$= 10 \cdot 1 = 10$

Periodic
Period = 2
even

Imp
$\boxed{P-8}$ (extension of P-3)

3:) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ ↳ (U.L - variable)

4:) $\int_a^b f(x)\,dx = \int_0^b f(a+b-x)\,dx$ ↳ (Sum of limit - variable)

~~S~~

Now,

$\int_{-10}^{10} f(x)\, dx$

dx

Steps: –

$$I = \int_a^b f(x)\,dx \quad \text{—(i)}$$

$$I = \int_a^b f(a+b-x)\,dx \quad \text{—(ii)}$$

Add: $$2I = \int_a^b \underbrace{f(x) + f(a+b-x)}_{\text{Integrable.}}\,dx$$

✱ If $f(x) + f(a+b-x) = 1$

then

$$2I = b-a$$

$$\therefore I = \frac{b-a}{2}$$

Ans: $\left(\frac{a.L - L.L}{2}\right)$

Asser 2d ? eg.) ~~State 1~~ :→ $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}}\,dx = \frac{\pi}{6}$

~~Stat~~ 2 : – $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

Here: –

$$\int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

Since, $a+b = \frac{\pi}{2}$

$$f(x) + f\left(\frac{\pi}{2} - x\right) = 1$$

$$\text{So. } I = \frac{\left(\frac{\pi}{3} - \frac{\pi}{6}\right)}{2} = \frac{\pi}{12}$$

eg:) $$\int_4^{10} \frac{x^2}{(x^2-28x+196)+x^2}\,dx = 3 \quad \boxed{\frac{10-4}{2}}$$

Reason:- $a+b=14$

$$\therefore f(x) = \frac{x^2}{(14-x)^2+x^2} \quad \because f(x)+f(14-x)=1$$

eg] $$\int_{1/n}^{(an-1)\frac{1}{n}} \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}\,dx \ ?$$

$$a+b = (an-1)\frac{1}{n}+\frac{1}{n} = a$$

Ans :-

$$\frac{a-\frac{1}{n}-\frac{1}{n}}{2}$$

$$= \frac{a}{2}-\frac{1}{n}$$

eg3] $$\int_{\sqrt{\log 2}}^{\log\sqrt{3}} \frac{x\cdot\sin x^2}{\sin x^2+\sin(\log 6-x^2)}\,dx \ ?$$

sol^n

$$x^2 = t$$

$$\frac{1}{2}\int_{\log 2}^{\log 3} \frac{\sin t\ dt}{\sin t+\sin(\log 6-t)}$$

$$a+b = \log 6$$

Ans :- $$\frac{1}{2}\cdot\frac{1}{2}(\log 6-\log 2)$$

$$\therefore \frac{\log\left(\frac{3}{2}\right)}{4}$$

* Removal of x:—

P-3

$$\int_0^a x\cdot f(x)\,dx = \frac{a}{2}\int_0^a f(x)\,dx$$

$\downarrow$

$f(a-x) = f(x)$

P-8

$$\int_a^b x\cdot f(x)\,dx = \frac{a+b}{2}\int_a^b f(x)\,dx$$

$\downarrow$

If $f(a+b-x) = f(x)$

Q) If $f(a+b-x) = f(x)$ than $\int_a^b x\cdot f(x)\,dx$ is

i) $\frac{a+b}{2}\int_a^b f(a+b-x)\,dx$

ii) $\frac{a+b}{2}\int_a^b f(b-x)\,dx$

iii) $\frac{a+b}{2}\int_a^b f(x)\,dx$

iv) $\frac{b-a}{2}\int_a^b f(x)\,dx$

Q2) Let $I_1 = \int_{1-k}^{k} x\cdot g(x)(1-x)\,dx$

and

$I_2 = \int_{1-k}^{k} g(x(1-x)\,dx$ then

$2I_1 = I_2$ (T/F)?

Sol $a+b = 1-k+k = 1$

$$I_1 = \frac{1}{2}\int_{1-k}^{k} g(x)(1-x)\,dx$$

$\therefore 2I_1 = I_2$ /

eg3) Let $I_1 = \int_{f(-a)}^{f(a)} x\cdot g(x)(1-x)\,dx$ and $I_2 = \int_{f(-a)}^{f(a)} g(x(1-x))\,dx$

and

$f(x) = \dfrac{e^x}{1+e^x}$ than $2I_1 = I_2$ (T/F)

How?

Sol

$f(-a) + f(a) \Leftarrow$ sum of limits

$$= \frac{e^{-a}}{1+e^{-a}} + \frac{e^a}{1+e^a}$$

$$= \frac{1}{e^a+1} + \frac{e^a}{1+e^a}$$

$$= \frac{\cancel{1+e^a}}{\cancel{1+e^a}} = 1$$

$\hbar$ $I_1 = \frac{1}{2}I_2$

$2I_1 = I_2$ /

H.W

$$\int_{\pi/4}^{3\pi/4} \frac{\phi}{1+\sin\phi}\,d\phi$$

Removal of exponential →

Concept:- $\int_{-k}^{k}$ (function contains exponential function) dx

→ even/odd (X)

→ $\boxed{P-8}$ ✓

like

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{(1+a^x)}\,dx \qquad \text{—(i)}$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+a^{-x}}\,dx$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{a^x\cdot\cos^2 x}{1+a^x}\,dx$$

$$2I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x(1+a^x)}{(1+a^x)}\,dx$$

$$I = \int_{0}^{\pi/2} \cos^2 x\,dx = \frac{1}{2}\cdot\frac{\pi}{2} = \pi/4$$

~~eg~~ good Q) Let $I_n = \int_{-\pi}^{\pi} \frac{\sin n\pi x}{(1+\pi^x)\sin x}\,dx$, $n=0,1,2,3\ldots$

then, i) $I_{n+2} = I_n$ ii) $\sum_{m=1}^{10} I_{2m+1} = 10\cdot\pi$

iii) $\sum_{m=1}^{10} I_{2m} = 0$ iv) $I_{n+1} = I_n$

sol

ii)

$$I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}\,dx \quad -(i)$$

$$I_n = \int_{-\pi}^{\pi} \frac{+\sin(nx)}{(1+\pi^{-x})(-1+\sin x)}\,dx$$

$$I_n = \int_{-\pi}^{\pi} \frac{\pi^x(\sin nx)}{(\pi^x+1)(\sin x)}\,dx \quad -(ii)$$

Add (i)

$$2I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{\sin x}\frac{(1+\pi^x)}{\cancel{(\pi^x+1)}}\,dx$$

$$\boxed{I_n = \int_0^{\pi} \frac{\sin nx}{\sin x}\,dx}$$

1).

$$I_{n+2} = I_n$$

$$I_{n+2} - I_n = 0$$

$$= \int_0^{\pi} \frac{\sin(n+2)x}{\sin x}\,dx - \int_0^{\pi} \frac{\sin nx}{\sin x}\,dx$$

$$= \int_0^{\pi} \frac{\sin(n+2)x - \sin nx}{\sin x}\,dx$$

$$= \int_0^{\pi} \frac{2\cos(n+1)x \cdot \cancel{\sin x}}{\cancel{\sin x}}\,dx$$

$$= \frac{2}{n+1}\left(\sin(n+1)x\right]_0^{\pi}$$

$$= \frac{2}{n+1}(0-0) = 0$$

iii) $\sum_{n=1}^{10} i_{2n+1}$

$= i_3 + i_5 + \dots + i_{21}$

$= 10 - i_7$

$= 10 - i_4 = 10$

iv)

