ND

Study Material

Based on

PHYSICS

(Part-I)

12

CONTENTS

Laser Typesetting by :
VIBHUTI COMPUTER
New Delhi

Printed at :
EAGLE OFFSET
New Delhi

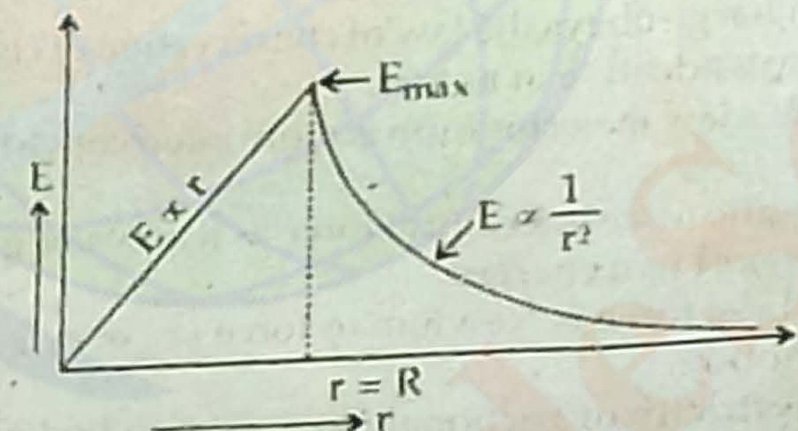
1

Electric Charges and Fields

Chapter at a Glance

- The charge on an electron and proton is called fundamental charge.
- Electric charge is quantized and charge on a body can be expressed as, $q = \pm ne$, where n is an integer and $e = 1.6 \times 10^{-19}$ C.
- Minimum value of dielectric constant is 1 for free space.
- Maximum value of dielectric constant is infinity for conductors *i.e.*, metals.
- Dielectric constant is a dimensionless number as it is the ratio of two similar quantities.
- Electric charge is a scalar quantity.
- Electric charge obeys the law of conservation of charge.
- It is always additive in nature.
- Coulomb's law in vector form is more informative than in its scalar form.
- Electrostatic force is a central force as it acts along the line joining the centres of two charges.
- Electrostatic force is Newtonian force *i.e.*, obey's Newton's third law of motion.
- Static electricity or frictional electricity on bodies occurs mainly due to the transfer of electrons from one body to another body.
- $1\ C = 3 \times 10^{9}$ stat Coulomb.
- Stat Coulomb is the C.G.S. unit of charge. It is also called electrostatic unit (e.s.u.) of charge.
- S.I. unit of electric field is NC^{-1}.
- Dielectric constant is also known as relative permitivity of the medium (ϵ_r).
- Two equal and opposite charges separated by a finite distance constitute an electric dipole.
- S.I. Unit of dipolemoment is Coulomb metre (C m).
- Electric dipolemoment is a vector quantity acting from – q to + q charge.

- In a uniform electric field, the net force on the dipole is zero and it experiences a torque only.
- In a uniform electric field, dipole has only rotatory motion.
- In a non-uniform electric field, the dipole experiences both the torque and force, hence it has rotatory as well as translatory motion.
- Electric lines of force never intersect each other. They always leave or enter the surface of the conductor perpendicularly.
- Electric field inside a charged or uncharged conductor placed in an external field is always zero.
- Electric flux is a scalar quantity and its S.I. unit is $Nm^{-2}c^{-1}$.
- The electric field is maximum at the surface of a charged spherical shell and zero inside it.
- The electric field due to a cloud of charge or due to a solid charged sphere is maximum at its surface and varies with distance from its centre as :



- Electric field lines are perpendicular to the equipotential surface.
- The surface of a charged conductor is an equipotential surface.
- Coulomb's law is valid only for point charges.
- Electric charge does not change with velocity.
- No point charge produces electric field at its own location.
- Electric charge resides only on the outer surface of a conductor.
- Coulomb's force between two charges is independent of the presence of other charges.
- $\vec{E}$ is independent of the shape of the conductor.
- $\vec{E}$ at a point on the surface of a conductor is directly proportional to the surface density of charge at that point.
- $\vec{E}$ at the centre of a charged circular ring is always zero.

