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PHYSICS

Target : JEE (Main)

**MAGNETIC EFFECT OF CURRENT AND
MAGNETIC FORCE ON CHARGE OR CURRENT**

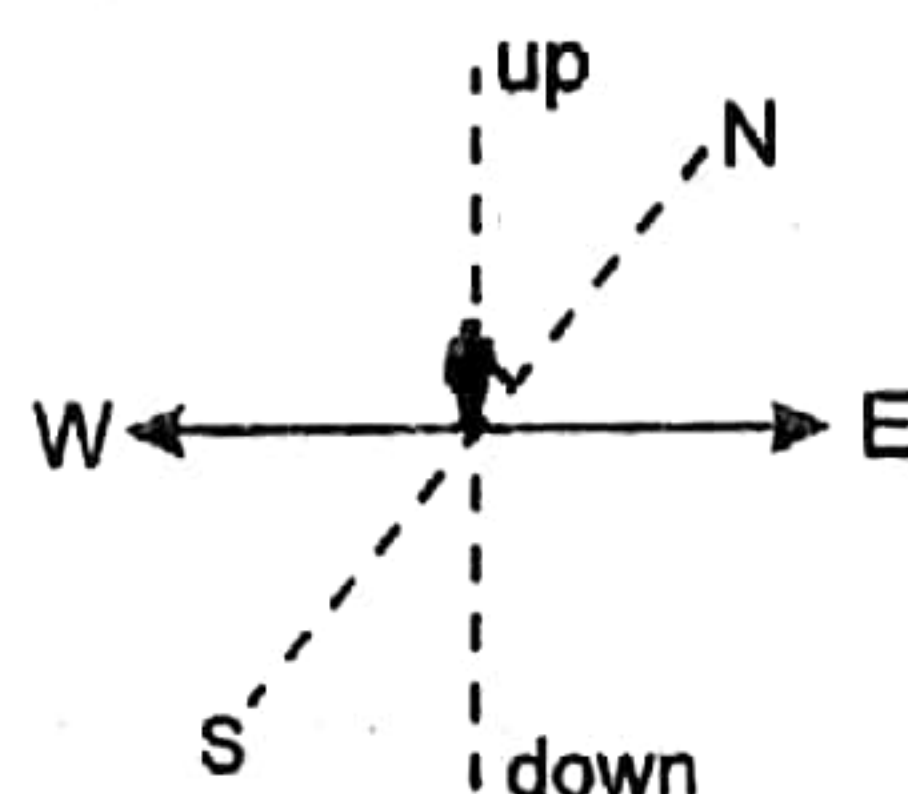
MAGNETIC EFFECT OF CURRENT AND MAGNETIC FORCE ON CHARGE OR CURRENT



1. Magnet :

Two bodies even after being neutral (showing no electric interaction) may attract / repel strongly if they have a special property. This property is known as magnetism. This force is called magnetic force. Those bodies are called magnets. Later on we will see that it is due to circulating currents inside the atoms. Magnets are found in different shape but for many experimental purposes, a bar magnet is frequently used. When a bar magnet is suspended at its middle, as shown, and it is free to rotate in the horizontal plane it always comes to equilibrium in a fixed direction.

One end of the magnet (say A) is directed approximately towards north and the other end (say B) approximately towards south. This observation is made everywhere on the earth. Due to this reason the end A, which points towards north direction is called NORTH POLE and the other end which points towards south direction is called SOUTH POLE. They can be marked as 'N' and 'S' on the magnet. This property can be used to determine the north or south direction anywhere on the earth and indirectly east



and west also if they are not known by other method (like rising of sun and setting of the sun). This method is used by navigators of ships and aeroplanes. The directions are as shown in the figure. All directions E, W, N, S are in the horizontal plane.

The magnet rotates due to the earth's magnetic field about which we will discuss later in this chapter.

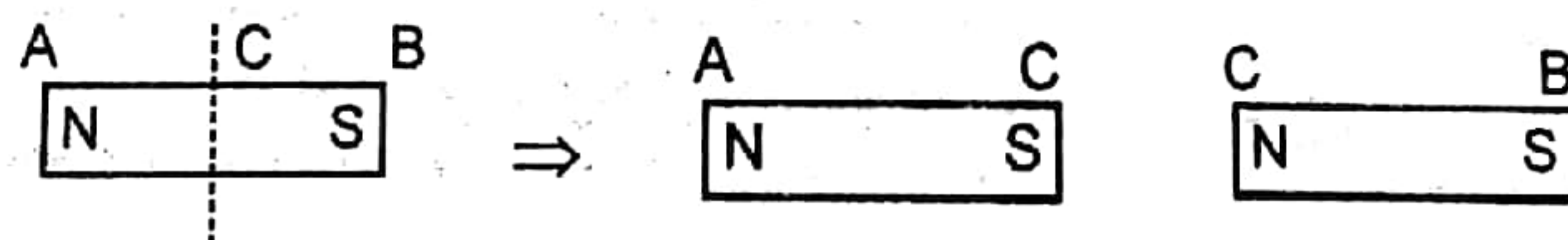
1.1 Pole strength magnetic dipole and magnetic dipole moment :

A magnet always has two poles 'N' and 'S' and like poles of two magnets repel each other and the unlike poles of two magnets attract each other they form action reaction pair.



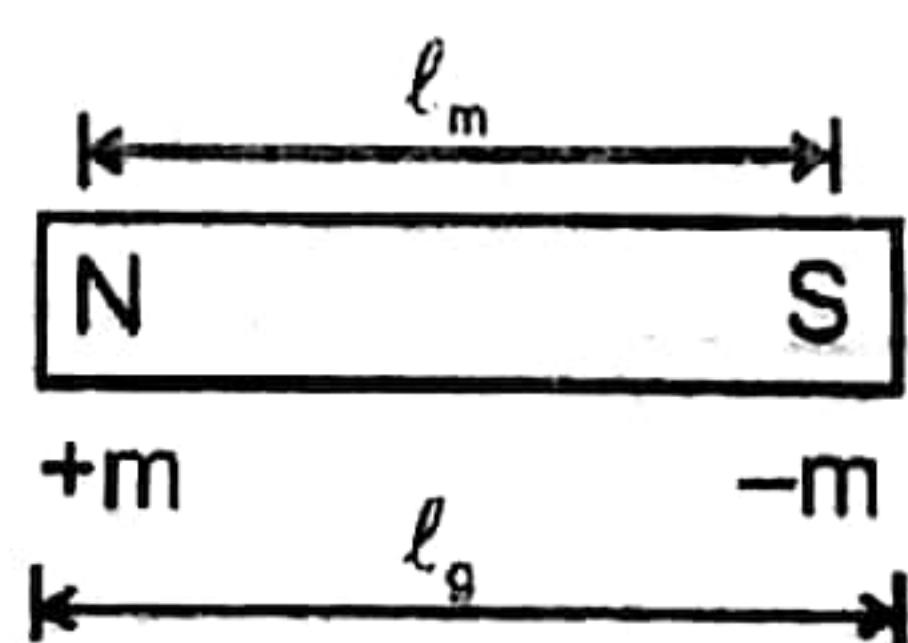
The poles of the same magnet do not come to meet each other due to attraction. They are maintained we cannot get two isolated poles by cutting the magnet from the middle. The other end becomes pole of opposite nature. So, 'N' and 'S' always exist together.

∴ they are



Known as +ve and -ve poles. North pole is treated as positive pole (or positive magnetic charge) and the south pole is treated as -ve pole (or -ve magnetic charge). They are quantitatively represented by their "POLE STRENGTH" $+m$ and $-m$ respectively (just like we have charges $+q$ and $-q$ in electrostatics). Pole strength is a scalar quantity and represents the strength of the pole hence, of the magnet also).

A magnet can be treated as a dipole since it always has two opposite poles (just like in electric dipole we have two opposite charges $-q$ and $+q$). It is called MAGNETIC DIPOLE and it has a MAGNETIC DIPOLE MOMENT. It is represented by \vec{M} . It is a vector quantity. Its direction is from $-m$ to $+m$ that means from 'S' to 'N')





$M = m \cdot \ell_m$ here ℓ_m = magnetic length of the magnet. ℓ_m is slightly less than ℓ_g (it is geometrical length of the magnet = end to end distance). The 'N' and 'S' are not located exactly at the ends of the magnet. For calculation purposes we can assume $\ell_m = \ell_g$ [Actually $\ell_m/\ell_g \approx 0.84$].

The units of m and M will be mentioned afterwards where you can remember and understand.

1.2 Magnetic field and strength of magnetic field.

The physical space around a magnetic pole has special influence due to which other pole experience a force. That special influence is called MAGNETIC FIELD and that force is called 'MAGNETIC FORCE'. This field is qualitatively represented by 'STRENGTH OF MAGNETIC FIELD' or "MAGNETIC INDUCTION" or "MAGNETIC FLUX DENSITY". It is represented by \vec{B} . It is a vector quantity.

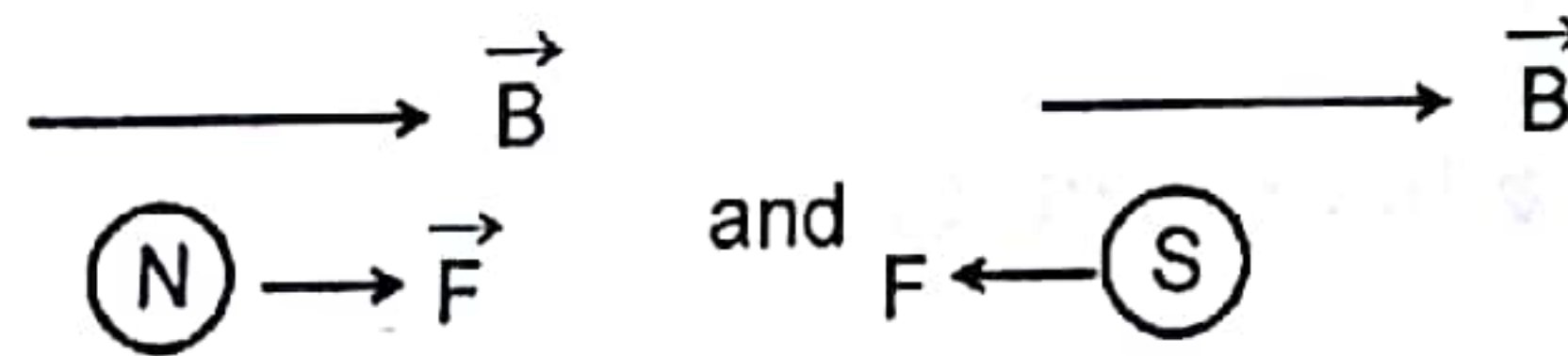
Definition of \vec{B} : The magnetic force experienced by a north pole of unit pole strength at a point due to some other poles (called source) is called the strength of magnetic field at that point due to the source.

Mathematically, $\vec{B} = \frac{\vec{F}}{m}$

Here \vec{F} = magnetic force on pole of pole strength m . m may be +ve or -ve and of any value.

S.I. unit of \vec{B} is **Tesla** or **Weber/m²** (abbreviated as T and Wb/m²).

We can also write $\vec{F} = m\vec{B}$. According to this direction of on +ve pole (North pole) will be in the direction of field and on -ve pole (south pole) it will be opposite to the direction of \vec{B} .

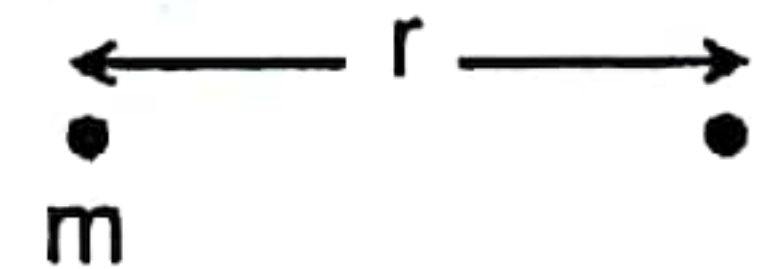


The field generated by sources does not depend on the test pole (for its any value and any sign).

(a) \vec{B} due to various source

(i) Due to a single pole :

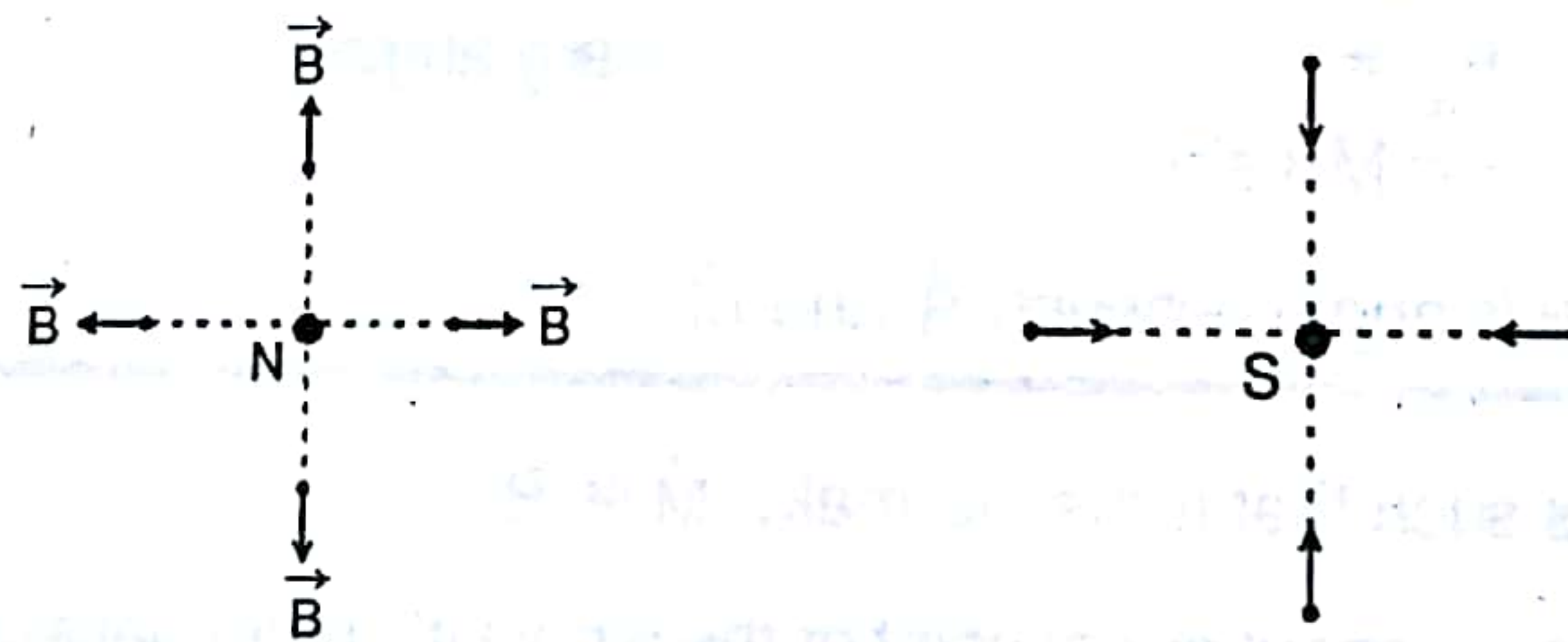
(Similar to the case of a point charge in electrostatics)



$$B = \left(\frac{\mu_0}{4\pi} \right) \frac{m}{r^2}$$

This is magnitude

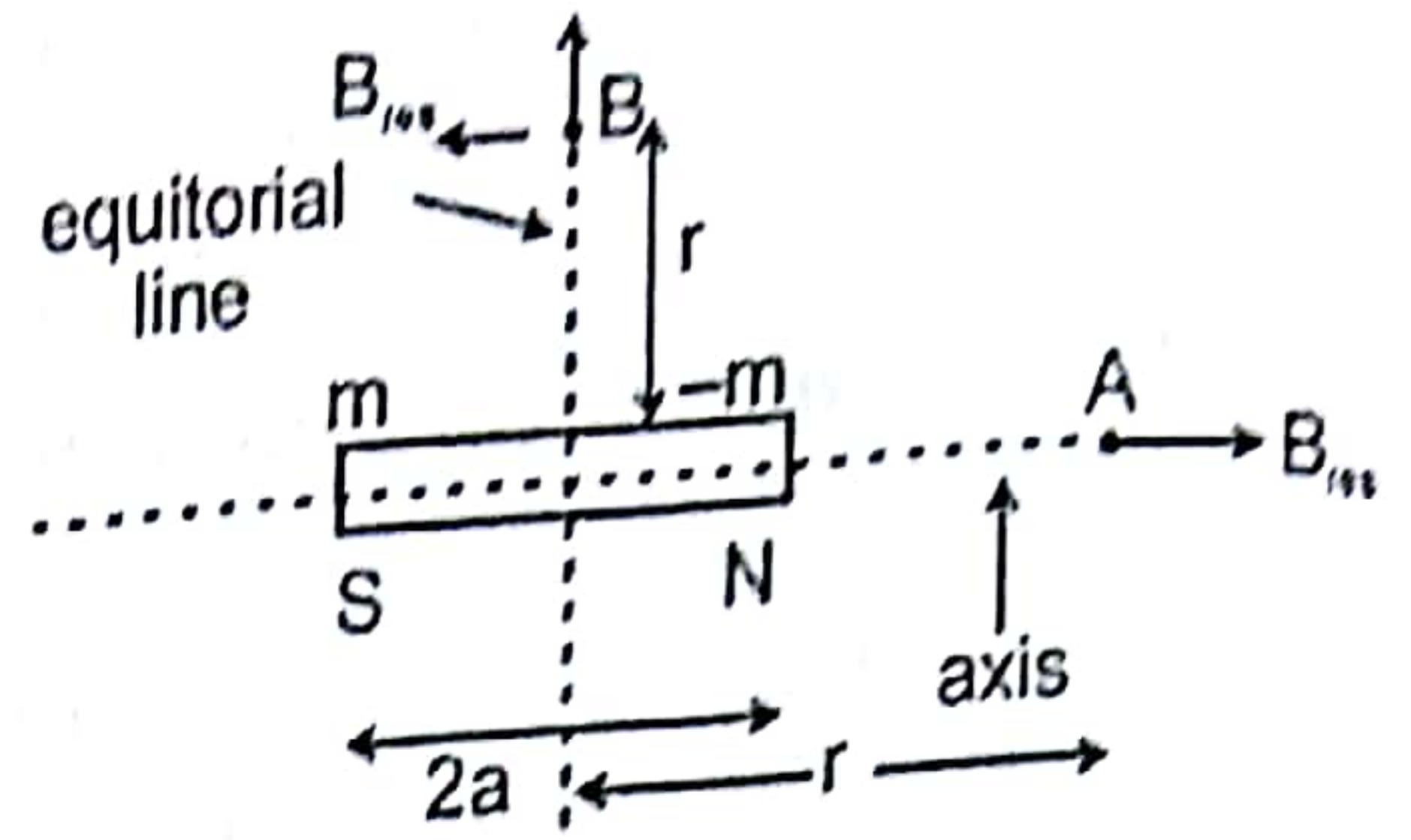
Direction of B due to north pole and due to south poles are as shown



in vector form $\vec{B} = \left(\frac{\mu_0}{4\pi} \right) \frac{m}{r^3} \vec{r}$

here m is with sign and \vec{r} = position vector of the test point with respect to the pole.

(II) **Due to a bar magnet :**
 (Same as the case of electric dipole in electrostatics)
 Independent case never found. Always 'N' and 'S' exist together as magnet.



at A (on the axis) = $\left(\frac{\mu_0}{4\pi}\right) \frac{\ddot{M}}{r^3}$ for $a \ll r$

at B (on the equatorial) = $-\left(\frac{\mu_0}{4\pi}\right) \frac{\ddot{M}}{r^3}$ for $a \ll r$

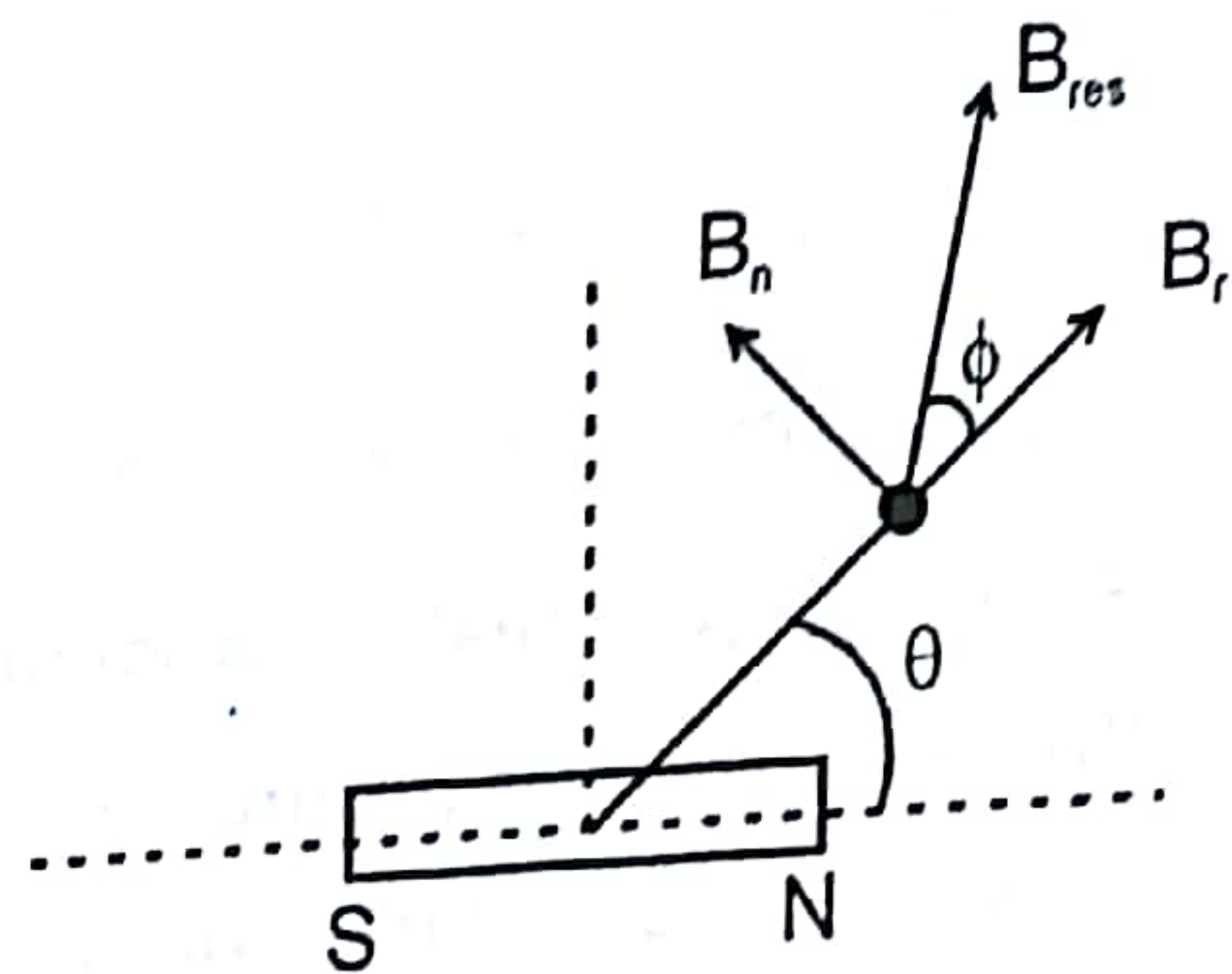
At General point :

$$B_r = 2\left(\frac{\mu_0}{4\pi}\right) \frac{M \cos \theta}{r^3}$$

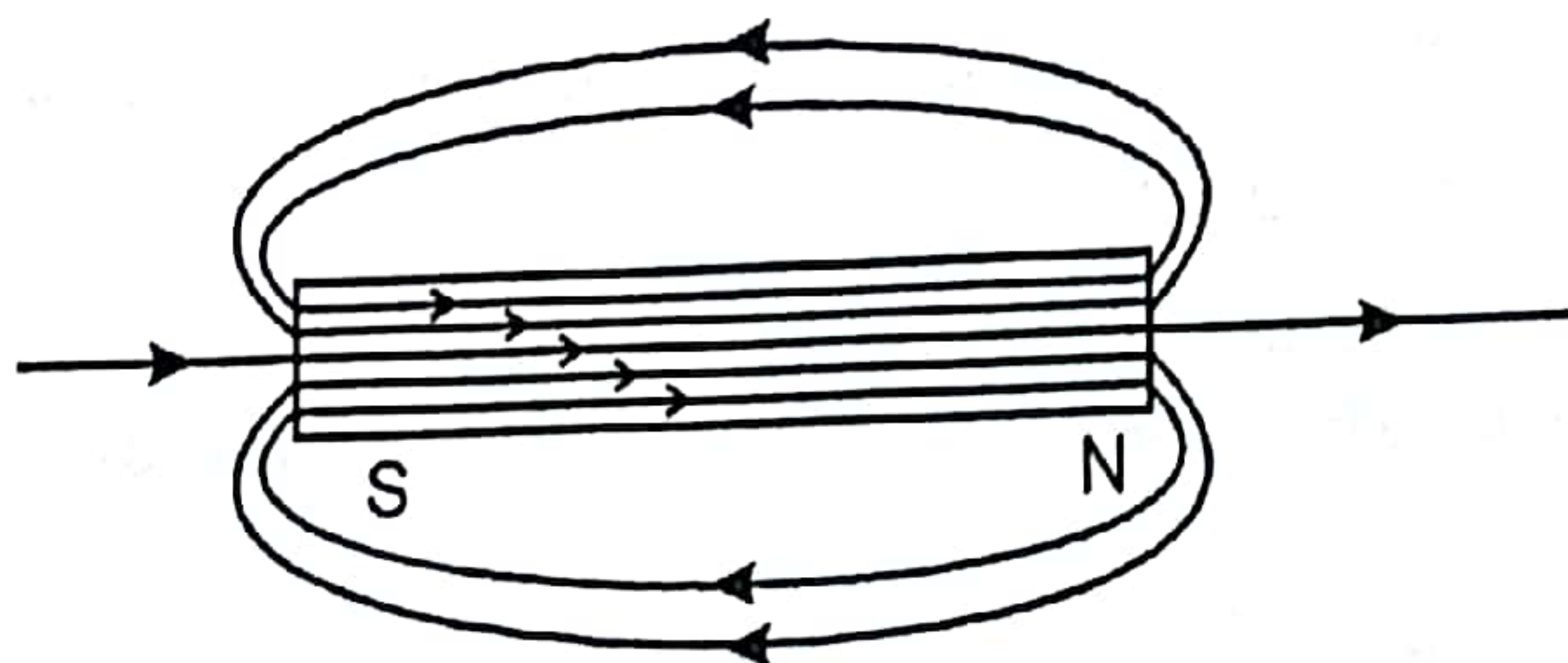
$$B_n = 2\left(\frac{\mu_0}{4\pi}\right) \frac{M \sin \theta}{r^3}$$

$$B_{res} = \frac{\mu_0 M}{4\pi r^3} \sqrt{1 + 3 \cos^2 \theta}$$

$$\tan \phi = \frac{B_n}{B_r} = \frac{\tan \theta}{2}$$



Magnetic lines of force of a bar magnet :

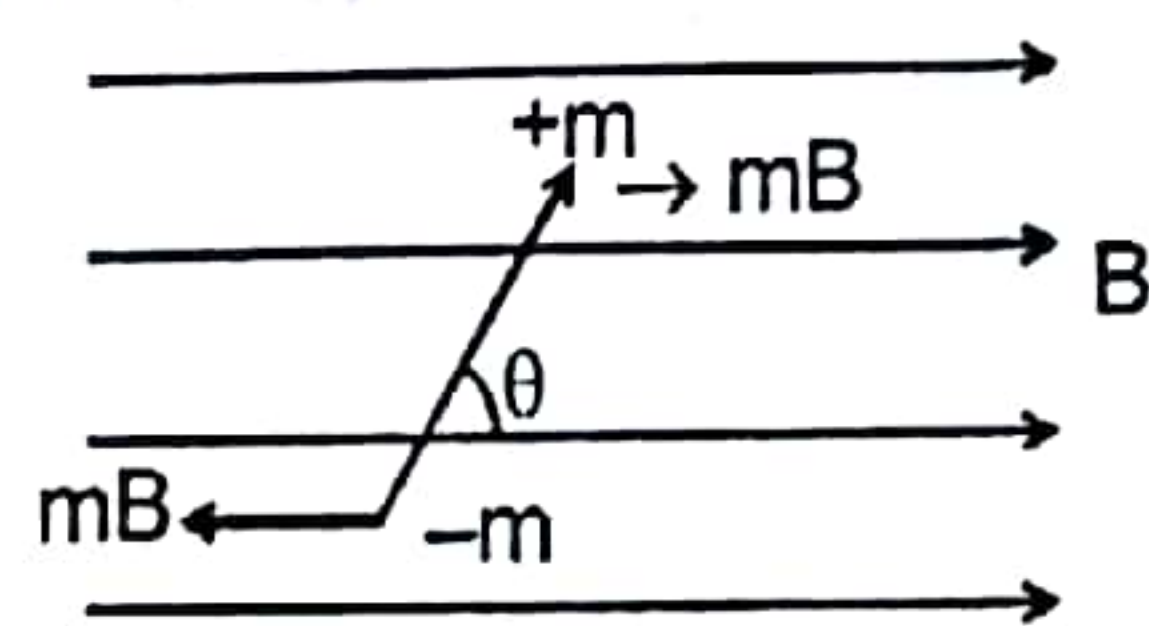


1.3 Magnet in an external uniform magnetic field :

(same as case of electric dipole)

$$F_{res} = 0 \quad (\text{for any angle})$$

$$\tau = MB \sin \theta$$



*here θ is angle between \vec{B} and \vec{M}

Note : • $\vec{\tau}$ acts such that it tries to make $\vec{M} \times \vec{B}$.

• $\vec{\tau}$ is same about every point of the dipole it's potential energy is

$$U = -MB \cos \theta = -\vec{M} \cdot \vec{B}$$

$\theta = 0^\circ$ is stable equilibrium

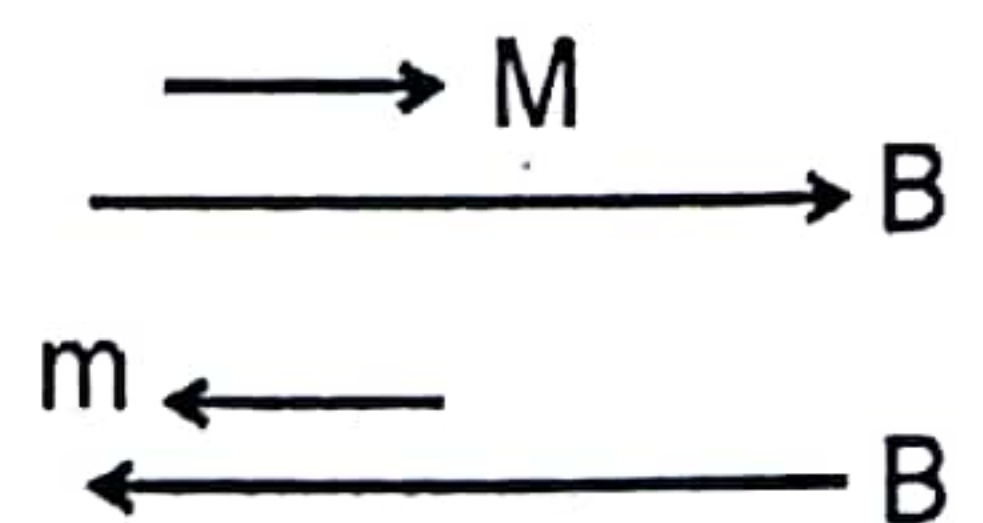
$\theta = \pi$ is unstable equilibrium

for small ' θ ' the dipole performs SHM about $\theta = 0^\circ$ position

$$\tau = -MB \sin \theta ;$$

$$I \alpha = -MB \sin \theta$$

for small θ , $\sin \theta \approx \theta$





$$\alpha = -\left(\frac{MB}{I}\right)\theta$$

Angular frequency of SHM

$$\omega = \sqrt{\frac{MB}{I}} = \frac{2\pi}{T}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{I}{MB}}$$

here $I = I_{cm}$ if the dipole is free to rotate
 $= I_{hinge}$ if the dipole is hinged

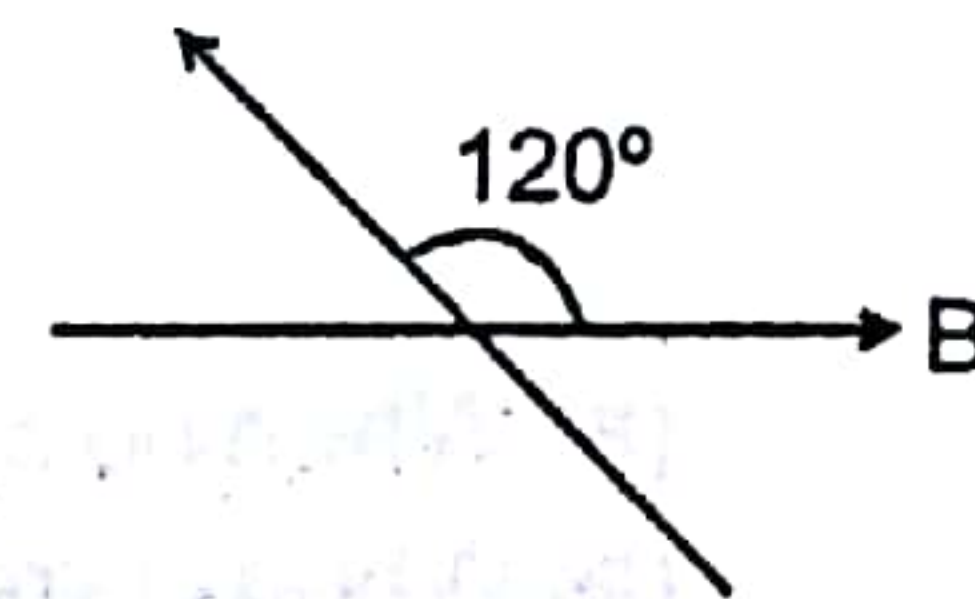
Solved Examples

Example 1. A bar magnet having a magnetic moment of 1.0×10^{-4} J/T is free to rotate in a horizontal plane. A horizontal magnetic field $B = 4 \times 10^{-5}$ T exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60° from the field.

Solution : The work done by the external agent = change in potential energy

$$\begin{aligned} &= (-MB \cos\theta_2) - (-MB \cos\theta_1) \\ &= -MB (\cos 60^\circ - \cos 0^\circ) \\ &= \frac{1}{2} MB = \frac{1}{2} \times (1.0 \times 10^{-4} \text{ J/T}) (4 \times 10^{-5} \text{ T}) = 0.2 \text{ J} \end{aligned}$$

Example 2. A magnet of magnetic dipole moment M is released in a uniform magnetic field of induction B from the position shown in the figure. Find :



- Its kinetic energy at $\theta = 90^\circ$
- its maximum kinetic energy during the motion.
- will it perform SHM? oscillation? Periodic motion? What is its amplitude?

Solution : (i) Apply energy conservation at $\theta = 120^\circ$ and $\theta = 90^\circ$

$$\begin{aligned} &- MB \cos 120^\circ + 0 \\ &= - MB \cos 90^\circ + (\text{K.E.}) \end{aligned}$$

$$\text{KE} = \frac{MB}{2} \quad \text{Ans.}$$

(ii) K.E. will be maximum where P.E. is minimum. P.E. is minimum at $\theta = 0^\circ$. Now apply energy conservation between $\theta = 120^\circ$ and $\theta = 0^\circ$.

$$\begin{aligned} &- mB \cos 120^\circ + 0 \\ &= -mB \cos 0^\circ + (\text{KE})_{\max} \end{aligned}$$

$$(\text{KE})_{\max} = \frac{3}{2} MB \quad \text{Ans.}$$

The K.E. is max at $\theta = 0^\circ$ can also be proved by torque method. From $\theta = 120^\circ$ to $\theta = 0^\circ$ the torque always acts on the dipole in the same direction (here it is clockwise) so its K.E. keeps on increases till $\theta = 0^\circ$. Beyond that τ reverses its direction and then K.E. starts decreasing

$\therefore \theta = 0^\circ$ is the orientation of M to here the maximum K.E.

(iii) Since ' θ ' is not small.

\therefore the motion is not S.H.M. but it is oscillatory and periodic amplitude is 120° .





1.4 Magnet in an External Nonuniform Magnetic Field :

No special formula are applied in such problems. Instead see the force on individual poles and calculate the resultant force torque on the dipole.