[4] ~~Let~~ in ~~a~~ $[a,b]$ or (a,b)

if

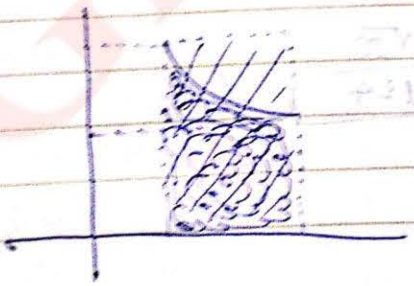
$f(x) > g(x)$

then

$$\int_a^b f(x)\,dx > \int_a^b g(x)\,dx$$

* If m be least value of f(x) and 'M' be the greatest value of f(x) in [a,b] then,

$$m(b-a) < \int_a^b f(x)\,dx < m(b-a)$$



q1) Prove that:-

$$1 < \int_0^1 e^{x^2} dx < e \,?$$

$$f(x) = e^{x^2} \uparrow$$

$$f'(x) = \underset{\oplus}{e^{x^2}} \cdot \underset{\oplus}{2x}$$

$$m = f(0) = 1$$

$$M = f(1) = e$$

$$b - a = 1$$

$$\therefore 1 < \int_0^1 e^{x^2} . dx < e$$

q2)

$$A < \int_1^2 \sqrt{3+x^2}\, dx < B$$

find $A + B^2$?

sol^n $f(x) = \sqrt{3+x^2} \uparrow$

$A = 4$

$B =$

$$f'(x) = \frac{1}{\underset{\oplus}{2\sqrt{3+x^2}}} \cdot \underset{\oplus}{3x^2} \quad \oplus$$

~~m = 2~~ $m = 2$, $M = \sqrt{30}$, $b - a = 2$

$$\therefore A = 4$$

$$B = 2\sqrt{30}$$

$$\overline{124} \text{ Ans}$$

eg) $C < \int_1^3 \frac{e^{-x}}{x} dx < D$, find C and D?

$f(x) = \frac{e^{-x}}{x}$

$f'(x) = \frac{-x \cdot e^{-x} - e^{x}}{x^2}$

$= -\underset{(+)}{e^{-x}} \frac{(x+1)}{\underset{(+)}{x^2}} < 0$

$m = f(3) = \frac{e^{-3}}{3} = \frac{1}{3e^3}$

$M = f(1) = \frac{1}{e}$

$\therefore C = \frac{2}{3e^3}$ /

$D = \frac{2}{e}$ /

★ Learn in (0,1) ; $\sin x < x$; $\tan x > x$; $\cos x < 1$

eg) $I = \int_0^1 \frac{\sin x \, dx}{\sqrt{x}}$, $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$

a) $I < \frac{2}{3}$, $J < 2$, (b) $I < \frac{2}{3}$, $J > 2$

(c) $I > \frac{2}{3}$, $J < 2$ (d) $I > \frac{2}{3}$, $J > 2$

Sol^n

$$\boxed{\begin{aligned} &\int \sqrt{x}\, dx = \frac{2}{3} x^{3/2} \\ &\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} \end{aligned}}$$

Sr. $\sin x < x$

$\frac{\sin x}{\sqrt{x}} < \sqrt{x}$
$\downarrow$

$\cos x < 1$

$\frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$
$\downarrow$

classmate

Date

Page

$$\int_0^1 \frac{\sin x}{\sqrt{x}}dx < \int_0^1 \sqrt{x}\,dx$$

$$\Downarrow$$

$$\frac{2}{3}(x^{3/2})_0^1$$

$$\boxed{I < \frac{2}{3}}$$

$$\int_0^1 \frac{\cos x}{\sqrt{x}}dx < \int_0^1 \frac{1}{\sqrt{x}}dx$$

$$\downarrow$$

$$(2\sqrt{x})_0^1$$

$$= 2^0$$

$$J < 2$$

eg.) $I = \int_0^1 \frac{1+x^8}{1+x^4}dx, \quad J = \int_0^1 \frac{1+x^9}{1+x^3}dx$

I_1 , $I > J$, ?

sol^n $0 < x < 1$

$x^8 > x^9$ & $x^3 > x^4$

$1+x^8 > 1+x^9$ $1+x^3 > 1+x^4$

$$\frac{1+x^8}{1+x^4} > \frac{1+x^9}{1+x^3}$$

$\therefore I > J$

eg) $I > \int_1^2 \frac{1}{\sqrt{1+x^2}}dx \; ; \; J > \int_1^2 \frac{1}{x}dx$

$\therefore \sqrt{x^2} = x$

$\sqrt{1+x^2} > x$

$\frac{1}{\sqrt{1+x^2}} < \frac{1}{x}$ $I < J$

★ Summation of series: -

from $\lim_{n\to\infty} \sum_{r=a}^{b} f\left(\frac{r}{n}\right) \frac{1}{n} = \int_{L.L}^{U.L} f(x)\, dx$

$\uparrow$ Integrate

method: -

(1) $\lim_{n\to\infty} \sum_{r=a}^{b} = \int$

(2) $\frac{r}{n} = x$

(3) $\frac{1}{n} = dx$

(4) $L.L = \lim_{n\to\infty} \frac{a}{n}$

(5) $U.P = \lim_{n\to\infty} \frac{b}{n}$

Q1) $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{r/n}$

$= \int_0^1 e^x dx$

$$\boxed{\begin{aligned} L.L &= \lim_{n\to\infty} \frac{1}{n} = 0 \\ U.P &= \lim_{n\to\infty} \frac{n}{n} = 1 \end{aligned}}$$

$= (e^x)_0^1$

$= e - 1$ Ans

Q2) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \frac{r}{\sqrt{n^2+r^2}} = \lim_{n\to\infty} \frac{1}{n} \frac{r/n}{\sqrt{1+\left(\frac{r}{n}\right)^2}}$

$= \int_0^2 \frac{x}{\sqrt{1+x^2}} dx$

$1+x^2 = t$

$2x\,dx = dt$

$$= \frac{1}{2}\int_1^5 \frac{dt}{\sqrt{t}}$$

$$= \frac{1}{2}\cdot 2\left(\sqrt{t}\right)_1^5$$

$$= \left(\sqrt{5} - 1\right)$$

eg 3) $\lim_{n\to\infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{5n}\right) \to n+4n$

Sol^n $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n+r} = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(\frac{1}{1+r/n}\right)$

$$= \int_0^4 \frac{1}{1+x}\,dx$$

$$= \left[\log(1+x)\right]_0^4$$

$$= \log 5 - \log 4$$

$$= \log 5 \quad \text{Ans}$$

eg 4) $\lim_{n\to\infty} \frac{1}{na} + \frac{1}{na+1} + \frac{1}{na+2} + \cdots + \frac{1}{nb}$

$\uparrow$

$na + (nb - na)$

$$\lim_{n\to\infty} \sum_{r=0}^{n(b-a)} \frac{1}{na+r}$$

$$\lim_{n\to\infty}\sum_{r=0}^{n(b-a)} \frac{1}{n}\left(\frac{1}{a+r/n}\right)$$

$$= \int_0^{b-a} \frac{1}{a+x}\,dx$$

$$= \left(\log(a+x)\right)_0^{b-a}$$

$$= \log(a+b-a) - \log a$$

$$= \log(b/a)$$

(Q5) $$\lim_{n\to\infty} \frac{1+2^4+3^4+4^4+\cdots n^4}{n^5} = \frac{1}{5}$$

$$\lim_{n\to\infty} \frac{1+2^{99}+3^{99}+\cdots+n^{99}}{n^{100}} = \frac{1}{100}$$

