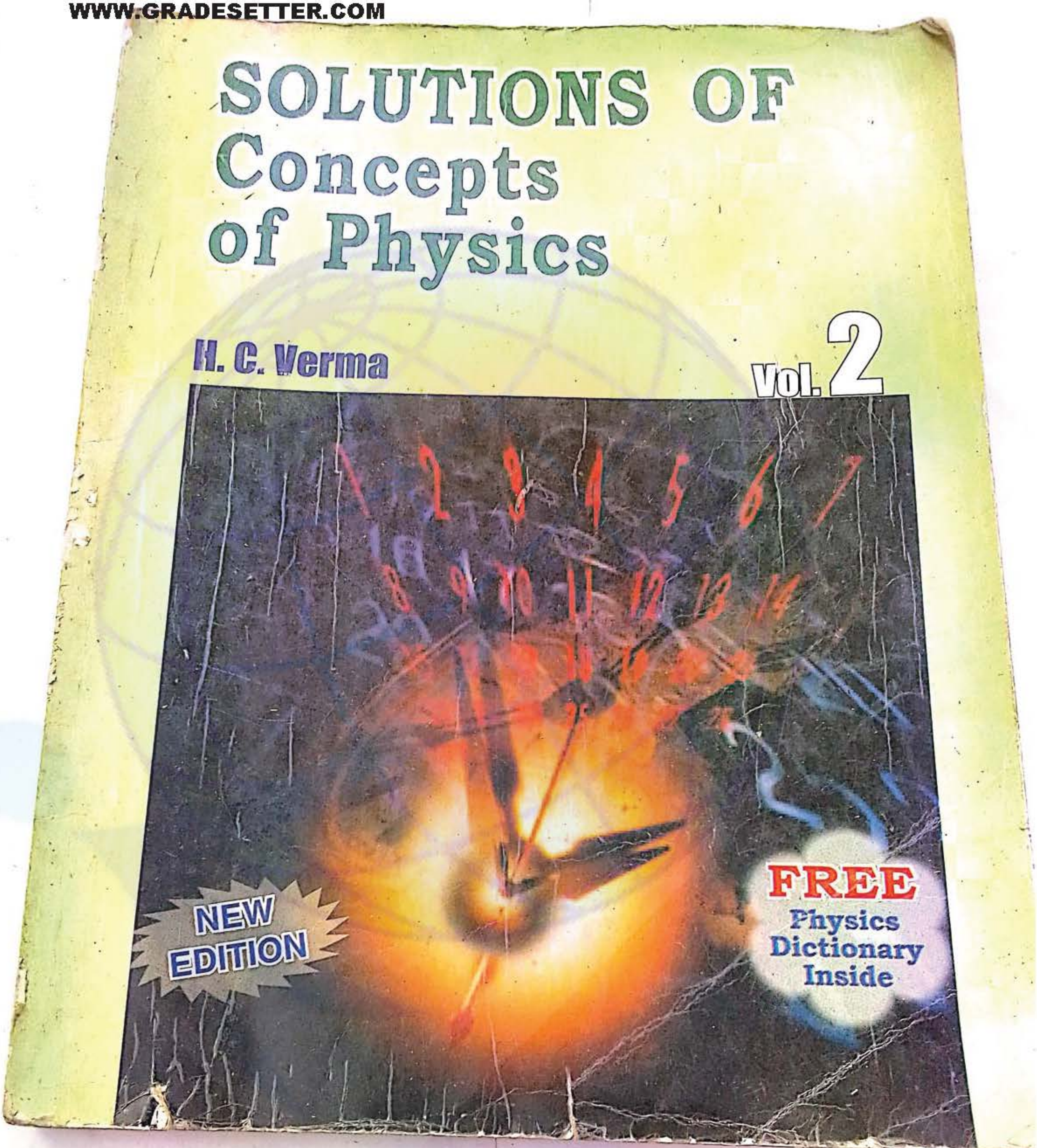

SOLUTIONS OF
Concepts
of Physics
H. C. Verma
Vol. 2
NEW EDITION
FREE
Physics
Dictionary
Inside

SOLUTIONS OF
CONCEPTS OF PHYSICS

[PART-2]

By
Govind Verma
Ph.D

RAJ PUBLICATIONS
Publishers & Distributors
Darya Ganj, New Delhi-110002

CONTETNS

SOLUTIONS OF CONCEPTS OF PHYSICS

CHAPTER 23 HEAT AND TEMPERATURE

EXERCISES

23.1 Ice point = 20° (T_0), $T_1 = 32°$ C
steam point = 80° (T_{100})

$$T = \frac{T_1 - T_0}{T_{100} - T_0} \times 100$$

$$= \frac{32 - 20}{80 - 20} \times 100$$

$$= \frac{12}{60} \times 100 = \frac{120}{6} = 20°C$$

23.2 Given $P_{tr} = 1.500 \times 10^4$ Pa
$P = 2.050 \times 10^4$ Pa

We know, for constant volume gas thermometer

$$T = \frac{P}{P_{tr}} \times 273.16 \text{ K}$$

$$= \frac{2.050 \times 10^4}{1.5 \times 10^4} \times 273.16$$

$$= 373.31$$

23.3 Pressure measured at
M_p = 2.2 × Pressure at triple point

$$T = \frac{P}{P_{tr}} \times 273.16$$

$$= \frac{2.2 \times P_{tr}}{P_{tr}} \times 273.16$$

$$= 2.2 \times 273.16 = 600.952 \text{ K}$$

$$= 601 \text{ K}$$

23.4 Given, $P_{tr} = 40 \times 10^3$ Pa, P = ?
$T = 100°C = 373$ K

$$T = \frac{P}{P_{tr}} \times 273.16$$

$$\Rightarrow \quad P = \frac{T \times P_{tr}}{273.16} = \frac{373 \times 40 \times 10^3}{273.16}$$

$$= 54620 \text{ Pa}$$

$$= 54.6 \times 10^3 \text{ Pa} = 55 \text{ KPa}$$

23.5 $P_1 = 70$ KPa, $P_2 = ?$
$T_1 = 273$ K $T_2 = 373$ K

$$T_1 = \frac{P_1}{P_{tr}} \times 273.16$$

$$\Rightarrow \quad 273 = \frac{70}{P_{tr}} \times 10^3 \times 273.16$$

$$\Rightarrow \quad P_{tr} = \frac{70 \times 273.16 \times 10^3}{273}$$

Again, $T_2 = \frac{P_2 \times 273.16}{P_{tr}}$

$$\Rightarrow \quad 373 = \frac{P_2 \times 273 \times 273.16}{70 \times 273.16 \times 10^3}$$

$$\Rightarrow \quad P_2 = \frac{373 \times 70 \times 10^3}{273} = 95.6 \text{ KPa}$$

23.6 Given

$P_{\text{ice point}} = P_0 = 80$ cm Hg

$P_{\text{steam point}} = P_{100} = 90$ cm Hg

$P_0 = 100$ cm

$$T = \frac{P - P_0}{P_{100} - P_0} \times 100°C$$

$$= \frac{80 - 100}{90 - 100} \times 100$$

$$= \frac{20}{10} \times 100 = 200°C$$

23.7 $$T' = \frac{V}{V - V'} \times T_{0'}$$

$T_0 = 273$ K
$V = 1800$ cc

$V' = 200\ cc$

$T' = \frac{1800}{1600} \times 273$

$= 307.125 \approx 307$

23.8 $R_t = 86\ \Omega,\quad R_0 = 80\ \Omega \quad R_{100} = 90\ \Omega$

$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100$

$= \frac{86 - 80}{90 - 80} \times 100$

$= \frac{6}{10} \times 100 = 60°C$

23.9 R at ice point $(R_0) = 20\ \Omega$

R at steam point $(R_{100}) = 27.5\ \Omega$

R at zinc point $(R_{420}) = 50\ \Omega$

$R_\theta = R_0 (1 + \alpha\,\theta + \beta\theta^2)$

$\Rightarrow \quad R_{100} = R_0 + R_0\,\alpha\,\theta + R_0\,\beta\theta^2$

$\Rightarrow \quad \frac{(R_{100} - R_0)}{R_0} = \alpha\theta + \beta\theta^2$

$\Rightarrow \quad \frac{27.5 - 20}{20} = \alpha\theta + \beta\theta^2$

$\Rightarrow \quad \frac{7.5}{20} = \alpha\,100 + \beta\,10000 \quad ..(i)$

Again $\quad R_{420} = R_0 (1 + \alpha\theta + \beta\theta^2)$

$\Rightarrow \quad \frac{50 - R_0}{R_0} = \alpha\theta + \beta\theta^2$

$\frac{50 - 20}{20} = 420\,\alpha + 176400\,\beta$

$\Rightarrow \quad \frac{3}{2} = 420\,\alpha + 176400\,\beta \,..(ii)$

After solving *(i)* and *(ii)*, we get

$\alpha = 3.8 \times 10^{-3} / °C,$

$\beta = -5.6 \times 10^{-7} / °C$

23.10 Given $\quad L_1 = ? \qquad L_0 = 10\ m$

$\alpha = 1 \times 10^{-5} / °C$

$t = 35°C$

$L_1 = L_0 (1 + \alpha\, t)$

$= 10 (1 + 10^{-5} \times 35)$

$= 10 + 35 \times 10^{-4}$

$= 10.0035\ m$

23.11 $t_1 = 20°C$

$t_2 = 10°C$

$L_1 = 1\ cm = 0.01\ m$

$L_2 = ?$

$\alpha_{steel} = 1.1 \times 10^{-5} / °C$

$L_2 = L_1 (1 + \alpha_{steel}\,\Delta T)$

$= 0.01 (1 + 1.1 \times 10^{-5} \times 10)$

$= 0.01 + 0.01 \times 1.1 \times 10^{-4}$

$= 10^{-2} + 10^{-2} \times 1.1 \times 10^{-4}$

$= 10^{-2} + 1.1 \times 10^{-6}$

$= 10^4 \times 10^{-6} + 1.1 \times 10^{-6}$

$= 10^{-6} (10000 + 1.1) = 10001.1$

$= 1.00011 \times 10^{-2}\ m$

$= 1.00011\ cm$

23.12 $L_0 = 12\ m \qquad \alpha = 11 \times 10^{-6} / °C$

$t_w = 18°C \qquad t_s = 48°C$

$L_w = L_0 (1 + \alpha\,\Delta t_w)$

$= 12 (1 + 11 \times 10^{-6} \times 18)$

$= 12.00237\ m$

$L_s = L_0 (1 + \alpha\,\Delta t_s)$

$= 12 (1 + 11 \times 10^{-6} \times 48)$

$= 12.006336\ m$

$\therefore \quad \Delta L = 12.006336 - 12.002376$

$= 0.00396\ m \approx 0.4\ cm$

23.13 $d_1 = 2\ cm = 2 \times 10^{-2}\ m, \qquad d_2 = ?$

$t_1 = 0°C,\ t_2 = 100°C$

$\alpha_{al} = 2.3 \times 10^{-5} / °C$

$\therefore \quad d_2 = d_1 (1 + \alpha\,\Delta t)$

$= 2 \times 10^{-2} (1 + 2.3 \times 10^{-5} \times 10^2)$

$= 2 \times 10^{-2} (1 + 2.3 \times 10^{-3})$

$= 2 \times 10^{-2} + 2.3 \times 2 \times 10^{-5}$

$= 0.02 + 0.000046$

$= 0.020046\ m \approx 2.0046\ cm$

23.14 $L_{st} = L_{al}$ at 20°C

$\alpha_{al} = 2.3 \times 10^{-5} / °C$

$\alpha_{st} = 1.1 \times 10^{-5} / °C$

So $\quad L_{0st} (1 - \alpha_{st} \times 20)$

$= L_{0al} (1 - \alpha_{al} \times 20)$

(a) $\frac{L_{0st}}{L_{0al}} = \frac{(1 - \alpha_{al} \times 20)}{(1 - \alpha_{st} \times 20)}$

$= \frac{1 - 2.3 \times 10^{-5} \times 20}{1 - 1.1 \times 10^{-5} \times 20}$

$= \frac{0.99954}{0.99978} = 0.999759$

(b) $\frac{L_{40\,Al}}{L_{40\,st}} = \frac{L_{0\,Al} (1 + \alpha_{Al} \times 40)}{L_{0\,st} (1 + \alpha_{st} \times 40)}$

$$\Rightarrow \frac{L_{40\,Al}}{L_{40\,st}} = \frac{L_{0\,Al}}{L_{0\,st}} \times \frac{(1 + 2.3 \times 10^{-5} \times 40)}{(1 + 1.1 \times 10^{-5} \times 40)}$$

$$= \frac{0.99977 \times 1.00092}{1.00044} = 1.0002496$$

(c) $$\frac{L_{100\,Al}}{L_{100\,st}} = \frac{L_{0\,Al}\ (1 + \alpha_{Al} \times 100)}{L_{0\,st}\ (1 + \alpha_{st} \times 100)}$$

$$= \frac{0.99977 \times 1.0023}{1.0023} = 1.00096$$

23.15 (a) Length at 16°C = L,

$t_1 = 16°C, \quad t_2 = 46°C$

$\alpha = 1.1 \times 10^{-5} / °C$

$\Delta L = L\,\alpha\Delta\theta = L \times 1.1 \times 10^{-5} \times 30$

$$\% \text{ of error} = \left(\frac{\Delta L}{L} \times 100\right)\%$$

$$= \left(\frac{L\alpha\ \Delta\theta}{L} \times 100\right)\%$$

$= [1.1 \times 10^{-5} \times 30 \times 100]\%$

$= 3.3 \times 10^{-2}\ \% = 0.033\%$

(b) $t_2 = 6°C$

$$\% \text{ error} = \left(\frac{\Delta L}{L} \times 100\right)\%$$

$$= \left(\frac{L\,\alpha\Delta\theta}{L} \times 100\right)\%$$

$= -1.1 \times 10^{-5} \times 10 \times 100\%$

$= -1.1 \times 10^{-2} = -0.011\%$

23.16 $T_1 = 20°C$

$\Delta L = 0.055$ mm

$= 0.055 \times 10^{-3}$ m

$T_2 = ?$

$\alpha_{st} = 11 \times 10^{-6} / °C$

we know $\Delta L = L_0\,\alpha\Delta T$

$\Rightarrow 0.055 \times 10^{-3} = 1 \times 11 \times 10^{-6} \times T_1 \pm T_2)$

$\Rightarrow 5 \times 10^{-3} = (20 \pm T_2) \times 10^{-3}$

$\Rightarrow 20 \pm T_2 = 5$

$\Rightarrow T_2 = 20 + 5 = 25°C$

or $20 - 5 = 15°C$

Hence the experiment can be performed from 15° to 25°C

23.17 $f_0\,°C = 0.998\ g / cm^3$

$f_4\,°C = 1\ g / cm^3$

We know, $f_0\,°C = \dfrac{f_4\,°C}{1 + \gamma\Delta t}$

$\Rightarrow 0.998 = \dfrac{1}{1 + \gamma.4}$

$\Rightarrow 1 + 4\gamma = \dfrac{1}{0.998}$

$\Rightarrow 4\gamma = \left(\dfrac{1}{0.998}\right) - 1$

$\Rightarrow \gamma = 0.0005 = 5 \times 10^{-4}$

As density decreases

$\gamma = -5 \times 10^{-4}$

23.18 ***Iron Rod*** ***Aluminium Rod***

L_{Fe} L_{Al}

$\alpha_{Fe} = 12 \times 10^{-6} / °C \quad \alpha_{Al} = 23 \times 10^{-6} / °C$

Since the difference in length is independent of temperature the difference always remains constant

$L'_{Fe} = L_{Fe}\,(1 + \alpha_{Fe} \times \Delta T)$ ...(1)

$L'_{Al} = L_{Al}\,(1 + \alpha_{Al} \times \Delta T)$ ...(2)

$L'_{Fe} - L'_{Al} = L_{Fe} - L_{Al} + L_{Fe} \times \alpha_{Fe}\,\Delta T - L_{Al} \times \alpha_{Al} \times \Delta T$

Given

$L'_{Fe} - L'_{Al} = L_{Fe} - L_{Al}$

Hence $L_{Fe}\,\alpha_{Fe} = L_{Al}\,\alpha_{Al}$

$\dfrac{L_{Fe}}{L_{Al}} = \dfrac{23}{12}$ **Ans.**

23.19 $g_1 = 9.8\ m/s^2$,

$g_2 = 9.788\ m/s^2$

$$T_1 = \frac{2\pi\sqrt{l_1}}{\sqrt{g_1}}$$

$$T_2 = \frac{2\pi\sqrt{l_2}}{\sqrt{g_2}}$$

$$= \frac{2\pi\sqrt{l_2\ (1 + \alpha T)}}{\sqrt{g_2}}$$

$\alpha_{steel} = 12 \times 10^{-6} / °C$

$Q_1 = 20°C$

$Q_2 = ?$

$T_1 = T_2$

$$\Rightarrow \frac{2\pi\sqrt{l_1}}{\sqrt{g_1}} = \frac{2\pi\sqrt{l_1(1 + \alpha\Delta T)}}{\sqrt{g_2}}$$

$$\Rightarrow \sqrt{\left(\frac{l_1}{g_1}\right)} = \frac{\sqrt{l_1\ (1 + \alpha\Delta T)}}{\sqrt{g_2}}$$

$\Rightarrow \frac{1}{9.8} = \frac{1 + 12 \times 10^{-6} \times \Delta T}{9.788}$

$\Rightarrow \frac{9.788}{9.8} = 1 + 12 \times 10^{-6} \times \Delta T$

$\Rightarrow \frac{9.788}{9.8} - 1 = 12 \times 10^{-6} \times \Delta T$

$\Rightarrow \Delta T = \frac{-0.00122}{12 \times 10^{-6}}$

$\Rightarrow T_2 - 20 = -102.4$

$\Rightarrow T_2 = -102.4 + 20$

$= -82.4 \approx -82°C$

23.20 Given $d_{st} = 2.005$ cm

$d_{Al} = 2.000$ cm

$\alpha_{st} = 11 \times 10^{-6} / °C$

$\alpha_{Al} = 23 \times 10^{-6} / °C$

we know,

$d'_{st} = 2.005 (1 + \alpha_{st} \Delta T)$

(where ΔT is change in temperature)

$\Rightarrow d'_{st} = 2.005 + 2.005 \times 11 \times 10^{-6} \times \Delta T$

$d'_{Al} = 2 (1 + \alpha_{Al} \times \Delta T)$

$= 2 + 2 \times 23 \times 10^{-6} \times \Delta T$

The steel ball will fall when both the diameters become equal.

So,

$\Rightarrow 2.005 + 2.005 \times 11 \times 10^{-6} \Delta T = 2 + 2 \times 23 \times 10^{-6} \Delta T$

$\Rightarrow (46 - 22.055) \times 10^{-6} \Delta T = 0.005$

$\Rightarrow \Delta T = \frac{0.005 \times 10^6}{23.945} = 208.81$

Now $\Delta T = T_2 - T_1 = T_2 - 10°C$

$[\because T_1 = 10°C$ given]

$\Rightarrow T_2 = \Delta T + T_1 = 208.81 + 10$

$= 218.81 = \sim 219\ °C$

23.21 The final length of Aluminium should be equal to the final length of Glass

Let the initial length of Aluminium = l

$l (1 - \alpha_{Al} \Delta\theta) = 20 (1 - \alpha_g \Delta\theta)$

$\Rightarrow l (1 - 24 \times 10^{-6} \times 40)$

$= 20 (1 - 9 \times 10^{-6} \times 40)$

$\Rightarrow l (1 - 0.00096) = 20 (1 - 0.0036)$

$\Rightarrow l = \frac{20 \times 0.99964}{1 - 0.00096}$

$= \frac{20 \times 0.99964}{0.99904} = 20.012$ cm

Let initial breadth of Aluminium = b

$b (1 - \alpha_{Al} \Delta\theta) = 30 (1 - \alpha_g \Delta\theta)$

$\Rightarrow b = \frac{30 \times (1 - 9 \times 10^{-6} \times 40)}{(1 - 24 \times 10^{-6} \times 40)}$

$= \frac{30 \times 0.99964}{0.99904}$

$= 30.018$ cm

23.22 $V_g = 1000$ cc $\quad V_{Hg} = ?$

$T_1 = 20°C$

$\gamma_{Hg} = 1.8 \times 10^{-4} / °C$

$\gamma_g = 9 \times 10^{-6} / °C$

Here ΔT remains constant

Volume of remaining space = $V'_g - V'_{Hg}$

Now $V'_g = V_g (1 + \gamma_g \Delta T)$ ...(1)

$V'_{Hg} = V_{Hg} (1 + \gamma_{Hg} \Delta T)$ ...(2)

Subtracting (2) from (1)

$V'_g - V'_{Hg} = V_g - V_{Hg} + V_g\gamma_g \Delta T - V_{Hg}\gamma_{Hg} \Delta T$

$\Rightarrow \frac{V_g}{V_{Hg}} = \frac{\gamma_{Hg}}{\gamma_g}$

$\Rightarrow \frac{1000}{V_{Hg}} = \frac{1.8 \times 10^{-4}}{9 \times 10^{-6}}$

$\Rightarrow V_{Hg} = \frac{9 \times 10^{-3}}{1.8 \times 10^{-4}} = 50$ cc

23.23. Volume of water = 500 cm^3

Area of cross-section of 'can' = 125 cm^2

Final volume of water = $500 (1 + \gamma\Delta\theta)$

$= 500 [1 + 3.2 \times 10^{-4} \times (80 - 10)]$

$= 500 [1 + 3.2 \times 10^{-4} \times 70]$

$= 511.2\ cm^3$

The Aluminium vessel in its length only. So area of expansion of base can be neglected

Increase in volume of water = 11.2 cm^3

Consider a cylinder of volume 11.2 cm^3

$\therefore$ height of water increased = $\frac{11.2}{125}$

$= 0.0896 = 0.089$ cm

23.24 Given,

$$V_g = 10 \times 10 \times 10 = 1000 \text{ cc.}$$
$$\Delta T = 10°C$$
$$V'_{Hg} - V'_g = 1.6 \text{ cm}^3$$
$$\alpha_g = 6.5 \times 10^{-6} / °C$$
$$\gamma_{Hg} = ?$$
$$\gamma_g = 3 \times 6.5 \times 10^{-6} / °C$$
$$V'_{Hg} = V_{Hg}(1 + \gamma_{Hg} \Delta T) \quad ...(1)$$
$$V'_g = V_g(1 + \gamma_g \Delta T) \quad ...(2)$$
$$V'_{Hg} - V'_g = V_{Hg} - V_g + V_{Hg}\gamma_{Hg}\Delta T - V_g \gamma_g \Delta T)$$
$$1.6 = 1000 \times \gamma_{Hg} \times 10 - 1000 \times 6.5 \times 3 \times 10^{-6} \times 10$$
$$\gamma_{Hg} = \frac{1.6 + 19.5 \times 10^{-2}}{10000}$$
$$= \frac{1.6 + 0.195}{10000} = \frac{1.795}{10000}$$
$$= 1.795 \times 10^{-4}$$
$$= 1.8 \times 10^{-4} / °C$$

23.25 Given $f_w = 880 \text{ kg/m}^3$,

$$f_b = 900 \text{ kg/m}^3$$
$$T_1 = 0°C,$$
$$\gamma_w = 1.2 \times 10^{-3} /°C,$$
$$\gamma_b = 1.5 \times 10^{-3} /°C$$

The sphere begins to sink when

$\Rightarrow (mg)_{sphere}$ = displaced water

$$Vf'_w g = Vf'_b g$$
$$\Rightarrow \frac{f_w}{1 + \gamma_w \Delta\theta} = \frac{f_b}{1 + \gamma_b \Delta\theta}$$
$$\Rightarrow \frac{880}{1 + 1.2 \times 10^{-3} \Delta\theta} = \frac{900}{1 + 1.5 \times 10^{-3} \Delta\theta}$$
$$\Rightarrow 880 + 880 \times 1.5 \times 10^{-3} \Delta\theta = 900 + 900 \times 1.2 \times 10^{-3} \Delta\theta$$
$$\Rightarrow (880 \times 1.5 \times 10^{-3} - 900 \times 1.2 \times 10^{-3}) \Delta\theta = 20$$
$$\Rightarrow (1320 - 1080) \times 10^{-3} \Delta\theta = 20$$
$$\Rightarrow \Delta\theta = 83.3°C$$
$$T_2 - T_1 = 83 \qquad \Rightarrow T_2 - 0° = 83$$
$$\Rightarrow T_2 = 83°C$$

23.26 Given, $\Delta T = 100°C$

A longitudinal strain develops if and only if there is an opposition to the expansion. Since there is no opposition in this case the longitudinal strain = zero.