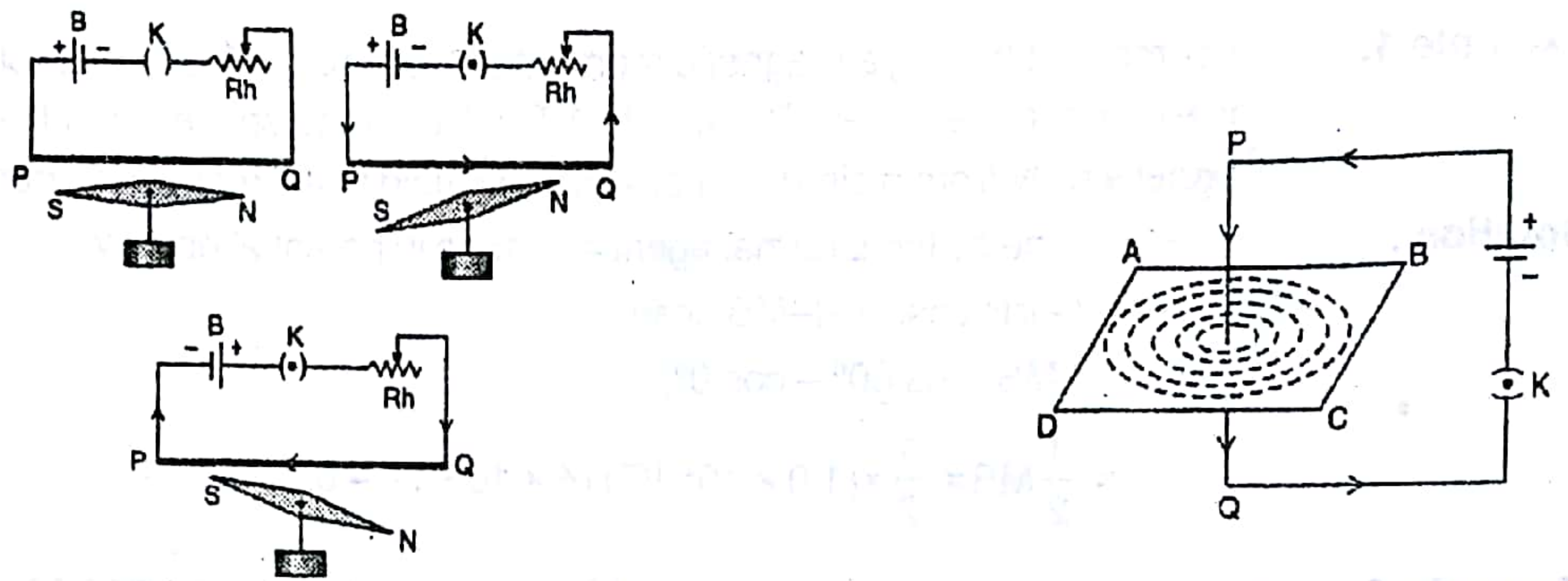


2. Magnetic effects of current (and moving charge)

It was observed by **OERSTED** that a current carrying wire produces magnetic field nearby it. It can be tested by placing a magnet in the near by space, it will show some movement (deflection or rotation or displacement). This observation shows that current or moving charge produces magnetic field.

OERSTED EXPERIMENT AND OBSERVATIONS

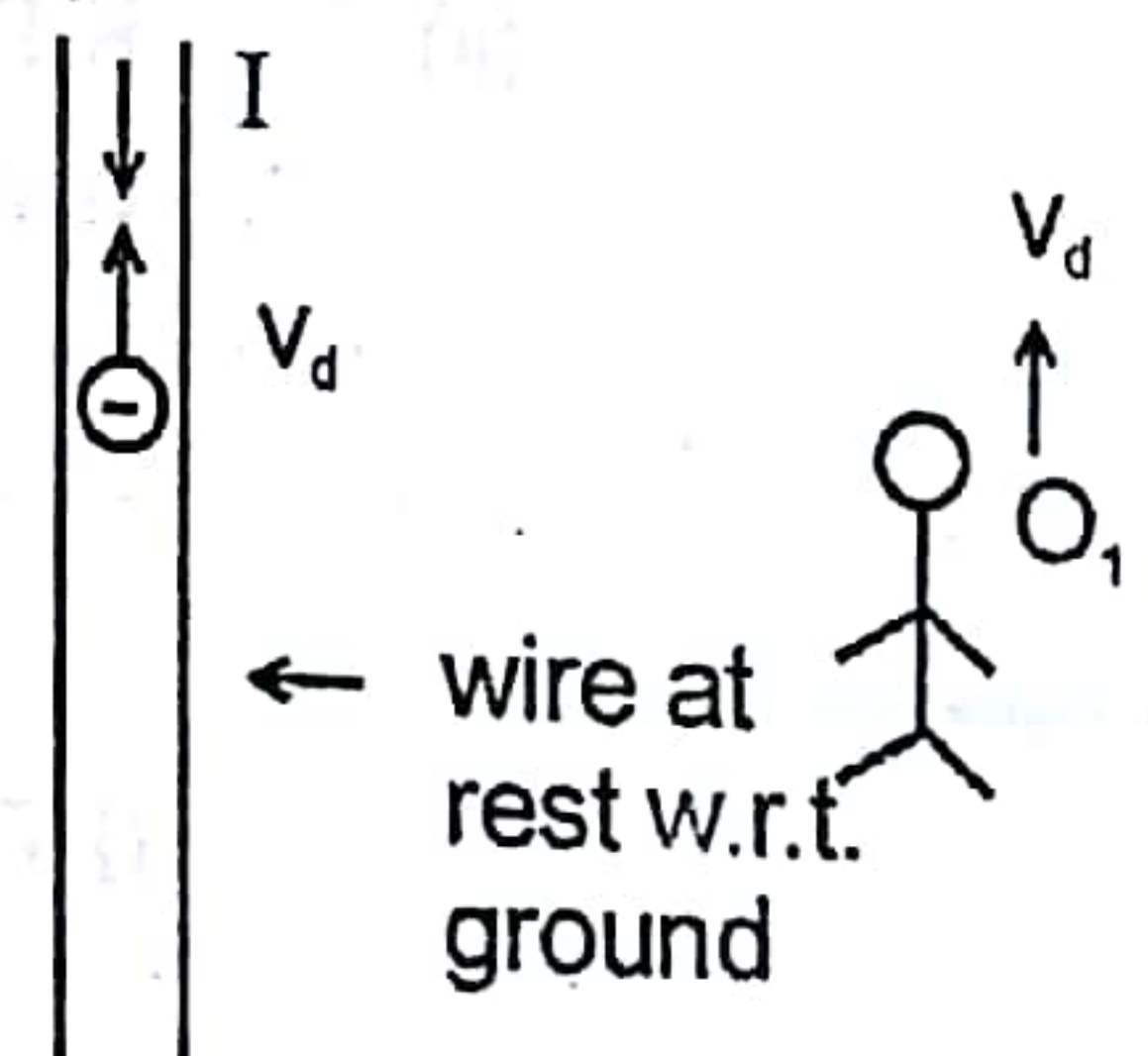
(i) Oersted performed an experiment in 1819 whose arrangement is shown in the following figure. Following observations were noted from this experiment.



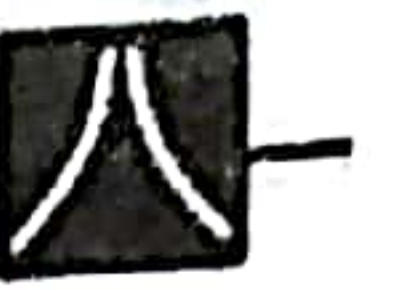
- (a) When no current is passed through the wire AB, the magnetic needle remains undeflected.
 - (b) When current is passed through the wire AB, the magnetic needle gets deflected in a particular direction and the deflection increases as the current increases.
 - (c) When the current flowing in the wire is reversed, the magnetic needle gets deflected in the opposite direction and its deflection increases as the current increases.
- (ii) Oersted concluded from this experiment that on passing a current through the conducting wire, a magnetic field is produced around this wire. As a result the magnetic needle is deflected. This phenomenon is called magnetic effect of current.
- (iii) From another experiment, it is found that the magnetic lines of force due to the current flowing in the wire are in the form of concentric circles around the conducting wire.

2.1 Frame Dependence of \vec{B} .

- (a) The motion of anything is a relative term. A charge may appear at rest by an observer (say O_1) and moving at same velocity \vec{v}_1 with respect to observer O_2 and at velocity \vec{v}_2 with respect to observers O_3 then \vec{B} due to that charge w.r.t. O_1 will be zero and w.r.t. O_2 and O_3 it will be \vec{B}_1 and \vec{B}_2 (that means different).



- (b) In a current carrying wire electrons move in the opposite direction to that of the current and +ve ions (of the metal) are static w.r.t. the wire. Now if some observer (O_1) moves with velocity V_d in the direction of motion of the electrons then electrons will have zero velocity and +ve ions will have velocity V_d in the downward direction w.r.t. O_1 . The density (n) of +ve ions is same as the density of free electrons and their charges are of the same magnitudes. So, w.r.t. O_1 electrons will produce zero magnetic field but +ve ions will produce +ve same \vec{B} due to the current carrying wire does not depend on the reference frame (this is true for any velocity of the observer).



(c) \vec{B} due to magnet :

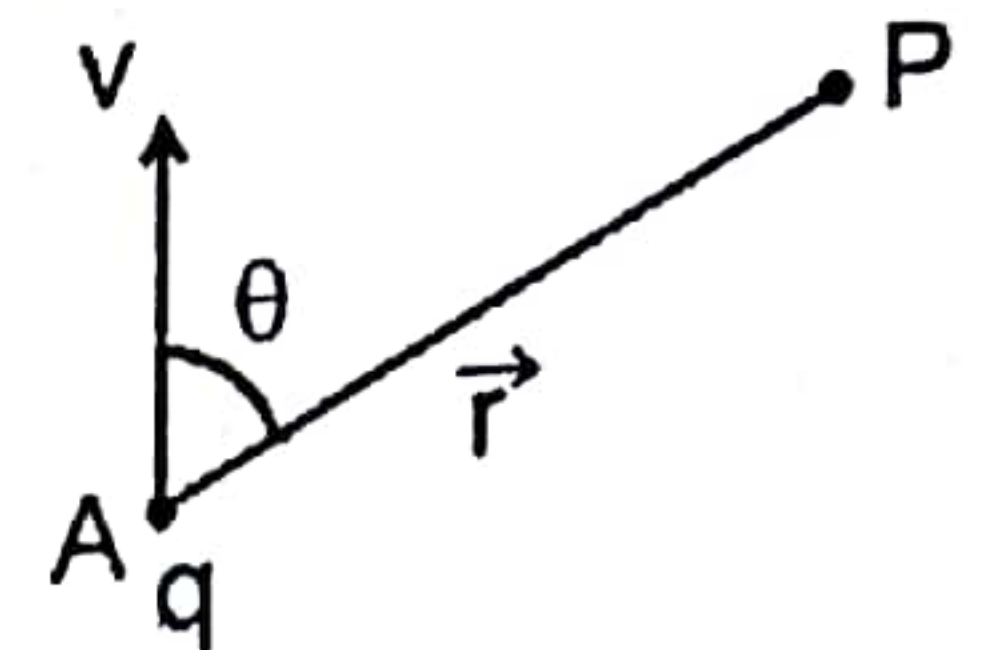
\vec{B} produced by the magnet does not contain the term of velocity

So, we can say that the \vec{B} due magnet does not depend on frame.

2.2 \vec{B} due to a point charge :

A charge particle 'q' has velocity v as shown in the figure. It is at position 'A' at some time. \vec{r} is the position vector of point 'P' w.r. to position of the charge. Then \vec{B} at P due to q is

$$B = \left(\frac{\mu_0}{4\pi} \right) \frac{qv \sin \theta}{r^2} ; \text{ here } \theta = \text{angle between } \vec{v} \text{ and } \vec{r}$$



$$\vec{B} = \left(\frac{\mu_0}{4\pi} \right) \frac{q\vec{v} \times \vec{r}}{r^3} ; q \text{ with sign } \vec{B} \perp \vec{v} \text{ and also } \vec{B} \perp \vec{r}.$$

Direction of \vec{B} will be found by using the rules of vector product.

2.3 Biot-savart's law (\vec{B} due to a wire)

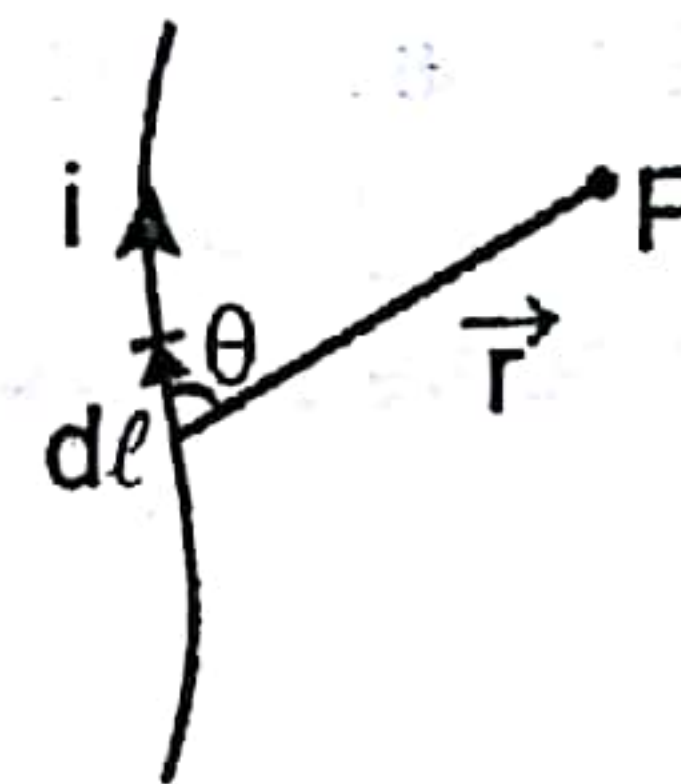
It is a experimental law. A current 'i' flows in a wire (may be straight or curved). Due to 'dℓ' length of the wire the magnetic field at 'P' is

$$dB \propto i d\ell$$

$$\propto \frac{1}{r^2}$$

$$\propto \sin \theta$$

$$\Rightarrow dB \propto \frac{id\ell \sin \theta}{r^2}$$



$$dB = \left(\frac{\mu_0}{4\pi} \right) \frac{id\ell \sin \theta}{r^2} \Rightarrow \vec{dB} = \left(\frac{\mu_0}{4\pi} \right) \frac{i d\vec{\ell} \times \vec{r}}{r^3}$$

here \vec{r} = position vector of the test point w.r.t. $d\vec{\ell}$

$$\theta = \text{angle between } d\vec{\ell} \text{ and } \vec{r} . \text{ The resultant } \vec{B} = \int d\vec{B}$$

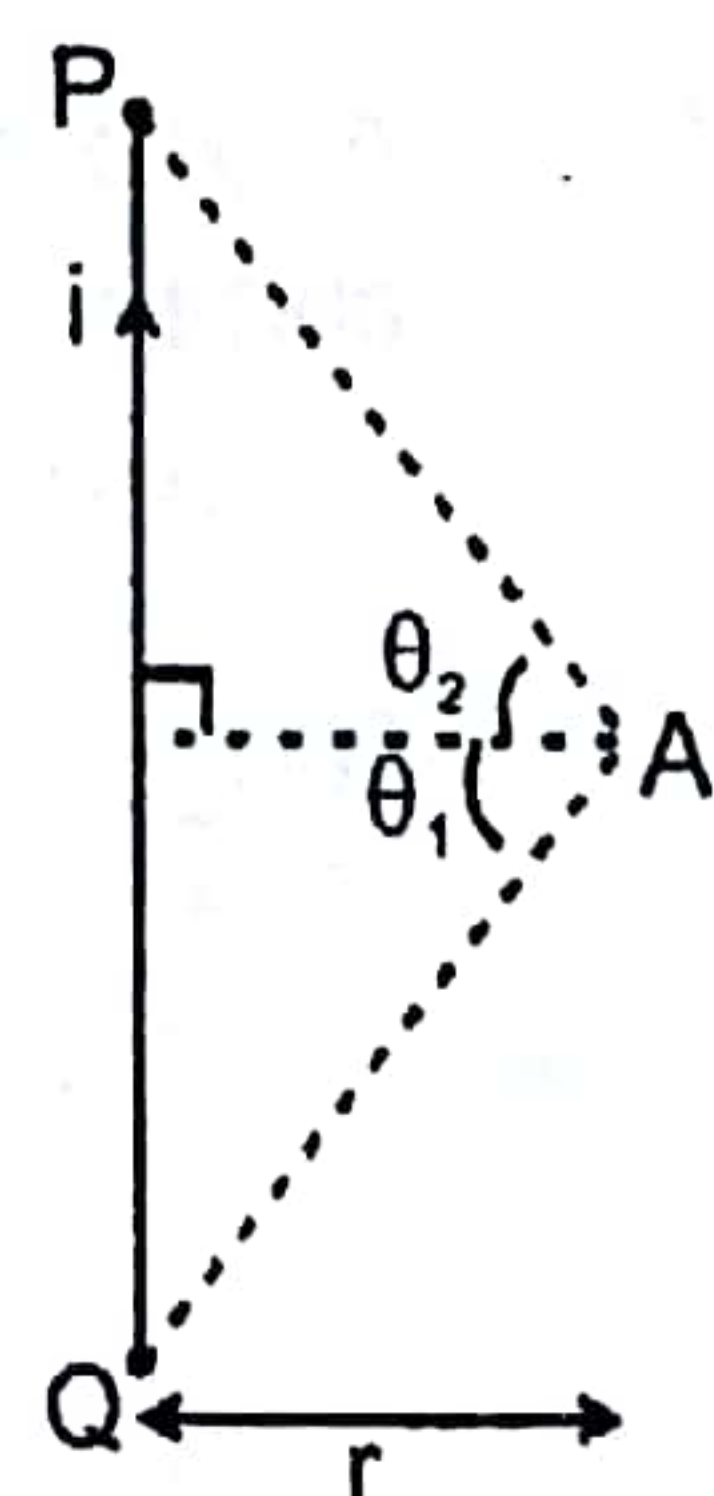
Using this fundamental formula we can derive the expression of \vec{B} due a long wire.

2.3.1 \vec{B} due to a straight wire :

Due to a straight wire 'PQ' carrying a current 'i' the \vec{B} at A is given by the formula

$$B = \frac{\mu_0 I}{4\pi r} (\sin \theta_1 + \sin \theta_2)$$

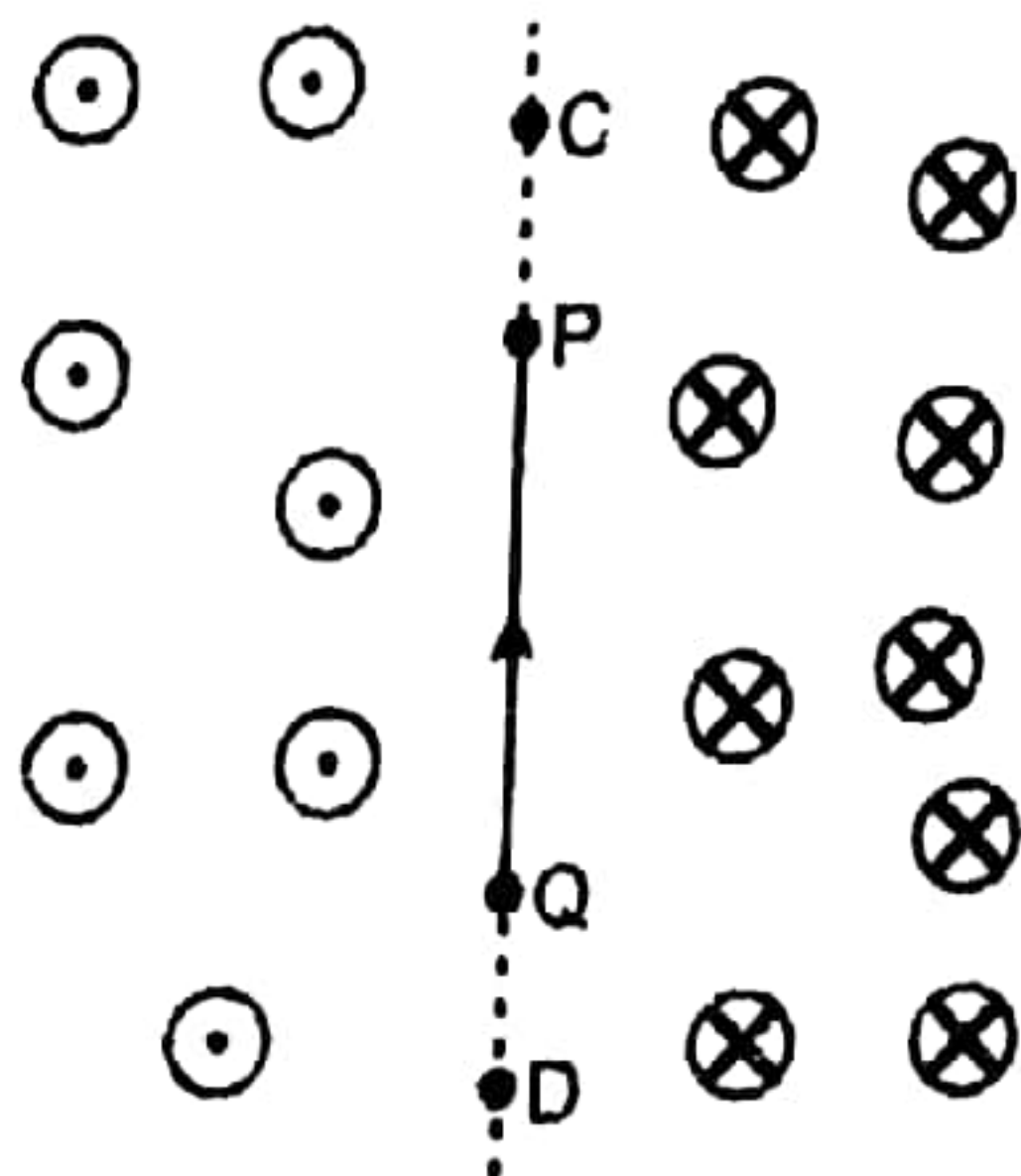
(Derivation can be seen in a standard text book like your school book or concept of physics of HCV part-II)



Direction :

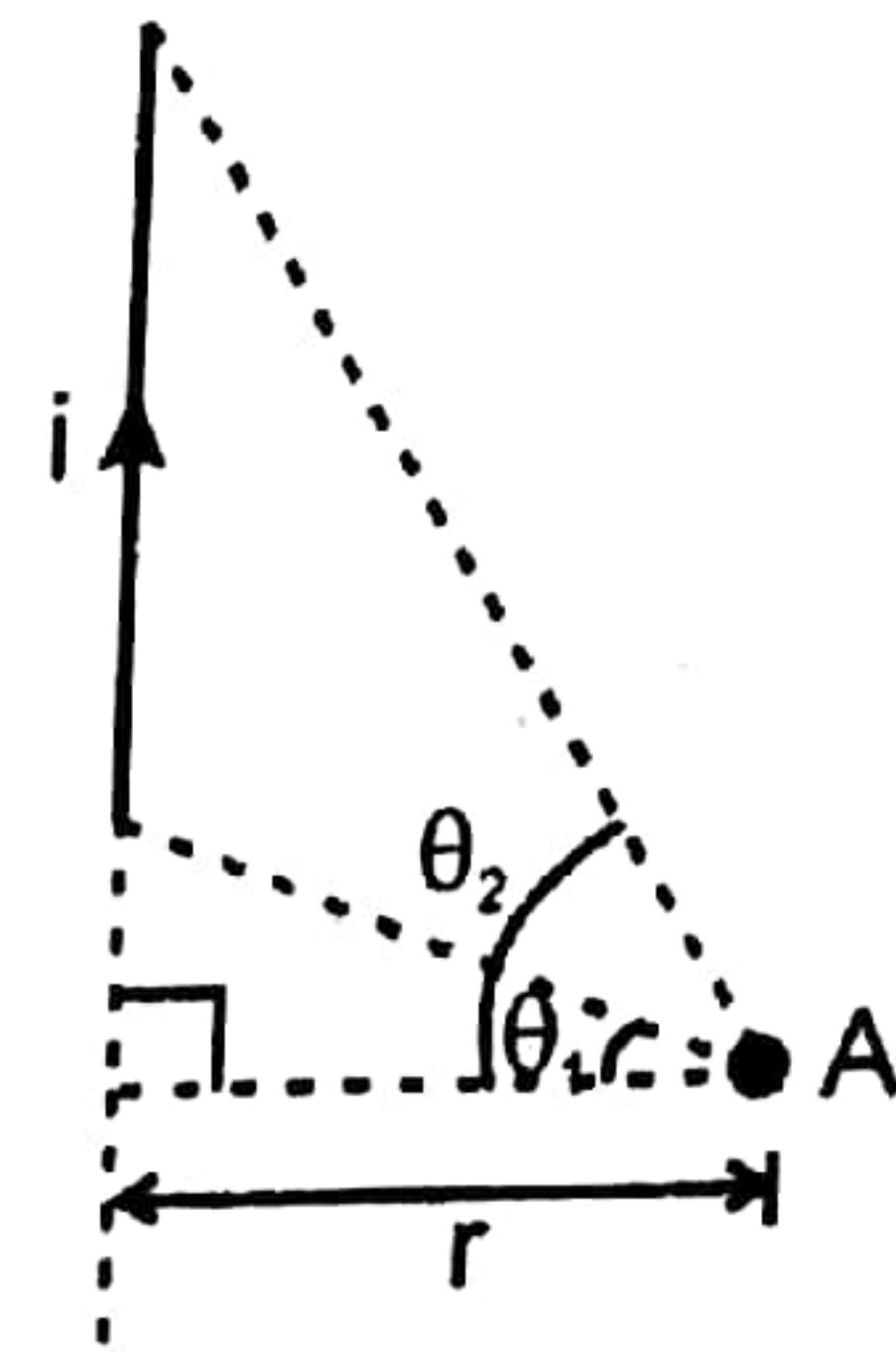
Due to every element of 'PQ' \vec{B} at A is directed inwards. So its resultant is also directed inwards. It is represented by (x)

The direction of \vec{B} at various points is shown in the figure shown.



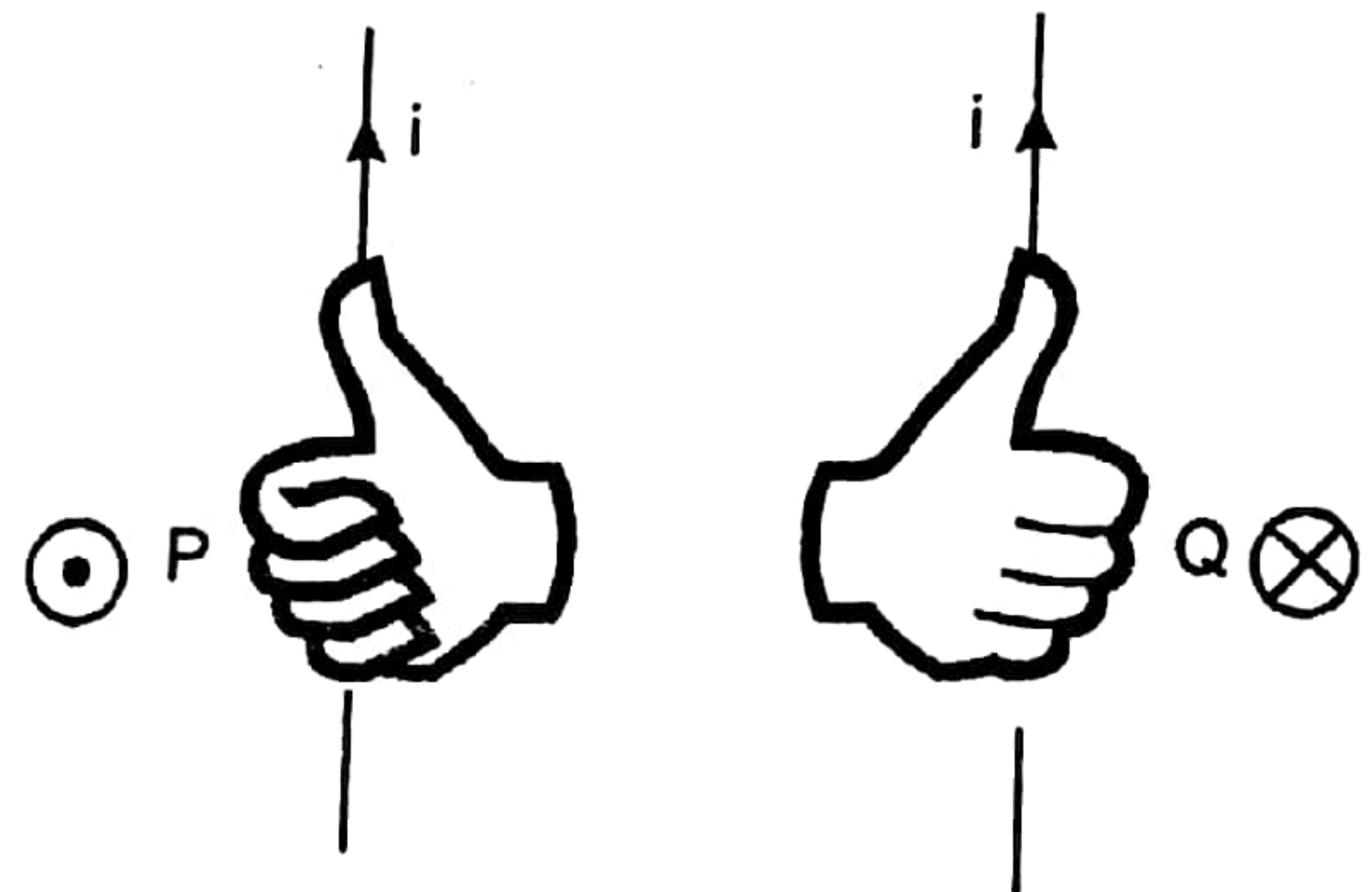
At points 'C' and 'D' $\vec{B} = 0$ (think how).
For the case shown in figure

$$B \text{ at } A = \frac{\mu_0 i}{4\pi r} (\sin \theta_2 - \sin \theta_1) (\otimes)$$

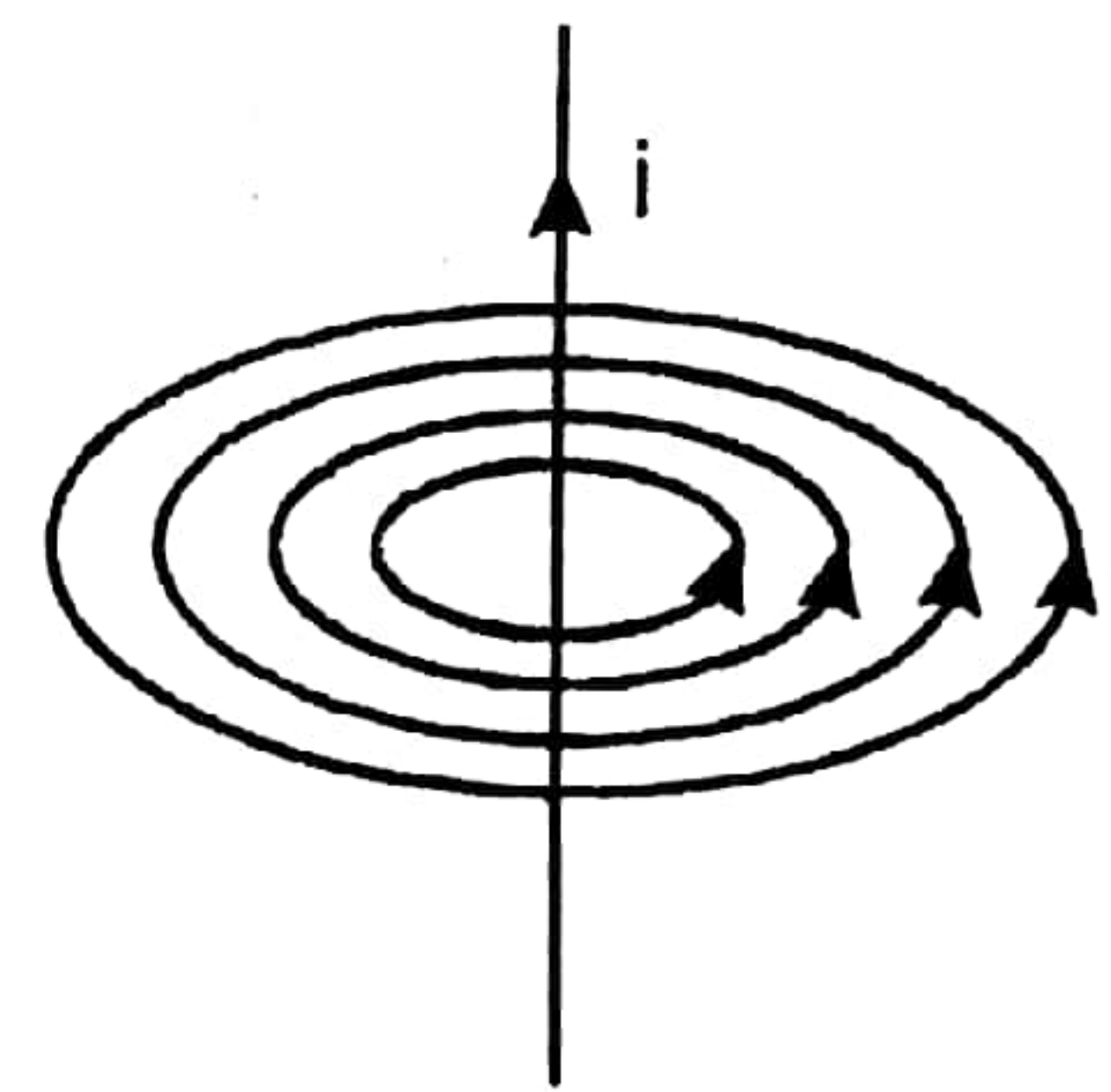


Shortcut for Direction :

The direction of the magnetic field at a point P due to a straight wire can be found by a slight variation in the right-hand thumb rule. If we stretch the thumb of the right hand along the current and curl our fingers to pass through the point P, the direction of the fingers at P gives the direction of the magnetic field there.



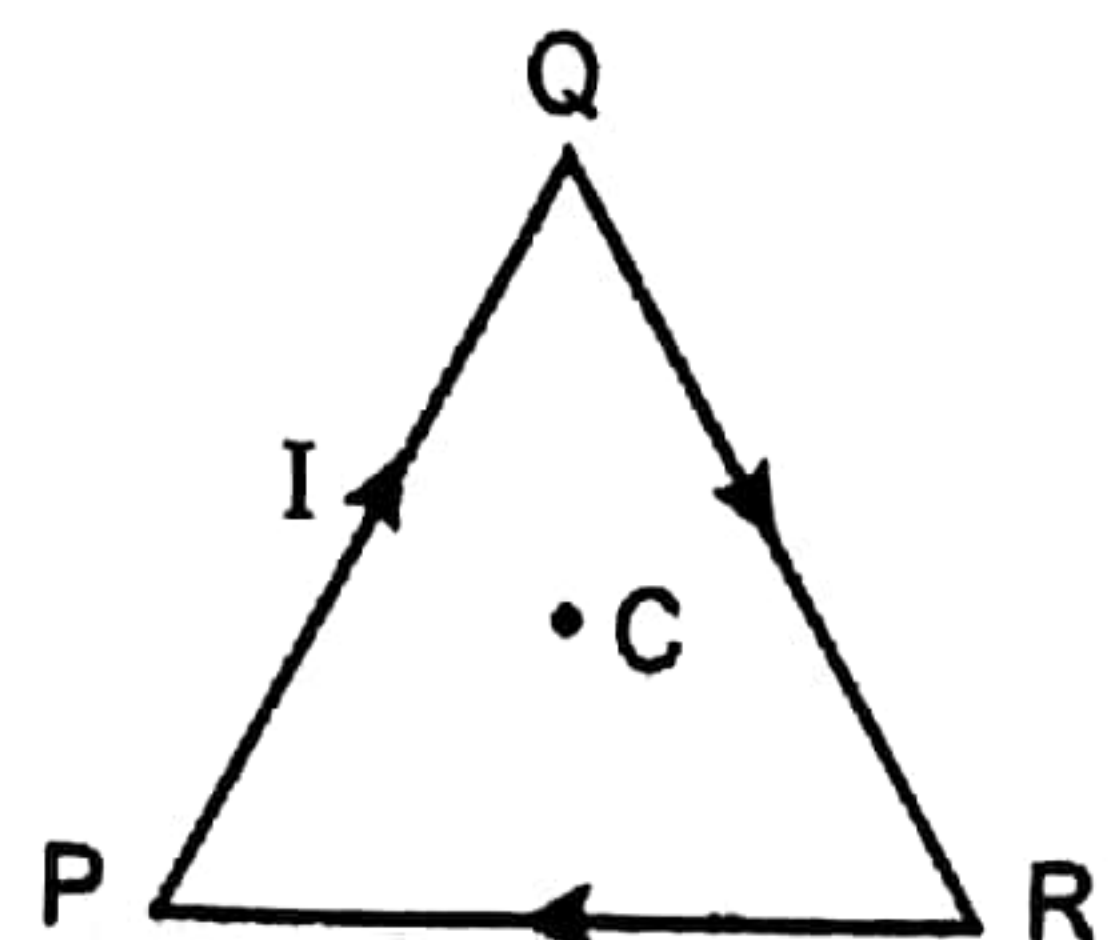
We can draw magnetic field lines on the pattern of electric field lines. A tangent to a magnetic field line gives the direction of the magnetic field existing at that point. For a straight wire, the field lines are concentric circles with their centres on the wire and in the plane perpendicular to the wire. There will be infinite number of such lines in the planes parallel to the above mentioned plane.

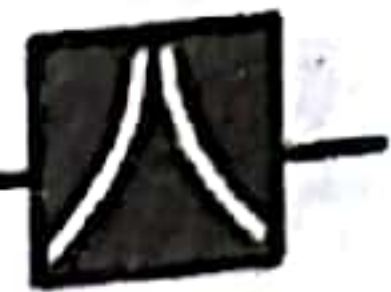


Solved Examples

Example 3. A loop in the shape of an equilateral triangle of side 'a' carries a current I as shown in the figure. Find out the magnetic field at the centre 'C' of the triangle.