Mean $$\lim_{n\to\infty} \frac{1+2^p+3^p+\cdots+n^p}{n^{p+1}} = \frac{1}{n+1}$$

$$\lim_{n\to\infty} \frac{1}{n^5}+\frac{2^4}{n^5}+\frac{3^4}{n^5}+\cdots+\frac{n^4}{n^5}$$

$$\lim_{n\to\infty} \left(\frac{1}{n}\right)^4\frac{1}{n}+\left(\frac{2}{n}\right)^4\frac{1}{n}+\cdots+\left(\frac{n}{n}\right)^4\frac{1}{n}$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^4\frac{1}{n}$$

$$\int_0^1 x^4\,dx = \left(\frac{x^5}{5}\right)_0^1 = \frac{1}{5}-0 = \frac{1}{5}$$ Ans

$$\lim_{n\to\infty} \sum_{r=0}^{n(b-a)} \frac{1}{n}\left(\frac{1}{a+r/n}\right)$$

$$= \int_0^{b-a} \frac{1}{a+x}\,dx$$

$$= \left(\log(a+x)\right)_0^{b-a}$$

$$= \log(\cancel{a}+b-\cancel{a}) - \log a$$

$$= \log(b/a)$$

(99s) $\lim_{n\to\infty} \frac{1+2^4+3^4+4^4+\cdots\ n^4}{n^5} = \frac{1}{5}$

$$\lim_{n\to\infty} \frac{1+2^{99}+3^{99}+\cdots+n^{99}}{n^{100}} = \frac{1}{100}$$

Mean $\lim_{n\to\infty} \frac{1+2^p+3^p+\cdots+n^p}{n^{p+1}} = \frac{1}{n+1}$

$$\lim_{n\to\infty} \frac{1}{n^5}+\frac{2^4}{n^5}+\frac{3^4}{n^5}+\cdots+\frac{n^4}{n^5}$$

$$\lim_{n\to\infty} \left(\frac{1}{n}\right)^4\frac{1}{n}+\left(\frac{2}{n}\right)^4\frac{1}{n}+\cdots+\left(\frac{n}{n}\right)^4\frac{1}{n}$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{r}{n}\right)^4\frac{1}{n}$$

$$\int_0^1 x^4\,dx = \left(\frac{x^5}{5}\right)_0^1 = \frac{1}{5}-0 = \frac{1}{5}$$ A

अगर deno. में और Numerator में Power का अंतर
"1 होतो $\frac{1}{\text{den. का Power}}$ (As) अगर अंतर [illegible]
होगा तो Answer will be zero

$$\text{But} \lim_{n\to\infty} \frac{1+2^4+3^4+---+n^4}{n^6=n^5 \cdot n} = \underline{\underline{0}}$$

$\hookrightarrow 6, 1$ —— original $\frac{1}{n}$ [illegible]

eg.)

$$\lim_{n\to\infty} \frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec\frac{2^2}{n^2} + \cdots \frac{n}{n^2}\sec^2\frac{n^2}{n^2}$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{r}{n^2}\sec^2\frac{r^2}{n^2}$$

$$\int_0^1 x \cdot \sec^2 x^2\, dx$$

$x^2 = t$

$2x \cdot dx = dt$

$$= \frac{1}{2}\int_0^1 \sec^2 t\, dt$$

$$= \frac{1}{2}\left(\tan t\right)_0^1$$

$$= \frac{1}{2}\left(\tan 1 - \tan 0\right)$$

$$= \frac{1}{2}\tan 1$$

$$\boxed{\begin{array}{c} \tan\frac{\pi}{4} = 1 \\ \tan 1 \neq \frac{\pi}{4} \end{array}}$$

eg.)

$$\lim_{n\to\infty} \frac{1}{2n} + \frac{1}{\sqrt{4x^2-1^2}} + \frac{1}{\sqrt{4x^2-2^2}} + ---+ \frac{1}{\sqrt{4x^2-(n-1)^2}}$$

$$\lim_{n\to\infty} \sum_{r=0}^{n-1} \frac{1}{\sqrt{4n^2-r^2}}$$

$\lim_{n\to\infty}$

$\int$

$= (\sin^{-1}$

$= \sin^{-1}$

$= \pi$

eg) $\lim_{n\to\infty}$

$$\lim_{n\to\infty} \sum_{r=0}^{n-1} \frac{1}{n} \frac{1}{\sqrt{4-\left(\frac{r}{n}\right)^2}}$$

$$\int_0^1 \frac{1}{\sqrt{2^2-x^2}}\,dx$$

$$= \left(\sin^{-1}\left(\frac{x}{2}\right)\right)_0^1$$

$$= \sin^{-1}\left(\frac{1}{2}\right) - 0$$

$$= \frac{\pi}{6}$$

eg) $$\lim_{n\to\infty} \frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+2^2} + \dots + \frac{n+n^{\swarrow 1/n}}{n^2+n^2}$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{n+r}{n^2+r^2}$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{n}{n^2} \frac{\left(1+r/n\right)}{1+\left(\frac{r}{n}\right)^2}$$

$$= \int_0^1 \frac{1}{1+x^2}\,dx + \int_0^1 \frac{x}{1+x^2}\,dx$$

$$= \left(\tan^{-1}x\right)_0^1 + \frac{1}{2}\left(\log\left(1+x^2\right)\right)_0^1$$

$$= \frac{\pi}{4} + \frac{1}{2}\log 2$$ Ans

S.imp Area Under the curve :-

Always Remember —

Graph : $y = e^x$, e^{-x}, x, x^2

$y = x^3$, $\frac{1}{x}$, $\log x$, $|x|$

$y = \langle x \rangle$, all trigonometric ratio

circle, Parabola, $x=0$, $y=0$, $y=c$, $x=c$
All polynomial, All the st. line.

(1) If constant term is absent in $f(x,y)=0$.
$\Rightarrow$ curve pass through origin

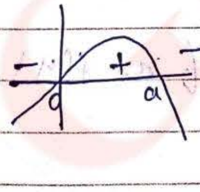
(2) If all the power of y are even
$\Rightarrow$ graph is symm. about x-axis.

(3) If all the power of x are even
$\Rightarrow$ graph is symm. about y-axis.

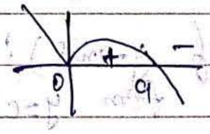
(4) To get point of intersection by curve on x-axis, put $y=0$

(5) To get point of intersection by the graph on y-axis, put $x=0$

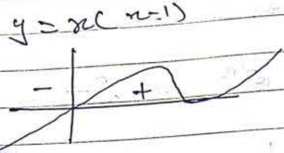
like – Graph of
$y = x^3(a-x)$, $a>0$



Graph of
$y = x^4(a-x)$



Graph of , $y = x(x-1)^2$



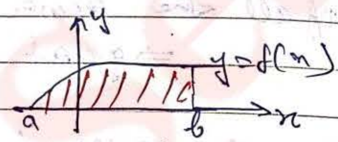
***

$$\begin{tikzcd}
 & f(x)+a & \\
f(x+a) & f(x) \arrow[l] \arrow[r] \arrow[u] \arrow[d] & f(x-a) \\
 & f(x)-a &
\end{tikzcd}$$

