Pre-Medical

Physics 3

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Contents

THERMAL PHYSICS

NEET SYLLABUS

Thermal equilibrium and definition of temperature (zeroth law of Thermodynamics). Heat, temperature, thermal expansion; therma expansion of solids, liquids, and gases. Anomalous expansion. Specific heat capacity: Cp, Cv- calorimetry; change of state - latent hea Heat transfer- conduction and thermal conductivity, convection and radiation. Qualitative ideas of Black Body Radiation, Wein's displaceme law Newton's law of cooling and Stefan's law. Kinetic theory of gases: Assumptions, concept of pressure. Kinetic energy and temperatu degrees of freedom, law of equipartition of energy (statement only) and application to specific heat capacities of gases; concept of m free path. Heat, work and internal energy. First law of thermodynamics. Isothermal and adiabatic processes. Second law of thermodynamics: Reversible and irreversible processes. Heat engines and refrigerators. Equation of state of a perfect gas, work dor compressing a gas.

RUDOLF CLAUSIUS (1822-1888)

Rudolf Clausius, born in Poland, is generally regarded as the discoverer of the Second Law of Thermodynamics. Based on the work of Carnot and Thomson, Clausius arrived at the important notion of entropy that led him to a fundamental version of the Second Law of Thermodynamics that states that the entropy of an isolated system can never decrease. Clausius also worked on the kinetic theory of gases and obtained the first reliable estimates of molecular size, speed, mean free path, etc

LUDWIG BOLTZMANN (1844 – 1906)

Ludwig Boltzmann, born in Vienna, Austria, worked on the kinetic theory of gases independently of Maxwell. A firm advocate of atomism, that is basic to kinetic theory, Boltzmann provided a statistical interpretation of the Second Law of thermodynamics and the concept of entropy. He is regarded as one of the founders of classical statistical mechanics. The proportionality constant connecting energy and temperature in kinetic theory is known as Boltzmann's constant in his honour.

TEMPERATURE & THERMAL EXPANSION

1. TEMPERATURE & TEMPERATURE SCALES

- **Temperature**

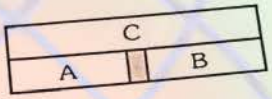
Temperature may be defined as the ***degree of hotness or coldness*** of a body. Heat energy flows from a body at higher temperature to that at lower temperature until their temperatures become equal. At this stage, the bodies are said to be in thermal equilibrium.

- **Thermal equilibrium**

Thermal equilibrium is a situation in which two objects would not exchange energy by heat or electromagnetic radiation if they were placed in thermal contact. Heat is the transfer of energy from one object to another object as a result of a difference in temperature between them.

- **Zeroth law of thermodynamics**

If objects A and B are separately in thermal equilibrium with a third object C (say thermometer), then objects A and B are in thermal equilibrium with each other. Zeroth law of thermodynamics introduces the concept of temperature. Two objects (or systems) are said to be in thermal equilibrium if their temperatures are same.



In measuring the temperature of a body, it is important that the thermometer should be in thermal equilibrium with the body whose temperature is to be measured.

- **Measurement of Temperature**

The branch of thermodynamics which deals with the measurement of temperature is called thermometry. A thermometer is a device used to measure the temperature of a body. The substances like liquids and gases which are used in the thermometer are called thermometric substances.

- **Different Scales of Temperature**

A thermometer can be graduated into following scales :

(a) The Centigrade or Celsius scale (°C)

(b) The Fahrenheit scale (°F)

(c) Kelvin scale (K)

- **Comparison between Different Temperature Scales**

The general formula for the conversion between different temperature scales is:

$$\frac{K-273}{100}=\frac{C}{100}=\frac{F-32}{180}=\frac{X-LFP}{UFP-LFP}$$

Where X→ Reading in unknown temperature scale, LFP → Lower Fixed Point, UFP → Upper Fixed Point

Change in temperature $\frac{\Delta K}{100}=\frac{\Delta C}{100}=\frac{\Delta F}{180}=\frac{\Delta X}{UFP-LFP}$

GOLDEN KEY POINTS

- Although the temperature of a body can be raised without limit, it can not be lowered without limit and theoretically limiting low temperature is taken to be zero of the Kelvin scale (i.e. no negative temperature on Kelvin scale is possible).
- Though when universe was created 10^{10} years ago, its temp. was about 10^{39} K which at present is about 3K. The highest laboratory temperature is about 10^{8} K (in fusion test reactor) while lowest 10^{-10} K (achieved in 1999 through nuclear spin cooling) Theory has established that zero Kelvin temperature can never be achieved practically.

Illustrations

Illustration 1.

Temperature of a patient is 40°C . Find the temperature on Fahrenheit scale ?

Solution :

$$\frac{F-32}{180} = \frac{40-0}{100} \Rightarrow F = 104°F$$

Illustration 2.

At what temperature is the Fahrenheit scale reading equal to twice of Celsius ?

Solution :

$$\frac{F-32}{180} = \frac{C-0}{100} \Rightarrow \frac{2x-32}{180} = \frac{x-0}{100} \Rightarrow x = 160$$

Illustration 3.

The lower and upper fixed points of a faulty thermometer are 5 W and 105 W. If the thermometer reads 25 W, what is the actual temperature in Celsius scale ?

Solution : $\frac{25-5}{100} = \frac{C-0}{100} \Rightarrow C = 20°C$

Illustration 4.

A thermometer with an arbitrary scale has the ice point at –20° and the steam point at 180°. When the thermometer reads 5°, a Centigrade thermometer will read

(1) 7.5 °C (2) 12.5 °C (3) 16.5 °C (4) –9.37 °C

Solution :

$$\frac{C-0}{100-0} = \frac{t-(-20)}{180-(-20)} \quad (\text{Here } t = 5°)$$

$$\Rightarrow \frac{C}{100} = \frac{5+20}{200} \Rightarrow C = 12.5\ °C$$

Illustration 5.

The temperature of an iron piece is raised from 30°C to 90°C. What is the change in its temperature on the Fahrenheit scale and on the Kelvin scale?

Solution

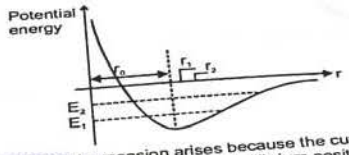
$\Delta C = 90° - 30° = 60°C$

Temperature difference on Fahrenheit Scale $\Delta F = \frac{9}{5}\Delta C = \frac{9}{5}(60°C) = 108°F$

Temperature difference on Kelvin Scale $\Delta K = \Delta C = 60K$

2. THERMAL EXPANSION

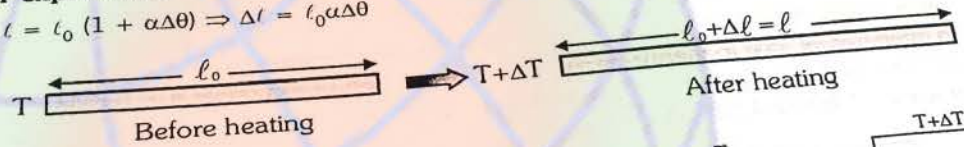
When matter is heated without any change in its state, it usually expands. According to atomic theory of matter, asymmetry in potential energy curve is responsible for thermal expansion. As with rise in temperature the amplitude of vibration increases and hence energy of atoms increases, hence the average distance between the atom increases. So the matter as a whole expands.



Thermal expansion arises because the curve is not symmetrical about the equilibrium position r_0. As the temperature rises the energy of the atom increases. The mean position when the energy is E_2 is not the same as that when the energy is E_1.

- Thermal expansion is minimum in case of solids but maximum in case of gases because intermolecular forces are maximum in solids but minimum in gases.
- Solids can expand in one dimension (Linear expansion), two dimensions (Superficial expansion) and three dimensions (Volumetric expansion) while liquids and gases usually suffers change in volume only.
- **Linear expansion :**

$\ell = \ell_0 (1 + \alpha\Delta\theta) \Rightarrow \Delta\ell = \ell_0\alpha\Delta\theta$

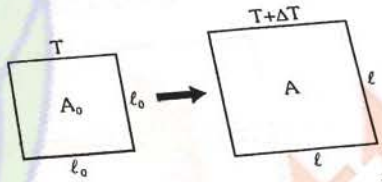


- **Superficial (areal) expansion :**

$A = A_0 (1 + \beta\Delta\theta)$

Also $A_0 = \ell_0^2$ and $A = \ell^2$

So $\ell^2 = \ell_0^2(1 + \beta\Delta\theta) = [\ell_0(1 + \alpha\Delta\theta)]^2 \Rightarrow \beta = 2\alpha$



- **Volumetric expansion :**

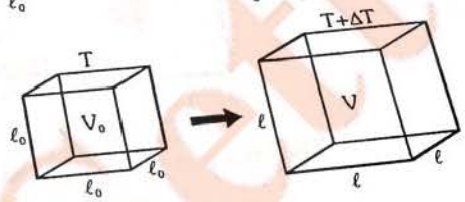
$V = V_0 (1 + \gamma\Delta\theta)$ Also $V = \ell^3$ and $V_0=\ell_0^3$ so $\gamma = 3\alpha$

$\ell^3=[\ell_0(1+\alpha\Delta\theta)]^3 \Rightarrow 6\alpha = 3\beta = 2\gamma$ or $\alpha : \beta : \gamma = 1 : 2 : 3$

α = coefficient of linear expansion

β = coefficient of superficial expansion

γ = coefficient of volumetric expansion

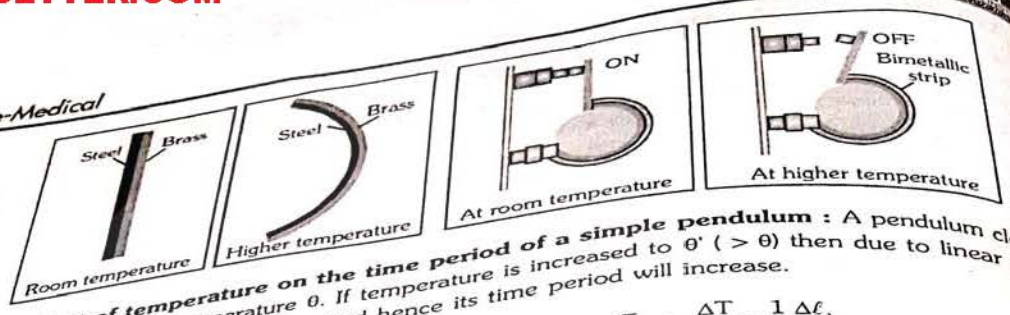


- **Contraction on heating :**

Some rubber like substances contract on heating because transverse vibration of atoms of substance dominate over longitudinal vibration which is responsible for expansion.

Applications of thermal Expansion in Solids

(a) Bi–metallic strip : When two strips of equal length but of different materials (different coefficient of linear expansion) are joined together, it is called "Bi–metallic strip" and can be used in thermostat to break or make electrical contact. This strip has the characteristic property of bending on heating due to unequal linear expansion of the two metals. The strip will bend with metal of greater α on outer side. Coefficient of expansion is more for brass than steel.

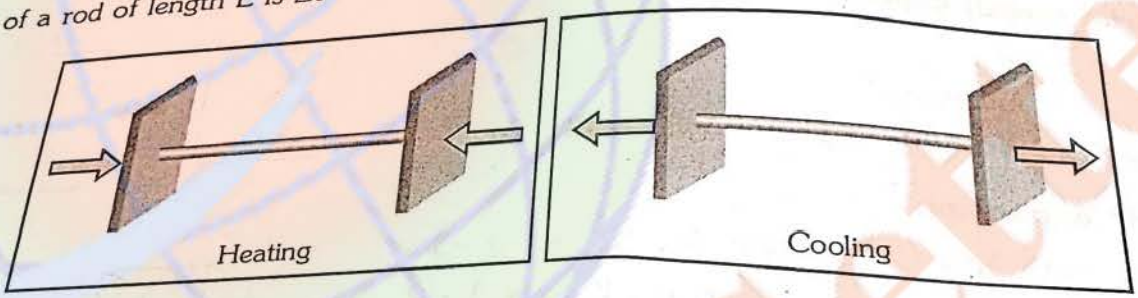


(b) **Effect of temperature on the time period of a simple pendulum :** A pendulum clock has proper time at temperature θ. If temperature is increased to θ' ($> \theta$) then due to linear expansion length of pendulum increases and hence its time period will increase.

Fractional change in time period $\frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta$ $(\because T \propto \sqrt{\ell} \quad \therefore \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta \ell}{\ell})$

- Due to increment in its time period, a pendulum clock becomes slow in summer and will time. Loss of time in a time period $\Delta T = \frac{1}{2}\alpha\,\Delta\theta T$
- The clock will lose time i.e. will become slow if $\theta' > \theta$ (in summer) and will gain time i.e. become fast if $\theta' < \theta$ (in winter).
- Since coefficient of linear expansion (α) is very small for invar, hence pendulums are made invar to show the correct time in all seasons.

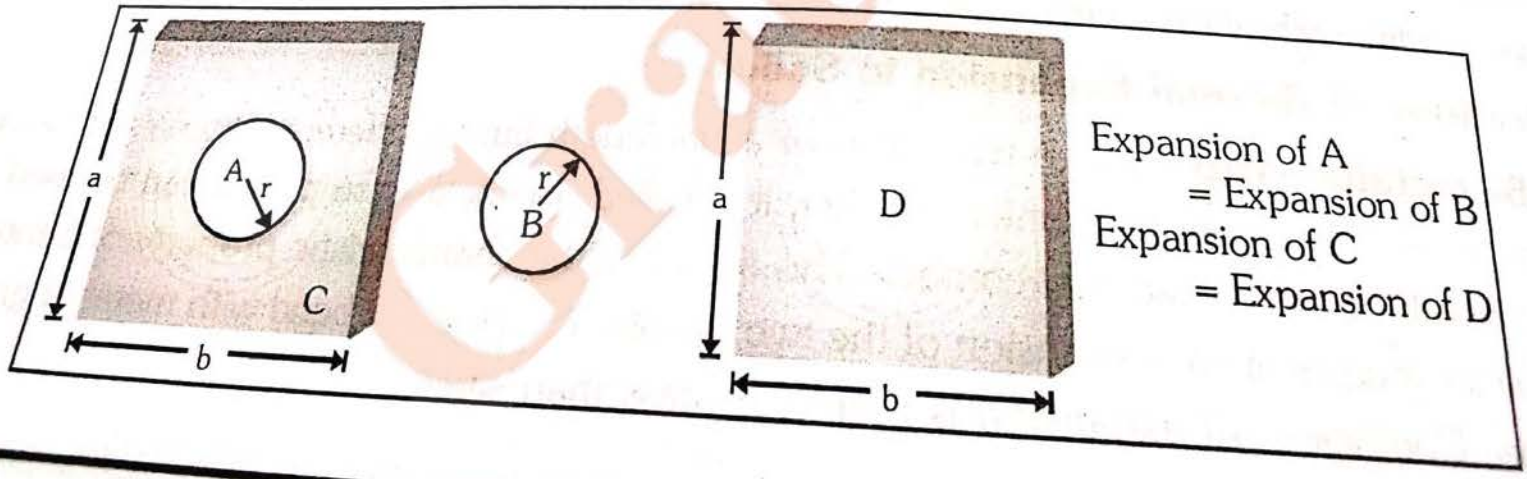
(c) When a rod whose ends are rigidly fixed so as to prevent expansion or contraction undergoes a change in temperature, due to thermal expansion or contraction, a compressive or tensile stress is developed in it. Due to this thermal stress the rod will exert a large force on the supports. If the change in temperature of a rod of length L is $\Delta\theta$ then :–



Thermal strain $= \frac{\Delta L}{L} = \alpha\Delta\theta$ $\quad \because \alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta\theta}$ So thermal stress $= Y\alpha\Delta\theta$ $\because Y = \frac{\text{stres}}{\text{stra}}$

So force on the supports $F = YA\alpha\Delta\theta$

(d) **Expansion of cavity:** Thermal expansion of an isotropic object may be imagined as a ph enlargement.



(e) Some other applications

- When rails are laid down on the ground, space is left between the ends of two rails
- The transmission cable are not tightly fixed to the poles
- Test tubes, beakers and cubicles are made up of pyrex–glass or silica because they have very low value of coefficient of linear expansion
- The iron rim to be put on a cart wheel is always of slightly smaller diameter than that of wheel
- A glass stopper jammed in the neck of a glass bottle can be taken out by warming the neck of the bottle.

- **Thermal Expansion in Liquids**
  - Liquids do not have linear and superficial expansion but these only have volumetric expansion.
  - Since liquids are always to be heated along with a vessel which contains them so initially on heating the system (liquid + vessel), the level of liquid in vessel falls (as vessel expands more since it absorbs heat and liquid expands less) but later on, it starts rising due to faster expansion of the liquid.

QP → represents expansion of vessel

QR → represents the real expansion of liquid.

  - The actual increase in the volume of the liquid

    = The apparent increase in the volume of liquid + the increase in the volume of the vessel.
  - Liquids have two coefficients of volume expansion.