Answer : $\frac{9\mu_0 i}{2\pi a}$

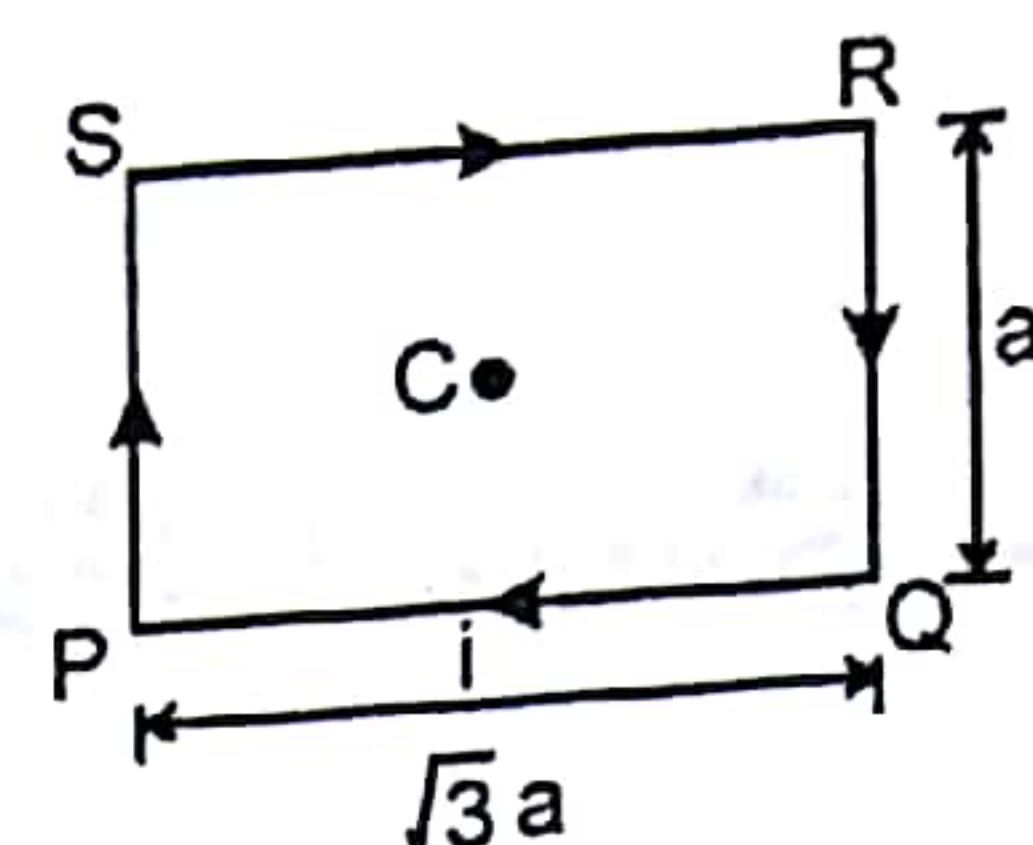




Solution : $B = B_1 + B_2 + B_3 = 3B_1$

$$= 3 \frac{\mu_0}{4\pi} \times \frac{i}{\left(\frac{a}{2\sqrt{3}}\right)} \times (\sin 60^\circ + \sin 60^\circ) = \frac{9\mu_0 i}{2\pi a}$$

Example 4. Find resultant magnetic field at 'C' in the figure shown.



Solution :

It is clear that 'B' at 'C' due all the wires is directed \otimes . Also B at 'C' due PQ and SR is same.

Also due to QR and PS is same

$$\therefore B_{res} = 2(B_{PQ} + B_{SP})$$

$$B_{PQ} = \frac{\mu_0 i}{4\pi \frac{a}{2}} (\sin 60^\circ + \sin 60^\circ),$$

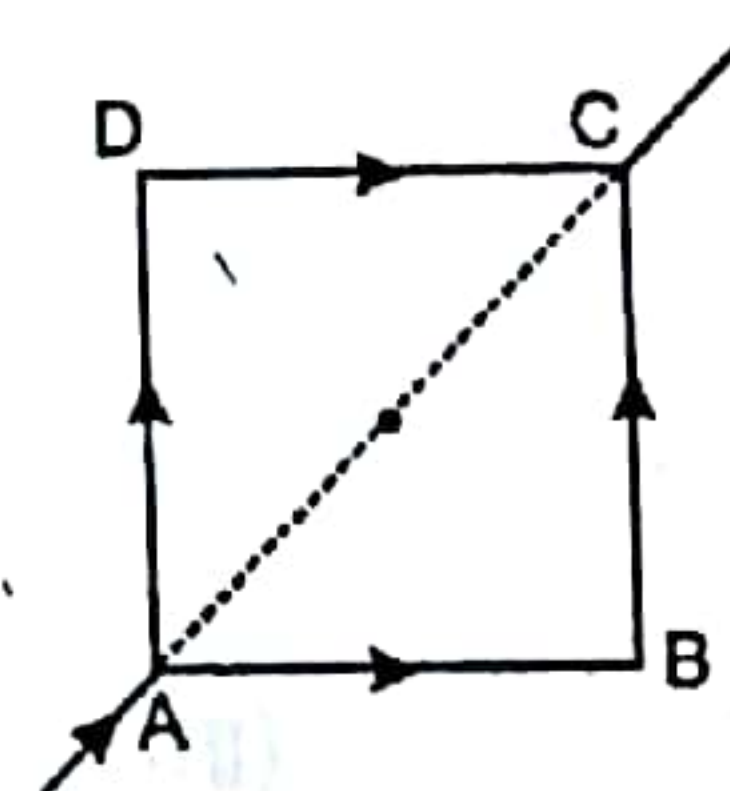
$$B_{sp} = \frac{\mu_0 i}{4\pi \frac{\sqrt{3}a}{2}} (\sin 30^\circ + \sin 30^\circ) \Rightarrow B_{res} = 2 \left(\frac{\sqrt{3} \mu_0 i}{2\pi a} + \frac{\mu_0 i}{2\pi a \sqrt{3}} \right) = \frac{4\mu_0 i}{\sqrt{3}\pi a}$$

Example 5. Two long wires are kept along x and y axes they carry currents I & I respectively in +ve x and +ve y directions respectively. Find \vec{B} at a point (0, 0, d).

Answer : $\frac{\mu_0 I}{2\pi d} (\hat{i} - \hat{j})$

Solution : $\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0}{2\pi} \frac{i}{d} ((-\hat{j}) + \frac{\mu_0}{2\pi d} (\hat{i})) = \frac{\mu_0 I}{2\pi d} (\hat{i} - \hat{j})$

Example 6. Figure shows a square loop made from a uniform wire. Find the magnetic field at the centre of the square if a battery is connected between the points A and C.



Solution :

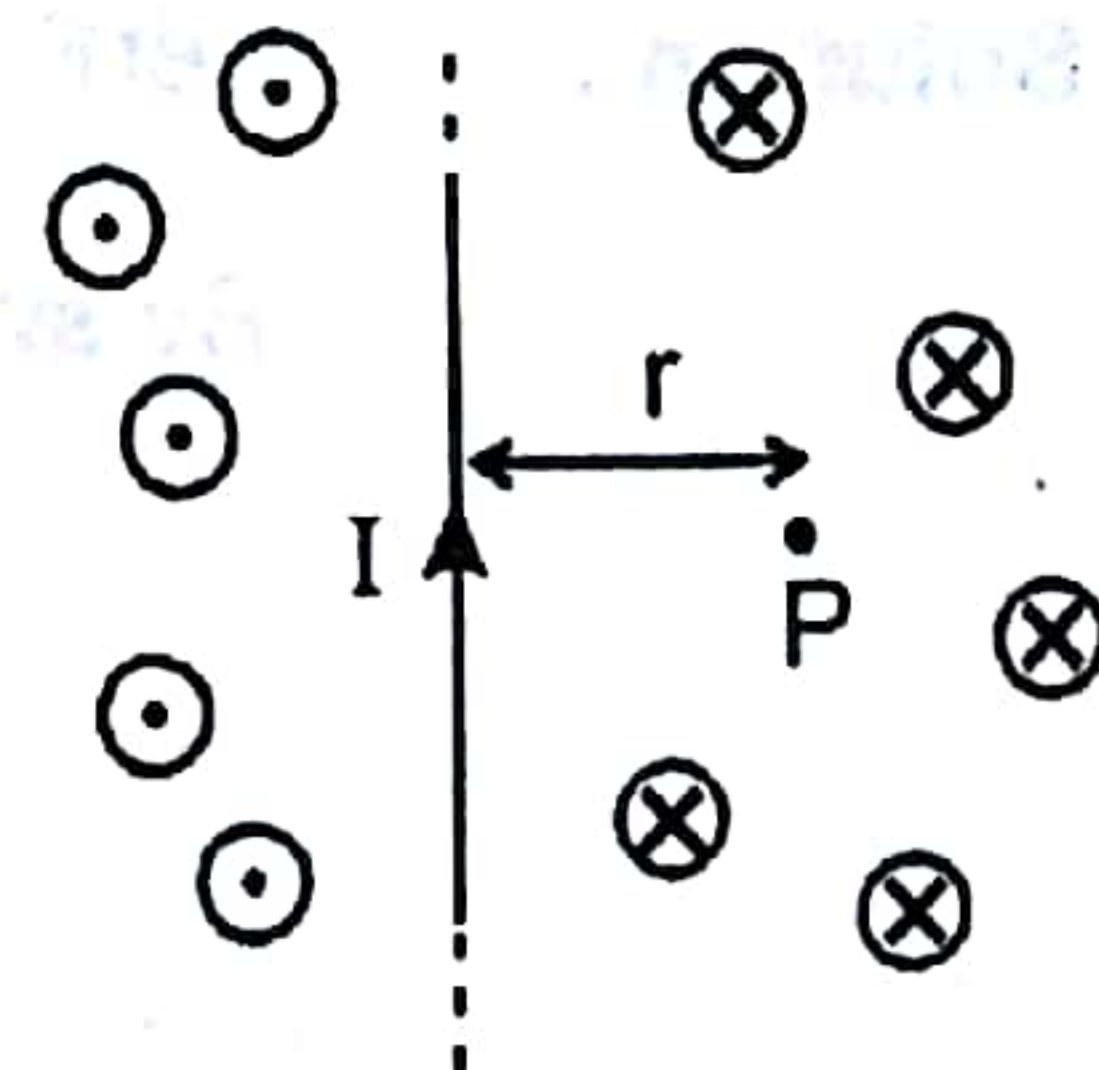
The current will be equally divided at A. The fields at the centre due to the currents in the wires AB and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AD and BC will be zero. Hence, the net field at the centre will be zero.



Special case :

(i) If the wire is infinitely long then the magnetic field at 'P' (as shown in the figure) is given by (using $\theta_1 = \theta_2 = 90^\circ$ and the formula of 'B' due to straight wire)

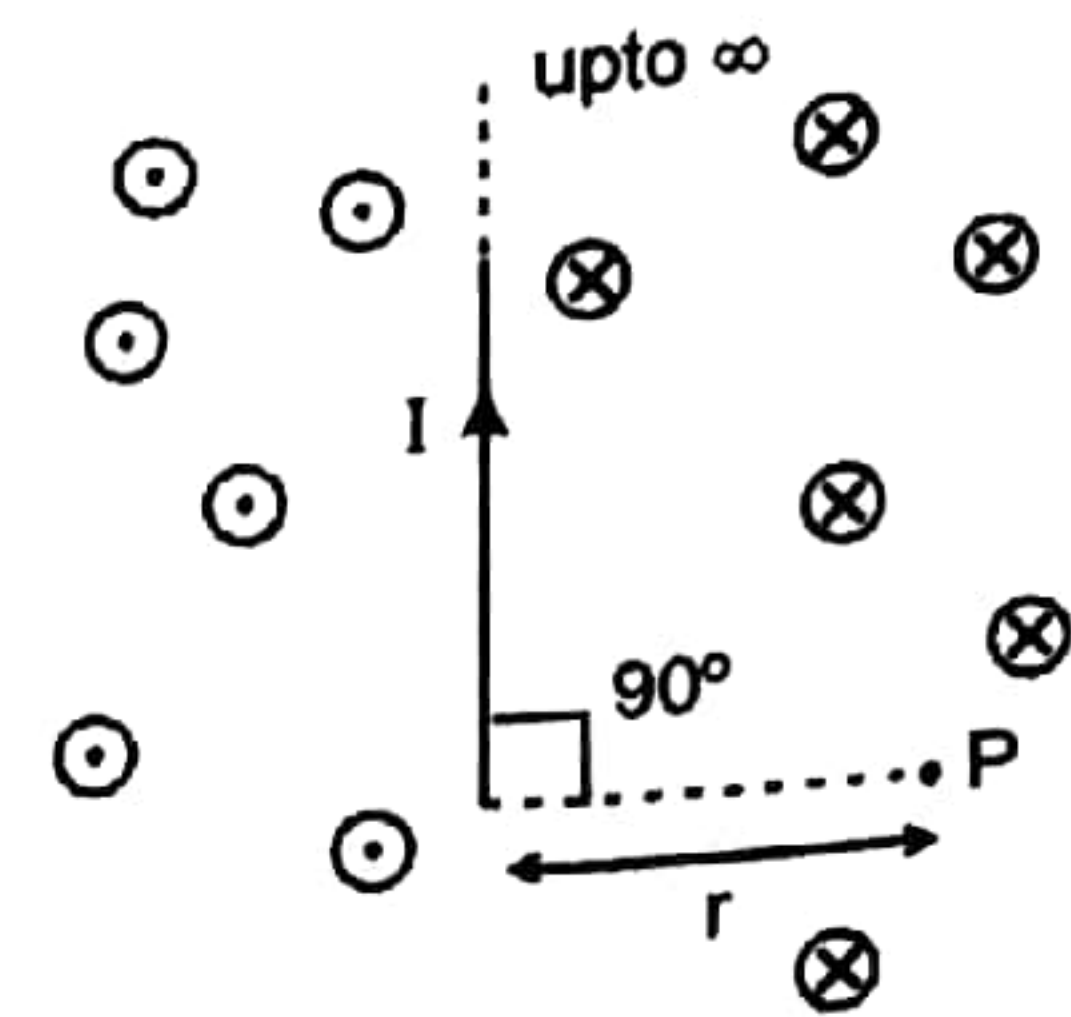
$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow B \propto \frac{I}{r}$$



The direction of \vec{B} at various is as shown in the figure. The magnetic lines of force will be concentric circles around the wire (as shown earlier)

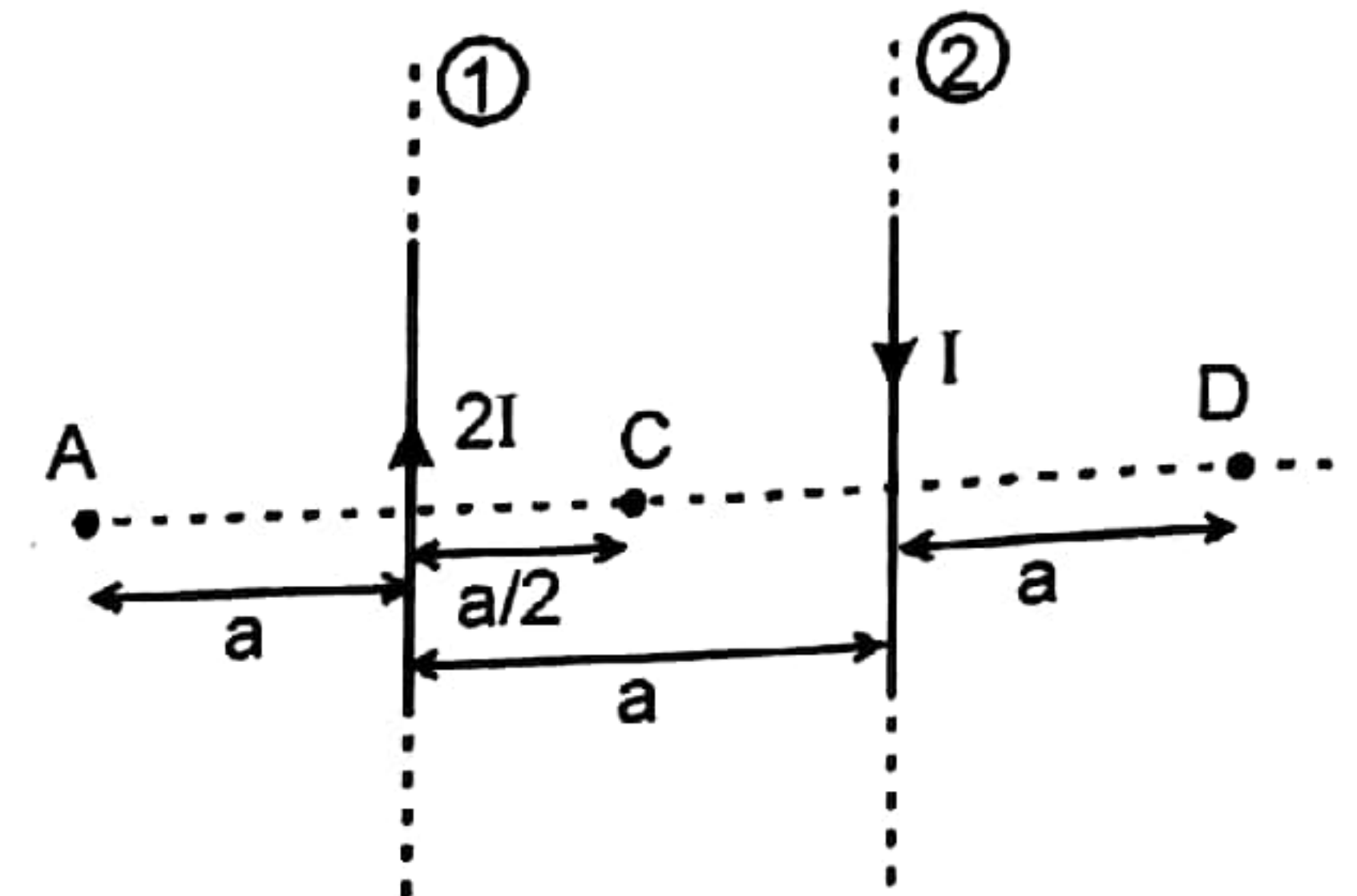
- (ii) If the wire is infinitely long but 'P' is as shown in the figure. The direction of \vec{B} at various points is as shown in the figure. At 'P'

$$B = \frac{\mu_0 I}{4\pi r}$$



Solved Examples

Example 7. In the figure shown there are two parallel long wires (placed in the plane of paper) are carrying currents $2I$ and I consider points A, C, D on the line perpendicular to both the wires and also in the plane of the paper. The distances are mentioned. Find (i) \vec{B} at A, C, D



- (ii) position of point on line A C D where \vec{B} is 0.

Solution : (i) Let us call \vec{B} due to (1) and (2) as \vec{B}_1 and \vec{B}_2 respectively. Then

at A: \vec{B}_1 is \odot and \vec{B}_2 is \otimes

$$B_1 = \frac{\mu_0 2I}{2\pi a} \text{ and } B_2 = \frac{\mu_0 I}{2\pi 2a}$$

$$\therefore B_{res} = B_1 - B_2 = \frac{3}{4} \frac{\mu_0 I}{\pi a} \odot$$

Ans.

at C: \vec{B}_1 is \otimes and \vec{B}_2 also \otimes

$$\therefore B_{res} = B_1 + B_2 = \frac{\mu_0 2I}{2\pi \frac{a}{2}} + \frac{\mu_0 I}{2\pi \frac{a}{2}} = \frac{6\mu_0 I}{2\pi a} = \frac{3\mu_0 I}{\pi a} \otimes$$

Ans.

at D: \vec{B}_1 is \otimes and \vec{B}_2 is \odot and both are equal in magnitude.

$$\therefore B_{res} = 0$$

Ans.

- (ii) It is clear from the above solution that $B = 0$ at point 'D'.

Example 8. In the figure shown two long wires W_1 and W_2 each carrying current I are placed parallel to each other and parallel to z-axis. The direction of current in W_1 is outward and in W_2 it is inwards. Find the \vec{B} at 'P' and 'Q'.

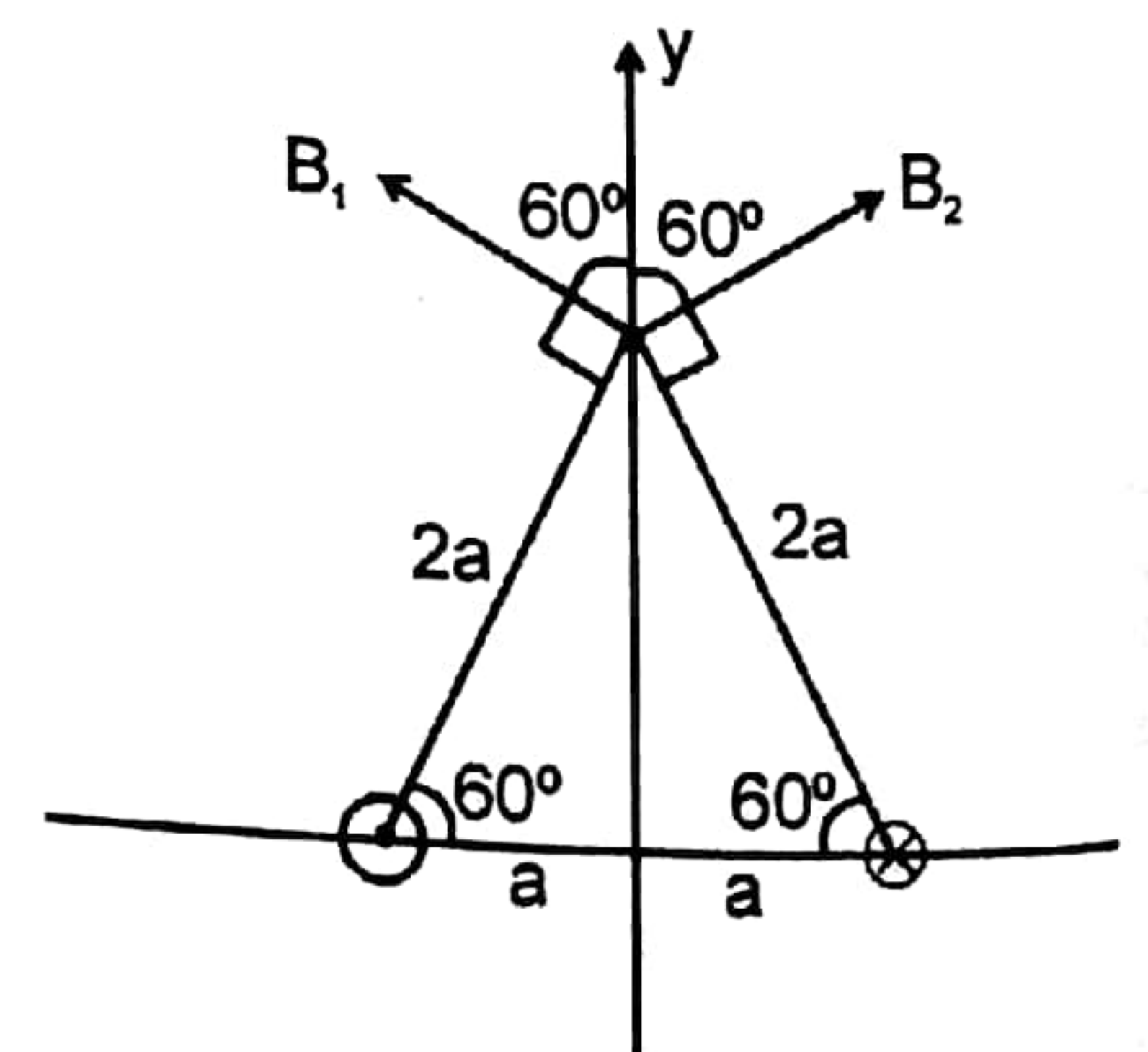
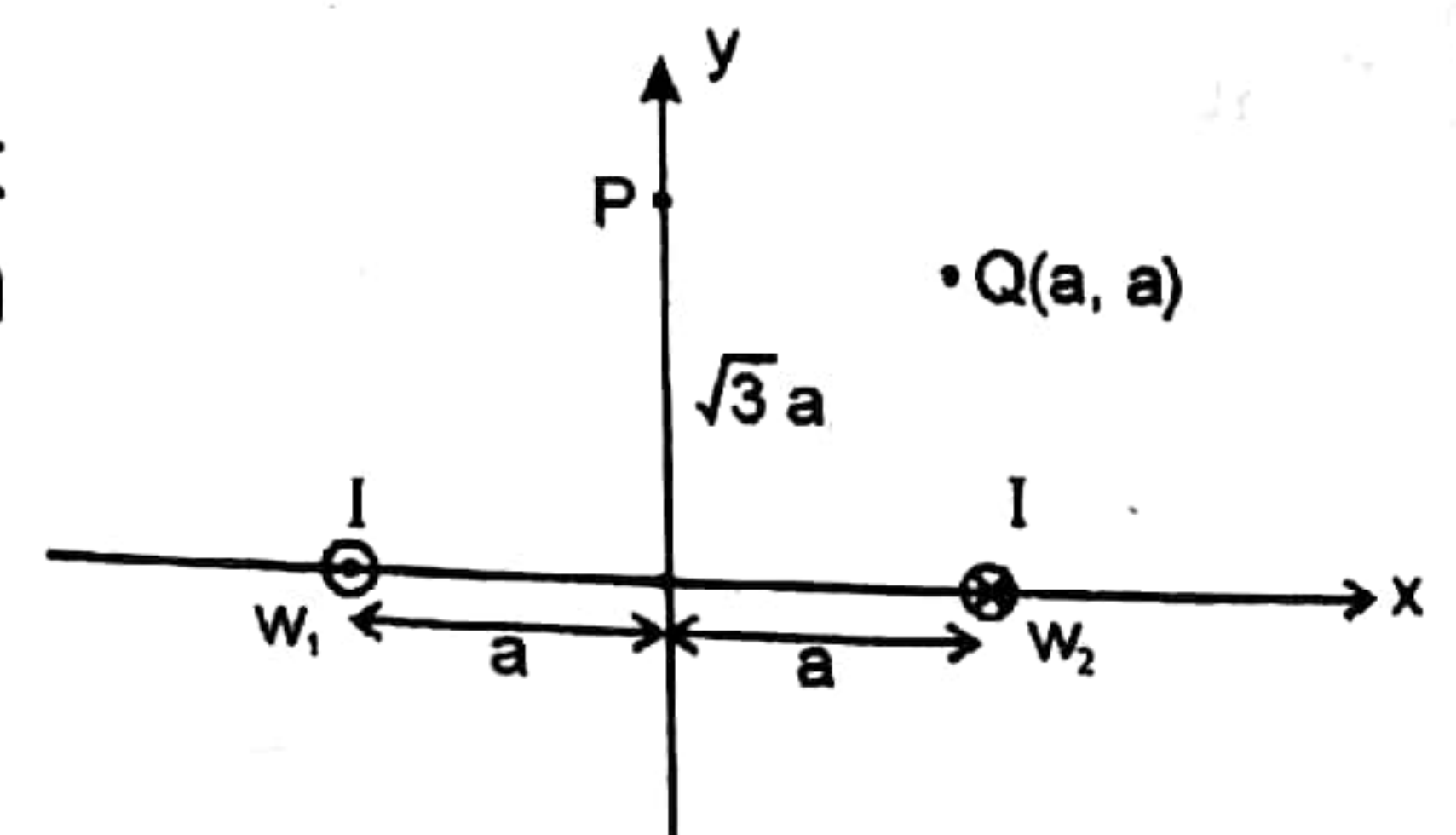
Solution : Let \vec{B} due to W_1 be \vec{B}_1 and due to W_2 be \vec{B}_2 .

By symmetry $|\vec{B}_1| = |\vec{B}_2| = B$

$$B_p = 2 B \cos 60^\circ = B = \frac{\mu_0 I}{2\pi 2a} = \frac{\mu_0 I}{4\pi a}$$

$$\therefore \vec{B}_p = \frac{\mu_0 I}{4\pi a} \hat{j} \quad \text{Ans.}$$

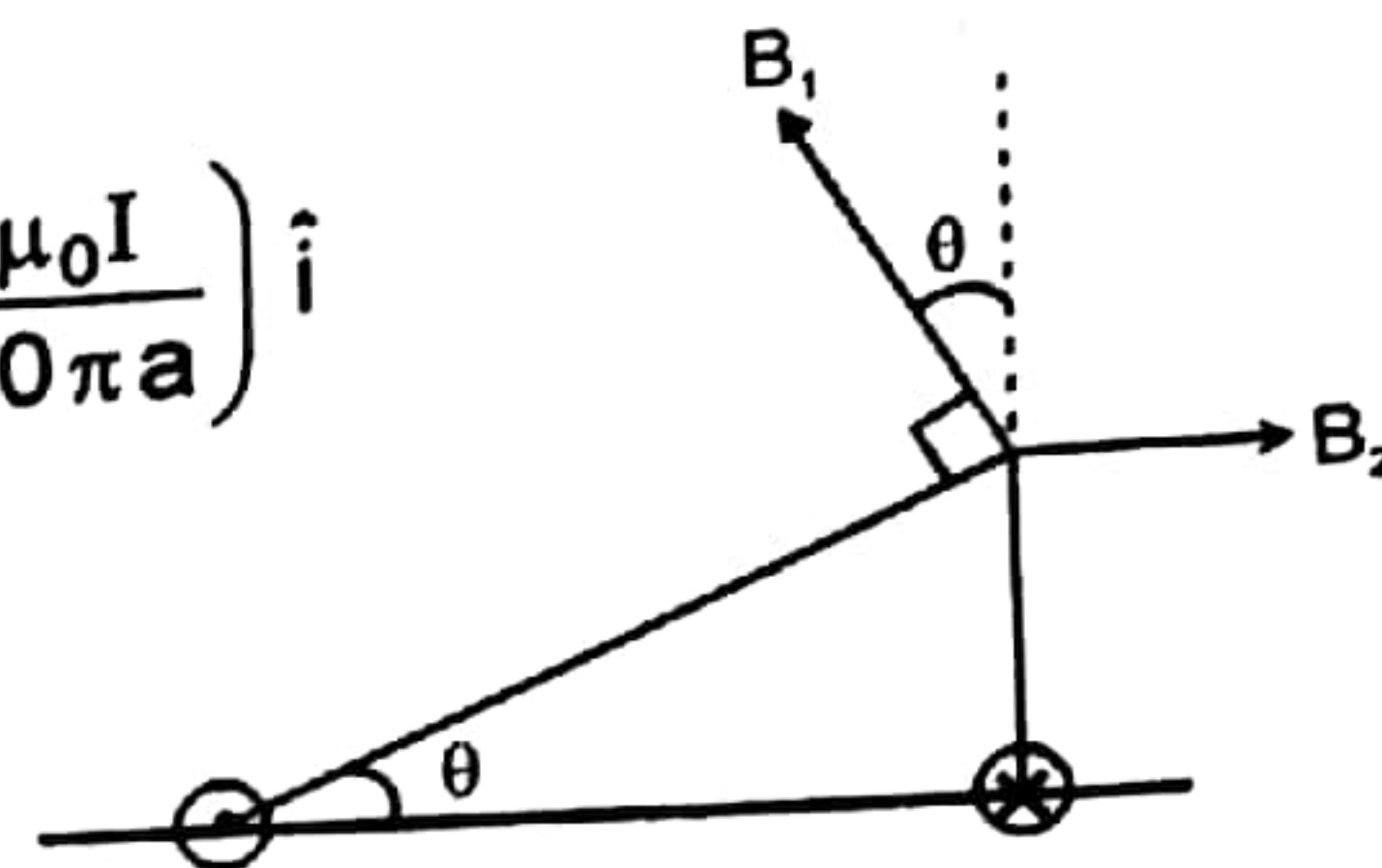
For θ $B_1 = \frac{\mu_0 I}{2\pi\sqrt{5}a} \Rightarrow B_2 = \frac{\mu_0 I}{2\pi a}$



$$\tan \theta = \frac{a}{2a} = \frac{1}{2} \Rightarrow \vec{B} = (B_1 \cos \theta \hat{j}) + (B_2 - B_1 \sin \theta) \hat{i}$$

$$\sin \theta = \frac{1}{\sqrt{5}} \Rightarrow \vec{B} = \frac{\mu_0 I}{5\pi a} \hat{j} + \left(\frac{\mu_0 I}{2\pi a} - \frac{\mu_0 I}{10\pi a} \right) \hat{i}$$

$$\cos \theta = \frac{2}{\sqrt{5}} \Rightarrow \vec{B} = \frac{2\mu_0 I}{5\pi a} \hat{i} + \frac{\mu_0 I}{5\pi a} \hat{j}$$



2.3.2 \vec{B} due to circular loop

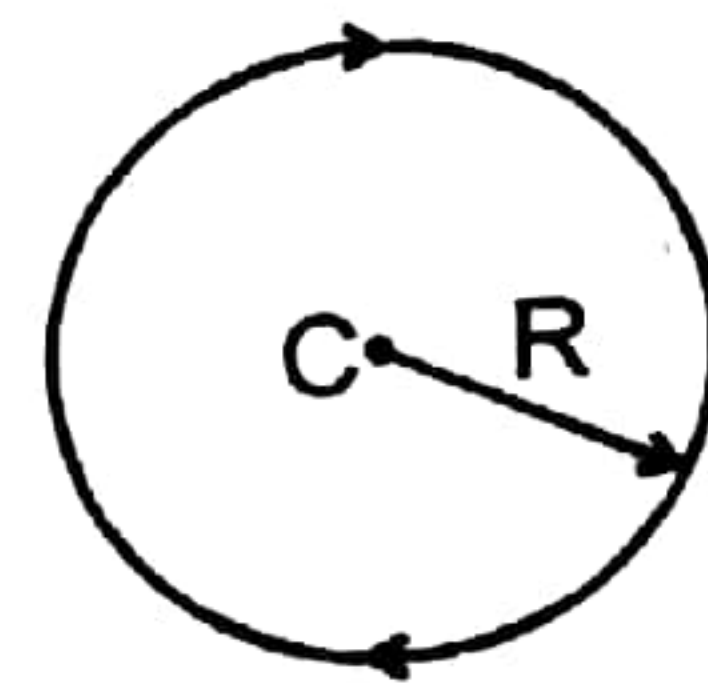
(a) **At centre** : Due to each $d\vec{\ell}$ element of the loop \vec{B} at 'c' is inwards (in this case).

$\therefore \vec{B}_{res}$ at 'c' is \otimes .

$$B = \frac{\mu_0 NI}{2R}$$

N = No. of turns in the loop.

$$= \frac{\ell}{2\pi R}; \ell = \text{length of the loop.}$$

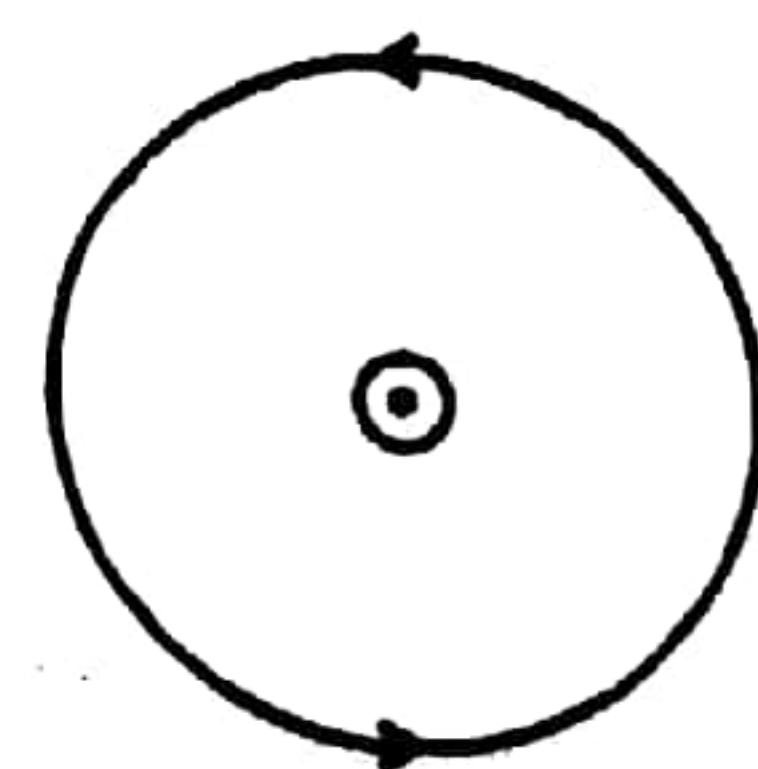
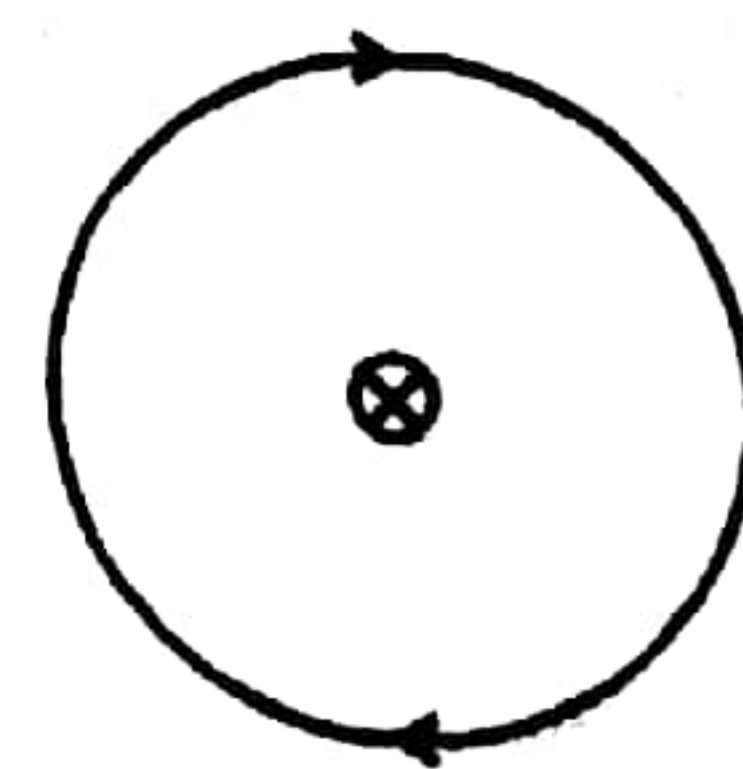


N can be fraction $\left(\frac{1}{4}, \frac{1}{3}, \frac{11}{3} \text{ etc.}\right)$ or integer.