Important Terms, Definitions and Laws

- **Coulomb's law in electrostatics**—Two point charges attract or repel each other with a force directly proportional to the product of magnitude of charges and inversely proportional to the square of distance between them.
- **Frictional electricity**—Electricity produced on bodies when they are rubbed against each other.
- **Additive nature of charge**—Total charge on an isolated system is equal to the algebraic sum of all individual charges of the system.
- **Law of conservation of charge**—Total charge on an isolated system always remains conserved.
- **Principle of superposition**—It states that the total force on a given point charge due to other interacting charges is the vector sum of the forces applied by the individual charges on it.
- **Test charge**—It is a small +ve charge. It is denoted by q_0.
- **Electric field**—It is defined as the space around a point charge in which its effect can be felt.

  Or

  It is the limiting value of electrostatic force per unit test charge

  i.e.,
  $$\overline{E} = \lim_{q_0 \to 0} \frac{\overline{F}}{q_0}$$
- **Electric dipole**—It is a system of two equal and opposite charges separated by a finite distance.
- **Electric dipole moment**—It is defined as the product of magnitude of either charge and the dipole length.
- **Electric line of force**—It is defined as the path straight or curved tangent at every point of which gives the direction of electric field.
- **Electric flux (ϕ)**—It is defined as the total number of electric lines of force passing through an area held normal to them around a given point.
- **Gauss's law or Theorem**—It states that the electric flux through a closed surface is $\frac{1}{\varepsilon_0}$ times the total charge enclosed inside it.

  i.e.
  $$\phi = \oint_S \overline{E}\,\overline{dS} = \frac{q}{\varepsilon_0}.$$
- **Gaussian Surface**—It is defined as any closed surface around the charge distribution enclosing some charge in it.

Important Formulae

- $$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

where $$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

and $$\varepsilon_0 = 8.854 \times 10^{-12} \text{ N}^{-1} \text{ m}^{-2} \text{ C}^2.$$

- $$\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|^3} (\vec{r_1} - \vec{r_2})$$

Electric field due to a point charge q is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \hat{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^3} \vec{r}$$

- **Dipole Moment of an electric dipole is**

$$\vec{p} = 2 \vec{a} q.$$

- Electric field at a point on the axis at a distance x from centre of a charged circular coil of radius r having charge q centre is given by

$$E = \frac{1}{4\pi\varepsilon_0} \frac{qx}{(r^2 - x^2)^{3/2}} \text{ along its axis.}$$

- Electric field at a point on the axial line of an electric dipole at a distance r from its centre is given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3} \text{ for a short dipole.}$$

- Electric field at a point on the equitorial line of an electric dipole at a distance r from its centre is given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2 + x^2)^{3/2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3} \text{ for a short dipole}$$

- Torque on an electric dipole in a uniform $\vec{E}$ is

$$\vec{\tau} = \vec{p} \times \vec{E}$$

or

$$\tau = pE\sin\theta$$

where θ is the angle between $\vec{p}$ and $\vec{E}$.

- Force on a charge due to n other charges is

$$\vec{F_i} = \vec{F_{i1}} + \vec{F_{i2}} + \ldots\ldots + \vec{F_{in}} = \sum_{\substack{j=1 \\ j\neq i}}^{n} \vec{F_{ij}}$$

$$= \frac{1}{4\pi\varepsilon_0} \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{q_i \cdot q_j}{|\vec{r_i} - \vec{r_j}|^3} (\vec{r_i} - \vec{r_j}).$$

- Electric flux,

$$\phi = \vec{E} \cdot \vec{dS}$$

When $\vec{dS}$ is the area vector acting along outward drawn normal.

-

$$\phi = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

- Electric field at a point due to an infinitely long straight conductor or wire of linear charge density is

$$E = \frac{1}{2\pi\varepsilon_0} \cdot \frac{\lambda}{r}$$

where r = perpendicular distance of the point from the wire,

- $\overline{E}$ due to an infinite plane sheet of charge having surface charge density σ is given by

$$E = \frac{\sigma}{2\varepsilon_0}$$

- $\overline{E}$ between two plane parallel sheets of charge is given by

$$E = \frac{\sigma}{\varepsilon_0}$$

- $\overline{E}$ at a point due to a spherical shell is

$$E = \frac{\sigma}{\varepsilon_0} \cdot \frac{R^2}{r^2} \quad \text{(for } r > R\text{)}$$

$$= \frac{\sigma}{\varepsilon_0} \quad \text{(for } r = R\text{)}$$

$$= 0 \text{ for } r < R.$$

When σ = surface charge density.
R = radius of shell.