(i) Co–efficient of apparent expansion (γ_a)

It is due to apparent (that appears to be, but not in real) increase in the volume of liquid if expansion of vessel containing the liquid is not taken into account.

$$\gamma_a = \frac{\text{Apparent expansion in volume}}{\text{Initial volume} \times \Delta\theta} = \frac{(\Delta V)}{V \times \Delta\theta}$$

(ii) Co–efficient of real expansion (γ_r)

It is due to the actual increase in volume of liquid due to heating.

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Initial volume} \times \Delta\theta} = \frac{(\Delta V)}{V \times \Delta\theta}$$

- Also coefficient of expansion of flask $\gamma_{Vessel} = \frac{(\Delta V)_{Vessel}}{V \times \Delta\theta}$

$\gamma_{Real} = \gamma_{Apparent} + \gamma_{Vessel}$

- Change (apparent change) in volume in liquid relative to vessel is

$\Delta V_{app} = V(\gamma_{Real} - \gamma_{Vessel})\,\Delta\theta = V(\gamma_r - 3\alpha)\Delta\theta$

α = Coefficient of linear expansion of the vessel.

- **Different level of liquid in vessel**

γ	ΔV	Level
$\gamma_{Real} > \gamma_{Vessel} (= 3\alpha) \Rightarrow \gamma_{app} > 0$	ΔV_{app} is positive	Level of liquid in vessel will rise on heating
$\gamma_{Real} < \gamma_{Vessel} (= 3\alpha) \Rightarrow \gamma_{app} < 0$	ΔV_{app} is negative	Level of liquid in vessel will fall on heating
$\gamma_{Real} = \gamma_{Vessel} (= 3\alpha) \Rightarrow \gamma_{app} = 0$	$\Delta V_{app} = 0$	Level of liquid in vessel will remain same

GOLDEN KEY POINTS

	K	C	F
Water boils	373.15	100	212
body temp.	310.2	37.0	98.6
Room temp.	300	27	80.6
Triple point of water	273.16	0.01	32
Water freezes	273.15	0	
Solid CO_2	195	-78	-109
Hydrogen boils	20.7	-252.5	-422.5
Absolute zero	0	-273.15	-489.67

- Actually thermal expansion is always 3-D expansion. When other two dimensions of object are negligible with respect to one, then observations are significant only in one dimension and it is known as linear expansion.
- Relation between α, β and γ

(i) For isotropic solids: $\alpha : \beta : \gamma = 1 : 2 : 3$ or $\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$

(ii) For non-isotropic solids $\beta = \alpha_1 + \alpha_2$ and $\gamma = \alpha_1 + \alpha_2 + \alpha_3$. Here α_1, α_2 and α_3 are coefficients of linear expansion in X, Y and Z direction.

Illustrations

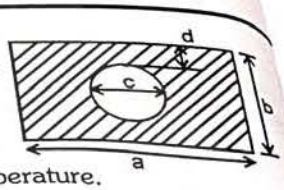


Illustration 6.

A rectangular plate has a circular cavity as shown in the figure. If we increase its temperature then which dimension will increase in following figure.

Solution :

Distance between any two point on an object increases with increase in temperature. So, all dimension a, b, c and d will increase

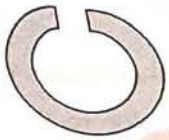


Illustration 7.

A small ring having small gap is shown in figure on heating what will happen to the size of gap.

Solution :

Gap will also increase. The reason is same as in above example.

Illustration 8.

What is the percentage change in length of 1m iron rod if its temperature changes by 100°C. α for iron is 2×10^{-5}/°C.

Solution :

percentage change in length due to temperature change

$$\%\ell = \frac{\Delta\ell}{\ell} \times 100 = \alpha\Delta\theta \times 100 = 2 \times 10^{-5} \times 100 \times 100 = 0.2\%$$

Illustration 9.

A concrete slab has a length of 10 m on a winter night when the temperature is 0°C. Find the length of the slab on a summer day when the temperature is 35°C. The coefficient of linear expansion of is 1.0×10^{-5} /°C.

Solution :

$$\ell_t = 10(1 + 1 \times 10^{-5} \times 35) = 10.0035 \text{ m}$$

Illustration 10.

A glass vessel of volume 100 cm^3 is filled with mercury and is heated from 25°C to 75°C. What volume of mercury will overflow? Coefficient of linear expansion of glass = 1.8×10^{-6}/°C and coefficient of volume expansion of mercury is 1.8×10^{-4}/°C.

Solution :

$\Delta V = V_0(\gamma_L - \gamma_C)\,\Delta T \quad = 100 \times (1.8 \times 10^{-4} - 3 \times 1.8 \times 10^{-6}) \times 50 \Rightarrow \Delta V = 0.87\ cm^3$ Ans.

Illustration 11.

There are two spheres of same radius and material at same temperature but one being solid while the other hollow. Which sphere will expand more if they are heated to the same temperature,

Solution

As thermal expansion of isotropic solids is similar to true photographic enlargement, expansion of a cavity is same as if it had been a solid body of the same material

i.e. $\Delta V = V\gamma\,\Delta\theta$

As here V, γ and $\Delta\theta$ are same for both solid and hollow spheres treated (cavity) ; so the expansion of both will be equal.

Illustration 12.

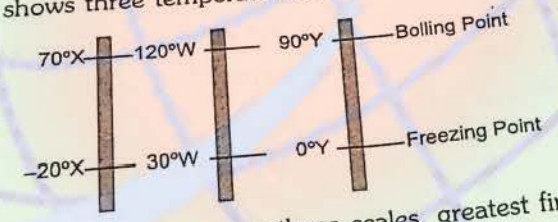
A steel wire of cross-sectional area 0.5 mm^2 is held between two fixed supports. If the wire is just taut at 20°C, determine the tension when the temperature falls to 0°C. Coefficient of linear expansion of steel is 1.2×10^{-5}/°C and its Young's modulus is 2.0×10^{11} N/m^2.

Solution : Here final length is less than the original length so that strain is tensile and tensile force is given by

$F = AY\alpha\Delta\theta = 0.5 \times 10^{-6} \times 2 \times 10^{11} \times 1.2 \times 10^{-5} \times 20 = 24$ N

BEGINNER'S BOX-1

1. Write down the following temperatures in the increasing order 50°F, 50°C and 50 K.
2. The figure shows three temperature scales with the freezing and boiling points of water indicated.



(a) Rank the size of a degree on these scales, greatest first.

(b) Rank the following temperatures, highest first : 50°X, 50°W and 50°Y.

3. What is the temperature at which we get the same reading on both the Centigrade and Fahrenheit scales?
4. A thin copper wire of length L increases in length by 1% when heated from temperature T_1 to T_2. What is the percentage change in area when a thin copper plate having dimensions 2L × L is heated from T_1 to T_2?

(A) 1% (B) 2% (C) 3% (D) 4%

5. A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0 °C. What is the change in the diameter of the hole when the sheet is heated to 227 °C ? Coefficient of linear expansion of copper = 1.70×10^{-5} °C^{-1},
6. If a bimetallic strip is heated, it will

(A) bend towards the metal with lower thermal expansion coefficient.

(B) bend towards the metal with higher thermal expansion coefficient.

(C) twist itself into helix.

(D) have no bending.

When a hot body is placed in contact with a cold one, the former gets colder and the latter warmer. this observation it is natural to conclude that a certain quantity of heat has passed from the hot body to cold one. Heat is a form of energy.

Heat is felt by its effects. Some of the effects of heat are :

(a) Change in the degree of hotness (b) Change in length, surface area and volume

(c) Change in state of a substance (d) Change in the resistance of a conductor

(e) Thermo e.m.f. effect

SI Unit : J (joule). Also measured in calorie.

- **Calorie**

It is defined as the amount of heat required to raise the temperature of 1 g water by 1°C or 1 K.

- **International calorie**

International calorie is the amount of heat required to raise the temperature of 1g water from 14.5 °C to 15.5 °C rise of temperature at pressure of 1 atm.

- **kilo Calorie**

kilo calorie is defined as the amount of heat required to raise the temperature of 1 kg water from 14.5 to 15.5 °C. (1 kcal = 1000 calorie) at pressure of 1 atm.

- **British thermal unit (B.T.U.)**

It is the amount of heat required to raise the temperature of one pound water by 1°F. (1 B.T.U. = 252 calorie)

Mechanical equivalent of heat

According to Joule, work may be converted into heat and vice–versa. The ratio of work done to the heat produced is always constant. $\frac{W}{H}$ = constant (J) $\Rightarrow$ W = J H

W must be in joule, irrespective of nature of energy or work and H must be in calorie.

J is called mechanical equivalent of heat. It is not a physical quantity but simply a conversion factor. It converts unit of work into that of heat and vice–versa.

J = 4.186 joule/cal or 4.186×10^3 joule/kcal. For rough calculations we take J = 4.2 joule/cal

Specific heat (s or c)

The amount of energy required to raise the temperature of unit mass of a substance by 1°C (or 1K) is called its specific heat. It is represented by s or c.

If the temperature of a substance of mass m changes from T to T + dT when it exchanges an amount of heat dQ with its surroundings then its specific heat is $c = \frac{1}{m}\frac{dQ}{dT}$

The specific heat depends on the pressure, volume and temperature of the substance.

For liquids and solids, specific heat measurements are most often made at a constant pressure as function of temperature, because constant pressure is quite easy to produce experimentally.

SI unit : joule/kg–K **CGS unit :** cal/g –°C

Specific heat of water : c_{water} = 1 cal/g–°C = 1 cal/g–K = 1 kcal/kg–K = 4200 joule/kg–K

When a substance does not undergo a change of state (i.e., liquid remains liquid or solid remains solid), the amount of heat required to raise the temperature of mass m of the substance by an amount $\Delta\theta$ is

$$\boxed{Q = ms\Delta\theta}$$

Variation in specific heat of water is less than 1% over the interval from 0 to 100°C. Such a small variation is typical for most solids and liquids, so their specific heats can generally be taken to be constant over fairly large temperature ranges.

- There are many possible processes to give heat to a gas.
- A specific heat can be associated to each such process which depends on the nature of process.
- Value of specific heats of gas can vary from zero (0) to infinity.
- Generally two types of specific heats are defined for a gas –
  (a) Specific heat at constant volume (C_v) (b) Specific heat at constant pressure (C_p)
- These specific heats can be molar or gram.

Molar heat capacity

The amount of energy needed to raise the temperature of one mole of a substance by 1°C (or 1K) is called molar heat capacity. The molar heat capacity is the product of molecular weight and specific heat i.e.,

Molar heat capacity C = Molecular weight (M) × Specific heat(c) ⇒ $C = \frac{1}{\mu}\left(\frac{dQ}{dT}\right)$

If the molecular mass of the substance is M and the mass of the substance is m then number of moles of the substance $\mu = \frac{m}{M} \Rightarrow C = \frac{M}{m}\left(\frac{dQ}{dT}\right)$

SI Unit : J/mol–K

Thermal capacity (Heat capacity)

The quantity of heat required to raise the temperature of the whole substance through 1°C is called thermal capacity. The thermal capacity of mass m of the whole substance of specific heat (s) = ms

Thermal capacity = mass × specific heat

Thermal capacity depends on property of material of the body and mass of the body.

SI Unit : cal/°C or cal/K, **Dimensions :** $ML^2 T^{-2}K^{-1}$

Water equivalent of a body

As the specific heat of water is unity so the thermal capacity of a body (ms) represents its water equivalent also.

- Mass of water having the same thermal capacity as the body is called the water equivalent of the body
- The water equivalent of a body is the amount of water that absorbs or gives out the same amount of heat as is done by the body when heated or cooled through 1°C.

Water equivalent= mass of body × specific heat of the material ⇒ (w = ms).

Latent heat or Hidden heat

When phase of a body changes, change of phase takes place at constant temperature [melting point or boiling point] and heat released or absorbed is Q = mL where L is latent heat. Heat is absorbed if solid converts into liquid (at melting point) or liquid convert into vapours (at boiling point) and heat is released if liquid converts into solid or vapours convert into liquid.

Latent heat of fusion

It is the quantity of heat (in kilocalories) required to change 1 kg mass of a substance from solid to liquid state at its melting point. Latent heat of fusion for ice : 80 kcal/kg = 80 cal /g.

Latent heat of vaporization

The quantity of heat required to change its 1 kg mass from liquid to vapour state at its boiling point.

Latent heat of vaporisation for water : 536 kcal/kg = 536 cal/g = 540 cal/g

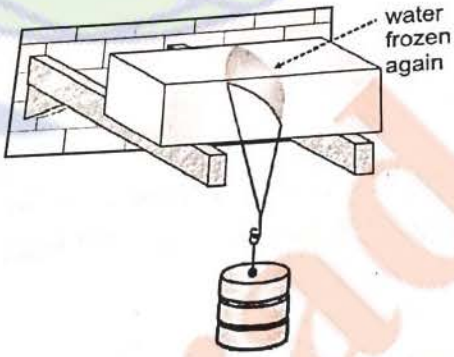
Change of State

- **Melting**
  Conversion of solid into liquid state at constant temperature is known as melting.
- **Boiling**
  Evaporation within the whole mass of the liquid is called boiling. Boiling takes place at a constant temperature known as boiling point. A liquid boils when the saturated vapour pressure on its surface is equal to atmospheric pressure. Boiling point reduces on decreasing pressure.
- **Evaporation**
  Conversion of liquid into vapours at all temperatures is called evaporation. It is a surface phenomenon. Greater the temperature, faster is the evaporation. Smaller the boiling point of liquid, more rapid is the evaporation. Smaller the humidity, more is the evaporation. Evaporation increases on decreasing pressure that is why evaporation is faster in vacuum.
- **Heat of evaporation**
  Heat required to change unit mass of a liquid into vapour at a given temperature is called heat of evaporation at that temperature.
- **Sublimation**
  Direct conversion of solid into vapour state is called sublimation.
- **Heat of sublimation**
  Heat required to change unit mass of solid directly into vapours at a given temperature is called heat of sublimation at that temperature.
- Camphor and ammonium chloride sublimates on heating in normal conditions.
- A block of ice sublimates into vapours on the surface of moon because of very–very low pressure on its surface
- **Condensation**
  The process of conversion from gaseous or vapour state to liquid state is known as condensation . These materials again get converted to vapour or gaseous state on heating.
- **Hoar frost**
  Direct conversion of vapours into solid is called hoar frost. This process is just reverse of the process of sublimation.

  Ex. : Formation of snow by freezing of clouds.
- **Regelation**

Regelation is the melting of ice caused by pressure and its resolidification when the pressure is removed. Ice shrinks when it melts, and if pressure is applied, deliberately promoting shrinkage, it is found that melting is thereby assisted. In other words, melting of cold ice is ordinarily effected by raising the temperature, but if pressure is present to help with the shrinkage the temperature need not be raised so much.

Ice heals up after being cut through by the wire. Melting takes place under the wire because pressure lowers the melting temperature. Refreezing (regelation) occurs above the wire when the water escapes to normal pressure again.



se of pressure lowers the melting (or freezing) point of water. Conversely, if a substance expands o
, the melting point is raised by pressure.

Phase of a substance

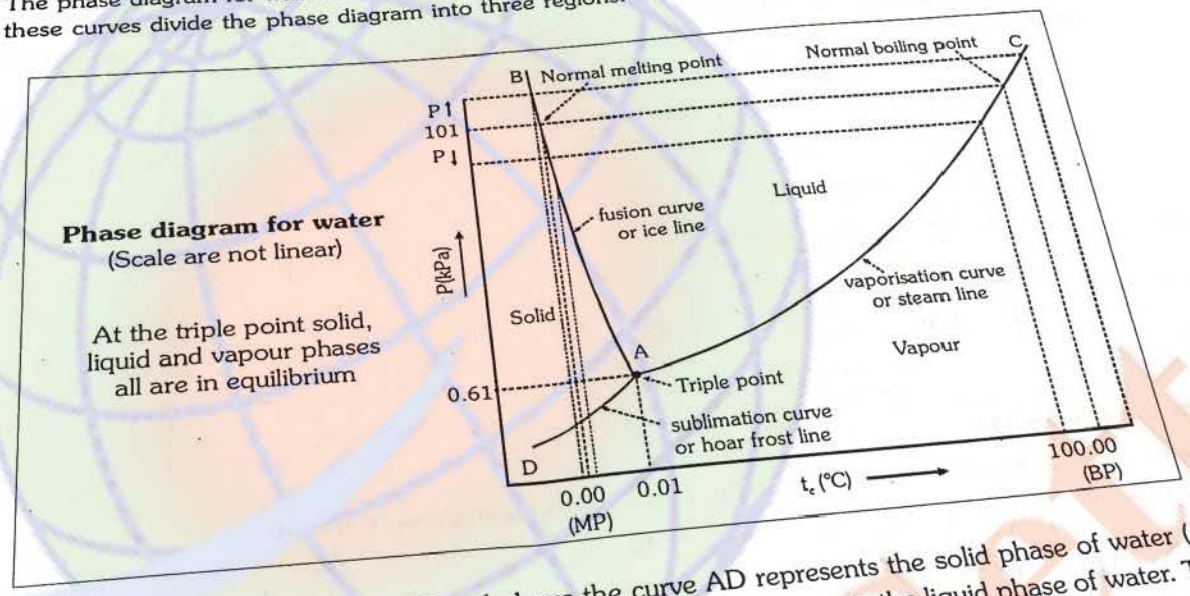
The phase of a substance is defined as its form which is homogeneous, physically distinct and mechanically separable from the other forms of that substance.