* How to calculate the Area :-

(1) (Above x-axis) :- If from $x=a$, to $x=b$, graph is above x-axis then area with x-axis is

$$A = \int_a^b f(x)dx$$,



$$A = \int_a^b f(x)\,dx$$

(2) (Below x-axis) :- If from $x=a$ to $x=b$ graph is below x-axis then

$$A = -\int_a^b f(x)\,dx$$

(3) (Right of y-axis) :- If graph of $x=f(y)$ is right of y-axis from $y=c$ to $y=d$

$$A = \int_c^d x = f(y)\,dy$$

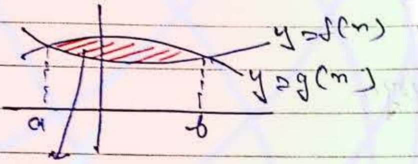
(4) left of y-axis

If graph of $x=f(y)$ is left of y-axis from $y=c$ to $y=a$

then

$$A = -\int_c^d f(y)\,dy$$

(5) Area b/w two curves

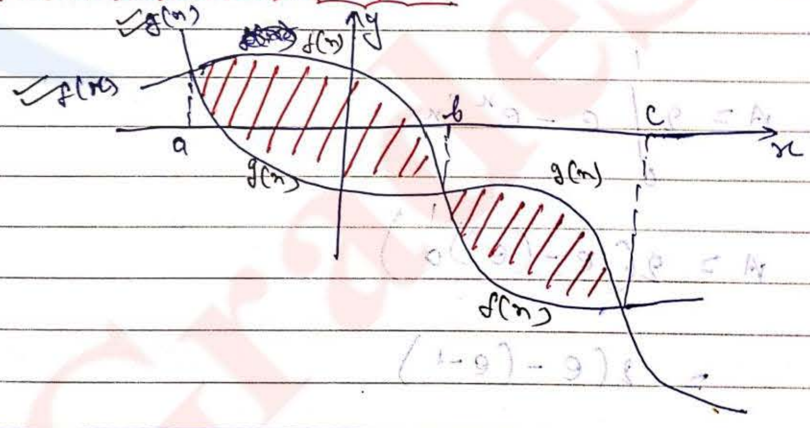


$$A = \int_a^b (f(x) - g(x))\,dx$$

$$\boxed{A = \int_a^b (\text{upper graph} - \text{lower graph})\,dx}$$

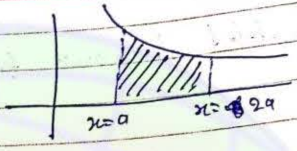
Imp.

(6) Area under the curve :- (जरूर से जरूर पढ़े)



$$\boxed{\text{Area} = \int_a^b f(x) - g(x)\,dx + \int_b^c g(x) - f(x)\,dx}$$

(6)



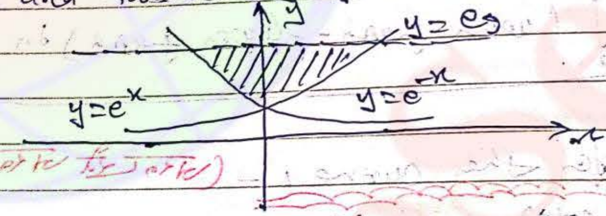
$A = \int_a^{2a} \frac{a^2}{x}\, dx$

$= a^2 (\log x)_a^{2a}$

$= a^2 \log 2a - \log a$

$= a^2 \log 2$

eg.) Find area bounded by $y=e^x$, $y=e^{-x}$, $y=e$.



([illegible])

$A = 2\int_0^1 e - e^x \, dx$

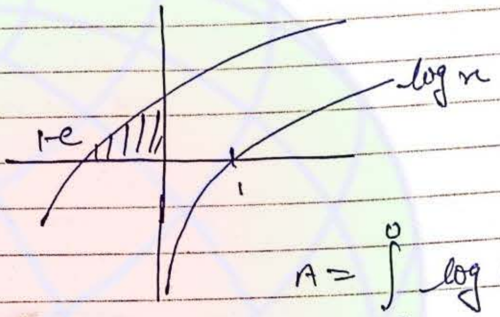
$A = 2\left(e - (e^x)_0^1\right)$

$= 2(e - (e-1)$

$= 2$ Ans

eg:-) Find

co-eg.) Find the Area bound by $y = \log(x+e)$ with co-ordinate axes?



$$A = \int_{1-e}^{0} \log(x+e)\,dx$$

$x+e = t$

$dx = dt$

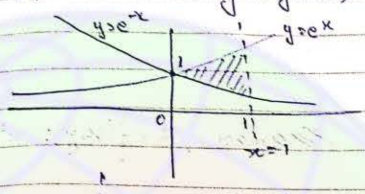
$$\int_{1}^{e} \log t \, dt = \left(t\log t - t\right)_{1}^{e}$$

$= (e-e) - (0-1)$

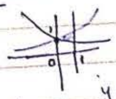
$= 1$

96) Find area bounded by $y=e^x$, $y=e^{-x}$ and $x=1$?

sol



own -



$y=$

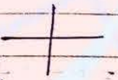
$$\int_0^1 e^x dx - \int_0^1 e^{-x} dx$$

$$A = \int_0^1 e^x - e^{-x} dx$$

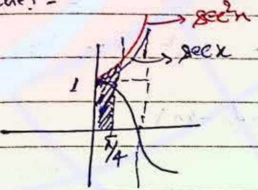
$$= e+e^{-1}-2$$

97) Find Area bounded by $y = \frac{1}{\cos^2 x}$ with co-ordinates axis and $x=\frac{\pi}{4}$?

sol own -



Teache: -

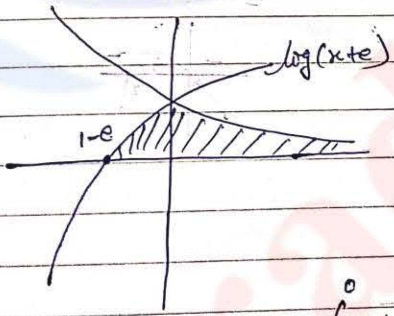


$$A = \int_0^{\pi/4} \sec^2 x dx$$

$$= (\tan x)_0^{\pi/4} = 1 \text{ Ans}$$

98) Find Area bounded by $y=\log(x+e)$, $x=\log\left(\frac{1}{y}\right)$ with x-axis?

sol



$x=\log\left(\frac{1}{y}\right)$

$e^x = \frac{1}{y}$

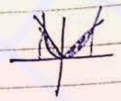
so, $y=e^{-x}$ /

$$\text{so Total Area} = \underbrace{\int_{1-e}^{0} \log(x+e) dx}_{1} + \int_0^{\infty} e^{-x} dx$$

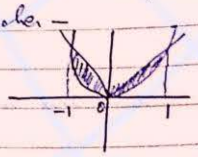
$$= 1-(e^{-x})_0^{\infty} = 1-(0-1) = 2$$

eg1) Find ~~total~~ ~~area~~ of the ~~Area~~region, $A = \{(x,y) : x^2 \leq y \leq |x|\}$.

So/n own -



Teacher -



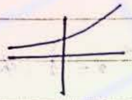
$$A = 2\int_0^1 (x - x^2)\,dx$$

$$= 2 \int \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= 2\left(\frac{1}{2} - \frac{1}{3}\right) = 2 \cdot \frac{1}{6} = \frac{1}{3}$$

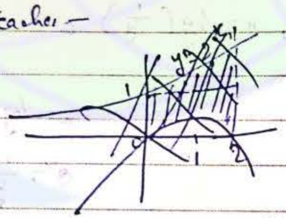
eg2) The area bounded by $y = 2^x$, $y = 2x - x^2$, $x = 0$, $x = 2$

So/n own -

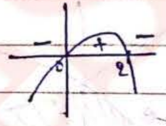


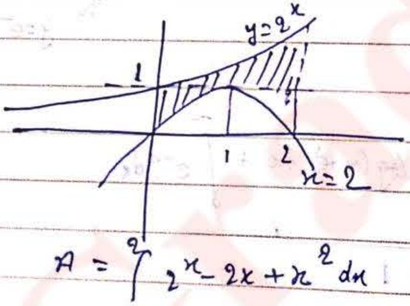
$y = 2^x$ $y = 2x$; $y = -(x^2 - 2x)$

Teacher -



$y = 2x - x^2$
$= -x(x-2)$





$$A = \int_0^2 2^x - 2x + x^2\,dx$$

$$A = \left(\frac{2^x}{\log 2}\right)_0^2 - \left(x^2\right)_0^2 + \left(\frac{x^3}{3}\right)_0^2$$

eg3) Find

So/n

eg4) fin

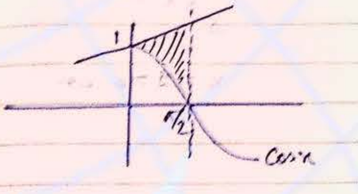
So/n

$$A = \frac{3}{\log 2} - 4 + \frac{8}{3}$$

$$A = \frac{3}{\log 2} - \frac{4}{3}$$

q3) Find Area bounded by $y = \cos x$; $y = 1 + \frac{2}{\pi} \cdot x$ and $x = \frac{\pi}{2}$?

