Fundamentals of
QUANTITATIVE ABILITY

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Contents

Number System and Algebra

Introduction:

Let us go back to our school days. This chapter is designed to give you a quick recap of what you had learnt during the school. A quick brush-up of the concepts shall help you in identifying and solving the problems at a greater pace.

Real Numbers:

Numbers which can be commonly seen and identified and can be represented on a number line.

e.g. $-14, 2.77, 0, 1, 7$

Number Line:

It is a line on which all the positive and negative numbers can be marked in a sequence.

Imaginary numbers:

Those numbers that CANNOT be represented on a number line are imaginary numbers.

e.g. $\sqrt{-1}, \sqrt{-5}$, etc.

$\sqrt{-1}$ is represented by i.

Rational Numbers:

All numbers that can be expressed in $\frac{p}{q}$ form, where p, q are integers and $q \neq 0$.

e.g. $\frac{-5}{7}, \frac{2}{3}$

Irrational Numbers:

Those numbers that CANNOT be expressed in $\frac{p}{q}$ form.

e.g. $\pi, \sqrt{2}, \sqrt{3}-1$

Fractions:

All rational numbers which are in $\frac{p}{q}$ form, where p, q are integers and p is not a multiple of q.
p is called numerator whereas q is known as denominator.

- Fractions are of the following types.
- Proper: $p < q$ e.g. $\frac{2}{7}, \frac{3}{8}$ etc.
- Improper: $p \geq q$ e.g. $\frac{6}{5}, \frac{5}{2}$ etc.
- Mixed: It is an integer plus a fraction e.g. $3\frac{1}{5}, 7\frac{1}{3}$, etc.

Integers:

All the rational numbers that do not have any decimal or fractional part.

$-\infty, \ldots, -3, -2, -1, 0, +1, +2, +3, \ldots, +\infty$

Whole Numbers:

All non negative integers are whole numbers.
W = {0, 1, 2, 3, ...}

Natural Numbers:

Whole numbers, except zero, are called natural numbers.
N = {1, 2, 3, 4, ...}

Odd Numbers:

All natural numbers which are not divisible by 2 are odd numbers.
Such numbers are expressed as $2k \pm 1$ (k is any natural number).

e.g.: 1, 3, 5, 7, ...

Even Numbers:

All natural numbers that are divisible by 2 are called even numbers.
Such numbers are expressed as 2k (k is any natural number).

e.g.: 2, 4, 6, 8, ...

The box given below exhibits the types of numbers obtained while carrying out arithmetic operations between two type of numbers.

odd ± odd = even
odd ± even = odd
even ± even = even
odd × odd = odd
odd × even = even
even × even = even

Prime Numbers:

All the natural numbers that are greater than 1, and are only divisible by 1 or the number itself, are called prime numbers.
The box given below depicts some of the characteristics of the prime numbers.

* There are 25 prime numbers upto 100
  2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.
* 2 is the only even prime number.
* 97 is the only prime number from 90 to 100.
* 91 is often mistaken as a prime, but it is not a prime
  $\because\ 91 = 7 \times 13$.

Co-primes:

Two numbers 'a' and 'b' are said to be co-prim they don't have any common factor other than 1

e.g.: (3, 5), (7, 12), etc.

Composite Numbers:

Numbers greater than 1 that are not prime are ca composite numbers.

e.g.: 4, 6, 8, 9, ...

* 1 is neither a prime number nor a composite number
  Reason : A prime number has two factors, 1 and the number itself; whereas a composite number has more than two factors. Since, 1 has only one factor i.e., 1, hence it is neither a prime nor a composite numbers.

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No.	Real	Imaginary	Rational	Irrational	Even	Odd	Prime	Composite	Whole	Natural	Integer	Frac
3	Y	N	Y	N	N	Y	Y	N	Y	Y	Y	[illegible]
$\sqrt{3}$	Y	N	N	Y	N	N	N	N	N	N	N	[illegible]
$\frac{7}{2}$	Y	N	Y	N	N	N	N	N	N	N	N	[illegible]
i	N	Y	N	N	N	N	N	N	N	N	N	[illegible]

Table 1: Classification of numbers

erfect Numbers:

number 'a' is said to be perfect if the sum of its ctors (excluding itself but including 1) is qual to 'a'.

g. 6, 28, etc.