Phase diagram

- A phase diagram is a graph in which pressure (P) is represented along the y–axis and temperature (T) is represented along the x–axis.
- **Characteristics of Phase diagram**
  - (i) Different phases of a substance can be shown on a phase diagram.
  - (ii) A region on the phase diagram represents a single phase of the substance, a curve represents equilibrium between two phases and a common point represents equilibrium between three phases.
  - (iii) A phase diagram helps to determine the condition under which the different phases are in equilibrium.
  - (iv) A phase diagram is useful for finding a convenient way in which a desired change of phase can be produced.

Phase diagram for water

The phase diagram for water consists of three curves AB, AC and AD meeting each other at the point A, these curves divide the phase diagram into three regions.



Region to the left of the curve AB and above the curve AD represents the solid phase of water (ice). The region to the right of the curve AB and above the curve AC represents the liquid phase of water. The region below the curves AC and AD represents the gaseous phase of water (i.e. water vapour). A curve on the phase diagram represents the boundary between two phases of the substance.

Along any curve the two phases can coexist in equilibrium

- Along curve AB, ice and water can remain in equilibrium.This curve is called fusion curve or ice line. This curve shows that the melting point of ice decreases with increase in pressure.
- Along the curve AC, water and water vapour can remain in equilibrium. The curve is called vaporisation curve or steam line. *The curve shows that the boiling point of water increases with increase in pressure.*
- Along the curve AD, ice and water vapour can remain in equilibrium. This curve is called sublimation curve or hoar frost line.

Triple point of water

The three curves in the phase diagram of water meet at a single point A, which is called the triple point of water. The triple point of water represents the co–existance of all the three phases of water ice water and water vapour in equilibrium. The pressure corresponding to triple point of water is 6.03×10^{-3} atmosphere or 4.58 mm of Hg and temperature corresponding to it is 273.16K.

- **Significance of triple point of water**

Triple point of water represents a unique condition and it is used to define the absolute temperature. While making Kelvin's absolute scale, upper fixed point is 273.16 K and lower fixed point is 0 K. One kelvin of temperature is $\frac{1}{273.16}$ part of the temperature of triple point of water.

Effect of change in pressure on M.P. and B.P. for water

- If $P \uparrow \xrightarrow{\text{then result}}$ M.P. ↓ & B.P. ↑
- If $P \downarrow \xleftarrow{\text{then result}}$ M.P. ↑ & B.P. ↓

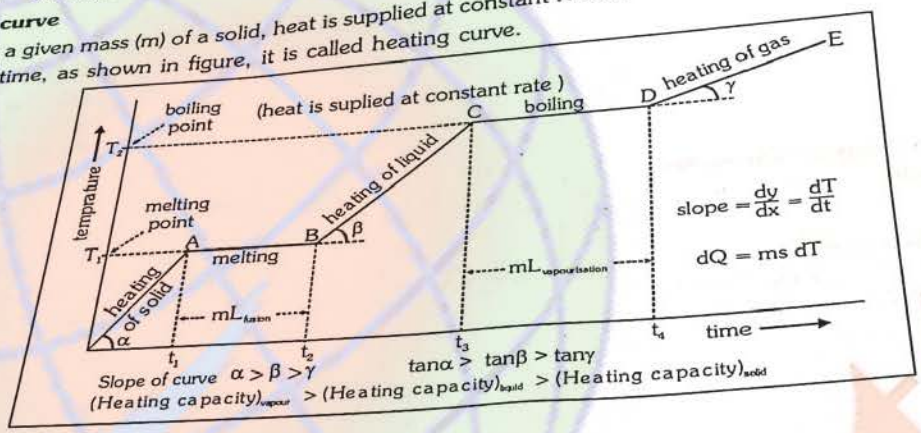
Example :

(1) A bottle is filled with water at 30°C on opening at moon then water will boil and vapourised

(2) At higher altitudes of mountain, food can not be cooked properly.

Heating curve

If to a given mass (m) of a solid, heat is supplied at constant rate and a graph is plotted between temperature and time, as shown in figure, it is called heating curve.



- ***In the region OA***

Rate of heat supply P is constant and temperature of solid is changing with time.

So, $Q = mc_S \Delta T \Rightarrow P\,\Delta t = mc_S\,\Delta T$ $[\because Q = P\,\Delta t]$

$\because \frac{\Delta T}{\Delta t} =$ The slope of temperature–time curve so specific heat of solid $c_S \propto \frac{1}{\text{slope of line OA}}$ specific heat (or thermal capacity) is inversely proportional to the slope of temperature–time curve.

- ***In the region AB***

Temperature is constant, so it represents change of state, i.e., melting of solid at melting point T_1. At point A melting starts and at point B all solid is converted into liquid. So between A and B substance is partly solid and partly liquid. If L_F is the latent heat of fusion then

$$Q = mL_F \Rightarrow L_F = \frac{P(t_2 - t_1)}{m} \quad [\text{as } Q = P(t_2 - t_1] \Rightarrow L_F \propto \text{length of line AB}$$

i.e., Latent heat of fusion is proportional to the length of line of zero slope.

[In this region specific heat $\propto \frac{1}{\tan 0°} = \infty$]

- **In the region BC**

  Temperature of liquid increases so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line BC, $c_L \propto \frac{1}{\text{slope of line BC}}$

- **In the region CD**

  Temperature is constant, so it represents change of state, i.e., liquid is boiling at boiling point T_2. At C all substance is in liquid state while at D is vapour state and between C and D partly liquid and partly gas. The length of line CD is proportional to latent heat of vaporisation, i.e., $L_V \propto$ Length of line CD.

  [In this region specific heat $\propto \frac{1}{\tan 0^\circ} = \infty$]

- **In the region DE**

  The line DE represents gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.

Law of Mixtures : Principle of calorimetry

- When two bodies at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature releases heat while body at lower temperature absorbs it, so that

  Heat lost = Heat gained (Principle of calorimetry)

  Principle of calorimetry represents the law of conservation of heat energy.

- Temperature of mixture (T) is always ≥ lower temperature (T_L) and ≤ higher temperature (T_H), $T_L \le T \le T_H$

  The temperature of mixture can never be lesser than lower temperature (as a body cannot be cooled below the temperature of cooling body) and greater than higher temperature (as a body cannot be heated above the temperature of heating body). Further more usually, rise in temperature of one body may not be equal to the fall in temperature of the other body though heat gained by one body is equal to the heat lost by the other.

Illustrations

Illustration 13.

5 g ice at 0°C is mixed with 5 g of steam at 100°C . What is the final temperature?

Solution

Heat required by ice to raise its temperature to 100°C,

$Q_1 = m_1L_1 + m_1c_1\Delta\theta_1 = 5 \times 80 + 5 \times 1 \times 100 = 400 + 500 = 900$ cal

Heat given by steam when condensed $Q_2 = m_2L_2 = 5 \times 536 = 2680$ cal

As $Q_2 > Q_1$. This means that whole steam is not even condensed.

Hence temperature of mixture will remain at 100°C.

Illustration 14.

A calorimeter of heat capacity 100 J/K is at room temperature of 30°C. 100 g of water at 40°C of spec heat 4200 J/kg–K is poured into the calorimeter. What is the temperature of water in calorimeter?

Solution

Let the temperature of water in calorimeter is t. Then heat lost by water = heat gained by calorim

$(0.1) \times 4200 \times (40 - t) = 100\,(t - 30) \Rightarrow 420 \times 40 - 420\,t = 100\,t - 3000 \Rightarrow t = 38.07$ °C

Illustration 15.

Find the quantity of heat required to convert 40 g of ice at –20°C into water at 20°C.
Given $L_f = 0.336 \times 10^6$ J/kg. Specific heat of ice = 2100 J/kg–K, specific heat of water = 4200 J/kg–K

Solution

Heat required to raise the temperature of ice from –20°C to 0°C = $0.04 \times 2100 \times 20 = 1680$ J

Heat required to convert the ice into water at 0°C = $mL = 0.04 \times 0.336 \times 10^6 = 13440$ J

Heat required to heat water from 0°C to 20°C = $0.04 \times 4200 \times 20 = 3360$ J

Total heat required = 1680 + 13440 + 3360 = 18480 J

Illustration 16.

Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. What is the mass of steam condensed? Latent heat of steam = 536 cal/g.

Solution

Heat required by (calorimeter + water)

$$Q = (m_1c_1 + m_2c_2)\,\Delta\theta = (0.02 + 1.1 \times 1)\,(80 - 15) = 72.8 \text{ kcal}$$

If m is mass of steam condensed, then heat given by steam

$$Q = mL + mc\,\Delta\theta = m \times 536 + m \times 1 \times (100 - 80) = 556\ m \quad \therefore\ 556\ m = 72.8$$

$$\therefore \text{ Mass of steam condensed } m = \frac{72.8}{556} = 0.130 \text{ kg}$$

Illustration 17.

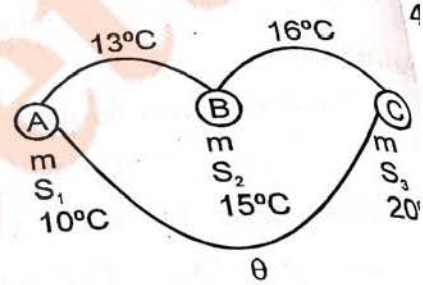
An iron block of mass 2 kg, fall from a height 10 m. After colliding with the ground it loses 25% energy to surroundings. Then find the temperature rise of the block. (Take specific heat of iron 470 J/kg °C)

Solution :

$$mS\Delta\theta = \frac{3}{4}\,mgh \quad \Rightarrow \quad \Delta\theta = \frac{3 \times 10 \times 10}{4 \times 470} = 0.159\ °C$$

Illustration 18.

The temperature of equal masses of three different liquids A, B, and C are 10°C 15°C and 20°C respectively. The temperature when A and B are mixed is 13°C and when B and C are mixed, it is 16°C. What will be the temperature when A and C are mixed?



Solution :

when A and B are mixed

$$mS_1 \times (13 - 10) = m \times S_2 \times (15 - 13) \quad .....(1)$$

$$3S_1 = 2S_2$$

when B and C are mixed

$$S_2 \times 1 = S_3 \times 4 \quad .....(2)$$

when C and A are mixed

$$S_1(\theta - 10) = S_3 \times (20 - \theta) \quad .....(3)$$

using equation (1), (2) and (3)

$$\text{Let } \theta = \frac{140}{11}\,°C$$

Illustration 19.
5 kg of steam at 100°C is mixed with 10 kg of ice at 0°C. Choose incorrect alternative
(Given s_{water} = 1 cal/g°C, L_F = 80 cal/g, L_V = 540 cal/g)
(A) Equilibrium temperature of mixture is 160°C
(B) Equilibrium temperature of mixture is 100°C
(C) At equilibrium, mixture contains $13\frac{1}{3}$ kg of water
(D) At equilibrium, mixture contains $1\frac{2}{3}$ kg of steam

Ans. (A)

Solution

Required heat
10 kg ice (0°C)
↓ 800 kcal
10 kg water (0°C)
↓ 1000 kcal
10 kg water (100°C)

Available heat
5 kg steam (100°C)
↓ 2700 Kcal
5 kg water (100°C)

So available heat is more than required heat therefore final temperature will be 100°C.

Mass of heat condensed $= \frac{800+1000}{540} = \frac{10}{3}$ kg. Total mass of water $= 10+\frac{10}{3} = \frac{40}{3} = 13\frac{1}{3}$ kg

Total mass of steam $= 5-\frac{10}{3} = \frac{5}{3} = 1\frac{2}{3}$ kg

BEGINNER'S BOX-2

1. A bullet of mass 10 g is moving with speed 400m/s. Find its kinetic energy in calories ?
2. Calculate amount of heat required to convert 1 kg steam from 100°C to 200°C steam ?
3. Calculate heat required to raise the temperature of 1 g of water by 1°C ?
4. 420 J of energy supplied to 10 g of water will raise its temperature by ?
5. The ratio of the densities of the two bodies is 3 : 4 and the ratio of specific heats is 4 : 3 . Find the ratio of their thermal capacities for unit volume ?
6. Heat releases by 1 kg steam at 150°C if it is converted into 1 kg water at 50°C.
7. 200 g water is filled in a calorimetry of negligible heat capacity. It is heated till its temperature is increase by 20°C. Find the heat supplied to the water.
8. A bullet of mass 5 gm is moving with speed 400 m/s strike a target. Then calculate rise of temperature of bullet. Assuming all the lose in kinetic energy is converted into heat energy of bullet if its specific heat is 500J/kg°C.
9. 1 kg ice at –10°C is mixed with 1 kg water at 100°C. Then find equilibrium temperature and mixture content.
10. 540 g of ice at 0°C is mixed with 540 g of water at 80°C. The final temperature of the mixture is
(Given latent heat of fusion of ice = 80 cal/g and specific heat capacity of water = 1 cal/g^{0}C)
(A) 0°C (B) 40°C (C) 80°C (D) less than 0°C

MODES OF HEAT TRANSFER

Heat is a form of energy which transfers from a body at higher temperature to a body at lower temperature. The transfer of heat from one body to another may take place by any one of the following modes :.

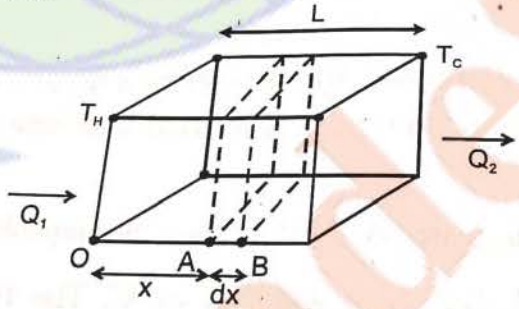
- **Conduction**
  The process in which the material takes an active part by molecular action and energy is passed from one particle to another is called conduction. It is predominant in solids.
- **Convection**
  The transfer of energy by actual motion of particle of medium from one place to another is called convection. It is predominant is fluids (liquids and gases).
- **Radiation**
  Quickest way of transmission of heat is known as radiation. In this mode of energy transmission, heat is transferred from one place to another without effecting the inter-venning medium.

Conduction	Convection	Radiation
Heat Transfer due to Temperature difference	Heat transfer due to density difference	Heat transfer without any medium
Due to free electron or vibration motion of molecules	Actual motion of particles	Electromagnetic radiation
Heat transfer in solids and Hg	Heat transfer in fluids (Liquid + gas)	No medium required
Slow process	Slow process	Fast process (3×10^8 m/sec)
Irregular path	Irregular path	Straight line (like light)

3.1 Thermal conduction

The process by which heat is transferred from hot part to cold part of a body through the transfer of energy from one particle to another particle of the body without the actual movement of the particles from their equilibrium positions is called conduction. The process of conduction takes place only in solid body (except Hg). Heat transfer by conduction from one part of body to another continues till their temperatures become equal.

For example if you hold an iron rod with one of its end on a fire for some time, the handle will become hot. The heat is transferred from the fire to the handle by conduction along the length of iron rod. The vibrational amplitude of atoms and electrons of the iron rod at the hot end takes relatively higher values due to the higher temperature of their environment. These increased vibrational amplitude are transferred along the rod, from atom to atom during collision between adjacent atoms. In this way a region of rising temperature extends itself along the rod to your hand.



nsider a slab of face area A, Lateral thickness L, whose faces have temperatures T_H and T_C(T_H >
consider two cross sections in the slab at positions A and B separated by a lateral distance of

Let temperature of face A be T and that of face B be T + ΔT. Then experiments show that Q, the amount of heat crossing the area A of the slab at position x in time t is given by

$$\boxed{\frac{Q}{t} = -KA\frac{dT}{dx}} \text{ or } \boxed{\frac{Q}{t} = \frac{KA(T_H - T_C)}{L}}$$

K → Thermal conductivity

→ It is the measure of the ability of material to conduct the heat.

Here K is a constant depending on nature of the material of the slab and is named thermal conductivity of the material, and the quantity $\left(\frac{dT}{dx}\right)$ is called temperature gradiant. The (–) sign shows heat flows from high temperature to low temperature (ΔT is a –ve quantity).

Steady State

If the temperature of a cross-section at any position x in the above slab remains constant with time (remember, it does vary with position x), the slab is said to be in steady state and temperature of rod is not same. Remember steady-state is distinct from thermal equilibrium for which temperature at any position (x) in the slab must be same.

For a conductor in steady state there is no absorption or emission of heat at any cross-section. (as temperature at each point remains constant with time). The left and right faces are maintained at constant temperatures T_H and T_C respectively, and all other faces must be covered with adiabatic walls so that no heat escapes through them and same amount of heat flows through each cross-section in a given interval of time.

Hence $Q_1 = Q = Q_2$. Consequently the temperature gradient is constant throughout the slab.