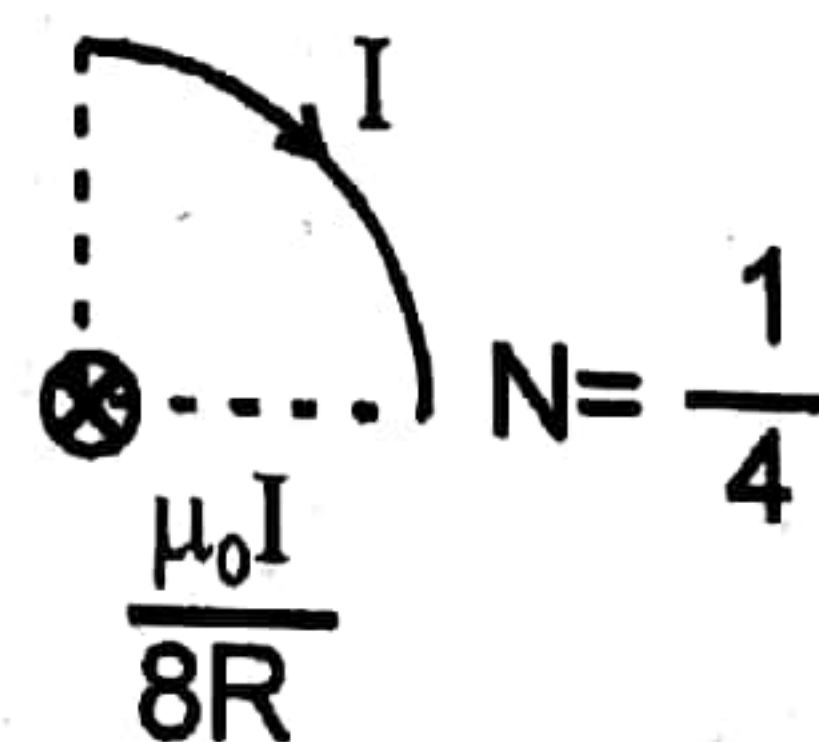
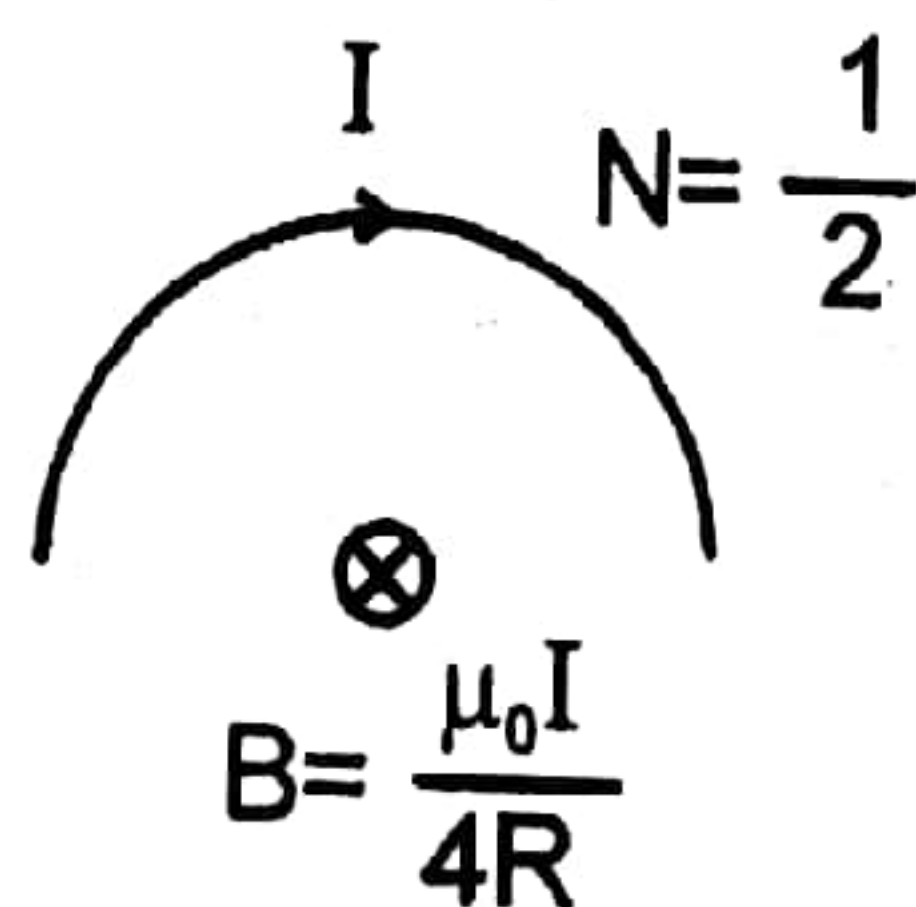
Direction of \vec{B} : The direction of the magnetic field at the centre of a circular wire can be obtained using the right-hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field (figure).



Another way to find the direction is to look into the loop along its axis. If the current is in anticlockwise direction, the magnetic field is towards the viewer. If the current is in clockwise direction, the field is away from the viewer.

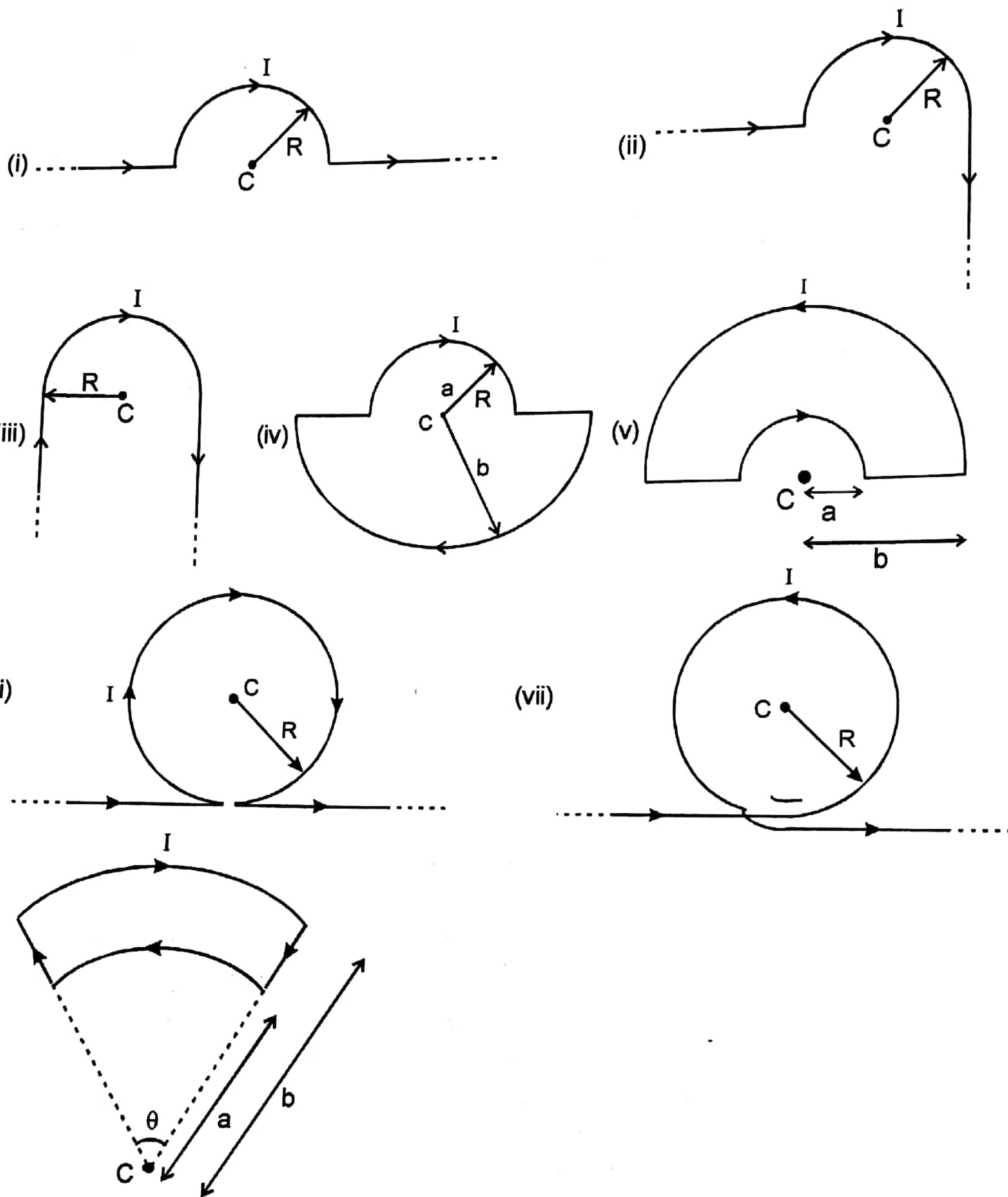


Semicircular and Quarter of a circle :



Solved Examples

Example 9. Find 'B' at centre 'C' in the following cases :



Answer :

- (i) $\frac{\mu_0 I}{4R} \otimes$
- (ii) $\frac{\mu_0 I}{4R} \left(1 + \frac{1}{\pi}\right) \otimes$
- (iii) $\frac{\mu_0 I}{2R} \left(\frac{1}{2} + \frac{1}{\pi}\right) \otimes$
- (iv) $\frac{\mu_0 I}{4} \left(\frac{1}{a} + \frac{1}{b}\right) \otimes$
- (v) $\frac{\mu_0 I}{4} \left(\frac{1}{a} - \frac{1}{b}\right) \otimes$
- (vi) $\frac{\mu_0 I}{2R} \left(1 - \frac{1}{\pi}\right) \otimes$
- (vii) $\frac{\mu_0 I}{2R} \left(1 + \frac{1}{\pi}\right) \otimes$
- (viii) $\frac{\mu_0 I \theta}{4\pi} \left(\frac{1}{a} - \frac{1}{b}\right) \otimes$

Solution :

(i) $B = \frac{\mu_0 I}{2R} \times \frac{1}{2} = \frac{\mu_0 I}{4R}$

(ii) $B = B_1 + B_2 = \left(\frac{\mu_0 I}{2R} \times \frac{1}{2}\right) + \left(\frac{\mu_0}{4\pi} \cdot \frac{I}{R}\right) = \frac{\mu_0 I}{4R} \left(1 + \frac{1}{\pi}\right)$

$$(iii) \quad B = B_1 + B_2 + B_3 = 2B_1 + B_2 = \left(2 \times \frac{\mu_0 I}{4\pi R}\right) + \left(\frac{\mu_0 I}{2R} \times \frac{1}{2}\right) = \frac{\mu_0 I}{2R} \left(\frac{1}{2} + \frac{1}{\pi}\right)$$

$$(iv) \quad B = B_1 + B_2 = \frac{\mu_0}{2a} \times \frac{1}{2} + \frac{\mu_0}{2b} \times \frac{1}{2} = \frac{\mu_0 I}{4} \left(\frac{1}{a} + \frac{1}{b}\right)$$

$$(v) \quad B = B_1 - B_2 = \left(\frac{\mu_0 I}{2a} \times \frac{1}{2} - \frac{\mu_0 I}{2b} \times \frac{1}{2}\right) = \frac{\mu_0 I}{4} \left(\frac{1}{a} - \frac{1}{b}\right)$$

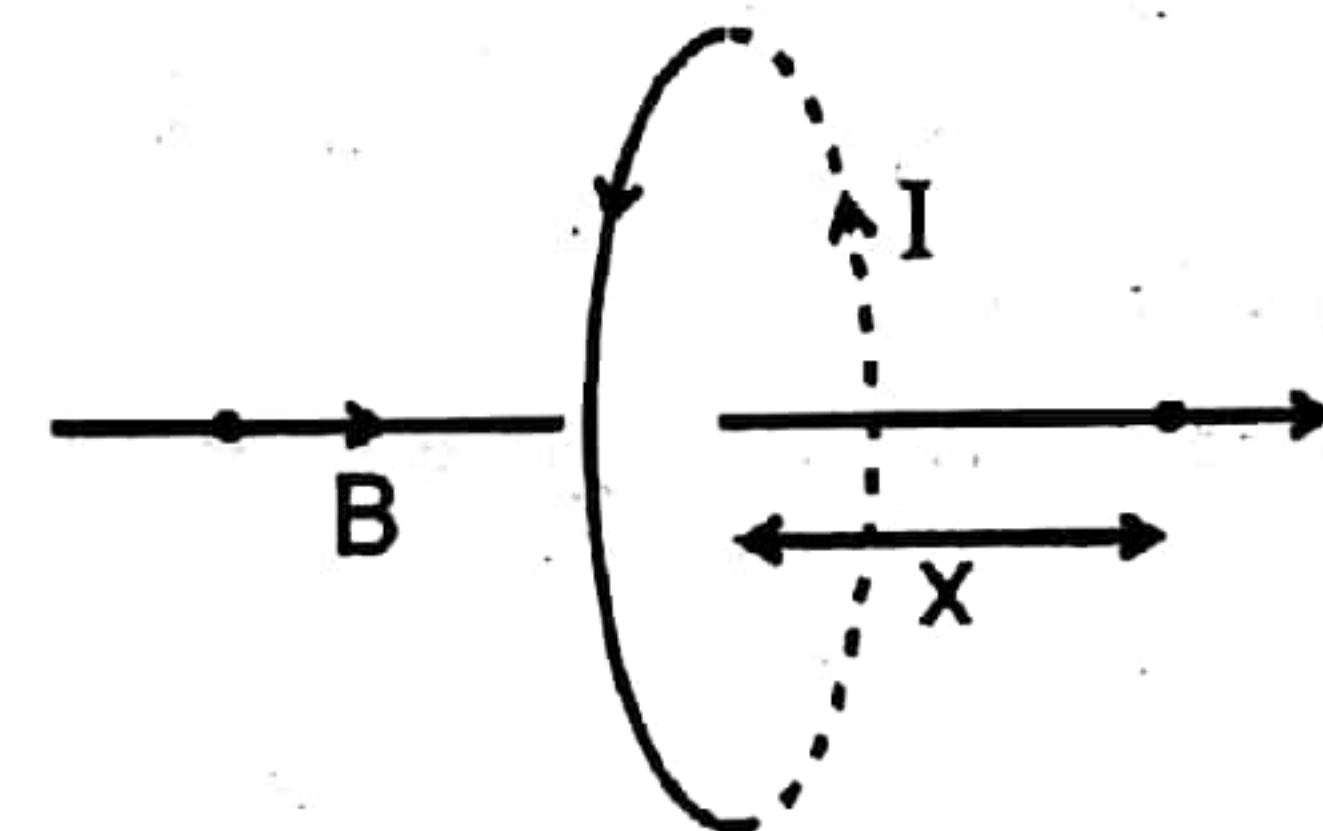
$$(vi) \quad B = B_1 - B_2 = \frac{\mu_0 I}{2R} - \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2R} \left(1 - \frac{1}{\pi}\right)$$

$$(vii) \quad B = B_1 + B_2 = \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2R} \left(1 + \frac{1}{\pi}\right)$$

$$(viii) \quad B = B_1 - B_2 = \frac{\mu_0 I}{2a} - \frac{\mu_0 I}{2b} \times \frac{\theta}{2\pi} = \frac{\mu_0 I \theta}{4\pi} \left(\frac{1}{a} - \frac{1}{b}\right)$$

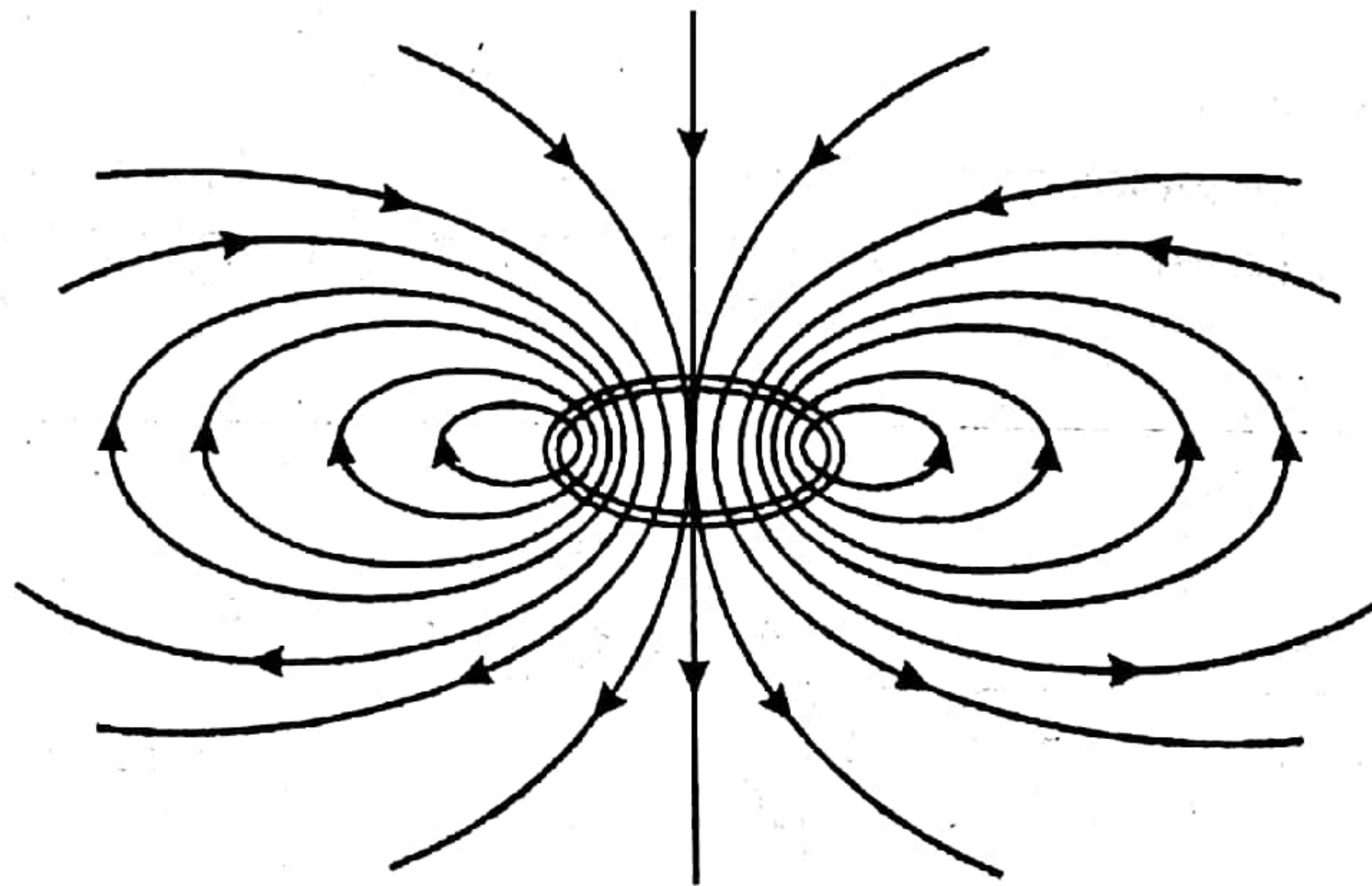


(b) On the axis of the loop : $B = \frac{\mu_0 N I R^2}{2(R^2 + x^2)^{3/2}}$
 N = No. of turns (integer)



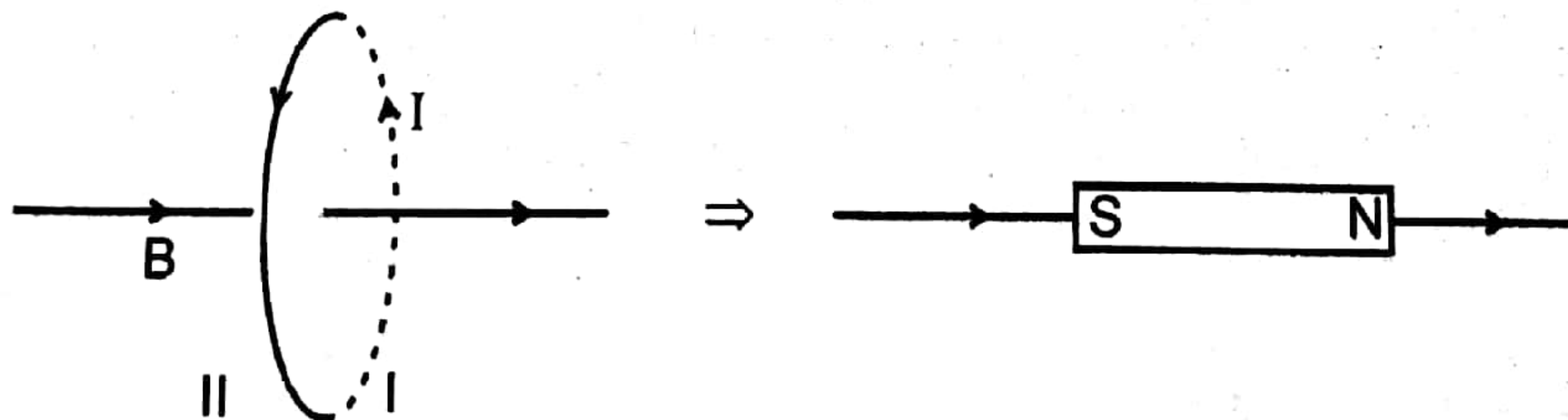
Direction can be obtained by right hand thumb rule. curl your fingers in the direction of the current then the direction of the thumb points in the direction of \vec{B} at the points on the axis.

The magnetic field at a point not on the axis is mathematically difficult to calculate. We show qualitatively in figure the magnetic field lines due to a circular current which will give some idea of the field.

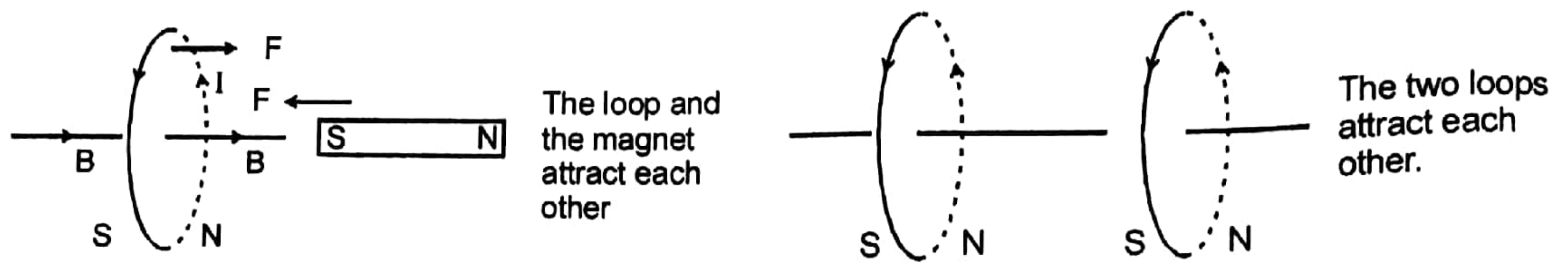


2.3.3 A loop as a magnet :

The pattern of the magnetic field is comparable with the magnetic field produced by a bar magnet.



The side 'I' (the side from which the \vec{B} emerges out) of the loop acts as 'NORTH POLE' and side II (the side in which the \vec{B} enters) acts as the 'SOUTH POLE'. It can be verified by studying force on one loop due to a magnet or a loop.



Mathematically :

$$B_{axis} = \frac{\mu_0 N I R^2}{2(R^2 + x^2)^{3/2}} \approx \frac{\mu_0 N I R^2}{2x^3} \text{ for } x \gg R$$

$$= 2 \left(\frac{\mu_0}{4\pi} \right) \left(\frac{I N \pi R^2}{x^3} \right)$$

It is similar to B_{axis} due to magnet = $2 \left(\frac{\mu_0}{4\pi} \right) \frac{m}{x^3}$

Magnetic dipole moment of the loop

$$M = I N \pi R^2$$

$M = I N A$ for any other shaped loop.

Unit of M is Amp. m^2

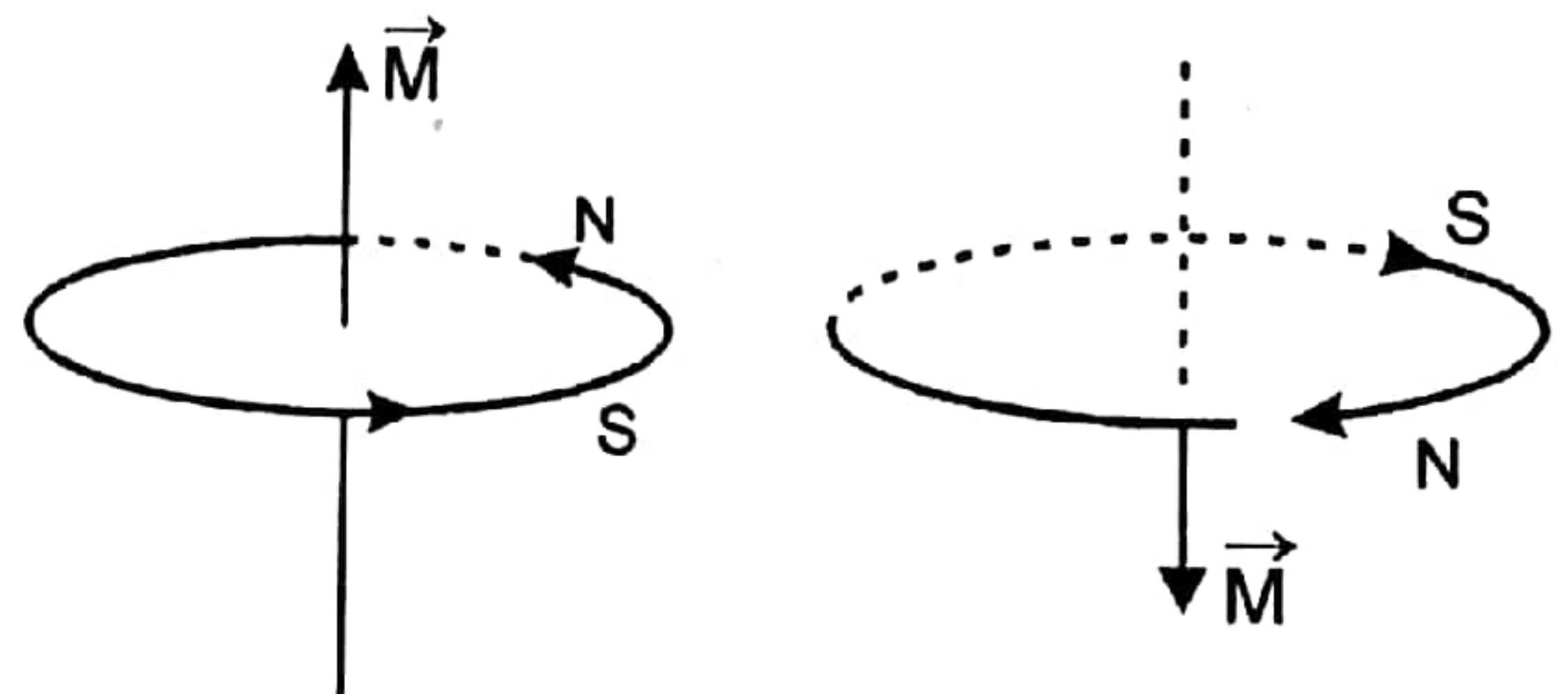
Unit of m (pole strength) = Amp. m

{ \because in magnet $M = m \ell$ }

$$\vec{M} = I N \vec{A}$$

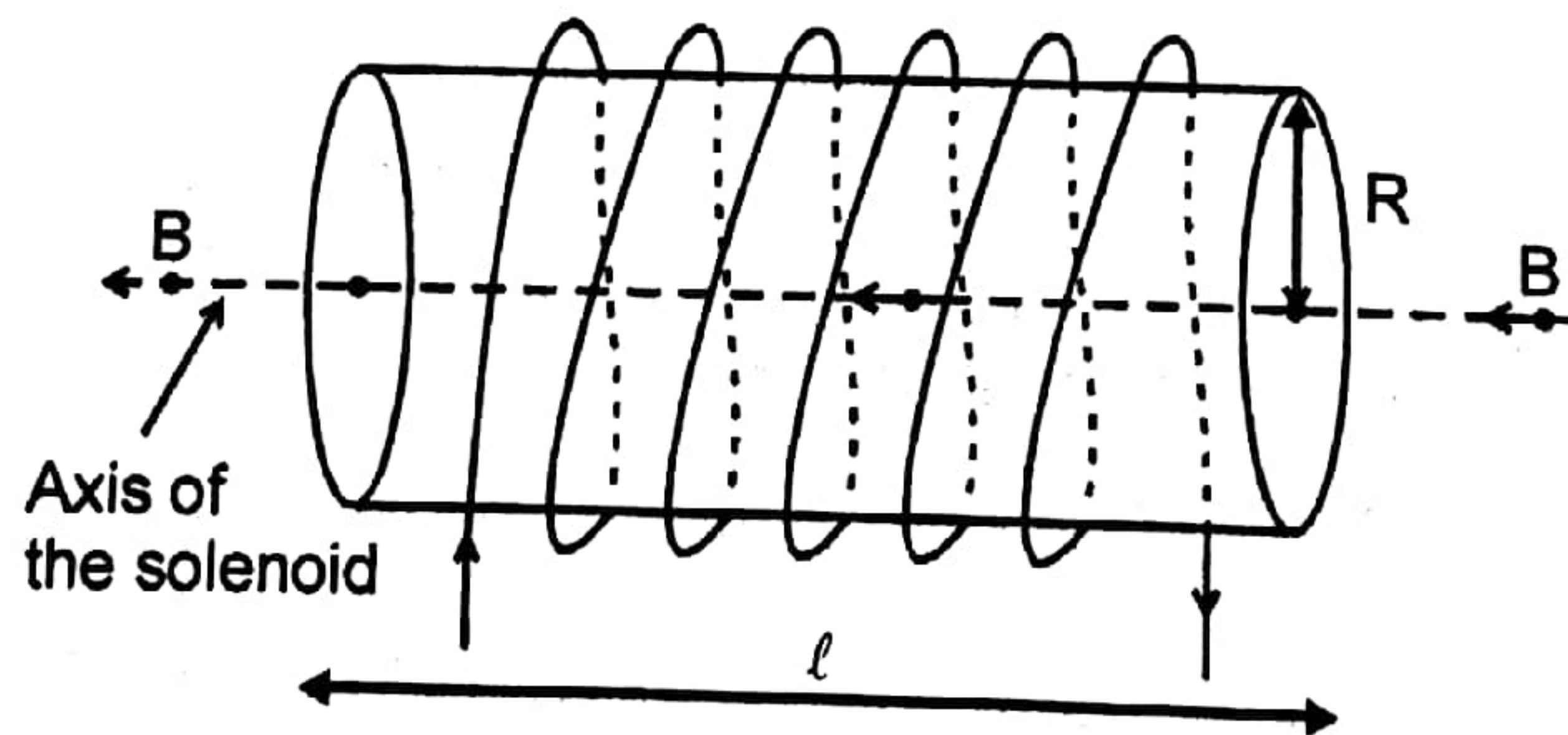
\vec{A} = unit normal vector for the loop.

To be determined by right hand rule which is also used to determine direction of \vec{B} on the axis. It is also from 'S' side to 'N' side of the loop.



2.3.4 Solenoid :

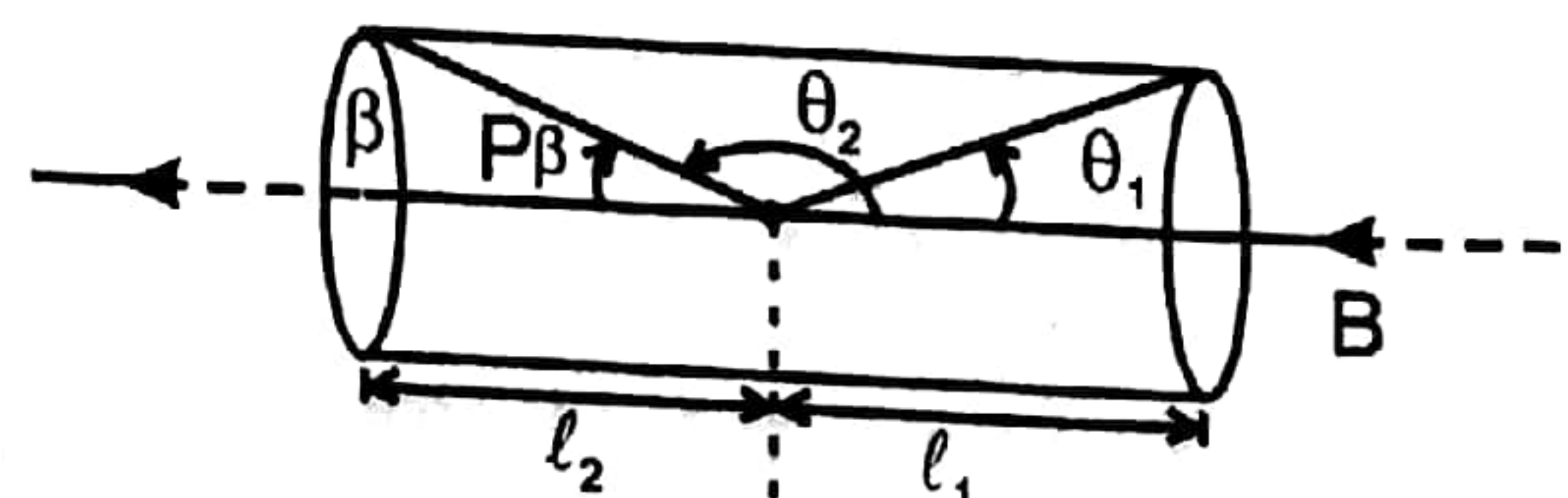
- (i) Solenoid contains large number of circular loops wrapped around a non-conducting cylinder. (it may be a hollow cylinder or it may be a solid cylinder)



- (ii) The winding of the wire is uniform direction of the magnetic field is same at all points of the axis.
- (iii) \vec{B} on axis (turns should be very close to each others).

$$B = \frac{\mu_0 n i}{2} (\cos \theta_1 - \cos \theta_2)$$

where n : number of turns per unit length.



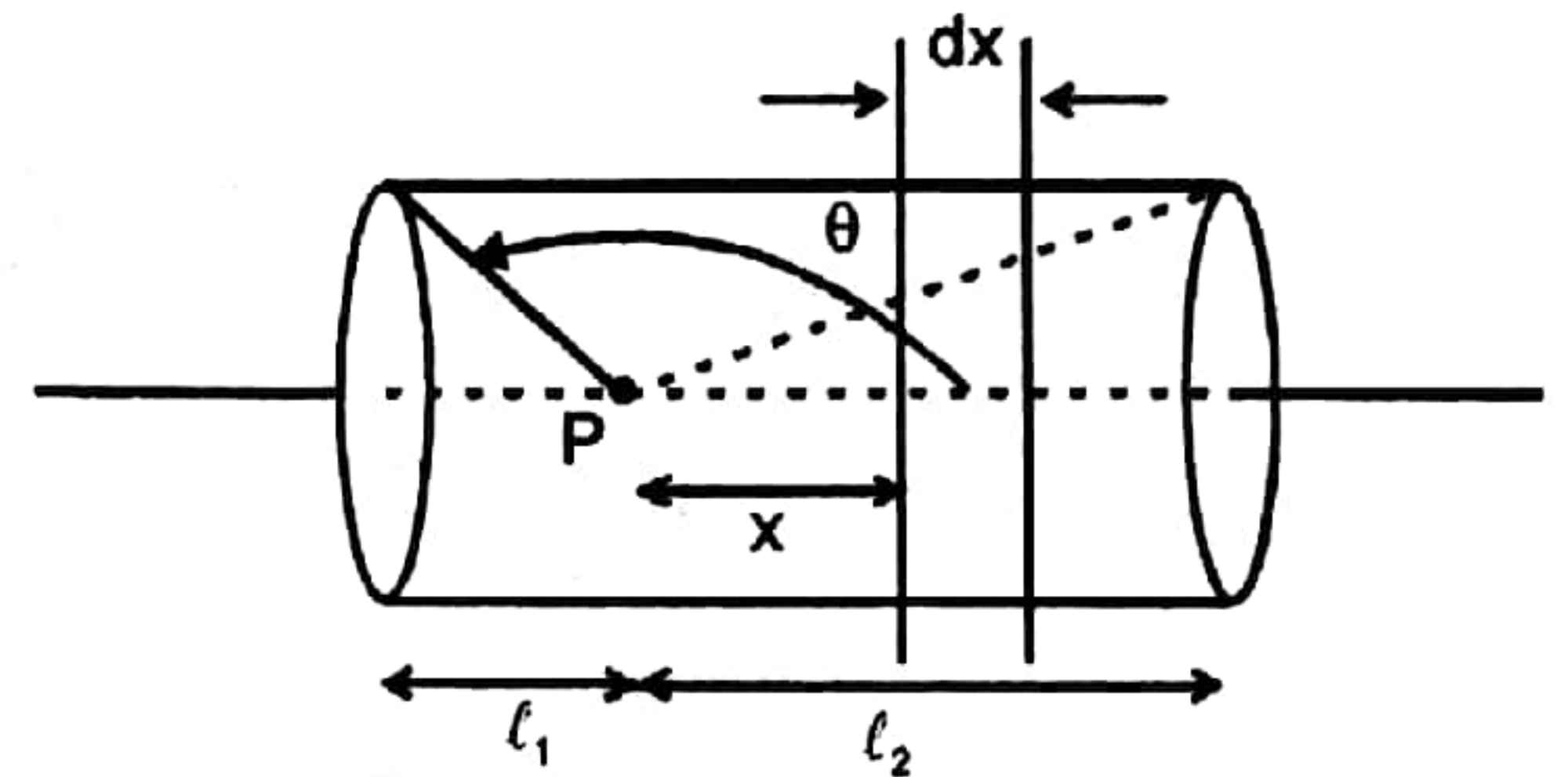
$$\cos \theta_1 = \frac{l_1}{\sqrt{l_1^2 + R^2}} \quad ; \quad \cos \beta = \frac{l_2}{\sqrt{l_2^2 + R^2}} = -\cos \theta_2$$

$$B = \frac{\mu_0 ni}{2} \left[\frac{l_1}{\sqrt{l_1^2 + R^2}} + \frac{l_2}{\sqrt{l_2^2 + R^2}} \right] = \frac{\mu_0 ni}{2} (\cos \theta_1 + \cos \beta)$$

Note : Use right hand rule for direction (same as the direction due to loop).