- $\overline{E}$ at a point due to a solid sphere of radius R volume charge density ρ at a point at a distance r is given by

$$E = \frac{\rho}{3\varepsilon_0} \cdot \frac{R^3}{r^2} \quad \text{(for } r > R\text{)}$$

$$= \frac{\rho R}{3\varepsilon_0} \quad \text{(for } r = R\text{)}$$

$$= \frac{\rho r}{3\varepsilon_0} \quad \text{(for } r < R\text{)}$$

EXERCISES

Q. 1.1. What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air?

Ans. Here, $q_1 = 2 \times 10^{-7}$ C, $q_2 = 3 \times 10^{-7}$ C, r = 30 cm = 0.30 m.

F = ?

Using the relation, $F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{d^2}$, we get

$$F = 9 \times 10^9 \times \frac{2 \times 10^{-7} \times 3 \times 10^{-7}}{(0.30)^2}$$

$$= 6 \times 10^{-3} \text{ N.}$$

Q. 1.2. The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge – 0.8 μC in air is 0.2 N. (a) What is the distance between the two spheres? (b) What is the force on the second sphere due to the first?

Ans. Here, $q_1 = 0.4\ \mu C$, $q_2 = -0.8\ \mu C = -0.8 \times 10^{-6}$ C $= 0.4 \times 10^{-6}$ C

F = electrostatic force between q_1 and q_2 = 0.2 N.

(a) r = ?

Using the relation, $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{F}$, we get

$$r^2 = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{F}$$

$$= 9 \times 10^9 \times \frac{0.4 \times 10^{-6} \times (0.8 \times 10^{-6})}{0.2}$$

$$= 16 \times 9 \times 10^{-4} = 144 \times 10^{-4} \text{ m}^2$$

$$\therefore \quad r = 12 \times 10^{-2} \text{ m} = 0.12 \text{ m.}$$

(b) Force on q_2 due to q_1 = ?

We know that electrostatic forces always, appear in priors and follow Newton's 3rd law of motion.

$\therefore |\overline{F_{21}}|$ = Force on q_2 due to q_1 = 0.2 N and is attractive in nature.

$$F_{21} = 9 \times 10^9 \times \frac{(0.4 \times 10^{-6}) \times (0.8 \times 10^{-6})}{(0.12)^2}$$

$$= 0.2 \text{ N.}$$

Q. 1.3. Checks that the ratio ke^2/Gm_em_p is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?

Ans. Dimensions of e^2 are $= [C^2]$

Dimensions of k are $= [Nm^2 C^{-2}] = [ML^3 T^{-2} C^{-2}]$

Dimensions of G are $= [M^{-1} L^3 T^{-2}]$

Dimensions of m_e $= [M]$

$\therefore$ Dimensions of $\frac{Ke^2}{Gm_em_p}$ are obtained as :

$$\frac{Ke^2}{Gm_em_p} = \frac{[ML^3T^{-2}C^{-2}][C^2]}{[M^{-1}L^3T^{-2}][M][M]}$$

$$= [M^{2-2} L^{3-3} T^{-2+2} C^{-2+2}]$$

$$= [M^0L^0T^0C^0] = [M^0L^0T^0]$$

$\therefore$ $\frac{Ke^2}{Gm_e m_p}$ is a dimensionless quantity as it has no units.

Now using $e = 1.6 \times 10^{-19}$ C, $k = 9 \times 10^9$ Nm^2 C^{-2}, $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}, $m_e = 9.1 \times 10^{-31}$ kg, $m_p = 1.66 \times 10^{-27}$ kg, we get

$$\frac{ke^2}{Gm_e m_p} = \frac{(9\times10^9)\times(1.6\times10^{-19})^2}{6.67\times10^{-11}\times9.1\times10^{-31}\times1.66\times10^{-27}}$$

$$= 2.29 \times 10^{39}.$$

This factor represents the ratio of electrostatic force and the gravitational force between an electron and a proton.

Q. 1.4. (a) Explain the meaning of the statement 'electric charge of a body is quantised'.

(b) Why can one ignore quantisation of electric charge when dealing with macroscopic *i.e.*, large scale charges ?