Hence, $$\frac{dT}{dx} = \frac{\Delta T}{L} = \frac{T_f - T_i}{L} = \frac{T_C - T_H}{L}$$

and $$\frac{Q}{t} = -KA\frac{\Delta T}{L} \Rightarrow \frac{Q}{t} = KA\left(\frac{T_H - T_C}{L}\right)$$

Here Q is the amount of heat flowing through a cross-section of slab at any position in a time interval t.

Thermal conductivity (K) :

- It depends on nature of material.

  Order of thermal conductivity $\boxed{Ag > Cu > Au > Al}$ K [For Ag maximum is (410 W/mK) / For Freon minimum is 12 (0.008 W/mK)]

- **SI Unit :** $J\ s^{-1}\ m^{-1}\ K^{-1}$ **Dimensions :** $M^1 L^1 T^{-3} \theta^{-1}$
- For an ideal or perfect conductor of heat the value of $K = \infty$
- For an ideal or perfect bad conductor or insulator the value of K = 0
- For cooking the food, low specific heat and high conductivity utensils are most suitable.

Application of Thermal Conduction

- In winter, the iron chairs appear to be colder than the wooden chairs.
- Cooking utensils are made of aluminium and brass whereas their handles are made of wood.
- Ice is covered in gunny bags to prevent melting of ice.
- We feel warm in woollen clothes and fur coat.
- Two thin blankets are warmer than a single blanket of double the thickness.
- Birds often swell their feathers in winter.
- A new quilt is warmer than old one.

Thermal Resistance to conduction

If you are interested in insulating your house from cold weather or for that matter keeping the meal h your tiffin-box, you are more interested in poor heat conductors, rather than good conductors. For this rea the concept of thermal resistance R has been introduced.

$$R = \frac{L}{KA}$$

For a slab of cross-section A, Lateral thickness L and thermal conductivity K,

$$\frac{Q}{t} = \frac{(T_H - T_L)}{R}$$

In terms of R, the amount of heat flowing though a slab in steady-state (in time t)

If we name $\frac{Q}{t}$ as thermal current i_T then,

$$\boxed{i_T = \frac{T_H - T_L}{R}}$$

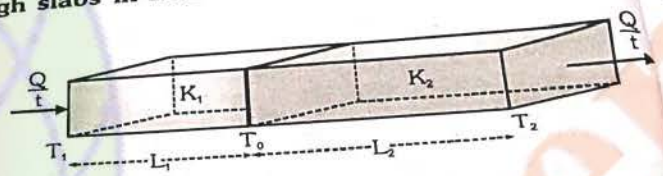
This is mathematically equivalent to OHM's law, with temperature doing the role of electric potential. Her results derived from OHM's law are also valid for thermal conduction.

More over, for a slab in steady state we have seen earlier that the thermal current i_L remains same at ea cross-section. This is analogous to Kirchhoff's current law in electricity, which can be very conveniently appl to thermal conduction.

- **Equivalent conductivity for Heat flow through slabs in series**

$$R_{eq} = R_1 + R_2$$

$$\frac{L_1 + L_2}{K_{eq}A} = \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A}$$

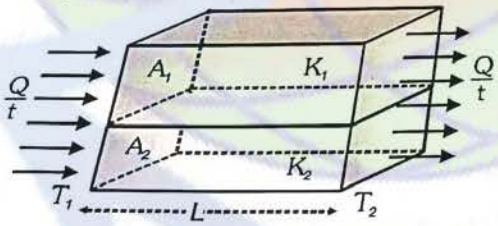


Equivalent thermal conductivity of the system is

$$K_{eq} = \frac{L_1 + L_2}{\frac{L_1}{K_1} + \frac{L_2}{K_2}} = \frac{\Sigma L_i}{\Sigma \frac{L_i}{K_i}}$$

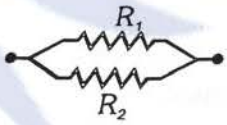
equivalent to

- **Equivalent thermal condctivity for Heat flow through slabs in parallel**



$$R = \frac{L}{KA}$$

equivalent to



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{K_{eq}}{L}(A_1 + A_2) = \frac{K_1 A_1}{L} + \frac{K_2 A_2}{L}$$

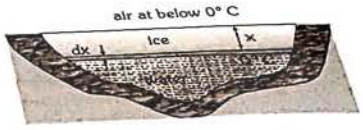
Equivalent thermal conductivity $$K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2} = \frac{\Sigma K_i A_i}{\Sigma A_i}$$

Growth of Ice on Lakes

In winter atmospheric temperature falls below 0°C and water in the lake start freezing. Let at time t thickness of ice on the surface of the lake = x and air temperature = $-\theta°$ C

The temperature of water in contact with the lower surface of ice = 0°C

Let area of the lake = A



Heat escaping through ice in time dt is $dQ = KA\frac{[0-(-\theta)]}{x}dt$

Due to escape of this heat increasing extra thickness of ice = dx

Mass of this extra thickness of ice is m = ρV = ρ A.dx

$$dQ = mL = (\rho\ A.dx)\ L$$

$$\therefore\quad KA\frac{\theta}{x}dt = (\rho\ A.dx)\ L \Rightarrow\ dt = \frac{\rho L}{K\theta}x\,dx$$

So time taken by ice to grow a thickness x is $t = \frac{\rho L}{K\theta}\int_0^x x\,dx = \frac{1}{2}\frac{\rho L}{K\theta}x^2$

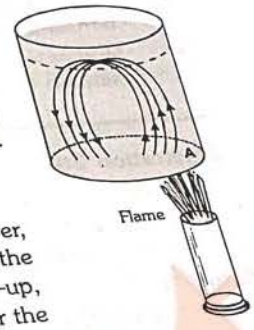
So time taken by ice to grow from thickness x_1 to thickness x_2 is

$$t = t_2 - t_1 = \frac{1}{2}\frac{\rho L}{KT}(x_2{}^2 - x_1{}^2) \quad \text{and} \quad \boxed{t \propto (x_2^2 - x_1^2)}$$

Time taken to double and triple the thickness ratio $t_1 : t_2 : t_3 :: 1^2 : 2^2 : 3^2$ So $t_1 : t_2 : t_3 :: 1 : 4 : 9$

3.2 Convection

Convection requires a medium and is the process in which heat is transferred from one place to other by actual movement of heated substance (usually fluid).The type of convection which results from difference in densities is called natural convection (for example, a fluid in a container heated through its bottom). However, if a heated fluid is forced to move by a blower, fan or pump, the process is called forced convection.



Phenomena Based on convection :

(i) Land and sea breezes :

The heat from the Sun is absorbed more rapidly by land than by sea–water. Moreover, the specific heat of land is low as compared to that of sea–water. Consequently, the rise in temperature of land is higher as compared to that of sea–water. To sum–up, land is hotter than the sea during day time. As a result of this, the colder air over the sea blows towards the land. This is called sea–breeze.

At night, air blows from land towards sea. This is called land breeze.

(ii) Formation of trade winds :

The surface of Earth near the equator gets heated strongly. So, the air in contact with the surface of Earth at the expands and rises upwards. As a result of this, a low pressure is created at the equator. At the poles, the air in the upper atmosphere gets cooled and comes down. So, a high pressure is created at the poles. Due to difference of pressures at the poles and equator, the air at the poles moves towards the equator, rises up, moves towards poles and so on. In this way, a wind is formed in the atmosphere. The rotation of the Earth also affects the motion of the wind. Due to anti–clockwise rotation of Earth the warm wind blowing from equator to north drifts towards east. The steady wind blowing from north–East to equator, near the surface of Earth, is called trade wind.

(iii) Monsoons :

In summer, the peninsular mass of central Asia becomes more strongly heated than the water of the Indian Ocean. This is due to the fact that the specific heat of water is much higher than that of the soil and rocks. Hot air from the heated land mass rises up and moves towards the Indian ocean. Air filled with moisture flows over the Indian ocean on the south towards heated land mass. When obstructed by mountains, the moist air rushes upwards to great height. In the process, it gets cooled. Consequently, the moisture condenses and falls as rain.

(iv) **Ventillation :**
Ventillator of exhaust fan in a room help of remove impure and warm air from a room. The fresh air fr
outside blows into the room. This is all due to the forced convection current set up in the room.

(v) **To regulate temperature in the human body :**
Heat transfer in the human body involves a combination of mechanisms. These together maintain a remarkab
uniform temperature in the human body inspite of large changes in environmental conditions. The chief interm
mechanism is forced convection. The heart serves as the pump and the blood as the circulating fluid.

Some important points :

- Natural convection takes place from bottom to top due to gravity while forced convection in any directio
- In case of natural convection, convection currents move warm air upwards and cool air downward
- This is why heating is done from base, while cooling from the top.
- Natural convection is not possible in a gravity free region such as a freely falling lift or an orbiting satellite
- Natural convection plays an important role in ventilation, in changing climate and weather and in formi
- land and sea breezes and trade winds.
- The forced convection of blood in our body by a pump (heart) helps in keeping the temperature
- body constant.
- Convection takes place in fluids (gases and liquid)

GOLDEN KEY POINTS

- For heat propagation via natural convection, temperature gradient exists in vertical direction and not
- horizontal direction.
- Most of heat transfer that is taking place on Earth is by convection, the contribution due to conduction an
- radiation is very small.

Illustrations

Illustration 20.

One face of an aluminium cube of edge 2 metre is maintained at 100 °C and the other end is maintaine
at 0 °C. All other surfaces are covered by adiabatic walls. Find the amount of heat flowing through the cu
in 5 seconds. (thermal conductivity of aluminium is 209 W/m–°C)

Solution

Heat will flow from the end at 100°C to the end at 0°C. $\frac{Q}{t} = KA\frac{(T_H - T_C)}{L}$

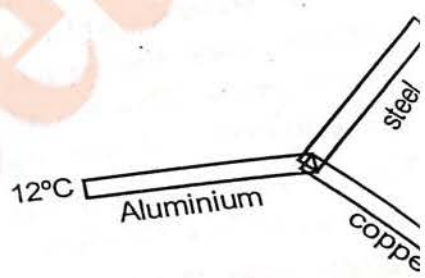
Area of cross-section perpendicular to direction of heat flow, $A = 4m^2$ then

$$Q = \frac{(209W/m°C)(4m^2)(100°C - 0°C)(5\,sec)}{2m} = 209\ kJ$$

Illustration 21.

Three identical rods of length 1m each, having cross-section area of $1cm^2$ each and made of Aluminium, copper and steel respectively are maintained at temperatures of 12°C, 4°C and 50°C respectively at their separate ends. Find the temperature of their common junction.

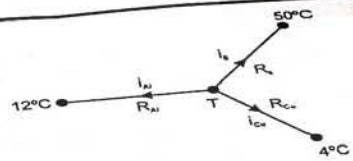
[K_{Cu}=400 W/m-K , K_{Al} = 200 W/m-K , K_{steel} = 50 W/m-K]



Solution

$$R_{Al} = \frac{L}{KA} = \frac{1}{200\times10^{-4}} = \frac{10^4}{200}$$

Similarly $R_{steel} = \frac{10^4}{50}$ and $R_{copper} = \frac{10^4}{400}$



Let temperature of common junction = T

then from Kirchhoff's current law, $i_{Al} + i_{steel} + i_{Cu} = 0$

$\Rightarrow \quad \frac{T-12}{R_{Al}} + \frac{T-50}{R_{steel}} + \frac{T-4}{R_{Cu}} = 0$

$\Rightarrow \quad (T-12)\,200 + (T-50)\,50 + (T-4)\,400 = 0$

$\Rightarrow \quad 4(T-12) + (T-50) + 8\,(T-4) = 0$

$\Rightarrow \quad 13T = 48 + 50 + 32 = 130 \quad \Rightarrow \quad T = 10°C$

Illustration 22.

The thermal conductivity of brick is 1.7 W m^{-1} K^{-1}, and that of cement is 2.9 W m^{-1} K^{-1}. What thickness of cement will have same insulation as the brick of thickness 20 cm ? Assuming their area to be same.

Solution

Since $Q = \frac{KA(T_1 - T_2)t}{L}$. For same insulation by the brick and cement ; Q, A $(T_1 - T_2)$ and t do not change.

Hence, $\frac{K}{L}$ remain constant. If K_1 and K_2 be the thermal conductivities of brick and cement respectively and L_1 and L_2 be the required thickness then $\frac{K_1}{L_1} = \frac{K_2}{L_2}$ or $\frac{1.7}{20} = \frac{2.9}{L_2}$ $\quad \therefore L_2 = \frac{2.9}{1.7} \times 20 = 34.12$ cm

Illustration 23.

Two vessels of different materials are identical in size and wall–thickness. They are filled with equal quantities of ice at 0°C. If the ice melts completely, in 10 and 25 minutes respectively then compare the coefficients of thermal conductivity of the materials of the vessels.

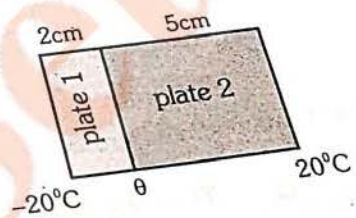
Solution

Let K_1 and K_2 be the coefficients of thermal conductivity of the materials, and t_1 and t_2 be the time in which ice melts in the two vessels. Since both the vessels are identical, so A and L in both the cases is same.

Now, $Q = \frac{K_1 A(\theta_1 - \theta_2)t_1}{L} = \frac{K_2 A(\theta_1 - \theta_2)t_2}{L} \quad \Rightarrow \frac{K_1}{K_2} = \frac{t_2}{t_1} = \frac{25 \text{ min}}{10 \text{ min}} = \frac{5}{2}$

Illustration 24.

Two plates of equal areas are placed in contact with each other. Their thickness are 2.0 cm and 5.0 cm respectively. The temperature of the external surface of the first plate is –20°C and that of the external surface of the second plate is 20°C. What will be the temperature of the contact surface if the plate (i) are of the same material, (ii) have thermal conductivities in the ratio 2 : 5.



Solution

Rate of flow of heat in the plates is $\frac{Q}{t} = \frac{K_1 A(\theta_1 - \theta)}{L_1} = \frac{K_2 A(\theta - \theta_2)}{L_2}$ ...(i)

(i) Here $\theta_1 = -20°C$, $\theta_2 = 20°C$,

$L_1 = 2$ cm $= 0.02$ m, $L_2 = 5$ cm $= 0.05$ m and $K_1 = K_2 = K$

$\therefore$ equation (i) becomes $\frac{KA(-20-\theta)}{0.02} = \frac{KA(\theta - 20)}{0.05}$

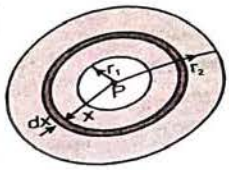
$\therefore 5(-20-\theta) = 2(\theta - 20) \Rightarrow -100 - 5\theta = 2\theta - 40 \Rightarrow 7\theta = -60 \Rightarrow \theta = -8.6°C$

(ii) $\frac{K_1}{K_2} = \frac{2}{5}$ or $K_1 = \frac{2}{5}K_2$

$\therefore$ from equation (i) $\frac{2/5\,K_2 A(-20-\theta)}{0.02} = \frac{K_2 A(\theta - 20)}{0.05} \Rightarrow -20 - \theta = \theta - 20 \quad \therefore \theta = 0°C$

Illustration 25.

Two thin concentric shells made of copper with radius r_1 and r_2 ($r_2 > r_1$) have a material of thermal conductivity K filled between them. The inner and outer spheres are maintained at temperatures T_H and T_C respectively by keeping a heater of power P at the centre of the two spheres. Find the value of P.



Solution : Heat flowing per second through each cross-section of the sphere = P = i.

Thermal resistance of the spherical shell of radius x and thickness dx,

$$dR = \frac{dx}{K.4\pi x^2}$$

$$\Rightarrow \quad R = \int_{r_1}^{r_2} \frac{dx}{4\pi x^2.K} = \frac{1}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

thermal current

$$i = P = \frac{T_H - T_C}{R} = \frac{4\pi K(T_H - T_C)r_1 r_2}{(r_2 - r_1)}.$$

Illustration 26.

Water in a closed tube is heated with one arm vertically placed above the lamp. In what direction water will begin the circulate along the tube ?

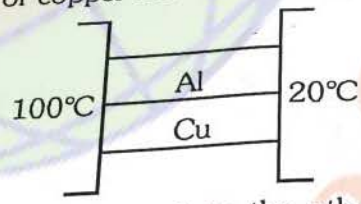
Solution

On heating the liquid at A will become lighter and will rise up. This will push the liquid in the tube upwards and so the liquid in the tube will move clockwise i.e. from B to A.

BEGINNER'S BOX-3

1. Explain why :
   (a) a brass tumbler feels much colder than a wooden tray on a chilly day
   (b) two layers of cloth of equal thickness provide warmer covering than a single layer of cloth of double thickness ?
   (c) mud-houses are colder in summer and warmer in winter?
   (d) In winter birds sit with their wings spread out ?
   (e) Woollen clothes are warmer than cotton clothes ?