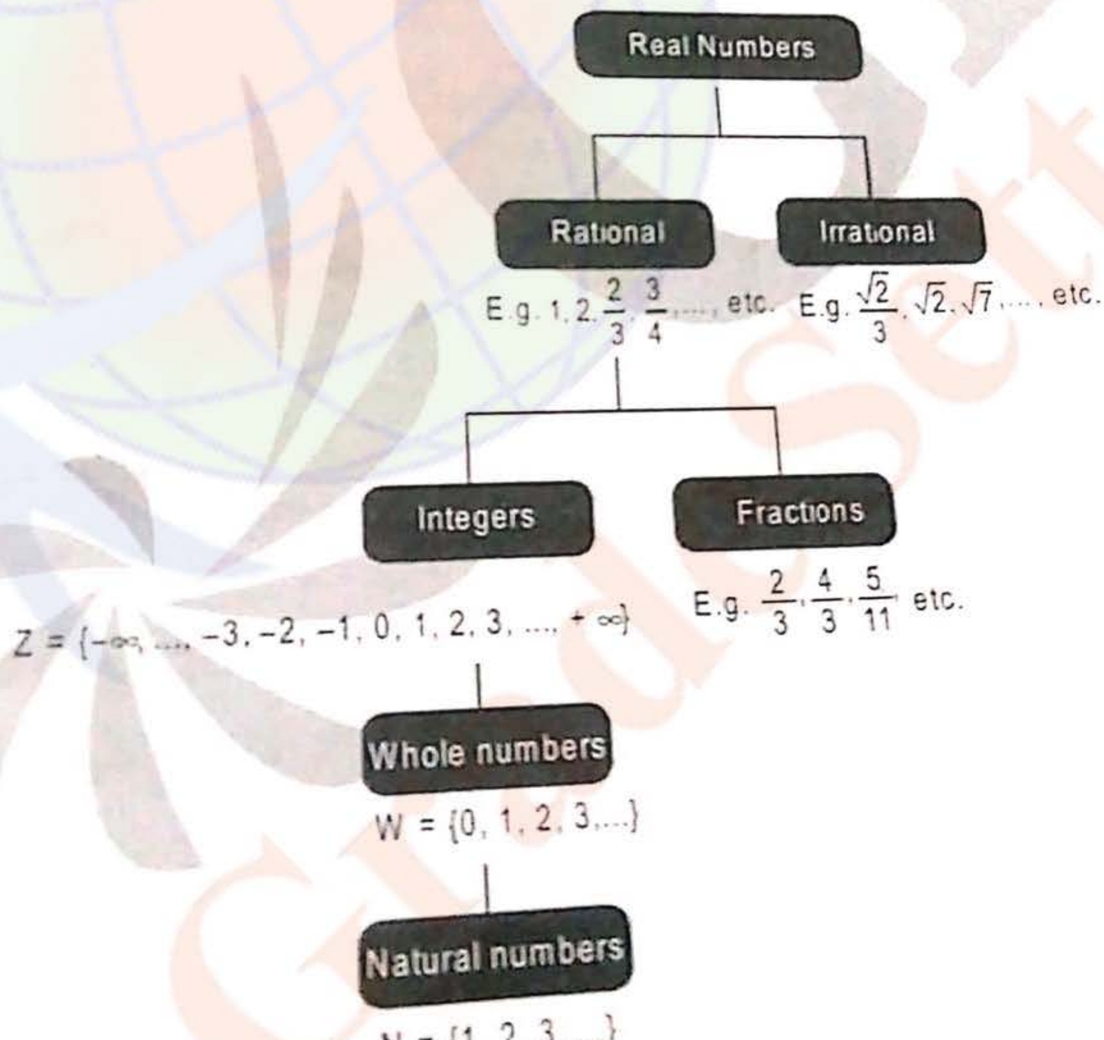
e perfect number 6 has 1, 2 and 3 as its factors, hich sum up to 6.
milarly, the perfect number 28 has 1, 2, 4, 7 and 4 as it's factors, which sum up to 28.