Ans. (a) The meaning of the statement 'electric charge of a body is quantised' is that the charge on it is always some integral multiple of elementary charge of an electron or a proton (= e in magnitude) *i.e.*, charge on a body never varies continuously but it varies in the form of discrete packets called quanta or packets of charge. Mathematically, the charge on a body can be expressed as

$$q = \pm ne$$

where n is an integer, e = magnitude of the charge of an electron or proton = 1.6×10^{-19} C. A fraction of the fundamental charge e has never been observed in free state.

(b) In practice, the charge on a charged body at macroscopic level is very large while the charge on an electron is very small. When electrons are added to or removed from a body, the change taking place in the total charge on the body is so small that the charge seems to be varying in a continuous manner. Thus quantisation of electric charge can be ignored at macroscopic level *i.e.*, when dealing with a large scale charged body.

Q. 1.5. When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservaton of charge.

Ans. Initially *i.e.*, before rubbing both the glass rod and silk cloth are electrically neutral. In other words, net charge on the glass rod and silk cloth is zero. When the glass rod is rubbed with silk cloth, a few electrons get transferred from the rod to the silk cloth, thus glass rod becomes positively charged and silk cloth negatively charged. The positive charge on the glass rod is exactly equal to the negative charge on the silk cloth, so net charge on the system is again zero. Thus the appearance of charge on the glass rod and silk cloth is in accordance with the law of conservation of charge as the total charge of the isolated system is constant. Similarly when ebonite rod is rubbed with fur, they acquire – ve and + ve charges respectively and net charge is zero again.

Thus we conclude that charge is neither created nor destroyed but it is merely transferred from one body to another which is consistent with the law of conservation of charge.

Q. 1.6. Four point charges $q_A = 2\ \mu C$, $q_B = -5\ \mu C$, $q_C = 2\ \mu C$, and $q_D = -5\ \mu C$ are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 μC placed at the centre of the square?

Ans. Consider the square ABCD of each side 10 cm and centre O.

The charge of 1 μC is placed at O.

Now clearly $OA = OB = OC = OD.$

$$AB = BC = 10\text{ cm} = 0.1\text{ m}.$$

$$\therefore \quad AO = \frac{1}{2}AC = \frac{1}{2}\sqrt{AB^2 + BC^2} = \frac{1}{2} \times \sqrt{2}\ AB$$

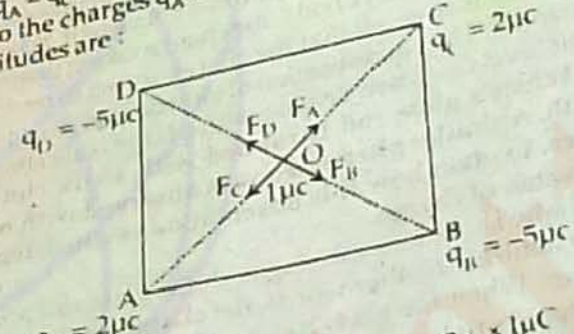
$$= \frac{1}{\sqrt{2}} \times 0.1\text{ m} = OB = OC = OD.$$

Here, $q_A = 2\,\mu C$, $q_B = -5\,\mu C$, $q_C = 2\,\mu C$, $q_D = -5\,\mu C$

Clearly $q_A = q_C = 2\mu C = 2 \times 10^{-6}$ C.

and $q_B = q_D = -5\,\mu C = -5 \times 10^{-6}$ C.

Since $q_A = q_C$, the charge of 1 μC will experience equal and opposite forces due to the charges q_A and q_C *i.e.*, along OC and OA respectively. Their magnitudes are :



$$F_A = F_C = \frac{1}{4\pi\varepsilon_0} \times \frac{q_A \times 1\mu C}{AO^2}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 10^{-6}}{\left(\frac{1}{\sqrt{2}} \times 0.1\right)^2}$$

$$= 3.6 \text{ N}$$

$$\overrightarrow{F_A} = -\overrightarrow{F_C}$$

Similarly $F_B = F_D$, the charge of 1 μC will experience equal and opposite forces due to the charge q_B and q_D *i.e.*, along OB and OD respectively. Thus $\overrightarrow{F_B} = -\overrightarrow{F_D}$.

Thus the net force on the charge of 1μC due to the given arrangement of charges is zero *i.e.*,

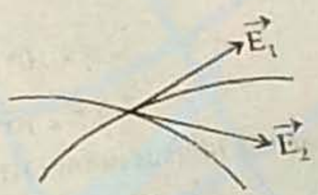
$$\overrightarrow{F} = \overrightarrow{F_A} + \overrightarrow{F_B} + \overrightarrow{F_C} + \overrightarrow{F_D} = 0.$$

Q. 1.7. (a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?

(b) Explain why two fields lines never cross each other at any point?