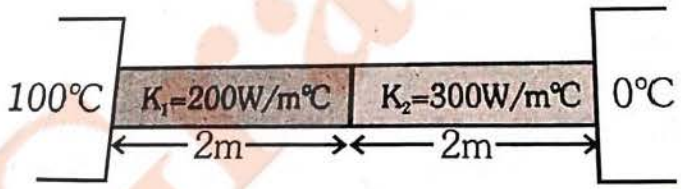
2. Two metal cubes with 3 cm-edges of copper and aluminium are arranged as shown in figure. Find



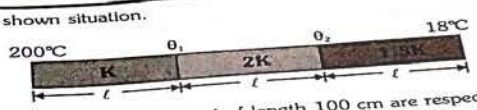
   (a) The total thermal current from one reservoir to the other.
   (b) The ratio of the thermal current carried by the copper cube to that carried by the aluminium cube.

   Thermal conductivity of copper is 60 W/m–K and that of aluminium is 40 W/m–K.

For shown situation, calculate the temperature of the common interface.

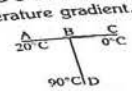


4. Calculate θ_1 and θ_2 in shown situation.



5. The temperature at the ends of a uniform rod of length 100 cm are respectively 95°C and 5°C. What will be the temperature at a point 30 cm far from the hotter end ? Also calculate the temperature gradient.

6. Three conducting rods of same material and cross-section are shown in figure. Temperature of A, D and C are maintained at 20°C, 90°C and 0°C. Find the ratio of length BD and BC if there is no heat flow in AB.



3.3 Thermal Radiation

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation. When a body is heated and placed in vacuum, it loses heat even when there is no medium surrounding it. The heat can not go out from the body by the process of conduction or convection since both of these process require the presence of a material medium between source and surrounding objects. The process by which heat is lost in this case is called radiation. This does not require the presence of any material medium.

It is radiation by which heat from the Sun reaches the Earth. Radiation has the following properties:

(a) Radiant energy travels in straight lines and when some object is placed in the path, its shadow is formed at the detector.

(b) It is reflected and refracted or can be made to interfere. The reflection or refraction are exactly as in case of light.

(c) It can travel through vacuum.

(d) Intensity of radiation follows the law of inverse square.

(e) Thermal radiation can be polarised in the same way as light by transmission through a nicol prism.

(f) Radiation takes place in solid, liquid & gases.

All these and many other properties establish that heat radiation has nearly all the properties possessed by light and these are also electromagnetic waves with the only difference of wavelength or frequency. The wavelength of thermal radiation is larger than that of visible light.

- When radiation passes through any medium then radiations slightly absorbed by medium according to its absorptive power so temperature of medium slightly increases.
- In order to obtain a spectrum of radiation, a special prism is used like KCℓ prism, Rock salt prism Flourspar prism. Normal glass prism or Quartz prism can not be used (because it absorbs some radiation).
- Radiation intensity is measured with a specific device named as Bolometer.
- Heat radiation are always obtained in infra–red region of electromagnetic wave spectrum so they are called Infra-red rays.
- Thermal radiation when incident on a surface, then exert pressure on the surface which is known as Radiation Pressure.

Basic Fundamental definitions

- **Absorptive power or absorptive coefficient (a) :** The ratio of amount of radiation absorbed by a surface (Q_a) to the amount of radiation incident (Q) upon it, is defined as the coefficient of absorption $a = \frac{Q_a}{Q}$. It is unitless and dimensionless.

- **Spectral absorptive power (a_λ)** $a_\lambda = \frac{Qa_\lambda}{Q_\lambda}$: Also called monochromatic absorptive coefficient

  At a given wavelength $a = \int_0^\infty a_\lambda d\lambda$. For ideal black body a_λ and $a = 1$, a and a_λ are unitless

- **Emissive power (e)** : The amount of heat radiation emitted by unit surface area in unit second at a particular temperature. **SI UNIT** : $J/m^2\text{-}s$ or $watt/m^2$

- **Spectral Emmisive power (e_λ)** : The amount of heat radiation emitted by unit area of the body in one second in unit spectral region at a given wavelength.

  Emissive power or total emissive power $e = \int_0^\infty e_\lambda \, d\lambda$

  SI UNIT : $W/m^2\text{-}\mathring{A}$

Emissivity (e)

- **Absolute emissivity or emissivity** : Radiation energy given out by a unit surface area of a body in unit time corresponding to unit temperature difference w.r.t. the surroundings is called Emissivity.

  S I unit : $W/m^2\,K$

- **Relative emissivity (e_r)** : $e_r = \frac{Q_{GB}}{Q_{IBB}} = \frac{e_{GB}}{E_{IBB}} = \frac{\text{emitted radiation by gray body}}{\text{emitted radiation by ideal black body}}$

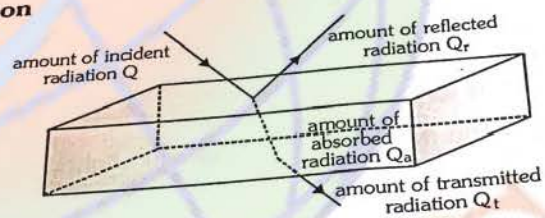
  GB = gray or general body, IBB = Ideal black body

  (i) No unit (ii) For ideal black body $e_r = 1$ (iii) range $0 < e_r < 1$

Spectral, emissive, absorptive and transmittive power of a given body surface

Due to incident radiations on the surface of a body following phenomena occur by which the radiation is divided into three parts. (a) Reflection (b) Absorption (c) Transmission

- **From energy conservation**



$$Q = Q_r + Q_a + Q_t \Rightarrow \frac{Q_r}{Q} + \frac{Q_a}{Q} + \frac{Q_t}{Q} = 1 \Rightarrow r + a + t = 1$$

Reflective Coefficient $r = \frac{Q_r}{Q}$, Absorptive Coefficient $a = \frac{Q_a}{Q}$,

Transmittive Coefficient $t = \frac{Q_t}{Q}$

$r = 1$ and $a = 0$, $t = 0$ $\Rightarrow$ Perfect reflector

$a = 1$ and $r = 0$, $t = 0$ $\Rightarrow$ Ideal absorber (ideal black body)

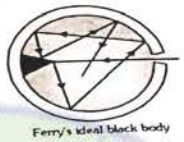
$t = 1$ and $a = 0$, $r = 0$ $\Rightarrow$ Perfect transmitter (diathermanous)

Reflection power (r) $= \left[\frac{Q_r}{Q} \times 100\right]\%$, Absorption power (a) $= \left[\frac{Q_a}{Q} \times 100\right]\%$

Transmission power (t) $= \left[\frac{Q_t}{Q} \times 100\right]\%$

Ideal Black Body

- A body surface which absorbs all incident thermal radiations at low temperature, irrespective of their wave length and emits out all these absorbed radiations at high temperature is assumed to be an ideal black body surface.
- The identical parameters of an ideal black body is given by $a = a_\lambda = 1$ and $r = 0 = t$, $e_r = 1$



Ferry's ideal black body

- The nature of emitted radiations from surface of ideal black body only depends on its temperature
- The radiations emitted from surface of ideal black body are called as either full or white radiations.
- At any temperature the spectral energy distribution curve for surface of an ideal black body is always continuous and according to this concept if the spectrum of a heat source obtained is continuous then it must be ideal black body like kerosene lamp; oil lamp, heating filament etc.
- There are two experimentally ideal black body
  (a) Ferry's ideal black body (b) Wien's ideal black body.
- At low temperature, surface of ideal black body is a perfect absorber and at a high temperature it proves to be a perfect emitter.
- An ideal black body need not be of black colour (eg. Sun).

Prevost's theory of heat energy exchange

According to Prevost, at every possible temperature (except zero kelvin temperature) there is a continuous heat energy exchange between a body and its surrounding and this exchange carry on for infinite time. The relation between temperature difference of body with its surrounding decides whether the body experience cooling effect or heating effect.

When a cold body is placed in the hot surrounding : The body radiates less energy and absorbs more energy from the surrounding, therefore the temperature of body increases. (Heating effect)

When a hot body placed in cooler surrounding : The body radiates more energy and absorbs less energy from the surroundings. Therefore temperature of body decreases. (cooling effect)

When the temperature of a body is equal to the temperature of the surrounding

The energy radiated per unit time by the body is equal to the energy absorbed per unit time by the body, therefore its temperature remains constant and the body is in thermal equillibrium with surrounding. Hence no heating and cooling effects are seen.

KIRCHHOFF'S LAW

At a given temperature for all bodies the ratio of their spectral emissive power (e_λ) to spectral absorptive power (a_λ) is constant and this constant is equal to spectral emissive power (E_λ) of the ideal black body at same temperature

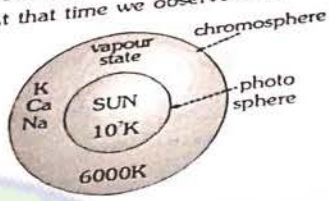
$$\frac{e_\lambda}{a_\lambda} = E_\lambda = \text{constant} \qquad \left[\frac{e_\lambda}{a_\lambda}\right]_1 = \left[\frac{e_\lambda}{a_\lambda}\right]_2 = \text{constant} \qquad \text{Hence} \qquad \boxed{e_\lambda \propto a_\lambda}$$

Good absorbers are good emitters and bad absorbers are bad emitters

Applications of Kirchoff Law

- **Fraunhoffer's lines**

Fraunhoffer lines are dark lines in the spectrum of the Sun. When white light emitted from the central core of the Sun (Photosphere) passes through its atmosphere (chromosphere) some of the radiations are absorbed by the gases present, resulting in dark lines in the spectrum of Sun. At the time of total solar eclipse direct light rays emitted from photosphere cannot reach on the Earth and only rays from chromosphere are able to reach on the Earth surface. At that time we observe bright fraunhoffer lines.



- **In deserts days are hot and nights cold**

Sand is rough and black, so it is a good absorber and hence in deserts, days (when radiation from Sun is incident on sand) will be very hot. Now in accordance with Kirchhoff's Law, good absorber is a good emitter. So nights (when send emits radiation) will be cold.

Stefan's Law

The amount of radiation emitted per second per unit area by ideal black body is directly proportional to the fourth power of its absolute temperature.

Amount of radiation emitted $\boxed{E \propto T^4}$ where T = temperature of ideal black body (in K)

$E = \sigma T^4$ (This law is true for only ideal black body)

SI Unit : $E = watt/m^2$ $\sigma \rightarrow$ Stefen's constant $= 5.67 \times 10^{-8}$ watt $/m^2 K^4$ (universal constant)

Dimensions of σ **:** $M^1 L^0 T^{-3} \theta^{-4}$

Total radiation energy emitted out by surface of area A in time t :

Ideal black body $Q_{IBB} = \sigma A T^4 t$ and for any other body $Q_{GB} = e_r \sigma A T^4 t$

Rate of emission of radiation

When temperature of surrounding T_0 (Let $T_0 < T$)

Rate of emission of radiation from per unit area of ideal black body surface $E_1 = \sigma T^4$

Rate of emission or absorption of radiation (per unit area) from surrounding $E_2 = \sigma T_0^4$

Net rate of loss of radiation per unit area from ideal black body surface is

$E = E_1 - E_2 = \sigma T^4 - \sigma T_0^4 = \sigma (T^4 - T_0^4)$

Net loss of radiation energy from entire surface area in time t is $Q_{IBB} = \sigma A (T^4 - T_0^4) t$

For any other body $Q_{GB} = e_r A\sigma (T^4 - T_0^4) t$

in time dt the net heat energy loss for ideal black body is dQ and because of this its temperature falls by dT

e of loss of heat (IBB) $\boxed{R_H = \frac{dQ}{dt} = \sigma A(T^4 - T_0^4) \text{ J/s}}$

also equal to emitted power or radiation emitted per second

fall in temperature (Rate of cooling) $\boxed{R_F = \frac{dT}{dt} = \frac{\sigma A}{ms}(T^4 - T_0^4)}$ $\left[\because \frac{dQ}{dt} = ms\frac{dT}{dt}\right]$

Body	R_H	R_F
(i) Two solid sphere (same material) (same T, T_0, s, ρ) (different radius r_1, r_2)	$R_H \propto r^2$	$R_F \propto \frac{1}{r}$
(ii) Two solid sphere (different material) (same T, T_0)	$R_H \propto r^2$	$R_F \propto \frac{1}{r\rho s}$
(iii) Different shape bodies (Cube, sphere, cylinder flat surface) (const. T, T_0, V, same materials) surface	$R_H \propto A$ • maximum for flat surface • minimum for spherical	$R_F \propto \frac{A}{V}$ • maximum for flat surface • minimum for spherical surface
(iv) Two sphere (one solid and another hollow) (T, T_0, s, A are same)	• R_H is same for both	R_F • maximum for hollow sphere • minimum for solid sphere
(v) Different bodies made of different material (T, T_0, M, A)(s-different)	• R_H is same for all bodies	$R_F \propto \frac{1}{s}$

Where T and T_0 absolute temperature of body and surrounding, M = mass of body, s = Specific heat, ρ = density, V = volume.

- **When a body cools by radiation, its cooling depends on :**

(i) Nature of radiating surface : greater the emissivity (e_r), faster will be the cooling.

(ii) Area of radiating surface : greater the area of radiating surface, faster will be the cooling.

(iii) Mass of radiating body : greater the mass of radiating body slower will be the cooling.

(iv) Specific heat of radiating body : greater the specific heat of radiating body slower will be the cooling.

(v) Temperature of radiating body : greater the temperature of radiating body faster will be the cooling.

NEWTON'S LAW OF COOLING

Rate of cooling $\left(\frac{dT}{dt}\right)$ is directly proportional to excess of temperature of the body over that of surrounding.

[(when $(T - T_0) \not> 35°C$] $\quad -\frac{dT}{dt} \propto (T - T_0)$

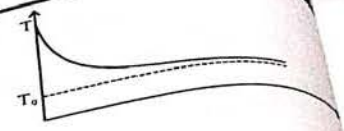
T = temperature of body [all temperatures in °C]

T_o = temperature of surrounding,

$T - T_0$ = excess of temperature ($T > T_0$)

If the temperature of body is decreased by dT in time dt then rate of fall of temperature $\boxed{-\frac{dT}{dt} = K(T - T_0)}$

Where negative sign indictates that the rate of cooling is decreasing with time.



For Numerical Problems, Newton's Law of cooling

If the temperature of body decreases from T_1 to T_2 and temperature of surroundings is T_0 then

average excess of temperature $= \left[\frac{T_1 + T_2}{2} - T_0\right]$

$$\Rightarrow \boxed{\left[\frac{T_1 - T_2}{t}\right] = +K\left[\frac{T_1 + T_2}{2} - T_0\right]}$$

Limitations of Newton's Law of Cooling

- Temperature difference should not exceed 35° C, $(T - T_0) \not> 35°$ C
- Loss of heat should only be by radiation.
- This law is an extended form of Stefan–Boltzman's law.

Derivation of Newton's law from Stefan's Boltzman law

$$\frac{dT}{dt} = \frac{\sigma A}{ms}(T^4 - T_0^4) \quad \begin{Bmatrix} T - T_0 = \Delta T \\ T = T_0 + \Delta T \end{Bmatrix}$$

$$\frac{dT}{dt} = \frac{\sigma A}{ms}\left[(T_0 + \Delta T)^4 - T_0^4\right]$$

If x <<< 1 then $(1 + x)^n = 1 + nx$ (Binomial theorem)

$$\frac{dT}{dt} = \frac{\sigma A}{ms}\left[T_0^4\left(1 + \frac{\Delta T}{T_0}\right)^4 - T_0^4\right] = \frac{\sigma A}{ms}T_0^4\left[\left(1 + \frac{\Delta T}{T_0}\right)^4 - 1\right] = \frac{\sigma A}{ms}T_0^4\left[1 + 4\frac{\Delta T}{T_0} - 1\right]$$

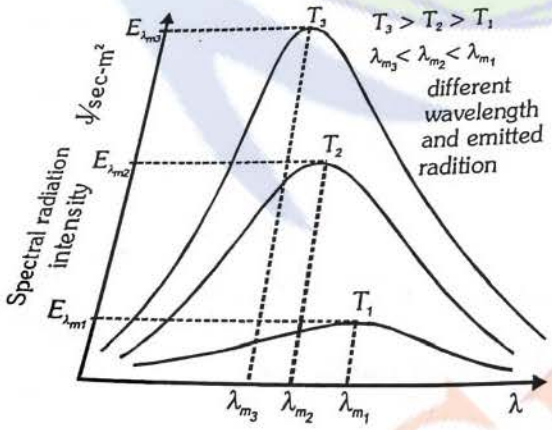
$$\frac{dT}{dt} = \left[4\frac{\sigma A}{ms}T_0^3\right]\Delta T \quad \Rightarrow \quad \frac{dT}{dt} = K\Delta T$$

here constant $K = \frac{4\sigma A T_0^3}{ms}$

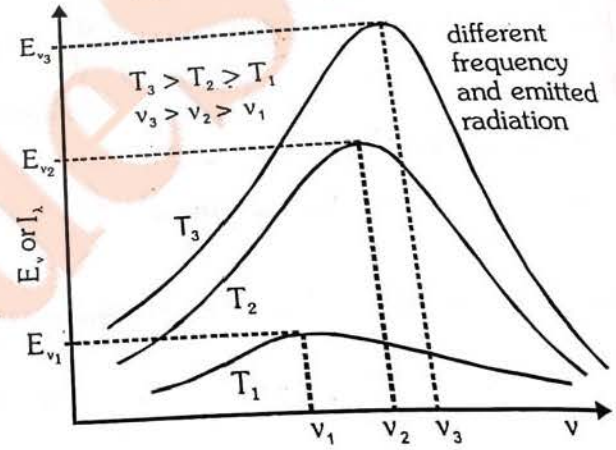
Newton's law of cooling $\frac{dT}{dt} \propto \Delta T$ (for small temperature difference)

Spectral Energy distribution curve of Black Body radiations

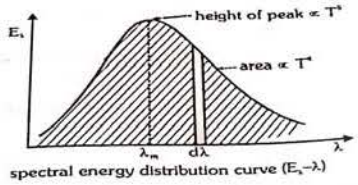
Practically given by : Lumers and Pringshem



Mathematically given by : Plank



Results from these Graphs



spectral energy distribution curve (E_λ–λ)

(i) $\lambda_m \propto \frac{1}{T}$ Wien's displacement law

$\boxed{\lambda_m T = b}$

$\boxed{\lambda_{m_1} T_1 = \lambda_{m_2} T_2}$

(ii) $E_{\lambda_m} \propto T^5$

(iii) i.e. Area $\int_0^\infty E_\lambda \, d\lambda = E = \sigma T^4$

Area between curve and λ axis gives the emissive power of body

Hence $\frac{A_1}{A_2} = \left[\frac{T_1}{T_2}\right]^4$

Wein's Displacement Law

The wavelength corresponding to maximum emission of radiation decrease with increasing temperature $\left[\lambda_m \propto \frac{1}{T}\right]$. This is known as Wein's displacement law.