23.27 Given, $\theta_1 = 20°C$, $\theta_2 = 50°C$

$$\alpha_{steel} = 1.2 \times 10^{-5} / °C$$

Longitudinal strain = ?

$$\text{Strain} = \frac{\Delta L}{L} - \frac{L\alpha\Delta\theta}{L} = \alpha\Delta\theta$$
$$= 1.2 \times 10^{-5} \times (50 - 20)$$
$$= 1.2 \times 10^{-5} \times 30$$
$$= 36 \times 10^{-5} = 3.6 \times 10^{-4}$$

The strain is opposite to the direction of expansion.

23.28 Given

$$A = 0.5 \text{ mm}^2 = 0.5 \times 10^{-6} \text{ m}^2$$
$$T_1 = 20°C, T_2 = 0°C$$
$$\alpha_s = 1.2 \times 10^{-5} / °C$$
$$\gamma = 2 \times 10^{11} \text{N/m}^2$$

Decrease in length due to compression

$$= L\alpha\Delta\theta \quad ...(1)$$
$$\gamma = \frac{\text{stress}}{\text{strain}} = \frac{F}{A} \times \frac{L}{\Delta L}$$
$$\Rightarrow \Delta L = \frac{FL}{AY}$$

Tension is developed due to (1) and (2).
Equating (1) and (2)

$$L\alpha\Delta\theta = \frac{FL}{AY}$$
$$\Rightarrow F = \alpha\Delta\ \theta AY$$
$$= 1.2 \times 10^{-5} \times (20 - 0) \times 0.5 \times 10^{-6} \times 2 \times 10^{11}$$
$$= 1.2 \times 20 = 24 \text{ N}$$

23.29

$$\theta_1 = 20°C, \qquad \theta_2 = 100°C$$
$$A = 2 \text{ mm}^2 = 2 \times 10^{-6} \text{ m}^2$$
$$\alpha_{steel} = 12 \times 10^{-6} / °C$$
$$Y_{steel} = 2 \times 10^{11} \text{ N/m}^2$$

Force exerted on the clamps = ?

$$\frac{F/A}{\text{strain}} = Y$$
$$F = \frac{Y \times \Delta L}{L} \times A$$
$$= \frac{YL\alpha\Delta\theta A}{L} = YA\alpha\Delta\theta$$
$$= 2 \times 10^{11} \times 2 \times 10^{-6} \times 12 \times 10^{-6} \times 80$$
$$= 48 \times 80 \times 10^{-1} = 384 \text{ N}$$

23.30 Let the initial length of the system at 0°C $= l_0$

When temperature changes by θ

Final length of the steel at system temperature

So, strain of the system = $\frac{(l - l_0)}{l_0}$

Steel
Aluminium
Steel

But the total strain of the system

$$= \frac{\text{total stress of system}}{\text{total Young's modulus of system}}$$

Now, total stress due to two steel rod + stress due to aluminium

$$= \gamma_s \alpha_s \theta + \gamma_s \alpha_s \theta + \gamma_{Al} \alpha_{Al} \theta$$
$$= 2\gamma_s \alpha_s \theta + \gamma_{Al} \alpha_{Al} \theta$$

Now Young's modulus system

$$= \gamma_s + \gamma_s + \gamma_{Al} = 2\gamma_s + \gamma_{Al}$$

$$\therefore \text{Strain of system} = \frac{2\gamma_s \alpha_s \theta + \gamma_{Al} \alpha_{Al} \theta}{2\gamma_s + \gamma_{Al}}$$

$$\Rightarrow \frac{l - l}{l_0} = \frac{2\gamma_s \alpha_s \theta + \gamma_{Al} \alpha_{Al} \theta}{2\gamma_s + \gamma_{Al}}$$

$$\Rightarrow l = l_0\left[1 + \frac{2\gamma_s \alpha_s \theta + 2\gamma_{Al} \alpha_{Al} \theta}{2\gamma_s + \gamma_{Al}}\right]$$

23.31 The ball tries to expand its volume but it is kept at a constant volume.
So the stress arises

$$\frac{P}{\Delta V / V} = B$$

$$\Rightarrow P = B\frac{\Delta V}{V} = B \times \gamma \Delta\theta$$
$$= B \times 3\,\alpha\Delta\theta$$
$$= 1.6 \times 10^{11} \times 3 \times 12 \times 10^{-6} \times (120 - 20)$$
$$= 1.6 \times 3 \times 12 \times 10^{11} \times 10^{-6} \times 10^2$$
$$= 57.6 \times 10^7 = 5.8 \times 10^8 \text{ pa}$$

23.32 Given, I_0 = moment of inertia at 0°C
α = Coeffcient of inertia expansion

To prove $I = I_0 (1 + 2\,\alpha\theta)$

Let the temperature changes to θ from 0°C

$\Delta T = \theta$

Let R_0 be the radius of gyration

Now $R_0 = R(1 + \alpha\theta)$

$I_0 = MR^2$ where M is the mass.

Now $I = MR^2 = MR^2 (1 + \alpha\theta)^2$

[by binomial expansion and neglecting $\alpha_2\theta_2$ which is a very small value]

$= MR^2 (1 + 2\,\alpha\theta)$

So, $I = I_0 (1 + 2\,\alpha\theta)$ (proved)

23.33 Let the initial length at 0°C be l_0

Time period at 0°C, $T = 2\pi\sqrt{\frac{l}{K}}$

$l = l_0 (1 + 2\,\alpha\Delta\theta)$

at 5°C,

$$T_1 = \frac{2\pi\sqrt{l_0 (1 + 2\,\alpha\Delta\theta)}}{\sqrt{K}}$$
$$= \frac{2\pi\sqrt{l_0 (1 + 2\alpha \times 45)}}{\sqrt{K}}$$
$$= \frac{2\pi\sqrt{l_0 (1 + 10\,\alpha)}}{\sqrt{K}}$$

at 45°C,

$$T_2 = \frac{2\pi\sqrt{l_0 (1 + 2\alpha \times 45)}}{\sqrt{K}}$$
$$= \frac{2\pi\sqrt{l_0 (1 + 90\alpha)}}{\sqrt{K}}$$

$$\frac{T_2}{T_1} = \frac{\sqrt{1 + 90\alpha}}{\sqrt{1 + 10\alpha}}$$
$$= \frac{\sqrt{1 + 90 \times 2.4 \times 10^{-5}}}{\sqrt{1 + 10 \times 2.4 \times 10^{-5}}}$$
$$= \frac{\sqrt{1.00216}}{\sqrt{1.00024}}$$

$$\% \text{ change} = \left(\frac{T_2}{T_1} - 1\right) \times 100$$
$$= 0.0959\% \approx 9.6 \times 10^{-2}\,\%$$

23.34 Given, $T_1 = 20°$, $T_2 = 50°C$

$\Delta T = 30°C$, $\alpha = 1.2 \times 10^{-5} /°C$

ω remains constant

(i) $\omega = \frac{V}{R}$ *(ii)* $\omega = \frac{V'}{R'}$

Now, $R' = R(1 + \alpha\Delta\theta)$

$= R + R \times 1.2 \times 10^{-5} \times 30$

$= 1.00036\,R$

From *(i)* and *(ii)*

$$\frac{V}{R} = \frac{V'}{R'} = \frac{V'}{1.00036\,R}$$

$$\Rightarrow V' = 1.00036\,V$$

$$\% \text{ change} = \frac{(1.00036\,V - V)}{V} \times 100$$
$$= 0.00036 \times 100$$
$$= 3.6 \times 10^{-2}\,\%$$

CHAPTER 24

KINETIC THEORY OF GASES

EXERCISES

24.1 Volume of 1 mole gas

$$PV = nRT$$

$$\Rightarrow \quad V = \frac{RT}{P} = 0.082 \times 273 \text{ (as P = 1)}$$

$$V = 22.386 = 22.4 \text{ L}$$

$$= 22.4 \times 10^{-3} \text{ m}^3$$

$$= 2.24 \times 10^{-2} \text{ m}^3$$

24.2 We know,

$$n = \frac{PV}{RT} = \frac{1 \times 1 \times 10^{-3}}{0.082 \times 273}$$

$$= \frac{10^{-3}}{22.4} = \frac{1}{22400}$$

No. of molecules

$$= 0.023 \times 10^{23} \times \frac{1}{22400}$$

$$= 2.688 \times 10^{-4} \times 10^{23}$$

$$= 2.688 \times 10^{19}$$

24.3 $V = 1 \text{ cm}^3$, $T = 0°C$,

$P = 10^{-5}$ mm of Hg

$$n = \frac{PV}{RT} = \frac{\rho gh \times V}{RT}$$

$$= \frac{13.6 \times 980 \times 10^{-6} \times 1}{8.31 \times 273}$$

$$= 5.874 \times 10^{-13}$$

No. of molecules

$$= N \times n$$

$$= 6.023 \times 10^{23} \times 5.874 \times 10^{-13}$$

$$= 35.384 \times 10^{10}$$

$$= 3.538 \times 10^{11}$$

24.4

$$n = \frac{PV}{RT} = \frac{1 \times 1 \times 10^{-3}}{0.082 \times 273} = \frac{10^{-3}}{22.4}$$

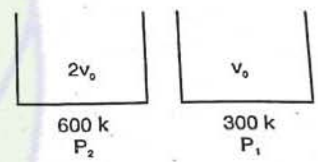
$$\text{mass} = \frac{(10^{-3} \times 32)g}{224}$$

$$= 1.428 \times 10^{-3} \text{ g}$$

$$= 1.428 \text{ mg}$$

24.5 Since mass is same

$$n_1 = n_2 = n$$



$$P_1 = \frac{nR \times 300}{V_0}$$

$$P_2 = \frac{nR \times 600}{2V_0}$$

$$\frac{P_1}{P_2} = \frac{nR \times 300}{V_0} \times \frac{2V_0}{nR \times 600} = 1:1$$

24.6

$$V = 250 \text{ cc} = 250 \times 10^{-3} . l$$

$$P = 10^{-3} \text{ mm} = 10^{-3} \times 10^{-3} \text{ m}$$

$$= (10^{-6} \times 13600 \times 10)$$

$$= 136 \times 10^{-3} \text{ Pascal}$$

$$T = 27°C = 300 \text{ K}$$

$$n = \frac{PV}{RT}$$

$$= \frac{136 \times 10^{-3} \times 250 \times 10^{-3}}{8.3 \times 300}$$

$$= \frac{136 \times 250 \times 10^{-6}}{8.3 \times 300}$$

No. of molecules

$$= \frac{136 \times 250}{8.3 \times 300} \times 10^{-6} \times 6 \times 10^{23}$$

$$= 81 \times 10^{17} = 0.81 \times 10^{15}$$

24.7 Given

$P_{max} = 1.0 \times 10^6$ Pa,
$P_0 = 8.0 \times 10^5$ Pa,
$T_0 = 300$ K

Volume is constant (Given)

So Gas law,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad (\because V_1 = V_2)$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

or $$T_2 = \frac{P_{max} \times T_0}{P_0}$$

$$= \frac{1.0 \times 10^6 \times 300}{8.0 \times 10^5}$$

$$= 375 \text{ K}$$

24.8 Given that:

$n = 1$ mole, $V = 0.02$ m^3
$T = 300$ K, $R = 8.3$

From the formula

$PV = nRT$

$$P = \frac{nRT}{V} = \frac{1 \times 8.3 \times 300}{0.02}$$

$$P = 1.24 \times 10^5 \text{ Pa}$$

24.9 From the formula

$$PV = nRT = \frac{m}{M} RT \qquad \text{...(1)}$$

where

m = mass of the gas
M = molecular weight of the gas

From equation (1),

$$P = \left(\frac{m}{M}\right)\frac{RT}{V} = \frac{dRT}{M}$$

$$\therefore \quad M = \frac{dRT}{P} = \frac{1.25 \times 10^3 \times 8.3 \times 273}{10^5}$$

$$= 28.3 \text{ g/mol}$$

24.10 For Simla

$T_1 = 15.0°C = 288$ K
$P_1 = 72.0$ cm of mercury

For Kalka

$T_2 = 35.0°C = 308$ K
$P_2 = 76.0$ cm of mercury

From the formula

$$PV = \frac{m}{M} RT$$

$$\Rightarrow \quad P = \left(\frac{m}{V}\right)\frac{1}{M} RT$$

$$\Rightarrow \quad P = \frac{d}{M} RT$$

$$\Rightarrow \quad d = \frac{PM}{RT}$$

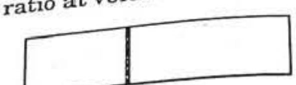
Thus

$$\frac{d_2}{d_1} = \frac{P_2}{T_2} \times \frac{T_1}{P_1} = \frac{76.0 \times 288}{308 \times 72.0}$$

$$\frac{d_2}{d_1} = .987$$

24.11 According to question, tube divided in the ratio 1 : 3.

So ratio at volume = 1 : 3



From the formula $PV = nRT$

$$\text{So} \quad \frac{P_2}{P_1} = \frac{V_2}{V_1} = 3:1$$

24.12 r.m.s. velocity of Hydrogen molecule = ?

$T = 300$K, $R = 8.3$
$M = 2 \text{ g} = 2 \times 10^{-3}$ kg

$$C = \frac{\sqrt{3RT}}{\sqrt{M}}$$

$$= \frac{\sqrt{3 \times 8.3 \times 300}}{\sqrt{2 \times 10^{-3}}}$$

$$= 1932.6 \text{ m/s}$$

$$= \sim 1930 \text{ m/s}$$

Let the temperature at the

$C = 2 \times 1932.6$ is T'

$$\text{Then } 2 \times 1932.6 = \frac{\sqrt{3 \times 8.3 \times T'}}{\sqrt{2 \times 10^{-3}}}$$

$$\Rightarrow (2 \times 1932.6)^2 = \frac{3 \times 8.3 \times T'}{2 \times 10^{-3}}$$

$$\Rightarrow \frac{(2 \times 1932.6)^2 \times 2 \times 10^{-3}}{3 \times 8.3} = T'$$

$$\Rightarrow \quad T' = 1199.98$$

$$\approx 1200 \text{ K}$$

the

24.13 $V_{r.m.s.} = \frac{\sqrt{3\rho}}{\sqrt{\rho}}$

Here $P = 10^5 \text{ Pa} = 1 \text{ atm}$

$= \frac{\sqrt{3 \times 10^5 \times 10^{-3}}}{\sqrt{1.77 \times 10^{-4}}}$

$= 1301.8 = 1302 \text{ m/s}.$

24.14 Average K.E. $= \frac{3}{2}$

$\frac{3}{2}$ KT $= 0.04 \times 1.6 \times 10^{-19}$

$\Rightarrow \frac{3}{2} \times 1.38 \times 10^{-23} \times T$

$= 0.04 \times 1.6 \times 10^{-19}$

$\Rightarrow T = \frac{2 \times 0.04 \times 1.6 \times 10^{-19}}{3 \times 1.38 \times 10^{-23}}$

$= 0.0309178 \times 10^4$

$= 309.178 = 310 \text{ K}$

24.15 $V_{avg} = \frac{\sqrt{8RT}}{\sqrt{\pi M}} = \frac{\sqrt{8 \times 8.83 \times 300}}{\sqrt{3.14 \times 0.032}}$

$= 445.25 \text{ m/s}$

$T = \frac{\text{distance}}{\text{speed}} = \frac{6400000 \times 2 \text{ sec}}{445.25}$

$= \frac{28747.83 \text{ hrs}}{3600}$

$= 7.985 \text{ hrs} = 8 \text{ hrs}.$

24.16 $M = 4 \times 10^{-3}$ kg

$V_{avg} = \frac{\sqrt{8RT}}{\sqrt{\pi M}}$

$= \frac{\sqrt{8 \times 8.3 \times 273}}{\sqrt{3.14 \times 4 \times 10^{-3}}}$

$= 1201.35$

Momentum

$= M \times V_{avg}$

$= 6.64 \times 10^{-27} \times 1201.35$

$= 7.97 \times 10^{-24}$

$= 8 \times 10^{-24}$ kg-m/s

24.17 $V_{avg} = \frac{\sqrt{8RT}}{\sqrt{\pi M}} = \sqrt{\frac{8 \times 8.3 \times 300}{3.14 \times 0.032}}$

Now, $\frac{8RT_1}{\pi \times 2} = \frac{8RT_2}{\pi \times 4}$

$\Rightarrow \frac{T_1}{T_2} = \frac{1}{2}$

24.18 Mean speed of the molecule $= \sqrt{\frac{8RT}{\pi M}}$

Escape velocity

$= \sqrt{2gr}$

$\frac{\sqrt{8RT}}{\sqrt{\pi M}} = \sqrt{2gr}$

$\Rightarrow \frac{8RT}{\pi M} = 2gr$

$\Rightarrow T = 2gr\frac{\pi M}{8R}$

$= \frac{2 \times 9.8 \times 6400000 \times 3.14 \times 2 \times 10}{8 \times 8.3}$

$= 11863.9 = 11800 \text{ m/s}$

24.19 We know

$V_{avg} = \frac{\sqrt{8RT}}{\sqrt{\pi M}}$

$\frac{V_{avg} H_2}{V_{avg} N_2} = \frac{\sqrt{8 \times RT}}{\sqrt{\pi \times 2}} \times \frac{\sqrt{\pi \times 28}}{\sqrt{8RT}}$

$= \sqrt{\frac{28}{2}} = \sqrt{14} = 3.74$

24.20 The leftside of the container has a gas having a molecular weight $= M_1$ and right part has molecular weight $= M_2$.
The temperature of both left and right chambers are equal since the separator is diathermic

$\frac{\sqrt{3RT}}{\sqrt{M_1}} = \frac{\sqrt{8RT}}{\sqrt{\pi M_2}}$

$\Rightarrow \frac{3RT}{M_1} = \frac{8RT}{\pi M_2}$

$\Rightarrow \frac{M_1}{\pi M_2} = \frac{3}{8}$

$\Rightarrow \frac{M_1}{M_2} = \frac{3\pi}{8} = 1.1775 = 1.18$

24.21 We have

$V_{mean} = \frac{\sqrt{8RT}}{\sqrt{\pi M}}$

$$= \frac{\sqrt{8 \times 8.3 \times 273}}{\sqrt{3.14 \times 2 \times 10^{-3}}}$$

$= 1698.96$

Total distance $= 1698.96$ m

$$\text{No. of collision} = \frac{1698.96}{1.38 \times 10^{-7}}$$

$= 1.23 \times 10^{10}$

24.22

$P = 1 \text{ atm} = 10^5$ Pascal

$T = 300K$

$M = 2g = 2 \times 10^{-3}$ kg

(a)
$$V_{avg} = \frac{\sqrt{8RT}}{\sqrt{\pi M}}$$

$$= \frac{\sqrt{8 \times 8.3 \times 300}}{\sqrt{3.14 \times 2 \times 10^{-3}}}$$

$$= \frac{\sqrt{8 \times 8.3 \times 300 \times 10^3}}{\sqrt{3.14 \times 2}}$$

$= 1781.004 = 1780$ m/s

(b) When the molecules strike at an angle 45°,

force exerted

$= mV\cos 45° - (-mV\cos 45°)$

$= 2mV\cos 45°$

$$= \frac{2mV}{\sqrt{2}} = \sqrt{2}\, mV$$

Number of molecules striking per unit area

$$= \frac{\text{force}}{\sqrt{2}\, mV \times \text{Area}}$$

$$= \frac{\text{Pressure}}{\sqrt{2}\, mV}$$

$$= \frac{\dfrac{10^5}{\sqrt{2} \times 2 \times 10^{-3} \times 1780}}{6 \times 10^{23}}$$

$$= \frac{3 \times 10^{31}}{2516.92}$$

$= 1.19 \times 10^{-3} \times 10^{31}$

$= 1.19 \times 10^{28} = 1.2 \times 10^{28}$

24.23 Given,

$P_1 = 200$ KPa

$= 2 \times 10^5$ Pa,

$T_1 = 20°C = 293$ K

$T_2 = 40°C = 313$ K

$V_2 = V_1 + 2\% \, V_1$

$= 1.02\, V_1$, $P_2 = ?$

We know

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow \frac{2 \times 10^5\, V_1}{293} = \frac{P_2 \times 1.02\, V_1}{100 \times 313}$$

$$\Rightarrow P_2 = \frac{2 \times 10^7 \times 313}{1.02 \times 293}$$

$= 209462$ Pa

$= 209.462$ KPa

24.24 Given,

$V_1 = 1 \times 10^{-3}$ m³

$P_1 = 1.5 \times 10^5$ Pa

$T_1 = 400$ K

We know,

$P_1V_1 = n_1RT_1$

$$\Rightarrow n_1 = \frac{P_1V_1}{RT_1}$$

$$= \frac{1.5 \times 10^5 \times 1 \times 10^{-3}}{8.3 \times 400}$$

$$\Rightarrow n_1 = \frac{1.5}{8.3 \times 4}$$

$$\Rightarrow m_1 = \frac{1.5 \times M}{8.3 \times 4} = \frac{1.5 \times 32}{4 \times 8.3}$$