Sol:



$$A = \int_0^{\pi/2} 1 + \frac{2}{\pi}x - \cos x \, dx$$

$$= \left(x + \frac{2}{\pi} \cdot \frac{x^2}{2} - \sin x \right)_0^{\pi/2}$$

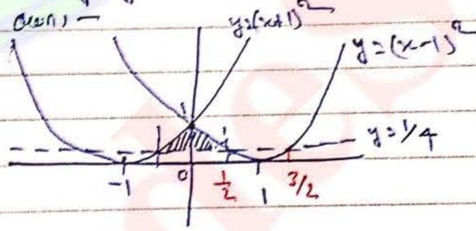
$$= \left(x + \frac{x^2}{4} - \sin x \right)_0^{\pi/2}$$

$$= \frac{\pi}{2} + \frac{2}{\pi}\left(\frac{x^2}{2}\right)_0^{\pi/2} - 1$$

$$= \frac{\pi}{2} + \frac{\pi}{4} - 1 = \frac{3\pi}{4} - 1 //$$

q4) find the Area bounded by $y = (x-1)^2$; $y = (x+1)^2$, and $y = \frac{1}{4}$?

Sol: ~~cos θ~~



$$A = 2\int_0^{1/2} (x-1)^2 - \frac{1}{4} \, dx$$

$$(x-1)^2 = \frac{1}{4}$$

$$x - 1 = \pm \frac{1}{2}$$

$$x = \frac{1}{2}, \frac{3}{2}$$

$$= 2 \left[\frac{(x-1)^3}{3} - \frac{1}{4}x \right]_0^{1/2}$$

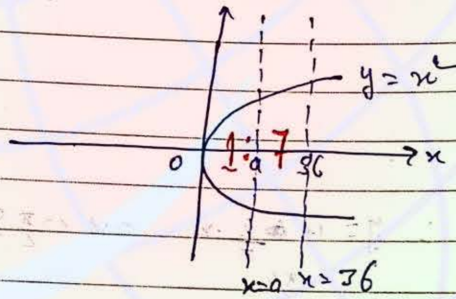
$$= 2 \left[\frac{(x-1)^3}{3} - \frac{1}{8} \right]$$

$$= 2 \left[-\frac{1}{24} + \frac{1}{3} - \frac{1}{8} \right] = \frac{1}{3} \text{ Ar}$$

$\boxed{L = 3, + 4}$ → Indefinite Integeraly

Q If the straight line $x=a$, divides the area enclosed by $y^2=x$ and $x=36$, in the ratio 1:7, find a?

Sol/



$$\Rightarrow \frac{\cancel{2}\int_0^a \sqrt{x}\,dx}{\cancel{2}\int_a^{36} \sqrt{x}\,dn} = \frac{1}{7}$$

$x = 9 = a$ A

Differential Equation

Def:- An eq^n involving the differentials of dependent variable 'y' w.r. to independent variable X

$$\left.\begin{aligned} dx + dy &= 0 \\ \frac{d^2y}{dx^2} + \sin x &= \frac{dy}{dx} \end{aligned}\right\} \rightarrow \text{ODE} \checkmark$$

(ordinary differ eq^n)

$$\frac{\partial f}{\partial x} + \frac{df}{dy} = 0 \leftarrow \text{PDE}$$

(Partial derivative eq^n)

Order of diff eq^n: $f\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}\right) = 0$

$O = 3$

The highest order of highest order derivative is called order of diff eq^n

Degree of diff eq^n — $f\left(\left(\frac{dy}{dx}\right)^4, \left(\frac{d^2y}{dx^2}\right)^3\right) = 0.$

$D = 3$

The highest power of highest order derivative involved in the eq^n is called degree of differential eq^n is free from fraction and radicals as far as derivatives concerned.

$\rightarrow$ Power like $\frac{1}{2}, \frac{2}{3}$.

ex) $\frac{dy}{dx} + \frac{x}{\frac{dy}{dx}} = 0 \longrightarrow$ order $= 1$

$\left(\frac{dy}{dx}\right)^2 + x = 0 \longrightarrow$ Degree $= 2$

ex) $c \cdot \frac{d^2y}{dx^2} = \left(1 + \frac{dy}{dx}\right)^{3/2}$ degree $= 2$

$c^2 \left(\frac{d^2y}{dx^2}\right)^2 = \left(1 + \frac{dy}{dx}\right)^3$

ex) $(y'')^2 - \sqrt{y'} = y^3 \longrightarrow$ order $= 2$

$\Rightarrow (y'')^2 - y^3 = \sqrt{y'}$

$(y'')^4 + y^6 - 2y^3(y'')^2 = y' \longrightarrow$ degree =

①

ex) $y_2^{3/2} + y_1^{1/2} + 2 = 0 \rightarrow$ order $= 2$

$\rightarrow$ degree $= 6$

$y_2^{3/2} + 2 = -y_1^{1/2}$

Squaring $y_2^3 + 4 + 4y_2^{3/2} = y_1$

$y_2^3 + 4 - y_1 = -4y_2^{3/2}$

Squaring

$y_2^6 - -$

ex) $\sin\left(\frac{d^2y}{dx^2}\right) = x$ order = 2

$\left(\frac{d^2y}{dx^2}\right) = \sin^{-1}x$ degree = 1

But $\sin\left(\frac{d^2y}{dx^2}\right) = \frac{dy}{dx}$ ⇒ order = 2

$\frac{d^2y}{dx^2} = \sin^{-1}\left(\frac{dy}{dx}\right)$ → degree = Not defined.

like $\log(1+y'') = x + y' + y$ → order = 2

$1 + y'' = e^{x+y'+y}$ → degree → Not defined.

Linear and Non linear Diff eqn.

1\. ① If the power of all the derivatives and y are one

and ⑪ No product of derivative and or y present then eqn is linear differential eqn.

$\frac{d^2y}{dx^2} + xy^2 = \frac{dy}{dx}$ ← N L

$\frac{dy}{dx} + \sin x = y\left(\frac{d^2y}{dx^2}\right)$ ← N.L

$\frac{d^2y}{dx^2} = \sqrt{1+y^2}$ ← Nonlinear

imp Formation of differential Equation

like $y = mx \rightarrow y - mx = 0, \Rightarrow f(x, y, m) =$

$(x-h)^2 + (y-k)^2 = 25 \Rightarrow f(x, y, h, k) = 0$

Let $f(x, y, c_1, c_2 \ldots c_n) = 0$ be n param family

Method ① given family is -- ①

① differentiate this family w.r.t x n times

we get n+1 total eqns

(ii) Now eliminate these n param we get nth order diff eqn

~~No of parater~~

$\boxed{\text{No of parameter} = \text{order}}$

ex) $y = e^{cx}$ be a family of plane curve then find diff. eqn represent this family.