$\boxed{\lambda_m T = b}$ where b is Wein's constant = 2.89×10^{-3} mK.

Dimensions of b : = $M^0 L^1 T^0 \theta^1$

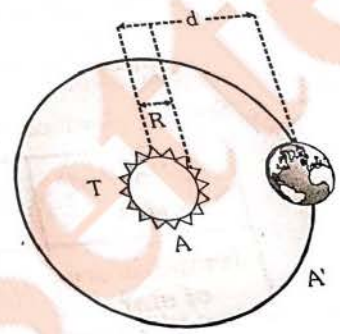
Relation between frequency and temperature $\nu_m = \frac{c}{b} T$ $[c = \lambda \times \nu]$

Solar constant 'S'

The Sun emits radiant energy continuously in space of which an in significant part reaches the Earth. The solar radiant energy received per unit area per unit time by a black surface held at right angles to the Sun's rays and placed at the mean distance of the Earth (in the absence of atmosphere) is called solar constant. The solar constant S is taken to be 1340 watt/m^2 or 1.937 Cal/cm^2–minute

Temperature of the Sun

Let R be the radius of the Sun and 'd' be the radius of Earth's orbit around the Sun. Let E be the energy emitted by the Sun per second per unit area. The total energy emitted by the Sun in one second = E.A = E × $4\pi R^2$. (This energy is falling on a sphere of radius equal to the radius of the Earth's orbit around the Sun i.e., on a sphere of surface area $4\pi d^2$)



So, The energy falling per unit time per unit area of Earth

$$= \frac{4\pi R^2 \times E}{4\pi d^2} = \frac{E R^2}{d^2}$$

$R = 7 \times 10^8$ m , $d = 1.5 \times 10^{11}$ m, $\sigma = 5.7 \times 10^{-8}$ W m^{-2} K^{-4}

Solar constant $S = \frac{E R^2}{d^2}$

By Stefan's Law $E = \sigma T^4$

$$S = \frac{\sigma T^4 R^2}{d^2} \Rightarrow T = \left[\frac{S \times d^2}{\sigma \times R^2}\right]^{\frac{1}{4}} = \left[\frac{1340 \times (1.5 \times 10^{11})^2}{5.7 \times 10^{-8} \times (7 \times 10^8)^2}\right]^{\frac{1}{4}} = 5732 \text{ K}$$

GOLDEN KEY POINTS

- At absolute zero temperature (zero kelvin) all atoms of a given substance remains in ground state, so, this temperature emission and absorption of radiation from any substance is impossible, so Prevost's energy exchange theory does not applied at this temperature, so it is called **limiting temperature** of prevost theory.
- With the help of Prevost's theory rate of cooling of any body w.r.t. its surroundings can be worked out (appl to Stefan Boltzman law, Newton's law of cooling.)
- For a constant temperature the spectral emmisive power of an ideal black body is a constant parameter
- The practical confirmation of Kirchhoff's law carried out by Rishi apparatus and the main base of this apparatus is a Lessilie container.
- The main conclusion predicted from Kirchhof's law can be expressed as

Good absorber $\rightleftharpoons$ Good emitter

Bad absorber $\rightleftharpoons$ Bad emitter

(at Low temperature) (at high temperature)

- If all of T, T_0, m, s, V, ρ, are same for different shape body then R_F and R_H will be maximum for the fl surface.
- If a solid and hollow sphere are taken with all the parameters same then hollow will cool down at fast ra
- Rate of temperature fall , $R_F \propto \frac{1}{s} \propto \frac{dT}{dt}$ so $dt \propto s$. If condition of specific heat is $s_1 > s_2 > s_3$ and if all cooled same temperature i.e. temperature fall is also identical for all then required time

$\boxed{t \propto s}$ $\therefore t_1 > t_2 > t_3$

- Spectral energy distribution curves are continuous. At any temperature in all possible wavelengt radiation between $(0 - \infty)$ are emitted but quantity of radiations are different for different wavelength.
- As the wave length increases, the amount of radiation emitted first increase, becomes maximum and the decreases.
- At a particular temperature the area enclosed between the spectral energy curve and wavelength axis sho the emissive power of the body. $\text{Area} = \int_0^\infty E_\lambda d\lambda = E = \sigma T^4$
- Spectral classification of stars

Colour of star	Blue	White	Yellow	Orange	Red
Temperature of star	20000 K-30000 K	10000K-20000K	6000K-8000K	4000K	3000K

Illustrations

tion 27.

λ_m for the moon is 14.5 micron, then find its temperature.

displacement law $\lambda_m T = b$ $\therefore T = \frac{b}{\lambda_m} = \frac{2.89 \times 10^{-3}}{14.5 \times 10^{-6}} = 199.3K$

Illustration 28.

Total radiation incident o by body. Then find out

Solution

$Q = Q_t + Q_r + Q_a \Rightarrow$

So transmittive powe

Illustration 29.

The operating temp Find the surface a

Solution

$\because$ Rate of emis

$\therefore W = Ae\sigma T$

Illustration 30.

Draw a gra

Solution

$E = \sigma T$

$\log E = l$

$\log E =$

Illustration

If tem emit

Solution

E

Illu

S

Illustration 28.

Total radiation incident on body is 400 J, If 20% of incident radiation reflected back and 120 J is absorbed by body. Then find out transmittive power in percentage.

Solution

$Q = Q_i + Q_r + Q_a \Rightarrow 400 = 80 + 120 + Q_t \Rightarrow Q_t = 200$

So transmittive power is $\frac{Q_t}{Q} \times 100\% = 50\%$

Illustration 29.

The operating temperature of a tungesten filament in an incandescent lamp is 2000 K and its emissivity is 0.3. Find the surface area of the filament of a 25 watt lamp. Stefan's constant $\sigma = 5.67 \times 10^{-8}\ Wm^{-2}\ K^{-4}$

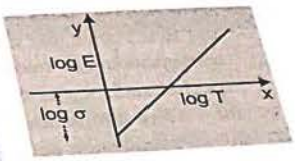
Solution

$\because$ Rate of emission = wattage of the lamp

$\therefore W = Ae\sigma T^4 \Rightarrow A = \frac{W}{e\sigma T^4} = \frac{25}{0.3 \times 5.67 \times 10^{-8} \times (2000)^4} = 0.918 \times 10^{-4}\ m^2$

Illustration 30.

Draw a graph between log E and log T



Solution

$E = \sigma T^4$ (taking log)

$\log E = \log(\sigma T^4)$

$\log E = 4\log T + \log\sigma$ This is equivalent to $y = mx - C$ ($\sigma < 1$ so its log is a negative quantity)

Illustration 31.

If temperature of ideal black body is increased by 50%, what will be percentage increase in quantity of radiations emitted from its surface.

Solution

$E \propto T^4$ and $\therefore$ $E' \propto (1.5)^4 T^4 \propto \left[\frac{15}{10}\right]^4 T^4 \propto \left[\frac{3}{2}\right]^4 T^4 \propto \frac{81}{16} T^4$

$$\frac{E' - E}{E} \times 100\% = \left[\frac{\frac{81}{16}T^4 - T^4}{T^4}\right] \times 100\% = 406\ \% \approx 400\ \%$$

Illustration 32.

If temperature of ideal black body is decreased from T to $\frac{T}{2}$ than find out percentage loss in emissive rate

Solution

$E \propto T^4, \quad E' \propto \left[\frac{T}{2}\right]^4 \quad \frac{T^4}{16}$

$$\left[\frac{E - E'}{E}\right] \times 100\% = \left[1 - \frac{1}{16}\right] \times 100\% = \frac{15}{16} \times 100\% \approx 94\%$$ Remaining is 6% (Approx.)

Illustration 33.

Calculate the temperature at which a perfect black body radiates at the rate of 5.67 W cm^{-2}. Stefan's c... is 5.67×10^{-8} J s^{-1} m^{-2} K^{-4}.

Solution

Given $E = 5.67$ W cm^{-2} $= 5.67 \times 10^{-4}$ W m^{-2}, $\sigma = 5.67 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4}

Using, $E = \sigma T^4$; $T^4 = \frac{E}{\sigma}$ or $T = \left[\frac{E}{\sigma}\right]^{\frac{1}{4}} = \left[\frac{5.67 \times 10^{-4}}{5.67 \times 10^{-8}}\right]^{\frac{1}{4}} = (10^{12})^{1/4} = 1000$ K

Illustration 34.

The temperature of furnace is 2000°C. in its spectrum the maximum intensity is obtained at about q... If the maximum intensity is at 2000Å calculate the temperature of the furnace in °C.

Solution

by using $\lambda_m T = b$, $4000\ (2000+273) = 2000(T) \Rightarrow T = 4546$K

The temperature of furnace $= 4546 - 273 = 4273$ °C

Illustration 35.

Two bodies A and B have thermal emissiviities of 0.01 and 0.81 respectively. The outer surface are... the two bodies are same, the two bodies emit total radiant power at the same rate. The wavelengt... corresponding to maximum spectral radiancy of B is shifted from the wavelength corresponding to maxi... spectral radiancy in the radiation of A by 1.0 μm. If the temperature of A is 5802K, Calculate :-

(a) The temperature of B

(b) Wavelength λ_B

Solution

(a) As both bodies A and B having same radiant power

$\therefore P_A = P_B \Rightarrow e_A \sigma A_A T_A^4 = e_B \sigma A_B T_B^4 \Rightarrow (0.01)\sigma A T_A^4 = (0.81)\sigma A T_B^4$

$$T_B = \left(\frac{0.01}{0.81}\right)^{1/4} T_A = \frac{T_A}{3} = \frac{5802}{3} = \boxed{1934\ K}$$

(b) According to wein's displacement law

$$\lambda_A T_A = \lambda_B T_B \Rightarrow \lambda_B = \left(\frac{5802}{1934}\right)\lambda_A = 3\lambda_A$$

$$\text{As } \lambda_B - \lambda_A = 1\ \mu m \Rightarrow \lambda_B - \frac{\lambda_B}{3} = 1\ \mu m \Rightarrow \frac{2\lambda_B}{3} = 1\ \mu m \Rightarrow \lambda_B = 1.5\ \mu m.$$

Illustration 36.

When a metallic body is heated in a furnace, then what colour will appear as temperature increases.

Solution. $T \propto \frac{1}{\lambda_m}$

As Temperature increases colour of body will appear from red, yellow, green, blue and then white.

ustration 37.

Define (i) Steady state and (ii) Temperature gradient in conduction of heat through a conducting rod.

ution

(i) When one end of a rod is heated, the temperature of various points of the rod changes continuo... but after some time a state is reached, when the temperature of each cross–section becomes ste... which is called steady state. In this state the heat received by any section will be totally transfere... the next section so no heat is absorbed by any cross section.

Temperature gradient is defined as the rate of change of temperature with distance in the direc... of flow of heat.

Illustration 38.

Assuming Newton's la... in 7 minutes. Temper...

Solution

According to Newto...

Since the temperatu...

$\frac{60-40}{7} = K\left(\frac{60}{}\right.$

If the temperatur...

$\Rightarrow 40 - T_2 =$

$\Rightarrow 5T_2 = 140$

1. Explain why
   (a) a bo...
   (b) an ...
   too
   th...
   (c) th...
   (d) ...
   (e)
   (f)
   (g)
2. Tw...
   If ...
3. If ...
   t...
4.
5.

Illustration 38.

Assuming Newton's law of cooling to be valid. The temperature of body changes from 60°C to 40°C in 7 minutes. Temperature of surroundings being 10°C. Find its temperature after next 7 minutes.

Solution

According to Newton's law of cooling $\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_0\right)$

Since the temperature decreases from 60°C to 40°C in 7 minutes

$\frac{60-40}{7} = K\left(\frac{60+40}{2} - 10\right) \Rightarrow \frac{20}{7} = K(50 - 10) \Rightarrow K = \frac{1}{14}$

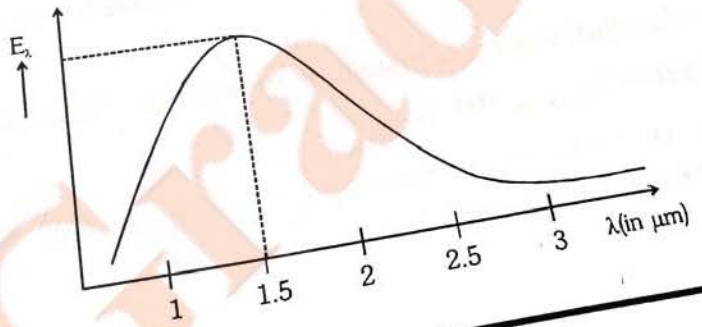
If the temperature of object becoms T_2 in next 7 minutes then $\frac{40 - T_2}{7} = \frac{1}{14}\left(\frac{40 + T_2}{2} - 10\right)$

$\Rightarrow 40 - T_2 = \frac{1}{4}(40 + T_2 - 20) \Rightarrow 160 - 4T_2 = 20 + T_2$

$\Rightarrow 5T_2 = 140 \Rightarrow T_2 = 28°C$

BEGINNER'S BOX-4

1. Explain why :
   (a) a body with large reflectivity is a poor emitter.
   (b) an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace
   (c) the earth without its atmosphere would be inhospitably cold
   (d) Heat is generated continuously in an electric heater but its temperature remains constant after some time.
   (e) The bottom of a cooking vessel is made black ?
   (f) On winter night you feel warmer when clouds cover the sky than when the sky is clear.
   (g) A thermos or vacuum flask can keep hot things hot and cold things cold for a long time, how?
2. Two spherical ideal black bodies of radii r_1 and r_2 are having surface temperature T_1 and T_2 respectively, If both radiate the same power. Then calculate the ratio of $\frac{T_1}{T_2}$.
3. If a liquid takes 30 sec. in cooling from 80°C to 70°C and 70 sec in cooling from 60°C to 50°C, then find the room temperature.
4. A body cools in 10 minutes from 60°C to 40°C. What will be its temperature after next 10 minutes? The temperature of the surrounding is 10°C.
5. Calculate the temperature of the black body from given graph.



KINETIC THEORY OF GASES

The properties of the gases are entirely different from those of solid and liquid. In case of gases, the expansion is very large as compared to solids and liquids. To state the conditions of a gas, its volume, pres and temperature must be specified.

Intermolecular force Solid > liquid > real gas > ideal gas (zero)

Potential energy Solid < liquid < real gas < ideal gas (zero)

At a given temperature for solid, liquid and gas:

(i) Internal kinetic energy : Same for all

(ii) Internal potential Energy : Maximum for ideal gas (PE = 0) and Minimum for solids (PE

(iii) Internal Energy : Maximum for Ideal gas and Minimum for solid

Notation :

μ = Molar amount of gas = $M/M_w = N/N_0$

N_0 = Avogadro constant = 6.023×10^{23} molecules/mole

R = Universal gas constant = 8.31 J/mole$^-$ k

≈ 2 cal/mole-k, [Dimension = $ML^2T^{-2}\mu^{-1}\theta^{-1}$]

r = Specific gas constant ($r = R/M_w$)

k = Boltzmann constant = ($k = R/N_0$) = 1.38×10^{-23} J/kelvin

m = mass of a gas molecule

M = mass of gas

M_w = molecular weight

N = No. of gas molecules

n = Molecular density ($n = N/V$)

ρ = gas density ($\rho = M/V = mn$)

	N.T.P. (Normal temperature and pressure)	S.T.P. (Standard Temperature and Pressure)
Temperature	0° C = 273.15 K	0.01° C = 273.16K
Pressure	1 atm = 1.01325×10^5 N/m^2 = 1.01325×10^5 pascal	1 atm
Volume	22.4 litre	22.4 litre

1 lit = 10^{-3} m^3 = 10^3 cm^3 = 10^3cc = 10^3ml

ρ_{water}=1000 kg/m^3 = 1 g/cc, ρ_{ice}=900 kg/m^3 = 0.9 g/cc

Ideal Gas Concept

- Volume of gas molecules is negligible as compared to volume of container.

