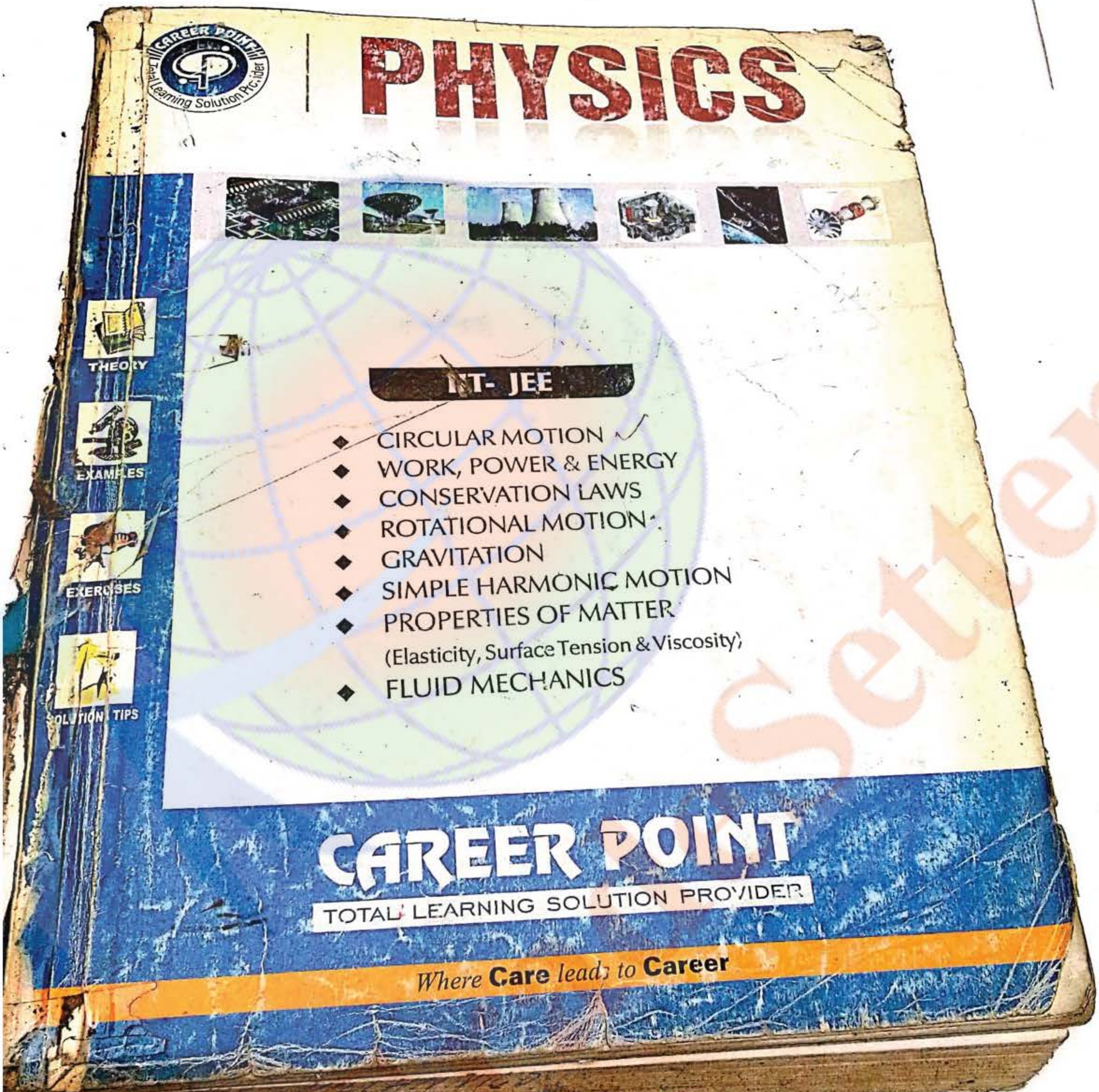
PHYSICS
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IIT- JEE
CIRCULAR MOTION
WORK, POWER & ENERGY
CONSERVATION LAWS
ROTATIONAL MOTION
GRAVITATION
SIMPLE HARMONIC MOTION
PROPERTIES OF MATTER
(Elasticity, Surface Tension & Viscosity)
FLUID MECHANICS
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Where Care leads to Career

प्रार्थना

इतनी शक्ति हमें देना दाता, मन का विश्वास कमजोर हो ना।
हम-चलें नेक रस्ते पे हमसे, भूलकर भी कोई भूल हो ना।

दूर अज्ञान के हों अंधेरे, तू हमें ज्ञान की रोशनी दे।
हर बुराई से बचके रहें हम, जितनी भी दे भली जिंदगी दे।

बैर हो ना किसी का किसी से, भावना मन में बदले की हो ना।
हम चलें नेक रस्ते पे हमसे, भूलकर भी कोई भूल हो ना।
इतनी शक्ति हमें देना दाता, मन का विश्वास कमजोर हो ना।

हम न सोचें हमें क्या मिला है, हम ये सोचें किया क्या है अर्पण।
फूल खुशियों के बाटें सभी को, सबका जीवन भी बन जाये मधुबन।
अपनी करुणा का जल तू बहाके, कर दे पावन हर एक मन का कोना।
हम चलें नेक रस्ते पे हमसे, भूलकर भी कोई भूल हो ना।

इतनी शक्ति हमें देना दाता, मन का विश्वास कमजोर हो ना।
हम-चलें नेक रस्ते पे हमसे, भूलकर भी कोई भूल हो ना।

CONTENTS

Krishna Kanhaiya

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FLUID MECHANICS

— CIRCULAR MOTION —

IIT-JEE Syllabus

1. *Kinematics in two dimension*
2. *Circular motion*

Total No. of questions in **Circular motion** are:

....15

Solved examples..20

Exercise # 1 29

Exercise # 2 31

Exercise # 3 07

Exercise # 4 12

Exercise # 5 114

Total No. of questions

*** Students are advised to solve the questions of exercises in the same sequence or as directed by the faculty members.

Index : Preparing your own list of Important/Difficult Questions

Instruction to fill

(A) Write down the Question Number you are unable to solve in **column A** below, by Pen.