Ans. (a) The electrostatic line of force is the path tangent at every point of which gives the direction of electric field at that point. The direction of electric field generally changes from point to point. So the lines of force are generally curved lines. Further they are continuous curves and cannot have sudden breaks because if it is so, then it will indicate the absence of electric field at the break points.

(b) The electric lines of force never cross each other because if they do so, then at the point of their intersection, we can draw two tangents which give two directions of electric field at that point which is not possible.

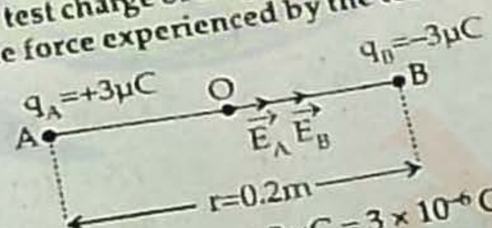


Q. 1.8. Two point charges $q_A = 3\ \mu C$ and $q_B = -3\ \mu C$ are located 20 cm apart in vacuum.

(a) What is the electric field at the midpoint O of the line AB joining the two charges?

(b) If a negative test charge of magnitude 1.5×10^{-29} C is placed at this point, what is the force experienced by the test charge?

Ans.



Here, charge at point A, $q_A = +3\ \mu C = 3 \times 10^{-6}$ C.

charge at point B, $q_B = -3\ \mu C = 3 \times 10^{-6}$ C.

$r = AB = 20\ cm = 0.2\ m.$

Let O be the mid-point of the line AB, then

$$OA = OB = \frac{r}{2} = \frac{0.2}{2} = 0.1\ m.$$

(a) If $\overline{E_A}$ and $\overline{E_B}$ be the electric fields at point O due to q_A and q_B respectively. Then

$$E_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A}{(OA)^2}$$

$$= 9 \times 10^9 \times \frac{3 \times 10^{-6}}{(0.1)^2}$$

$$= 2.7 \times 10^{-6}\ NC^{-1} \text{ along OB.}$$

and

$$E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_B}{(OB)^2}$$

$$= 9 \times 10^9 \times \frac{3 \times 10^{-6}}{(0.1)^2}$$

$$= 2.7 \times 10^{-6}\ NC^{-1} \text{ along OB.}$$

If $\overline{E}$ be the net electric field at point O due to q_A and q_B, then

$$\overline{E} = \overline{E_A} + \overline{E_B}$$

$$= 2.7 \times 10^{-6} + 2.7 \times 10^{-6}$$

$$= 5.4 \times 10^{-6}\ NC^{-1} \text{ along OB.}$$

(b) Force on a negative charge of magnitude 1.5×10^{-9} C is given by the formula.

$$\overline{F} = q_0 \cdot \overline{E}$$

Here $q_0 = -1.5 \times 10^{-9}$ C, $\overline{E} = 5.4 \times 10^{-6}\ NC^{-1} (\hat{OB})$

$$\overline{F} = -1.5 \times 10^{-9} \times 5.4 \times 10^{-6}$$

$$= -8.1 \times 10^{-3}\ N.$$

– ve sign shows that $\overline{F}$ acts opposite to $\overline{E}$ i.e. along OA.

Q. 1.9. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A (0, 0, – 15 cm) and B (0, 0, + 15 cm) respectively. What is the total charge and electric dipole moment of the system?

Ans. The charges q_A and q_B are located at points A (0, 0, – 15 cm) and B (0, 0, + 15 cm) on z axis as shown in the figure here. They form an electric dipole

q = total chrage = ?

p = electric dipole moment of the system = ?

p = q.2a.

Now $q = q_A + q_B = 2.5 \times 10^{-7} + (-2.5 \times 10^{-7}) = 0$

Also 2a = AB = dipole length = OA + OB

= 15 + 15 = 30 cm

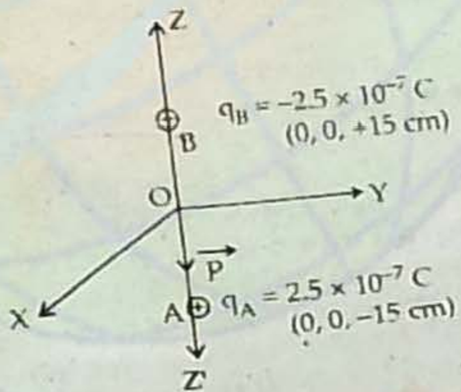
= 0.30 m.