Derivation :

Take an element of width dx at a distance x from point P. [point P is the point on axis at which we are going to calculate magnetic field. Total number of turns in the element $dn = ndx$ where n : number of turns per unit length.



$$dB = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}} (ndx)$$

$$B = \int dB = \int_{-l_1}^{l_2} \frac{\mu_0 i R^2 ndx}{2(R^2 + x^2)^{3/2}} = \frac{\mu_0 ni}{2} \left[\frac{l_1}{\sqrt{l_1^2 + R^2}} + \frac{l_2}{\sqrt{l_2^2 + R^2}} \right] = \frac{\mu_0 ni}{2} [\cos \theta_1 - \cos \theta_2]$$

(iv) For 'Ideal Solenoid' :

*Inside (at the mid point)

$l \gg R$ or length is infinite

$\theta_1 \rightarrow 0$

$\theta_2 \rightarrow \pi$

$$B = \frac{\mu_0 ni}{2} [1 - (-1)]$$

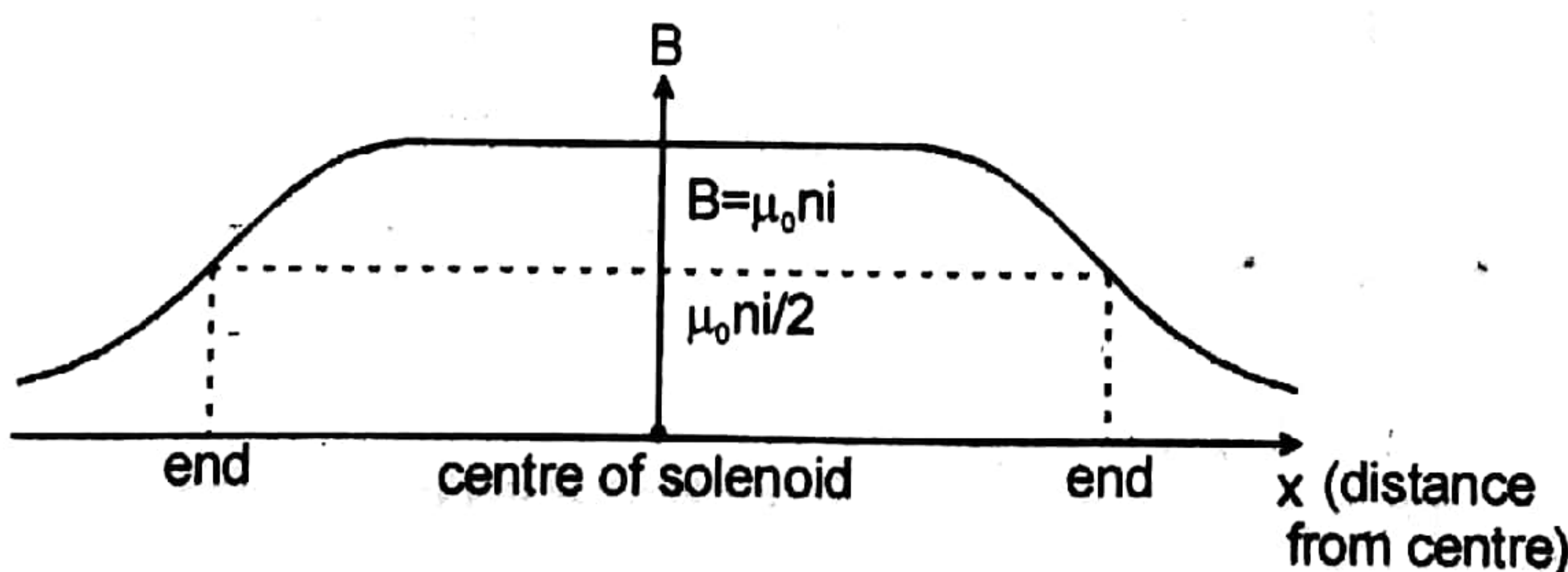
$$B = \mu_0 ni$$

If material of the solid cylinder has relative permeability ' μ_r ' then $B = \mu_0 \mu_r ni$

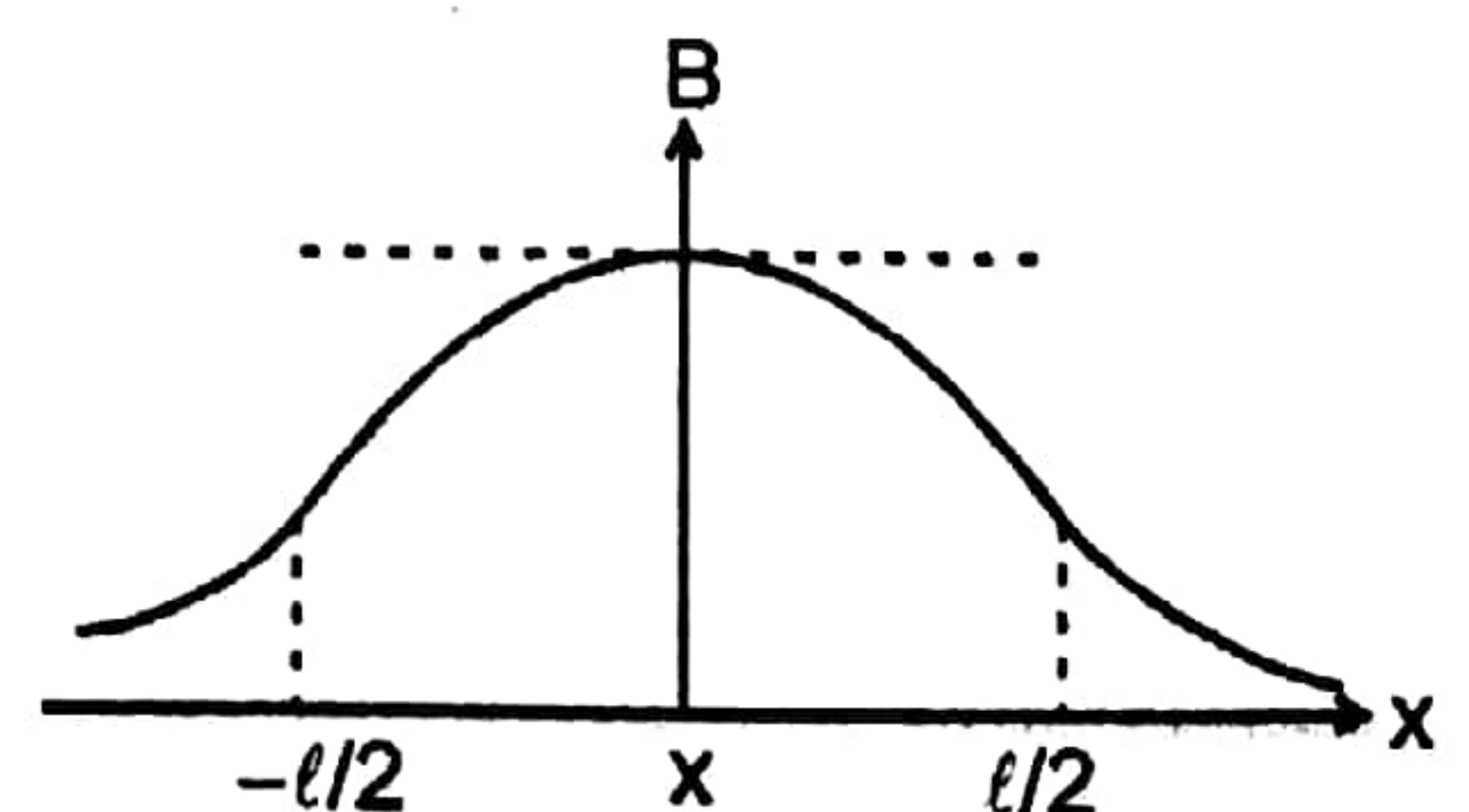
At the ends $B = \frac{\mu_0 ni}{2}$

(v) Comparison between ideal and real solenoid :

(a) Ideal Solenoid



Real Solenoid





2.4 AMPERE'S CIRCUITAL LAW :

The line integral $\oint \vec{B} \cdot d\vec{\ell}$ on a closed curve of any shape is equal to μ_0 (permeability of free space) times the net current I through the area bounded by the curve.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma I$$

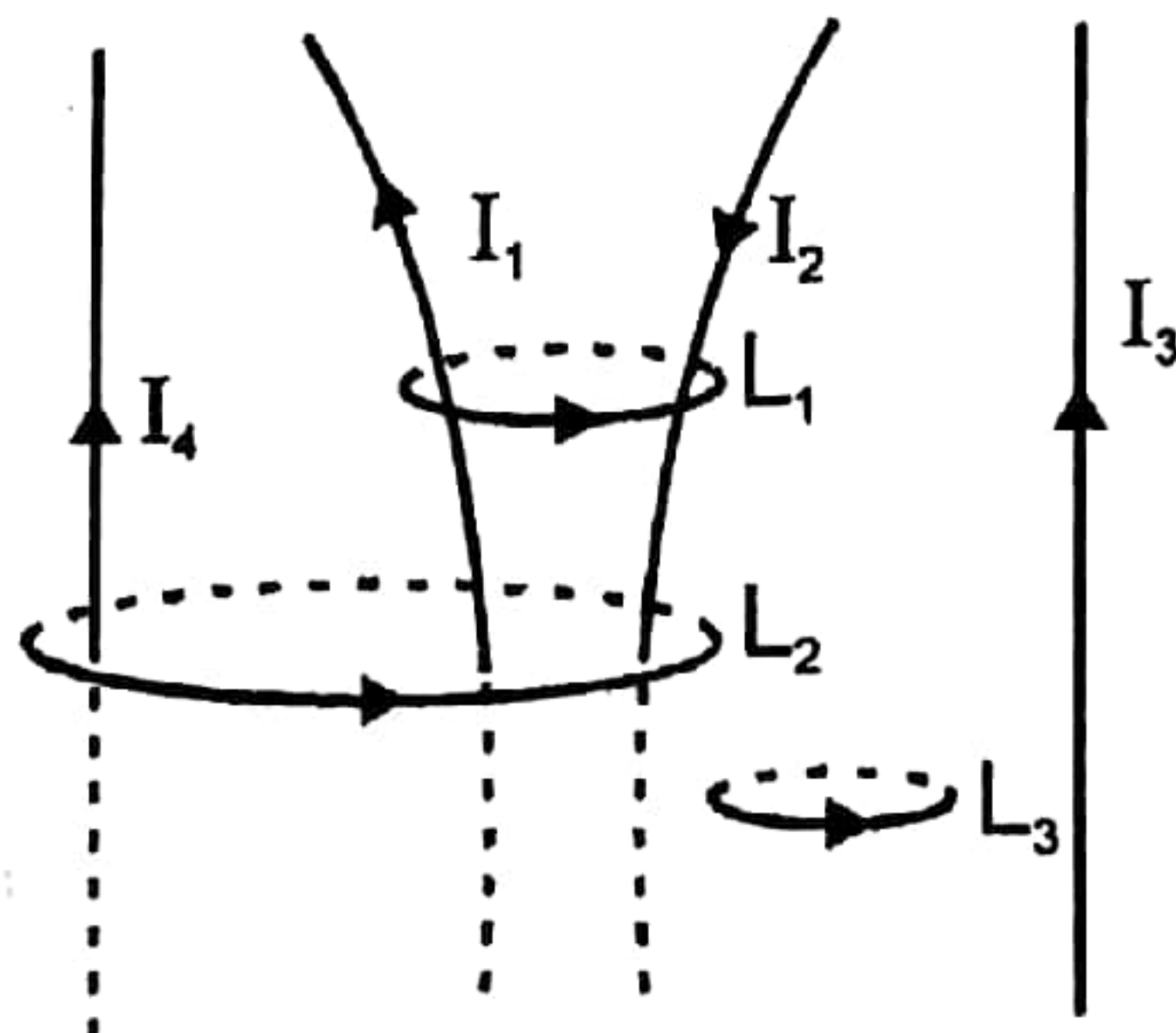
Note :

- Line integral is independent of the shape of path and position of wire with in it.
- The statement $\oint \vec{B} \cdot d\vec{\ell} = 0$ does not necessarily mean that $\vec{B} = 0$ everywhere along the path but only that no net current is passing through the path.
- **Sign of current :** The current due to which \vec{B} is produced in the same sense as $d\vec{\ell}$ (i.e. $\vec{B} \cdot d\vec{\ell}$ positive will be taken positive and the current which produces \vec{B} in the sense opposite to $d\vec{\ell}$ will be negative.

Solved Examples

Example 10. Find the values of $\oint \vec{B} \cdot d\vec{\ell}$ for the loops L_1, L_2, L_3 in the figure shown.

The sense of $d\vec{\ell}$ is mentioned in the figure.



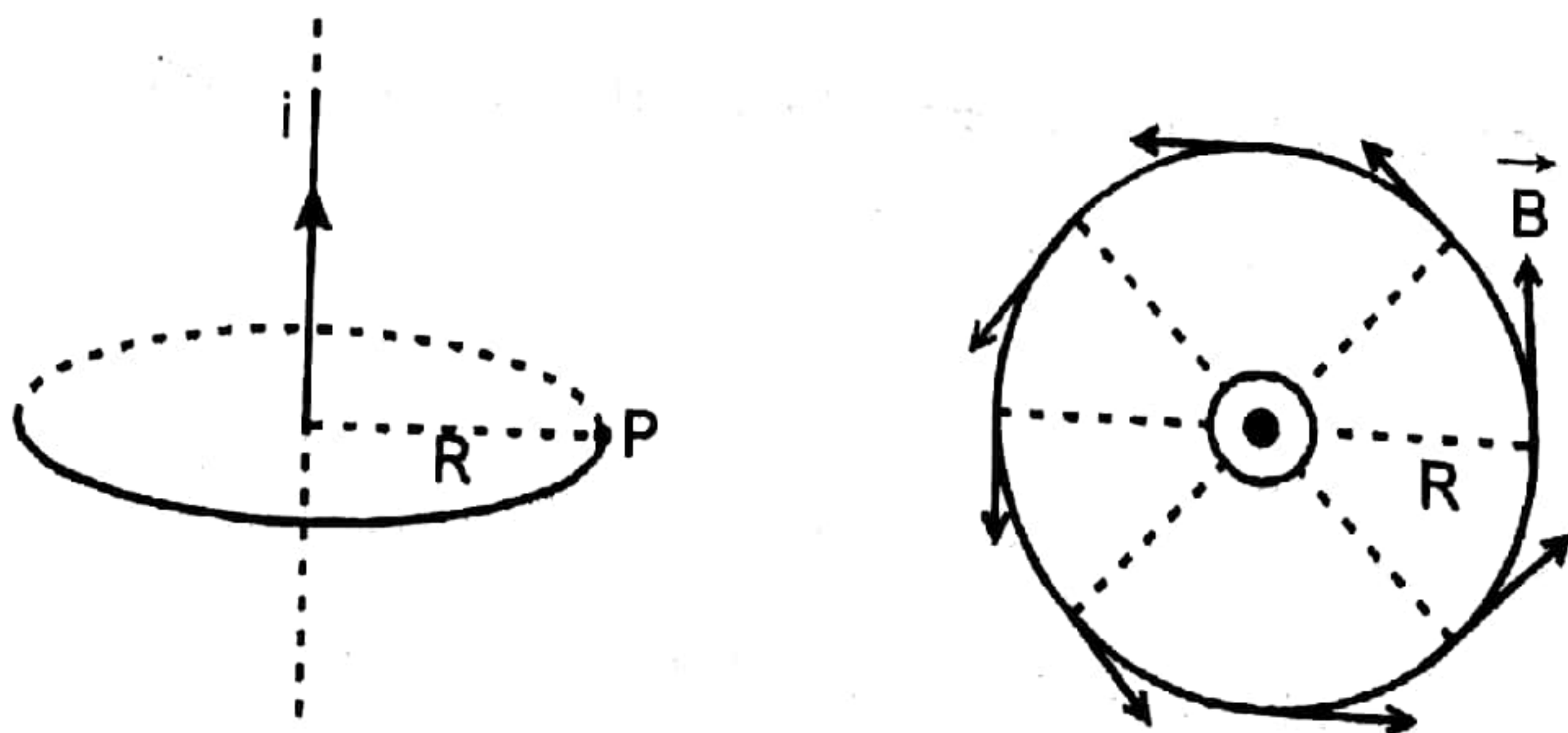
Solution : for L_1 $\oint \vec{B} \cdot d\vec{\ell} = \mu_0(I_1 - I_2)$

here I_1 is taken positive because magnetic lines of force produced by I_1 is anti clockwise as seen from top. I_2 produces lines of \vec{B} in clockwise sense as seen from top. The sense of $d\vec{\ell}$ is anticlockwise as seen from top.

for L_2 : $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 (I_1 - I_2 + I_4)$ for L_3 : $\oint \vec{B} \cdot d\vec{\ell} = 0$



Uses : 2.4.1 To find out magnetic field due to infinite current carrying wire



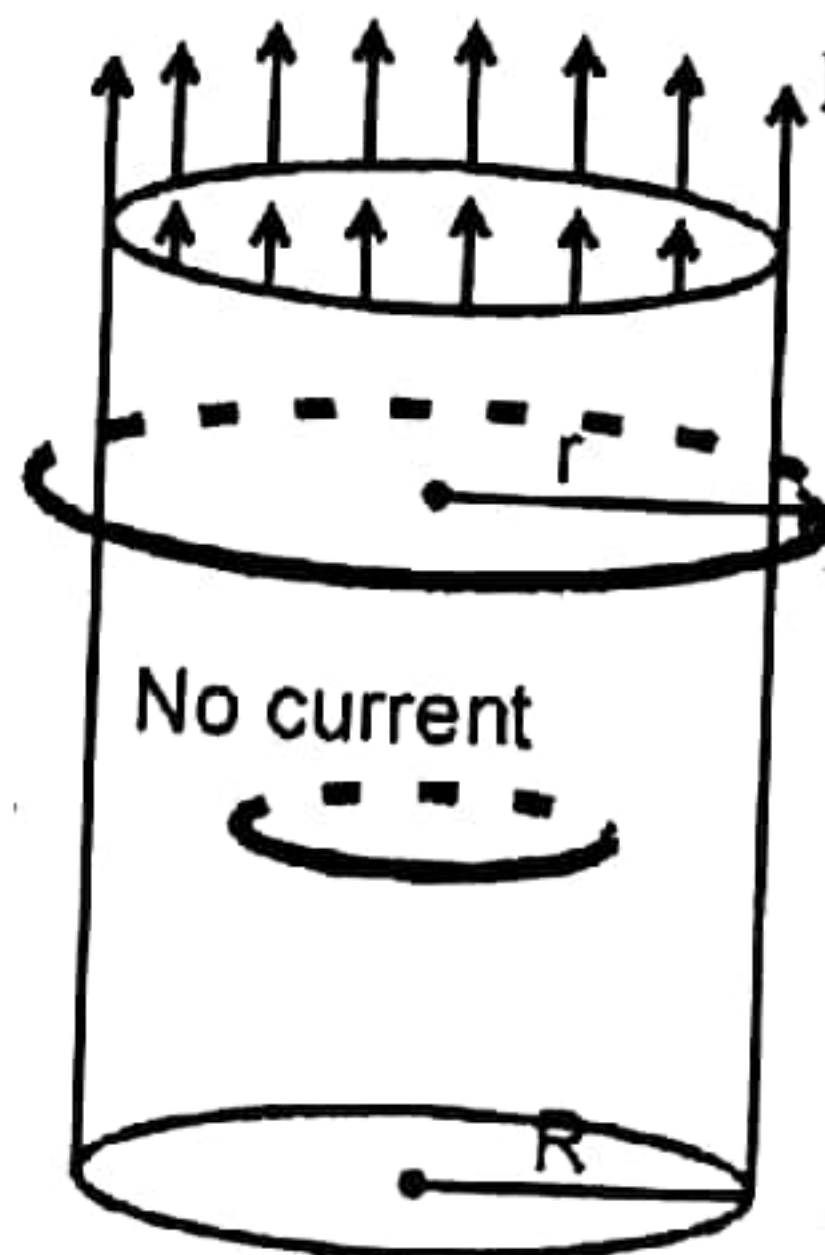
By B.S.L. \vec{B} will have circular lines. $d\vec{\ell}$ is also taken tangent to the circle.

$$\oint \vec{B} \cdot d\vec{\ell} = \oint B \cdot d\ell \quad \because \theta = 0^\circ \text{ so } B \oint d\ell = B 2\pi R \quad (\because B = \text{const.})$$

Now by amperes law :

$$B 2\pi R = \mu_0 I \quad \therefore B = \frac{\mu_0 I}{2\pi R}$$

2.4.2. Hollow current carrying infinitely long cylinder : (I is uniformly distributed on the whole circumference)



(i) for $r \geq R$

By symmetry the amperian loop is a circle.

$$\oint \vec{B} \cdot d\vec{\ell} = \oint B d\ell \quad \because \theta = 0$$

$$= B \int_0^{2\pi} d\ell \quad \because B = \text{const.} \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

(ii) $r < R$

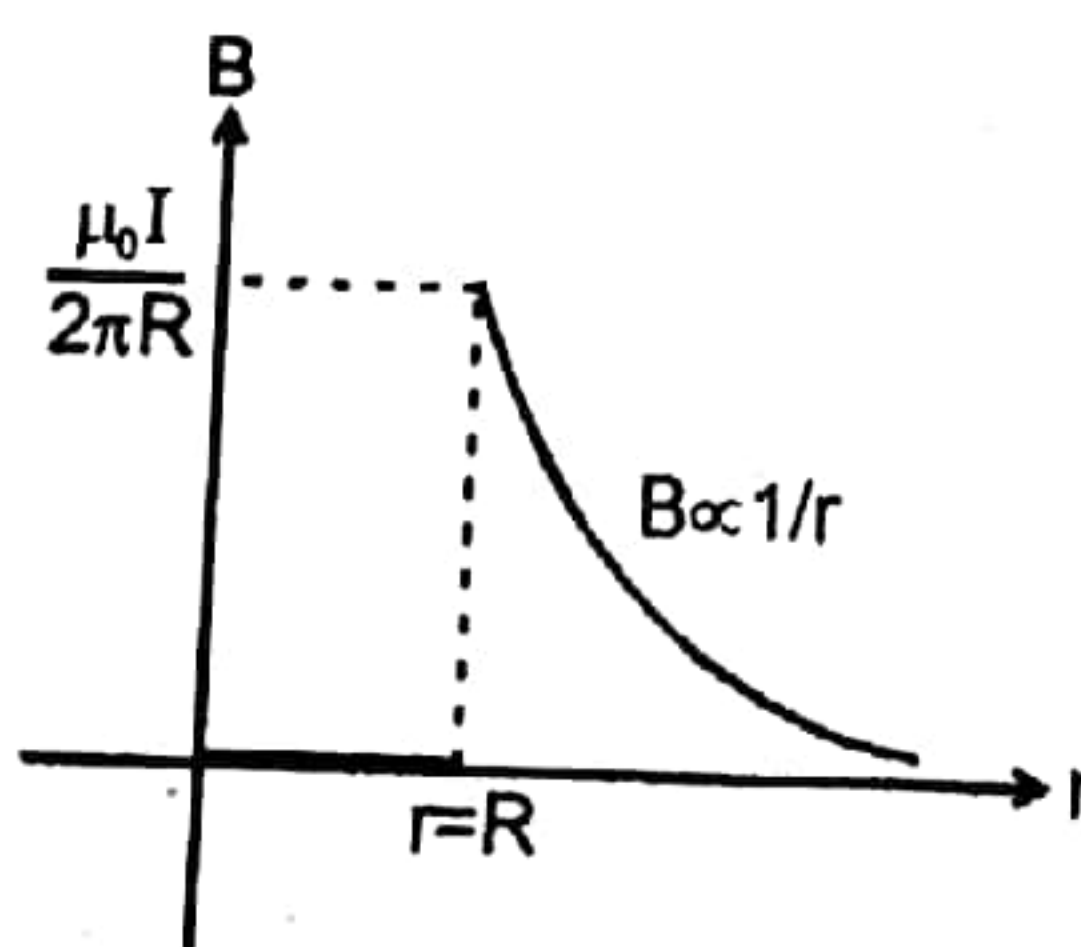
$$\oint \vec{B} \cdot d\vec{\ell}$$

$$= \oint B d\ell$$

$$= B(2\pi r) = 0$$

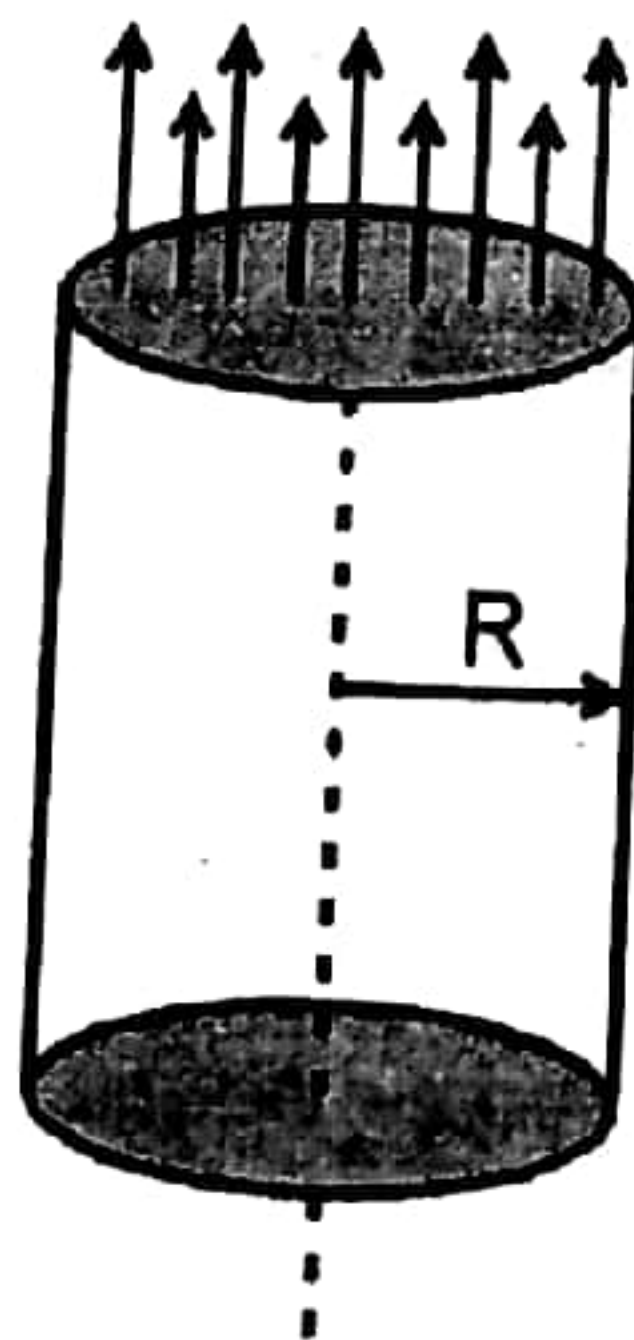
$$\Rightarrow B_{in} = 0$$

Graph :



2.4.3 Solid infinite current carrying cylinder :

Assume current is uniformly distributed on the whole cross section area



current density $J = \frac{I}{\pi R^2}$

Case (I) : $r \leq R$

take an amperian loop inside the cylinder. By symmetry it should be a circle whose centre is on the axis of cylinder and its axis also coincides with the cylinder axis on the loop.

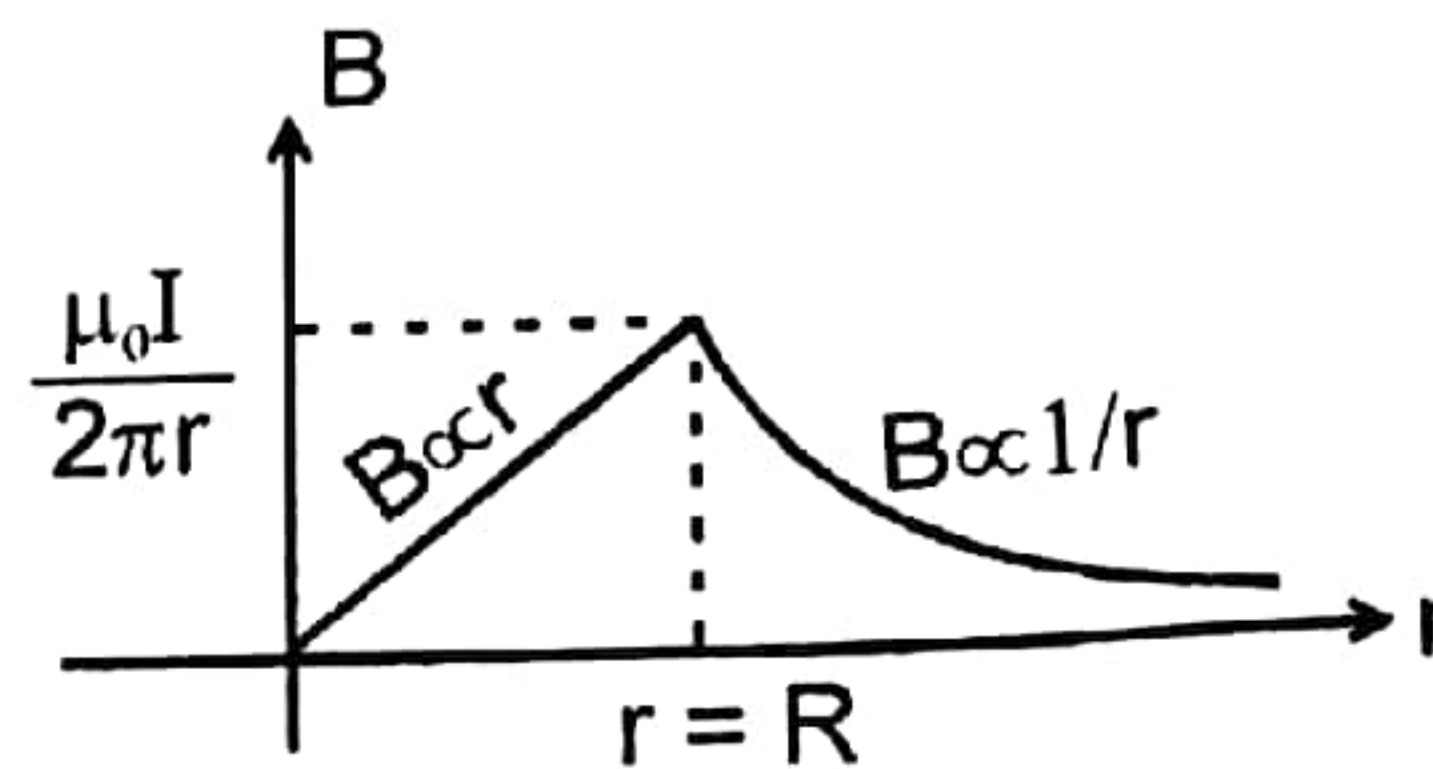
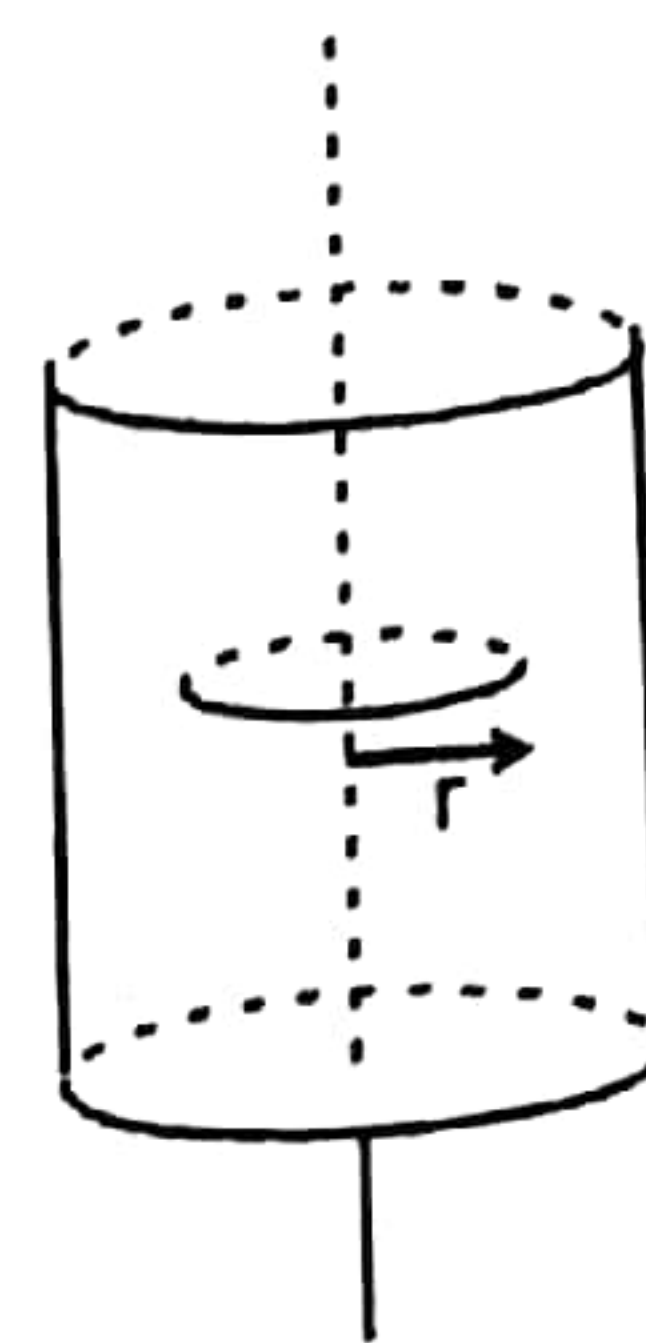
$$\oint \vec{B} \cdot d\vec{\ell} = \int B \cdot d\ell = B \int d\ell = B \cdot 2\pi r = \mu_0 \frac{I}{\pi R^2} \pi r^2$$

$$B = \frac{\mu_0 I r}{2\pi R^2} = \frac{\mu_0 J r}{2} \Rightarrow \vec{B} = \frac{\mu_0 (\vec{J} \times \vec{r})}{2}$$

Case (II) : $r \geq R$ $\oint \vec{B} \cdot d\vec{\ell} = \int B d\ell = B \int d\ell = B \cdot (2\pi r) = \mu_0 \cdot I$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r} \text{ also } \vec{B} = \frac{\mu_0 I}{2\pi r} (\hat{J} \times \hat{r}) = \frac{\mu_0 J \pi R^2}{2\pi r}$$

$$\vec{B} = \frac{\mu_0 R^2}{2r^2} (\vec{J} \times \vec{r})$$

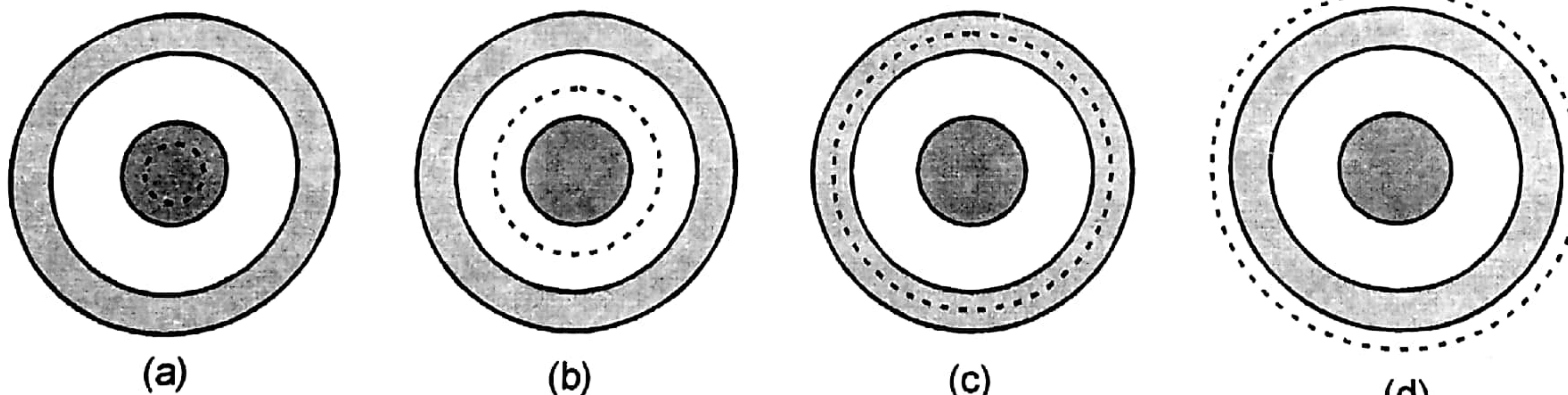


Solved Examples

Example 11.

Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c respectively. The inner wire carries an electric current i_0 and the outer shell carries an equal current in same direction. Find the magnetic field at a distance x from the axis where (a) $x < a$, (b) $a < x < b$ (c) $b < x < c$ and (d) $x > c$. Assume that the current density is uniform in the inner wire and also uniform in the outer shell.

Solution :



A cross-section of the cable is shown in figure. Draw a circle of radius x with the centre at the axis of the cable. The parts a, b, c and d of the figure correspond to the four parts of the problem. By symmetry, the magnetic field at each point of a circle will have the same magnitude and will be tangential to it. The circulation of B along this circle is, therefore,

$$\oint \vec{B} \cdot d\vec{\ell} = B 2\pi x$$

in each of the four parts of the figure.

(a) The current enclosed within the circle in part b is i_0 so that

$$\frac{i_0}{\pi a^2} \cdot \pi x^2 = \frac{i_0}{a^2} x^2$$

Ampere's law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i \text{ gives}$$

$$B \cdot 2\pi x = \frac{\mu_0 i_0 x^2}{a^2} \text{ or, } B = \frac{\mu_0 i_0 x}{2\pi a^2}$$

The direction will be along the tangent to the circle.



(b) The current enclosed within the circle in part b is i_0 so that

$$B 2\pi x = \mu_0 i_0 \quad \text{or,} \quad B = \frac{\mu_0 i_0}{2\pi x}$$

(c) The area of cross-section of the outer shell is $\pi c^2 - \pi b^2$. The area of cross-section of the outer shell within the circle in part c of the figure is $\pi x^2 - \pi b^2$.

Thus, the current through this part is $\frac{i_0(x^2 - b^2)}{(c^2 - b^2)}$. This is in the same direction to the current i_0 in the inner wire. Thus, the net current enclosed by the circle is

$$i_{\text{net}} = i_0 + \frac{i_0(x^2 - b^2)}{c^2 - b^2} = \frac{i_0(c^2 + x^2 - 2b^2)}{c^2 - b^2}$$

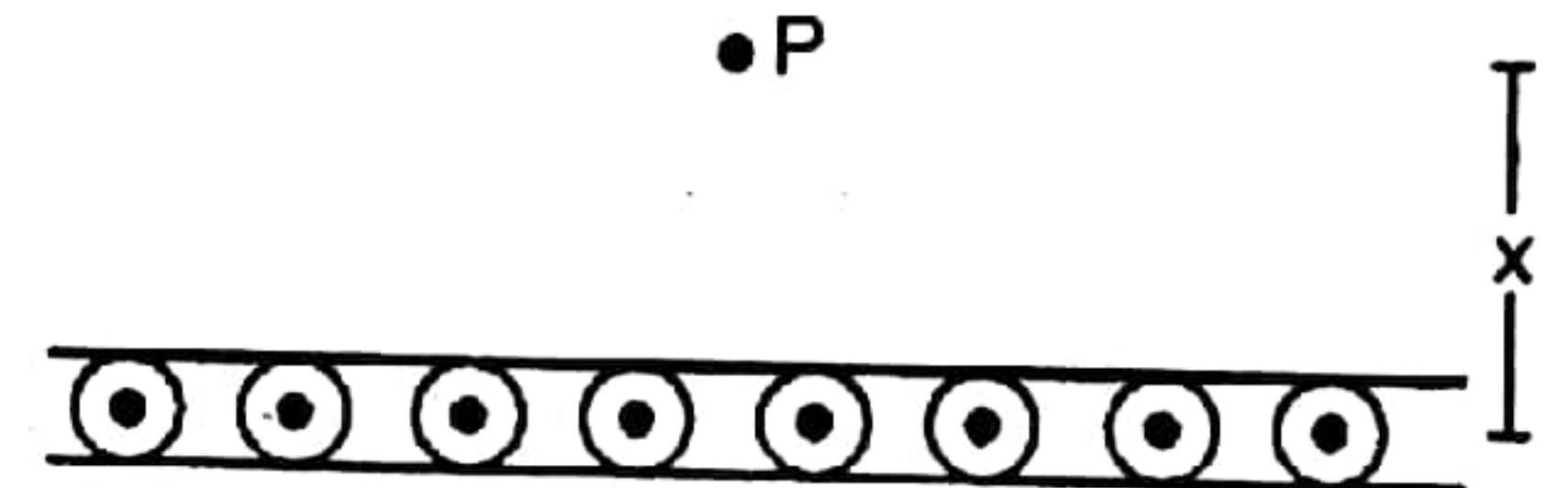
From Ampere's law,

$$B 2\pi x = \frac{i_0(c^2 + x^2 - 2b^2)}{c^2 - b^2} \quad \text{or,} \quad B = \frac{\mu_0 i_0 (c^2 + x^2 - 2b^2)}{2\pi x (c^2 - b^2)}$$

(d) The net current enclosed by the circle in part d of the figure is $2i_0$ and hence

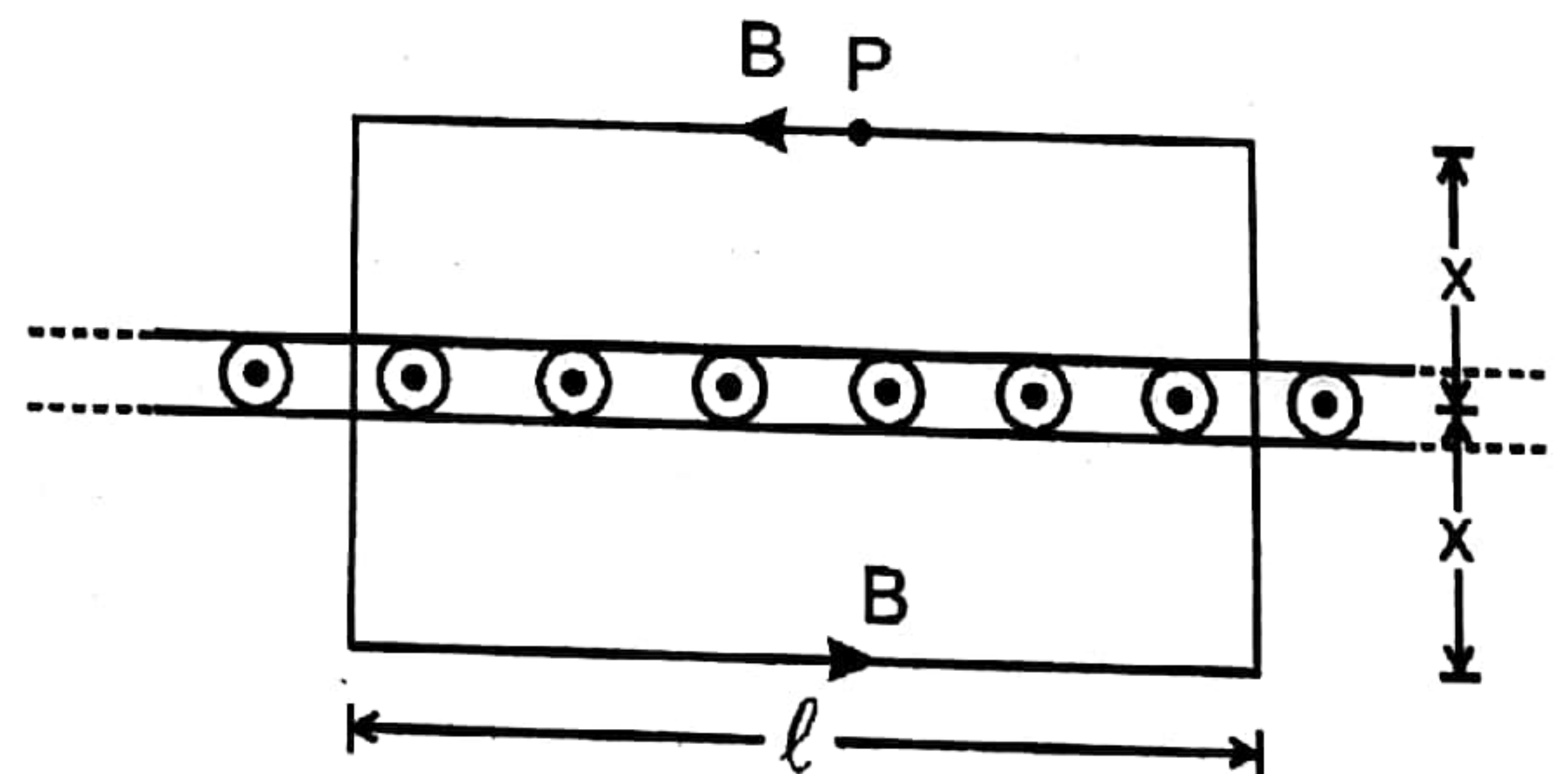
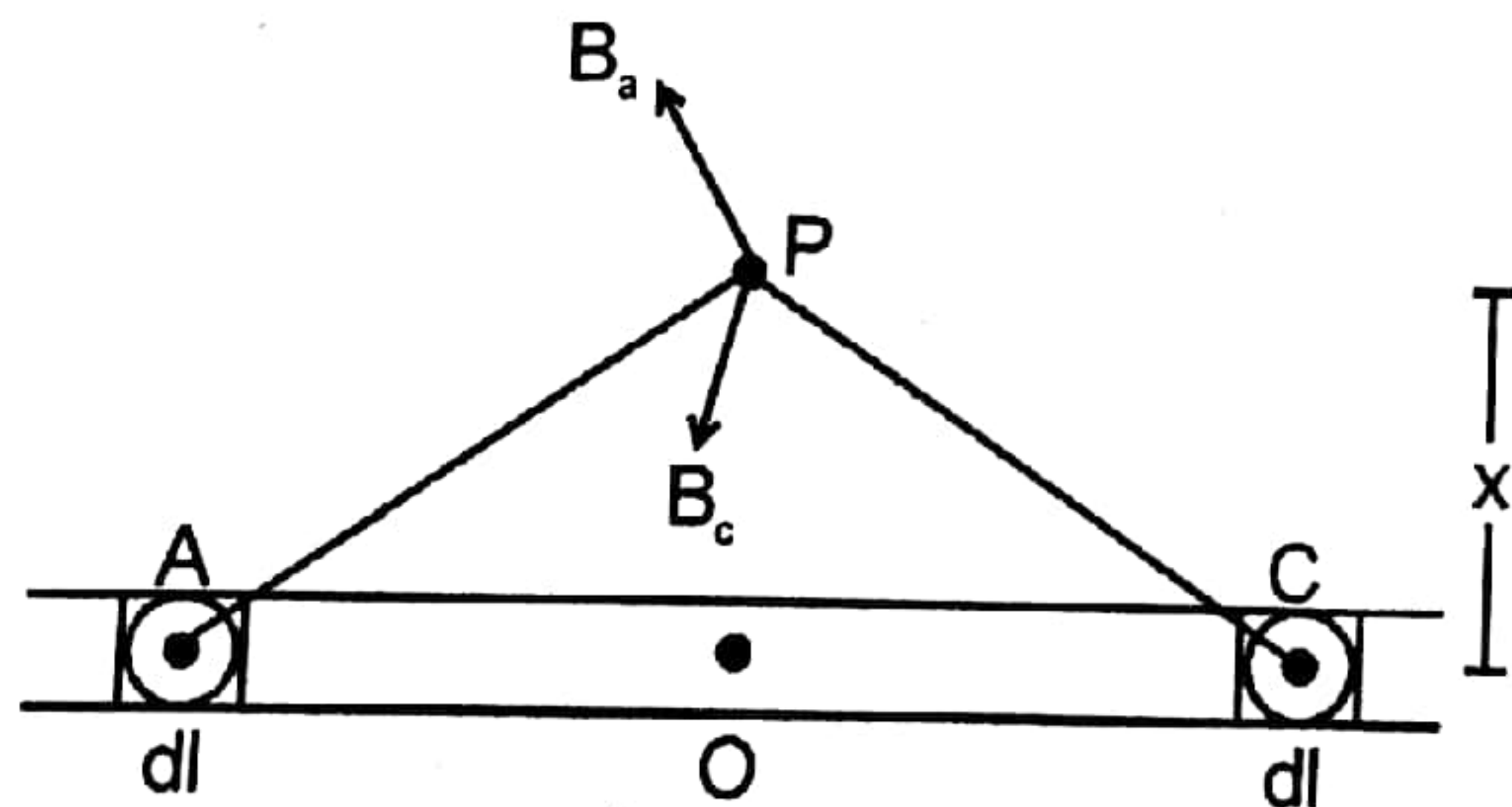
$$B 2\pi x = \mu_0 2i_0 \quad \text{or,} \quad B = \frac{\mu_0 i_0}{\pi x}$$

Example 12. Figure shows a cross-section of a large metal sheet carrying an electric current along its surface. The current in a strip of width dl is Kdl where K is a constant. Find the magnetic field at a point P at a distance x from the metal sheet.



Solution :

Consider two strips A and C of the sheet situated symmetrically on the two sides of P (figure). The magnetic field at P due to the strip A is B_a perpendicular to AP and that due to the strip C is B_c perpendicular to CP. The resultant of these two is parallel to the width AC of the sheet. The field due to the whole sheet will also be in this direction. Suppose this field has magnitude B .



The field on the opposite side of the sheet at the same distance will also be B but in opposite direction. Applying Ampere's law to the rectangle shown in figure.

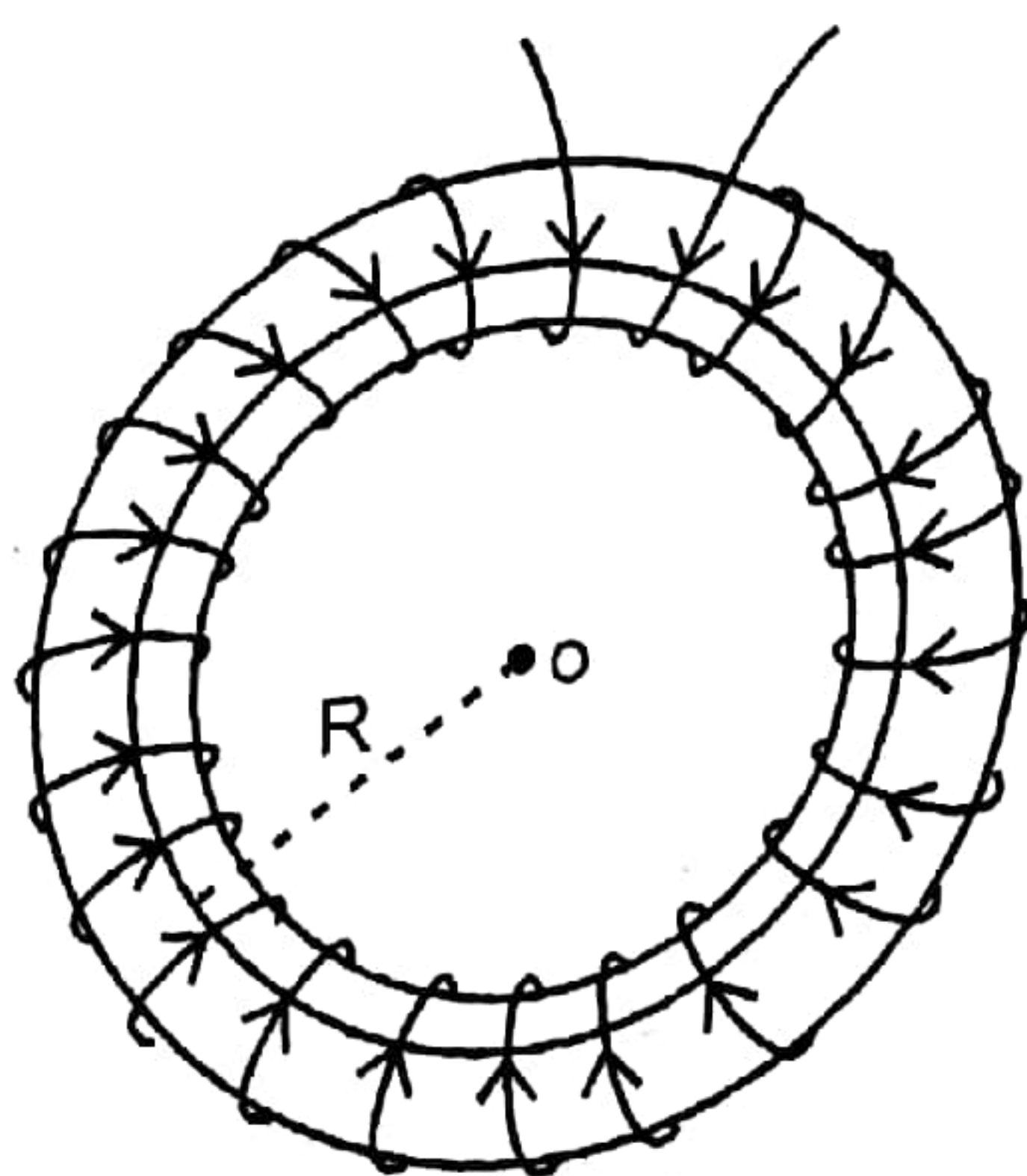
$$B \cdot l = \mu_0 K l \quad \text{or,} \quad B = \frac{1}{2} \mu_0 K$$

Note that it is independent of x .



Magnetic Field in a Toroid

(i) Toroid is like an endless cylindrical solenoid, i.e. if a long solenoid is bent round in the form of a closed ring, then it becomes a toroid.



(ii) Electrically insulated wire is wound uniformly over the toroid as shown in the figure.

(iii) The thickness of toroid is kept small in comparison to its radius and the number of turns is kept very large.

(iv) When a current i is passed through the toroid, each turn of the toroid produces a magnetic field along the axis at its centre. Due to uniform distribution of turns this magnetic field has same magnitude at their centres.

Thus the magnetic lines of force inside the toroid are circular.

(v) The magnetic field inside a toroid at all points is same but outside the toroid it is zero.

(vi) If total number of turns in a toroid is N and R is its radius, then number of turns per unit length of the toroid will be

$$n = \frac{N}{2\pi R}$$

(vii) The magnetic field due to toroid is determined by Ampere's law.

(viii) The magnetic field due to toroid is

$$B_0 = \mu_0 ni \quad \text{or} \quad B_0 = \mu_0 \left(\frac{N}{2\pi R} \right) i$$

(ix) If a substance of permeability μ is placed inside the toroid, then

$$B = \mu ni$$

If μ_r is relative magnetic permeability of the substance, then

$$B = \mu_r \mu_0 ni$$

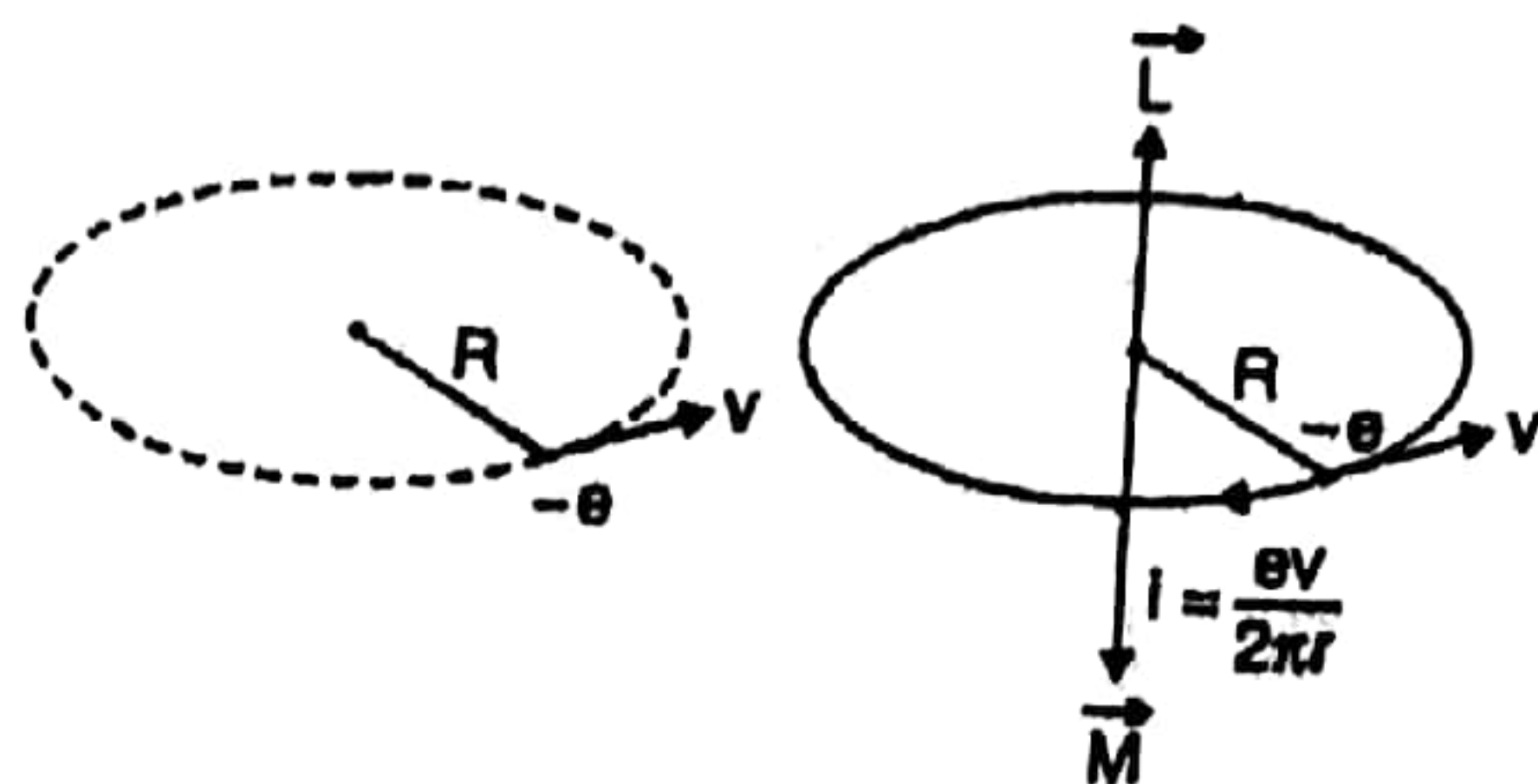
CURRENT AND MAGNETIC FIELD DUE TO CIRCULAR MOTION OF A CHARGE

(i) According to the theory of atomic structure every atom is made of electrons, protons and neutrons. protons and neutrons are in the nucleus of each atom and electrons are assumed to be moving in different orbits around the nucleus.

(ii) An electron and a proton present in the atom constitute an electric dipole at every moment but the direction of this dipole changes continuously and hence at any time the average dipole moment is zero. As a result static electric field is not observed.

(iii) Moving charge produces magnetic field and the average value of this field in the atom is not zero.

(iv) In an atom an electron moving in a circular path around the nucleus. Due to this motion current appears to be flowing in the electronic orbit and the orbit behaves like a current carrying coil. If e is the electron charge, R is the radius of the orbit and f is the frequency of motion of electron in the orbit, then



(a) current in the orbit = charge \times frequency = ef

If T is the period, then $f = \frac{1}{T}$

$$\therefore i = \frac{e}{T}$$

(b) Magnetic field at the nucleus (centre)

$$B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 ef}{2R} = \frac{\mu_0 e}{2RT}$$

(c) If the angular velocity of the electron is ω , then

$$\omega = 2\pi f \text{ and } f = \frac{\omega}{2\pi}$$

$$\therefore i = ef = \frac{e\omega}{2\pi}$$

$$\therefore B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 e\omega}{4\pi R}$$

(d) If the linear velocity of the electron is v , then

$$v = R\omega = R(2\pi f)$$

or
$$f = \left(\frac{v}{2\pi R} \right)$$

$$\therefore i = ef = \frac{ev}{2\pi R}$$

$$\therefore B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 ev}{4\pi R^2}$$

(v) Magnetic moment due to motion of electron in an orbit

$$M = iA = ef\pi R^2 = \frac{e\pi R^2}{T}$$

or
$$M = \frac{e\omega\pi R^2}{2\pi} = \frac{e\omega R^2}{2}$$

or
$$M = \frac{ev\pi R^2}{2\pi R} = \frac{evR}{2}$$

If the angular momentum of the electron is L , then

$$L = mvR = m\omega R^2$$

Writing M in terms of L

$$M = \frac{e\omega R^2}{2} = \frac{e\omega R^2}{2} \cdot \frac{m}{m} = \frac{eL}{2m}$$

According to Bohr's second postulate

$$mvR = n \frac{h}{2\pi}$$

In ground state $n = 1$

$$L = \frac{h}{2\pi}$$

$$\therefore M = \frac{eh}{4\pi m}$$

(vi) If a charge q (or a charged ring of charge q) is moving in a circular path of radius R with a frequency f or angular velocity ω , then



(a) current due to moving charge

$$i = qf = q\omega / 2\pi$$

(b) magnetic field at the centre of ring

$$B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 qf}{2R} \quad \text{or} \quad B_0 = \frac{\mu_0 q\omega}{4\pi R}$$

(c) magnetic moment

$$M = i(\pi R^2) = qf\pi R^2 = \frac{1}{2}q\omega R^2$$

(vii) If a charge q is distributed uniformly over the surface of plastic disc of radius R and it is rotated about its axis with an angular velocity ω , then

(a) the magnetic field produced at its centre will be

$$B_0 = \frac{\mu_0 q\omega}{2\pi R}$$

(b) the magnetic moment of the disc will be

$$dM = (di) \pi x^2$$

$$= \frac{\omega}{2\pi} dq \pi x^2$$

$$= \frac{\omega q}{R^2} x^3 dx$$

$$\Rightarrow M = \int dM = \frac{\omega q}{R^2} \int_0^R x^3 dx$$

$$M = \frac{q\omega R^2}{4}$$

$$M = \frac{q\omega R^2}{4}$$

3. Magnetic force on moving charge

When a charge q moves with velocity \vec{v} , in a magnetic field \vec{B} , then the magnetic force experienced by moving charge is given by following formula :

$$\vec{F} = q(\vec{v} \times \vec{B}) \quad \text{Put } q \text{ with sign.}$$

\vec{v} : Instantaneous velocity

\vec{B} : Magnetic field at that point.

Note :

- $\vec{F} \perp \vec{v}$ and also $\vec{F} \perp \vec{B}$
- $\because \vec{F} \perp \vec{v} \therefore$ power due to magnetic force on a charged particle is zero. (use the formula of power $P = \vec{F} \cdot \vec{v}$ for its proof).
- Since the $\vec{F} \perp \vec{B}$ so work done by magnetic force is zero in every part of the motion. The magnetic force cannot increase or decrease the speed (or kinetic energy) of a charged particle. Its can only change the direction of velocity.



- On a stationary charged particle, magnetic force is zero.
- If $\vec{v} \parallel \vec{B}$, then also magnetic force on charged particle is zero. It moves along a straight line if only magnetic field is acting.

Solved Examples

Example 13. A charged particle of mass 5 mg and charge $q = +2\mu\text{C}$ has velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Find out the magnetic force on the charged particle and its acceleration at this instant due to magnetic field $\vec{B} = 3\hat{j} - 2\hat{k}$. \vec{v} and \vec{B} are in m/s and Wb/m^2 respectively.

Solution : $\vec{F} = q\vec{v} \times \vec{B} = 2 \times 10^{-6} (2\hat{i} - 3\hat{j} + 4\hat{k}) \times (3\hat{j} - 2\hat{k}) = 2 \times 10^{-6} [-6\hat{i} + 4\hat{j} + 6\hat{k}] \text{ N}$

By Newton's Law $\vec{a} = \frac{\vec{F}}{m} = \frac{2 \times 10^{-6}}{5 \times 10^{-6}} (-6\hat{i} + 4\hat{j} + 6\hat{k})$
 $= 0.8 (-3\hat{i} + 2\hat{j} + 3\hat{k}) \text{ m/s}^2$

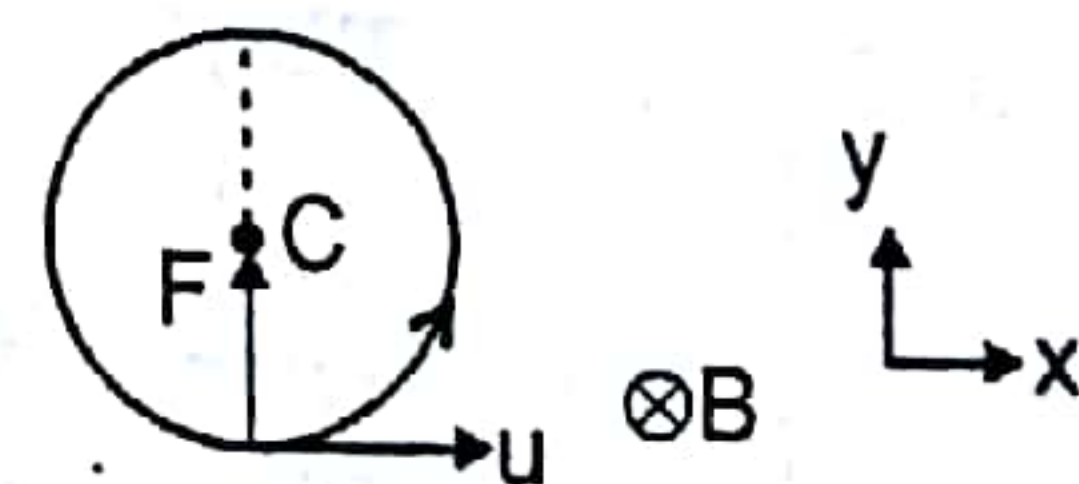
Example 14. A charged particle has acceleration $\vec{a} = 2\hat{i} + x\hat{j}$ in a magnetic field $\vec{B} = -3\hat{i} + 2\hat{j} - 4\hat{k}$. Find the value of x .

Solution : $\therefore \vec{F} \perp \vec{B}$
 $\therefore \vec{a} \perp \vec{B} \quad \therefore \vec{a} \cdot \vec{B} = 0$
 $\therefore (2\hat{i} + x\hat{j}) \cdot (-3\hat{i} + 2\hat{j} - 4\hat{k}) = 0$
 $\Rightarrow -6 + 2x = 0 \quad \Rightarrow x = 3.$



3.1 Motion of charged particles under the effect of magnetic force

- Particle released if $v = 0$ then $f_m = 0$
 \therefore particle will remain at rest
- $\vec{V} \parallel \vec{B}$ here $\theta = 0$ or $\theta = 180^\circ$
 $\therefore F_m = 0 \quad \therefore \vec{a} = 0 \quad \therefore \vec{V} = \text{const.}$
 \therefore particle will move in a straight line with constant velocity
- Initial velocity $\vec{u} \perp \vec{B}$ and $\vec{B} = \text{uniform}$



In this case $\therefore B$ is in z direction so the magnetic force in z -direction will be zero ($\therefore \vec{F}_m \perp \vec{B}$).

Now there is no initial velocity in z -direction.

\therefore particle will always move in xy plane.

\therefore velocity vector is always $\perp \vec{B} \therefore F_m = qvB = \text{constant}$

now $quB = \frac{mu^2}{R} \Rightarrow R = \frac{mu}{qB} = \text{constant.}$

The particle moves in a curved path whose radius of curvature is same every where, such curve in a plane is only a circle.

\therefore path of the particle is circular.

$$R = \frac{mu}{qB} = \frac{p}{qB} = \frac{\sqrt{2mk}}{qB}$$

here $p = \text{linear momentum ; } k = \text{kinetic energy}$

now $v = \omega R \Rightarrow \omega = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f$

Time period $T = 2\pi m/qB$

frequency $f = qB/2\pi m$

Note :

- ω, f, T are independent of velocity.

Solved Examples

Example 15. A proton (p), α -particle and deuteron (D) are moving in circular paths with same kinetic energies in the same magnetic field. Find the ratio of their radii and time periods. (Neglect interaction between particles).

Solution : $R = \frac{\sqrt{2mK}}{qB}$

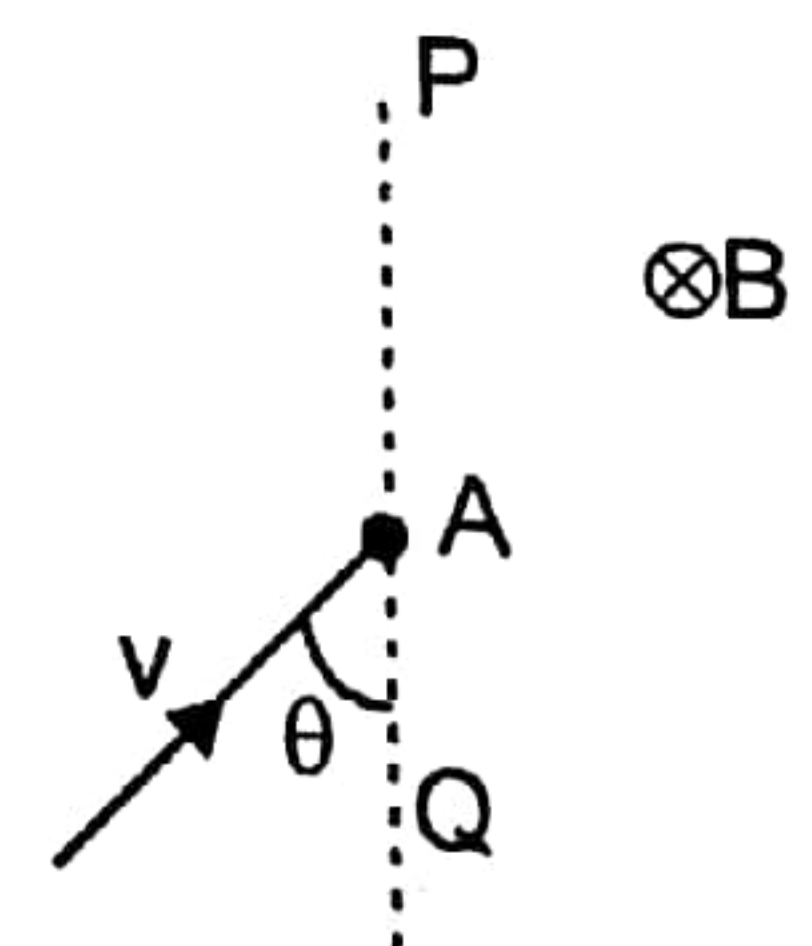
$$\therefore R_p : R_\alpha : R_D = \frac{\sqrt{2mK}}{qB} : \frac{\sqrt{2.4mK}}{2qB} : \frac{\sqrt{2.2mK}}{qB} = 1 : 1 : \sqrt{2}$$

$T = 2\pi m/qB$

$$\therefore T_p : T_\alpha : T_D = \frac{2\pi m}{qB} : \frac{2\pi 4m}{2qB} : \frac{2\pi 2m}{qB} = 1 : 2 : 2 \text{ Ans.}$$

Example 16. A positive charge particle of charge q , mass m enters into a uniform magnetic field with velocity v as shown in the figure. There is no magnetic field to the left of PQ.

- Find (i) time spent,
 (ii) distance travelled in the magnetic field
 (iii) impulse of magnetic force.