Try Yourself:

Classify the given numbers in all the types we just studied (real, integer, rational, prime, even, odd, natural, whole, etc).

23, $\frac{-7}{9}$, $\sqrt{31}$, $\sqrt{4}$, $\frac{\sqrt{8}}{8}$, 11, −7, $\frac{-7}{17}$, $\frac{-7}{12}$, $\frac{2}{8}$, $\frac{8}{2}$, 1, 8, 18, 29, $\frac{31}{15}$, $\frac{15}{10}$, 2.76, 3.33, 10.82, 5, 11, 19, 91, 27, −8, −8.3, $\frac{-3}{6}$, $\frac{-6}{3}$, 12.5

The Number Tree



Conversion of recurring decimal into fractions:

What is the $\frac{p}{q}$ form of 0.5555... (also represented as $0.\overline{5}$)?

Let, $x = 0.55555...$

$\Rightarrow 10x = 5.55555...$

$\therefore 9x = 10x - x = (5.5555...) - (0.55555...) = 5$

$\Rightarrow x = \frac{5}{9}$

If $x = 0.232323...$

$\Rightarrow 100x = 23.232323...$

$\therefore 99x = 100x - x = (23.232323...) - (0.232323...)$
$= 23$

$\Rightarrow x = \frac{23}{99}$

For a purely recurring number (all digits after decimal point recur) we can identify the procedure as:

The $\frac{p}{q}$ form of a purely recurring number

$$= \frac{\text{The recurring part written once}}{\text{As many 9's as the number of digits in the recurring part}}$$

In the number like 0.14333... i.e. $0.14\overline{3}$

Let $x = 0.143333...$

$\Rightarrow 100x = 14.3333...$

$\Rightarrow 1000x = 143.3333...$

$\Rightarrow 900x = 1000x - 100x$

$= (143.3333...) - (14.3333...) = 129$

$\Rightarrow x = \frac{129}{900}$

Thus for any recurring number we can identify the procedure as:

The $\frac{p}{q}$ form of any recurring number

$$= \frac{\text{(The non-recurring and recurring part written ...)} - \text{(the non-recurring part)}}{\text{As many 9's as the number of digits in the re... followed by as many 0's as digits in non-recurr...}}$$

Example 1:

Express $0.\overline{643}$ as a fraction.

Solution:

Let $x = 0.\overline{643}$, then

$1000x = 643.\overline{643}$

$\therefore 1000x - x = 643.\overline{643} - 0.\overline{643}$

$\Rightarrow 999x = 643$

$\Rightarrow x = \frac{643}{999}$

Example 2:

Express $6.\overline{43}$ as a fraction.

Solution:

Let $x = 6.\overline{43}$, then

$100x = 643.\overline{43}$

$\therefore 100x - x = 643.\overline{43} - 6.\overline{43}$

$\Rightarrow 99x = 637$

$\Rightarrow x = \frac{637}{99}$

Example 3:

Arrange the following rational numbers in ascend... order:

$\frac{-7}{10}, \frac{5}{-8}, \frac{2}{-3}$

Solution:

$\frac{-7}{10} = -0.7, \frac{5}{-8} = -0.625$ and $\frac{2}{-3} = -0.666$

Clearly, $-0.7 < -0.666 < -0.625$.

So, $\frac{-7}{10} < \frac{2}{-3} < \frac{5}{-8}$

Example 4:

Arrange the following fractions in descending order

$\frac{31}{27}, \frac{43}{39}, \frac{57}{53}, \frac{27}{23}$ and $\frac{29}{25}$

Solution:

As the difference between the numerator and the denominator is same, the fraction with smallest denominator, i.e., $\frac{27}{23}$ is largest and $\frac{57}{53}$ is smallest.