∴ p = either charge × dipole length

$= q_A \times AB = 2.5 \times 10^{-7} \times 0.30$

$= 7.5 \times 10^{-8}$ cm

The electric dipole moment is directed from B to A *i.e.*, along negative Z-axis.



Q. 1.10. An electric dipole with dipole moment 4×10^{-9} C m is aligned at 30° with the direction of a uniform electric field of magnitude 5×10^4 NC^{-1}. Calculate the magnitude of the torque acting on the dipole.

Ans. Here, $p = 4 \times 10^{-9}$ Cm, $E = 5 \times 10^4$ NC^{-1}, $\theta = 30°$, $\tau = ?$

Using the formula, $\tau = pE \sin\theta$, we get

$$\tau = 4 \times 10^{-9} \times 5 \times 10^4 \times \sin 30°$$

$$= 20 \times 10^{-5} \times \frac{1}{2}$$

$$= 10^{-4} \text{ Nm.}$$

Q. 1.11. A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.

(a) Estimate the number of electrons transferred (from which to which?)

(b) Is there a transfer of mass from wool to polythene?

Ans. (a) Here, q = Total charge transferred = -3×10^{-7} C. Charge on an electron, e = -1.6×10^{-19} C.

n = no. of electrons transferred = ?

As the polythene piece rubbed with wool is found to attain – ve charge, so the electrons are transferred from wool to polythene piece.

From quantisation of charge, we know that q = ne.

$$n = \frac{q}{e} = \frac{-3\times10^{-7}}{-1.6\times10^{-19}} = 1.875\times10^{12}.$$

$$= 2\times10^{12}$$

(b) Yes, there is a transfer of mass from wool to polythene as electrons are material particles and are transferred from wool to polythene piece.

m = mass of each electron = 9.1×10^{-31} kg, n = 1.875×10^{12}

M = total mass transferred to polythene = ?

= m × n

= $9.1\times10^{-31}\times1.875\times10^{12} = 1.71\times10^{-18}$ kg ≈ 2×10^{-18} kg.

Q. 1.12. (a) Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrosatic repulsion if the charge on each is 6.5×10^{-7} C? The radii of A and B are negligible compared to the distance of seperation.

(b) What is the force of repulsion of each sphere is charged double the above amount, and the distance between them is halved?

Ans. (a) Charge on 1st sphere, A = $q_A = 6.5\times10^{-7}$C

Charge on 2nd sphere, B = $q_B = 6.5\times10^{-7}$C

Distance between sphere A and B = 50 cm = 0.5 m = d

We know that,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_B}{r^2}$$

$$= \frac{9\times10^{9}\times6.5\times10^{-7}\times6.5\times10^{-7}}{(0.5)^2}$$

$$= \frac{9\times10^{-5}\times(6.5)^2}{0.25}$$

$$= 1.521\times10^{-2}\text{ N} = 1.5\times10^{-2}\text{ N}.$$

(b) If each sphere is charged double the amount, then

$$q_A = q_B = 2 \times 6.5 \times 10^{-7} C = 13 \times 10^{-7} C$$

and $r = \frac{1}{2} \times 50 \text{ cm} = 0.25 \text{ m}$

We know that,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A q_B}{r^2} = \frac{9\times10^9 \times 13\times10^{-7} \times 13\times10^{-7}}{(0.25)^2}$$

$$= 0.24 \text{ N}$$

Q. 1.13. Suppose the spheres A and B of previous exercise have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the force of repulsion between A and B?

Ans. Initial charges on spheres A and B

$= q_A = q_B = 6.5 \times 10^{-7}$ C

r = Distance between the spheres A and B = 0.5 m.

All the spheres will have equal charges on being brought in contact because they are of the same size.