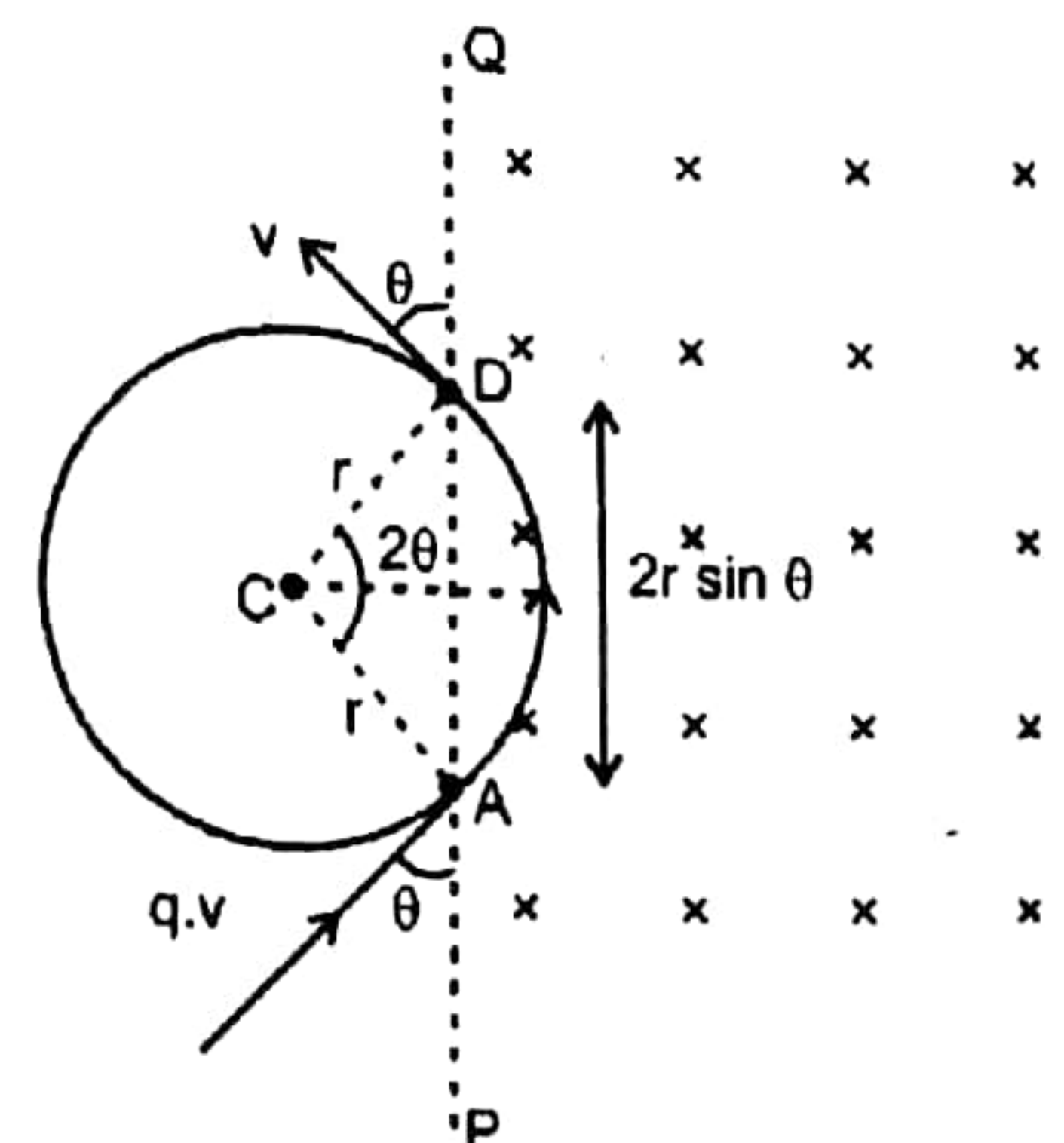
Solution : The particle will move in the field as shown
 Angle subtended by the arc at the centre = 2θ

(i) Time spent by the charge in magnetic field

$$\omega t = 2\theta \Rightarrow \frac{qB}{m} t = 2\theta \Rightarrow t = \frac{m2\theta}{qB}$$

(ii) Distance travelled by the charge in magnetic field :

$$= r(2\theta) = \frac{mv}{qB} \cdot 2\theta$$





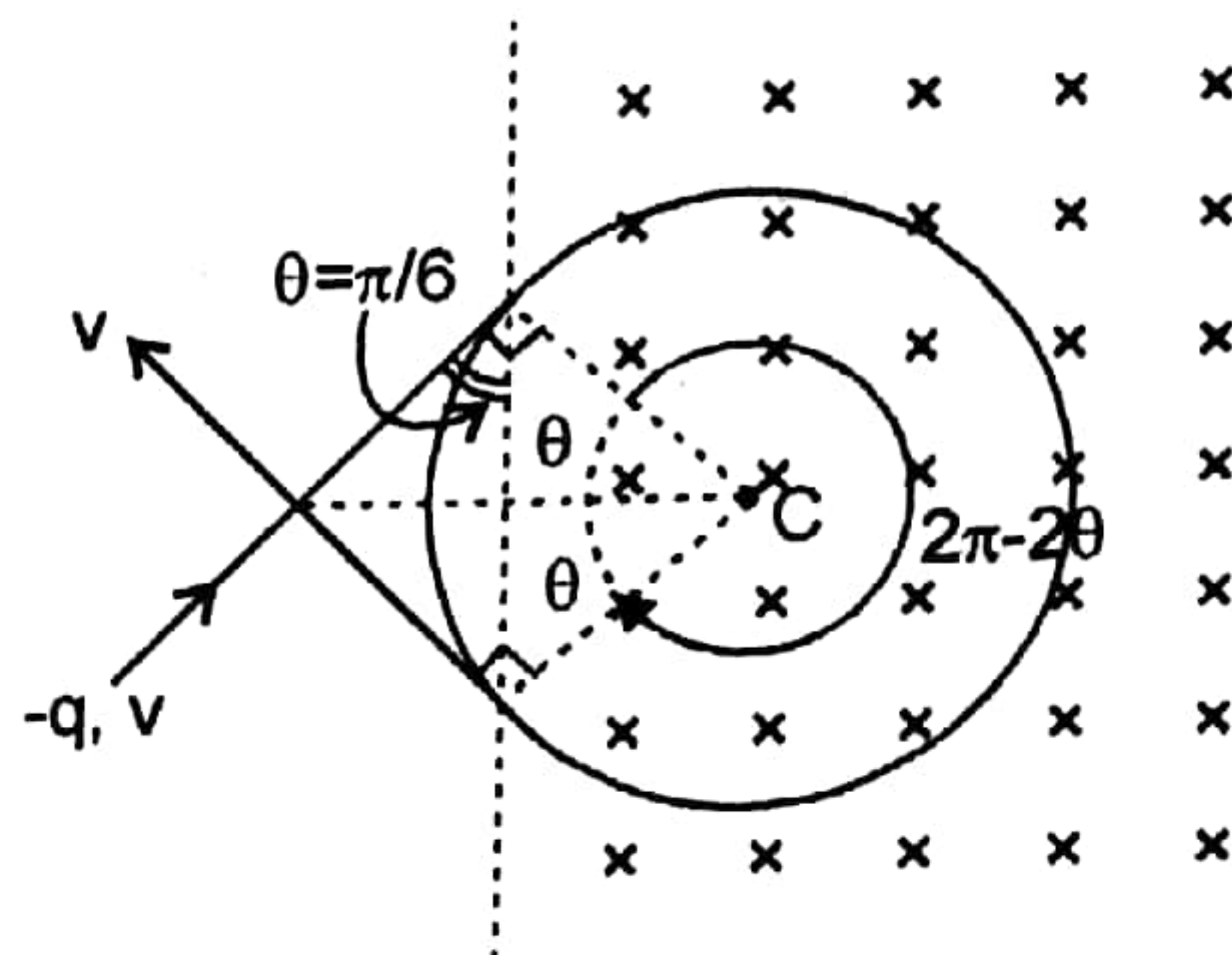
(iii) Impulse = change in momentum of the charge

$$= (-mv \sin \theta \hat{i} + mv \cos \theta \hat{j}) - (mv \sin \theta \hat{i} + mv \cos \theta \hat{j}) = -2mv \sin \theta \hat{i}$$

Example 17. Repeat above question if the charge is $-ve$ and the angle made by the boundary with the velocity is $\frac{\pi}{6}$.

Solution :

(i) $2\pi - 2\theta = 2\pi - 2 \cdot \frac{\pi}{6} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$
 $= \omega t = \frac{qBt}{m} \Rightarrow t = \frac{5\pi m}{3qB}$



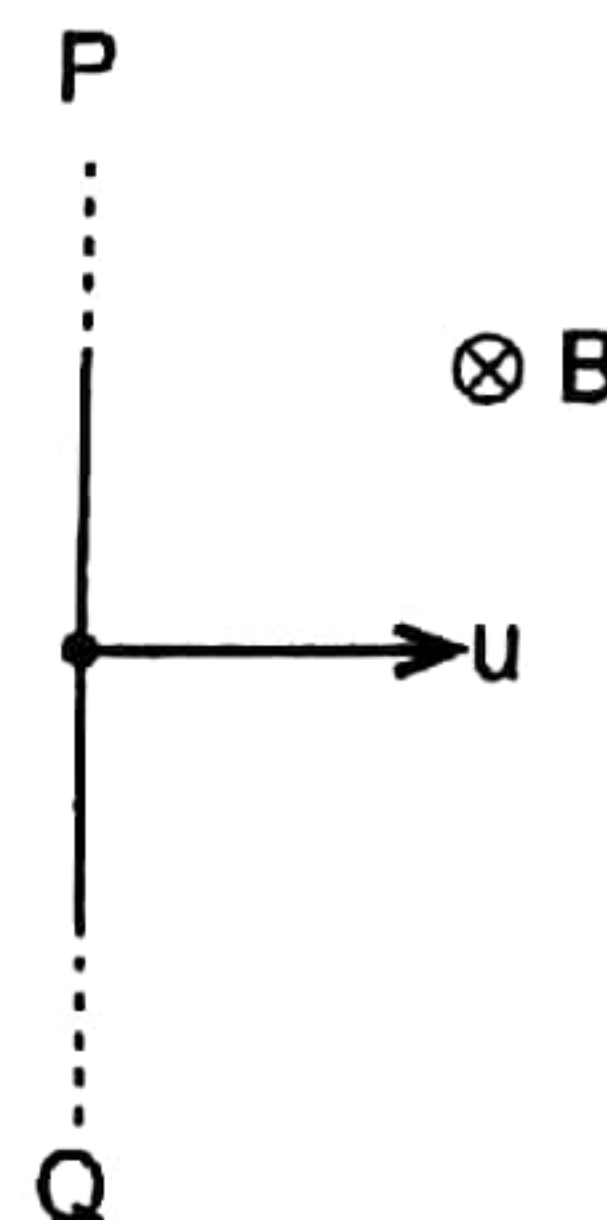
(ii) Distance travelled $s = r(2\pi - 2\theta) = \frac{5\pi r}{3}$

(iii) Impulse = change in linear momentum

$$= m(-v \sin \theta \hat{i} + v \cos \theta \hat{j}) - m(v \sin \theta \hat{i} + v \cos \theta \hat{j})$$

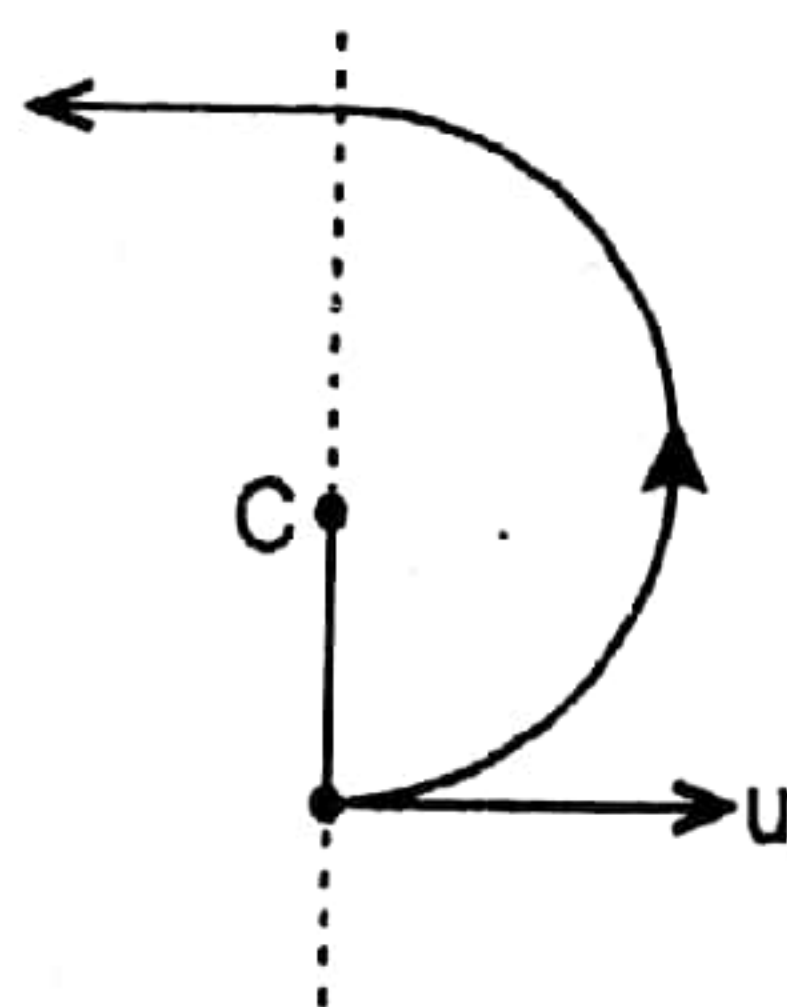
$$= -2mv \sin \theta \hat{i} = -2mv \sin \frac{\pi}{6} \hat{i} = -mv \hat{i}$$

Example 18. In the figure shown the magnetic field on the left of 'PQ' is zero and on the right of 'PQ' it is uniform. Find the time spent in the magnetic field.



Solution :

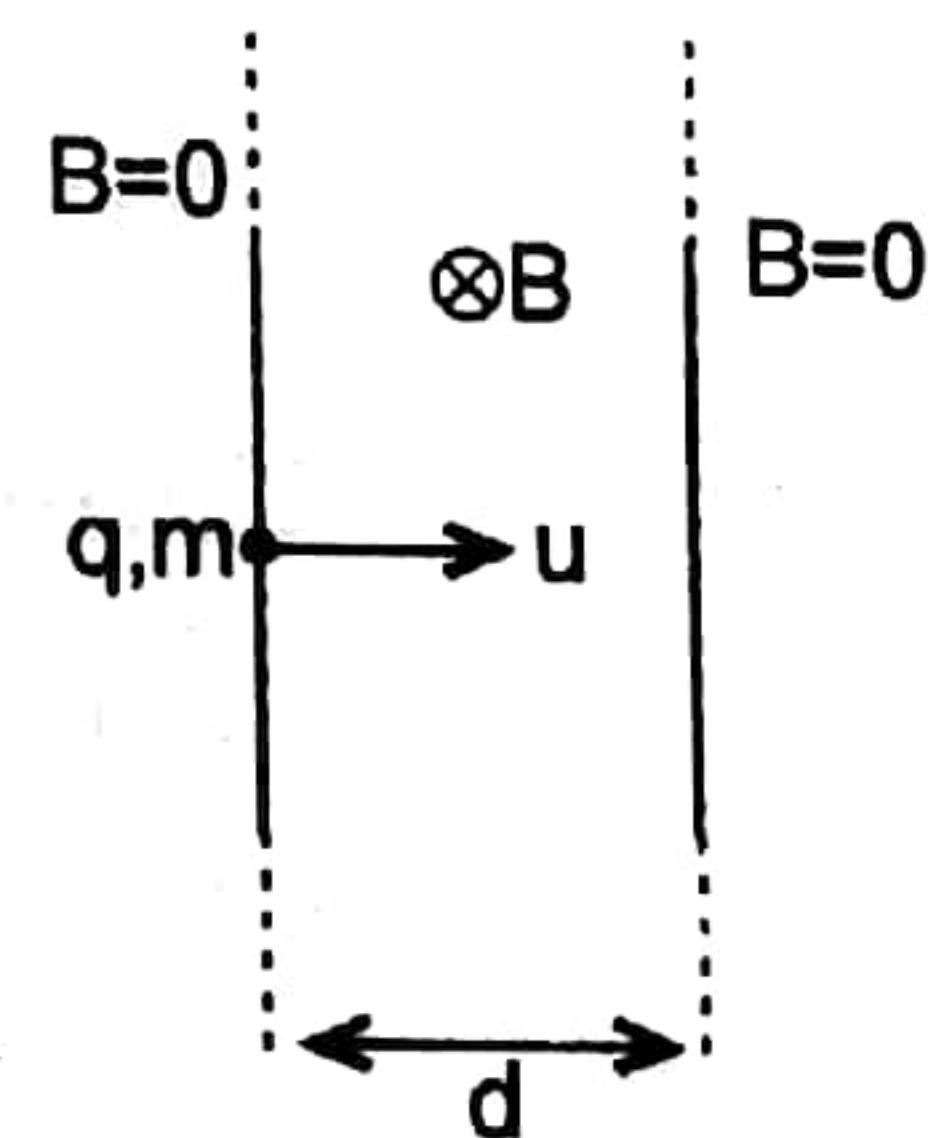
The path will be semicircular time spent = $T/2 = \pi m/qB$



Example 19. A uniform magnetic field of strength 'B' exists in a region of width 'd'. A particle of charge 'q' and mass 'm' is shot perpendicularly (as shown in the figure) into the magnetic field. Find the time spent by the particle in the magnetic field if

(i) $d > \frac{mu}{qB}$

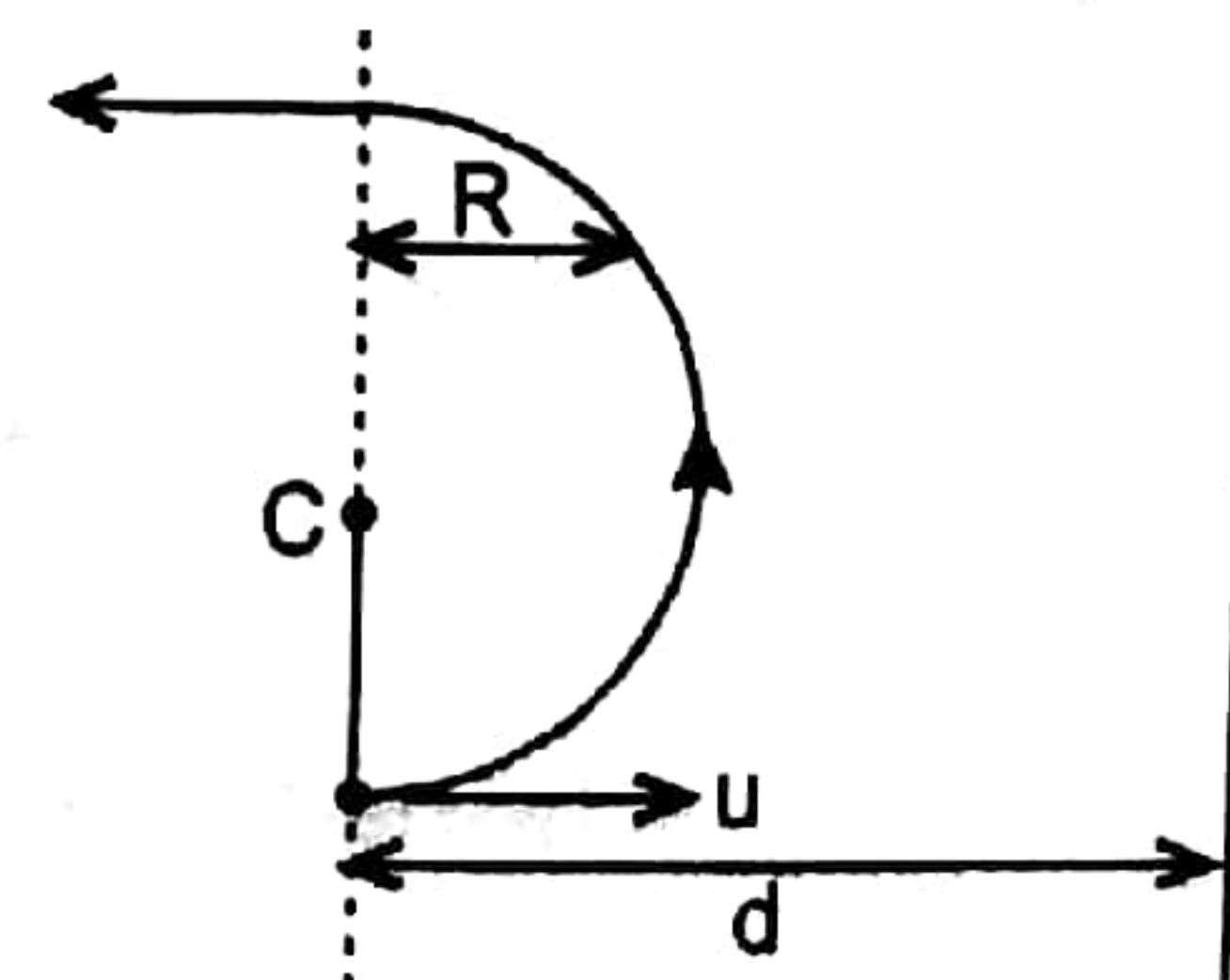
(ii) $d < \frac{mu}{qB}$



Solution :

(i) $d > \frac{mu}{qB}$ means $d > R$

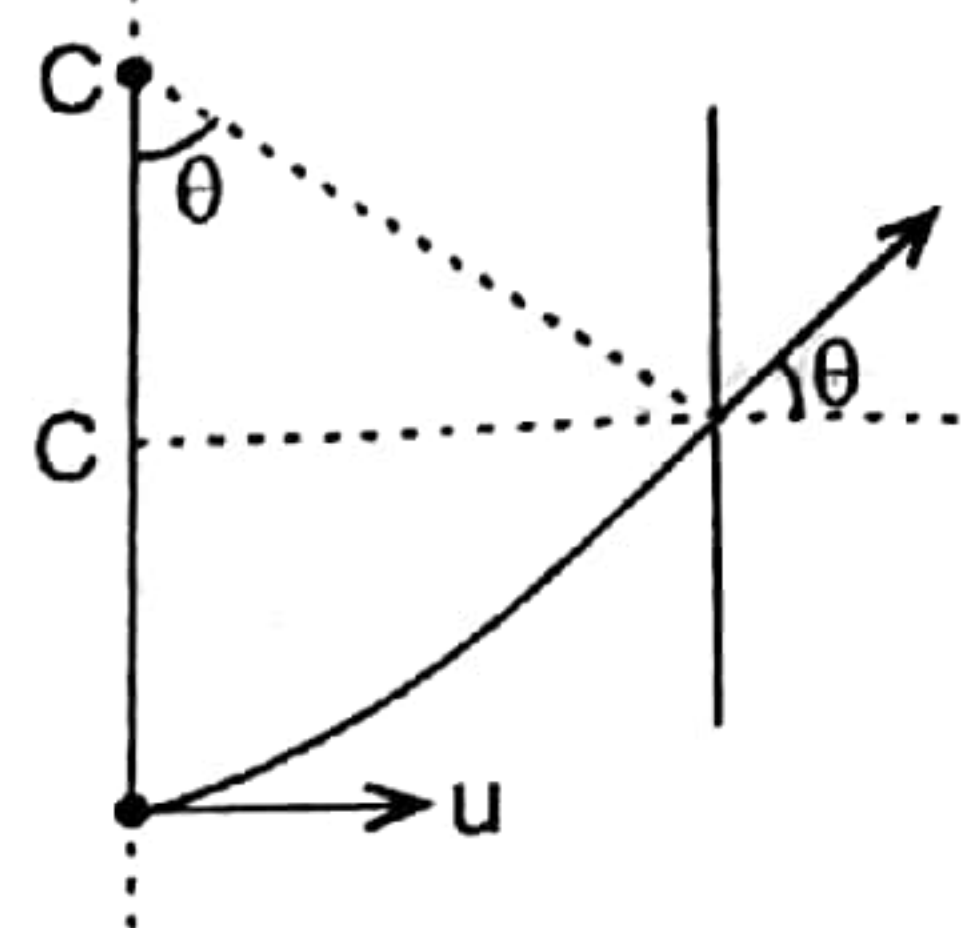
$$\therefore t = \frac{T}{2} = \frac{\pi m}{qB}$$



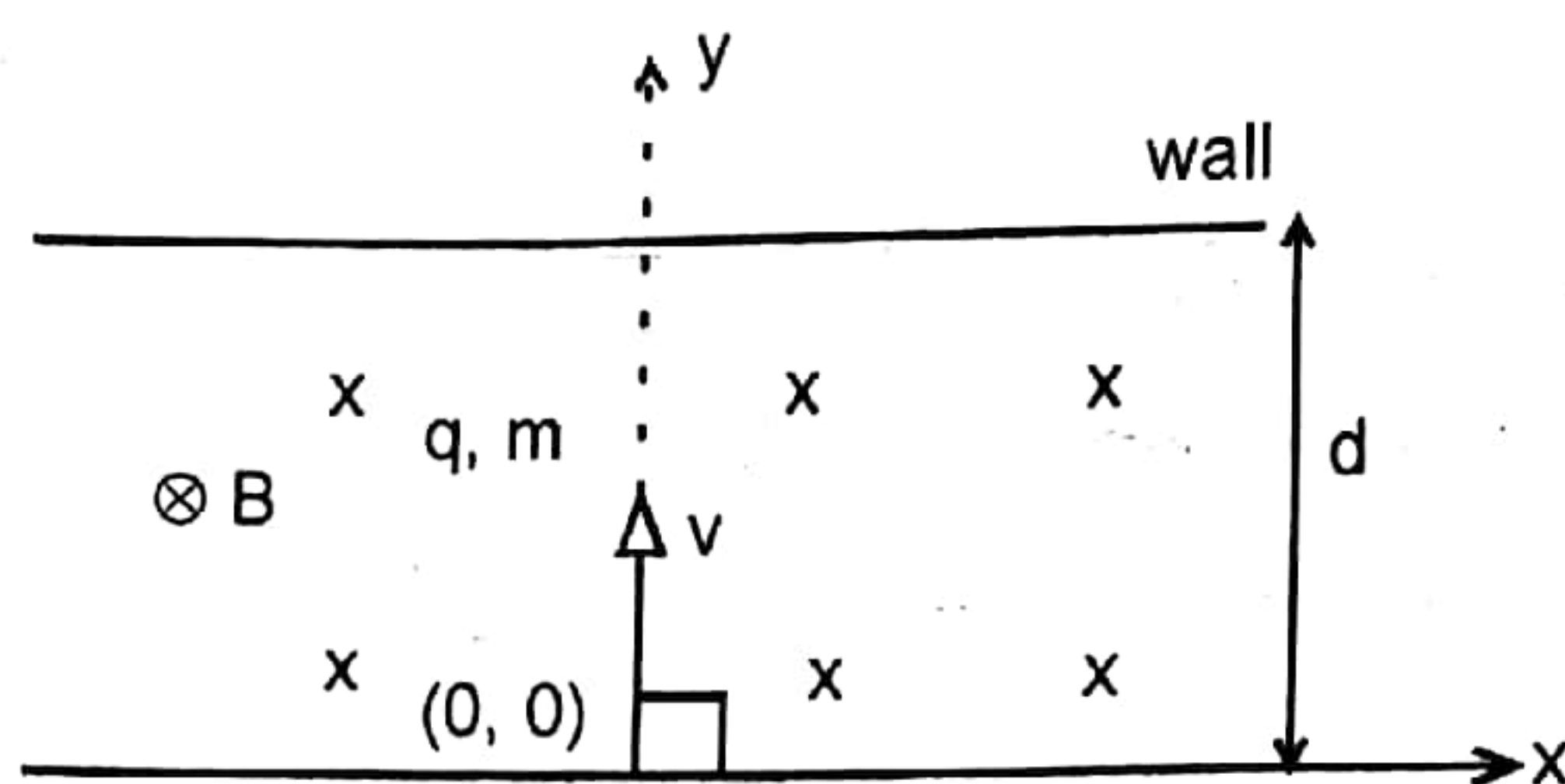
(ii) $\sin \theta = \frac{d}{R}$

$\theta = \sin^{-1} \left(\frac{d}{R} \right)$

$\omega t = \theta \Rightarrow t = \frac{m}{qB} \sin^{-1} \left(\frac{d}{R} \right)$



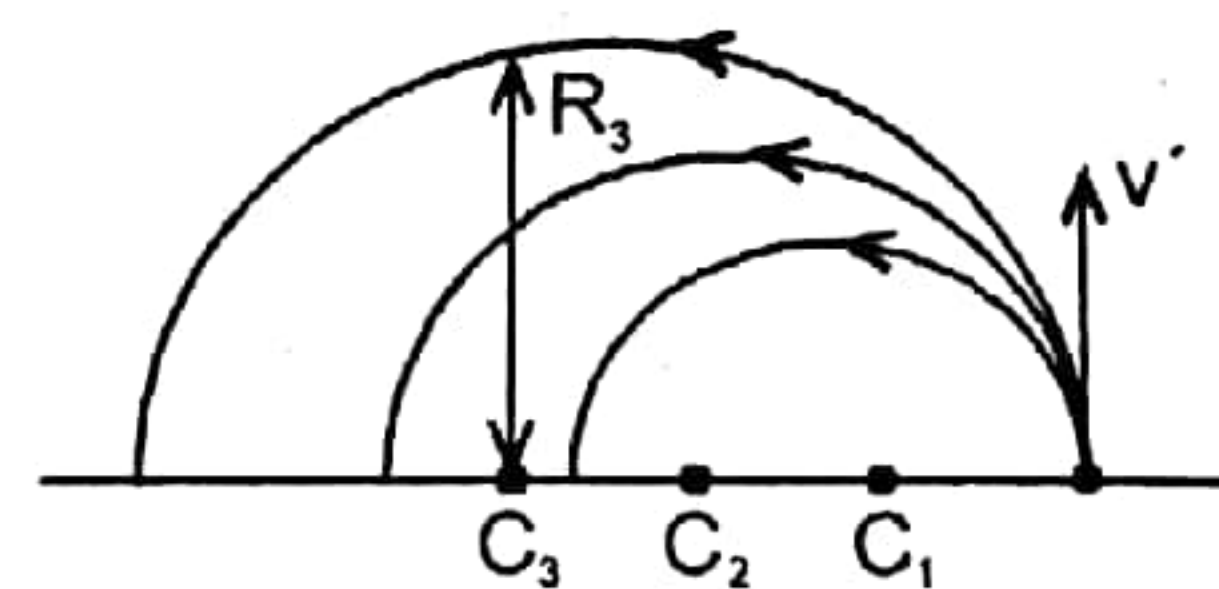
Example 20. What should be the speed of charged particle so that it can't collide with the upper wall? Also find the coordinate of the point where the particle strikes the lower plate in the limiting case of velocity.



Solution : (i) The path of the particle will be circular larger the velocity, larger will be the radius.

For particle not to strike $R < d$

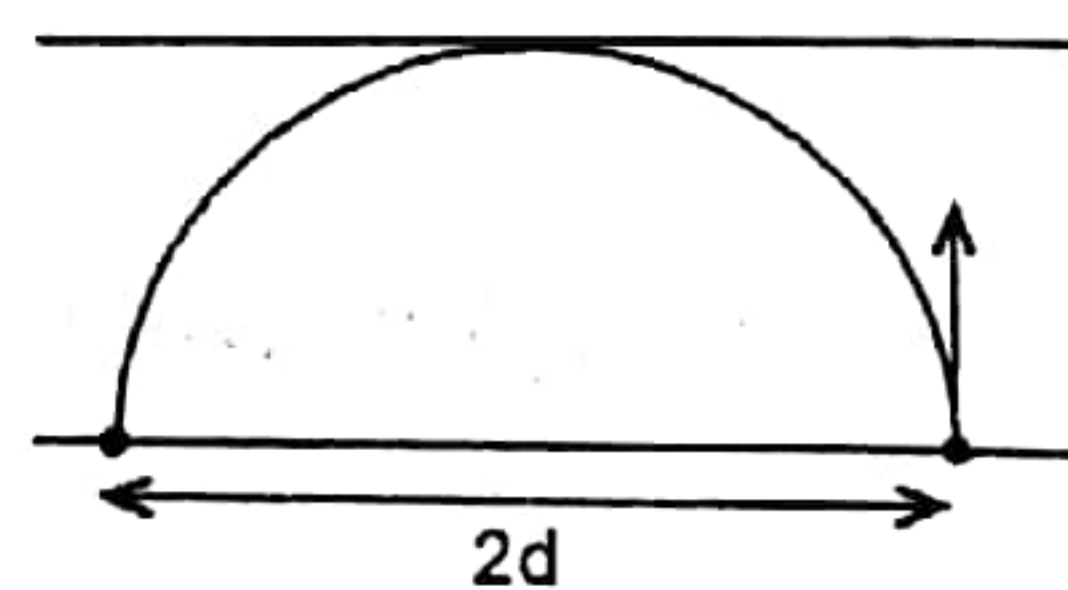
$\therefore \frac{mv}{qB} < d \Rightarrow v < \frac{qBd}{m}$



(ii) for limiting case $v = \frac{qBd}{m}$

$R = d$

\therefore coordinate = $(-2d, 0, 0)$

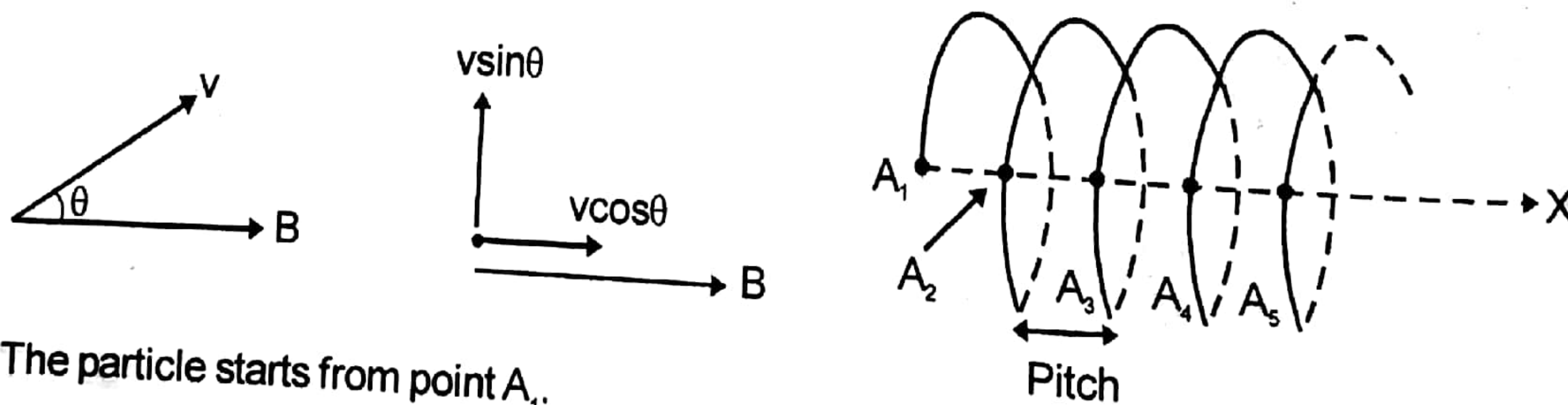


3.2 Helical path :

If the velocity of the charge is not perpendicular to the magnetic field, we can break the velocity in two components – $v_{||}$, parallel to the field and v_{\perp} , perpendicular to the field. The components $v_{||}$ remains unchanged as the force $q\vec{v} \times \vec{B}$ is perpendicular to it. In the plane perpendicular to the field, the particle traces a circle of radius $r = \frac{mv_{\perp}}{qB}$ as given by equation. The resultant path is helix.

Complete analysis :

Let a particle have initial velocity in the plane of the paper and a constant and uniform magnetic field also in the plane of the paper.



The particle starts from point A_1 .