$= 1.457 \approx 1.446$

Again, $P_2 = 1 \times 10^5$ Pa,

$V_2 = 1 \times 10^{-3}$ m³

$T_2 = 300$ K

We know,

$P_2V_2 = n_2RT_2$

$$\Rightarrow n_2 = \frac{P_2V_2}{RT_2} = \frac{10^5 \times 10^{-3}}{8.3 \times 300}$$

$$= \frac{1}{3 \times 8.3} = 0.040$$

$\Rightarrow m_2 = 0.04 \times 32 = 1.285$

$\Delta m = m_1 - m_2 = 1.446 - 1.285$

$= 0.1608$ g ≈ 0.16 g

24.25 Given,

$P_1 = 10^5 + \rho gh = 10^5 + 1000 \times 10 \times 3.3$

$= 1.33 \times 10^5$ Pa

$P_2 = 10^5$ Pa
$T_1 = T_2 = T$

$V_1 = \frac{4}{3\pi}(2 \times 10^{-3})^3,$

$V_2 = 4/3\pi r^3$ $r = ?$

We know, $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$$\Rightarrow \frac{1.33 \times 10^5 \times 4/3\pi \times (2 \times 10^{-3})^3}{T_1}$$

$$= \frac{10^5 \times 4/3\pi r^3}{T_2}$$

$$\Rightarrow = 1.33 \times 8 \times 10^5 \times 10^{-9}$$
$$= 10^5 \times r^3$$

$$\Rightarrow r = \sqrt[3]{10.64 \times 10^{-9}}$$

$$= 2.19 \times 10^{-3} = 2.2 \text{ mm}$$

24.26 $P_1 = 2 \text{ atm} = 2 \times 10^5$ Pa
$V_1 = 0.002 \text{ m}^3$, $T_1 = 300$ K
$P_1V_1 = n_1RT_1$

$$\Rightarrow n_1 = \frac{P_1V_1}{RT_1} = \frac{2 \times 10^5 \times 0.002}{8.3 \times 300}$$

$$= \frac{2 \times 10^5 \times 2 \times 10^{-3}}{8.3 \times 10^2 \times 3}$$

$$= \frac{4}{8.3 \times 3} = 0.1606$$

Again, $P_2 = 1 \text{ atm} = 10^5$ Pa
$V_2 = 0.0005 \text{ m}^3$, $T_2 = 300$ K
$P_2V_2 = n_2RT_2$

$$\Rightarrow n_2 = \frac{P_2V_2}{RT_2} = \frac{10^5 \times 0.0005}{8.3 \times 300}$$

$$= \frac{10^5 \times 5 \times 10^{-4}}{8.3 \times 3 \times 10^2}$$

$$= \frac{5}{3 \times 83} = 0.02$$

Δm = moles leaked out
$= 0.16 - 0.02 = 0.14$

24.27 Given, $m = 0.040$ g,
$T = 100°C$, $M_{He} = 0.04$ g

$$U = \frac{3}{2}nRT = \frac{3}{2} \times \frac{m}{MRT}$$

$T' = ?$

Given, $\frac{3}{2} \times \frac{m}{M} \times R \times T + 12$

$$= \frac{3}{2} \times \frac{m}{M} \times R \times T'$$

$\Rightarrow 1.5 \times 0.01 \times 8.3 \times 373 + 12$
$= 1.5 \times 0.01 \times 8.3 \ T'$

$\Rightarrow 58.4385 = 1245 \ T'$

$$\Rightarrow T' = \frac{58.4385}{0.1245}$$

$= 469.3855$ K
$= 196.3°C \approx 196°C$

24.28 PV^2 = Constant

$\Rightarrow P_1V_1^2 = P_2V_2^2$

$$\Rightarrow \frac{mRT_1}{V_1} \times V_1^2 = \frac{mRT_2}{V_2} \times V_2^2$$

$\Rightarrow T_1V_1 = T_2V_2$

$\Rightarrow TV = T_2 \times 2V$

$$\Rightarrow T_2 = \frac{T}{2}$$

24.29

$$P_{O_2} = \frac{n_{O_2}RT}{V}$$

$$P_{N_2} = \frac{n_{N_2}RT}{V}$$

$$n_{O_2} = \frac{m}{M_{O_2}} = \frac{1.60}{32}$$

$= 0.05$

$$n_{N_2} = \frac{m}{M_{N_2}} = \frac{2.80}{28} = 0.1$$

Now, $P_{mix} = \left(\frac{n_{O_2} + n_{N_2}}{V}\right)RT$

$$P_{mix} = \frac{(0.05 + 0.1) \times 8.3 \times 300}{0.168}$$

$= 2250 \text{ N/m}^2$

24.30 Given, P_1 = Atmospheric pressure
$= 75 \times \rho g,$
$V_1 = 100 \times A$
P_2 = Atmospheric pressure
+ Mercury pressure
$= 75 \rho g + h \rho g$
(if h = height of mercury)

$$V_2 = (100-h)$$

$$P_1V_1 = P_2V_2$$

$$\Rightarrow 75\,\rho g\,(100\,A) = (75+h)\,\rho g\,(100-h)\,A$$

$$\Rightarrow 75 \times 100 = (75+h)(100-h)$$

$$\Rightarrow 7500 = 7500 - 75h + 100h - h^2$$

$$\Rightarrow h^2 - 100h + 75h = 0$$

$$\Rightarrow h^2 - 25h = 0$$

$$\Rightarrow h^2 = 25h$$

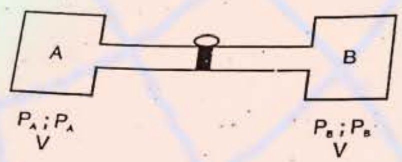
$$\Rightarrow h = 25 \text{ cm}$$

Height of Mercury that can be poured $= 25$ cm

24.31 Let the final pressure, volume and temperature after connection are

$P'_A : \rightarrow$ partial pressure of A

$P'_B : \rightarrow$ partial pressure of B



Now, $$\frac{P'_A \times 2V}{T} = \frac{P_A \times V}{T_A}$$

or, $$\frac{P'_A}{T} = \frac{P_A}{2T_A} \quad ...(i)$$

Similarly $$\frac{P'_B}{T} = \frac{P_B}{2T_B} \quad ...(ii)$$

Addiding (i) and (ii)

$$\frac{P'_A}{T} + \frac{P'_B}{T} = \frac{P_A}{2T_A} + \frac{P_B}{2T_B}$$

$$= \frac{1}{2}\left[\frac{P_A}{T_A} + \frac{P_B}{T_B}\right]$$

$$\Rightarrow \frac{P}{T} = \frac{1}{2}\left[\frac{P_A}{T_A} + \frac{P_B}{T_B}\right]$$

$$[\because P'_A + P'_B = P]$$

24.32 Given, $V = 50$ cc. $= 50 \times 10^{-6}$ m^3

$P = 100$ KPa $= 10^5$ Pa,

$M = 28.8$ g

(a) $PV = nRT_1$

$$\Rightarrow PV = \frac{m}{M}RT_1$$

$$\Rightarrow m = \frac{PMV}{RT_1}$$

$$= \frac{10^5 \times 28.8 \times 50 \times 10^{-6}}{8.3 \times 273}$$

$$= \frac{50 \times 28.8 \times 10^1}{8.3 \times 273}$$

$$= 0.0635 \text{ g}$$

(b) When the vesse! is kept on boiling water

$$PV = \frac{nRT_2}{M}$$

$$\Rightarrow m = \frac{PVM}{RT_2}$$

$$= \frac{10^5 \times 28.8 \times 50 \times 10^{-6}}{8.3 \times 373}$$

$$= 0.0465 \text{ gm.}$$

(c) When the vessel is closed

$$P \times 50 \times 10^{-6} = \frac{0.0465}{28.8} \times 8.3 \times 273$$

$$\Rightarrow P = \frac{0.0465 \times 8.3 \times 273}{28.8 \times 50 \times 10^{-6}}$$

$$= 0.07316 \times 10^6 \text{ Pa}$$

$$= 73.16 \text{ KPa}$$

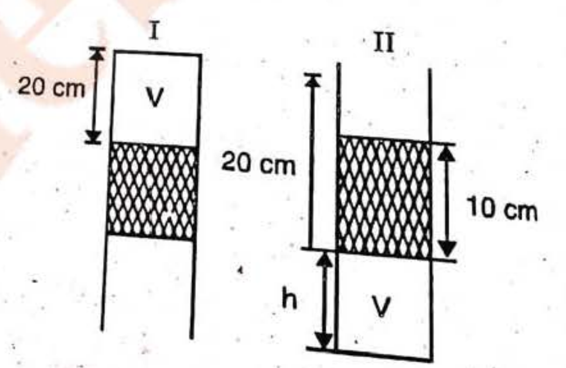
$$= 73 \text{ KPa}$$

24.33 **Case 1 :** Net pressure on air in volume

$$V = P_{atm} - h\rho g$$

$$= 75 \times \rho_{Hg} \times g - 10\,\rho_{Hg} \times g$$

$$= \rho_{Hg} \times g \times 65$$



Case 2 : Net pressure on air in volume 'V'

$$= P_{atm} + \rho_{Hg} \times g \times h$$

$$P_1V_1 = P_2V_2$$

$$\Rightarrow \rho_{Hg} \times g \times 65 \times A \times 20 = (\rho_{Hg} \times g \times 75 + \rho_{Hg} \times g \times 10)\,A \times h$$

24.34

$$\Rightarrow \quad 65 + 20 = 85\,h$$

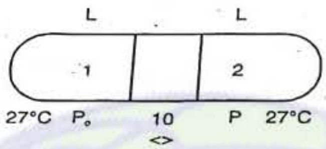
$$\Rightarrow \quad h = \frac{65 \times 20}{85} = 15.2 \text{ cm}$$

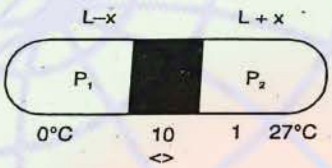
$$= 15 \text{ cm}$$

24.34 Here $2L + 10 = 100$ cm

$$\Rightarrow \quad 2L = 90 \text{ cm}$$

$$\Rightarrow \quad L = 45 \text{ cm}$$





Applying combined gas equation to part 1 of the tube,

$$\frac{(45A)\,P_0}{300} = \frac{(45 - x)\,AP_1}{273}$$

$$\Rightarrow \quad P_1 = \frac{273 \times 45 \times P_0}{300\,(45 - x)}$$

Applying combined gas equation to part 2 of the tube,

$$\frac{(45A)\,P_0}{300} = \frac{(45 + x)\,AP_2}{400}$$

$$\Rightarrow \quad P_2 = \frac{400 \times 45 \times P_0}{300\,(45 + x)}$$

$$P_1 = P_2$$

$$\Rightarrow \frac{273 \times 45 \times P_0}{300\,(45 - x)} = \frac{400 \times 45 \times P_0}{300\,(45 + x)}$$

$$\Rightarrow \quad \frac{45 \times 273}{45 - x} = \frac{400 \times 45}{45 + x}$$

$$\Rightarrow \quad (45 - x)\,400 = (45 + x)\,273$$

$$\Rightarrow 18000 - 400\,x = 12285 + 273\,x$$

$$\Rightarrow (400 + 273)\,x = 18000 - 12285$$

$$\Rightarrow \quad x = 8.49$$

$$\therefore P_1 = \frac{273 \times 45 \times 76}{300 + 36.51}$$

$$= 85.25 \text{ cm of Hg}$$

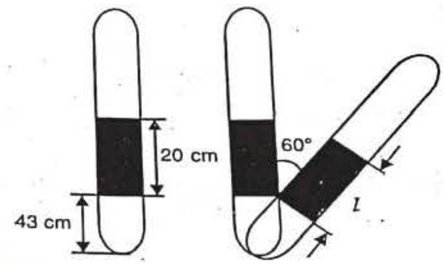
Length of air column on the air cooler side

$$= L - x$$

$$= 45 - 8.49 = 36.51 \text{ cm.}$$

24.35 Case 1 : Atmospheric pressure + Pressure due to mercury column

Case 2 : Atmospheric pressure + Component of the pressure due to mercury column



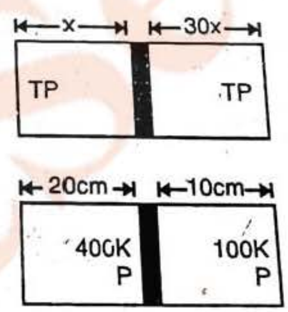
$$P_1 V_1 = P_2 V_2$$

$$\Rightarrow (76 \times \rho_{Hg} \times g + \rho_{Hg} \times g \times 20) \times A \times 43 = (76 \times \rho_{Hg} \times g + \rho_{Hg} \times g \times 20 \times \cos 60°)\,A \times 1$$

$$\Rightarrow \quad 96 \times 43 = 86 \times L$$

$$\Rightarrow \quad L = \frac{96 \times 43}{86} = 48 \text{ cm}$$

24.36 The middle wall is weakly conducting. Thus after a long time the temperature of both the parts will be equal. The final position of the separating wall be at a distance x from the left end. So it is at a distance $(30 - x)$ from right end.



Putting combined gas equation of one side of the separating wall

$$\frac{P_1 \times V_1}{T_1} = \frac{P_2 \times V_2}{T_2}$$

$$\Rightarrow \quad \frac{P \times 20\,A}{400} = \frac{P' \times x\,A}{T} \qquad \ldots(i)$$

$$\Rightarrow \quad \frac{P \times 10A}{100} = \frac{P' \times (30A - x)A}{T} \quad ...(ii)$$

using (i) and (ii),

$$\frac{P \times 20A \times 100}{400 \times P \times 10A} = \frac{P' \times xA}{TP'(30-x)A}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{x}{30-x}$$

$$\Rightarrow \quad 30 - x = 2x$$

$$\Rightarrow \quad 30 = 3x$$

$$\Rightarrow \quad x = 10 \text{ cm}$$

The separator will be at a distance 10 cm from the left end.

24.37 We have,

$$\frac{dV}{dt} = r$$

$$\Rightarrow \quad dV = r\,dt$$

Let the pressure pumped out gas = dp

Volume of container = V_0

At a pump dV amount of gas has been pumped out

$$P\,dV = -V_0\,dP$$

$$\Rightarrow \quad P r\,dt = -V_0\,dP$$

$$\Rightarrow \quad \frac{dp}{P} = -\frac{r\,dt}{V_0}$$

On integration, we get

$$P = e^{-\frac{rt}{V_0}}$$

Half of the gas been pumped out, pressure will be half

$$i.e. \quad 1 = \frac{1}{2} e^{-\frac{rt}{V_0}}$$

$$\Rightarrow \quad \ln 2 = \frac{rt}{V_0}$$

$$\Rightarrow \quad t = \ln 2 \times \frac{V_0}{r}$$

24.38 Here,

$$P = \frac{P_0}{1 + \left(\frac{V}{V_0}\right)^2}$$

$$\Rightarrow \quad \frac{nRT}{V} = \frac{P_0}{1 + \left(\frac{V}{V_0}\right)^2}$$

[PV = nRT according to ideal gas equation]

$$\Rightarrow \quad \frac{RT}{V} = \frac{P_0}{1 + \left(\frac{V}{V_0}\right)^2} \quad \text{[Since } n = 1\text{]}$$

$$\Rightarrow \quad \frac{RT}{V_0} = \frac{P_0}{1 + \left(\frac{V}{V_0}\right)^2} \quad [\text{at } V_0]$$

$$\Rightarrow \quad P_0V_0 = RT(1+1)$$

$$\Rightarrow \quad P_0V_0 = RT \times 2$$

$$T = \frac{P_0V_0}{2R}$$

24.39 Internal energy $= nRT$

Now, $PV = nRT$

$nT = PV$

here P and V constant So, nT is constant.

$\therefore$ Internal energy = R × constant

= constant

24.40 Frictional force = μN

Let the cork move to a distance = dl

$\therefore$ Workdone by frictionalforce = $\mu N dl$

Before that the work will not start, that means volume remain constant

$$\Rightarrow \quad \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\Rightarrow \quad \frac{1}{300} = \frac{P_2}{600}$$

$$\Rightarrow \quad P_2 = 2 \text{ atm}$$

$\therefore$ Extra pressure = 2 atm − 1 atm

= 1 atm

Workdone by 1 atm ($A\,dl$)

$$\mu N\,dl = [1 \text{ atm}]\,[A\,dl]$$

$$N = \frac{1 \times 10^5 \times \pi (5 \times 10^{-2})^2}{0.2}$$

$$= \frac{1 \times 10^5 \times \pi \times 25 \times 10^{-4}}{0.2}$$

Total circumference of work

$$= 2\pi r \frac{dN}{dl} = \frac{N}{2\pi r}$$

$$= \frac{1 \times 10^5 \times \pi \times 25 \times 10^{-4}}{0.2 \times 2\pi r}$$

$$= \frac{1 \times 10^5 \times 25 \times 10^{-4}}{0.2 \times 2 \times 5 \times 10^{-2}}$$

$$= 1.25 \times 10^4 \text{ N/m}$$

24.41 We ...

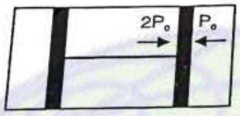
24.41 We gave,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow \frac{P_0V}{T_0} = \frac{P'V}{2T_0}$$

$$\Rightarrow P' = 2P_0$$

Net pressure = P_0 outward



$\therefore$ Tension in wire = P_0A

Where A is area of tube.

24.42 *(a)* $2P_0 = (h_2 + h_0)\rho g$

[$\therefore$ since liquids at same level have same pressure]

$$\Rightarrow 2P_0 = h_2\rho g + h_0\rho g$$

$$\Rightarrow h_2\rho g = 2P_0 - h_0\rho g$$

$$\Rightarrow h_2 = \frac{2P_0}{\rho g} - \frac{h_0\rho g}{\rho g}$$

$$= \frac{2P_0}{\rho g} - h_0$$

(b) K.E. of the water = pressure energy of the water at that layer

$$\Rightarrow \frac{1}{2}mV^2 = m \times \frac{P}{\rho}$$

$$\Rightarrow V^2 = \frac{2P}{\rho} = \frac{2}{\rho}[P_0 + \rho g(h_1 - h_0)]$$

$$\Rightarrow V = \left[\frac{2}{\rho}\{P_0 + \rho g(h_1 - h_0)\}\right]^{\frac{1}{2}}$$

(c) From question,

$$2P_0 + \rho g(h_1 - h_0) = P_0 + \rho gX$$

$$\Rightarrow X = \frac{P_0}{\rho g} + (h_1 - h_0) = h_2 + h_1$$

i.e. X is h_1 meter below the top

$\Rightarrow$ X is $-h_1$ above the top

24.43 Given, $A = 10\text{ cm}^2 = 10^{-3}\text{ m}^2$, $m = 1\text{ kg}$

$p = 100\text{ kPa} = 10^5\text{ Pa}$, $l = 20\text{ cm}$

Case 1 : External pressure exists

Case 2 : Internal pressure dose not exists

$$P_1V_1 = P_2V_2$$

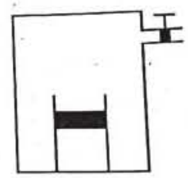
$$\Rightarrow \frac{10^5 + 1 \times 9.8}{10^{-3}} V = \frac{1 \times 9.8}{10^{-3}} \times V_1$$

$\Rightarrow (10^5 + 9.8 \times 10^3)A \times l$

$= 9.8 \times 10^3 \times A \times l'$

$\Rightarrow 10^5 \times 2 \times 10^{-1} + 2 \times 9.8 \times 10^2$

$= 9.8 \times 10^3 l'$



$$\Rightarrow l' = \frac{2 \times 10^4 + 19.6 \times 10^2}{9.8 \times 10^3}$$

$= 2.24081$ m

24.44 $P_1V_1 = P_2V_2$

$$\Rightarrow \left(\frac{mg}{A} + P_0\right)Al = P_0Al'$$

$$\Rightarrow \left(\frac{1 \times 9.8}{10 \times 10^{-4}} + 10^5\right)0.2 = 10^5 l'$$

$\Rightarrow (9.8 \times 10^3 + 10^5) \times 0.2 = 10^5 l'$

$= 109.8 \times 10^3 \times 0.2 = 10^5 l'$

$$\Rightarrow l' = \frac{109.8 \times 0.2}{10^2}$$

$= 0.2196\text{ m} = 0.22\text{ m} \approx 22\text{ cm}$

24.45 When the bulbs are maintained at two different temperatures.

The total heat gained by 'B' is the heat lost by 'A'. Let the final temperature be X

So, $m_1 SDt = m_2 SDt$

$\Rightarrow n_1M \times S \times (X - 0) = n_2M \times S \times (62 - X)$

$\Rightarrow n_1X = 62n_2 - n_2X$

So, $X = 31°C = 304\text{ K}$

[Since, Initial temperature = 0°C

P = 76 cm of Hg, $V_1 = V_2$ Hence $n_1 = n_2$]

for a single ball

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow \frac{76 \times V}{273} = \frac{P_2 \times V}{304}$$

$$\Rightarrow P_2 = \frac{304 \times 76}{273}$$

$= 84.630 = 84°C$

24.46 Given, Temperature = 20°C
Relative humidity = 100%
So the air is saturated at 20°C
Dew point is the temperature at which saturated vapour pressure is equal to present vapour pressure
So, 20°C is the dew point.

24.47 T = 25°C,
P = 104 kPa

$$R_H = \frac{VP}{SVP}$$

[SVP = 3.2 KPa, R_H = 0.6]

$$VP = 0.6 \times 3.2 \times 10^3$$
$$= 1.92 \times 10^3 = 2 \times 10^3$$

When vapour pressure removed VP reduces to zero
Net pressure inside the room now

$$= 104 \times 10^3 - 2 \times 10^3$$
$$= 102 \times 10^3 = 102 \text{ KPa}$$

24.48 Temperature = 20°C, dew point = 10°C
The place is saturated at 10°C even if the temperature drop dew point remains unaffected
The air has the vapour pressure which is the saturation VP at 10°C if saturated vapour pressure does not change on temperature.