~~$\log y = cx$~~

~~$\frac{1}{y}\frac{dy}{dx} = x$~~

~~$\log y = \frac{1}{y}\frac{dy}{dx}$~~

~~$c = \frac{\log y}{x}$~~

~~$\log y = \log y$~~

$y = e^{cx}$ — (I)

$y' = c e^{cx}$ — (II)

$y' = cy$

$y' = \frac{\log y}{x} y$

$x y' = y \log y$ (Ans)

ex) form a differential eqn of the family of curve $y = c_1 e^{c_2 x}$.

$y = c_1 e^{c_2 x}$ — (I)

$y' = c_1 c_2 e^{c_2 x}$ — (II)

$y'' = c_1 c_2^2 e^{c_2 x}$ — (III)

$\frac{(I)}{(II)}$: $\frac{y}{y'} = \frac{1}{c_2}$

$\frac{(II)}{(III)}$ $\frac{y'}{y''} = \frac{1}{c_2}$

$$\frac{y}{y'} = \frac{y'}{y''}$$

$$\boxed{y'^2 = y.y''}$$

Q) find order and degree of diff eqn whose solution is $y^2 = 2c(x+\sqrt{c})$

$$y^2 = 2c(x+\sqrt{c}) \longleftarrow ①$$

$$\cancel{2}y\frac{dy}{dx} = \cancel{2}c(1) \longrightarrow ⑪$$

$$y^2 = 2yy'(x+\sqrt{yy'})$$

$$y^2 = 2xyy' + 2(yy')^{3/2}$$

$$y^2 - 2xyy' = 2(yy')^{3/2}$$

Squaring $y^2 + 4x^2y^2y'^2 = 4(yy')^3$

Order = 1, degree = 3

Q) form
centre
of fine

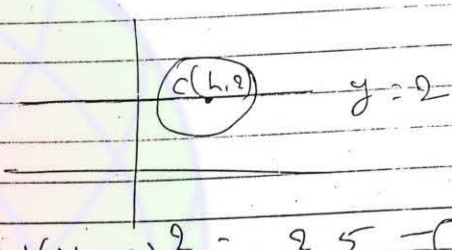
Solution

ex) fi

fami

Q) form the diff eqn of set of circles whose centre is on the st line y = 2 and of fixed radius 5 unit

Solution



$$\cancel{y}\Rightarrow (x-h)^2+(y-2)^2 = 25 \quad ①$$

~~x2~~

$$\cancel{2}(x-h) + \cancel{2}(y-2)y' = 0 \quad ⑪$$

$$(x-h) = -(y-2)y'$$

$$(y-2)^2 y'^2 + (y-2)^2 = 25$$

$$\boxed{(y-2)^2(1+y'^2) = 25}$$

ix) find the differential eqn of set of circle in xy-plane with the fixed radius 'a' units

family $(x-h)^2+(y-k)^2 = a^2$ — ①

$2(x-h) + 2(y-k)y' = 0$ —— (ii)

$\cancel{2} + \cancel{2}y'^2 + \cancel{2}(y-k)y'' = 0$ —— (iii)

$y-k = \frac{-(1+y'^2)}{y''}$

$(x-h) = -(y-k)y' = \frac{(1+y'^2)y'}{y''}$

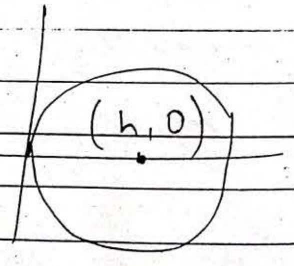
$\frac{(1+y'^2)^2 y'^2}{y''^2} + \frac{(1+y'^2)^2}{y''^2} = a^2$

$\frac{(1+y'^2)^2}{(y'')^2}(y'^2+1) = a^2$

$$\boxed{(1+y'^2)^3 = a^2(y'')^2}$$

ex) find a differential eqⁿ of all the circl whose centre is on x-axis and passi Through origion

$(x-h)^2 + y^2 = h^2$



$x^2 + \cancel{h^2} - 2hx + y^2 = \cancel{h^2}$

$2x - \cancel{2}h + \cancel{2}y\frac{dy}{dx} = 0$

$2x + 2y\frac{dy}{dx} = 2h$

x^2-2

y'

1) Vari

form

o

like –

$x^2 - 2x^2 - 2xyy' + y^2 = 0$

$y' = \frac{y^2 - x^2}{2xy}$

Solution of Diff Eq^n (Method)

1) Variable separable method

form: (function of x) dx = (function of y) dy

or (function of x) dx + (function of y) dy = 0

like – $\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} = 0$, solution

$\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$\int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$

$\sin^{-1} y = -\sin^{-1} x + C$

$\boxed{\sin^{-1} y + \sin^{-1} x = C}$

$= \sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right) = C$

$\boxed{x\sqrt{1-y^2}+y\sqrt{1-x^2} = K}$ (Ans)

when $x=0,\ y=2$

Q) Let $\frac{dy}{dx} = y+3$ given $\underline{y(0)=2}$, then $y(\ln$

1) 3 (ii) 5, (iii) 7 (iv) None of t

$\frac{dy}{dx} = y+3$

$\int \frac{dy}{y+3} = \int dx$

$\log(y+3) = x+C$

$\log(2+3 = 0+C$

$C = \log 5$

~~y0~~ $\log(y+3) = x+\log 5$

$\cancel{\log}(y+3) = \log 2 + \log 5 = \cancel{\log} 10$

$y+3 = 10$

$\boxed{y = 7}$

)

= 2

then

(ln 2) =

m of the

ex) Let $y = y(x)$ satisfies the eqn

$$\frac{2+\sin X}{1+y}\frac{dy}{dx} = -\cos X \text{ and } y(0) = 1$$

Then $y(\pi/2)$ is

$$\frac{2+\sin x}{1+y}\frac{dy}{dx} = -\cos X$$

$$\frac{dy}{1+y} = \frac{-\cos X\, dX}{2+\sin X}$$

$$\Rightarrow \log(1+y) = -\log(2+\sin X) + C$$

$$\log 2 = -\log 2 + C$$

$$C = \log 4$$

$$\log(1+y) = -\log(2+\sin X) + \log 4$$

$$\cancel{\log}(1+y) = \cancel{\log}\frac{4}{3}$$

$$1+y = \frac{4}{3}$$

$$y = \frac{1}{3}$$

ex) The solnⁿ of differential eqⁿ $\sin\frac{dy}{dx}$

when $y(0) = 1$ is

1) $\sin^{-1}\left(\frac{y-1}{x}\right) = a$ (ii) $\sin\left(\frac{y-1}{x}\right) = a$

(iii) $\sin\left(\frac{1-y}{x+1}\right) = a$

$$\frac{dy}{dx} = \sin^{-1} a$$

$$\int dy = \int \sin^{-1} a \; dx.$$

$$y = \sin^{-1}(a) \cdot x + C$$

$$\therefore y = (\sin^{-1} a) x + 1$$

$$\sin\left(\frac{y-1}{x}\right) = a$$

ex) The solution of diff eqⁿ $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ deter

a family of circles with

1) Variable radii with centre at (0,1).

