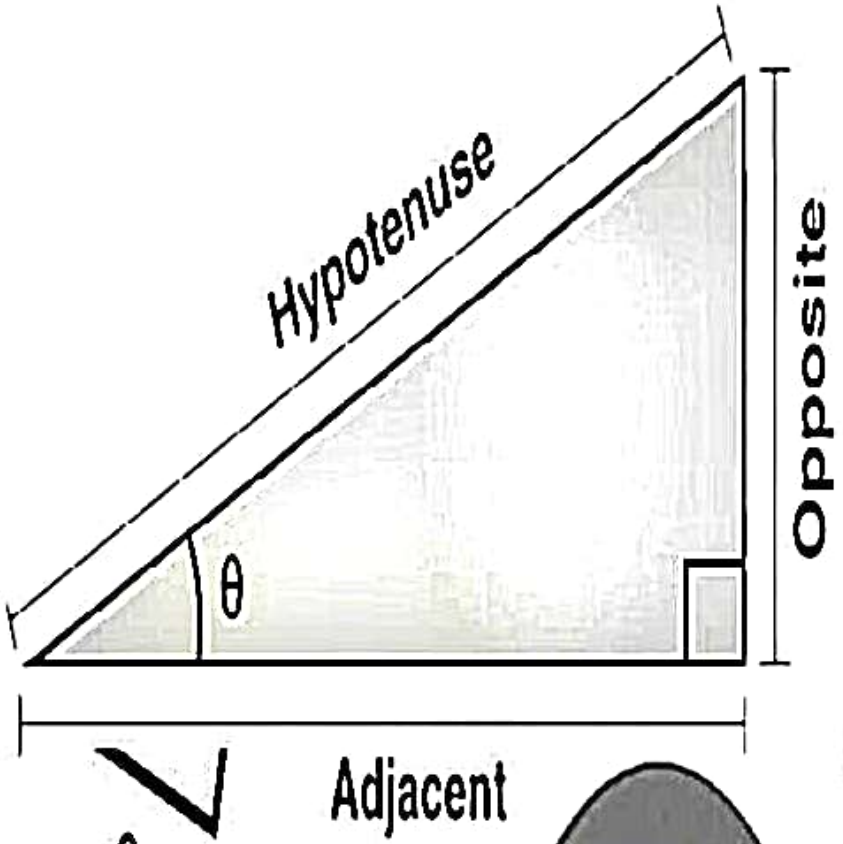
TRIGONOMETRY



cot θ

$\sin^2\theta + \cos^2\theta = 1$

tan θ

sin θ

cosec θ

cos (90° - θ)