Hence, the order is:

$$\frac{27}{23} > \frac{29}{25} > \frac{31}{27} > \frac{43}{39} > \frac{57}{53}$$

Example 5:

What is the divisor if dividend is 15968, quotient is 89 and the remainder is 37?

Solution:

$$\text{Divisor} = \left(\frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}\right)$$

$$= \left(\frac{15968 - 37}{89}\right) = 179$$

Divisibility Rules:

(1) A number is divisible by 2 when its unit's digit is even or 0.

(2) A number is divisible by 3 when the sum of its digits is divisible by 3.

(3) A number is divisible by 4 when the number formed by the last two digits is divisible by 4 or the last two digits are 0.

(4) A number is divisible by 5 when its unit's digit is 5 or 0.

(5) A number is divisible by 6 when it is divisible by both 2 and 3.

(6) A number is divisible by 8 when the number formed by the last three right-hand digits is divisible by 8, or when the last three digits are 0.

(7) A number is divisible by 9 when the sum of its digits is divisible by 9.

(8) A number is divisible by 10 when its unit's digit is 0

(9) A number is divisible by 11 when the difference between the sum of the digits in the odd and the even places is 0 or a multiple of 11. For example, if a number is abcd then [(a + c) – (b + d)] = k, where k = 0 or k is the multiple of 11

(10) A number is divisible by 12 when it is divisible by both 3 and 4

Example 6:

What least number must be subtracted from 2000 to get a number which is exactly divisible by 17?

Solution:

On dividing 2000 by 17, we get 11 as remainder.

∴ Required number to be subtracted = 11.

Example 7:

What least number must be added to 3000 to obtain a number exactly divisible by 19?

Solution:

On dividing 3000 by 19, we get 17 as remainder

∴ Number to be added = (19 – 17) = 2.

Example 8:

Find the number which is nearest to 3105 and exactly divisible by 21.

Solution:

On dividing 3105 by 21, we get 18 as remainder.

∴ Number to be added to 3105 is (21 – 18) = 3.

∴3108 is the required number.

Example 9:

A number when divided by 342 gives a remainder 47. When the same number is divided by 19, what would be the remainder?

Solution:

On dividing the given number by 342, let k be the quotient and 47 the remainder.

Then, number = 342k + 47

= [(19 × 18k) + (19 × 2 + 9)] = [19 (18k + 2) + 9]

∴ The given number when divided by 19 gives (18k + 2) as quotient and 9 as remainder.

Alternate method:
342 is a multiple of 19, divide the remainder by the second dividend to get the remainder. 47 when divided by 19 gives 9 as remainder.

How to find whether a number is prime or not?

For small numbers, we could find by checking, if that number is divisible by any other prime number till that number itself.

But for the larger numbers like, say 631, there is an alternate method.

Step 1: Find the approximate square root of the given number, i.e. 25.

Step 2: Check if any prime number from 2 to 25 divides 631.

The prime numbers from 2 to 25 are 2, 3, 5, 7, 11, 13, 17, 19 and 23. Since none of these numbers divide 631 exactly, 631 must be a prime number.

Factorial

The continued product of first n natural number is called 'n factorial' and is denoted by n! or $\lfloor n$.

$n! = 1 \times 2 \times 3 \times \ldots \times (n-1) \times n$

e.g.: $6! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$
By definition $0! = 1$.

Highest Common Factor (HCF) and Lowest Common Multiple (LCM):

HCF and LCM are one of the basic concepts of mathematics which is having a variety of applications in our daily life.

To understand this topic let us first look at certain terms:

Factor:

Factors of a number are those numbers which when divide the original number, leave no remainder. When talking about the factor we consider only the positive integral factor.

For example,
Factors of 20 = 20, 10, 5, 2, 1
Factors of 100 = 100, 50, 25, 20, 10, 5, 2, 1

Factors of a number are always countable.

Multiple:

Multiples of a number are those numbers when divided by the number leaves no remai... When talking about the multiples we consider the positive integral multiples.