It completes its one revolution at A_2 and 2nd revolution at A_3 and so on. X-axis is the tangent to the helix points

A_1, A_2, A_3, \dots all are on the x-axis.

distance $A_1 A_2 = A_3 A_4 = \dots = v \cos \theta$. $T =$ pitch

where $T =$ Time period

Let the initial position of the particle be $(0, 0, 0)$ and $v \sin \theta$ in $+y$ direction. Then

in x: $F_x = 0, a_x = 0, v_x = \text{constant} = v \cos \theta, x = (v \cos \theta)t$

In y-z plane :

From figure it is clear that

$$y = R \sin \beta, v_y = v \sin \theta \cos \beta$$

$$z = -(R - R \cos \beta)$$

$$v_z = v \sin \theta \sin \beta$$

acceleration towards centre $= (v \sin \theta)^2 / R = \omega^2 R$

$$\therefore a_y = -\omega^2 R \sin \beta, a_z = -\omega^2 R \cos \beta$$

At any time : the position vector of the particle (or its displacement w.r.t. initial position)

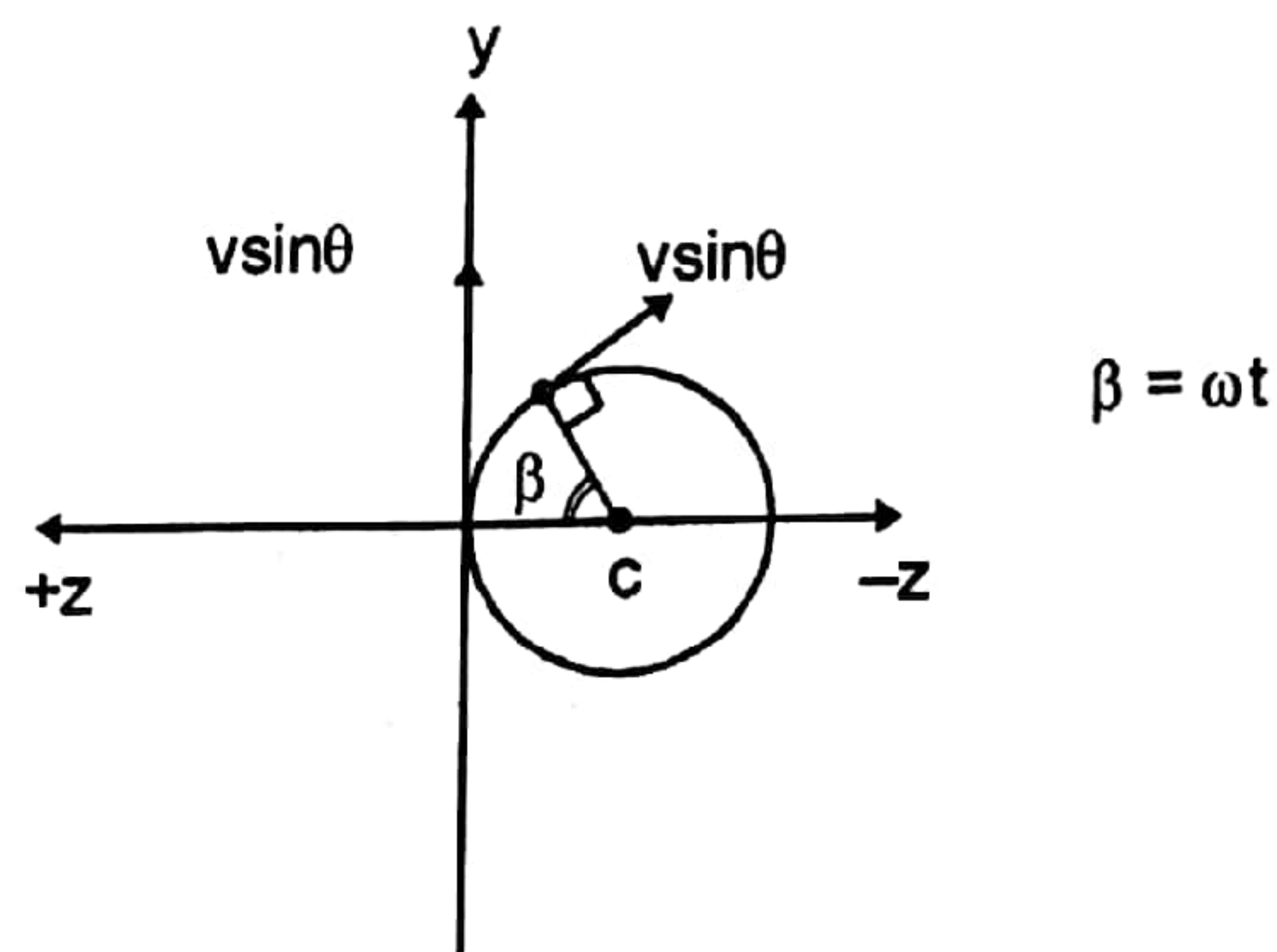
$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, x, y, z \text{ already found}$$

velocity $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}, v_x, v_y, v_z$ already found

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}, a_x, a_y, a_z \text{ already found}$$

Radius $q(v \sin \theta)B = \frac{m(v \sin \theta)^2}{R} \Rightarrow R = \frac{mv \sin \theta}{qB}$

$$\omega = \frac{v \sin \theta}{R} = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f.$$



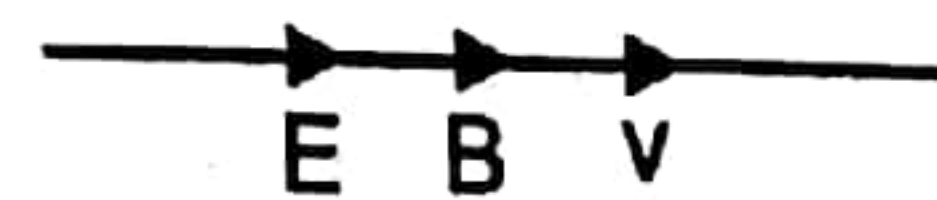
3.3 Charged Particle in \vec{E} & \vec{B}

When a charged particle moves with velocity \vec{v} in an electric field \vec{E} and magnetic field \vec{B} , then. Net force experienced by it is given by following equation.

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

Combined force is known as Lorentz force.

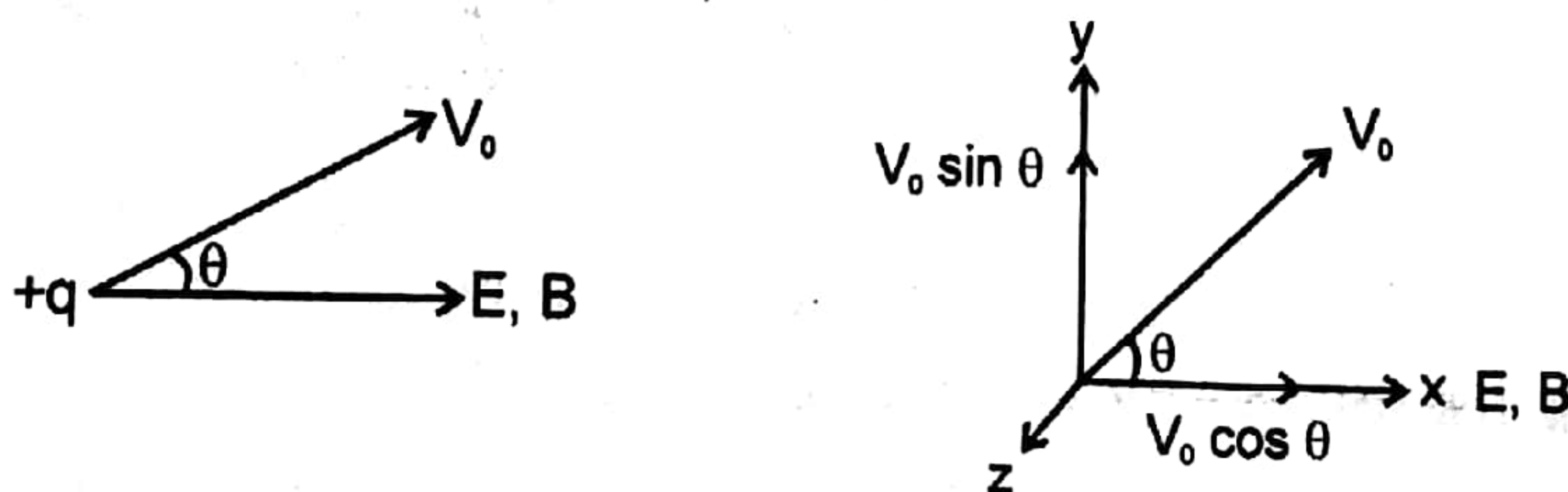
$$\vec{E} \parallel \vec{B} \parallel \vec{v}$$

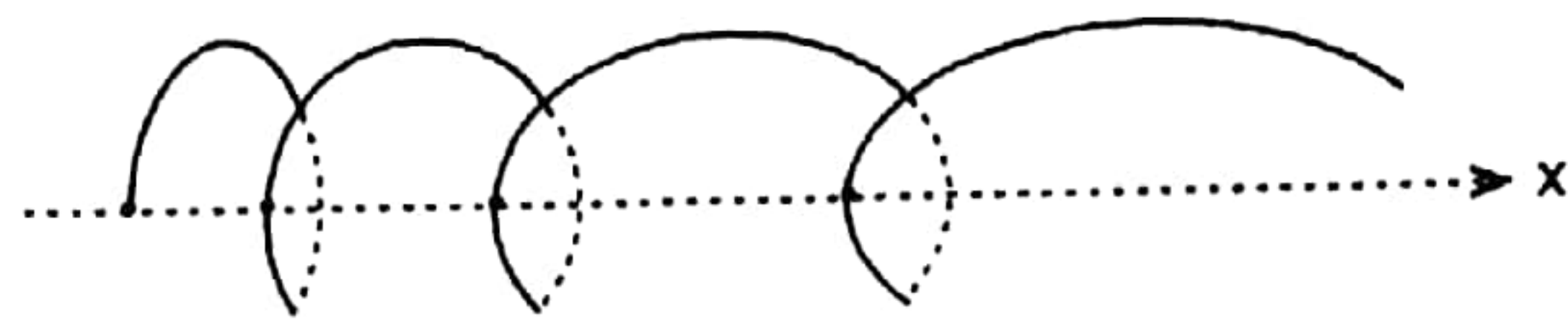


In above situation particle passes undeviated but its velocity will change due to electric field. Magnetic force on it = 0.

Case(i) :

- $\vec{E} \parallel \vec{B}$ and uniform $\theta \neq 0, 180^\circ$ (\vec{E} and \vec{B} are constant and uniform)





in x : $F_x = qE$, $a_x = \frac{qE}{m}$, $v_x = v_0 \cos \theta + a_x t$, $x = v_0 t + \frac{1}{2} a_x t^2$

in yz plane :

$qv_0 \sin \theta B = m(v_0 \sin \theta)^2 / R$

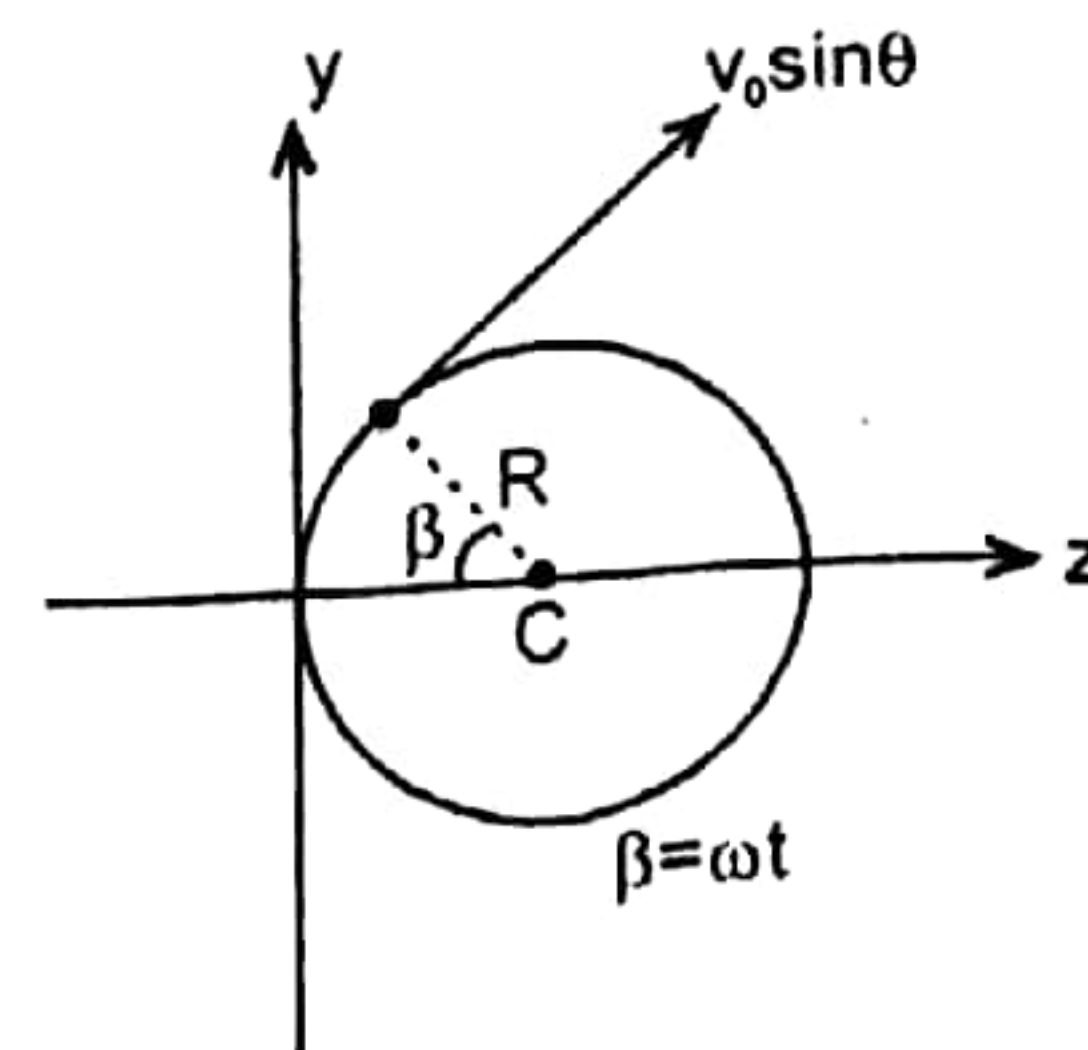
$\Rightarrow R = \frac{mv_0 \sin \theta}{qB}$

$\omega = \frac{v_0 \sin \theta}{R} = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f$

$\vec{r} = \{(V_0 \cos \theta)t + \frac{1}{2} \frac{qE}{m} t^2\} \hat{i} + R \sin \omega t \hat{j} + (R - R \cos \omega t) (-\hat{k})$

$\vec{v} = \left(V_0 \cos \theta + \frac{qE}{m} t \right) \hat{i} + (V_0 \sin \theta) \cos \omega t \hat{j} + V_0 \sin \theta \sin \omega t (-\hat{k})$

$\vec{a} = \frac{qE}{m} \hat{i} + \omega^2 R [-\sin \beta \hat{j} - \cos \beta \hat{k}]$



Solved Examples

Example 21. A particle of charge q and mass m is projected in a uniform and constant magnetic field of strength B . The initial velocity vector \vec{v} makes angle ' θ ' with the \vec{B} . Find the distance travelled by the particle in time ' t '.

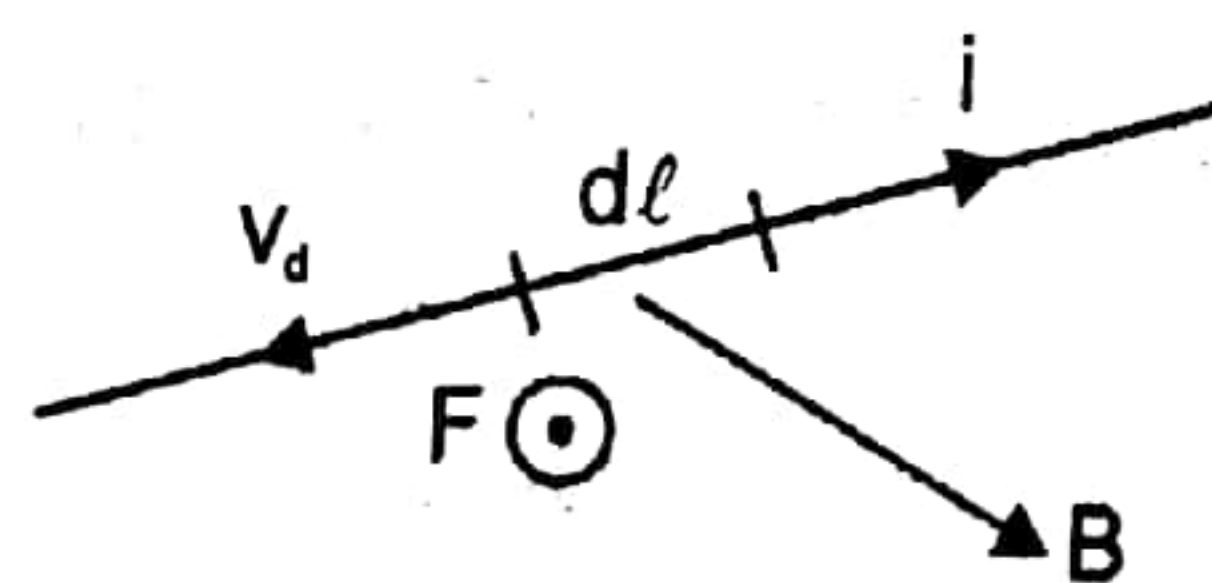
Answer : vt

Solution : Speed of the particle does not change therefore distance covered by the particle is $s = vt$



3.4 Magnetic force on a current carrying wire :

Suppose a conducting wire, carrying a current i , is placed in a magnetic field \vec{B} . Consider a small element $d\ell$ of the wire (figure). The free electrons drift with a speed v_d opposite to the direction of the current. The relation between the current i and the drift speed v_d is



$i = jA = nev_d A$ (i)

Here A is the area of cross-section of the wire and n is the number of free electrons per unit volume. Each electron experiences an average (why average?) magnetic force

$\vec{f} = -e\vec{v}_d \times \vec{B}$

The number of free electrons in the small element considered is $nAd\ell$. Thus, the magnetic force on the wire of length $d\ell$ is

$d\vec{F} = (nAd\ell)(-e\vec{v}_d \times \vec{B})$

If we denote the length $d\ell$ along the direction of the current by $d\vec{\ell}$, the above equation becomes

$$d\vec{F} = nAev_d \vec{d\ell} \times \vec{B}$$

Using (i), $d\vec{F} = i d\vec{\ell} \times \vec{B}$

The quantity $i d\vec{\ell}$ is called a *current element*.

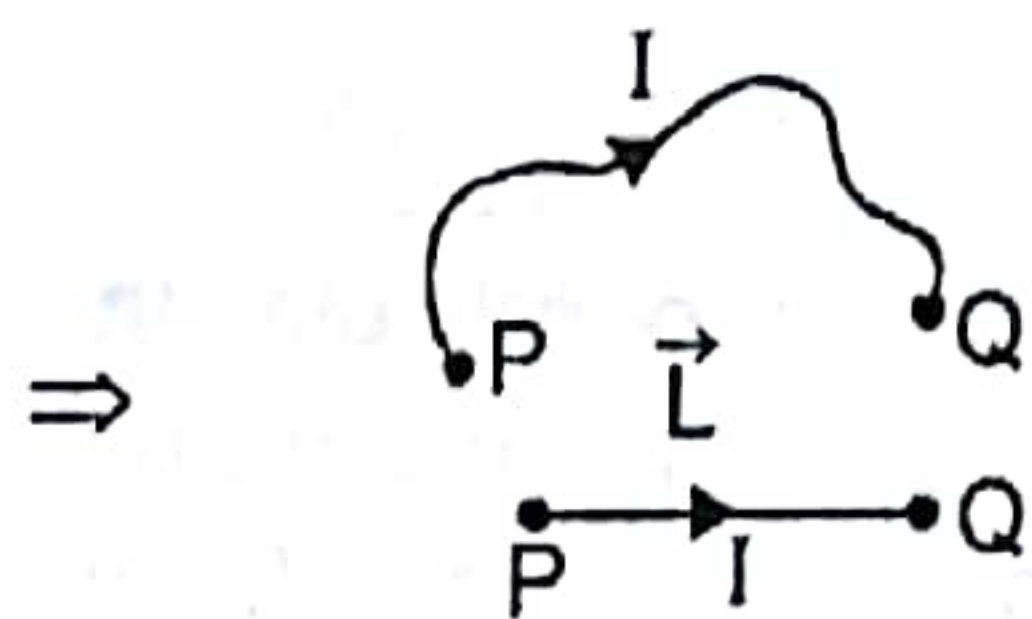
$$\vec{F}_{res} = \int d\vec{F} = \int i d\vec{\ell} \times \vec{B} = i \int d\vec{\ell} \times \vec{B}$$

(\because i is same at all points of the wire.)

If \vec{B} is uniform then $\vec{F}_{res} = i(\int d\vec{\ell}) \times \vec{B}$

$$\vec{F}_{res} = i\vec{L} \times \vec{B}$$

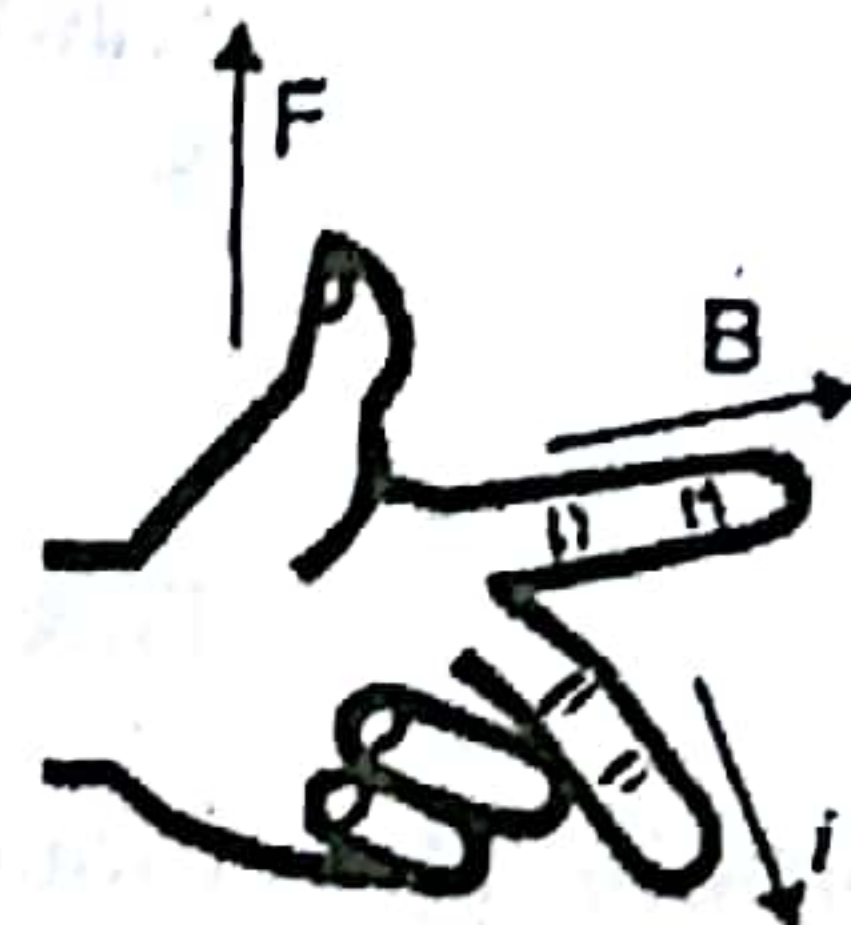
Here $\vec{L} = \int d\vec{\ell}$ = vector length of the wire = vector connecting the end points of the wire.



The direction of magnetic force is perpendicular to the plane of \vec{L} and \vec{B} according to right hand screw rule. Following two rules are used in determining the direction of the magnetic force.

(a) **Right hand palm rule** : If the right hand and the palm are stretched such that the thumb points in the direction of current and the stretched fingers in the direction of the magnetic field, then the force on the conductor will be perpendicular to the palm in the outward direction.

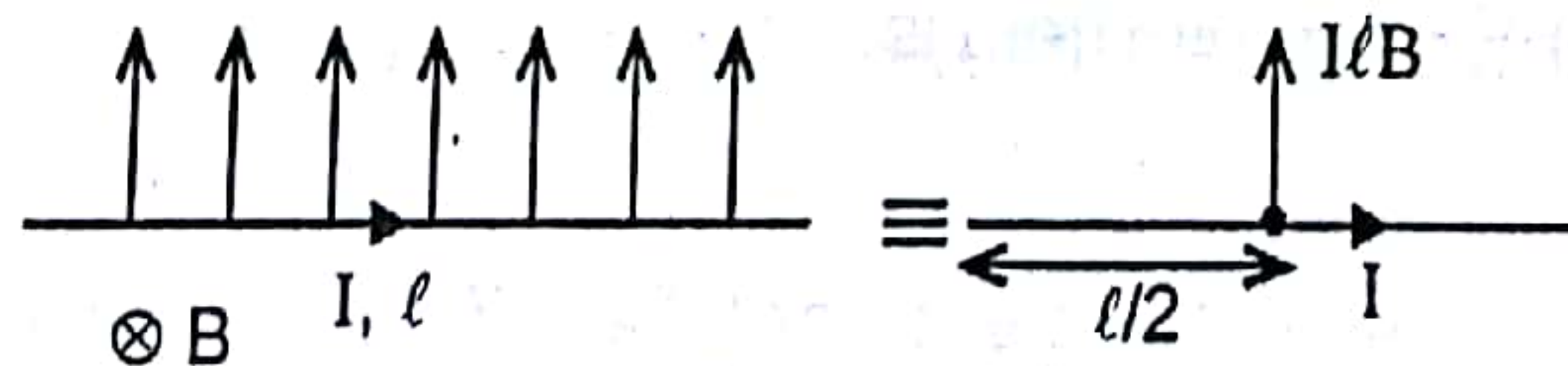
(b) **Fleming left hand rule** : If the thumb, fore finger and central finger of the left hand are stretched such that first finger points in the direction of magnetic field and the central finger in the direction of current, then the thumb will point in the direction of force acting on the conductor.



Note : If a current loop of any shape is placed in a uniform \vec{B} then $\vec{F}_{res} \Big|_{magnetic}$ on it = 0 ($\because \vec{L} = 0$).

3.5 Point of application of magnetic force :

On a straight current carrying wire the magnetic force in a uniform magnetic field can be assumed to be acting at its mid point.



This can be used for calculation of torque.

Solved Examples

Example 22. A wire is bent in the form of an equilateral triangle PQR of side 10 cm and carries a current of 5.0 A. It is placed in a magnetic field B of magnitude 2.0 T directed perpendicularly to the plane of the loop. Find the forces on the three sides of the triangle.

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Solution :

Suppose the field and the current have directions as shown in figure. The force on PQ is

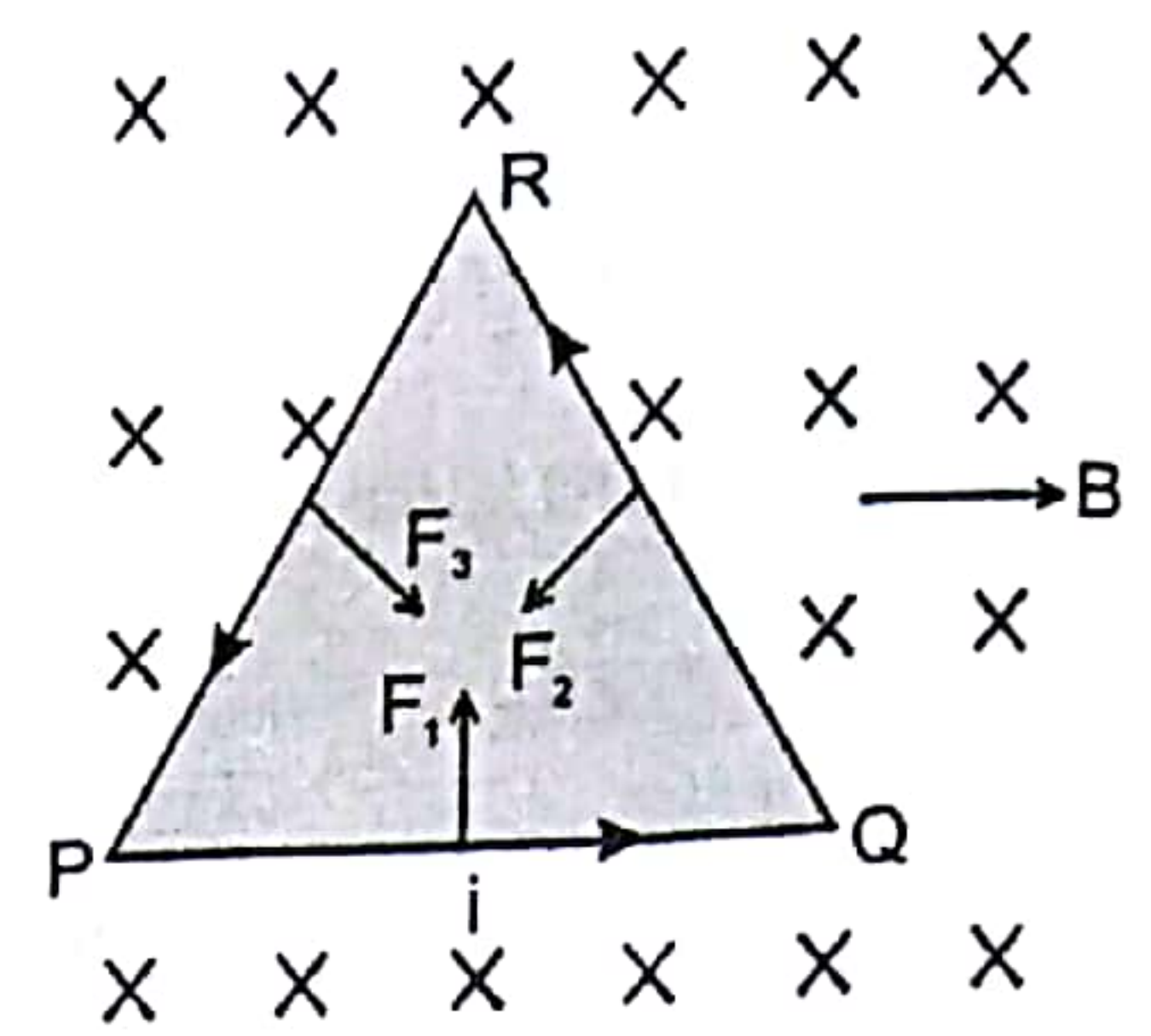
$$\vec{F}_1 = i\vec{\ell} \times \vec{B}$$

or, $F_1 = 5.0 \text{ A} \times 10 \text{ cm} \times 2.0 \text{ T} = 1.0 \text{ N}$

The rule of vector product shows that the force F_1 is perpendicular to PQ and is directed towards the inside of the triangle.

The forces \vec{F}_2 and \vec{F}_3 on QR and RP can also be obtained similarly. Both the forces are 1.0 N directed perpendicularly to the respective sides and towards the inside of the triangle.

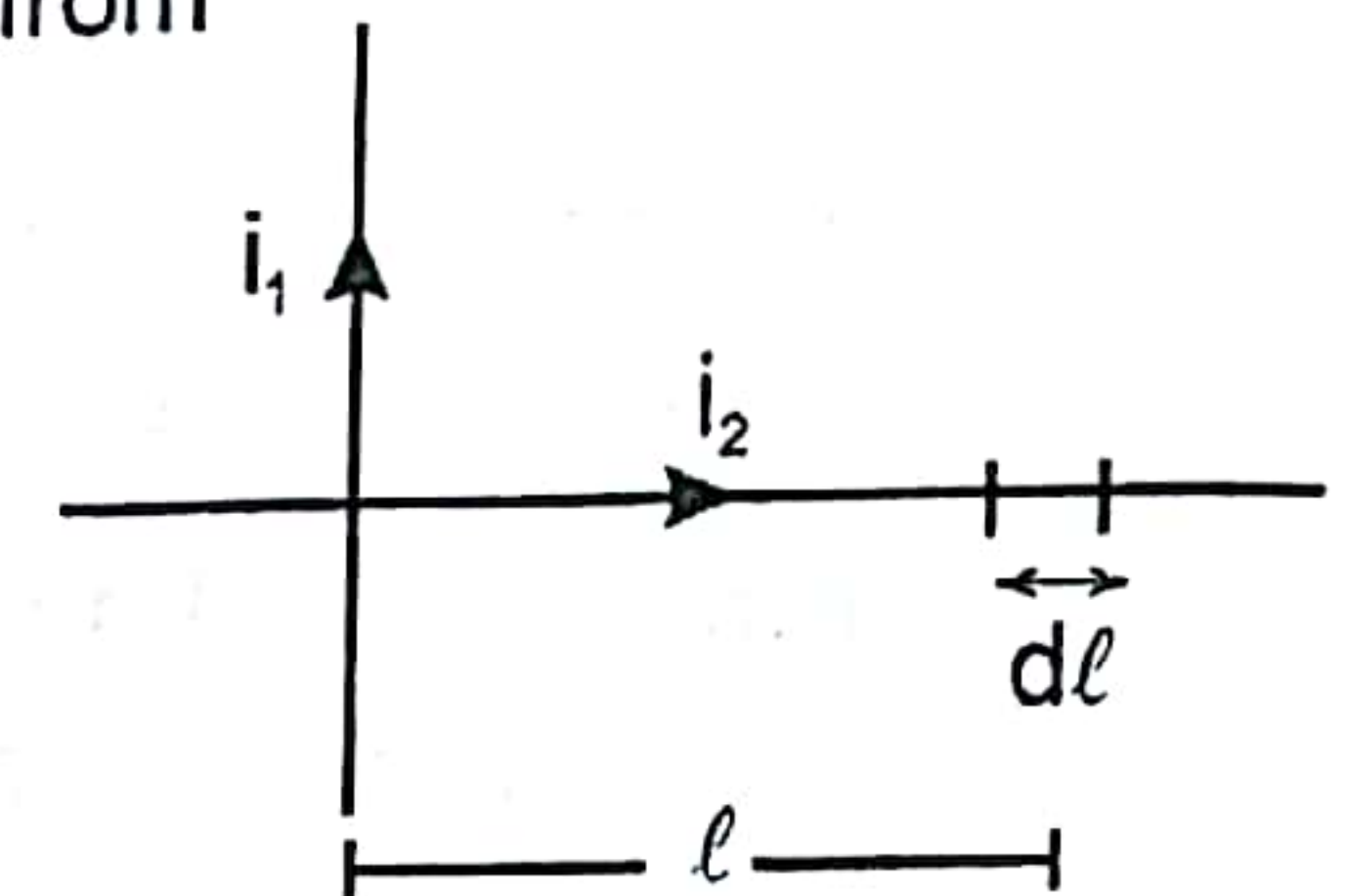
The three forces \vec{F}_1 , \vec{F}_2 and \vec{F}_3 will have zero resultant, so that there is no net magnetic force on the triangle. This result can be generalised. Any closed current loop, placed in a homogeneous magnetic field, does not experience a net magnetic force.



Example 23. Two long wires, carrying currents i_1 and i_2 , are placed perpendicular to each other in such a way that they just avoid a contact. Find the magnetic force on a small length $d\ell$ of the second wire situated at a distance ℓ from the first wire.

Solution : The situation is shown in figure. The magnetic field at the site of $d\ell$, due to the first wire is ,

$$B = \frac{\mu_0 i_1}{2\pi\ell}$$



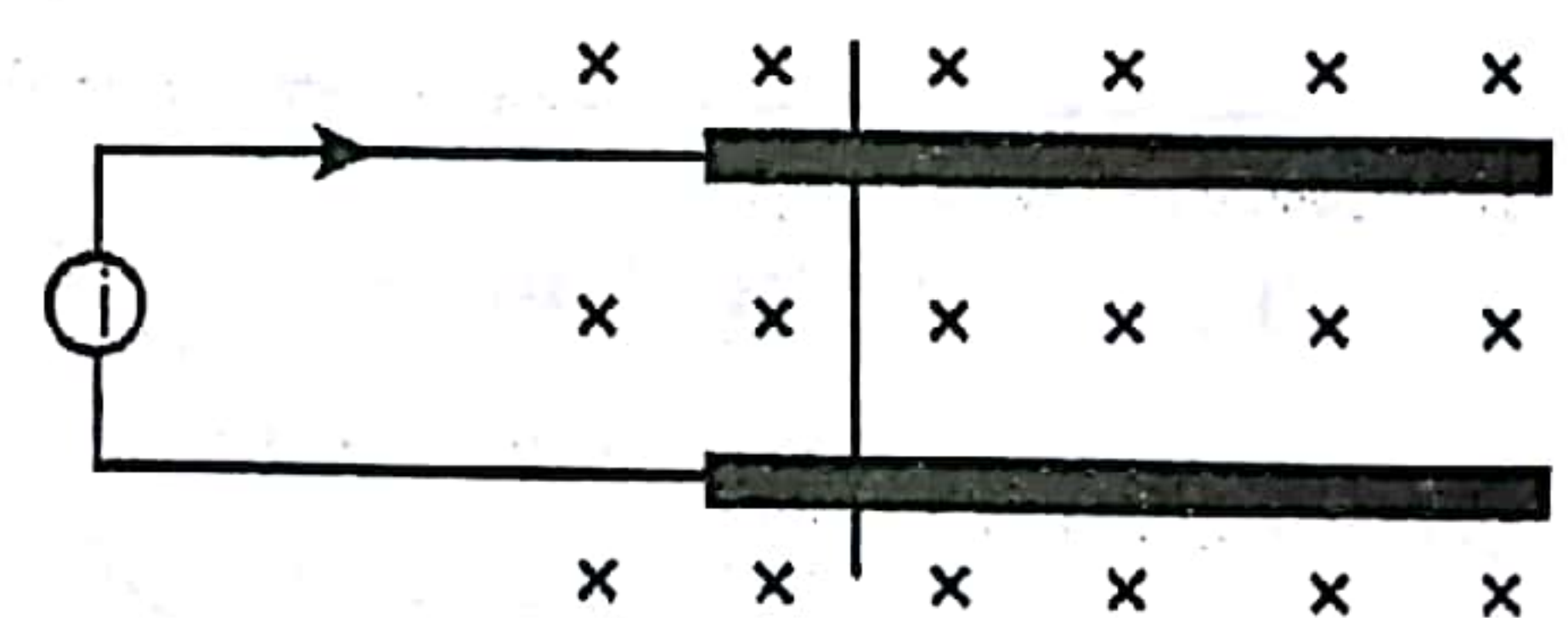
This field is perpendicular to the plane of the figure going into it. The magnetic force on the length $d\ell$ is,

$$dF = i_2 d\ell B \sin 90^\circ = \frac{\mu_0 i_1 i_2 d\ell}{2\pi\ell}$$

This force is parallel to the current i_1 .

Example 24. Figure shows two long metal rails placed horizontally and parallel to each other at a separation ℓ . A uniform magnetic field B exists in the vertically downward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is μ .

- (a) What is the minimum value of μ which can prevent the wire from sliding on the rails?
- (b) Describe the motion of the wire if the value of μ is half the



value found in the previous part

Sol. (a) The force on the wire due to the magnetic field is

$$\vec{F} = i\vec{\ell} \times \vec{B}$$

or, $F = i\ell B$

It acts towards right in the given figure. If the wire does not slide on the rails, the force of friction by the rails should be equal to F . If μ_0 be the minimum coefficient of friction which can prevent sliding, this force is also equal to $\mu_0 mg$. Thus,

$$\mu_0 mg = i\ell B$$

or, $\mu_0 = \frac{i\ell B}{mg}$

- (b) If the friction coefficient is $\mu = \frac{\mu_0}{2} = \frac{i\ell B}{2mg}$, the wire will slide towards right. The frictional force by the rails is