24.49
$$R_H = \frac{VP}{SVP}$$

The point where the vapour start condensing,

$$VP = SVP$$

We know $P_1V_1 = P_2V_2$

$$R_H \times SVP \times 10 = SVP \times V_2$$
$$\Rightarrow V_2 = 10\,R_H$$
$$= 10 \times 0.4$$
$$= 4 \text{ cm}^3$$

24.50 Atmospheric pressure = 76 cm of Hg
When water is introduced the water vapour exerts some pressure which counter acts at the atmospheric pressure
The pressure drops to 75.4 cm
Pressure of vapour

$$= (76 - 75.4) \text{ cm}$$
$$= 0.6 \text{ cm}$$

Relative Humidity

$$= \frac{VP}{SVP} = \frac{0.6 \text{ cm}}{1 \text{ cm}} = 60\%$$

24.51 From Fig. 24.6 we draw perpendicular from Y-axis to meet the graphs.
Hence we find the temperature to be approximately 65°C and 45°C.

24.52 The temperature of body is 98°F = 37°C
At 37°C from the graph SVP = just less than 50 mm
B.P. is the temperature when atmospheric pressure equal the vapour pressure of the blood. Thus minimum, pressure to prevent boiling is 50 mm of Hg.

24.53 Given :
SVF at the dew point = 8.9 mm
SVP at room temperature = 17.5 mm
Dew point = 10°C as at this temperature the condensation starts
Room temperature = 20°C

$$R_H = \frac{\text{SVP at dew point}}{\text{SVP at room temp.}}$$
$$= \frac{8.9}{17.5} = 51\%$$

24.54 50 cm³ of saturated vapour is cooled from 30° to 20°. The absolute humidity of saturated H_2O vapour 30 g / m³. Absolute humidity is the mass of water vapour present in a given volume. At 30°C it contains 30 g/m³
at 50 m³ it contains 30 × 50 = 1500g
at 20°C it contains 16 × 50 = 800 g
water condense = 1500 – 800 = 700 gm

24.55 Pressure is minimum when the vapour present inside is at saturation vapour pressure. As this is the maximum pressure which the vapours can exert.
Hence the normal level of mercury drops down by 0.8 cm
∴The height of Hg column
(Given SVP at atmospheric temperature = 0.08 cm of Hg)

$$= 76 - 0.80 \text{ cm}$$
$$= 75.2 \text{ cm of Hg}$$

24.56 Pressure inside the tube
= Atmospheric pressure
= 99.4 KPa

Pressure
If num
Then u
96 × 1
⇒
24.57 Let
Hei
Pr
fo
=
p
24.5

Pressure exerted by O_2 vapour

= Atmospheric pressure-V.P.

= 99.4 KPa − 3.4 KPa

= 96 KPa

If number of mole of $O_2 = n$

Then using gas equation

$96 \times 10^3 \times 50 \times 10^{-6}$

$= n \times 8.3 \times 300$

$\Rightarrow \quad n = \frac{96 \times 50 \times 10^3}{8.3 \times 300}$

$= 1.9277 \times 10^{-3}$

24.57 Let the barometer has a length = X

Height of air above the air mercury column

$= (X - 74 + 1) = (X - 73)$

Pressure of air = 76 −74 −1 = 1 cm

for 2nd case, height of air above

$\Rightarrow \quad (X - 72.1 + 1) = (X - 71.1)$

pressure of air = (74 − 72.1 − 1) = 0.90

$(X - 73)(1) = \frac{9}{10}(X - 71.1)$

$\Rightarrow 10(X - 73) = 9(X - 71.1)$

$\Rightarrow \quad X = 10 \times 73 - 9(71.1)$

$= 730 - 639.9$

$X = 90.1$

Height of air = 90.1

Height of barometer tube above the mercury column

= 90.1 + 1 = 91.1 mm

24.58 Relative humidity = 40%

SVP = 4.6 mm of Hg

$0.4 = \frac{VP}{4.6}$

$\Rightarrow \quad VP = 0.4 \times 4.6 = 1.84$

$\frac{P_1 V}{T_1} = \frac{P_2 V}{T_2}$

$\Rightarrow \quad \frac{1.84}{273} = \frac{P_2}{293}$

$\Rightarrow \quad P_2 = \frac{1.84 \times 293}{273}$

Relative humidity at 20°C

$= \frac{VP}{SVP} = \frac{1.83 \times 293}{273 \times 18}$

$= 0.109 = 10.9\%$

24.59 $R_H = \frac{VP}{SVP}$

Given, $0.50 = \frac{VP}{3600}$

$\Rightarrow \quad VP = 3600 \times 0.50$

Let the extra pressure needed be P

So, $P = \frac{m}{M} \times \frac{RT}{V}$

$= \frac{m}{18} \times \frac{8.3 \times 300}{1}$

Now, $\frac{m}{M} \times 8.3 \times 300 + 3600 \times 0.50$

$= 3600$

[air is saturated *i.e.* RH = 100% = 1 or VP = SVP]

$\Rightarrow \quad m = \frac{(36 - 18)\,6}{8.3}$

$= \frac{18 \times 6}{8.3} = 13$ gm

24.60 Given, T = 300 K,

Relative humidity = 20%,

V = 50 m³

SVP at 300 K = 3.3 KPa

VP = Relative humidity× SVP

$= (0.2 \times 3.3 \times 10^3)$

$PV = \frac{mRT}{M}$

$\Rightarrow 0.2 \times 3.3 \times 10^3 \times 50$

$= \frac{m \times 8.3 \times 300}{18}$

$\Rightarrow \quad m = \frac{0.2 \times 3.3 \times 50 \times 18 \times 10^3}{8.3 \times 300}$

$= 238.55 \text{ g} \approx 238 \text{ g}$

Mass of water present in the room = 238 g

24.61 $RH = \frac{VP}{SVP}$

$\Rightarrow \quad 0.20 = \frac{VP}{3.3 \times 10^3}$

$\Rightarrow \quad VP = 0.20 \times 3.3 \times 10^3$

$= 660$

$\Rightarrow$ Again $PV = nRT$

$$\Rightarrow \quad P = \frac{nRT}{V} = \frac{m}{M} \times \frac{R \times T}{V}$$

$$= \frac{500 \times 8.3 \times 300}{18 \times 50}$$

$$= 1383.3$$

Net $P = 1383.3 + 660 = 2043.3$

Now $RH = \frac{2043.3}{3300} = 0.619 = 62\%$

24.62 (a) Relative humidity

$$= \frac{VP}{SVP \text{ at } 15^\circ C}$$

$$\Rightarrow \quad 0.4 = \frac{VP}{1.6 \times 10^3}$$

$$\Rightarrow \quad VP = 0.4 \times 1.6 \times 10^3$$

The evaporation occurs as long as the atmosphere does not saturated

Net pressure change

$$= 1.6 \times 10^3 - 0.4 \times 1.6 \times 10^3$$

$$= (1.6 - 0.4 \times 1.6)\, 10^3$$

$$= 0.96 \times 10^3$$

Mass of water evaporated = m

$$\Rightarrow 0.96 \times 10^3 \times 50 = \frac{m \times 8.3 \times 288}{18}$$

$$\Rightarrow \quad m = \frac{0.96 \times 50 \times 18 \times 10^3}{8.3 \times 288}$$

$$= 361.45 \approx 361 \text{ g}$$

(b) At 20°C, SVP = 2.4 KPa,

At 15°C, SVP = 1.6 KPa

Net pressure change

$$= (2.4 - 1.6) \times 10^3 \text{ Pa}$$

$$= 0.8 \times 10^3 \text{ Pa}$$

Mass of water evaporated

$$= m = \frac{m' \times 8.3 \times 293}{18}$$

$$\Rightarrow \quad m' = \frac{0.8 \times 50 \times 18 \times 10^3}{8.3 \times 293}$$

$$= 296.06 \approx 296 \text{ grams.}$$

CHAPTER 25
CALORIMETRY

EXERCISES

25.1 Given Mass of aluminium = 0.5 kg

Mass of water = 0.2 kg

Mass of iron = 0.2 kg

Temperature of aluminium and water $= 20° = 293°K$

Temperature of iron $= 100°C = 373$ k.

Specific heat of Al = 910 J / kg-K

Heat gain $= 0.5 \times 910\,(T-293) + 0.2 \times 4200 \times (T-293)$

$= (T-293)\,[0.5 \times 910 + 0.2 \times 4200]$

Heat lost $= 0.2 \times 470 \times (373 - T)$

We know Heat gain = Heat lost.

$\Rightarrow (T-293)\,[0.5 \times 910 + 0.2 \times 4200] = 0.2 \times 470 \times (373 - T)$

$\Rightarrow (T-293)\,(455 + 840) = 94\,(373 - T)$

$\Rightarrow (T-293)\frac{1295}{94} = (373 - T) \quad \left[\frac{1295}{94} \approx 14\right]$

$(T-293) \times 14 = 373 - T$

$\Rightarrow 14\,T - 293 \times 14 = 373 - T$

$\Rightarrow 15\,T = 373 + 4102 = 4475$

$\Rightarrow T = \frac{4475}{15} = 298$ k

$\therefore T = (298 - 273)°C = 25°C$

$\therefore$ The final temp. = 25°C

25.2 Given, Mass of iron = 100g

Water equivalent of calory meter = 10 g

Mass of water = 240 gm

Let the temp. of surface = 0°C

$S_{iron} = 470$-J / kg

Total heat gained = Total heat lost

So, $\frac{100}{1000} \times 470 \times (\theta - 60°)$

$= \frac{(240+10)}{1000} \times 4200 \times (60 - 20)$

$\Rightarrow 47\theta - 47 \times 60 = 25 \times 42 \times 40$

$\Rightarrow \theta = \frac{42000 + 2820}{47}$

$= \frac{44820}{47} = 953.61°C$

25.3 Given, The temperature of A = 12°C,

The temperature of B = 19°C

The temperature of C = 28°C

$\Rightarrow$ The temperature of A + B = 16°C

$\Rightarrow$ The temperature of B + C = 23°C

In accordance with the principle of calorimetry, when A and B are mixed,

$M_{CA}\,(16 - 12) = M_{CB}\,(19 - 16)$

$\Rightarrow 4\,M_{CA} = 3\,M_{CB}$

$\Rightarrow M_{CA} = \left(\frac{3}{4}\right) M_{CB}$

and when B and C are mixed

$M_{CB}\,(23 - 19) = M_{CC}\,(28 - 23)$

$\Rightarrow 4\,M_{CB} = 5\,M_{CC}$

$\Rightarrow M_{CC} = \left(\frac{4}{5}\right) M_{CB}$

When A and C are mixed, if T is the common temperature of the mixture,

$M_{CA}\,(T - 12) = M_{CC}\,(28 - T)$

$\Rightarrow \left(\frac{3}{4}\right) M_{CB} (T - 12) = \left(\frac{4}{5}\right) M_{CB} (28 - T)$

$\Rightarrow \left(\frac{3}{4}\right) (T - 12) = \left(\frac{4}{5}\right) (28 - T)$

$\Rightarrow (3 \times 5)(T - 12) = (4 \times 4)(28 - T)$

$\Rightarrow 15T - 180 = 448 - 16T$

$\Rightarrow 31T = 628$

$\Rightarrow T = \frac{628}{31} = 20.258°C$

$= 20.3°C$

25.4 Here, Volume of ice $= 2 \times 2 \times 2 = 8\ cm^3$

Total mass of ice $= 8 \times 0.9 \times 4 = 28.8$ g

$\therefore$ Density of ice $= 0.9\ gm/cm^3$

Ice

mass = 28.8 g. temp = 0°C

Drink

mass = 200g., temp. = 10°C

Heat required for melting

$= 28.8 \times 80 = 2304$ cal.

Heat released to drop to 0°C

$= 200 \times 10 \times 1$

$= 2000$ cal.

Hence 2000 cal. released by drink must Have been utilised for melting of ice.

So, amount of ice melted $= \frac{2000}{80} = 25$ gm.

Since the total ice is not melted, hence the temp. of the system remains at 0°C.

25.5 Energy required to decreases the temp. of 10 kg. of water to 5°C.

$U = 10 \times 4200\ J/kg°C \times 5°C$

$= 210{,}000 = 21 \times 10^4$ J.

Energy required for evaporation of water to 0.2g. / sec.

$= 2 \times 10^{-4} \times 2.27 \times 10^6$

$= 454.$

454 J energy losing system per second

$= \frac{21 \times 10^4}{454}$

21×10^4 J energy losing system during one minute

$= \frac{21 \times 10^4}{454 \times 60} = 7.7$ minute

$\therefore$ The time required to decrease temp. by 5°C is 7.7 minute

25.6 Let the volume of cube = V

So, volume of ice displaced = V

Let the initial temp. be T° K,

Mass of cube = 8000 V / kg.

We know, heat gained = heat lost

$\Rightarrow 8000\ V \times 470 \times (T - 273)$

$= 900\ V \times 3.38 \times 10^3$

$\Rightarrow 376\ T - 376 \times 273 = 30420$

$\Rightarrow T = \frac{30420 + 102648}{376}$

$= \frac{133068}{376}$

$= 353.9042°$ K

$\approx 80°C.$

25.7 Heat absorbed by the ice to raise the temperature 100°C

$Q_1 = 1 \times 3.36 \times 10^5 + 1 \times 4200 \times 100$

$= 3.36 \times 10^5 + 4.2 \times 10^5$

$= (3.36 + 4.2) \times 10^5$

$= 7.56 \times 10^5 = 0.756 \times 10^6$

Q_2 heat released by steam

$= 1 \times 2.26 \times 10^6$ J

$= 2.26 \times 10^6$ J.

The extra heat $= Q_2 - Q_1$

$= (2.26 - 0.756) \times 10^6$

$= 1.506 \times 10^6$

$\therefore$ The amount of steam condensed

$= \frac{1.506 \times 10^6}{2.26 \times 10^6}$

$= 0.665$ kg = 665 gm

The extra ice = (1000 – 665) = 335 gm

$\therefore$ The amount of ice formed

$= 1000 + 335 = 1335$ g.

25.8 Total heat required to raise the temperature of 20 kg. of water from 10°C to 35°C

$= 20 \times 4200 \times 25$

$= 20 \times 4200 \times 25 = 21 \times 10^5$ J

Energy utilized $= t \times (0.80) \times 1000$

Equating now, we get

$t = \frac{21 \times 10^5}{800} = 44$ min.

25.9 Here $m = 0.5\ m^2 = 500\ L = 500\ kg$

So the heat liberated during the water changes 20°C to 5°C

$$= 500 \times 4200 \times 15$$
$$[\Delta\theta = 20 - 5 = 15]$$
$$= 500 \times 4200 \times 15$$
$$= 75 \times 420 \times 1000$$
$$= 31500 \times 1000$$

Let the height = h

the required work

$$= mgh = 10 \times 10 \times h = 100\,h$$

But, $100\,h = 31500000$

$\Rightarrow$ $h = 315000\ m = 315\ km$

25.10 Since the bullet enters and stops inside the block, the total K.E. change by the bullet is responsible for the internal changes of the block.

$\therefore$ Change in internal energy

= K.E. of bullet

$$= \frac{1}{2} \times \frac{20}{1000} \times 40 \times 40 = 16\ J$$

25.11 K.E. of the man $= \frac{1}{2} mV^2$

$$= \left(\frac{1}{2}\right) 50 \times 5^2$$
$$= 25 \times 25 = 625\ J$$

The amount of heat required to raise the temperature of water from 20°C to 30°C

$$= ms\Delta\theta = m \times 4200 \times (30 - 20)$$
$$= 42000\ m$$

But, $42 \times 10^3\ m = 625$

$$\Rightarrow \quad m = \frac{625}{42} \times 10^{-3}$$
$$= 14.88 \times 10^{-3}\ kg$$
$$= 15\ g$$

25.12 Total P.E. $= 4 \times 10 \times 3 = 120\ J$

Energy utilized $= 120 \times \frac{80}{100} = 96\ J$

$$= \frac{96}{4.2} = 22.857\ cal.$$
$$\approx 23\ cal.$$

25.13 Mass of van = 1500 kg

Speed V = 54 km / h

$$= 54 \times \left(\frac{5}{18}\right) = 15\ m/s.$$

$\therefore$ Total K.E. $= \frac{1}{2} mv^2$

$$= \frac{1}{2} \times 1500 \times (15)^2$$
$$= 750 \times 225$$
$$= 168750\ J$$
$$= 40178\ cal.$$

Loss in the total energy = 40178 cal.

In 1 second loss in the total energy

$$= \frac{40178}{10} = 4017.8$$
$$= 4000\ cal. / sec$$

$\therefore$ The average thermal energy cal/sec

$$\approx 4000\ cal. / sec$$

25.14 Thermal energy developed

= Change in K.E.

$$= \frac{1}{2} \times \frac{100}{1000} \times 100 - \frac{1}{2} \times \frac{100}{1000} \times 25$$
$$= 5 - \left(\frac{5}{4}\right) = \frac{15}{4} = 3.75\ J$$

25.15 $m_1 = 10\ kg,\ V_1 = 10\ m/s$

$m_2 = 20\ kg,\ V_2 = 20\ m/s$

Here, $m_2 V_2 - m_1 V_1 = (m_1 + m_2)V$

$\Rightarrow 20 \times 20 - 10 \times 10 = (10 + 20)\ V$

$\Rightarrow$ $400 - 100 = 30\ V$

$\Rightarrow$ $300 = 30\ V$

$\Rightarrow$ $V = 10\ m/s$

Initial kinetic energy

$$= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$
$$= \frac{1}{2} \times 10 \times (10)^2 + \frac{1}{2} \times 20 \times (20)^2$$
$$= 500 + 4000 = 4500$$

Final kinetic energy $= \frac{1}{2} (m_1 + m_2)V^2$

$$= \frac{1}{2} (10 + 20)(10)^2$$
$$= \left(\frac{30}{2}\right) \times 100 = 1500$$

∴ The total change in K.E.

$= 4500 - 1500 = 3000$ J

∴ The thermal energy developed in the process = 3000 J

25.16 Let the mass of ball be m kg.

and $V_1 = \sqrt{2gh} = \sqrt{40}$

$V_2 = \sqrt{2gh} = \sqrt{30}$

So change in K.E.

$= \frac{1}{2} \times m \times 40 - \left(\frac{1}{2}m\right) \times 30 = \left(\frac{10}{2}\right)m$

$= 5\,m$

This is utilised to increase in temperature of the ball

From question,

$$\left(\frac{40}{100}\right) \times \frac{10}{2}\, m = m \times 800 \times \Delta t$$

$$\Rightarrow \quad \Delta t = \frac{1}{400} = 0.0025$$

$$= 2.5 \times 10^{-3}\,°C$$

25.17 $m = 200\,g = 0.2$ kg

$l = 60$ cm $= 0.6\,m$, $f = mg$

Work done by an energy down by the copper block to a distance 60 cm

$W = mg\,l \sin\theta$

$= 0.2 \times 10 \times 0.6 \sin 37° \in$

$= 1.2 \times \left(\frac{3}{5}\right) = 0.72$

The thermal energy gained by block

$= ms\,\Delta\theta$

$= 0.2 \times 420\,\Delta\theta = 84\Delta\theta$

But, $84\Delta\theta = 0.72$

$\Rightarrow \quad \Delta\theta = \frac{0.72}{84} = 0.00857$

$= 0.0086 = 8.6 \times 10^{-3}\,°C$

25.18 Volume of the block $= \frac{1.2}{6000} = 2 \times 10^{-4}\,m^3$

When the mass is diped in water the block experiences a buoyant force and spring experience PE which is counteracted by its own weight.

$Kx + V\rho g = mg$

$\Rightarrow 200x + 2 \times 10^{-4} \times 1000 \times 10 = 12$

$\Rightarrow \quad x = \frac{(12-2)}{200}$

$= \frac{10}{200} = 0.05$

Now the heat is equally transfered to both block and water.

So, $\frac{1}{2}kx^2 + mgh - V\rho gh = m_1 s_1 \Delta\theta + m_2 s_2 \Delta\theta$

$$\Rightarrow \frac{1}{2} \times 200 \times 0.0025 + 1.2 \times 10 \times \left(\frac{40}{100}\right) - 2 \times 10^{-4} \times 1000 \times 10 \times \left(\frac{40}{100}\right)$$

$$= \left(\frac{260}{1000}\right) \times 4200 \times \Delta t + 1.2 \times 250 \times \Delta t$$

$\Rightarrow 0.25 + 4.8 - 0.8 = 1092\,\Delta t + 300\,\Delta t$

$\Rightarrow \quad 1392\,\Delta t = 4.25$

$\Rightarrow \quad \Delta t = \frac{4.25}{1392} = 0.003053$

$= 3 \times 10^{-3}\,°C$

CHAPTER 26
LAWS OF THERMODYNAMICS

EXERCISES

26.1 $t_1 = 15°C, t_2 = 17°C,$

$\Delta t = t_2 - t_1$

$= 17°C - 15°C = 2°C = 275$ K

$m_v = 100$ g $= 0.1$ kg.,

$m_w = 200$ g $= 0.2$ kg

Sp. heat capacity of

$c_u = 420$ J / kg-K

Sp. heat capacity of water

$= 4200$ J / kg-K

(a) The heat transfered to the liquid vessel system is 0. The internal heat is shared in between the vessel and water.

(b) Work done on the system

= Heat product unit

$\Rightarrow \quad dw = 100 \times 10^{-3} \times 420 \times 2 + 200 \times 10^{-3} \times 4200 \times 2$

$= 84 + 84 \times 20 = 84 \times 21$

$= 1764$ J

(c) $dQ = 0, dU = -dw = 1764$

since dw = –ve work done in the system

26.2 (a) Heat is not given to the liquid Instead, the mechanical workdone is converted to heat.