(B) After discussing the Questions written in **column A** with faculties, strike off them in the manner so that you can see at the time of Revision also, to solve these questions again.

(C) Write down the Question Number you feel are important or good in the **column B**.

EXERCISE NO.	COLUMN :A Questions I am unable to solve in first attempt	COLUMN :B Good/Important questions
1		
2		
3		
4		
5		

Advantages

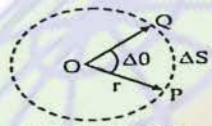
1. It is advised to the students that they should prepare a question bank for the revision as it very difficult to solve all the questions at the time of revision.
2. Using above index you can prepare and maintain the questions for your revision.

KEY CONCEPT

1. Rotational kinematics

1.1 Angular Displacement

Introduction : Angle substended by the position vector of a particle moving along any arbitrary path w.r.t. some fixed point is called angular displacement.



Particle moving in circular path

$$\text{angle} = \frac{\text{arc}}{\text{radius}} = \frac{\text{linear displacement}}{\text{radius}}$$

Note : $1 \text{ radian} = \frac{360°}{2\pi} \Rightarrow \pi \text{ radian} = 180°$

If a body makes n revolutions, its angular displacement $\theta = 2n\pi$ radians

1.2 Angular Velocity

It is defined as the rate of change of angular displacement of a body or particle moving in circular path.

Its direction is same as that of angular displacement

i.e. perpendicular to plane of rotation

Note : If the particle is revolving in the clockwise direction then the direction of angular velocity is perpendicular to the plane downwards. Whereas in case of anticlockwise direction the direction will be upwards.

(i) Average Angular Velocity :

$$\omega_{av} = \frac{\text{Total angular displacement}}{\text{Total time taken}}$$

(ii) Instantaneous Angular velocity :

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

2. Relation between linear velocity and angular velocity

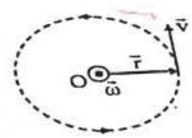
$$v = r\omega$$

$v = \frac{ds}{dt}$ = linear velocity and $\omega = \frac{d\theta}{dt}$

In vector form, $\vec{v} = \vec{\omega} \times \vec{r}$

Note :

(i)



⊙ → Outward normal to plane of paper. (direction of $\vec{\omega}$)

(ii) When a particle moves along a curved path, its linear velocity at a point is along the tangent drawn at that point

(iii) When a particle moves along curved path, its velocity has two components. One along the radius, which increases or decreases the radius and another one perpendicular to the radius, which makes the particle to revolve about the point of observation.

3. Angular acceleration

(i) The rate of change of angular velocity is defined as angular acceleration.

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

(ii) Its direction is that of change in angular velocity

4. Equations related to motion of a particle with constant angular acceleration

$$\omega = \omega_0 + \alpha t$$

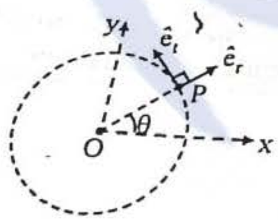
$$\omega = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\,\theta$$

5. Equation of linear motion and rotational motion

S.N.		Linear Motion	Rotational Motion
			$\alpha = 0,\ \theta = \omega t$
(i)	With constant velocity	$a = 0,\ s = ut$	(i) Average angular velocity $\omega_{av} = \frac{\omega_1 + \omega_2}{2}$
(ii)	With constant acceleration	(i) Average velocity $v_{av} = \frac{v+u}{2}$ (ii) Average acceleration $a_{av} = \frac{v-u}{t}$ (iii) $s = v_{av}\, t = \frac{v+u}{2}\, t$ (iv) $v = u + at$ (v) $s = ut + \frac{1}{2} at^2$ (vi) $s = ut - \frac{1}{2} at^2$ (vii) $v^2 = u^2 + 2as$ (viii) $S_n = u + \frac{1}{2}(2n-1)a$ displacement in nth sec.	(ii) Average angular acceleration $\alpha_{av} = \frac{\omega_2 - \omega_1}{t}$ (iii) $\theta = \omega_{av}.\, t = \frac{\omega_1 + \omega_2}{2}\, t$ (iv) $\omega = \omega_0 + \alpha t$ (v) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ (vi) $\theta = \omega_0 t - \frac{1}{2}\alpha t^2$ (vi) $\omega^2 = \omega_0^2 + 2\alpha\theta$ (viii) $\theta_n = \omega_0 + \frac{1}{2}(2n-1)\alpha$ Angular displacement in nth sec
(iii)	With variable acceleration	(i) $v = \frac{ds}{dt}$ (ii) $\int ds = \int v dt$ (iii) $a = \frac{dv}{dt} = v\frac{dv}{ds}$ (iv) $\int dv = \int a\, dt$; if $a = f(t)$ (v) $\int v dv = \int a\, ds$; if $a = f(s)$	(i) $\omega = d\theta/dt$ (ii) $\int d\theta = \int \omega dt$ (iii) $\alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$ (iv) $\int d\omega = \int \alpha\, dt$; if $\alpha = f(t)$ (v) $\int \omega\, d\omega = \int \alpha\, d\theta$; if $\alpha = f(\theta)$

6. Acceleration in circular motion



$\hat{e}_r$: unit vector along the outward radius

$\hat{e}_t$: unit vector along the tangent in the direction of increasing θ.

$\hat{e}_r = \hat{i}(\cos\theta) + \hat{j}(\sin\theta)$ and

$\hat{e}_t = -\hat{i}(\sin\theta) + \hat{j}(\cos\theta)$

where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axes respectively.

velocity of particle

$\vec{v} = \frac{d\vec{r}}{dt}$, where $\vec{r} = r(\hat{i}\cos\theta + \hat{j}\sin\theta)$

r is the radius of circle

acceleration of particle :

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = (-\omega^2 r)\hat{e}_r + \left(\frac{dv}{dt}\right)\hat{e}_t$$

$$\vec{a} = \vec{a}_c + \vec{a}_t$$

where $\vec{a}_c$ (centripetal acceleration) $= \omega^2 r(\hat{e}_r)$

and $\vec{a}_t$ (tangential acceleration) $= \frac{dv}{dt}(\hat{e}_t)$

Tangential Acceleration ($\vec{a}_t$)

It is defined as the rate of change of speed.

$$|\vec{a}_t| = \frac{d|\vec{v}|}{dt}$$

Its direction is along the tangent to the path.