When 3rd sphere C having charge ε_3 = zero is brought in contact with 1st sphere A, then;

Charge on A = Charge on C = q'_1 say

$$\therefore \quad q'_1 = \frac{q_A + q_3}{2} = \frac{6.5 \times 10^{-7} + 0}{2} = 3.25 \times 10^{-7} C$$

or $q'_1 = 3.25 \times 10^{-7}$ C

When 3rd sphere C having charge 3.25×10^{-7} C is brought in contact with 2nd sphere B, then; charge left on B say q'_2 is given by

Charge on B = Charge on C

$$\text{or} \quad q'_2 \frac{q_B + q'_1}{2} = \frac{6.5 \times 10^{-7} + 3.25 \times 10^{-7}}{2}$$

or $q'_2 = 4.875 \times 10^{-7}$ C

If F be the force of repulsion between spheres A and B and when sphere C is removed, then according to Coulomb's law of electrostatic forces,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q'_1 q'_2}{r^2}$$

Here,

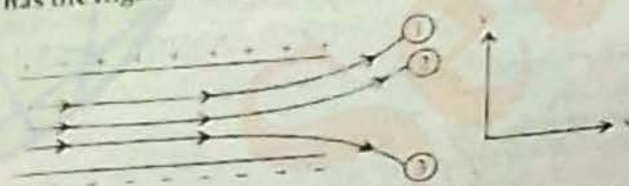
$$q'_1 = 3.25 \times 10^{-7}\ C$$
$$q'_2 = 4.875 \times 10^{-7}\ C$$
$$r = 0.5\ m$$

$$F = \frac{9 \times 10^{9} \times 3.25 \times 10^{-7} \times 4.875 \times 10^{-7}}{(0.5)^2}$$

$$= \frac{9 \times 3.25 \times 4.875 \times 10^{-5}}{0.25}$$

$$= 5.704 \times 10^{-3}\ N$$
$$= 5.7 \times 10^{-3}\ N$$

Q. 1.14. Figure below shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?



Ans. Charges on the two plates are shown in the figure. Since charged particles are deflected towards the oppositely charged plates, therefore (1) and (2) are –vely charged while particle (3) is +vely charged.

Since all the three particles are crossing the same electric field with same speed, so they remain under the action of $\vec{E}$ for same time t (say). The deflection produced in the path of a charged particle along vertical direction is given by, $y = \frac{1}{2} at^2 = \frac{1}{2} \frac{eE}{m} t^2$

As E and t are same, so the displacement $y \propto \left(\frac{e}{m}\right)$. As the charged particle 3 suffers maximum deflection along vertical so value of y is maximum for it, hence it has the highest charge to mass ratio.

Q. 1.15. Consider a uniform electric field $E = 3 \times 10^3\ \hat{i}$ N/C. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane? (b) What is the flux through the same square if the normal to its plane makes at 60° angle with the x-axis?

Ans. Here, $\vec{E} = 3 \times 10^3\ \hat{i}\ NC^{-1}$ *i.e.*, the electric field acts along positive direction of x-axis.

Side of square = 10 cm

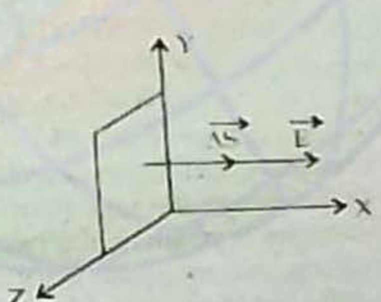
$\therefore$ its surface area, $\Delta S = (10\text{ cm})^2 = 10^{-2}\ m^2$

or $\vec{\Delta S} = 10^{-2}\ \hat{i}\ m^2$

as normal to the square is along x-axis.

(a) If ϕ be the electric flux through the square, then

$$\phi = \vec{E} \cdot \vec{\Delta S}$$
$$= (3 \times 10^3\ \hat{i}).(10^{-2}\ \hat{i})$$
$$= 3 \times 10^3 \times 10^{-2}\ \hat{i}.\hat{i}$$
$$= 3 \times 10 = 30\ Nm^2 C^{-1}.$$

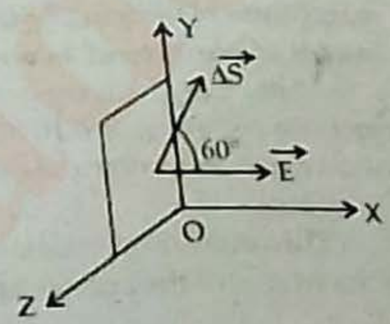


(b) Here, angle between normal to the square *i.e.*, area vector and the electric field is 60°.

i.e., $\theta = 60°$

$\therefore$ $\phi = \vec{E} \cdot \vec{\Delta S} = E.\ \Delta S \cos 60°$

$$= 3 \times 10^3 \times 10^{-2} \times \frac{1}{2}$$
$$= 15\ Nm^2 C^{-1}.$$



Q. 1.16. What is the net flux of the uniform electric field of Exercise 1.15 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

Ans. Net flux over the cube is zero becuase the number of lines entering the cube of side 20 cm is same as the number of lines leaving the cube.