So, heat given to liquid is zero

(b) Work done on the liquid is the PE lost by the 12 kg mass $= mgh$

$= 12 \times 10 \times 0.70$

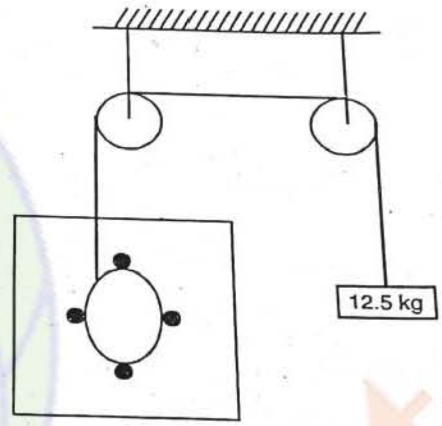
$= 84$ J

(c) Let rise in temperature be Δt

We know, $84 = ms\Delta t$

$\Rightarrow \quad 84 = 1 \times 4200 \times \Delta t$ (for 'm' = 1 kg)

$\Rightarrow \quad \Delta t = \frac{84}{4200} = \frac{1}{50} = 0.02$ K



26.3 Mass of block = 100 kg., $u = 2$ m/s

$\mu = 0.2, v = 0$

We know $\quad dQ = du + dw$

In this case $\quad dQ = 0$

$\Rightarrow \quad -du = dw$

$\Rightarrow \quad du = -\left(\frac{1}{2}mv^2 - \frac{1}{2}mu^2\right)$

$= -\frac{1}{2} \times 100 \times 2 \times 2$

$= 200$ J

26.4 Here, $\quad Q = 100$ J

We know, $\quad \Delta U = \Delta Q - \Delta W$

Here, since the container is rigid,

$\Delta V = 0$

Hence the $\Delta W = P\Delta V = 0$

So, $\quad \Delta U = \Delta Q = 100$ J.

26.5

$P_1 = 10$ kPa
$= 10 \times 10^3$ Pa
$P_2 = 50 \times 10^3$ Pa
$V_1 = 200$ CC, $V_2 = 50$ CC

(i) work done on the gas

$= \left[\left(\frac{1}{2}\right)(10+50)\times 10^3 \times (50-200)\times 10^{-6}\right]$

$= -4.5$ J

(ii) $\Delta Q = 0$,

So, $\Delta U = -\Delta W = 4.5$ J

26.6 Initial state '*i*'

Given : $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

Final state '*f*'

where, P_1 = Initial pressure,
P_2 = Final pressure
T_1, T_2 = Absolute temperatures

So, $\Delta V = 0$

Work done by gas $= P\Delta V = 0$

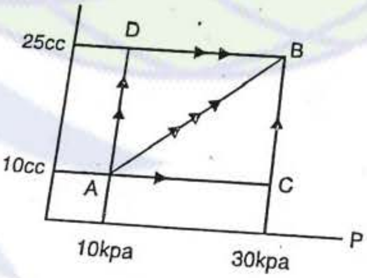
26.7 In path ACB,

$W_{AC} + W_{BC} = 0 + P\Delta V$
$= 30 \times 10^3 \times (25-10)\times 10^{-6}$
$= 0.45$ J

In path AB,

$W_{AB} = \frac{1}{2} \times (10+30)\times 10^3 \times 15 \times 10^{-6}$

$= \frac{1}{2} \times 40 \times 15 \times 10^{-3} = 0.30$ J



In path ADB,

$W = W_{AD} + W_{DB}$
$= 10 \times 10^3 (25-10)\times 10^{-6} + 0$
$= 10 \times 15 \times 10^{-3} = 0.15$ J

26.8

$\Delta Q = \Delta U + \Delta W$

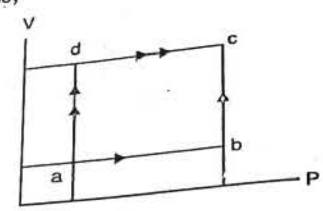
In *abc*, $\Delta Q = 80$ J, $\Delta W = 30$ J

So, $\Delta U = (80-30)$ J $= 50$ J

Now in *abc*,

$\Delta W = 10$ J

So, $\Delta Q = 10$ J $+ 50$ J$= 60$ J [$\because \Delta U = 50$ J]



26.9 In path ACB,

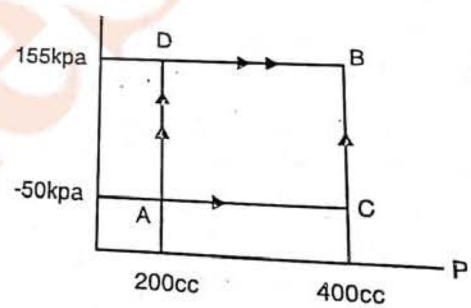
$\Delta Q = 50$ cal. $= (50 \times 4.2)$ J
$= 210$ J
$\Delta W = W_{AC} + W_{CB}$
$= 50 \times 10^{-3} \times 200 \times 10^{-6}$
$= 10$ J
$\Delta Q = \Delta U + \Delta W$
$\Rightarrow \Delta U = \Delta Q - \Delta W = (210-10)$ J
$= 200$ J

Inpath ADB, $\Delta Q = ?$

$\Delta U = 200$ J

(Internal energy change between two points is always same)

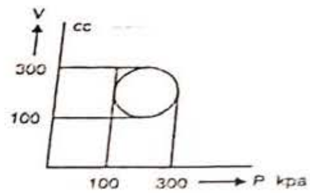
$\Delta W = W_{AD} + W_{DB} = \Delta W$
$= 0 + 155 \times 10^3 \times 200 \times 10^{-6}$
$= 31$ J
$\Delta Q = \Delta U + \Delta W$
$= (200 + 31)$ J $= 231$ J
$= 55$ cal.



26.10 Heat absorbed = Work done
= Area under the graph in the given case

Heat absorbed = area of the circle
$= \pi \times 10^4 \times 10^{-6} \times 10^3$ J
$= 3.14 \times 10 = 31.4$J

26.11



26.11 $\Delta Q = 2.4$ cal.

$\Delta W = W_{AB} + W_{BC} + W_{AC}$

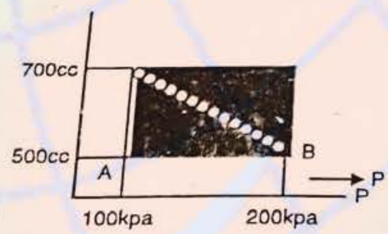
$= 0 + \frac{1}{2} \times (100 + 200) \times 10^3 \times 200 \times 10^{-6} - 100 \times 10^3 \times 200 \times 10^{-6}$

$= \frac{1}{2} \times 300 \times 10^3 \times 200 \times 10^{-6} - 20$

$= 30\ J - 20\ J = 10\ J$

$\Delta U = 0$ (in a cyclic process)

$\Delta Q = \Delta U + \Delta W$



$\Rightarrow 2.4J = 10$

$\Rightarrow \quad J = \frac{10}{2.4} = \frac{100}{24} = \frac{25}{6} = 4.17$ J/cal

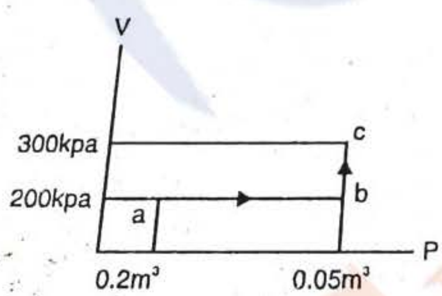
26.12 Now, $\Delta Q = 2625$ cal.

$\Delta U = 5000$ J

From graph,

$\Delta W = 200 \times 10^3 \times 0.03$

$= 6000$ J



Now, $\Delta Q = \Delta W + \Delta U$

$\Rightarrow \quad 2625$ cal. $= 6000\ J + 5000\ J$

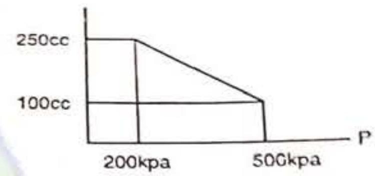
$\Rightarrow \quad J = \frac{11000}{2625} = 4.19$ J/cal.

26.13 $\Delta Q = 70$ cal $= (70 \times 4.2)$ J

$\Delta W = \frac{1}{2} \times (200 + 500) \times 10^3 \times 150 \times 10^{-6}$

$= \frac{1}{2} \times 700 \times 150 \times 10^{-3}$

$= 525 \times 10^{-1} = 52.5$ J



$\Delta U = ?$

$\Delta Q = \Delta U + \Delta W$

$\Rightarrow -294 = \Delta U + 52.5$

$\Rightarrow \quad \Delta U = -294 - 52.5 = -346.5$ J

26.14 $\Delta U = 1.5\,pV$

$\Delta V = (200 - 100)$ cm³ $= 100$ cm³

$= 10^{-4}$ m³

$P = 1 \times 10^5$ Pa

$\Delta U = 1.5 \times 10^5 \times 10^{-4} = 15$ J

$\Delta W = 10^5 \times 10^{-4} = 10$ J

$\Delta Q = \Delta U + \Delta W = 10 + 15 = 25$ J

26.15 $\Delta Q = 10$ J, $\Delta V = A \times 10$ cm

$= (4 \times 10)$ cm³ $= 40 \times 10^{-6}$ m³

$\Delta W = P\Delta V = 100 \times 10^3 \times 40 \times 10^{-6}$ m³

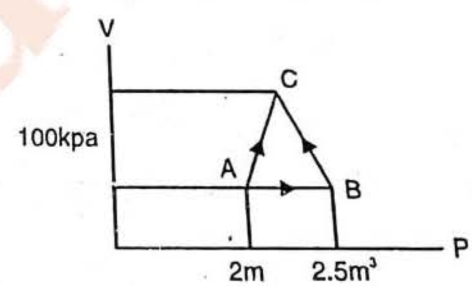
$= 4$ m³

$\Delta U = ?$

$10 = \Delta U + \Delta W$

$\Rightarrow \quad 10 = \Delta U + 4$

$\Rightarrow \quad \Delta U = 6$ J



26.16 *(a)* $P_1 = 100$ kPa, $V_1 = 2$ m³

$\Delta V_1 = 0.5\ m^3$, $\Delta P_1 = 100$ kPa

From the graph, we find that area under AC is greater than area under AB. So, We see that heat is extracted from the system

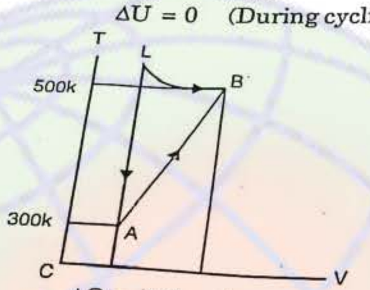
(b) Amount of heat = area under ABC

$$= \frac{1}{2} \times \frac{5}{10} \times 10^5 = 25000\ J$$

26.17 $n = 2$ mole

$\Delta Q = -1200$ J

$\Delta U = 0$ (During cyclic process)



$$\Delta Q = \Delta U + \Delta W$$

$$\Rightarrow \quad -1200 = W_{AB} + W_{BC} + W_{CA}$$

$$\Rightarrow \quad -1200 = nR\Delta T + W_{BC} + \text{'0'}$$

$$\Rightarrow \quad -1200 = 2 \times 8.3 \times 200 + W_{BC}$$

$$W_{BC} = -400 \times 8.3 - 1200$$

$$= -4520\ J$$

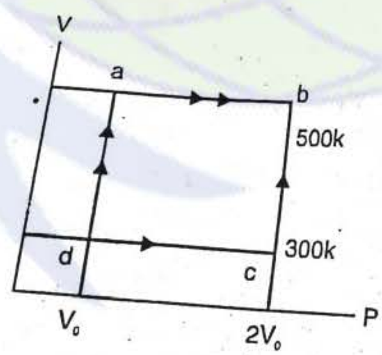
26.18 Given, $n = 2$ moles.

$\Delta V = 0$

In ad and bc

Hence $\Delta W = \Delta Q$

$\Delta W = \Delta W_{AB} + \Delta W_{CD}$



$$= nRT_1 \, ln \left(\frac{2\,V_0}{V_0}\right) + nRT_2 \, ln \left(\frac{V_0}{2\,V_0}\right)$$

$$= nR \times 2.303 \times \log 2 \times (500 - 300)$$

$$= 2 \times 8.314 \times 2.303 \times 0.301 \times 200$$

$$= 2305.31\ J$$

26.19 Given, $M = 2$kg.

$\Delta\theta = 4°C$ $s_w = 4200$ J/kg-K,

$\rho_0 = 999.9\ kg/m^3$

$\rho_4 = 1000\ kg/m^3$, $P = 10^5$ kPa

Let internal energy $= \Delta V$

$$\Delta Q = \Delta U + \Delta W$$

$$\Rightarrow \quad ms\Delta\theta = \Delta U + P\,(V_0 - V_4)$$

$$\Rightarrow 2 \times 4200 \times 4 = \Delta U + 10^5\,(V_0 - V_4)$$

$$\Rightarrow \quad 33600 = \Delta U + 10^5 \left(\frac{m}{\rho_0} - \frac{m}{\rho_4}\right)$$

$$\Rightarrow \quad = \Delta U + 10^5 \times 0.0000002$$

$$\Rightarrow \quad 33600 = \Delta U + 0.02$$

$$\Delta U = (33600 - 0.02)\ J$$

26.20 Mass $= 10$ g $= 0.01$ kg.

$P = 10^5$ kPa

$$\Delta Q = QH_2O\ 0° - 100° + QH_2O - \text{steam}$$

$$= 0.01 \times 4200 \times 100 + 0.01 \times 2.5 \times 10^6$$

$$= 4200 + 25000 = 29200$$

$$\Delta W = P\Delta V$$

$$\Delta V = \left(\frac{0.01}{0.6}\right) - \left(\frac{0.01}{1000}\right) = 0.01699$$

$$\Delta W = P\Delta V = 0.01699 \times 10^5 = 1699\ J$$

$$\Delta Q = \Delta W + \Delta U$$

$$\text{or } \Delta U = \Delta Q - \Delta W = 29200 - 1699$$

$$= 27501 = 2.75 \times 10^4\ J.$$

26.21 (a) Since the wall cannot be moved

thus, $\Delta U = 0$

and $\Delta Q = 0$

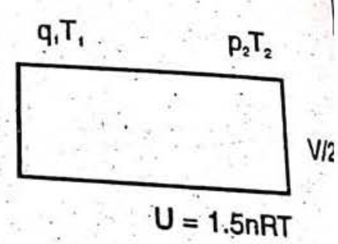
hence, $\Delta W = 0$

(b) Let final pressure in LHS $= P_1$

that in RHS $= P_2$

Since No. of moles remains constant.

$$\text{So } \frac{P_1 V}{2RT_1} = \frac{P_2 V}{2RT}$$



$$\Rightarrow \quad P_1 = \frac{P_1 T}{T_1}$$

$$= \frac{P_1 T_1 T_2 (P_1 + P_2)}{T_1 . \lambda} \quad \text{As } T = \frac{T_2 T_1 (P_1 + P_2)}{\lambda}$$

$$= \frac{P_1 T_2 (P_1 + P_2)}{\lambda}$$

Similarly $P_2 = \frac{P_2T_1(P_1+P_2)}{\lambda}$

(c) Let $T_2 > T_1$ and 'T' be the common temperature

Initially $\frac{P_1V}{2} = n_1RT_1$

$$\Rightarrow n_1 = \frac{P_1V}{2RT_1}$$

Hence $\Delta Q = 0, \Delta W = 0$

Hence $\Delta U = 0$

In case LHS,

$$\Delta U_1 = 1.5\,n_1 R\,(T - T_1)$$

In case RHS

$$= \Delta U_2 = 1.5\,n_2 R\,(T_2 - T)$$

But $\Delta U_1 - \Delta U_2 = 0$

$$\Rightarrow 1.5\,n_1R\,(T-T_1) = 1.5\,n_2R\,(T_2-T)$$

$$\Rightarrow n_1T - n_1T_1 = n_2T_2 - n_2T$$

$$\Rightarrow T(n_1+n_2) = n_1T_1 + n_2T_2$$

$$\Rightarrow T = \frac{n_1T_1+n_2T_2}{n_1+n_2}$$

$$= \frac{\frac{P_1V\times T_1}{2RT_1} + \frac{P_2V\times T_2}{2RT_2}}{\frac{P_1V}{2RT_1} + \frac{P_2V}{2RT_1}}$$

$$= \frac{(P_1+P_2)\,T_1T_2}{\lambda}$$

as, $P_1T_2 + P_2T_1 = \lambda$

(d) For RHS

$\Delta Q = \Delta U = $ (as $\Delta W = 0$)

$$= 1.5\,n_2R\,(T_2 - T)$$

$$= \frac{1.5P_2V}{2RT_2}R\left[T_2 - \frac{(P_1+P_2)\,T_1T_2}{P_1T_2+P_2T_1}\right]$$

$$= \frac{1.5P_2V}{2T_2}\times\frac{P_1T_2^2 - P_1T_1T_2}{\lambda}$$

$$= \frac{1.5P_2V}{2T_2}\times\frac{T_2P_1\,(T_2-T_1)}{\lambda}$$

$$= \frac{3P_2P_1V\,(T_2-T_1)}{4\lambda}$$

26.22 *(a)* As the conducting wall is fixed the work done by the gas on the left part during the process is zero.

(b) For left side,

Let pressure = P

Volume = V

No. of moles = n (1 mole)

Let initial temperature = T_1

$$\frac{PV}{2} = nRT_1$$

$$\Rightarrow \frac{PV}{2} = (1)\,RT$$

$$\Rightarrow T_1 = \frac{PV}{(2\text{ moles})\,R}$$

For Rightside

No. of moles = n (2 moles).

Let initial temperature = T_2

$$\frac{PV}{2} = nRT_2$$

$$\Rightarrow T_2 = \frac{PV}{(4\text{ moles})\,R}$$

(c) Let final temperature = t

Final pressure = P

No. of moles = 1 mole + 2 moles

= 3 moles.

$$PV = nRT$$

$$\Rightarrow T = \frac{PV}{nR}$$

$$= \frac{PV}{(3\text{ moles})\times R}$$

(d) For RHS,

$\Delta Q = \Delta U$ as $\Delta W = 0$

$$\Delta U = 1.5\,n_2\,R\,(T - T_2)$$

$$= 1.5\times 2\times R\,(T-T_2)$$

$$= 1.5\times 2\times\frac{4PV - 3PV}{4\times 3\text{ mole}}$$

$$= \frac{3\times PV}{4\times 3\text{ mole}} = \frac{PV}{4}$$

(e) As $dQ = -dU$

$$\Rightarrow dU = -dQ = \frac{-PV}{4}$$

■■

CHAPTER 27
SPECIFIC HEAT CAPACITIES OF GASES

EXERCISES

27.1 $N = 1$ mole, $W = 20$ g/mole, $V = 50$ m/s

K.E. of the vessel = internal energy of the gas

$$K.E. = \frac{1}{2} mV^2$$

$$= \frac{1}{2} \times 20 \times 10^{-3} \times 50 \times 50$$

$$= 25 \text{ J.}$$

So, $25 = \frac{3}{2} n R (\Delta T)$

$\Rightarrow \quad 25 = 1 \times \frac{3}{2} \times 8.31 \times \Delta T$

$\Rightarrow \quad \Delta T = \frac{50}{(3 \times 8.3)} = 2 \text{ K.}$

27.2 $m = 5$ g,

$\Delta T = 25 - 15°C = 10°C$

$C_v = 0.172 \text{ cal / g} - °C$,

$J = 4.2$ J/cal.

We know,

$$dQ = dU + dW$$

Now, $V = 0$ (for a rigid body)

So, $dW = 0$

So, $dQ = dU$

$$Q = msdT = 5 \times 0.172 \times 10$$

$$= 8.6 \text{ cal} = 8.6 \times 4.2 \text{ J}$$

$$= 36.12 \text{ J.}$$

27.3 $\gamma = 1.4$, Weight of piston $(w) = 50$ kg,

Area of cross section of piston (A) $= 100 \text{ cm}^2$

Atmospheric pressure $(P_0) = 100$ kPa,

$g = 10 \text{ m/s}^2$, $x = 20$ cm,

$$dW = Pdv = \left(\frac{mg}{A} + P_0\right).Adx$$

$$= \left(\frac{50 \times 10}{100 \times 10^{-4}} + 10^5\right).(100 \times 10^{-4} \times 20 \times 10^{-2})$$

$$= (5 \times 10^4 + 10^5) \times 20 \times 10^{-4}$$

$$= 1.5 \times 10^5 \times 20 \times 10^{-4}$$

$$= 300 \text{ J.}$$

Hence $nRdT = 300$

$\Rightarrow \quad dT = \frac{300}{nR}$

So, $\quad dQ = nCpdT = nc_p \times \left(\frac{300}{nR}\right)$

$$= \frac{n\gamma R\, 300}{(\gamma - 1)\, nR}$$

$$= \left(\frac{300 \times 1.4}{0.4}\right) = 1050 \text{ J}$$

27.4 $C_v(H_2) = 2.4 \text{ cal./ g °C}$,

$C_p(H_2) = 3.4 \text{ cal. / g °C}$

$M = 2$ g/Mol.

$R = 8.3 \times 10^7$ erg/mol.°C

We know, $C_p - C_v = 1$ cal/g°C

So, difference of molar specific heat,

$\Rightarrow C_p \times M - C_v \times M = 1$ cal/g°C

Now, $\quad 2 \times J = R$

$\Rightarrow \quad 2 \times J = 8.3 \times 10^7$ erg/mol.-°C

$\Rightarrow \quad J = 4.15 \times 10^7$ erg/cal.