- $\vec{a}_t = \vec{\alpha} \times \vec{r}$

  where $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$

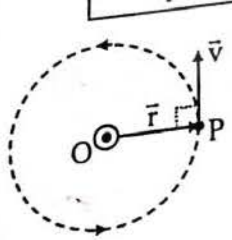
7. Centripetal acceleration and centripetal force

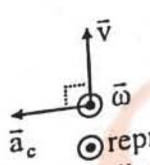
(i) The velocity of the particle changes while moving on the circular path, this change in velocity is brought by a force known as centripetal force and the acceleration so produced in the body is known as centripetal acceleration.

(ii) The direction of centripetal force or acceleration is always towards the centre of circular path.

(iii) Expression for centripetal acceleration:

$$\Rightarrow a_c = \frac{v^2}{r} = r\omega^2$$



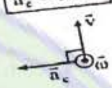


⊙ represents direction of $\vec{\omega}$

This is the magnitude of centripetal acceleration of particle. It is a vector quantity.

In vector form

$$\vec{a}_c = \vec{\omega} \times \vec{v}$$



(iv) The direction of $\vec{a}_c$ would be the same as that of $\Delta\vec{v}$ (change in velocity vector)

(v) Expression for Centripetal force :

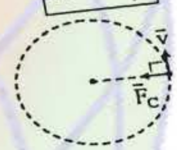
If v = velocity of particle ,

r = radius of curvature of path

Then necessary centripetal force

F_c = mass × acceleration

$$F_c = m\frac{v^2}{r}$$

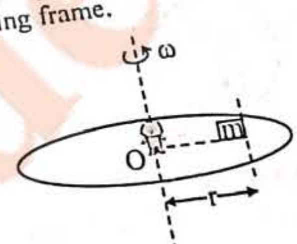


Note :

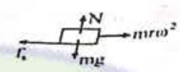
- Centripetal force is not a real force. It is only the requirement for circular motion.
- It is not a new kind of force. Any of the forces found in nature such as gravitational force, electrostatic force friction force, tension in string, reaction force etc may act as centripetal force.

8. Centrifugal force

It is a sufficient pseudo force, only if we are anlaysing the particles at rest in a uniformly rotating frame.



In the given figure, the block of mass 'm' is at rest with respect to the rotating platform (as observed by the observer O on the rotating platform).



centrifugal force = m rω²

centrifugal force acts (or is assumed to act) because we describe the particle from a rotating frame which is non-inertial and still use Newton's laws.

9. Type of circular motion

9.1 Uniform circular motion

9.2 Non Uniform Circular Motion :

9.1 Uniform Circular Motion :

If m = mass of body ,

r = radius of circular orbit,

v = magnitude of velocity

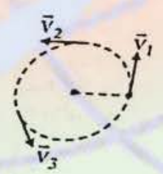
a_c = centripetal acceleration,

a_t = tangential acceleration

In uniform circular motion :

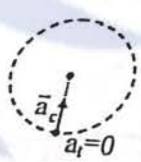
(i) $|\vec{v}_1| = |\vec{v}_2| = |\vec{v}_3|$ = constant

i.e. speed is constant

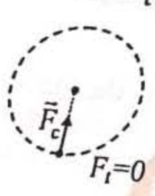


(ii) As $|\vec{v}|$ is constt.

so tangential acceleration $a_t = 0$



(iii) Tangential force $F_t = 0$



(iv) Total acceleration $a = \sqrt{a_c^2 + a_t^2} = a_c$ (towards the centre)

(v) In uniform circular motion $F_t = 0$, as $a_t = 0$, work done will be zero by tangential force.

But in non-uniform circular motion $F_t \neq$ thus there will be a work done by tangential force in this case.

Rate of work done by net force in non-uniform circular motion = rate of work done by tangential force

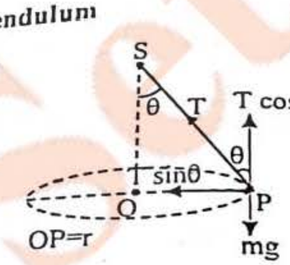
$$\Rightarrow P = \frac{dW}{dt} = \vec{F}_t \cdot \vec{v}$$

Note:

- Because F_c is always perpendicular velocity or displacement, hence the work done by this force will always be zero.
- There is an important difference between projectile motion and circular motion.

In projectile motion, both the magnitude and the direction of acceleration (g) remain constant, while in circular motion the magnitude remains constant but the direction continuously changes.

9.1.1 Motion In Horizontal Circle : Conical pendulum



$$\therefore T\sin\theta = \frac{mv^2}{r} \quad \text{and } T\cos\theta = mg$$

From these equation

$$T = mg\sqrt{1 + \frac{v^4}{r^2g^2}} \quad \text{....(i)}$$

$$\text{and } \tan\theta = \frac{v^2}{rg} \quad \text{....(ii)}$$

9.2 Non-uniform Circular Motion :

(i) In non-uniform circular motion :

$|\vec{v}| \neq$ constant $\Rightarrow \omega \neq$ constant

i.e. speed ≠ constant

i.e. angular velocity ≠ constant

(ii) Tangential acceleration : $a_t = \frac{dv}{dt}$,

where $v = \frac{ds}{dt}$ and s = arc length

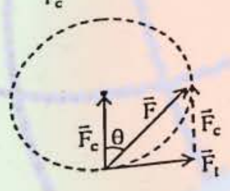
(iii) Tangential force : $F_t = ma_t$

(iv) Centripetal force : $F_c = \frac{mv^2}{r} = m\omega^2 r$

(v) Net force on the particle :

$\vec{F} = \vec{F_c} + \vec{F_t} \Rightarrow F = \sqrt{F_c^2 + F_t^2}$

$\tan\theta = \frac{F_t}{F_c}$

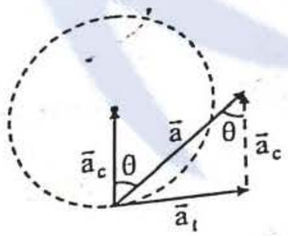


(vi) Net acceleration,

$$a = \sqrt{a_c^2 + a_t^2} = \frac{F_{net}}{m}$$

The angle made by 'a' with a_c,

$$\tan\theta = \frac{a_t}{a_c} = \frac{F_t}{F_c}$$



9.2.1 Motion in Vertical Circle : Motion of a body suspended by string

When the body rises from the bottom to the height h, a part of its kinetic energy converts into potential energy

(a) **Velocity at a point P :**

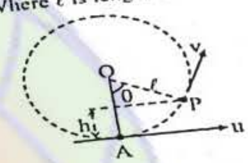
∵ Total mechanical energy remains conserved at point A and point P.