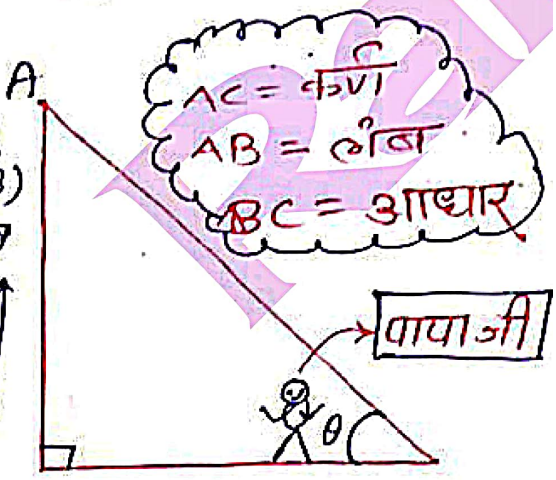
Trigonometry

Tri → तीन gon → भुजा metron → मापन

अर्थ :- तीनों भुजाओं का मापन

समकोण त्रिभुज :→ ऐसा त्रिभुज जिसका एक कोण 90° का हो उसे समकोण त्रिभुज कहते हैं।

ΔABC
$\angle B = 90°$



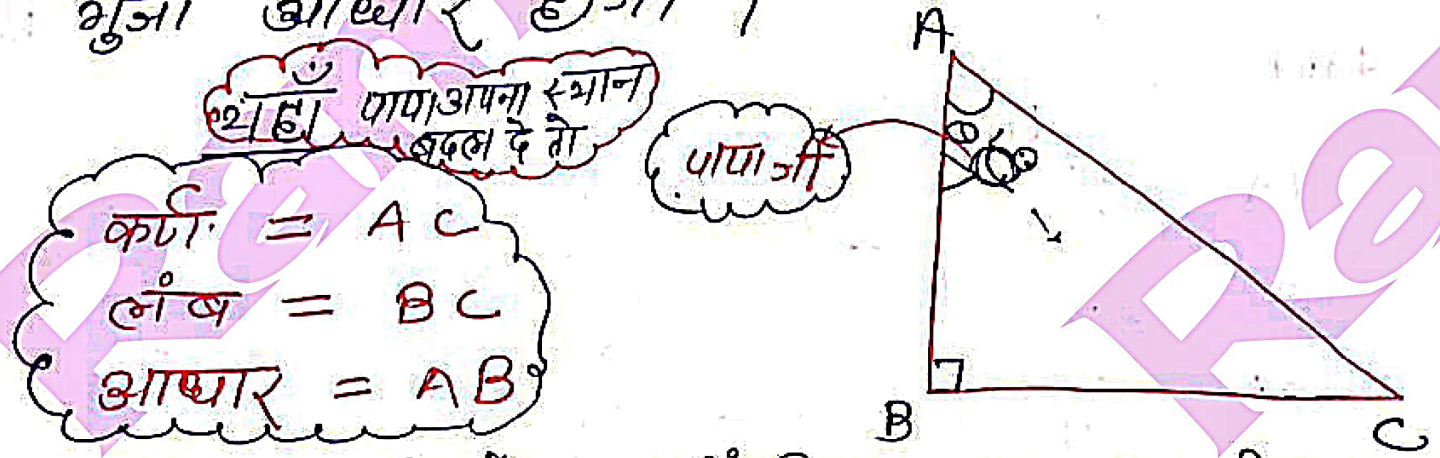
Note :→ (i) समकोण त्रिभुज मे कोण 90° के सामने वाली भुजा को कर्ण कहते हैं

(ii) कोण θ के सामने की भुजा को लंब कहते हैं यानी (θ) पापाजी जिसको देख रहे हैं वह लंब होगा।

(iii) θ जिसके साथ होता वह आधार होगा यानी (θ) कोण या पापाजी जिसपर पैर रखकर खड़े हैं वह भुजा आधार होगा।

यदि पापा अपना स्थान बदल दे तो

कर्ण = AC
लंब = BC
आधार = AB

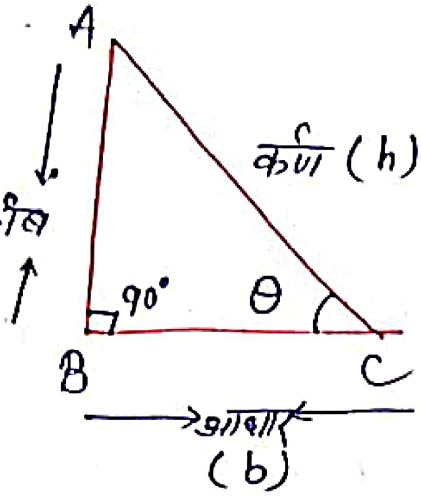


→ यानी पापा जी को पता है किसका नाम क्या होगा।

पापाजी = कोण (θ)

Ram Sir

त्रिकोणमितिय अनुपात :-



* $\sin\theta = \frac{\text{लंब}}{\text{कर्ण}} = \frac{AB}{AC} = \frac{P}{h}$

* $\cos\theta = \frac{\text{आधार}}{\text{कर्ण}} = \frac{BC}{AC} = \frac{b}{h}$

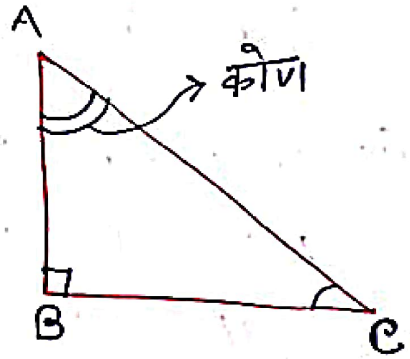
* $\tan\theta = \frac{\text{लंब}}{\text{आधार}} = \frac{AB}{BC} = \frac{P}{b}$

* $\cot\theta = \frac{\text{आधार}}{\text{लंब}} = \frac{BC}{AB} = \frac{b}{P}$

* $\text{cosec}\,\theta = \frac{\text{कर्ण}}{\text{लंब}} = \frac{AC}{AB} = \frac{h}{P}$