27

27.5 $\left(\frac{C_p}{C_v}\right) = \frac{7}{6}$, $n = 1$ mol., $\Delta T = 50$ K

(a) Keeping the pressure constant,

$dQ = dU + dW$,

$\Delta T = 50$ K, $\gamma = \frac{7}{6}$, $n = 1$ mol.

$$dQ = dU + dW$$
$$\Rightarrow nC_p\, dT = dU + RdT$$
$$\Rightarrow dU = nC_p\, dT - RdT$$
$$= 1 \times \frac{R\gamma}{(\gamma - 1)} \times dT - RdT$$
$$= 7\, RdT - RdT$$
$$= 7\, RdT - RdT = 6\, RdT$$
$$= 6 \times 8.3 \times 50 = 2490 \text{ J.}$$

(b) Keeping volume constant,

$$dU = nC_v\, dT$$
$$= 1 \times \frac{R}{\gamma - 1} \times dT$$
$$= 1\left(\frac{8.3}{7/6} - 1\right) \times 50$$
$$= 8.3 \times 50 \times 6 = 2490 \text{ J.}$$

(c) Adiabatically, $dQ = 0$,

$$dU = -dW$$
$$= \left[\frac{n \times R}{\gamma - 1}(T_1 - T_2)\right]$$
$$= \frac{1 \times 8.3}{7/6 - 1} = (T_2 - T_1)$$
$$= 8.3 \times 6 \times 50 = 2490 \text{ J.}$$

27.6

$$m = 1.18 \text{ g}$$
$$V = 1 \times 10^3 \text{ cm}^3 = 1L$$
$$T = 300 \text{ K}, P = 10^5 \text{ Pa}$$
$$PV = nRT$$

or $n = \frac{PV}{RT}$ $[10^5 \text{ Pa} = 1 \text{ atm.}]$

$$\Rightarrow n = \frac{1}{8.2 \times 10^{-2} \times 3 \times 10^2}$$
$$\Rightarrow = \frac{1}{8.2 \times 3} = \frac{1}{24.6}$$

Now, $C_v = \frac{1}{n} \times 2 = 24.6 \times 2 = 49.2$

$$C_p = R + C_v = 1.987 + 49.2$$
$$= 51.187$$
$$Q = nC_p\, dT$$
$$= \frac{1}{24.6} \times 51.187 \times 1$$
$$= 2.08 \text{ cal.}$$

27.7 $V_1 = 100 \text{ cm}^3$, $V_2 = 200 \text{ cm}^3$,

$P = 2 \times 10^5$ Pa $dQ = 50$ J

(a) $dQ = dU + dW$

$$\Rightarrow 50 = dU + 2 \times 10^5 (200 - 100) \times 10^{-6}$$
$$\Rightarrow 50 = dU + 2 \times 10$$
$$\Rightarrow dU = 30 \text{ J.}$$

(b) $30 = n \times \frac{3}{2} \times 8.3 \times 300$

$[U = \frac{3}{2} nRT$ for monoatomic$]$

$$\Rightarrow n = \frac{2}{(83 \times 3)} = \frac{2}{249} = 0.008$$

(c) $dU = nC_v dT$

$$\Rightarrow C_v = \frac{dU}{ndT} = \frac{30}{0.008 \times 300} = 12.5$$
$$C_p = C_v + R = 12.5 + 8.3 = 20.8$$

(d) $C_v = 12.5$

27.8 Q = Amount of heat given

Work done $= \frac{Q}{2}$,

$$\Delta Q = \Delta W + \Delta U$$
$$\Rightarrow \Delta U = Q - \frac{Q}{2} = \frac{Q}{2}$$

For monoatomic gas

$$\Delta V = \frac{3}{2} nRT$$
$$\Rightarrow \frac{Q}{2} = nT \times \frac{3}{2} R$$

or $Q = 3\, nRT$

Again $Q = nc_p dT$

where C_p is molar heat capacity at constant pressure

$$3\, RnT = ndTC_p$$
$$\Rightarrow C_p = 3\, R$$

27.9

$$P = KV$$

$$\Rightarrow \frac{nRT}{V} = KV \quad \Rightarrow RT = KV^2$$

$$\Rightarrow R\Delta T = 2KV\Delta V$$

$$\Rightarrow \frac{R\Delta T}{2KV} = \Delta V$$

$$dQ = dU + dW$$

$$\Rightarrow msdT = C_v\,dT + PdV$$

$$\Rightarrow msdT = C_v\,dT + \frac{PRdT}{2KV}$$

$$\Rightarrow ms = C_v + \frac{RKV}{2KV}$$

$$\therefore C_p = C_v + \frac{R}{2}$$

27.10

$$\left(\frac{C_p}{C_v}\right) = \gamma;\ C_p - C_v = R,\ C_v = C_p = 0$$

$$pdv = \frac{1}{(b+1)}\,(Rdt)$$

$$\Rightarrow 0 = (b+1)\,C_v\,dT + (RdT)$$

$$\Rightarrow \frac{1}{(b+1)} = \frac{-C_v}{R}$$

$$\Rightarrow b + 1 = \frac{-R}{C_v} = \frac{-(C_p - C_v)}{C_v}$$

$$= -\gamma + 1$$

$$\Rightarrow b = -\gamma$$

27.11 Considering two gases, in gas (1) we have, Cp_1 (Sp. heat at constant P), Cv_1 (Sp. heat at constant V), n_1 (No. of moles)

$$\frac{Cp_1}{Cp_1} = \gamma \text{ and } Cp_1 - Cv_1 = R$$

$$\Rightarrow \gamma Cv_1 - Cv_1 = R$$

$$\Rightarrow Cv_1\,(\gamma - 1) = R$$

$$\Rightarrow Cv_1 = \frac{R}{(\gamma-1)} \text{ and } Cp_1 = \gamma\,\frac{R}{(\gamma-1)}$$

In gas (2), we have

Cp_2 (Sp. heat at constant P), Cv_2 (Sp. heat at constant V), n_2 (No. of moles)

$$\frac{Cp_2}{Cv_2} = \gamma \text{ and } Cp_2 - Cv_2 = R$$

$$\Rightarrow \gamma Cv_2 - Cv_2 = R$$

$$\Rightarrow Cv_2\,(\gamma - 1) = R$$

$$\Rightarrow Cv_2 = \frac{R}{(\gamma-1)} \text{ and}$$

$$Cp_2 = \gamma\,\frac{R}{(\gamma-1)}$$

Given, $n_1 = n_2 = 1:2$

$dU_1 = nCv_1dt$ and $dU_2 = 2nCv_2\,dT$

When gases are mixed,

$$nCv_1dT + 2nCv_2dT = 3nCvdT$$

$$Cv = \frac{Cv_1 + 2Cv_2}{3}$$

$$= \frac{\frac{R}{(\gamma-1)} + \frac{2R}{(\gamma-1)}}{3}$$

$$= \frac{3R}{(\gamma-1)\,3} = \frac{R}{\gamma - 1}$$

Hence Cp / Cv in the mixture $= \gamma$.

27.12

$Cp' = 2.5\,R$, $Cp'' = 3.5\,R$

$Cv' = 1.5\,R$ $Cv'' = 2.5\,R$

$n_1 = n_2 = 1$ mol.,

$$[n_1 + n_2]\,CvdT = n_1C'vdT + n_2\,C''vdT$$

$$\Rightarrow Cv = \frac{n_1C'_v + n_2C''_v}{n_1 + n_2}$$

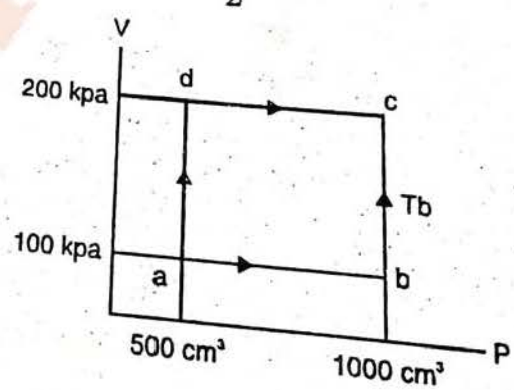
$$= \frac{1.5R + 2.5R}{2} = 2R$$

$$Cp = Cv + R = 2\,R + R = 3\,R$$

$$\gamma = \frac{Cp}{Cv} = \frac{3R}{2R} = 1.5$$

27.13

$$n = \frac{1}{2}\text{ mol.},$$



$$R = \frac{25}{3}\ J/mol.\text{-}K$$

$$\gamma = \frac{5}{3}$$

(a) Temperature at a = Ta,

$$PaVa = nRTa$$

$$\Rightarrow \quad Ta = \frac{PaVa}{nR} = 120\ K$$

Similarly, temperature at b = 240 K, at c it is 480 K and at d it is 240 K..

(b) For ab process

$$dQ = nc_p dT$$

[Since ab is isometric]

$$= \frac{1}{2} \times \frac{R\gamma}{\gamma - 1}\ (Tb - Ta)$$

$$= \frac{1}{2} \times \frac{\frac{25 \times 5}{3 \times 3}}{\frac{5}{3} - 1} \times (240 - 120)$$

$$= \frac{1}{2} \times \frac{125}{9} \times \frac{3}{2} \times (120)$$

$$= 1250\ J.$$

For bc, $dQ = dU + dW$

[$dW = 0$, Isochoric process]

$$dQ = dU = nCvdT$$

$$= nCv\ (Tc - Tb)$$

$$= \frac{1}{2} \times \frac{\left(\frac{25}{3}\right)}{\left[\left(\frac{5}{3}\right) - 1\right]} \times (240)$$

$$= \frac{1}{2} \times \frac{25}{3} \times \frac{3}{2} \times 240 = 1500\ J.$$

(c) Heat liberated in $cd = -nCpdT$

$$= -\frac{1}{2} \times \frac{\gamma R}{(\gamma - 1)} \times \left(\frac{Td}{Tc}\right)$$

$$= -\frac{1}{2} \times \frac{125}{9} \times \frac{3}{2} \times (240 - 480)$$

$$= -\frac{1}{2} \times \frac{125}{6} \times 240 = 2500\ J.$$

Heat liberated in da

$$= -nCvdT$$

$$= -\frac{1}{2} \times \frac{R}{\gamma - 1}\ (Ta - Td)$$

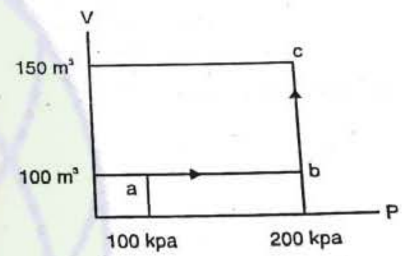
$$= -\frac{1}{2} \times \frac{25}{2} \times (120 - 240)$$

$$= \frac{25}{4} \times 120 = 750\ J.$$

27.14 (a) For a, b 'V' is constant

So, $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$$\Rightarrow \quad \frac{100}{300} = \frac{200}{T_2}$$



$$T_2 = \left(\frac{200 \times 300}{100}\right) = 600\ K$$

For b, c 'p' is constant

So, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$$\Rightarrow \quad \frac{100}{600} = \frac{150}{T_2}$$

$$\Rightarrow \quad T_2\ \left(\frac{600 \times 150}{100}\right) = 900\ K$$

(b) Work done = Area enclosed under the graph 50 cc × 20 KPa

$$= 50 \times 10^{-6} \times 200 \times 10^3\ J$$

$$= 10\ J.$$

(c) 'Q' supplied = $nCvdT$

Now,

$$Qbc = \left(\frac{PV}{RT}\right) \times \left(\frac{R}{\gamma - 1}\right) \times dT$$

$$= \frac{200 \times 10^3 \times 100 \times 10^{-6} \times 300}{600 \times 0.67}$$

$$= 14.925 \qquad (\because \gamma = 1.67)$$

'Q' supplied to $bc = nCpdT$ $\left(Cp = \frac{\gamma R}{\gamma - 1}\right)$

$$= \frac{PV}{RT} \times \frac{\gamma R}{\gamma - 1} \times dT$$

$$= \frac{200 \times 10^3 \times 150 \times 10^{-6}}{600 \times 0.67} \times 300$$

$$= 10 \times \frac{1.67}{0.67} = \frac{16.7}{0.67} = 24.925$$

(d) $Q = \Delta U + W$

Now,

$\Delta U = Q - W$

= Heat supplied – Work done

$= (24.925 + 14.925) - 10$

$= 39.850 - 10 = 29.850$ J.

27.15 In Jolly's differential steam calorimeter,

$$Cv = \frac{m_2 L}{m_1 (\theta_2 - \theta_1)}$$

m_2 = Mass of steam condensed

$= 0.095$ g

$L = 540$ cal/g $= 540 \times 4.2$ J/g.

m_1 = Mass of gas present = 3 g

$\theta_1 = 20°C$, $\theta_2 = 100°C$.

$$\Rightarrow Cv = \frac{0.095 \times 540 \times 4.2}{3 \times (100 - 20)}$$

$= 0.89 = 0.9$ J/g-K.

27.16 $\gamma = 1.5$

Since it is an adiabatic process,

So, PV^γ = constant.

(a) $P_1 V_1^\gamma = P_2 V_2^\gamma$

Given, $V_1 = 4$ L, $V_2 = 3$ L, $\frac{P_2}{P_1} = ?$

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma$$

$$= \left(\frac{4}{3}\right)^{1.5} = 1.5396 = 1.54$$

(b) $TV^{\gamma-1}$ = Constant

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{4}{3}\right)^{0.5} = 1.1[illegible]$$

27.17 $P_1 = 2.5 \times 10^5$ Pa,

$T_1 = 300$K

$V_1 = 100$ CC,

(a) $P_1 V_1^\gamma = P_2 V_2^\gamma$

$$\Rightarrow 2.5 \times 10^5 \times V^{1.5} = \left(\frac{V}{2}\right)^{1.5} \times P_2$$

$\Rightarrow P_2 = 7.07 \times 10^5$

$= 7.1 \times 10^5$

(b) $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$\Rightarrow 300 \times (100)^{1.5-1} = T_2 \times \left(\frac{100}{2}\right)^{1.5-1}$$

$= T_2 \times (50)^{1.5-1}$

$\Rightarrow 300 \times 10 = T_2 \times 7.07$

$\Rightarrow T_2 = 424.32$ K $= 424$ K.

(c) Work done by the gas in the process

$$W = \frac{mR}{(\gamma - 1)} [T_2 - T_1]$$

$$= \frac{P_1 V_1}{(\gamma - 1)} [T_2 - T_1]$$

$$= \frac{2.5 \times 10}{300 \times 0.5} \times 124$$

$= 20.67 = 21$ J.

27.18 $\gamma = 1.4$. $T_1 = 20°C = 293$ K,

$P_1 = 2$ atm, $P_2 = 1$ atm

We know for adiabatic process

$P_1^{1-\gamma} \times T_1^{1-\gamma} = P^{1-\gamma} \times T_2^\gamma$

or $(2)^{1-1.4} \times (293)^{1.4} = (1)^{1-1.4} \times T_2^{1.4}$

$\Rightarrow (2)^{-0.4} \times (293)^{1.4} = T_2^{1.4}$

$\Rightarrow 2153.78 = T_2^{1.4}$

$\Rightarrow T_2 = (2153.78)^{1/1.4}$

$= 240.3$ K.

27.19 $P_1 = 100$ KPa,

$V_1 = 400$ cm^3

$= 400 \times 10^{-6}$ m^3,

$T_1 = 300$ K,

$$\gamma = \frac{Cp}{Cv} = 1.5$$

(a) Suddenly compressed to $V_2 = 100\ cm^3$

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$\Rightarrow \quad 10^5 \times (400)^{1.5} = P_2\,(100)^{1.5}$$

$$\Rightarrow \quad P_2 = 10^5\,(4)^{1.5} = 800\ KPa$$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\Rightarrow \quad 300 \times (400)^{1.5-1} = T_2\,(100)^{1.5-1}$$

$$\Rightarrow \quad 300 \times (400)^{0.5} = T_2\,(100)^{0.5}$$

$$\Rightarrow \quad T_2 = 600\ K.$$

(b) Even if the container is slowly compressed the walls are adiabatic so heat transferred is zero.

Thus the values remain,

$$P_2 = 800\ KPa$$

$$T_2 = 600\ K$$

27.20 Given, $\frac{Cp}{Cv} = \gamma$

P_0 (Initial pressure), V_0 (Initial volume)

(a) (i) Isothermal compression,

$$P_1V_1 = P_2V_2$$

or $$P_0V_0 = P_2\frac{V_0}{2} \Rightarrow P_2 = 2P_0$$

(ii) Adiabatic compression

$$P_1V_1^\gamma = P_2V_2^\gamma$$

or $$2P_0\left(\frac{V_0}{2}\right)^\gamma = P_2\left(\frac{V_0}{4}\right)^\gamma$$

$$\Rightarrow \quad P_2 = \frac{V_0^\gamma}{2^\gamma} \times 2P_0 \times \frac{4^\gamma}{V_0^\gamma}$$

$$= 2^\gamma \times 2P_0 = P_0 2^{\gamma+1}$$

(b) (i) Adiabatic compression

$$P_1V_1^\gamma = P_2V_2^\gamma$$

or $$P_0V_0^\gamma = P'V_1^\gamma$$

$$\Rightarrow \quad P' = P_0 2^\gamma$$

(ii) Isothermal compression

or $$P_1V_1 = P_2V_2$$

$$2^\gamma P_0 \times \frac{V_0}{2} = P''\left(\frac{V_0}{2}\right)$$

$$P'' = P_0 2^\gamma \times 2$$

$$\Rightarrow \quad = P_0 2^{\gamma+1}$$

27.21 *(a)* Given that

$$P_1 = P_0,\ V_1 = V_0$$

For isothermal process,

$$P_2 = \frac{\frac{P_0V_0}{\frac{P_0}{2}}}{} = 2V_0$$

For adiabatic process $P_3 = \frac{P_0}{4}$, $V_3 = ?$

$$P_2V_2^\gamma = P_3V_3^\gamma$$

$$\Rightarrow \quad \left(\frac{V_3}{V_2}\right)^\gamma = \left(\frac{P_2}{P_3}\right)$$

$$\Rightarrow \quad \left(\frac{V_3}{V_2}\right)^\gamma = \left(\frac{P_0/2}{P_0/4}\right) = 2$$

$$\Rightarrow \quad \frac{V_3}{V_2} = 2^{\frac{1}{\gamma}}$$

$$\therefore \quad V_3 = V_2\,2^{\frac{1}{\gamma}} = 2V_0\,2^{\frac{1}{\gamma}}$$

$$= 2^{\frac{\gamma+1}{\gamma}}\,V_0$$

(b) $$P_1V_1^\gamma = P_2V_2^\gamma$$

or $$\left(\frac{V_2}{V_1}\right) = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}}$$

or $$V_2 = V_0\,2^{\frac{1}{\gamma}}$$

Again isothermal process,

$$P_2V_2 = P_3V_3$$

$$\therefore \quad V_3 = \frac{P_2V_2}{P_3} = 2.2^{\frac{1}{\gamma}}\,V_0$$

$$= 2^{\frac{\gamma+1}{\gamma}}\,V_0$$

27.22 $$PV = nRT$$

Given, $P = 150\ KPa = 150 \times 10^3\ Pa$,

$V = 150\ cm^3 = 150 \times 10^{-6}\ m^3$

$T = 300\ K$

(a) $$n = \frac{PV}{RT} = 9.036 \times 10^{-3}$$

$$= 0.009\ \text{moles.}$$

(b) $$\frac{Cp}{Cv} = \gamma,\ Cp - Cv = R$$

So, $$Cv = \frac{R}{\gamma - 1} = \frac{8.3}{0.5} = 16.65/\text{moles.}$$

(c) Given,

$P_1 = 150\ KPa = 150 \times 10^3\ Pa$,
$P_2 = ?\ V_1 = 150\ cm^3$
$= 150 \times 10^{-6}\ m^3$
$\gamma = 1.5$
$V_2 = 50\ cm^3 = 50 \times 10^{-6}\ m^3$,
$T_1 = 300\ K,\ T_2 = ?$

Since the process is adiabatic hence –

$P_1V_1^\gamma = P_2V_2^\gamma$

$\Rightarrow 150 \times 10^3 \times (150 \times 10^{-6})^\gamma = P_2 \times (50 \times 10^{-6})^\gamma$

$$\Rightarrow P_2 = 150 \times 10^3 \times \frac{(150 \times 10^6)^{1.5}}{(50 \times 10^{-6})^{1.5}}$$

$= 150000 \times (3)^{1.5}$
$= 779.422 \times 10^3\ Pa$
$= 780\ KPa$

Again,

$P_1^{1-\gamma} T_1^\gamma = P_1^{1-\gamma} T_2^\gamma$

$\Rightarrow (150 \times 10^3)^{1-1.5} \times (330)^{1.5} = (780 \times 10^3)^{1-1.5} \times T_2^{1.5}$

$\Rightarrow T_2^{1.5} = (150 \times 10^3)^{1-1.5} \times (300)^{1.5} \times 300^{1.5}$
$= 11849.050$

$\Rightarrow T_2 = (11849.050)^{1/1.5}$
$= 519.74 = 520.$

(d) $dQ = W + dU$

or $W = -dU$ [$dQ = 0$, in adiabatic]
$= -nCvdT$
$= -0.009 \times 16.6 \times (520 - 300)$
$= -0.009 \times 16.6 \times 220$
$= -32.8\ J = -33\ J.$

(e) $dU = nCvdT$
$= 0.009 \times 16.6 \times 220 = 33\ J.$

27.23 $V_A = V_B = V_C$ $T_A = T_B = T_C$

For A the process is isothermal

$P_AV_A = P'_A 2V_A$

$$\Rightarrow P'_A = P_A \times \frac{1}{2}$$

For B, the process is adiabatic

$P_BV_B^\gamma = P_B'(2V_B)^\gamma$

$$\Rightarrow P_B' = \frac{P_B}{2^{1.5}}$$

For C the process is adiabatic

$$\frac{V_C}{T_C} = \frac{V'_C}{T'_C} \Rightarrow \frac{V_C}{T_C} = \frac{2V_C}{T'_C}$$

$\Rightarrow T_C' = 2\ T_C$

Final Pressures are equal

$$\frac{P_A}{2} = \frac{P_B}{2^{1.5}} = P_C$$

$\Rightarrow P_A : P_B : P_C = 2 : 2^{1.5} : 1$

27.24 P_1 = Initial pressure,
V_1 = Initial volume
P_2 = Final pressure,
V_2 = Final volume

Given, $V_2 = 2\ V_1$

Isothermal work done

$$= nRT_1\ ln\ \frac{V_2}{V_1}$$

Adiabatic work done

$$= \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

Given that work done at both cases are same.