$\Rightarrow 0 + \frac{1}{2}mu^2 = mgh + \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{u^2 - 2gh} = \sqrt{u^2 - 2g\ell(1-\cos\theta)}$;

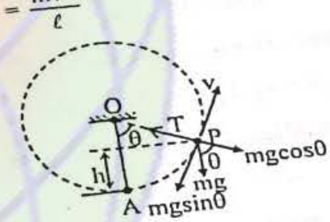
as $h = \ell - \ell\cos\theta$

[Where ℓ is length of the string]



(b) **Tension at a point P :**

At point P required centripetal force $= \frac{mv^2}{\ell}$



Net force towards the centre = T − mg cosθ

This net force provides required centripetal force.

$$\therefore \quad T - mg\cos\theta = \frac{mv^2}{\ell}$$

$$T = m\left[g\cos\theta + \frac{v^2}{\ell}\right]$$

$$T = \frac{m}{\ell}\left[u^2 - g\ell(2 - 3\cos\theta)\right] \quad ...(1)$$

(c) **Tangential force for the motion**

$F_t = mg\sin\theta$

This force retards the motion.

(d) **Different cases :**

(i) If $u > \sqrt{5g\ell}$

In this case tension in the string will not be zero at any of the point, which implies that the particle will continue the circular motion.

(ii) Condition of looping the loop : $u = \sqrt{5g\ell}$

In this case the tension at the top most point (B) will be zero, which implies that the particle will just complete the circular motion.

Critical Velocity : The minimum velocity at which the circular motion is possible

The critical velocity at A = $\sqrt{5g\ell}$

The critical velocity at B = $\sqrt{g\ell}$

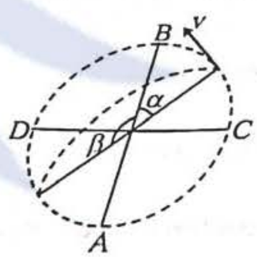
The critical velocity at C = $\sqrt{3g\ell}$

Also $T_A = 6mg$, $T_B = 0$, $T_C = 3mg$

(iii) Condition of leaving the circular path

$\sqrt{2g\ell} < u < \sqrt{5g\ell}$

In this case particle will not follow circular motion. Tension in string becomes zero somewhere between points C & B whereas velocity remain positive. Particle leaves circular path and follow parabolic trajectory



(iv) Condition of oscillation :

$0 < u \le \sqrt{2g\ell}$

(a) If $u = \sqrt{2g\ell}$

In this case both velocity and tension in the string becomes zero at point C and D and the particle will oscillate along semi circular path.

(b) If $u < \sqrt{2g\ell}$

The velocity of particle will become zero between A and C but tension will not be zero and the particle will oscillate about the point A.

10. Dynamics of circular motion in horizontal plane

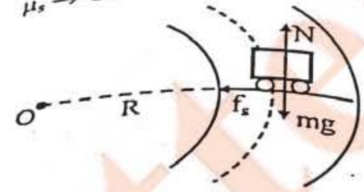
- Maximum speed of vehicle for safe turning on rough horizontal circular turn

$$v_{max} = \sqrt{\mu_s Rg}$$

where

$R \rightarrow$ radius of curvature of circular turn

$\mu_s \rightarrow$ Coefficient of static friction

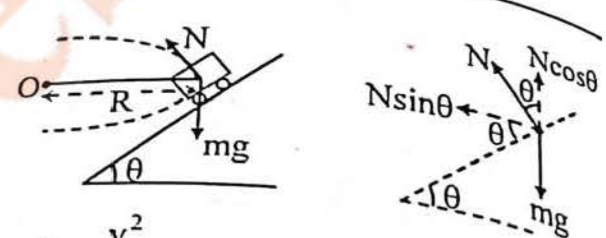


$f_s \le \mu_s mg$

$$f_s = \frac{mv^2}{R}$$

- Static friction provides the required centripetal force.

11. Banking of road



$$\tan\theta = \frac{v^2}{Rg}$$

$$v = \sqrt{Rg\tan\theta}$$

$v \rightarrow$ safe speed on banked road.

d tension in
int C and ti
along semi

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will not b
late abou

rning on

turn

SOLVED EXAMPLES

Ex.1 The magnitude of the linear acceleration of the particle moving in a circle of radius of 10cm with uniform speed completing the circle in 4s, will be -

Sol. The distance covered in completing the circle is $2\pi r = 2\pi \times 10$cm

The linear speed is

$$v = \frac{2\pi r}{t} = \frac{2\pi \times 10}{4} = 5\pi \text{ cm/s}$$

The linear acceleration is,

$$a = \frac{v^2}{r} = \frac{(5\pi)^2}{10} = 2.5\pi^2 \text{ cm/s}^2$$

This acceleration is directed towards the centre of the circle

Ex.2 The length of second's hand in a watch is 1cm. The change in velocity of its tip in 15 seconds is -

Sol. $$\text{Velocity} = \frac{\text{Circumference}}{\text{Time of revolution}}$$

$$= \frac{2\pi r}{60} = \frac{2\pi \times 1}{60} = \frac{\pi}{30} \text{ cm/s}$$

Change in velocity

$$\Delta v = \sqrt{\left(\frac{\pi}{30}\right)^2 + \left(\frac{\pi}{30}\right)^2}$$

$$= \frac{\pi}{30}\sqrt{2} \text{ cm/s}$$

Ex.3 An electron is moving in a circular orbit of radius 5.3×10^{-11} metre around the atomic nucleus at a rate of 6.6×10^{15} revolutions per second. The acceleration of the electron and centripetal force acting on it. will be - (The mass of the electron is 9.1×10^{-31}kg.)