For example,
Multiples of 20 = 20, 40, 60, 80, etc.
Multiples of 100 = 100, 200, 300, 400, etc.

Understanding HCF:
Let us take two numbers 15 and 20
Factors of 15 are = 15, 5, 3, 1
Factors of 20 are = 20, 10, 5, 1

To find the HCF, check what is the highest fa... common to both the numbers. We can find it a...

Understanding LCM:
Let us take two numbers 15 and 20.
Multiples of 15 = 15, 30, 45, 60, 75, 90, 105, 1... 135, etc.
Multiples of 20 = 20, 40, 60, 80, 100, 120, 140, ...

To find the LCM of these two numbers, check w... is the lowest number common to the set... multiples of both the numbers: We can find it as...

How to find HCF of two numbers?

There are two methods:

a. Division method
b. Prime factorisation method.

a. Division method:

In this method divisor becomes dividend... remainder becomes
divisor and this process continues till... can divide.
The last divisor is your answer.
Now, try to understand the following illust... examples.

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Example 10:
To find the HCF of 15 and 20.

```
15) 20 (1
   -15
    5) 15 (3
      -15
        0
```

So, HCF of 15 and 20 is 5.

Example 11:
To find the HCF of 20 and 28.

```
20) 28 (1
   -20
    8) 20 (2
      -16
       4) 8 (2
         -8
          0
```

So, the HCF of 20 and 28 is 4.

Example 12:
To find the HCF of 20, 28 and 45
We have seen that HCF of 20 and 28 is 4.
So, we will take HCF of 4 and 45.

```
4) 45 (11
  -44
   1) 4 (4
     -4
      0
```

So, HCF of 20, 28 and 45 is 1.

Note: The HCF of an odd number and an even number is always 1.

Prime factorization method:

Write the number in terms of prime factors.
$20 = 2^2 \times 5^1$
$45 = 2^0 \times 3^2 \times 5^1$
For finding out their HCF, take the lowest power of all prime numbers. The HCF of 20 and 45 is $2^0 \times 3^0 \times 5^1$ i.e. 5.

How to find LCM of two or more numbers?

There are two methods

Division method
Prime factorisation method

i. **Division method:**

LCM of 18, 27 and 30.

3	18,	27,	30
3	6,	9,	10
	2,	3,	5

LCM = 3 × 3 × 3 × 2 × 5 = 270

ii. **Prime factorisation method:**

Take two numbers 20 and 45.
Write the numbers in terms of prime factors.
$20 = 2^2 \times 5^1$
$45 = 2^0 \times 3^2 \times 5^1$

For finding out their LCM, take the highest power of all prime numbers.
The LCM of 20 and 45 is $2^2 \times 3^2 \times 5^1$ i.e. 180.

Example 13:
Find the HCF of 24 and 72.

Solution:
24 = 2 × 2 × 2 × 3
72 = 2 × 2 × 2 ×3 × 3
HCF = 2 × 2 × 2 × 3 = 24
Similarly, you can find the HCF of sets containir more than 2 numbers.

Example 14:
Find the largest number that can exactly divide 5 783 and 1107.

Solution:
Required number = HCF of 513, 783 and 110
Now, $513 = 3^3 \times 19$, $783 = 3^3 \times 29$, 1107 = 3
$\therefore$ HCF = 3^3 = 27.
Hence, the required number is 27.

Example 15:
Find the least number exactly divisible by 1 and 27.

Solution:
Required number = LCM of 12, 15, 20, 2
$\therefore$LCM = 3 × 4 × 5 × 9 = 540

Example 16:
Find the least number which when divided by 6, 7, 8, 9 and 12 leaves the same remainder 1 in each case.

Solution:
Required number = (LCM of 6, 7, 8, 9, 12) + 1
$\therefore$ LCM = 3 × 2 × 2 × 7 × 2 × 3 = 504
Hence, required number = (504 + 1) = 505

Example 17:
The traffic lights at three different road-crossings, change after every 24 sec, 72 sec and 120 sec respectively. If they all change simultaneously at 10 : 54 : 00 hr, then at what time will they change next simultaneously?