* $\sec\theta = \frac{\text{कर्ण}}{\text{आधार}} = \frac{AC}{BC} = \frac{h}{b}$

MVVI अगर कोण 'A' लिया जाए तो



$\sin A = \frac{BC}{AC}$

$\cos A = \frac{AB}{AC}$

$\tan A = \frac{BC}{AB}$

$\cot A = \frac{AB}{BC}$

$\sec A = \frac{AC}{AB}$

$\text{cosec}\,A = \frac{AC}{BC}$

MVVI

अगर कोण 'C' लिया जाए तो

$\sin C = \frac{AB}{AC}$

$\cos C = \frac{BC}{AC}$

$\tan C = \frac{AB}{BC}$

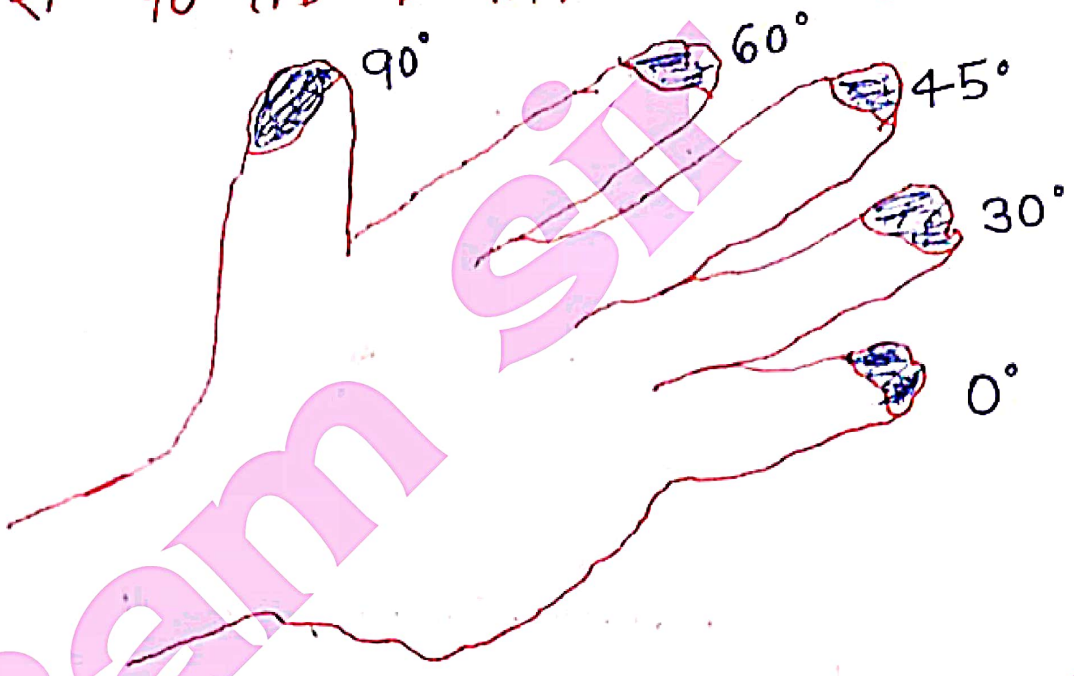
$\cot C = \frac{BC}{AB}$

$\text{cosec}\,C = \frac{AC}{AB}$

$\sec C = \frac{AC}{BC}$

Rani sir

0° से 90° तक के त्रिकोणमितीय अनुपात के माप :-



Trick :→

$$\sin\theta = \frac{\sqrt{\text{नीचे वाले अंगुली की संख्या}}}{2}$$

$\sin 0 = \frac{\sqrt{0}}{2} = \frac{0}{2} = 0$ ✓

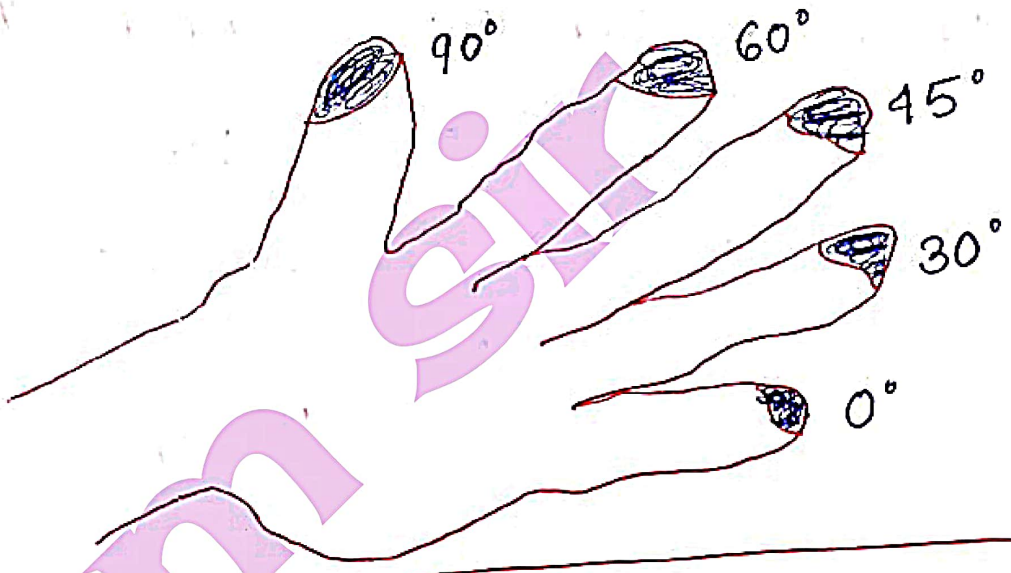
$\sin 30 = \frac{\sqrt{1}}{2} = \frac{1}{2}$ ✓

$\sin 45 = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{1}{\sqrt{2}}$ ✓

$\sin 60 = \frac{\sqrt{3}}{2}$

$\sin 90 = \frac{\sqrt{4}}{2} = \frac{2}{2} = 1$ ✓

Ram sir



Trick

$$\cos\theta = \frac{\sqrt{\text{ऊपर वाली अंगुली की संख्या}}}{2}$$