Sol. Let the radius of the orbit be r and the number of revolutions per second be n. Then the velocity of electron is given by

$v = 2\pi nr$,

$$\therefore \text{acceleration } a = \frac{v^2}{r} = \frac{4\pi^2 r^2 n^2}{r}$$

$$= 4\pi^2 r n^2$$

Substituting the given values, we have

$a = 4 \times (3.14)^2 \times (5.3 \times 10^{-11})(6.6 \times 10^{15})^2$

$= 9.1 \times 10^{22}$ m/s² towards the nucleus.

The centripetal force is

$F_C = ma = (9.1 \times 10^{-31})(9.1 \times 10^{22})$

$= 8.3 \times 10^{-8}$ N towards the nucleus.

Ex.4 An air craft executes a horizontal loop of radius 1km with a steady speed of 900km/h. The ratio of centripetal acceleration to that gravitational acceleration will be-

Sol. Given that radius of horizontal loop

r = 1 km = 1000 m

$$\text{Speed } v = 900 \text{ km/h} = \frac{900 \times 5}{18} = 250 \text{ m/s}$$

Centripetal acceleration

$$a_c = \frac{v^2}{r} = \frac{250 \times 250}{1000} = 62.5 \text{ m/s}^2$$

$$\therefore \frac{\text{Centripetal acceleration}}{\text{gravitational acceleration}} = \frac{a_c}{g} = \frac{62.5}{9.8}$$

$= 6.38 : 1$

Ex.5 Write an expression for the position vector r for a particle describing uniform circular motion, using rectangular coordinates and the unit vectors i and j. The vector expressions for the velocity v and acceleration a will be -

Sol. $\vec{r} = \hat{i}\,x + \hat{j}\,y, x = r\cos\theta, y = r\sin\theta$

where $\theta = \omega t$

$\vec{r} = \hat{i}\,(r\cos\omega t) + \hat{j}(r\sin\omega t)$

$v = d\vec{r}/dt = -\hat{i}\,(\omega r\sin\omega t) + \hat{j}(\omega r\cos\omega t)$

$a = d^2\vec{r}/dt^2 = -\omega^2\vec{r}$

Ex.6 The vertical section of a road over a ca[nal] bridge in the direction of its length is in form of circle of radius 8.9 metre. Fin[d the] greatest speed at which the car can cro[ss the] bridge without losing contact with the [road at] its highest point, the center of gravit[y of the] car being at a height h = 1.1 metre [from the] ground. (Take g = 10m/sec²)

Sol. Let R be the normal reaction exerted by the road on the car. At the highest point, we have

$\frac{mv^2}{(r+h)} = mg - R$, R should not be negative.

Therefore $v^2 \le (r+h)g = (8.9 + 1.1) \times 10$

or $v^2 \le 10 \times 10 \Rightarrow v \le 10$

$\therefore$ $v_{max} = 10$ m/sec

Ex.7 The angular speed with which the earth would have to rotate on it axis so that a person on the equator would weigh $(3/5)^{th}$ as much as present will be (Take the equatorial radius as 6400km)

Sol. Let v be the speed of earth's rotation.

We know that $W = mg$

Hence $\frac{3}{5}W = mg - \frac{mv^2}{r}$

or $\frac{3}{5}mg = mg - \frac{mv^2}{r}$

$\therefore$ $\frac{2}{5}mg = \frac{mv^2}{r}$ or $v^2 = \frac{2gr}{5}$

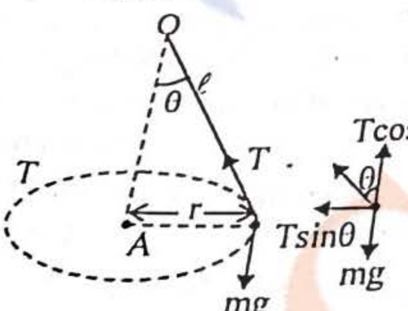
Now $v^2 = \frac{2 \times 9.8 \times (6400 \times 10^3)}{5}$

Solving, we get $v = 5 \times 10^9$ m/sec,

$\omega = \sqrt{\left(\frac{2g}{5r}\right)} = 7.8 \times 10^4$ radian/sec.

Ex.8 A man whirls a stone round his head on the end of a string 4.0metre long. Can the string be in a horizontal, plane? If the stone has a mass of 0.4kg and the string will break, if the tension in it exceeds 8N. The smallest angle the string can make with the horizontal and the speed of the stone will respectively be (Take $g = 10 m/sec^2$.)

Sol. Form figure



$T\cos\theta = mg$ ......(A)

$T\sin\theta = \frac{mv^2}{r} = \frac{mv^2}{\ell\sin\theta}$ ......(B)

Form eq. (A) $T = \frac{mg}{\cos\theta}$

When the string is horizontal, θ must be, i.e., $\cos 90° = 0$

$\therefore$ $T = \frac{mg}{0} = \infty$

Thus the tension must be infinite which impossible, so the string can not be horizontal plane.

The maximum angle θ is given by breaking tension of the string in the equation $T\cos\theta = m.g.$

Here T (Maximum) = 8N and m = 0.4 kg

$\therefore$ $8\cos\theta = 0.4 \times g = 0.4 \times 10 = 4$

$\cos\theta = (4/8) = \frac{1}{2}$, $\theta = 60°$

The angle with horizontal = 90° – 60° = 30°

From equation (B), $8\sin 60° = \frac{0.4 \times v^2}{4\sin 60°}$

$v^2 = \frac{32\sin^2 60°}{0.4} = 80\sin^2 60°$

$\Rightarrow v = \sqrt{80}\ \sin 60° = 7.7$ m/sec.