Solution:
Interval of change = LCM of (24, 72, 120) sec = 360 sec.

The lights will change simultaneously after every 360s i.e., 6 min 00 sec.

Next simultaneous change will take place at 11 : 00 : 00 hr.

Example 18:
How many three-digit numbers are divisible by 6?

Solution:
There are 16 numbers before 100 which are divisible by 6.
There are 166 numbers before 999 which are divisible by 6.
Total three-digit numbers divisible by 6 are 166 – 16 =150.

Important results:

If 2 numbers a and b are given, and their LCM and HCF are L and H respectively, then L × H = a × b.

LCM and HCF of fractions:

$$\text{LCM of fractions} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

$$\text{HCF of fractions} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

e.g.: Find the LCM and HCF of $\frac{25}{12}$ and $\frac{35}{18}$

$$\text{LCM} = \frac{\text{LCM of 25 and 35}}{\text{HCF of 12 and 18}} = \frac{175}{6}$$

$$\text{HCF} = \frac{\text{HCF of 25 and 35}}{\text{LCM of 12 and 18}} = \frac{5}{36}$$

Note: Do not directly apply the formula if the fractions are not in their simplest form.

Example 19:
The HCF of two numbers is 11 and their LCM is 693. If one of the numbers is 77, find the other.

Solution:

$$\text{The other number} = \frac{11 \times 693}{77} = 99$$

Decimal system:

A number 78324 can be represented as
7 × 10000 + 8 × 1000 + 3 × 100 + 2 × 10 + 4 = 78324
So, what could be the place value of 8 in the above number?
Since, it is in the thousand's place it can be represented as
8 × 1000 = 8000 which is the place value.

Note: A two-digit number xy can be represented as 10x + y and a three-digit number xyz can be represented as 100x + 10y + z.

Example 20:
The difference between a two-digit number and the number obtained by interchanging the digits is 27. What is the difference between the digits of the number?

Solution:
Let ten's digit be x and unit's digit be y.
Then (10x + y) – (10y + x) = 27
⇒ 9 (x – y) = 81
⇒ x – y = 9

[illegible] place digit of a number:

[illegible] digit at the unit's place of any number is the [illegible] when the number is divided by 10. [illegible] example, [illegible] the number [illegible]. The [illegible] when [illegible] is divided by 10 is [illegible] is the unit's digit of the number [illegible]. [illegible] find the unit's digit of a number which is the [illegible] of two or more numbers, multiply the unit's digit of the numbers and find the units digit of the [illegible] number. For example, [illegible], the product [illegible] the units digit of [illegible] and [illegible] is [illegible] and the unit's [illegible] of [illegible], hence the unit's digit of [illegible] is [illegible]

[illegible]'s digit of higher powers of any number

$2^1 = 2$	$2^2 = 4$	$2^3 = 8$	$2^4 = 16$
$2^5 = 32$	$2^6 = 64$	$2^7 = 128$	$2^8 = 256$
$2^9 = 512$	$2^{10} = 1024$	$2^{11} = 2048$	$2^{12} = 4096$

[illegible] can see that the unit's digit of 2^1, 2^5, 2^9 is 2, [illegible] digit of 2^2, 2^6, 2^{10} is 4, units digit of 2^3, 2^7, 2^{11} [illegible] and units digit of 2^4, 2^8, 2^{12} is 6

[illegible]erefore after every four powers of 2, the units digit [illegible] the number starts repeating. Thus we say that [illegible]clicity of unit's digit of higher powers of 2 is 4. [illegible]ilarly the digits whose cyclicity is 4 are 2, 3, 7 [illegible] 8. The digits whose cyclicity is 2 are 4 and 9. [illegible]ny power of numbers whose unit's digit 1, 5 or 6 [illegible]ways ends in 1, 5 and 6 respectively.

[illegible] example, $11^2 = 121$, $25^2 = 625$ and $16^2 = 256$.