So volume of gas = volume of container (Except 0 K)

No intermoleculer forces act between gas molecules.

perties of Ideal Gas

A gas which follows all gas laws and gas equation at every possible temperature and pressure is known ideal or perfect gas.

'deal gas molecules can do only translational motion, so their kinetic energy is only translational kinetic ene

'eal gas can not be liquified because IMF is zero

tential energy of ideal gas is zero so internal energy of ideal gas is perfectly translational K.E. of g

directly proportional to absolute temperature.

So, internal energy depends on

$E_{tran} \propto T$

For a substance U = U

U_{KE} : depends only on

- Specific heat of Ideal gas is
- All real gases behaves as a
- Gas molecules have point is no effect of gravity on

Equation of state for Ideal

$PV = \mu RT$ ⇒ PV

⇒ P

Gas Laws

- **Boyle's Law**

According to it

proportional

- **Charle**

Accord

to its

Gay-

So, internal energy depends only and only on its temperature.

$E_{trans} \propto T$

For a substance $U = U_{KE} + U_{PE}$

U_{KE} : depends only on T, U_{PE} : depends upon intermolecular forces (Always negative)

- Specific heat of ideal gas is constant quantity and it does not change with temperature
- All real gases behaves as an ideal gas at high temperature and low pressure and low density.
- Gas molecules have point mass and negligible volume and velocity is very high (10^7 cm/s). That's why there is no effect of gravity on them.

Equation of state for Ideal gas

$\boxed{PV = \mu RT}$ $\Rightarrow PV = \frac{M}{M_w}RT = \left[\frac{mN}{mN_0}\right]RT = \left[\frac{R}{N_0}\right]NT = NkT$ where μ = number of moles of gas

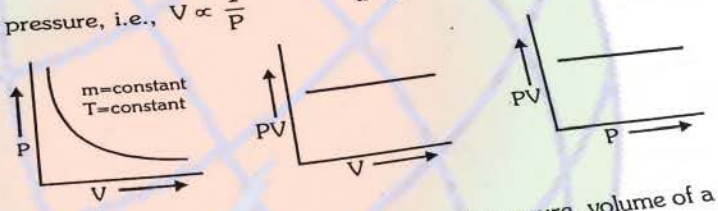
k = Boltzamann constant

$$\Rightarrow \boxed{\frac{P}{\rho} = \frac{RT}{M_w} = \frac{kT}{m}}$$

Gas Laws

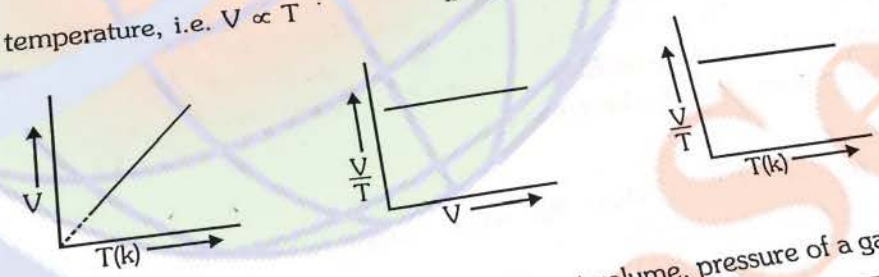
- **Boyle's Law**

According to it for a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure, i.e., $V \propto \frac{1}{P}$ if m and T = Constant $\Rightarrow \boxed{P_1V_1 = P_2V_2}$



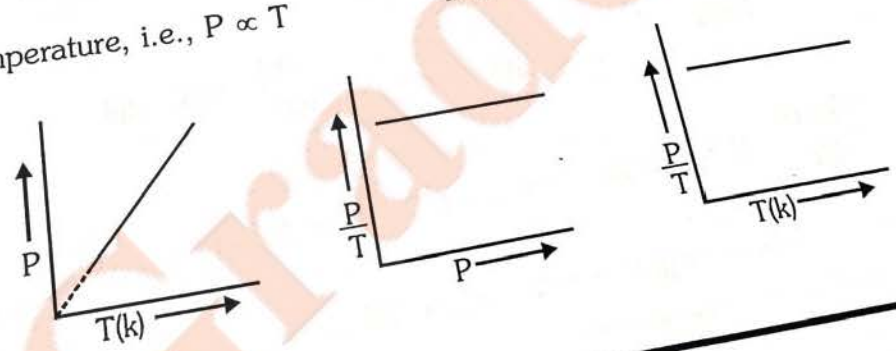
- **Charle's Law**

According to it for a given mass of an ideal gas at constant pressure, volume of a gas is directly proportional to its absolute temperature, i.e. $V \propto T$ if m and P = Constant $\Rightarrow \boxed{\frac{V_1}{V_2} = \frac{T_1}{T_2}}$



Gay–Lussac's Law

According to it, for a given mass of an ideal gas at constant volume, pressure of a gas is directly proportional to its absolute temperature, i.e., $P \propto T$ if m and V = constant $\Rightarrow \boxed{\frac{P_1}{P_2} = \frac{T_1}{T_2}}$



Avogadro's Law

According to it, at same temperature and pressure, equal volume of all gases contain equal number of molecules.

i.e., $N_1 = N_2$ if P,V and T are same.

Dalton's Partial Pressure Mixture Law :

According to it, the pressure exerted by mixture of non-reactive gases is equal to the sum of partial pressures of each component gases present in the mixture, ie., $P = P_1 + P_2 + ...$

Illustrations

Illustration 39.

By increasing temperature of gas by 5° C its pressure increases by 0.5% from its initial value at constant volume then what is initial temperature of gas ?

Solution

$\because$ At constant volume $T \propto P$ $\quad \therefore \frac{\Delta T}{T} \times 100 = \frac{\Delta P}{P} \times 100 = 0.5 \Rightarrow T = \frac{5 \times 100}{0.5} = 1000K$

Illustration 40.

Calculate the value of universal gas constant at STP.

Solution

Universal gas constant is given by $R = \frac{PV}{T}$

One mole of all gases at S.T.P. occupy volume V = 22·4 litre = $22{\cdot}4 \times 10^{-3}$ m^3

$P = 760$ mm of Hg $= 760 \times 10^{-3} \times 13{\cdot}6 \times 10^3 \times 9.80$ N m^{-2}, T = 273.16 K

$$\therefore R = \frac{760 \times 10^{-3} \times 13.6 \times 10^3 \times 9.80 \times 22.4 \times 10^{-3}}{273.16} = 8{\cdot}314 \text{ J mol}^{-1} \text{ K}^{-1}$$

Illustration 41.

A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases at a temperature of 27° and pressure of 1×10^5 N/m^2. The total mass of the mixture is 28 g. If the gram molecular weights of neon and argon are 20 and 40 respectively, find the mass of the individual gases in the container, assuming them to be ideal. Given : R = 8.314 J/mol-K.

Solution

Let m gram be the mass of neon. Then, the mass of argon is (28 – m)g.

Total number of moles of the mixture, $\mu = \frac{m}{20} + \frac{28-m}{40} = \frac{28+m}{40}$ ...(i)

Now, $\mu = \frac{PV}{RT} = \frac{1 \times 10^5 \times 0.02}{8.314 \times 300} = 0.8$ ...(ii)

(i) and (ii), $\frac{28+m}{40} = 0.8 \Rightarrow 28 + m = 32 \Rightarrow m = 4$ gram

or mass of argon = (28 – 4)g = 24 g

Illustration 42.

Calculate the temperature of the Sun if density is 1.4 g cm^{-3}, pressure is 1.4×10^9 atmosphere and average molecular weight of gases in the Sun in 2 g/mole. [Given R = 8.4 J $mol^{-1}K^{-1}$]

Solution

$PV = \mu RT \Rightarrow T = \frac{PV}{\mu R}$ ...(i) But $\mu = \frac{M}{M_w}$ and $\rho = \frac{M}{V}$ $\therefore \mu = \frac{\rho V}{M_w}$

From equation (i) $T = \frac{PVM_w}{\rho VR} = \frac{PM_w}{\rho R} = \frac{1.4 \times 10^9 \times 1.01 \times 10^5 \times 2 \times 10^{-3}}{1.4 \times 1000 \times 8.4} = 2.4 \times 10^7$ K

Illustration 43.

At the top of a mountain a thermometer reads 7°C and barometer reads 70 cm of Hg. At the bottom of the mountain they read 27°C and 76 cm of Hg respectively. Compare the density of the air at the top with that at the bottom.

Solution

By gas equation $PV = \frac{M}{M_w}RT \Rightarrow \frac{P}{\rho T} = \frac{R}{M_w}\left[\because \mu = \frac{M}{M_w} \text{ and } \frac{M}{V} = \rho\right]$

Now as M_W and R are same for top and bottom $\left[\frac{P}{\rho T}\right]_T = \left[\frac{P}{\rho T}\right]_B$ So $\frac{\rho_T}{\rho_B} = \frac{P_T}{P_B} \times \frac{T_B}{T_T} = \frac{70}{76} \times \frac{300}{280} = \frac{75}{76} = 0.9868$

Illustration 44.

A sample of oxygen of volume of 500 cc at a pressure of 2 atm is compressed to a volume of 400 cc. What pressure is needed to do this if the temperature is kept constant ?

Solution

Temperature is constant, so $P_1 V_1 = P_2 V_2 \therefore P_2 = P_1 \frac{V_1}{V_2} = 2\left[\frac{500}{400}\right] = 2.5$ atm

Illustration 45.

A vessel of volume 8.0×10^{-3} m^3 contains an ideal gas at 300 K and 200 k Pa. The gas is allowed to leak till the pressure falls to 125 kPa. Calculate the amount of the gas leaked assuming that the temperature remains constant.

Solution

As the gas leaks out, the volume and the temperature of the remaining gas do not change. The number of moles of the gas in the vessel in given by $\mu = \frac{PV}{RT}$.

The number of moles in the vessel before the leakage is $\mu_1 = \frac{P_1 V}{RT}$ and that after the leakage is $\mu_2 = \frac{P}{}$

The amount leaked is $\mu_1 - \mu_2 = \frac{(P_1 - P_2)V}{RT} = \frac{(200-125) \times 10^3 \times 8.0 \times 10^{-3}}{8.3 \times 300} = 0.24$ mole

Illustration 46.

1500 ml of a gas at a room temperature of 23°C is inhaled by a person whose body temperature is 37°C. if the pressure and mass stay constant, what will be the volume of the gas in the lungs of the person?

Solution

$T_1 = 273 + 23 = 296$ K; $T_2 = 273 + 37 = 310$ K. Pressure and amount of the gas are kept constant.

So $\frac{V_1}{T_1} = \frac{V_2}{T_2} \therefore V_2 = V_1 \times \frac{T_2}{T_1} = 1500 \times \frac{310}{296} = 1570.95$ ml

Illustration 47.

A sample of O_2 is at a pressure of 1 atm when the volume is 100 ml and its temperature is 27°C. What will be the temperature of the gas if the pressure becomes 2 atm and volume remains 100 ml.

Solution

$T_1 = 273 + 27 = 300$ K

For constant volume $\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow T_2 = T_1 \times \frac{P_2}{P_1} = 300 \times \frac{2}{1} = 600$ K $= 600 - 273 = 327$°C

BEGINNER'S BOX-5

1. A vessel of volume 8.3×10^{-3} m^3 contains an ideal gas at temperature 27°C and pressure 200 kPa. The gas is allowed to leak till the pressure falls to 100 kPa and temperature increases to 327°C. What is the amount of gas in moles will be leaked out ?
2. Two closed vessels of equal volume contain air at 105 kPa, 300 K and are connected through a narrow tube. If one of the vessels is now maintained at 300 K and other at 400 K, What will be the pressure in the vessels.
3. The volume of a gas is 1 litre at the pressure 1.2×10^7 Nm^{-2} and temperature 400 K. Calculate the number of molecules in the gas.
4. A vessel contains two non- reactive gases : neon (monatomic) and oxygen (diatomic) The ratio of their partial pressures is 3:2. Estimate the ratio of
   (a) number of molecules and
   (b) mass density of neon and oxygen in the vessel.

   Atomic mass of Ne = 20.2 u. molecular mass of O_2 = 32.0 u.

The kinetic theory of gases

- Rudolph Claussius (1822–88) and James Clark Maxwell (1831–75) developed the kinetic theory of gases in order to explain gas laws in terms of the motion of the gas molecules. The theory is based on following assumptions regarding the motion of molecules and the nature of the gases.

Basic postulates of Kinetic theory of gases

- Every gas consists of extremely small particles known as molecules. The molecules of a given gas are all identical but are different than those of another gas.
- The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses.
- The size is negligible in comparision to inter molecular distance (10^{-9} m)

Assumptions regarding motion :

- Molecules of a gas keep on moving randomly in all possible direction with all possible velocities.
- The speed of gas molecules lie between zero and infinity (very high speed).
- The number of molecules moving with most probable speed is maximum.

Assumptions regarding collision:

- The gas molecules keep colliding among themselves as well as with the walls of containing vessel. These collision are perfectly elastic. (ie., the total energy before collision = total energy after the collision.)

Assumptions regarding force:

- No attractive or repulsive force acts between gas molecules.
- Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.

Assumptions regarding pressure:

- Molecules constantly collide with the walls of container due to which their momentum changes. This change in momentum is transferred to the walls of the container. Consequently pressure is exerted by gas molecules on the walls of container.

Assumptions regarding density:

- The density of gas is constant at all points of the container.

Properties/Assumptions of Ideal gas

- The molecules of a gas are in a state of continuous random motion. They move with all possible velocities in all possible directions. They obey Newton's law of motion.
- Mean momentum = 0; Mean velocity $< \vec{v} > = 0$; $< v^2 > \neq 0$ (Non zero); $< v^3 > = < v^5 > = 0$
- The average distance travelled by a molecule between two successive collisions is called as mean free path (λ_m) of the molecule.
- The time during which a collision takes place is negligible as compared to time taken by the molecule to cover the mean free path. At NTP ratio of time of collision to time of motion to cover mean free path is $10^{-8} : 1$.
- When a gas is taken into a vessel it is uniformly distributed in entire volume of vessel such that its mass density, moleculer density, motion of molecules etc. all are identical for all direction, therefore root mean velocity

  $\overline{v}_x^2 = \overline{v}_y^2 = \overline{v}_z^2$ are equal. Pressure exerted by the gas in all direction $P_x = P_y = P_z = P \rightarrow$ equal
- All those assumptions can be justified, if number of gas molecules are taken very large i.e., 10^{23} molecules/cm^3.

Different speeds of gas molecules

- **Average velocity**

Because molecules are in random motion in all possible direction with all possible velocity. Therefore, the average velocity of the gas molecules in container is zero. $< \vec{v} > = \dfrac{\vec{v}_1 + \vec{v}_2 + \ldots\ldots\ldots\ldots \vec{v}_N}{N} = 0$

RMS speed of molecules $v_{rms} = \sqrt{\dfrac{3P}{\rho}} = \sqrt{\dfrac{3RT}{M_w}} = \sqrt{\dfrac{3kT}{m}} = 1.73\sqrt{\dfrac{kT}{m}}$

Average speed or Mean speed of molecules :

By maxwell's velocity distribution law v_M or $<|\vec{v}|> \equiv v_{mean}$

$$<|\vec{v}|> = v_{mean} = \frac{|\vec{v}_1| + |\vec{v}_2| + \ldots\ldots\ldots |\vec{v}_n|}{N} = \sqrt{\frac{8P}{\pi\rho}} = \sqrt{\frac{8RT}{\pi M_w}} = \sqrt{\frac{8kT}{\pi m}} = 1.59\sqrt{\frac{kT}{m}}$$

Most probable speed of molecules (v_{mp})

At a given temperature, the speed to which maximum number of molecules belongs is called as most pro

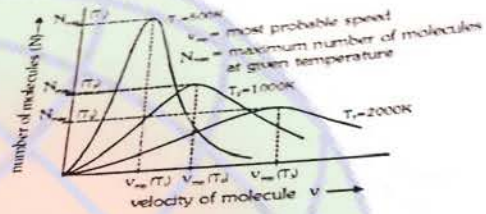
speed (v_{mp}) $v_{mp} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2RT}{M_w}} = \sqrt{\frac{2kT}{m}} = 1.41\sqrt{\frac{kT}{m}}$

Velocity of sound in gas medium (v_s) $v_{sound} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M_w}} = \sqrt{\frac{\gamma kT}{m}}$

- At any temperature $v_{rms} > v_{Mean} > v_{mp} > v_{sound}$ (always)
- For a gas at any temperature (T) $\frac{v_{rms}}{v_{sound}} = \sqrt{\frac{3}{\gamma}}$, $\frac{v_{rms}}{v_{mp}} = \sqrt{\frac{3}{2}}$
- A temperature is not possible at which above order can be changed

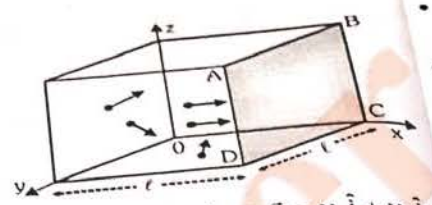
$v_{rms} \neq v_{Mean} \neq v_{mp} \neq v_{sound}$ (always)

Maxwell's law of distribution of velocities



$T_3 > T_2 > T_1$

Expression for Pressure of An Ideal gas



Consider an ideal gas enclosed in a cubical vessel of length ℓ.