Ex.9 A smooth table is placed horizontally. A spring of unstreched length ℓ_0 and force constant k has one end fixed to its centre. The other end of the spring is attached a mass m which is making n revolutions per second around the centre. Tension in the spring will

Sol. Let T be the tension produced in the spring. The centripetal force required for mass m to move in a circle is provided by tension T. The stretched length of the spring is r (radius of the circle). Now,

Elongation produced in the spring = $(r - \ell_0)$

Tension produced in the spring,

$T = k(r - \ell_0)$

Where k is the force constant ......(A)

Linear velocity of the motion $v = 2\pi r n$

∴ Centripe

Equating e

$k(r - \ell_0) =$

$\Rightarrow kr - k$

$r(k - 4\pi^2$

$\Rightarrow r =$

Substitu have

T =

or T =

.10 A 40 lengt angu in th vert is am wi Th

$\therefore$ Centripetal force $= \frac{mv^2}{r} = \frac{m(2\pi rn)^2}{r}$

$= 4\pi^2 r n^2 m$ ....(B)

Equating equation. (A) and (B), we get

$k(r - \ell_0) = 4\pi^2 r n^2 m$ $(\therefore T = mv^2/r)$

$\Rightarrow kr - k\ell_0 = 4\pi^2 r n^2 m$

$r(k - 4\pi^2 n^2 m) = k\ell_0$

$\Rightarrow r = \frac{k\ell_0}{(k - 4\pi^2 n^2 m)}$ ....(C)

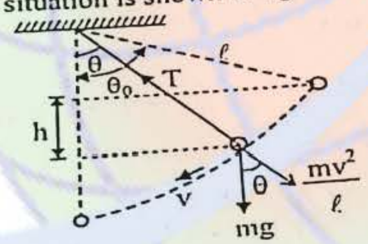
Substituting the value of r in eqn. (A) we have

$$T = k\left[\frac{k\ell_0}{(k - 4\pi^2 n^2 m)} - \ell_0\right]$$

or $T = \frac{4\pi^2 n^2 m \ell_0 k}{(k - 4\pi^2 n^2 m)}$ .... (D)

Ex.10 A 40 kg mass, hanging at the end of a rope of length ℓ, oscillates in a vertical plane with an angular amplitude of θ_0. What is the tension in the rope, when it makes an angle θ with the vertical? If the breaking strength of the rope is 80 kgf, what is the maximum angular amplitude with which the mass can oscillate without the rope breaking?

Sol. The situation is shown in fig



(a) From figure $h = \ell(\cos\theta - \cos\theta_0)$

and $v^2 = 2gh = 2g\ell(\cos\theta - \cos\theta_0)$ .... (A)

Again $T - mg\cos\theta = mv^2/\ell$ .... (B)

Substituting the value of v^2 from eq. (A) in eq. (B) we get

$T - mg\cos\theta = m\{2g\ell(\cos\theta - \cos\theta_0)/\ell\}$

or $T = mg\cos\theta + 2mg(\cos\theta - \cos\theta_0)$

or $T = 40g(3\cos\theta - 2\cos\theta_0)$ newton

or $T = 40(3\cos\theta - 2\cos\theta_0)$ kg f.

(b) Let θ_0 be the maximum amplitude. The maximum tension T will be at mean position where $\theta = 0$

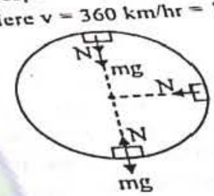
$\therefore T_{max} = 40(3 - 2\cos\theta_0)$,

But $T_{max} = 80$ kgf

Solving we get $\theta_0 = 60°$

Ex.11 An aircraft loops the loop of radius R = 500 m with a constant velocity v = 360 km/hour. The weight of the flyer of mass m = 70 kg in the lower, upper and middle points of the loop will respectively be-

Sol. See fig, Here v = 360 km/hr = 100 m/sec.



At lower point, $N - mg = \frac{mv^2}{R}$,

N = weight of the flyer $= mg + \frac{mv^2}{R}$.

$N = 70 \times 10 + \frac{70 \times (10000)}{500} = 2100N$

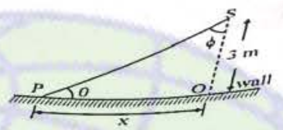
At upper point, $N + mg = \frac{mv^2}{R}$,

$N = \frac{mv^2}{R} - mg = 1400 - 700 = 700N$

At middle point, $N = \frac{mv^2}{R} = 1400N$

Ex.12 A spot light S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light P moves along, the wall at a distance 3 m. What is the velocity of the spot P when $\theta = 45°$ -

Sol. If x is the distance of point P from O then from fig.



or $\tan\phi = (x/h)$

or $x = h\tan\phi$

or $\frac{dx}{dt} = h(\sec^2\phi)\frac{d\phi}{dt}$

i.e. $v = h\sec^2\phi\,\omega$

[as $(dx/dt) = v$ and $(d\phi/dt) = \omega$]

Here $h = 3$ m, $\phi = 180 - (45 + 90) = 45°$

and $\omega = 0.1$ rad/s.

So $v = 3 \times (\sqrt{2})^2 \times 0.1 = 0.6$ m/s.

Ex.13 *A planet P revolves around the sun in a circular orbit, with the sun at the centre, which is coplanar and concentric to the circular orbit of earth E around the sun. P and E revolve in the same direction. The time required for the revolution of P and E around the sun are 3 years and 1 year respectively. What is the time required for P to make one revolution around the sun relative to E -*

Sol. As $T_P > T_E$ and $T = 2\pi/\omega$ so $\omega_P < \omega_E$ and hence with respect to sun the difference in their angular displacement per unit time will be

$(\omega_E - \omega_P)$. So they will be at same position with respect to the sun again for the first time when their relative angular displacement becomes 2π. So if T is the required time

$$(\omega_E - \omega_P)\,T = 2\pi \qquad [\text{as } \theta = \omega t]$$

or $$\left[\frac{2\pi}{T_E} - \frac{2\pi}{T_P}\right] = \frac{2\pi}{T}\left[\text{as } T = \frac{2\pi}{\omega}\right]$$

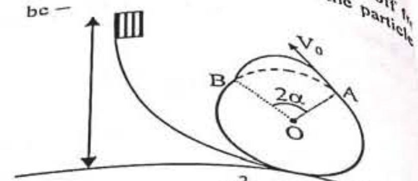
or $$\frac{1}{T} = \frac{1}{T_E} - \frac{1}{T_P},$$

i.e., $$T = \frac{T_P T_E}{T_P - T_E}$$

So $$T = \frac{3\times 1}{3-1} = 1.5 \text{ year}$$

Ex.14 A small object loops a vertical loo[p] which a symmetrical section of angle 2[α] been removed as shown in fig. Fin[d the] maximum and the minimum heights [from] which the object, after losing contact w[ith the] loop at point A and flying through a[ir, will] reach point B. Find the corresponding an[gle] of the section removed for which [it is] possible–