[illegible]erfect square:

[illegible] number is said to be a perfect square if and only if [illegible] square root of that number is an integer.

[illegible]me important facts about perfect squares:

(i) The square of an even number is always even.

(ii) The square of an odd number is always odd.

(iii) Square of an integer cannot end in 2, 3, 7 or 8.

(iv) The square of a real number (negative or positive) is always positive.

[illegible]me important formulae used in simplification:

(i) $(a + b)^2 = a^2 + b^2 + 2ab$

(ii) $(a - b)^2 = a^2 + b^2 - 2ab$

(iii) $(a + b)^2 = (a - b)^2 + 4ab$

(iv) $a^2 - b^2 = (a - b)(a + b)$

(v) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(vi) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example 21:

Simplify $\frac{527 \times 527 \times 527 + 183 \times 183 \times 183}{527 \times 527 - 527 \times 183 + 183 \times 183}$

Solution:

The given expression is equivalent to

$$\frac{(527)^3 + (183)^3}{(527)^2 - 527 \times 183 + (183)^2}$$

We know that, $\frac{a^3 + b^3}{a^2 - ab + b^2} = a + b$

In the above example a = 527 and b = 183

∴ The expression is equal to (527 + 183) = 710

Example 22:

Simplify $\left(\frac{(614 + 168)^2 - (614 - 168)^2}{614 \times 168} \right)$

Solution:

Let a = 614 and b = 168, then the expression becomes $\frac{(a + b)^2 - (a - b)^2}{ab} = \frac{4ab}{ab} = 4$

Example 23:

Find the square of 1605.

Solution:

$(1605)^2 = (1600 + 5)^2$

$= (1600)^2 + 2 \times 1600 \times 5 + (5)^2$

$= 2560000 + 16000 + 25 = 2576025$

Example 24:

Find the value of 896 × 896 – 204 × 204

Solution:

$a^2 - b^2 = (a + b)(a - b)$

(where a = 896 and b = 204)

$= (896 + 204)(896 - 204) = 1100 \times$ [illegible]

Example 25:
Evaluate: $(57)^2 + (43)^2 + 2 \times 57 \times 43$

Solution:
$a^2 + b^2 + 2ab = (a + b)^2 = (57 + 43)^2 = 100^2 = 10000$

Example 26:
Simplify $(81)^2 + (68)^2 - 2 \times 81 \times 68$

Solution:
$(81 - 68)^2 = 13^2 = 169$

Example 27:
Evaluate: $(313 \times 313 + 287 \times 287)$

Solution:
$a^2 + b^2 = \frac{1}{2}[(a+b)^2 + (a-b)^2]$
(where a = 313 and b = 287)

$$= \frac{1}{2}[(313+287)^2 + (313-287)^2]$$

$$= \frac{1}{2}\left[(600)^2 + (26)^2\right] = 180338$$

Rules of counting numbers:

1. Sum of first n natural numbers = $\frac{n(n+1)}{2}$
2. Sum of first n odd numbers = n^2
3. Sum of first n even numbers = n (n + 1)
4. Sum of the squares of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$
5. Sum of the cubes of first n natural numbers $= \left[\frac{n(n+1)}{2}\right]^2$

Example 28:
If square root of 15 = 3.88, the value of squ... $\frac{5}{3}$ is

Solution:
$$\sqrt{\frac{5}{3}} = \sqrt{\frac{5 \times 3}{3 \times 3}} = \frac{\sqrt{15}}{3} = \frac{3.88}{3} = 1.29$$

Example 29:
A four-digit number divisible by 7 becomes ... by 3, when 10 is added to it. Find the larg... number.

Solution:
Largest four-digit number is 9999.
On dividing 9999 by 7, we get 3 as remainder.
Largest four-digit number divisible by 7 is 9996.
Let 9996 – x + 10 be divisible by 3.
By trial and error, we find that x = 7
Required number = (9996 – 7) = 9989.