Suppose there are 'N' molecules in a gas which are moving with velocities $\vec{v}_1, \vec{v}_2 ..... \vec{v}_N$.

If we consider any single molecule then its instantaneous velocity $\vec{v}$ can be expressed as $\vec{v} = v_x\hat{i} + v_y\hat{j} + v$

Due to random motion of the molecule $v_x = v_y = v_z$; $|\vec{v}| = v_x\sqrt{3} = v_y\sqrt{3} = v_z\sqrt{3} = \sqrt{v_x^2 + v_y^2 + v_z^2}$

Suppose a molecule of mass m is moving with a velocity v_x towards the face ABCD. It strikes the fa the cubical vessel and returns back to strike the opposite face.

Change in momentum of the molecule per collision $\Delta p = -mv_x - mv_x = -2mv_x$

Momentum transferred to the wall of the vessel per molecule per collision $\Delta p = 2mv_x$

The distance travelled by the molecule in going to face ABCD and coming back is 2ℓ.

So, the time between two successive collision is $\Delta t = \frac{2\ell}{v_x}$

Number of collision per sec per molecule is $f_c = \frac{v_x}{2\ell} = \frac{\text{molecule velocity}}{\text{mean free path}}$; $f_c = \frac{v_{rms}}{\lambda_m}$ or $f_c = \frac{v_m}{\lambda_m}$

Hence momentum transferred to the wall per second by the molecule is equal to force applied

therefore, force $F = (2mv_x)\frac{v_x}{2\ell} = \frac{mv_x^2}{\ell} = \frac{mv^2}{3\ell}$ $[\text{As } V = \sqrt{3}V_x]$

Pressure exerted by gas molecule $P = \frac{F}{A} = \frac{1}{3}\frac{mv^2}{l \times A}$ $\Rightarrow P = \frac{1}{3}\frac{mv^2}{V}$ $[\because A \times l = V]$

pressure exerted by gas $P = \sum \frac{1}{3}\frac{mv^2}{V} = \sum \frac{1}{3}\frac{mv^2}{V} \times \frac{N}{N} = \frac{1}{3}\frac{mN}{V}\frac{\sum v^2}{N} = \frac{1}{3}\frac{mN}{V}v^2_{rms}$

$$v^2_{rms} = \frac{3PV}{M} = \frac{3\mu RT}{\mu M_w} \Rightarrow v_{rms} = \sqrt{\frac{3RT}{M_w}}, \; P = \frac{1}{3}\frac{M}{V}v^2_{rms} = \frac{1}{3}\rho v^2_{rms}$$

- Average number of molecules for each wall $= \frac{N}{6}$.

No. of molecules along each axis $= \frac{N}{3}$ $(N_x = N_y = N_z)$

- $\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2 = \frac{v^2_{rms}}{3}$ Root mean square velocity along any axis for gas molecule is $(v_{rms})_x = (v_{rms})_y = (v_{rms})_z = \frac{v_{rms}}{\sqrt{3}}$

All gas laws and gas equation can be obtained by expression of pressure of gas (except Joule's law)

Degree of freedom (f)

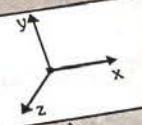
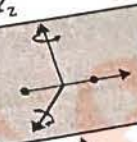
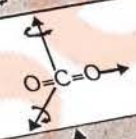
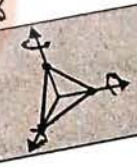
- The number of independent ways in which a molecule or an atom can exhibit motion or have energy is called its degrees of freedom.
- The number of independent coordinates required to specify the dynamic state of a system is called its degrees of freedom.
- The degrees of freedom are of three types :

(a) **Translational Degree of freedom :** There are maximum three degree of freedom corresponding to translational motion.

(b) **Rotational Degree of freedom** : The number of degrees of freedom in this case depends on the structure of the molecule.

(c) **Vibrational Degree of freedom :** It is exhibited at high temperature.

Degrees of freedom for different gases according to atomicity of gas at low temperature

Atomicity of gas	Translational	Rotational	Total	
Monoatomic **Ex.** Ar, Ne, Ideal gas etc	3	0	3	
Diatomic **Ex.** O_2, Cl_2, N_2 etc.	3	2	5	
Triatomic (linear) **Ex.** CO_2, C_2H_2	3	2	5	
Triatomic (Non-linear) or Polyatomic **Ex.** H_2O, NH_3, CH_4	3	3	6	

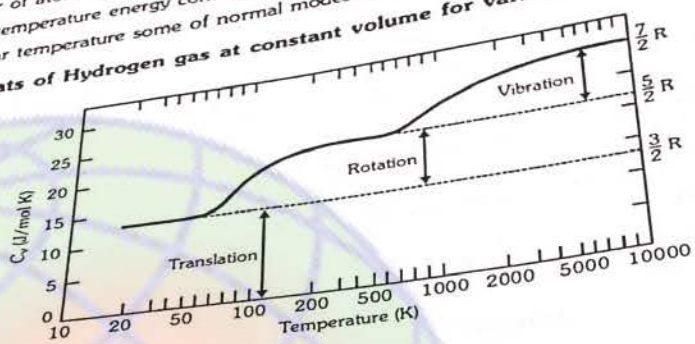
At high temperature a diatomic molecule has 7 degrees of freedom. (3 translational, 2 rotational and 2 vibrational)

Vibrational degree of freedom

For each normal mode of vibration there will be two degrees of freedom.

- For linear molecule there are (3N – 5) normal modes of vibration.
- For non linear molecule there are (3N – 6) normal modes of vibration.
- N = Number of atoms in the molecule
- At very low temperature energy contribution to vibration is negligible.
- At a particular temperature some of normal modes of vibration are active and rest are frozen.

Molar specific heats of Hydrogen gas at constant volume for various temperatures



Note : Hydrogen liquifies at 20 K.

Maxwell's law of equipartition of energy

The total kinetic energy of a gas molecules is equally distributed among its all degree of freedom and energy associated with each degree of freedom at absolute temperature T is $\frac{1}{2}kT$

For one molecule of gas

Energy related with each degree of freedom = $\frac{1}{2}kT$

Energy related with all degree of freedom = $\frac{f}{2}kT$ $\quad \because \overline{v}_x^2 = \overline{v}_y^2 = \overline{v}_z^2 = \frac{v_{rms}^2}{3} \Rightarrow \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

So energy related with one degree of freedom = $\frac{1}{2}m\frac{v_{rms}^2}{3} = \frac{3}{2}\frac{kT}{3} = \frac{1}{2}kT$

Different K.E. of gas (Internal Energy)

- **Translational kinetic energy (E_T)** $\quad E_T = \frac{1}{2}Mv_{rms}^2 = \frac{3}{2}PV$

Kinetic energy of volume V is = $\frac{1}{2}Mv_{rms}^2$ [Note : Total internal energy of ideal gas is total kinetic ene

Energy per unit volume or energy density (E_V) $\quad E_V = \frac{\text{Total energy}}{\text{Volume}} = \frac{E}{V}$; $E_V = \frac{1}{2}\left[\frac{M}{V}\right]v_{rms}^2 = \frac{1}{}$

$\because P = \frac{2}{3}\left[\frac{1}{2}\rho v_{rms}^2\right] \therefore E_V = \frac{3}{2}P$

- **Molar K.E. or Mean Molar K.E. (E)** $E = \frac{1}{2}M_w v^2_{rms}$ for N_0 molecules or M_w (gram) :

$$E = \frac{3}{2}RT = \frac{3}{2}N_0 kT$$

- **Molecular kinetic energy or mean molecular K.E. ($\bar{E}$)** $E = \frac{1}{2}M_w v^2_{rms}$, $\bar{E} = \frac{E}{N_0} = \frac{3RT}{2N_0} = \frac{3}{2}kT$

- **Kinetic energy of 1 g mass (E_m)** $= \frac{3}{2}\frac{RT}{M_w} = \frac{3}{2}\frac{N_0 \times KT}{N_0 m} = \frac{3}{2}\frac{KT}{m}$

SPECIFIC HEAT CAPACITY

Monoatomic Gases

The molecule of a monoatomic gas has only three translational degrees of freedom. Thus, the average energy of a molecule at temperature T is (3/2)kT.

The total internal energy of a mole of such a gas is

$$U = \frac{3}{2}kT \times N_A = \frac{3}{2}RT$$

The molar specific heat at constant volume, C_v, is

$$C_v \text{ (monoatomic gas)} = \frac{dU}{dT} = \frac{3}{2}R$$

Diatomic Gases

As explained earlier, a diatomic molecule treated as a rigid rotator like a dumbbell has 5 degrees of freedom : 3 translational and 2 rotational. Using the law of equipartition of energy, the total internal energy of a mole of such a gas is

$$U = \frac{5}{2}kT \times N_A = \frac{5}{2}RT$$

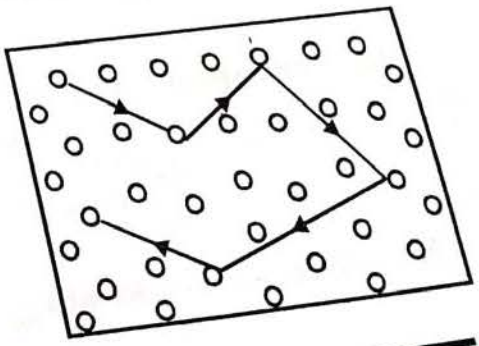
The molar specific heats are then given by

$$C_v \text{ (rigid diatomic)} = \frac{5}{2}R$$

MEAN FREE PATH

A molecule in its path undergoes a number of collisions so the path traversed by it is not a straight line but somewhat zigzag. Between two successive collisions a molecule travels in a straight line with uniform velocity. As motion is random thus the distance travelled by molecule between two succesive collisions is not always equal.

The average distance travelled by a molecule between two succesive collisions is called as mean free path (λ_m) of the molecule.



$$\boxed{\lambda_m = \frac{1}{\sqrt{2}\pi d^2 n}}$$

Here $n = \frac{N}{V}$ = No. of molecules per unit volume

d = diameter of a molecule

GOLDEN KEY POINTS

- At a given temperature graph drawn between molecular velocity and number of molecules is known as ve distribution curve.
- The velocities of molecules of a gas are in between zero and infinity $(0 - \infty)$
- With the increase in the temperature, the most probable velocity and maximum molecule velocity increases.
- The number of molecules within certain velocity range is constant although the velocity of molecule cha continuously at particular temperature.
- The area enclosed between the $(N - v)$ curve and the velocity axis represents the total number of molec
- On the basis of velocity distribution Maxwell established the law of equipartition of energy for gases at temperature.
  - Except 0 K, at any temperature T, $E > E_m > \bar{E}$
  - At a common temperature, for all ideal gas

    E and $\bar{E}$ are same while E_m is different and depends upon nature of gas (M_w or m)
- For thermal equilibrium of gases, temperature of each gas is same and this temperature called as tempera of mixture (T_m) which can be find out on basis of conservation of energy (All gases are of same atomic

$$T_m = \frac{\sum NT}{\sum N} = \frac{N_1T_1 + N_2T_2 + \ldots\ldots\ldots + N_nT_n}{N_1 + N_2 \ldots\ldots N_n}$$

- **1 mole gas :** Mean translational kinetic energy $= \frac{3}{2}RT$; Total kinetic energy $= \frac{f}{2}RT$

  1 molecule of gases : Mean kinetic energy $= \frac{3}{2}kT$; Total kinetic energy $= \frac{f}{2}kT$; $f \rightarrow$ Degree freedom
- Specific heats of all substances approach zero as $T \rightarrow 0$. This is related to the fact that degrees of freed get frozen and ineffective at low temperatures.

Illustrations

Illustration 48.

The velocities of ten particles in ms^{-1} are 0, 2, 3, 4, 4, 4, 5, 5, 6, 9. Calculate

(i) average speed and (ii) rms speed (iii) most probable speed.

lution

(i) average speed, $v_{av} = \frac{0+2+3+4+4+4+5+5+6+9}{10}$

$$= \frac{42}{10} = 4{\cdot}2\ ms^{-1}$$

rms speed, $v_{rms} = \left[\frac{(0)^2+(2)^2+(3)^2+(4)^2+(4)^2+(4)^2+(5)^2+(5)^2+(6)^2+(9)^2}{10}\right]^{1/2}$

$$= \left[\frac{228}{10}\right]^{1/2} = 4.77\ ms^{-1}$$

most probable speed $v_{mp} = 4$ m/s

Illustration 49.

At what temperature root mean square velocity of hydrogen becomes double of its value at S.T.P., pressure remaining constant ?

Solution

Let v_1 be the r.m.s. velocity at S.T.P. and v_2 be the r.m.s. velocity at unknown temperature T_2.

$$\therefore \quad \frac{v_1^2}{v_2^2} = \frac{T_1}{T_2}$$

$$\text{or} \quad T_2 = T_1\left[\frac{v_2}{v_1}\right]^2 = 273 \times (2)^2 = 273 \times 4 = 1092\ K = (1092 - 273) = 819°C$$

Illustration 50.

Calculate rms velocity of oxygen molecule at 27°C

Solution

Temperature, T = 27° C

$\Rightarrow$ 273 + 27 = 300 K,

Molecular weight of oxygen = 32×10^{-3} kg and $R = 8.31\ J\ mol^{-1}\ K^{-1}$

rms velocity is $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$= \sqrt{\frac{3 \times 8.31 \times 300}{32 \times 10^{-3}}} = 483.5\ ms^{-1}$$

Illustration 51.

Calculate the kinetic energy of a gram moelcule of argon at 127°C.

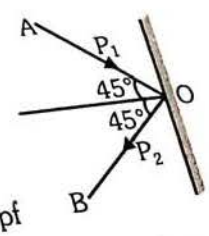
Solution

Temperature, T = 127°C = 273 + 127 = 400 K, R = 8.31 J/mol K

K.E. per gram molecule of argon = $\frac{3}{2}RT = \frac{3}{2} \times 8.31 \times 400 = 4986$ J

Illustration 52.

The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} molecules are colliding per second on a stationary wall of area 2 cm^2 at an angle of 45° to the normal to the wall and reflected elastically with a speed 10^3 m/s. Find the pressure exerted on the wall. (in N/m^2)



Solution

As the impact is elastic $\therefore |\vec{p}_1| = |\vec{p}_2| = p = mv = 3.32 \times 10^{-24}$ kg m/s

The change in momentum along the normal $\Delta p = |\vec{p}_2 - \vec{p}_1| = 2p\cos 45° = \sqrt{2}p$

If f is the collision frequency then force applied on the wall $F = \frac{\Delta p}{\Delta t} = \Delta p \times f = \sqrt{2}pf$

$$\therefore \quad \text{Pressure } P = \frac{F}{A} = \frac{\sqrt{2}pf}{A} = \frac{\sqrt{2} \times 3.32 \times 10^{-24} \times 10^{23}}{2 \times 10^{-4}} = 2.347 \times 10^3\ N/m^2$$

BEGINNER'S BOX-6

1. Given : Avogadro number $N = 6.02 \times 10^{23}$ molecules/mole and Boltzmann's consta $k = 1.38 \times 10^{-23}$ J/(molecule-K).

   Calculate :

   (a) The average kinetic energy of translation of an oxygen molecule at 27°C

   (b) The total kinetic energy of an oxygen molecule at 27°C

   (c) The total kinetic energy of 1 mole of oxygen gas at 27°C.

2. The temperature of gas is –73°C. To what temperature should it be heated so that

   (a) average kinetic energy of the molecule is doubled ?

   (b) the root mean square velocity of the molecules is doubled ?

3. For a gas $\frac{R}{C_P} = 0.4$. For this gas calculate the following –

   (a) atomicity and degree of freedom

   (b) value of C_v and γ

   (c) mean gram - molecular kinetic energy at 300 K temperature

4. If three molecules are having speeds v_1, v_2 and v_3 respectively, then what will be their average speed and mean square speed ?

5. Four molecules of a gas are having speeds of 1, 4, 8 and 16 ms^{-1}. Find the root mean square veloci the gas molecules ?

6. A flask contains argon and chlorine in the ratio of 2 : 1 by mass. The temperature of the mixture is 2 Obtain the ratio of

   (a) Average translational kinetic energy per molecule, and

   (b) root mean square speed (v_{rms}) of the molecules of the two gases.

Atomic mass of argon = 39.9 u :

Molecular mass of chlorine = 70.9 u.