Sol. In order that the particle may fly off fr[om A] and land at B, the range of the particle [must] be –



$AB = 2R\sin\alpha = \frac{v_0^2}{g}\sin 2\alpha \Rightarrow v_0^2 = \frac{\ldots}{c\ldots}$

Applying the law of conservation of ene[rgy]

$mgH = mg\,R\,(1 + \cos\alpha) + \frac{1}{2}mv_0^2$

$\Rightarrow mgH = mgR\,(1 + \cos\alpha) + \frac{1}{2}m\frac{gR}{\cos\ldots}$

$\Rightarrow 2\cos^2\alpha - 2\,(k-1)\cos\alpha + 1 = 0$

where $k = \frac{H}{R}$

$$\Rightarrow \cos\alpha = \frac{2(k-1) \pm \sqrt{4(k-1)^2 - 8}}{2\times 2}$$

$$= \frac{1}{2}(k-1) \pm \frac{1}{2}\sqrt{(k-1)^2 - 2}$$

Since $\cos\alpha$ is real

$(k-1)^2 \;\; 2 \Rightarrow k \;\; 1 + \sqrt{2}$

Since $0 < \cos\alpha \le 1$,

$\frac{1}{2}(k-1) \pm \frac{1}{2}\sqrt{(k-1)^2 - 2} \le 1$

$\Rightarrow (k-1)^2 - 2 \le (3-k)^2 \Rightarrow k \le 2.5$

Thus $1 + \sqrt{2} \le k \le 2.5$

$\Rightarrow (1 + \sqrt{2})\,R \le H \le 2.5\,R$

$\therefore H_{max} = 2.5\,R$ and

$H_{min} = (1 + \sqrt{2})\,R = 2.4\,R$

When $H = H_{min}$ $k = 1 + \sqrt{2}$

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Ex.15 A p

X =

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Sol.

and $\cos\alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45°$

When $H = H_{max}$ $k = \frac{5}{2}$

and $\cos\alpha = 1, \frac{1}{2} \Rightarrow \alpha = 0°, 60°$

The solution $\alpha = 0$ is not acceptable as that will mean no cut,

$\therefore \quad \alpha = 60°$ for H_{max}.

Ex.15 A particle moves in a plane according to
$X = R\sin\omega t + R$and
$y = R\cos\omega t + R$
Where ω and R are constant. This curve, called a cycloid, is the path traced out by a point on the rim of a wheel which rolls without slipping along the x-axis. Find the instantaneous velocity and acceleration when the particle is at its maximum & minimum value of y.

Sol. $\frac{dx}{dt} = R\omega\cos\omega t$ and $\frac{dy}{dt} = -R\omega\sin\omega t$

$y_{min} = 0$, when $\omega t = \pi$

Now $\frac{dx}{dt} = -R\omega$ and $\frac{dy}{dt} = 0$,

$\therefore\ v = -R\omega$

$y_{max} = 2R$, when $\omega t = 2\pi$

$\frac{dx}{dt} = \omega R$ and $\frac{dy}{dt} = 0$

Now $v = \sqrt{\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right]} = \omega R$

$a_x = \frac{d^2y}{dt^2} = -R\omega^2 \sin\omega t$

and $a_y = \frac{d^2y}{dt^2} = -R\omega^2\cos\omega t$

For $y = y_{min}$, when $\omega t = \pi$,

then $\frac{d^2x}{dt^2} = 0$

and $\frac{d^2y}{dt^2} = R\omega^2$.

$\therefore \quad a = R\omega^2$

$y = y_{max}$, when $\omega t = 2\pi$,

then $\frac{d^2x}{dt^2} = 0$ and $\frac{d^2y}{dt^2} = -R\omega^2$.

$\therefore \quad a = -R\omega^2$

EXERCISE # 1

Questions based on: Kinematics of circular motion

Q.1 A point moves along a circle with velocity $v = at$ where $a = 0.5$ m/sec^2. Then the total acceleration of the point at the moment when it covered $(1/10)^{th}$ of the circle after beginning of motion-

(A) 0.5 m/sec^2 (B) 0.6 m/sec^2
(C) 0.7 m/sec^2 (D) 0.8 m/sec^2

Q.2 Angular position of a line of a disc of radius $r = 6$ cm is given by $\theta = 10 - 5t + 4t^2$ rad. the average angular speed between 1 and 3 s is-

(A) π rad/s (B) 11 rad/s
(C) 22 rad/s (D) 5.5 rad/s

Q.3 A car is moving in a circular path of radius 500m with a speed of 30m/sec. If its speed is increasing at the rate of 2m/sec^2, the resultant acceleration will be -

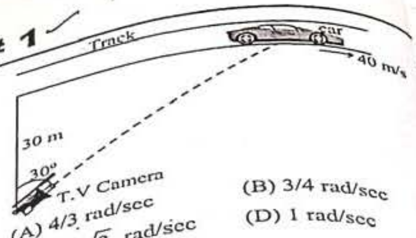
(A) 2 m/sec^2 (B) 2.5 m/sec^2
(C) 2.7 m/sec^2 (D) 4 m/sec^2

Q.4 An electric fan has blades of length 30 cm as measured from the axis of rotation. If the fan is rotating at 1200 r.p.m. The acceleration of a point on the tip of the blade is about-

(A) 1600 m/sec^2 (B) 4740 m/sec^2
(C) 2370 m/sec^2 (D) 5055 m/sec^2

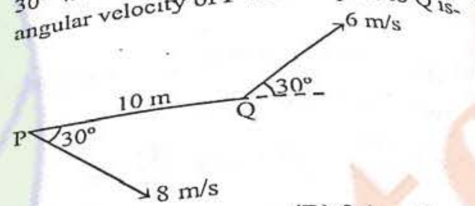
Questions based on: General motion along curved path

Q.5 A racing car is travelling along a track at a constant speed of 40 m/s. A T.V. camera men is recording the event from a distance of 30m directly away from the track as shown in figure. In order to keep the car under view in the position shown, the angular speed with which the camera should be rotated, is-