Hence,

$$nRT_1\ ln = \frac{V_2}{V_1} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} \quad ...(1)$$

At adiabatic process,

$$P_2 = P_1\left(\frac{V_2}{V_1}\right)^\gamma = P_1\left(\frac{1}{2}\right)^\gamma$$

From the equation (1)

$$nRT_1\ ln\ 2 = \frac{P_1V_1\left(1 - \frac{1}{2^\gamma}.2\right)}{\gamma - 1}$$

and $nRT_1 = P_1V_1$

So
$$ln\ 2 = \frac{1 - \frac{1}{2^\gamma}.2}{\gamma - 1}$$

or $(\gamma - 1)\ ln\ 2 = 1 - 2^{1-\gamma}$

27.25 $\gamma = 1.5$, T = 300K, V_1 = 1L, $V_2 = \frac{1}{2}$ L

(a) The process is adiabatic because volume is suddenly change

$P_1V_1^\gamma = P_2V_2^\gamma$

or $P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma = P_1(2)^\gamma$

or $\frac{P_2}{P_1} = 2^{1.5} = 2\sqrt{2}$

(b) $P_1 = 100\text{ KPa} = 10^5\text{ Pa}$

and $P_2 = 2\sqrt{2} \times 10^5\text{ Pa}$

Work done by adiabatic process

$$= \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$= \frac{10^5 \times 10^{-3} - 2\sqrt{2} \times 10^5 \times \frac{1}{2} \times 10^{-3}}{1.5 - 1}$$

$= -82\text{ J}$

(c) Internal energy,

$dQ = 0$

$\Rightarrow dU = -dW = -(-82\text{ J}) = 82\text{J}$

(d) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$= 300\,(2)^{0.5}$

$= 300\times \sqrt{2} \times = 300 \times 1.414$

$T_2 = 424\text{ K.}$

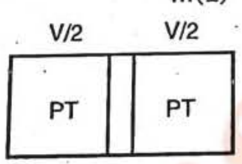
(e) The pressure is kept constant. The process is isobaric work done $= nRdT$,

Here, $n = \frac{PV}{RT} = \frac{10^5 \times 10^{-3}}{R \times 300} = \frac{1}{3R}$

So work done $= \frac{1}{3R} \times R\,(300 - 424)$

$= -41.4\text{ J.}$

(f) $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ ...(1)



$V_1 = V_2\,\frac{T_1}{T_2}$

Work done in this process

$$= nRT\,ln\,\frac{V_1}{V_2}$$

$$= \frac{1}{3R} \times R \times T \times ln\,2$$

$= 100 \times ln\,2 = 100 \times 1.039$

$= 103$

(g) Net work done

$= -82 - 41.4 + 103$

$= -20.4\text{J.}$

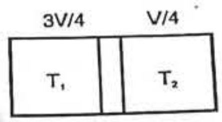
27.26 Given $\gamma = 1.5$

We know for adiabatic process $TV^{\gamma-1}$ = Constant

So, $T_1\,V_1^{\gamma-1} = T_2\,V_2^{\gamma-1}$ ...(equation)

As, it is an adiabatic process and all the other conditions are same, hence the above equation can be applied.

So, $T_1 \times \left(\frac{3v}{4}\right) = T_2 \times \left(\frac{v}{4}\right)$



So, $T_1 : T_2 = 1 : 3$

27.27 $V = 200\text{ cm}^3$, $Cv = 12.5$ J/mol-K,

$T = 300$ K, $P = 75$ cm.

(a) No. of moles of gases in each vessels,

$$n = \frac{PV}{RT}$$

$$= \frac{75 \times 13.6 \times 980 \times 200}{8.3 \times 10^7 \times 300}$$

$= 0.008$

(b) Heat is supplied to the gas but $dV = 0$

So, $dQ = dU$

$\Rightarrow 5 = nC_v dT$

$\Rightarrow 5 = 0.008 \times 12.5 \times dT$

$\Rightarrow dT = 50$ for (A)

Since $\frac{P}{T} = \frac{P_A}{T_A}$

$$\Rightarrow \frac{75}{300} = \frac{P_A\,0.008 \times 12.5}{5}$$

$$\Rightarrow P_A = \frac{75 \times 5}{300 \times 0.008 \times 12.5}$$

$= 12.5$ cm of Hg

again $\frac{P}{T} = \frac{P_B}{T_B}$ [For container B]

$$\Rightarrow \quad \frac{75}{300} = \frac{P_B \; 0.008 \times 12.5}{10}$$

$P_B = 25$ cm of Hg.

Mercury moves by a distance

$P_B - P_A = 25 - 12.5 = 12.5$ cm.

27.28 $m_{He} = 0.1g$,

$\gamma_1 = 1.67$, $M_{He} = 4$ g / mol.,

$M_{H2} = ?$; $M_{H2} = 2g$ / mol,

$\gamma_2 = 1.4$

Since it is an adiabatic surroundings

For He, $dQ = nCvdT$

$$= \frac{m}{2} \times \frac{R}{\gamma - 1} \times dT$$

$$= \frac{0.1}{4} \times \frac{R}{(1.67-1)} \times dT$$

For H_2, $dQ = nCvdT$

$$= \frac{m}{2} \times \frac{R}{\gamma - 1} \times dT$$

$$= \frac{m}{2} \times \frac{R}{1.4-1} \times dT$$

[where m is the required mass of H_2.]

Since equal amount of heat is given to both, so, dQ is same in both Equations ... *(i)* and ... *(ii)*, we get

$$\frac{0.1}{4} \times \frac{R}{0.67} \times dT$$

$$= \frac{m}{2} \times \frac{R}{0.4} \times dT$$

$$\Rightarrow \quad m = \frac{0.1}{2} \times \frac{0.4}{0.67}$$

$= 0.0298 = 0.03g$.

27.29 Initial pressure $= P_0$

Initial temperature $= T_0$

Initial Volume $= V_0 \quad \frac{Cp}{Cv} = \gamma$

(a) For diathermic vessel the temperature inside remains constant.

$$\Rightarrow \quad P_1V_1 = P_2V_2$$

$$P_0V_0 = P_2 \times 2V_0$$

$$\Rightarrow \quad P_2 = \frac{P_0}{2},$$

Temperature $= T_0$

For adiabatic vessels the temperature does not remain constant. The process is adiabatic.

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T_0V_0^{\gamma-1} = T \times (2V_0)^{\gamma-1}$$

$$\Rightarrow \quad T_2 = T_0 \times 2^{1-\gamma}$$

$$\Rightarrow \quad P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\Rightarrow \quad P_0V_0^{\gamma} = P_2 \times (2V_0)^{\gamma}$$

$$\Rightarrow \quad P_2 = \left(\frac{P_0}{2^{\gamma}}\right)$$

(b) When the valves are open, the temperature remains T_0 through out

$P_1 = P_2$ and $T_1 = T_1 = T_0$

So, $P_0 = P_1 + P_2$

$= 2P_1 = 2P_2$

So, $P_1 = P_2 = \frac{P_0}{2}$

27.30 For an adiabatic process, $PV^{\gamma} =$ Consta[nt]

So $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

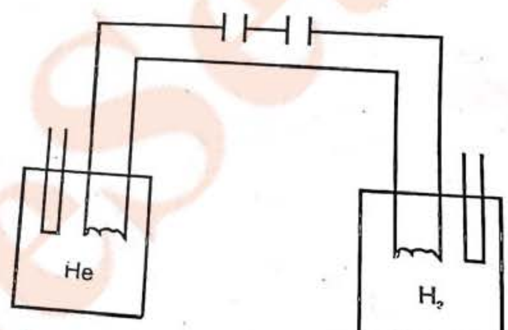
According to the problem

$$V_1 + V_2 = V_0$$

Then the equation *(i)*

$$P_1V_2^{\gamma} = P_2(V_0 - V_1)^{\gamma}$$

$$\text{or} \quad \left(\frac{P_1}{P_2}\right)^{1/\gamma} = \frac{V_0 - V_1}{V_1}$$



$$\text{or} \quad V_1 - P_1^{\frac{1}{\gamma}} = V_0 - P_2^{\frac{1}{\gamma}} - V_1P_2^{\frac{1}{\gamma}}$$

$$\text{or} \; V_1\left(P_1^{\frac{1}{\gamma}} + P_2^{\frac{1}{\gamma}}\right) = V_0 \; P_2^{\frac{1}{\gamma}}$$

$$\text{or} \quad V_1 = \frac{P_2^{\frac{1}{\gamma}} V_0}{P_1^{\frac{1}{\gamma}} + P_2^{\frac{1}{\gamma}}}$$

(b)

(c

F

27.31

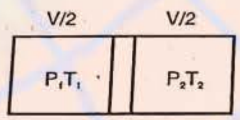
$$V_2 = \frac{P_1^{\frac{1}{\gamma}} V_0}{P_1^{\frac{1}{\gamma}} + P_2^{\frac{1}{\gamma}}}$$

(b) Since the whole process takes place in adiabatic surroundings, the separator is adiabatic.
Hence heat given to the gas in the left part = 0

(c) There will be a common pressure 'P' when the equilibrium is reached.

$$P_1 V_1^{\gamma} + P_2 V_2^{\gamma} = PV_0^{\gamma}$$

For equilibrium, $V_1 = V_2 = \frac{V_0}{2}$



Hence,

$$P_1\left(\frac{V_0}{2}\right)^{\gamma} + P_2\left(\frac{V_0}{2}\right)^{\gamma} = P(V_0)^{\gamma}$$

or $$P = \left(\frac{P_1^{\frac{1}{\gamma}} + P_2^{\frac{1}{\gamma}}}{2}\right)^{\gamma}$$

27.31 $A = 1\text{ cm}^2 = 1 \times 10^{-4}\text{ m}^2$,
$M = 0.03\text{ g} = 0.03 \times 10^{-3}\text{ kg}$.
$P = 1\text{ atm} = 10^5$ Pascal
$L = 40\text{ cm} = 0.4\text{ m}$, $L_1 = 80\text{ cm} = 0.8\text{ m}$,
$P' = 0.355$ atm
The process is adiabatic

$$P(V)^{\gamma} = P'(V')^{\gamma}$$

$\Rightarrow$ $1 \times (A \times 0.4)^{\gamma} = 0.355 \times (A \times 0.8)^{\gamma}$

$\Rightarrow$ $1 \times 1 = 0.355 \times 2^{\gamma}$

$\Rightarrow$ $\frac{1}{0.355} = 2\gamma$

$$\gamma \log \gamma = \log\left(\frac{1}{0.355}\right)$$

$\Rightarrow$ $\gamma = 1.4941$

Hence $V = \frac{\gamma p}{\rho}$

$$= \frac{1.4941 \times 105}{m / v}$$

$= 446.33$

$= 447$ m/s.

27.32 $V = 1280$ m/s, $T = 0°C = 273°K$
Density of $H_2 = 0.089\text{ kg/m}^3$,
$R = 8.3$ J/mol-K
At STP, $P = 10^5$ Pa,

we know $V_{sound} = \sqrt{\frac{\gamma p}{\rho}}$

$$1280 = \sqrt{\frac{\gamma \times 10^5}{0.089}}$$

or $$\gamma = \frac{1280 \times 1280 \times 0.089}{10^5}$$

$= 1.48$

$$\frac{C_p}{C_v} = \gamma \text{ or } C_p - C_v = R$$

$$C_v = \frac{R}{\gamma - 1} = \frac{8.31}{1.48 - 1}$$

$= 18.0$ J/mol-K

$C_p = \gamma C_v = 1.48 \times 18.0$

$= 26.3$/mol-K.

27.33 Given that

$C_p = 5.0$ cal/mol-K
$= 5.0 \times 4.2$ J/mol-K
$= 21$ J /mol-K

$$v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma PV}{M}}$$

$$C_p = \frac{R\gamma}{\gamma - 1}$$

or $21(\gamma - 1) = 8.3\gamma$
or $21\gamma - 8.3\gamma = 21$
or $12.7\gamma = 21$

$\therefore$ $\gamma = \frac{21}{12.7} = 1.653$

$$v = \sqrt{\frac{1.653 \times 1.0 \times 10^5 \times 0.0221}{4 \times 10^{-3}}}$$

$= 960$ m/s

27.34 given $\rho = 1.7 \times 10^{-3}\text{ g/cm}^3$
$= 1.7\text{ k/gm}^3$
$P = 1.5 \times 10^5$ Pa
$R = 8.3$ J/mo1-K,
$f = 3.0$ KHz.

Node separation in a Kundt's tube

$$= \frac{l}{2} = 6 \text{ cm}$$

$$l = 12 \text{ cm} = 12 \times 10^{-2} \text{ m}$$

So, $V = fl = 3 \times 10^3 \times 12 \times 10^{-2}$

$= 360$ m/s

We know, speed of sound $= \sqrt{\frac{\gamma p}{\rho}}$

or $V^2 = \frac{\gamma p}{\rho}$

$$\therefore \quad \gamma = \frac{v^2 \rho}{\rho} = \frac{(360)^2 \ 1.7}{1.5 \times 10^5}$$

$= 1.4688$

But $C_v = \frac{R}{\gamma - 1} = \frac{8.3}{0.4688}$

$= 17.71$ J/mol-K.

27.35 $f = 5 \times 10^3$ Hz,

$T = 300$ K, $\frac{l}{2} = 3.3$ cm

$l = 6.6 \times 10^{-2}$ m

$\therefore \quad V = fl = 5 \times 10^3 \times 6.6 \times 10^{-2}$

$= (66 \times 5) = 330$ m/s

$$v = \sqrt{\frac{\gamma RT}{M}} \text{ or } v^2 = \frac{\gamma RT}{M}$$

$$\therefore \quad \gamma = \frac{330 \times 330 \times 32}{8.3 \times 300 \times 100} = 1.3995$$

$$C_v = \frac{R}{\gamma - 1} = \frac{8.3}{0.3995}$$

$= 20.7$J mol-K

$C_p = C_v + R$

$= 20.7 \text{ J} + 8.3$

$= 29.07$ J/mo1-K.

CHAPTER 28 HEAT TRANSFER

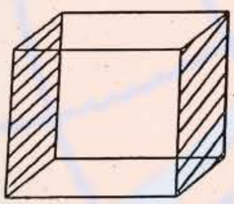
EXERCISES

28.1 $T_1 = 90°C, T_2 = 10°C$

$l = 1\text{ cm} = 0.01\text{ m}$

$A = 10\text{ cm} \times 10\text{ cm}$

$= 0.1 \times 0.1\text{ m}^2 = 1 \times 10^{-2}\text{ m}^2$



$k = 0.80\text{ W/m-°C}$

$$\frac{Q}{t} = \frac{kA(T_1 - T_2)}{l}$$

$$= \frac{8 \times 10^{-1} \times 10^{-2} \times (90 - 10)}{1 \times 10^{-2}}$$

$= 64\text{ J/s} = 3840\text{ J/min.}$

28.2 $d = 1\text{ cm} = 0.01\text{m} \quad A = 0.2\text{ m}^2$

$T_1 = 300°C$

$T^2 = 80°C$

$K = 0.025\text{ W/m–°C}$

$$\frac{Q}{t} = \frac{kA(T_1 - T_2)}{l}$$

$$= \frac{0.025 \times 0.2 \times (300 - 80)}{0.01}$$

$= 110$ watt.

28.3 $K = 0.04\text{ J/m-S°C},$

$A = 1.6\text{ m}^2$

$T_1 = 97°F = 36.10°C$

$T_2 = 47°F = 8.330°C$

$l = 0.5\text{ cm} = 0.005\text{ m}$

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l}$$

$$= \frac{0.04 \times 1.6 \times (27.7)}{0.005} = 356\text{ J/s}$$

28.4 $A = 25\text{ cm}^2 = 25 \times 10^{-4}\text{ m}^2,$

$L = 1\text{ mm} = 10^{-3}\text{ m}, K = 50\text{ W/m°C}$

$\frac{Q}{t}$ = rate of conversion of water in to steam

$$\frac{Q}{t} = \frac{100 \times 10^{-3} \times 2.26 \times 10^6}{1\text{ min.}}$$

$$= \frac{100 \times 10^3 \times 2.26}{1\text{ min.}}$$

$$= \frac{2.26}{6} \times 10^4$$

$= 0.376 \times 10^4\text{ J/s}$

$$\frac{Q}{t} = \frac{kA(T_1 - T_2)}{l}$$

$$\Rightarrow 0.376 \times 10^4 = \frac{50 \times 25 \times 10^{-4} \times (T - 100)}{10^{-3}}$$

$$\Rightarrow (T - 100) = \frac{10^{-3} \times 0.376 \times 10^4}{50 \times 25 \times 10^{-4}}$$

$$= \frac{10^{-3} \times 0.376}{50 \times 25}$$

$\Rightarrow (T - 100) = 3.008 \times 10^{-4} \times 10^{-3} \times 10^8$

$\Rightarrow (T - 100) = 3.008 \times 10$

$= 30°C = 30$

$\therefore \quad T = 100 + 30 = 130°C.$

28.5 $K = 46\ J/m\text{-}s\text{-}°C,\ l = 1\ m,$

$A = 0.04\ cm^2 = 4 \times 10^{-6}\ m^2$

$$\frac{Q}{t} = \frac{kA(T_1 - T_2)}{l}$$

$$= \frac{46 \times 4 \times 10^{6} \times (100 - 0)}{1}$$

$$= 184 \times 10^{-4}$$

$$mL = 184 \times 10^{-4}$$

$$m = \frac{184 \times 10^{-4}}{L}$$

$$m = \frac{184 \times 10^{-4}}{3.36 \times 10^{5}}$$

$$= 5.5 \times 10^{-5}\ g.$$

28.6 $A = 2400\ cm^2 = 2400 \times 10^{-4}\ m^2,$

$L = 2\ mm = 2 \times 10^{-3}\ m,\ K = 0.06\ W/m\text{-}°C$

$T_1 = 20°C,\ T_2 = 0°C$

$$\frac{Q}{t} = \frac{kA(T_1 - T_2)}{l}$$

$$= \frac{0.06 \times 2400 \times 10^{-4} \times 20}{2 \times 10^{-3}}$$

$$= \frac{6 \times 10^{-2} \times 24 \times 10^{2} \times 10^{-4} \times 20}{2 \times 10^{-3}}$$

$$= 24 \times 6 \times 10^{-1} \times 101$$

$$= 24 \times 6 = 144\ J/see.$$

Rate in which ice melts

$$= \frac{m}{t} = \frac{Q}{t \times l} = \frac{144 \times 3600}{3.4 \times 10^{5}}\ kg/h$$

$$= 1.52\ kg/h.$$

28.7 $l = 1\ mm = 1 \times 10^{-3}\ m,$ $M = 10\ kg$

$A = 200\ cm^2 = 2 \times 10^{-2}\ m^2$

$L_{vapour} = 2.27 \times 10^{-2}\ J/kg$

$K = 0.80\ J/m\text{-}s\text{-}°C,\ dQ = 2.27 \times 10^{6} \times 10;$

$$\frac{dQ}{t} = \frac{2.27 \times 10^{7}}{10^{5}}$$

$$= 2.27 \times 10^{2}\ J/s$$

Again we know

$$\frac{dQ}{t} = \frac{0.80 \times 2 \times 10^{-2} \times (42 - T)}{10^{-3}}$$

So, $\dfrac{0.80 \times 2 \times 10^{-2} \times (42 - T)}{10^{-3}} = 2.27 \times 10^{2}$

$$\Rightarrow 16 \times 42 - 16T = 227$$

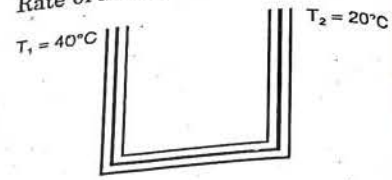
$$\Rightarrow T = 27.8 = 28°C.$$

28.8 $K = 45\ W/m\text{-}°C,$

$l = 60\ cm = 60 \times 10^{-2}\ m$

$A = 0.2\ cm^2 = 0.2 \times 10^{-4}\ m^2$

Rate of heat flow,



$$\frac{kA(T_1 - T_2)}{l} = \frac{45 \times 0.2 \times 10^{-4} \times 20}{60 \times 10^{-2}}$$

$$= 30 \times 10^{-3} = 0.03\ W$$

28.9 $A = 10\ cm^2;\ H = 10\ cm$

$$\frac{\Delta Q}{\Delta t} = \frac{kA(T_1 - T_2)}{l}$$

$$= \frac{200 \times 10 \times 10^{-4} \times 30}{1 \times 10^{-3}}$$

$$= 6000\ J/s$$

Since heat goes out from both surfaces hence net heat coming out

$$\frac{\Delta Q}{\Delta t} = 6000 \times 2 = 12000$$

$$\frac{\Delta Q}{\Delta t} = ms\frac{\Delta T}{\Delta l}$$

$$\Rightarrow 6000 \times 2 = 10^{-3} \times 10^{-1} \times 1000 \times 4200 \times \frac{\Delta T}{\Delta t}$$

$$\Rightarrow \frac{\Delta Q}{\Delta t} = \frac{12000}{420} = 28.57$$

So in one sec 28.57°C is dropped

Hence time to drop of $1°C = \dfrac{1}{28.57}$ sec

$= 0.035$ sec is required.

28.10 $l = 20\ cm = 20 \times 10^{-2}\ m,$

$A = 0.2\ cm^2 = 0.2 \times 10^{-4}\ m^2$

$T_1 = 80°C$ $T_2 = 20°C$

$k = 385$

(a) $\frac{Q}{t} = \frac{kA(T_1 - T_2)}{l}$

$= \frac{385 \times 0.2 \times 10^{-4}(80-20)}{20 \times 10^{-2}}$

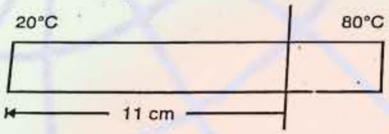
$= \frac{385 \times 2 \times 10^{-5} \times 60}{20 \times 10^{-2}}$

$= 385 \times 6 \times 10^{-4} \times 10$

$= 2310 \times 10^{-3} = 2.31$ J/s.

(b) Let the temperature of the 1 point at 11 cm be T.

$\frac{\Delta T}{l} = \frac{Q}{tkA}$



$\Rightarrow \quad \frac{\Delta T}{l} = \frac{2.31}{385 \times 0.2 \times 10^{-4}}$

$\Rightarrow \frac{T-20}{11 \times 10^{-2}} = \frac{2.31}{385 \times 0.2} \times 10^4$

$\Rightarrow \quad T - 20 = \frac{2.31 \times 10^4}{385 \times 0.2} \times 11 \times 10^{-2}$

$= 33$

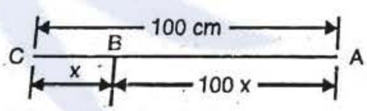
$\Rightarrow \quad T = 33 + 20 = 53°C$

28.11 Let the point to be touched be B

No Heat will flow when the temperature at that point is also 25°C

That means

$Q_{AB} = Q_{BC}$



So, $\frac{kA(100-25)}{100-x} = \frac{kA(25-0)}{x}$

$\Rightarrow \quad 75x = 2500 - 25x$

$\Rightarrow \quad 100x = 2500$

$x = 25$ cm

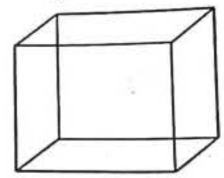
From the end with 0°C

28.12 $V = 216$ cm^3, $a = 6$ cm,

Surface area $= 6a^2 = 6 \times 36 \times 10^{-4}$ m^2

$l = 0.1$ cm

$\frac{Q}{t} = 100$ W



$\frac{Q}{t} = \frac{kA(T_1 - T_2)}{l}$

$\Rightarrow \quad 100 = \frac{k \times 6 \times 36 \times 10^{-4} \times 5}{0.1 \times 10^{-2}}$

$\Rightarrow \quad 100 = k \times 6 \times 36 \times 10^{-1} \times 5$

$\Rightarrow \quad k = \frac{100}{6 \times 36 \times 5 \times 10^{-1}}$

$= \frac{100 \times 10}{6 \times 36 \times 5}$

$= 0.9259$ W / m°C.