Q. 1.17. Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8.0 × 10^3 Nm^2/c

(i) What is the net charge inside the box?

(ii) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or why not?

Ans.(i) Given, $\phi = 8 \times 10^3\ Nm^2 C^{-1}$, $\varepsilon_0 = 8.854 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

If the net charge inside the black box is q, then using formula

$$\phi = \frac{q}{\varepsilon_0} \text{ or we get, } q = \varepsilon_0 \phi$$

$$\text{or } q = 8.854 \times 10^{-12} \times 8 \times 10^3 C$$
$$q = 8.854 \times 8 \times 10^{-9} C$$
$$q = 70.832 \times 10^{-9} C = 0.070832 \times 10^{-6} C$$
$$= 0.071\ \mu C$$

(ii) We cannot conclude that the net electric charge inside the box is zero if the outward flux through the surface of black box is zero because there might be equal amounts of positive and negative charges cancelling each other and thus making the resultant charge equal to zero. Thus, we can only conclude that the net charge inside the box is zero.

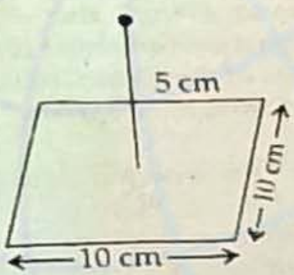
Q. 1.18. A point charge + 10 μC is a distance 5 cm directly above the centre of a square of side 10 cm as shown in figure. What is the magnitude of the electric flux through the square? (Hint : Think of the square as one face of a cube with edge 10 cm.)

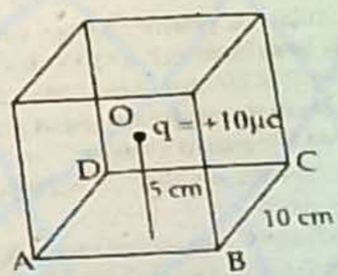
Ans. The given square ABCD can be imagined as one of the side faces of a cube of side 0.10 m. The given charge can be imagined to be at the centre of this cube at a distance of 5 cm.

Here, $q = +10\ \mu C = 10^{-5}$ C.

Then according to Gauss's Theorem, the total electric flux through all the 6 faces of the cube is given by

$$\phi = \frac{q}{\varepsilon_0}$$





If ϕ' be the electric flux through the square ABCD, then

$$\phi' = \frac{1}{6}\phi = \frac{1}{6}\cdot\frac{q}{\varepsilon_0}$$

$$= \frac{1}{6} \times \frac{10^{-5}}{8.854 \times 10^{-12}} \text{ NC}^{-1}\text{ m}^2.$$

$$= 1.88 \times 10^5 \text{ NC}^{-1}\text{ m}^2.$$

Q.1.19. A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?

Ans. Here, charge at the centre of the Gaussian surface,

$$q = 2\mu C = 2 \times 10^{-6} \text{ C}.$$

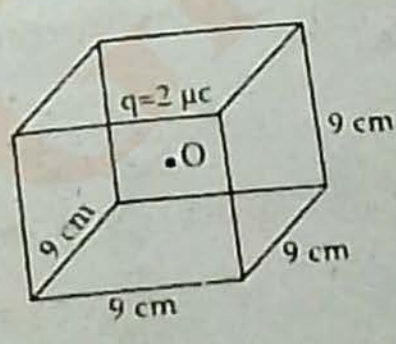
$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\phi = \text{electric flux through it} = ?$$

According to Gauss's Theorem, the electric flux through the six faces of the cubes *i.e.*, through the gaussian surface is given by

$$\phi = \frac{q}{\varepsilon_0} = \frac{2 \times 10^{-6}}{8.854 \times 10^{-12}}$$

$$= 2.26 \times 10^5 \text{ Nm}^2\text{C}^{-1}.$$



Published by

NAND LAL DAYA RAM

(Educational Publishers & Booksellers)

4159, Nai Sarak, Delhi-110 006

Price : Rs. 108.00