(A) 4/3 rad/sec (B) 3/4 rad/sec
(C) $8/3\sqrt{3}$ rad/sec (D) 1 rad/sec

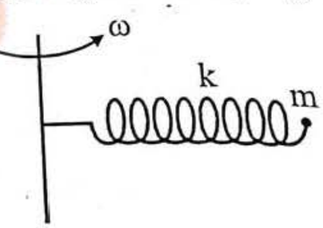
Q.6 Two moving particles P and Q are 10 m apart at a certain instant. The velocity of P is 8m/s making an angle 30° with the line joining P and Q and that of Q is 6m/s making an angle 30° with PQ as shown in the figure. Then angular velocity of P with respect to Q is-



(A) Zero (B) 0.1 rad/sec
(C) 0.4 rad/sec (D) 0.7 rad/sec

Questions based on: Dynamics of circular motion

Q.7 A particle of mass m is fixed to one end of a light spring of force constant k and unstretched length ℓ. The system is rotated about the other end of the spring with an angular velocity ω, in gravity free space. The increase in length of the spring will be-



(A) $\frac{m\omega^2\ell}{k}$ (B) $\frac{m\omega^2\ell}{k - m\omega^2}$

(C) $\frac{m\omega^2\ell}{k + m\omega^2}$ (D) None of these

Q.8 A railway track is ... making the height ... than that of th... between the rails ... of the track is r-

(A) $\frac{h}{d} = \frac{v^2}{rg}$

(B) tan(sin^{-1} ...

(C) tan^{-1}($\frac{h}{d}$ ...

(D) $\frac{h}{r} = \frac{v^2}{dg}$ ...

Q.9 The tube ... vertical ... cross-sec... tube, an... it. B is ... will-

Q.8 A railway track is banked for a speed v, by making the height of the outer rail (h) higher than that of the inner rail. The distance between the rails is d. The radius of curvature of the track is r-

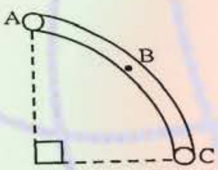
(A) $\frac{h}{d} = \frac{v^2}{rg}$

(B) $\tan\left(\sin^{-1}\frac{h}{d}\right) = \frac{v^2}{rg}$

(C) $\tan^{-1}\left(\frac{h}{d}\right) = \frac{v^2}{rg}$

(D) $\frac{h}{r} = \frac{v^2}{dg}$

Q.9 The tube AC forms a quarter circle in a vertical plane. The ball B has an area of cross-section slightly smaller than that of the tube, and can move without friction through it. B is placed at A and displaced slightly. It will-



(A) always be in contact with the inner wall of the tube

(B) always be in contact with the outer wall of the tube

(C) initially be in contact with the inner wall and later with the outer wall

(D) initially be in contact with the outer wall and later with the inner wall

Q.10 A particle is acted upon by a constant force always normal to the direction of motion of the particle. It is therefore inferred that-

(i) Its velocity is constant

(ii) It moves in a straight line

(iii) Its speed is constant

(iv) It moves in circular path

(A) i, iv (B) iii, iv (C) i, ii (D) i, ii, iii

Q.11 A particle of mass m is observed from an inertial frame of reference and is found to move in a circle of radius r with a uniform speed v. The centrifugal force on it is-

(A) $\frac{mv^2}{R}$ towards centre

(B) $\frac{mv^2}{R}$ away from centre

(C) $\frac{mv^2}{R}$ along tangent

(D) zero

Q.12 A car moves at a constant speed on a road as shown in figure. The normal force by the road on the car is N_A and N_B when it is at the points A and B -

(A) $N_A = N_B$

(B) $N_A > N_B$

(C) $N_A < N_B$

(D) insufficient information

Q.13 Three identical cars A, B and C are moving at the same speed on three bridges. The car A goes on plane bridge. B on a bridge convex upwards and car C on a bridge concave upwards. Let F_A, F_B and F_C be the normal forces exerted by the cars on the bridges when they are at the middle of bridge -

(A) F_A is maximum

(B) F_B is maximum

(C) F_C is maximum

(D) $F_A = F_B = F_C$

Q.14 A cylindrical bucket filled with water is whirled around in a vertical circle of radius r. What can be the minimum speed at the top of the path if water does not fall out from the bucket. If it continues with this speed. What normal force the bucket exerts on water at the lowest point of path ?

(A) $\sqrt{Rg}$, 2mg

(B) $\sqrt{2Rg}$, 2mg

(C) $\sqrt{3Rg}$, 3mg

(D) $\sqrt{Rg}$, 3mg

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Q.15 A bucket tied at the end of a 1.6 m long string is whirled in a vertical circle with constant speed. What should be the minimum speed so that the water from the bucket does not spill, When the bucket is at the highest position -
(Take $g = 10\ m/sec^2$)

(A) 4 m/sec (B) 6.25 m/sec
(C) 16 m/sec (D) None of the above

Q.16 A smooth hollow cone whose vertical angle is 2α, with it axis vertical and vertex downwards, revolves about its axis n times per second. Find distances from axis of rotation where a particle may be placed on the inner surface of cone so that it rotates with same speed -

(A) $\frac{g\cot\alpha}{4\pi^2 n^2}$ (B) $\frac{g\sin\alpha}{4\pi^2 n^2}$
(C) $\frac{4\pi^2 n^2}{g}$ (D) $\frac{g\sin\alpha}{4\pi^2 n^2}$

Q.17 A 2 kg stone at the end of a string 1m long is whirled in a vertical circle at a constant speed. The speed of the stone is 4 m/sec. The tension in the string will be 52N. when the stone is-

(A) At the top of the circle
(B) At the bottom of the circle
(C) Half way down
(D) None of the above

Q18 A particle, moving along a circular path has equal magnitudes of linear and angular acceleration. The diameter of path is (in meters) -

(A) 2 (B) 1
(C) π (D) 2π

Q.19 The vertical section of a road over a bridge in the direction of its length is in form of circle of radius 8.9 metre. The greatest speed at which the car can cross bridge without losing contact with the road at its highest point, the centre of gravity of car being at a height h = 1.1 metre from ground is- (Take $g = 10\ m/sec^2$)

(A) 5 m/sec (B) 10 m/sec
(C) 15 m/sec (D) 20 m/sec

Fill in the blanks type question

Q.20 A particle is projected with a speed u at angle θ with the horizontal. Considering small part of its path near the highest position, the approximate radius of curvature is given by R = ...

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