$\cos 0 = \frac{\sqrt{4}}{2} = \frac{2}{2} = 1$ ✓

$\cos 30 = \frac{\sqrt{3}}{2}$ ✓

$\cos 45 = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{1}{\sqrt{2}}$ ✓

$\cos 60 = \frac{\sqrt{1}}{2} = \frac{1}{2}$ ✓

$\cos 90 = \frac{\sqrt{0}}{2} = \frac{0}{2} = 0$ ✓

$$\tan\theta = \frac{\sqrt{\text{नीचे वाली अंगुली की संख्या}}}{\sqrt{\text{ऊपर वाली अंगुली की संख्या}}}$$

$\tan 0 = \frac{\sqrt{0}}{\sqrt{4}} = \frac{0}{2} = 0$ ✓

$\tan 30 = \frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$ ✓

~~$\tan 30$~~

$\tan 45 = \frac{\sqrt{2}}{\sqrt{2}} = 1$ ✓

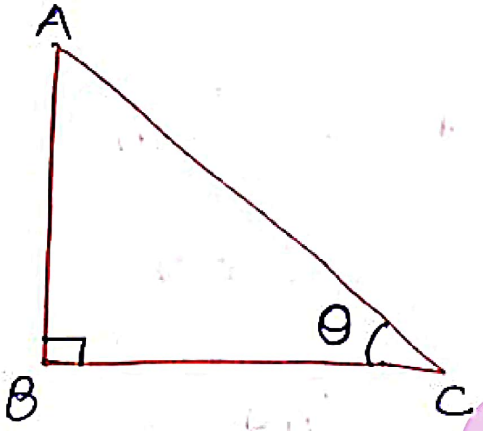
$\tan 60 = \frac{\sqrt{3}}{\sqrt{1}} = \sqrt{3}$ ✓

$\tan 90 = \frac{\sqrt{4}}{0} = \frac{2}{0} = \infty$ ✓

Ram sir

त्रिकोणमितिय सर्वसमिकाएँ

(i) ∈ $\boxed{\sin^2\theta + \cos^2\theta = 1}$



पाइथागोरस प्रमेय

(कर्ण)^2 = (लंब)^2 + (आधार)^2

$AC^2 = AB^2 + BC^2$

Proof :→ $AC^2 = AB^2 + BC^2$

दोनो तरफ 'AC^2' से भाग करने पर

$$\frac{AC^2}{AC^2} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$\boxed{1 = \sin^2\theta + \cos^2\theta}$$