2) Variable radii with centre at (0,-1)

③ fixed radius with center along X-a

④ fixed radius with centres along y-a

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y} \qquad 1-y^2 = t$$

$$-2y\,dy = dt$$

$$\int \frac{y\,dy}{\sqrt{1-y^2}} = \int dx$$

$$-\frac{1}{2}\int \frac{dt}{\sqrt{t}} = x+C$$

$$= -\frac{1}{\cancel{2}} \times \cancel{2}\sqrt{1-y^2} = x+C$$

$$1-y^2 = x^2+C^2+2Cx$$

$$x^2+y^2+2Cx+C^2-1=0$$

Centre $(-C,0)$

$$r = \sqrt{\cancel{C^2}+0-\cancel{C^2}+1} \qquad r=1$$

$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ $\qquad 1-y^2 = t$

$\qquad\qquad\qquad\qquad -2y\,dy = dt$

$\int \frac{y\,dy}{\sqrt{1-y^2}} = \int dx$

$-\frac{1}{2}\int \frac{dt}{\sqrt{t}} = x+C$

$-\frac{1}{\cancel{2}} \times \cancel{2}\sqrt{1-y^2} = x+C$

$1-y^2 = x^2+C^2+2Cx$

$x^2+y^2+2Cx+C^2-1=0$

Centre $(-C, 0)$

$r = \sqrt{\cancel{C^2}+0-\cancel{C^2}+1}$ $\quad r=1$

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⟨2⟩ Reducible to Variable seperable

form :- $\frac{dy}{dn} = f(ax+by+c)$

$$ax+by+c = t$$

$$a + b\frac{dy}{dn} = \frac{dt}{dt}$$

$$\frac{dy}{dn} = \frac{1}{b}\left(\frac{dt}{dn} - a\right)$$

$$\frac{1}{b}\left(\frac{dt}{dx} - a\right) = f(t)$$

$$\frac{dt}{dx} = b\,f(t) + a$$

Variable seperate $\int \frac{dt}{bf(t)+a} = \int dx$

ex) $(n+y)^2 = a^2 \frac{dn}{dy}$ solve it?

$$\frac{dy}{dn} = \frac{a^2}{(n+y)^2} = f(n+y)$$

$$\int \frac{dt}{\frac{a^2}{t^2}+1} = \int dn. \qquad = \frac{1}{\left(\frac{x+y}{a}\right)^2}$$

$$\int \frac{t^2+a^2-a^2}{a^2+t^2}\,dt = x+C$$

$$\int 1\,dt - a^2\int \frac{1}{a^2+t^2}\,dt = x+C$$

$$(\cancel{x}+y) - a^{\cancel{2}}\frac{1}{\cancel{a}}\tan^{-1}\left(\frac{x+y}{a}\right) = \cancel{x}+C$$

iv) Solve $\frac{dy}{dx} = \cot^2(x+y)$

$$\int \frac{dt}{\cot^2 t+1} = \int dx$$

$$\int \sin^2 t\,dt = x+C$$

$$= \frac{1}{2}\left[t - \frac{\sin 2t}{2}\right] = x+C$$

where $t = x+y$

ex) $\frac{dy}{dx} = \sin^2(x+3y) + 5 = f(x+3y)$

$$\int \frac{dt}{3(\sin^2 t + 5) + 1} = \int dx$$

$$\int \frac{dt}{3\sin^2 \cancel{t} + 16} = x + C$$

$$\int \frac{dt}{19\sin^2 t + 16\cos^2 t} = x + C$$

$$= \int \frac{\sec^2 t}{19\tan^2 t + 16}\, dt = x + C$$

$\tan t = p$

$\sec^2 t \, dt = dp$

$$\frac{1}{19}\int \frac{dp}{p^2 + \left(\frac{4}{\sqrt{19}}\right)^2} = x + C$$

$$= \frac{1}{19}\,\frac{\sqrt{19}}{4}\tan^{-1}\left(\frac{\sqrt{19}}{4}\left(\tan(x+3y)\right) = x + C\right.$$

ex) $(2x+3y-1)\,dx + (4x+6y-5)\,dy = 0$

$$\frac{dy}{dx} = -\frac{(2x+3y-1)}{(4x+6y-5)} \longleftarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\frac{dy}{dx} = -\frac{(2x+3y-1)}{2(2x+3y-1)-3} = f(2x+3y-1)$$

$$\int \frac{dt}{3\left(-\frac{t}{2t-3}\right)+2} = \int dx$$

$$\int \frac{2t-3}{t-6}\,dt = x+C$$

$$2\int \frac{\cancel{2t-3}\;(t-6)+6-\frac{3}{2}}{t-6}\,dt = x+C$$

$$= 2\left[\int 1+\frac{9}{2}\frac{1}{t-6}\,dt\right] = x+C$$

$$= 2\left[t+\frac{9}{2}\log(t-6)\right] = x+C$$

where $t = 2x+3y-1$

Imp

3) Homogeneous Diff eq^n

↓

every term has same degree

like $X^3, y^3, 4X^2y \longrightarrow$ degree = 3

or if $\frac{dy}{dx}$ can be expressed as $f(y/x)$

$$\frac{dy}{dx} = f(y/x) \longrightarrow \text{H.D.E}$$

$$\frac{dy}{dx} = \frac{1+(y/x)^2}{3(y/x)} = f(y/x)$$

Method $\quad y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = f(v)$$

$$x\frac{dv}{dx} = f(v) - v$$ (Variab
separ

$$\int \frac{dv}{f(v)-v} = \int \frac{dX}{X}$$

ex) The solution to the eqⁿ $(x^2-y^2)dx+2xy\,dy=$ is

$$(x^2-y^2)\,dx+2xy\,dy=0$$

$$\frac{dy}{dx}=-\left(\frac{x^2-y^2}{2xy}\right)$$

$$\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$$

$$y=vx$$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$v+x\frac{dv}{dx}=\frac{v^2-1}{2v}$$

$$x\frac{dv}{dx}=\frac{v^2-1-2v^2}{2v}$$

$$\frac{2v}{1+v^2}\,dv=-\frac{1}{x}\,dx$$

$$\log(1+v^2)=-\log x+\log C$$

$$\log\left(\frac{x^2+y^2}{x^2}\right)=-\log x+\log C=\log\frac{C}{X}$$

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$$\frac{x^2+y^2}{x\cancel{x}} = \frac{c}{\cancel{x}}$$

$$x^2+y^2 = cx$$

ex) Let $x\sin\left(\frac{y}{x}\right)dy = y\sin\left(\frac{y}{x}\right) - x)\, d$

and $y(1) = \frac{\pi}{2}$ then $\cos\left(\frac{y}{x}\right) =$ is

$$\frac{dy}{dx} = \frac{y\sin(y/x) - x}{x(\sin(y/x))}$$

$$v + x\frac{dv}{dx} = \frac{v\sin v - 1}{\sin v}$$

$$x\frac{dv}{dx} = \frac{v\sin v - 1}{\sin v} - v = \frac{-1}{\sin}$$

$$\sin v\, dv = -\frac{1}{x}\, dx$$

$$+\cos v = +\log x + c$$

$$\cos\left(\frac{y}{x}\right) = \log x + \cancel{c}$$

$$\cos\left(\frac{y}{x}\right) = \log x$$

ex)

ix) Let $(x^2+y^2)\,dy = xy\cdot dx$, if $x>0, y>0$ and $y(1)=1$. and $y(X_0)=0$, then find X_0=?

$$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$$

$$v + x\frac{dv}{dx} = \frac{v}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{v}{1+v^2} - v = \frac{\cancel{v} - \cancel{v} - v^3}{1+v^2}$$

$$\int \frac{1+v^2}{v^3}\,dv = -\int \frac{1}{x}\,dx$$

$$\int \left(v^{-3} + \frac{1}{v}\right) dv = -\log X + C$$

$$= \frac{-1}{2v^2} + \log v = -\log X + C$$

$$= -\frac{1}{2}\frac{x^2}{y^2} + \log y = C$$

$$-\frac{1}{2} + 0 = C$$

$$C = -\frac{1}{2}$$

$$-\frac{1}{2}\frac{x^2}{y^2} + \log y = -1/2$$

$$\cancel{+}\frac{1}{\cancel{2}}\frac{x_0^2}{e^2} = \cancel{+}\frac{3}{\cancel{2}}$$

$$x_0^2 = 3e^2$$

$$x_0 = \pm\sqrt{3}e$$

(-ve X) ← Not include

$$x_0 = \sqrt{3}e$$

4) Reducible to H.D.E →

form: $\frac{dy}{dx} = \frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$ $\left(\frac{a_1}{a_2} \ne\right.$