28.13 Given, $\theta_1 = 1°C, \theta_2 = 0°C$

$K = 0.5$ W / m-°C, $d = 2$ mm

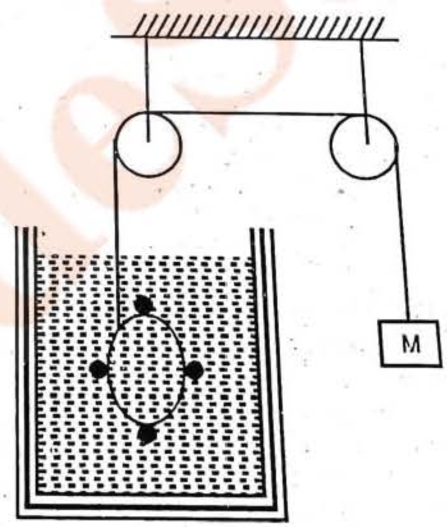
$= 2 \times 10^3$ m

$A = 5 \times 10^{-2}$ m^2,

$Q = 10$ cm/s $= 0.1$ m/s

Power = Force × Velocity

$= mg \times \theta$



Again, power $= \frac{dQ}{dt} = \frac{KA(\theta_1 - \theta_2)}{d}$

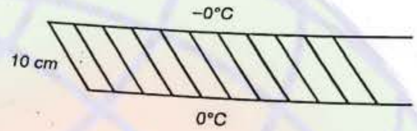
So, $mgQ = \frac{KA(\theta_1 - \theta_2)}{d}$

$\Rightarrow m = \frac{KA(\theta_1 - \theta_2)}{dQg}$

$= \frac{5 \times 10^{-1} \times 5 \times 10^{-2} \times 1}{2 \times 10^{-3} \times 10^{-1} \times 10}$

$= \frac{25}{2} = 12.5$ kg

28.14 $A = 1.7$ W/m-°C



$\rho_w = 1000$ kg/m³

$L_{ice} = 3.36 \times 10^5$ J/kg

$L = 10 \times 10^{-2}$ m

(a) $\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l}$

$\Rightarrow \frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{Q}$

$= \frac{KA(\theta_1 - \theta_2)}{mL}$

$= \frac{KA(\theta_1 - \theta_2)}{Al\rho_w L}$

$= \frac{1.7 \times |(0 - 10)|}{10 \times 1000 \times 3.36 \times 10^5 \times 10^{-2}}$

$= \frac{17 \times 10}{3.36 \times 10^7}$

$= \frac{17}{3.36} = 5.059 \times 10^{-7}$ m/sec.

$= 5 \times 10^{-7}$ m/sec.

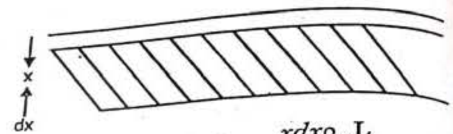
(b) Let us assume that x length of ice has been formed. To form a small strip of ice of length dx, dt time is required

$\frac{dQ}{dt} = \frac{KA(\Delta\theta)}{x}$

$\Rightarrow \frac{dmL}{dt} = \frac{KA(\Delta\theta)}{x}$

$\Rightarrow \frac{Adx\rho_w L}{dt} = \frac{KA(\Delta\theta)}{x}$

$\Rightarrow \frac{dx\rho_w L}{dt} = \frac{K\Delta\theta}{x}$



$\Rightarrow dt = \frac{xdx\rho_w L}{K\Delta\theta}$

$\Rightarrow \int_0^1 dt = \frac{\rho_w L}{K\Delta\theta} \int_0^1 x dx$

$\Rightarrow t = \frac{\rho_w L}{K\Delta\theta}\left[\frac{x^2}{2}\right]_0^1 = \frac{\rho_w L}{K\Delta\theta}$

Putting values

$t = \frac{1000 \times 3.36 \times 10^5 \times (10 \times 10^{-2})}{1.7 \times 10 \times 2}$

$= \frac{3.36}{2 \times 17} \times 10^{-2} \times 10^5 \times 10^3 \times 10$

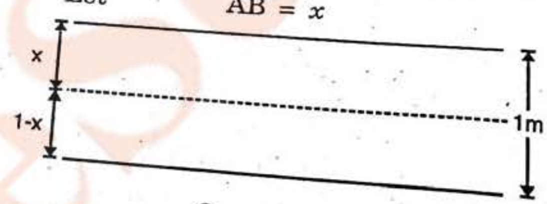
$= \frac{3.36}{2 \times 17} \times 10^7$ sec.

$= \frac{3.36 \times 10^7}{2 \times 17 \times 3600}$ hrs.

$= 27.45$ hr $= 27.5$ hrs.

28.15 Let B be the maximum level up to which ice is formed. Hence the heat conducted at that point from both the levels is same.

Let AB $= x$



i.e. $\frac{Q_{ice}}{t} = \frac{Q_{water}}{t}$

$\Rightarrow \frac{K_{ice} \times A \times 10}{x} = \frac{K_{water} \times A \times 10}{1 - x}$

$\Rightarrow \frac{1.7 \times 10}{x} = \frac{5 \times 10^{-1} \times 4}{1 - x}$

$\Rightarrow \frac{17}{x} = \frac{2}{1 - x}$

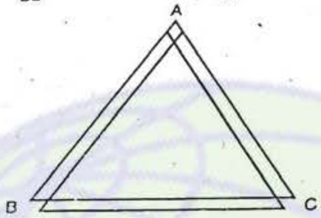
$$\Rightarrow \quad 17 - 17x = 2x$$

$$\Rightarrow \quad 19x = 17$$

$$\Rightarrow \quad x = \frac{17}{19} = 0.894 \text{ m} = 89 \text{ cm}$$

28.16 $K_{AB} = 50$ J/m-s-°C, $Q_A = 40°C$

$K_{BC} = 200$ J/m-s-°C, $Q_B = 80°C$



$K_{CA} = 400$ J/m-s-°C, $Q_C = 80°C$

Length = 20 cm = 20×10^{-2} m

$A = 1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$

(a) $$\frac{Q_{AB}}{t} = \frac{K_{AB}A(\theta_1 - \theta_2)}{l}$$

$$= \frac{50 \times 1 \times 10^{-4} \times 40}{20 \times 10^{-2}} = 1 \text{ W}$$

(b) $$\frac{Q_{AC}}{t} = \frac{K_{AC}A(\theta_A - \theta_C)}{l}$$

$$= \frac{400 \times 1 \times 10^{-4} \times 40}{20 \times 10^{-2}}$$

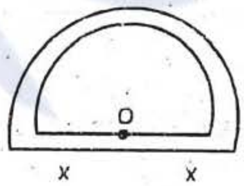
$$= 800 \times 10^{-2} = 8 \text{ W}$$

$$\frac{Q_{BC}}{t} = \frac{K_{BC}A(\theta_B - \theta_C)}{l}$$

$$= \frac{200 \times 1 \times 10^{-4} \times 0}{20 \times 10^{-2}}$$

28.17 We know $Q = \dfrac{kA(\theta_1 - \theta_2)}{d}$

$$Q_1 = \frac{kA(\theta_1 - \theta_2)}{d_1}$$



$$Q_2 = \frac{kA(\theta_1 - \theta_2)}{d_2}$$

$$\frac{Q_1}{Q_2} = \frac{\dfrac{KA(\theta_1 - \theta_2)}{\pi r}}{\dfrac{KA(\theta_1 - \theta_2)}{2r}} = \frac{2r}{\pi r} = \frac{2}{\pi}$$

where $d_1 = \pi r$, $d_2 = 2r$

28.18 The rate of heat flow per sec

$$\frac{dQ_A}{dt} = KA\frac{d\theta_A}{dl}$$

The rate of heat flow per sec

$$\frac{d\theta_B}{dt} = KA\frac{d\theta_B}{dl}$$

The part of heat is absorbed by the rod.

$$\frac{Q}{t} = \frac{ms\Delta\theta}{dt}$$

where $\dfrac{d\theta}{dt}$ = Rate of temperature variation

$$\Rightarrow \quad \frac{ms\Delta\theta}{dt} = KA\frac{d\theta_A}{dl} - KA\frac{d\theta_B}{dl}$$

$$\Rightarrow \quad ms\frac{d\theta}{dt} = KA\left(\frac{d\theta_A}{dl} - \frac{d\theta_B}{dl}\right)$$

$$\Rightarrow \quad 0.4\frac{d\theta}{dt} = 200 \times 1 \times 10^{-4}$$

$$(5 - 2.5)°\text{C/cm}$$

$$\Rightarrow \quad \frac{d\theta}{dt} = \frac{200 \times 2.5 \times 10^{-4}}{0.4 \times 10^{-2}} \text{ °C/m}$$

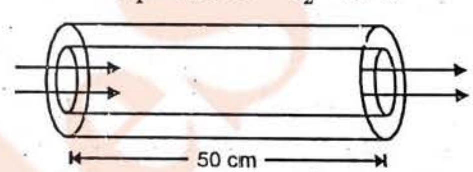
$$= \frac{200 \times 2.5 \times 10^{-4}}{0.4 \times 10^{-2}}$$

$$= 1250 \times 10^{-2} = 12.50 \text{ C/s.}$$

28.19 Given

$K_{rubber} = 0.15$ J/m-s-°C

$T_1 = 120°C \quad T_2 = 30°C$



We know for radial conduction in a cylinder

$$\frac{Q}{t} = \frac{2\pi Kl(T_2 - T_1)}{\ln(R_2/R_1)}$$

$$= \frac{2 \times 3.14 \times 15 \times 10^{-2} \times 50 \times 10^{-1} \times 90}{\ln(1.2/1)}$$

$$= 232.5 = 233 \text{ J/s}$$

28.20 $\frac{dQ}{dt}$ = Rate of flow of heat

Let us consider a strip at a distance r from the centre of thickness dr.

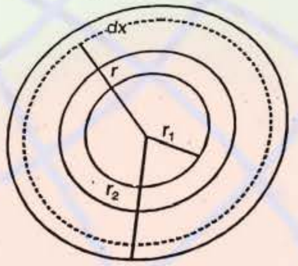
$$\frac{Q}{dt} = \frac{K \times 2\pi \times rd \times d\theta}{dr}$$

[$d\theta$ = temperature difference across the thickness dr]

$$C = \frac{K \times 2\pi \times rd \times d\theta}{dr} \quad \left[c = \frac{\theta}{t}\right]$$

$$C \times \frac{dr}{r} = K\, 2\pi\, d\, d\theta$$

Integrating



$$\Rightarrow C\int_{r_1}^{r_2} \frac{dr}{r} = K\, 2\pi\, d \int_{\theta_2}^{\theta_1} d\theta$$

$$\Rightarrow C[\log r]_{r_1}^{r_2} = K\, 2\pi\, d\, (\theta_1 - \theta_2)$$

$$C(\log r_2 - \log r_1) = K\, 2\pi\, d\, (\theta_1 - \theta_2)$$

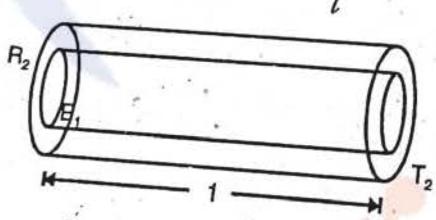
$$\Rightarrow \quad C = \frac{K\, 2\pi\, d\, (\theta_1 - \theta_2)}{\log (r_2 / r_1)}$$

28.21

$$T_2 > T_1$$
$$A = \pi (R_2^2 - R_1^2)$$

So, $$\frac{dQ}{dt} = \frac{KA(T_2 - T_1)}{l}$$

$$= \frac{K\pi (R_2^2 - R_1^2)(T_2 - T_1)}{l}$$



Considering a concentric cylindrical cell of radius R and thickness dR. The radial heat flow through the cell.

$$H = \frac{dQ}{dt} = -KA\frac{d\theta}{dr}$$

[– ve because as r increases Q decreases]

$$A = 2\pi rl,$$

$$H = -2\pi rl\, K\frac{d\theta}{dr}$$

or, $$\int_{r_1}^{r_2}\frac{dr}{r} = -\frac{2\pi l K}{H}\int_{T_1}^{T_2} d\theta$$

Integrating and simplifying we get

$$H = \frac{dQ}{dt} = \frac{2\pi Kl(T_2 - T_1)}{\ln(R_2/R_1)}$$

28.22 Here the thermal conductivity are in series,

$$\therefore \quad \frac{\frac{K_1A(\theta_1 - \theta_2)}{l_2} \times \frac{K_2A(\theta_1 - \theta_2)}{l_1}}{\frac{K_1A(\theta_1 - \theta_2)}{l_1} + \frac{K_2A(\theta_1 - \theta_2)}{l_2}} = \frac{KA(\theta_1 - \theta_2)}{l_1 + l_2}$$

$$\Rightarrow \quad \frac{\frac{K_1}{l_1} \times \frac{K_2}{l_2}}{\frac{K_1}{l_1} + \frac{K_2}{l_2}} = \frac{K}{l_1 + l_2}$$

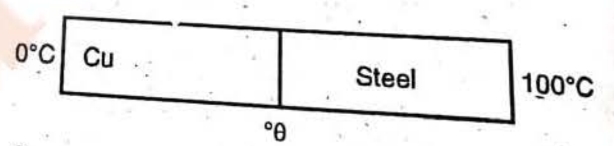
$$\Rightarrow \quad \frac{K_1 K_2}{K_1 l_2 + K_2 l_1} = \frac{K}{l_1 + l_2}$$

$$K = \frac{K_1K_2(l_1 + l^2)}{K_1l_2 + K_2 + l_1}$$

28.23

$$K_{Cu} = 390\ W/m\text{-}^\circ C$$
$$K_{st} = 46\ W/m\text{-}^\circ C$$

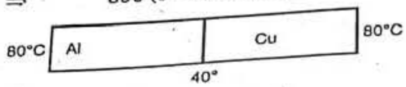
Now since they are in series connection, the heat passed through the cross section is the same



So, $$Q_1 = Q_2$$

or $$\frac{K_{cu} \times A \times (\theta - 0)}{l} = \frac{K_{st} \times A \times (100 - \theta)}{l}$$

$$\Rightarrow \quad 390(\theta - 0) = 46 \times 100 - 460$$



$$\Rightarrow \quad 436\theta = 4600$$

$$\theta = \frac{4600}{436}$$

$$= 10.55 = 10.6°C.$$

28.24 As the aluminium rod and copper rod joined are in parallel

$$\frac{Q}{t} = \left(\frac{Q}{t}\right)_{Al} + \left(\frac{Q}{t}\right)_{Cu}$$

$$\Rightarrow \frac{KA(\theta_1 - \theta_2)}{l}$$

$$= \frac{K_1 A(\theta_1 - \theta_2)}{l} + \frac{K_2 A(\theta_1 - \theta_2)}{l}$$

$$\Rightarrow \quad K = K_1 + K_2 = (390 + 200)$$

$$= 590$$

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l}$$

$$= \frac{590 \times 1 \times 10^{-4} \times (60 - 20)}{1}$$

$$= 590 \times 1 \times 10^{-4} \times 40$$

$$= 59 \times 4 \times 10^{-2} = 2.36 \text{ watt.}$$

28.25

$$K_{Al} = 200 \text{ W/m°C},$$

$$K_{Cu} = 400 \text{ W/m-°C},$$

$$A = 0.2 \text{ cm}^2 = 2 \times 10^{-5} \text{ m}^2$$

$$l = 20 \text{ cm} = 2 \times 10^{-1} \text{ m}$$

Heat drawn per sec.

$$Q_{Al} + Q_{Cu} = \frac{K_{Al} \times A \times (80 - 40)}{l} + \frac{K_{Cu} \times A \times (80 - 40)}{l}$$

$$= \frac{2 \times 10^{-5} \times 40}{2 \times 10^{-1}} [200 + 400]$$

$$= 2.4 \text{ J}$$

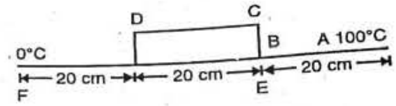
Heat drawn per minute = 2.4 × 60 = 144 J

28.26

$$\left(\frac{Q}{t}\right)_{AB} = \left(\frac{Q}{t}\right)_{Bent} + \left(\frac{Q}{t}\right)_{Straight}$$

$$\left(\frac{Q}{t}\right)_{Bent} = \frac{KA(\theta_1 - \theta_2)}{70}$$

$$\left(\frac{Q}{t}\right)_{Straight} = \frac{KA(\theta_1 - \theta_2)}{60}$$



$$\frac{\left(\frac{Q}{t}\right)_{Bent}}{\left(\frac{Q}{t}\right)_{Straight}} = \frac{60}{70} = \frac{6}{7}$$

$$\Rightarrow \quad \left(\frac{Q}{t}\right)_{Bent} + \left(\frac{Q}{t}\right)_{Straight} = 130$$

$$\left(\frac{Q}{t}\right)_{Bent} + \left(\frac{Q}{t}\right)_{Bent} \times \frac{7}{6} = 130$$

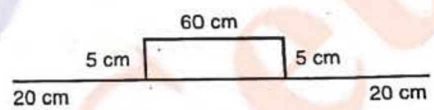
$$\Rightarrow \quad \left(\frac{7}{6} + 1\right)\left(\frac{Q}{t}\right)_{Bent} = 130$$

$$\Rightarrow \quad \frac{13}{6} \times \left(\frac{Q}{t}\right)_{Bent} = 130$$

$$\Rightarrow \quad \left(\frac{Q}{t}\right)_{Bent} = \frac{130 \times 6}{13} = 60 \text{ J/s.}$$

28.27

$$\left(\frac{Q}{t}\right)_{Bent} = \frac{780 \times A \times 100}{70}$$



$$\left(\frac{Q}{t}\right)_{Straight} = \frac{390 \times A \times 100}{70}$$

$$\frac{\left(\frac{Q}{t}\right)_{Bent}}{\left(\frac{Q}{t}\right)_{Straight}} = \frac{780 \times A \times 100}{70} \times \frac{60}{390 \times A \times 100} = \frac{12}{7}$$

28.28 *(a)*

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{1}$$

$$= \frac{1 \times 2 \times (40 - 32)}{2 \times 10^{-3}}$$

$$= 8000 \text{ J/sec.}$$

(b) Resistance of glass $= \frac{1}{ak_g} + \frac{1}{ak_g}$

Resistance of air $= \frac{1}{ak_a}$

Net resistance $= \frac{1}{ak_g} + \frac{1}{ak_a} + \frac{1}{ak_g}$

$$= \frac{1}{a}\left(\frac{2}{k_g} + \frac{1}{k_a}\right)$$

$$= \frac{1}{a}\left(\frac{2k_a + kg}{k_g k_a}\right)$$

$$= \frac{1 \times 10^{-3}}{2}\left(\frac{2 \times 0.025 + 1}{0.025}\right)$$

$$= \frac{1 \times 10^{-3} \times 1.05}{0.05}$$

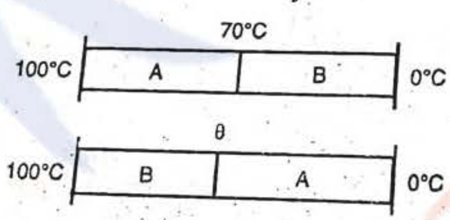
$$\frac{Q}{t} = \frac{(\theta_1 - \theta_2)}{R}$$

$$= 385.9 = 381 \text{ W.}$$

28.29 Now, $\frac{Q}{t}$ remains same in both cases

In case I, $\frac{K_A \times A \times (100 - 70)}{l}$

$$= \frac{K_B \times A \times (70 - 0)}{l}$$



$\Rightarrow \quad 30\,K_A = 70\,K_B$

In case II, $\frac{K_B \times A \times (100 - \theta)}{l}$

$$= \frac{K_A \times A \times (\theta - 0)}{l}$$

$$\Rightarrow 100\,K_B - K_B\,\theta = K_A\theta$$

$$\Rightarrow 100\,K_B - K_B\,\theta = \frac{70}{30}\,K_B \times \theta$$

$$\Rightarrow \quad 100 = \frac{7}{3}\,\theta + \theta$$

$$\Rightarrow \quad 100 = \frac{10}{3}\theta$$

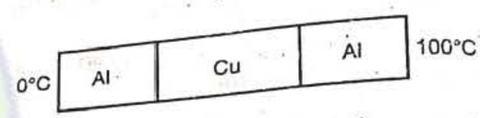
$$\Rightarrow \quad \theta = \frac{300}{10} = 30°C.$$

28.30 $\theta_1 = \theta_2 = 100$

$$\frac{Q}{t} = \frac{Ka\,(\theta_1 - \theta_2)}{t}$$

$$R = R_1 + R_2 + R_3$$

$$= \frac{1}{ak_{Al}} + \frac{1}{ak_{cu}} + \frac{1}{ak_{Al}}$$



$$= \frac{1}{a}\left(\frac{2}{200} + \frac{1}{400}\right)$$

$$= \frac{1}{a}\left(\frac{4+1}{400}\right) = \frac{1}{a}\frac{5}{400}$$

$$= \frac{1}{a}\frac{1}{80}$$

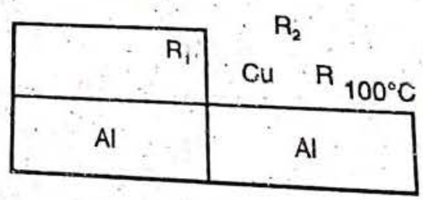
$$\frac{Q}{t} = \frac{100}{\frac{1}{a}\frac{1}{80}} \quad \left[\because \frac{Q}{t} = \frac{\theta_1 - \theta_2}{R}\right]$$

$$40 = 80 \times 100 \times \frac{a}{l}$$

[From question $\frac{Q}{t} = 40$W]

$$\frac{a}{l} = \frac{1}{200}$$

For, B $\quad R = R_1 + R_2 + R_3$



$$= R_{Al} + \frac{R_{Cu}\,R_{Al}}{R_{Cu} + R_{Al}}$$

RAJ PUBLICATIONS

Rs. 162.00