→ $\boxed{\sin^2\theta + \cos^2\theta = 1}$

→ $\boxed{\sin^2\theta = 1 - \cos^2\theta}$

→ $\boxed{\cos^2\theta = 1 - \sin^2\theta}$

(ii) $\text{Cosec}^2\theta - \cot^2\theta = 1$

proof :⇒ $AC^2 = AB^2 + BC^2$

दोनो तरफ 'AB^2' से भाग करने पर

$$\frac{AC^2}{AB^2} = \frac{AB^2}{AB^2} + \frac{BC^2}{AB^2}$$

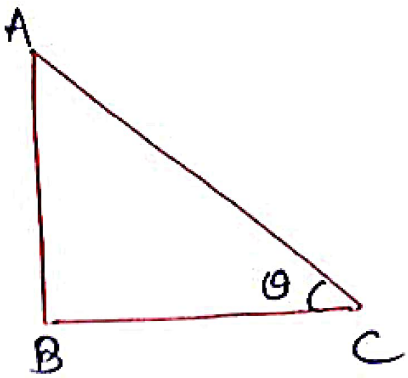
$$\text{Cosec}^2\theta = 1 + \cot^2\theta$$

→ $\boxed{\text{Cosec}^2\theta - \cot^2\theta = 1}$

→ $\boxed{\text{Cosec}^2\theta = 1 + \cot^2\theta}$ → $\boxed{\cot^2\theta = \text{Cosec}^2\theta - 1}$

Rani sir

(iii) $\sec^2\theta - \tan^2\theta = 1$



Proof :→ $AC^2 = AB^2 + BC^2$

दोनो तरफ BC^2 से भाग करने पर

$$\frac{AC^2}{BC^2} = \frac{AB^2}{BC^2} + \frac{BC^2}{BC^2}$$

$$\sec^2\theta = \tan^2\theta + 1$$

→ $\sec^2\theta - \tan^2\theta = 1$

→ $\sec^2\theta = 1 + \tan^2\theta$

→ $\tan^2\theta = \sec^2\theta - 1$

→ $\sin(90-\theta) = \cos\theta$

→ $\cos(90-\theta) = \sin\theta$

→ $\operatorname{cosec}(90-\theta) = \sec\theta$

→ $\sec(90-\theta) = \operatorname{cosec}\theta$

→ $\tan(90-\theta) = \cot\theta$

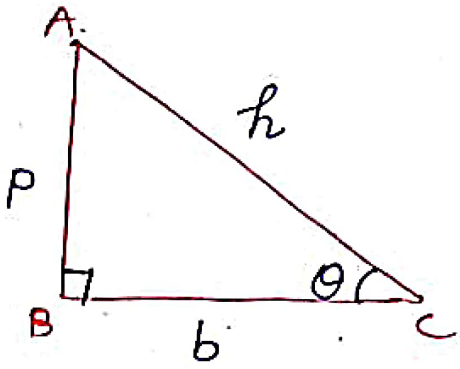
→ $\cot(90-\theta) = \tan\theta$

The end

Ram sir

$\rightarrow$ $\boxed{\operatorname{cosec}\theta \cdot \sin\theta = 1}$

proof $\Rightarrow$ $\frac{h}{p} \times \frac{p}{h} = 1$



$\rightarrow$ $\boxed{\operatorname{cosec}\theta = \frac{1}{\sin\theta}}$

$\boxed{\sin\theta = \frac{1}{\operatorname{cosec}\theta}}$

$\rightarrow$ $\boxed{\sec\theta \cdot \cos\theta = 1}$

proof $\Rightarrow$ $\frac{h}{b} \times \frac{b}{h} = 1$

$\boxed{\sec\theta = \frac{1}{\cos\theta}}$

$\boxed{\cos\theta = \frac{1}{\sec\theta}}$

$\rightarrow$ $\tan\theta \cdot \cot\theta = 1$

proof $\Rightarrow$ $\frac{p}{b} \times \frac{b}{p} = 1$

$\rightarrow$ $\boxed{\tan\theta = \frac{1}{\cot\theta}}$

$\rightarrow$ $\boxed{\cot\theta = \frac{1}{\tan\theta}}$

Note $\Rightarrow$

$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$

$\boxed{\cot\theta = \frac{\cos\theta}{\sin\theta}}$

Ram sir