Example 30:
A three-digit number 4a3 is added to another ... digit number 984 to give the four-digit number ... which is divisible by 11. Find the value of (a+...

Solution:

```
  4 a 3
 +9 8 4
 ------
 13 b 7
```

Here a + 8 = b, if 13b7 is divisible by 11 then ... – (b + 1) = 0; b = 9 and a + 8 = b or a = 1. He... + b = 9 + 1 = 10

Example 31:
Of the three numbers, the sum of the first tw... the sum of the second and the third is 55, a... sum of the third and thrice the first is 90. Fi... third number.

Solution:
Let the numbers be x, y and z. Then, x + ...
y + z = 55 and
3x + z = 90.

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... = 45 − x and ...

... = 20 ... = (45 − x) = 10 + x

... = (45 − 20) = 25 and z = (10 + 20) = 30

Third number = 30

Polynomials:

The word 'poly' means **many** and the word 'nomial' means **terms.** So, polynomial is an algebraic expression which consists of many terms involving powers of the variable.

For example, $5x - 5$, $x^2 + 5x + 6$, $y^3 + y^2 + 2z$

The general form of a polynomial is $a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_{n-1}x^{n-1} + a_nx^n$, where $a_0, a_1, a_2, \ldots a_n$ are real numbers and n is a non-negative integer.

A polynomial may be in more than one variable.

e.g., $x^2 + y^2 - 4$, $2x^2 + y^2 + z$.

Types of polynomials:

Polynomials can be classified as follows:

1. **By coefficients:** The basis of classification here is nature of the coefficients.

   a. In $5x^2 + 3x + 6$, the coefficients are 5, 3 and 6 and all are integers. So, this is called polynomial over the set of integers.

   b. In $\frac{2}{3}x^2 - \frac{4}{5}x + 2x - 3$, the coefficients are $\frac{2}{3}$, $-\frac{4}{5}$, 2 and −3, and all are rational numbers. So, this is called polynomial over the rational numbers.

   c. In $\sqrt{3}x^2 - 11x + \sqrt{3}$, the coefficients are real numbers. So, this is called polynomial over the real numbers.

2. **By the number of terms:** The basis of classification here is number of the coefficients.

   a. **Monomial:** consisting of a single term.

   For example, $\sqrt{5}x$, $-8y^2$, $15mn^2$.

   b. **Binomials:** consisting of two terms. For example, $2x + 3y$, $x - 4$, $x^2 + 3y$ [illegible]

   c. **Trinomials:** consisting of three terms. For example, $5x^2 - 3x + 2$, $x + y - 2$, [illegible]

   d. **Polynomials:** consisting of more than three terms.

   e.g. $x + y + z + m$, [illegible]

   $a + 2b + 2c + 3d + 4e$

3. **By degree:** The highest exponent of any monomial of the given polynomial is called the degree of the polynomial. For example $x^5 - 3x^4 + 12$. In this example the highest exponent of a term is 5. So the polynomial is of degree 5.

   $5y^2$ is a monomial of degree 2

   $4x^2y^3z^2$ is a monomial of degree $2 + 3 + 2 = 7$

   $\sqrt{5}x^3y^3$ is a monomial of degree $3 + 3 = 6$

   5 is a monomial of degree 0 (because 5 can be written as $5x^0$).

   $-\frac{4}{15}mn^3$ is a monomial of degree $3 + 1 = 4$.

   $y^7 - 5x^2 + 3y^2 = 8$ is a **polynomial** of degree 7.

Linear polynomial:

A polynomial of degree one is called linear polynomial.

For example, $5x + 3y$, $5x - 4$, $\frac{1}{3}x$, $x - \frac{5}{2}$ are linear polynomials.

Linear equation:

In the earlier topic we have learnt about linear polynomials. An equation consisting of only linear polynomials is called a linear equation.

For example, $3x - 2 = 7x$; $\frac{1}{2}y - 1 = 3y - \frac{2}{3}$ are linear equations in one variable, and $3x + 4y = 20$; $5x + 20 = 3y + 5$ are linear equations in two variables.