Method

Put $x = X+h$, $y = Y+k$ $\Big\}$ $\frac{dy}{dx} = \frac{dY}{dX}$.

$$\boxed{\frac{dY}{dX} = \frac{a_1X+b_1Y}{a_2X+b_2Y}}\;\frac{+a_1h\xcancel{+b_1k}+c_1}{+a_2h\xcancel{+b_2k}+c_2}$$

H.D.E

) $a_1 h + b_1 k + c_1 = 0$ — ①

$a_2 h + b_2 k + c_2 = 0$ — ⑪

(h, k) find the value of (h, k) using ~~eq~~ ① and ⑪ eq

5) Linear differential eq^n

1st form $$1\frac{dy}{dx} + \underbrace{Py = Q}_{\text{function of } x}$$

$$IF = e^{\int P dx}$$

Solution $y \cdot IF = \int (Q \cdot IF) dx + C$

IInd form $$1\frac{dx}{dy} + \underbrace{Px = Q}_{\text{function of } y}$$

$$\int IF = e^{\int P dy}$$

Sol^n $x \cdot IF = \int (Q \cdot IF) dy + C$

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ex) Let $(1+t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$

$y(1)$ is

$$\frac{dy}{dt} - \frac{t}{1+t} \cdot y = \frac{1}{1+t}$$

$$IF = e^{-\int \frac{t+1-1}{1+t} dt}$$

$$= e^{-\int 1 - \frac{1}{1+t} dt}$$

$$= e^{-(t - \log(1+t))}$$

$$= e^{-t + \log(1+t)}$$

$$= e^{-t} e^{\log(1+t)}$$

$$= (1+t) e^{-t}$$

$$y(1+t) e^{-t} = \int \frac{1}{\cancel{(1+t)}} \cancel{(1+t)} e^{-t} dt + C$$

$$y(1+t) e^{-t} = -e^{-t} + 0$$

$$y(1+t) = -1$$

$$y = -\frac{1}{1+t}$$

at $t = 1$, $y = -1/2$

Q) Let $(2x-10y^3)\frac{dy}{dx}+y=0$, then sol^n is $xy^2=2y^5+c$ (Prove it)

$$\frac{dy}{dx}+\frac{y}{2x-10y^3}=0$$

$$\frac{dx}{dy}=-\left(\frac{2x-10y^3}{y}\right)$$

$$\frac{dx}{dy}+\frac{2}{y}x\pm 10y^2$$

$$IF = e^{\int\frac{2}{y}dy} = e^{\log y^2} = \cancel{x}y^2$$

$$\cancel{yx^2 = \int 10y^2 x^2\,dx}$$

$$\cancel{yx^2 = 10y^2 x}$$

$$xy^2 = \int 10y^2 y^2\,dy + C$$

$$\underline{xy^2 = 2y^5+C}$$

*(iv) Let f be a diff-able function on $(0,\infty)$ such that $f(1)=1$. $\lim_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x}=1$, $x>0$, then find $f(x)$

$$\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$$

$\downarrow$ ($\frac{0}{0}$ → L Hospital

$$\text{So} \lim_{t \to x} \frac{2t f(x) - x^2 f'(t)}{1 - 0} = 1$$

$$2x f(x) - x^2 f'(x) = 1$$

$$2xy - x^2 \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} - \frac{2}{x} y = -\frac{1}{x^2}$$

$$IF = e^{\int \frac{-2}{x} dx} = e^{-2 \log x} = \frac{1}{x^2}$$

$$y \cdot \frac{1}{x^2} = -\int \frac{1}{x^2} \cdot \frac{1}{x^2} dx + C$$

$$\frac{y}{x^2} = \frac{1}{3x^3} + C$$

$$1 = \frac{1}{3} + C$$

$$C = \frac{2}{3}$$

$$\frac{y}{x^2} = \frac{1}{3x^2} + \frac{2}{3}$$

$$y = \frac{1}{3x} + \frac{2}{3} x^2$$

ex) let $f : [1, \infty] \rightarrow [2, \infty]$ such that $f(1) = 2$

and $6\int_1^x f(t)\,dt = 3x f(x) - x^3$

find $f(2) = ?$

diff

$$6 f(x) = 3x f'(x) + 3 f(x) - 3x^2$$

$$3 f(x) = 3x f'(x) - 3x^2$$

$$y = x \frac{dy}{dx} - x^2$$

$$\frac{dy}{dx} - \frac{1}{x} y = x$$

$$IF = e^{\int -\frac{1}{x} dx} = e^{-\log x} = x^{-1} = \frac{1}{x}$$

Sol

$$y \frac{1}{x} = \int x \cdot \frac{\cancel{1}}{x} dx + C$$

$$\frac{y}{x} = x + C$$

$$\frac{2}{1} = 1 + C \Rightarrow C = 1$$

$$y = x(x+1)$$

At $x = 2$

$$y = 2.3 = 6$$

6) Bernoullis diff eqn.

1st form $\frac{dy}{dx} + Py = Qy^n$

$$y^{-n}\frac{dy}{dx} + Py^{1-n} = Q$$

$$\downarrow$$

$$y^{1-n} = t$$

$$(1-n)y^{-n}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{1}{(1-n)}\frac{dt}{dx} + Pt = Q$$

$$\Rightarrow \frac{dt}{dx} + P(1-n)t = Q(1-n)$$

IF ___ and solution

IInd form $\frac{dx}{dy} + Px = Qx^n$

$$\boxed{\begin{array}{c}\frac{dt}{dy} + P(1-n)t = Q(1-n) \\ \text{where} = t = x^{1-n}\end{array}}$$

$\boxed{\frac{dt}{dx} + P(1-n)t = Q(1-n)}$ ← using Ber

y $\boxed{t = y^{1-n}}$

ex) Solution of D E

$\cos x\, dy = y(\sin x - y)\, dx.$

$$\frac{dy}{dx} = \frac{y \sin x - y^2}{\cos x}$$

$$\frac{dy}{dx} - \tan x\, y = -\sec x\, y^2$$

$n = 2$

$1 - n = -1$

$t = y^{-1}$

$$\frac{dt}{dx} + \tan x\, t = \sec x$$

$IF = e^{\int \tan x\, d} = e^{\log \sec x} = \sec x.$

$t \cdot \sec x = \int \sec^2 x\, dx + C$

$$\frac{\sec x}{y} = \tan x + C$$

$\sec x = y(\tan x + C)$

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1. $d(xy) = x\,dy + y\,dx$
2. $d\left(\frac{x}{y}\right) = \frac{y\,dx - x\,dy}{y^2}$
3. $d\left(\frac{y}{x}\right) = \frac{x\,dy - y\,dx}{x^2}$
4. $d\left(\frac{1}{xy}\right) = \frac{-1}{(xy)^2}\left(x\,dy + y\,dx\right)$

Q) The solution of $\frac{y\,dx - x\,dy}{y^2} + 3x^2 e^{x^3} dx$

$$\int d\left(\frac{x}{y}\right) + \int 3x^2 e^{x^3} dx = \int 0$$

$$\frac{x}{y} + e^{x^3} = c$$

Q) The solution of $x\,dy - y\,dx = y^2\,dy$?

$$= -\left(\frac{y\,dx - x\,dy}{y^2}\right) = dy$$

$$= -\int d\left(\frac{x}{y}\right) = \int dy$$

$$-\frac{x}{y} = y + C$$